Beyond Standard Model :

N=1 supersymmetry in 4D superspace

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IPHC, DRS, Theory group

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Introduction

The properties of elementary particles are associated to the underlying symmetry groups, what are the symmetries compatible with quantum mechanics and special relativity ?

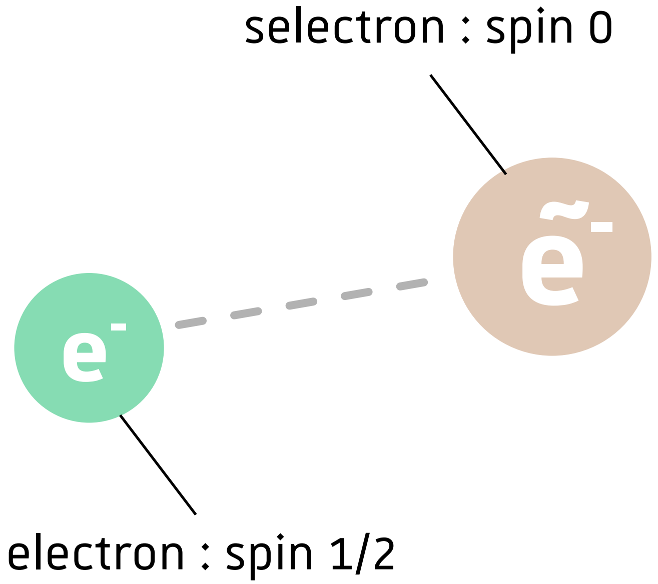
1967

- Coleman-Mandula : Lie algebra with bosonic generators

$$\mathfrak{g} = \mathfrak{iso}(1,3) \times \mathfrak{g}_c$$

1975

- Haag-Lopuszanski-Sohnius : Lie superalgebra with bosonic and fermionic generators



Poincaré superalgebra

Casimir operators for the Poincaré algebra : $\mathcal{C}_1 = P_\mu P^\mu$ $\mathcal{C}_2 = W_\mu W^\mu$

$$W_\mu = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}P^\nu M^{\alpha\beta}$$

N=1 means we introduce one Majorana spinor → 7 more relations to investigate :

$$\mathfrak{g}_{\text{Poincaré}} = \underbrace{\{M_{\mu\nu}, P_\mu, \mu,\nu = 0,\ldots,3\}}_{\mathfrak{g}_0 = \mathfrak{iso}(1,3)} \oplus \underbrace{\left(\{Q_\alpha, \alpha = 1,2\} \oplus \{\bar{Q}^{\dot\alpha}, \dot\alpha = 1,2\}\right)}_{\mathfrak{g}_1}$$

$$[P^\mu, P^\nu] = 0$$
$$[M^{\mu\nu}, P^\alpha] = \eta^{\nu\alpha}P^\mu - \eta^{\mu\alpha}P^\nu$$
$$\left[M^{\mu\nu}, M^{\alpha\beta}\right] = \eta^{\nu\beta}M^{\alpha\mu} - \eta^{\mu\beta}M^{\alpha\nu} + \eta^{\nu\alpha}M^{\mu\beta} - \eta^{\mu\alpha}M^{\nu\beta}$$

$$[M_{\mu\nu}, Q_\alpha] = \sigma_{\mu\nu\alpha}{}^\beta Q_\beta$$
$$\left[M_{\mu\nu}, \bar{Q}^{\dot\alpha}\right] = \bar{\sigma}_{\mu\nu}{}^{\dot\alpha}{}_{\dot\beta}\bar{Q}^{\dot\beta}$$
$$[P_\mu, Q_\alpha] = \left[P_\mu, \bar{Q}^{\dot\alpha}\right] = 0$$
$$\left\{Q_\alpha, \bar{Q}_{\dot\alpha}\right\} = -2i\sigma^\mu{}_{\alpha\dot\alpha}P_\mu$$
$$\{Q_\alpha, Q_\beta\} = \left\{\bar{Q}_{\dot\alpha}, \bar{Q}_{\dot\beta}\right\} = 0$$

means that particles in a given supermultiplet share the same mass

Poincaré superalgebra

In the massless case, multiplets are labelled as : $\underbrace{|p^\mu, \lambda\rangle}_{\text{contains states}}$

Matter supermultiplets :

Quark	Squark
Lepton	Slepton
Higgsino	Higgs
$\lambda = 1/2$	$\lambda = 0$

Gauge supermultiplets :

Photon	Photino
W, Z	Wino, Zino
Gluons	Gluinos
$\lambda = 1$	$\lambda = 1/2$

N=1 supergravity multiplet :

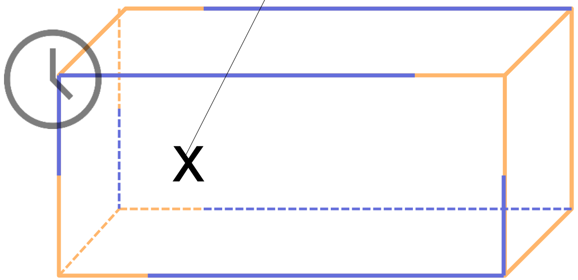
Graviton	Gravitino
$\lambda = 2$	$\lambda = 3/2$

We now deal with supermultiplets containing particles of different spins !

Superspace

Superspace is an extension of ordinary spacetime by fermionic coordinates ($\theta\bar{\theta} = -\bar{\theta}\theta$) :

$(x^\mu, \theta^\alpha, \bar{\theta}_{\dot{\alpha}})$



$$\mathcal{X}(x, \theta, \bar{\theta}) = e^{x^\mu P_\mu} \times e^{i(\theta^\alpha Q_\alpha + \bar{Q}_{\dot{\alpha}}\bar{\theta}^{\dot{\alpha}})}$$

$$Q_\alpha = -i\left(\partial_\alpha + i\sigma^\mu{}_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu\right)$$

$$\bar{Q}_{\dot{\alpha}} = i\left(\bar{\partial}_{\dot{\alpha}} + i\theta^\alpha\sigma^\mu{}_{\alpha\dot{\alpha}}\partial_\mu\right)$$

In superspace, the component fields of a supermultiplets are united into a superfield :

$$\delta\Phi(x, \theta, \bar{\theta}) = i(\epsilon Q + \bar{Q}\bar{\epsilon})\Phi(x, \theta, \bar{\theta})$$

infinitesimal parameter

shift the problem of finding representations

$$\bar{D}_{\dot{\alpha}} = \left(\bar{\partial}_{\dot{\alpha}} - i\theta^\alpha\sigma^\mu{}_{\alpha\dot{\alpha}}\partial_\mu\right)$$

$$D_\alpha = \left(\partial_\alpha - i\sigma^\mu{}_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu\right)$$

Superfields

complex scalar — R-Weyl spinor — complex vector — L-Weyl spinor

$$\Phi(x,\theta,\bar\theta) = f(x) + \sqrt{2}\theta\zeta(x) + \sqrt{2}\bar\theta\bar\chi(x) + \theta\theta m(x) + \bar\theta\bar\theta n(x) + \theta\sigma^\mu\bar\theta v_\mu(x) + \theta\theta\bar\theta\bar\lambda(x) + \bar\theta\bar\theta\theta\xi(x) + \frac{1}{2}\theta\theta\bar\theta\bar\theta d(x)$$

Chirality

$$D_\alpha\Phi^\dagger(x,\theta,\bar\theta) = 0 \qquad \bar D_{\dot\alpha}\Phi(x,\theta,\bar\theta) = 0$$

$$\Phi(y,\theta) = \phi(y) + \sqrt{2}\theta\psi(y) - \theta\theta F(y)$$

Chiral superfield ⇔ matter supermultiplet

$$y^\mu = x^\mu - i\theta\sigma^\mu\bar\theta$$

Reality

$$\Phi(x,\theta,\bar\theta) = \Phi^\dagger(x,\theta,\bar\theta)$$

$$V_{W-Z}(x,\theta,\bar\theta) = \theta\sigma^\mu\bar\theta v_\mu(x) + i\theta\theta\bar\theta\bar\lambda(x) - i\bar\theta\bar\theta\theta\lambda(x) + \frac{1}{2}\theta\theta\bar\theta\bar\theta D(x)$$

Vector superfield ⇔ gauge supermultiplet

Lagrangian formalism

How to describe interactions between chiral and vector superfields ?

Consider a gauge group G and chiral superfields in a representation R generated by the hermitian matrices T :

- transformation of the chiral superfield : $\Phi' = e^{i\Lambda^a T_a}\Phi$

$$\mathcal{L}_c = \Phi^\dagger\Phi\Big|_{\theta\theta\bar{\theta}\bar{\theta}} + \underbrace{\mathcal{W}(\Phi)\Big|_{\theta\theta}} + \mathcal{W}^\star(\Phi^\dagger)\Big|_{\bar{\theta}\bar{\theta}} \longrightarrow \mathcal{L}_c = \Phi^\dagger e^{2gV}\Phi\Big|_{\theta\theta\bar{\theta}\bar{\theta}} + \mathcal{W}(\Phi)\Big|_{\theta\theta} + \mathcal{W}^\star(\Phi^\dagger)\Big|_{\bar{\theta}\bar{\theta}}$$

holomorphic function of chiral **only**

The superpotential can take various forms : $\underbrace{\mathcal{W}(\Phi)} = k_i\Phi^i + \frac{1}{2}m_{ij}\Phi^i\Phi^j + \frac{1}{6}\lambda_{ijk}\Phi^i\Phi^j\Phi^k$

mass dimension = 3

Lagrangian formalism

For the vector part, the problem arises when we go to non-abelian gauge ($[T_a, T_b] = i f_{ab}{}^c T_c$):

$$W_\alpha = -\frac{1}{4}\bar{D}\bar{D}e^{2gV}D_\alpha e^{-2gV} \qquad \bar{W}_{\dot{\alpha}} = -\frac{1}{4}DDe^{-2gV}\bar{D}_{\dot{\alpha}}e^{2gV}$$

The most general renormalizable Lagrangian is then :

$$\mathcal{L} = \Phi^\dagger e^{-2gV}\Phi\Big|_{\theta\theta\bar{\theta}\bar{\theta}} + \frac{1}{16g^2\tau_{\mathcal{R}}}\mathrm{tr}\left(W^\alpha W_\alpha\right)\Big|_{\theta\theta} + \frac{1}{16g^2\tau_{\mathcal{R}}}\mathrm{tr}\left(\bar{W}_{\dot{\alpha}}\bar{W}^{\dot{\alpha}}\right)\Big|_{\bar{\theta}\bar{\theta}} + \mathcal{W}\left(\Phi\right)\big|_{\theta\theta} + \mathcal{W}^\star\left(\Phi^\dagger\right)\Big|_{\bar{\theta}\bar{\theta}}$$

$\mathrm{tr}\left\{T_a T_b\right\} = \tau_{\mathcal{R}}\delta_{ab}$

- application of abelian : SQED
- application of non-abelian : SQCD

SUSY breaking

$$\mathcal{L} = -\frac{1}{4}F_a^{\mu\nu}F_{\mu\nu}^a + \frac{i}{2}\left(\lambda^a\sigma^\mu D_\mu\bar{\lambda} - D_\mu\lambda^a\sigma^\mu\bar{\lambda}_a\right) + \frac{1}{2}D_aD^a$$
$$+ D_\mu\phi^\dagger D^\mu\phi - \frac{i}{2}\left(D_\mu\bar{\psi}\bar{\sigma}^\mu\psi - \bar{\psi}\bar{\sigma}^\mu D_\mu\psi\right) + F^\dagger F$$
$$- gD^a\phi^\dagger T_a\phi + i\sqrt{2}g\bar{\lambda}^a\bar{\psi}T_a\phi - i\sqrt{2}g\phi^\dagger T_a\psi\lambda^a$$
$$- m_{ij}\left(\phi^iF^j + \frac{1}{2}\psi^i\psi^j\right) - \frac{1}{2}\lambda_{ijk}\left(\phi^i\phi^jF^k + \phi^i\psi^j\psi^k\right) + h.c$$

$F_{\mu\nu} = F^0_{\mu\nu} - ig\left[v_\mu, v_\nu\right]$ $\qquad$ $D_\mu\lambda = \partial_\mu\lambda - ig\left[v_\mu, \lambda\right]$

We can hide these contributions into the scalar potential :

$$V(\phi, \phi^\dagger) = F^\dagger F + \frac{1}{2}D^2$$

SUSY breaking is required by phenomenology : no scalar electron discovered !

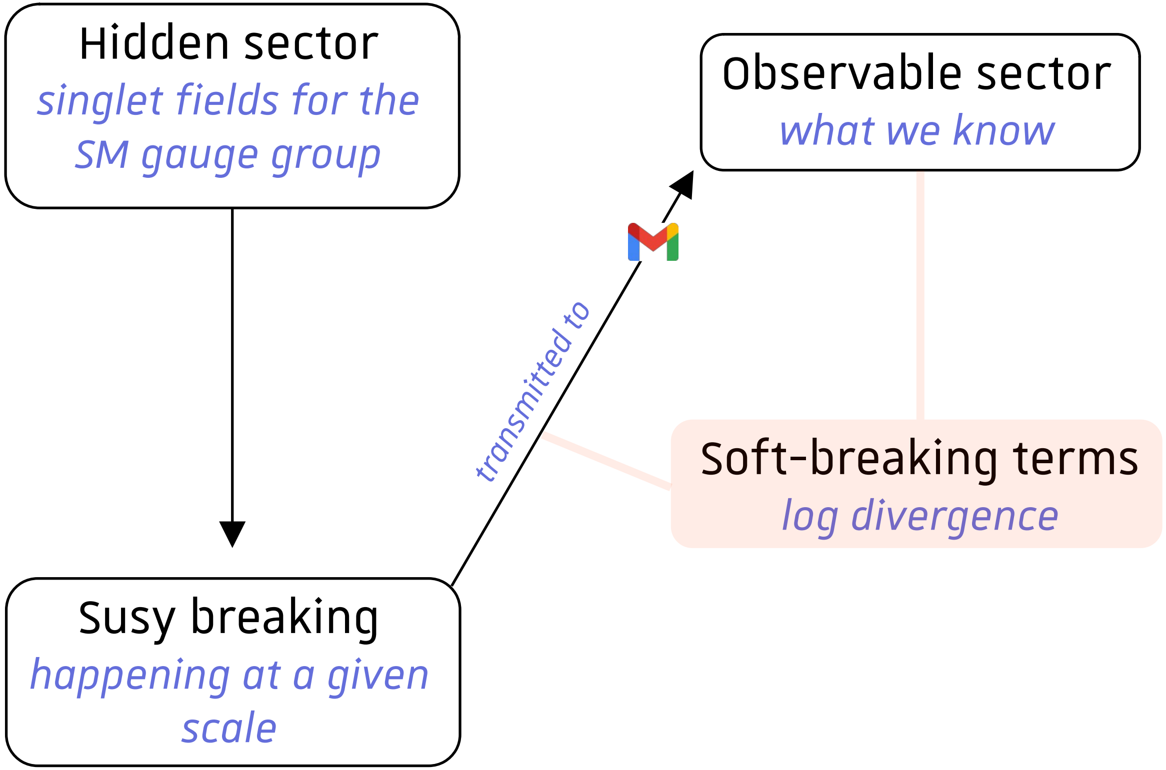
- spontaneously broken if the scalar potential admits a non-vanishing vev

O'Raifeartaigh mechanism
F-term breaking

Fayet-Iliopoulos mechanism
D-term breaking

SUSY breaking

Do we have a solution ? Yes, but at the cost of harder computations



We need :

- non-renormalization
- identify superfields for N=1 supergravity
- curved geometry

COMPUTATIONS

MSSM : Higgs sector

We supersymmetrize the SM with fields that differ in spin by half. The tool box is :

Matter sector

$Q^I \quad L^I$

$\bar{U}^I \quad \bar{D}^I \quad \bar{E}^I \quad \bar{N}^I$

Gauge sector

$V_g^a \quad V_{\vec{W}} \quad V_B$

a = 1,…,8

$$\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$$

Higgs sector

$$H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix} \quad H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}$$

$$\tilde{H}_1 = \begin{pmatrix} \tilde{H}_1^0 \\ \tilde{H}_1^- \end{pmatrix} \quad \tilde{H}_2 = \begin{pmatrix} \tilde{H}_2^+ \\ \tilde{H}_2^0 \end{pmatrix}$$

+

$$\begin{aligned} \mathcal{W} = &-y_{eIJ} L^I H_1 \bar{E}^J \\ &+ y_{uIJ} Q^I H_2 \bar{U}^J \\ &- y_{dIJ} Q^I H_1 \bar{D}^J \\ &+ y_{nIJ} L^I H_2 \bar{N}^J \\ &+ \mu H_1 H_2 \\ &+ m_{IJ} \bar{N}^I \bar{N}^J \end{aligned}$$

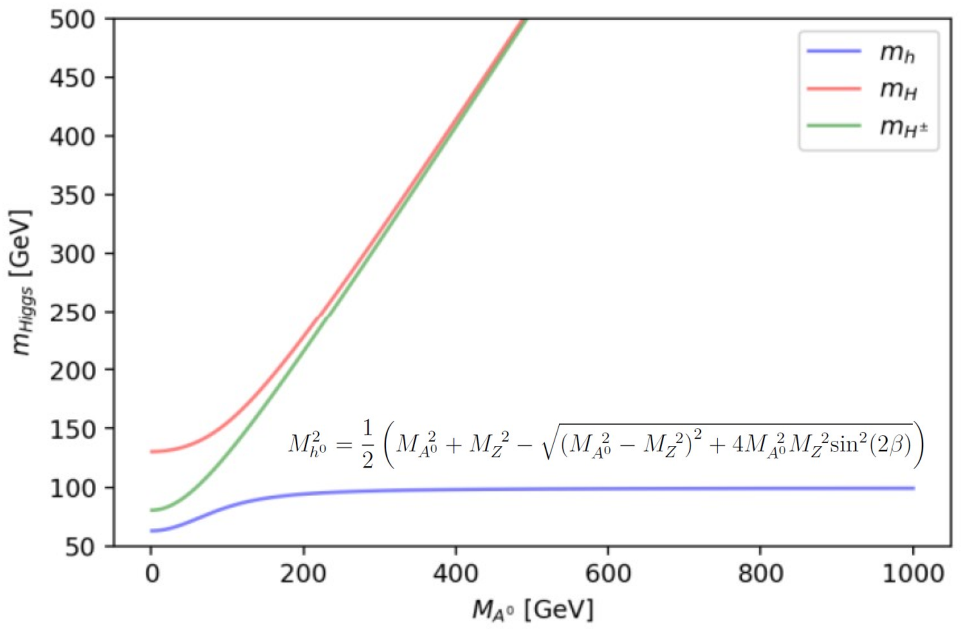
MSSM : Higgs sector

The Higgs sector is far richer :

- scalar : h^0, H^0
- pseudo-scalar : G^0, A^0
- charged : $G^{\pm}$, $H^{\pm}$

G are the Goldstone bosons generated by the breaking and are eaten by the gauge bosons

Superpotential + soft-breaking
possible to compute all the mass matrices for the MSSM



Higgs masses as a function of M_{A0} for tan(β)=3 (tree level)

Conclusion

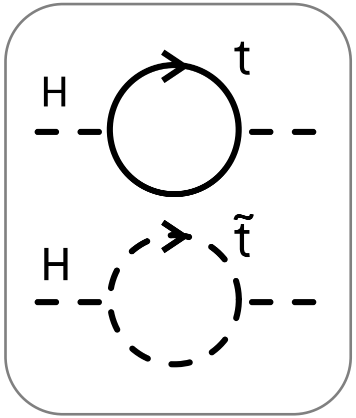
How to pursue the study ?

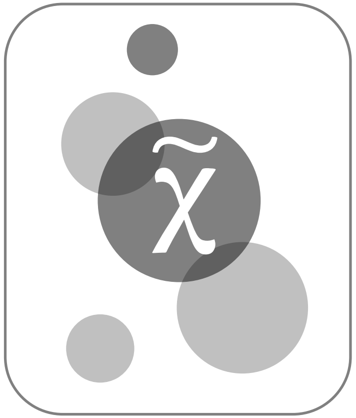
- local version : supergravity
- N>1 supersymmetry

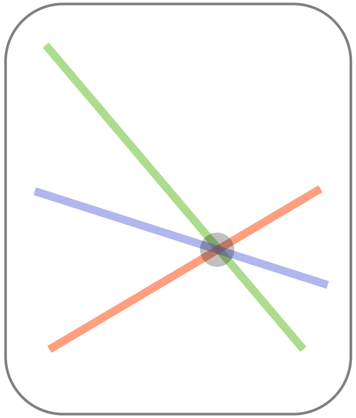
Even if there are no experimental evidence (yet) :

- still investigated
- any deviation from the SM would be a sign of new physics

Thanks to M. Rausch and M. Bonnefoy for having endured my 5-hour long presentation at the lab !







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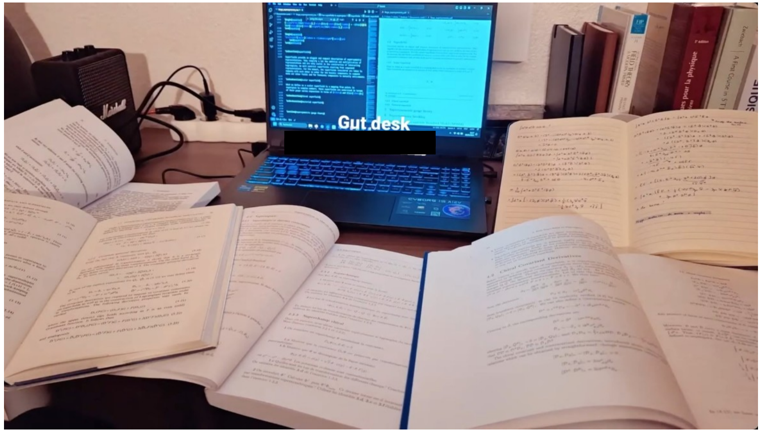


Average Simon experiment

A) Coleman-Mandula

Let G be a symmetry group of the S-matrix, and let the following conditions hold :

a) G contains a subgroup locally isomorphic to the Poincaré group P

b) All particle types correspond to positive-energy representations of P. For any finite M, there are only a finite number of particle types with mass less than M

c) Elastic-scattering amplitudes are analytic functions of center-of-mass energy, s, and invariant momentum transfer, t, in some neighborhood of the physical region, except at normal thresholds

d) The generators of G, considered as integral operators in momentum space, have distributions for their kernels.

A) Conventions

Majorana spinors (denoted ψ_M) are a peculiar kind of four-component spinors that can be obtained from the Dirac one by imposing a constraint :

$$\psi_M = \begin{pmatrix} \psi_L \\ \bar{\psi}_R \end{pmatrix} = \begin{pmatrix} \psi_L \\ -i\sigma^2 \psi_L^* \end{pmatrix} \tag{2.14}$$

Writing the expression of the Dirac spinor allow us to find another way to define it :

$$\psi_D = \begin{pmatrix} \lambda_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix} \qquad \Longrightarrow \psi_M = \begin{pmatrix} \lambda_\alpha \\ \bar{\lambda}^{\dot{\alpha}} \end{pmatrix} \tag{2.15}$$

and another formula comes from the charge conjugation matrix :

$$C = \begin{pmatrix} \epsilon_{\alpha\beta} & 0 \\ 0 & \epsilon^{\dot{\alpha}\dot{\beta}} \end{pmatrix}, \qquad C\bar{\psi}_M^t = \psi_M \tag{2.16}$$

We thus see that Majorana fermions are similar to Weyl fermions as they also have two components, but they must satisfy a reality condition and they must be invariant under charge conjugation.

A) Conventions

GRASSMANN variables provide a powerful and elegant framework for dealing with fermionic degrees of freedom. In our study, they are in fact the new coordinates that we need in order to describe supersymmetry in the superspace we will build later. Because we deal with fermionic fields that anticommute, the first definition we can take for these variables is :

$$\epsilon_i \epsilon_j = -\epsilon_j \epsilon_i, \qquad i, j = 1, \ldots, n \tag{2.17}$$

where ϵ_i are GRASSMANN elements that anticommute. In particular, we directly see that $\epsilon_i^2 = 0$, and we can take these variables to have a commutative product law with any ordinary number. This concept can be extended to that of an anticommuting variable θ, so let's consider conjugate anticommuting variables θ, $\bar{\theta}$ with the following properties :

$$\theta\bar{\theta} = -\bar{\theta}\theta, \qquad \theta^2 = \bar{\theta}^2 = 0, \qquad \bar{\bar{\theta}} = \theta \tag{2.18}$$

If one considers a generic analytic function in θ developped as a power series, the fact that θ squares to zero cancels all the terms except for two :

$$f(\theta) = \sum_{n=0}^{\infty} f_n \theta^n = f_0 + f_1\theta + f_2\theta^2 + \cdots = f_0 + f_1\theta \tag{2.19}$$

B) Lie superalgebra

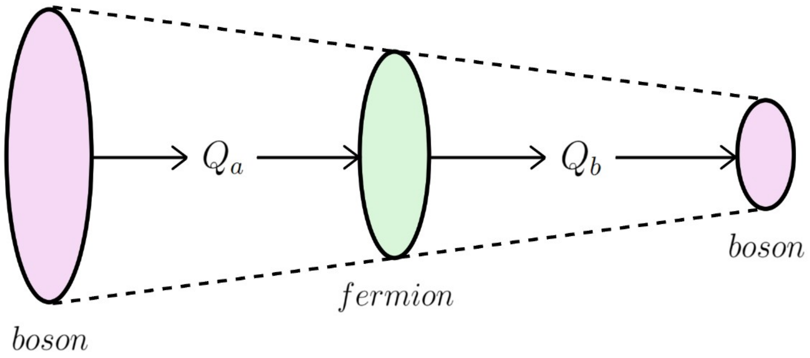
Basic idea: consider a Lagrangian with bosonic and fermionic fields, and symmetries generated by bosonic or fermionic charges

$$\begin{aligned}
\delta_A \mathcal{L}(\phi^a, \psi^i, \partial_\mu \phi^a, \partial_\mu \psi^i) &= \frac{\partial \mathcal{L}}{\partial \phi^a}\delta_A \phi^a + \frac{\partial \mathcal{L}}{\partial \psi^i}\delta_A \psi^i + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi^a)}\delta_A(\partial_\mu \phi^a) + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi^i)}\delta_A(\partial_\mu \psi^i) \\
&= \frac{\partial \mathcal{L}}{\partial \phi^a}\delta_A \phi^a + \frac{\partial \mathcal{L}}{\partial \psi^i}\delta_A \psi^i + \partial_\mu \left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi^a)}\delta_A \phi^a + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi^i)}\delta_A \psi^i\right] \\
&\quad - \underbrace{\partial_\mu \left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi^a)}\right]}_{\frac{\partial \mathcal{L}}{\partial \phi^a}} \delta_A \phi^a - \underbrace{\partial_\mu \left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi^i)}\right]}_{\frac{\partial \mathcal{L}}{\partial \psi^i}} \delta_A \psi^i \\
&= \partial_\mu \left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi^a)}\delta_A \phi^a + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi^i)}\delta_A \psi^i\right] = 0
\end{aligned}$$

then find the conserved current and then charge

$$[\mathcal{B}_A, \mathcal{B}_B] = f_{AB}{}^C \mathcal{B}_C \qquad [\mathcal{B}_A, \mathcal{F}_I] = r_{AI}{}^J \mathcal{F}_J \qquad \{\mathcal{F}_I, \mathcal{F}_J\} = q_{IJ}{}^A \mathcal{B}_A$$

C) Poincaré superalgebra



What would happen if not the same number of degrees of freedom ?

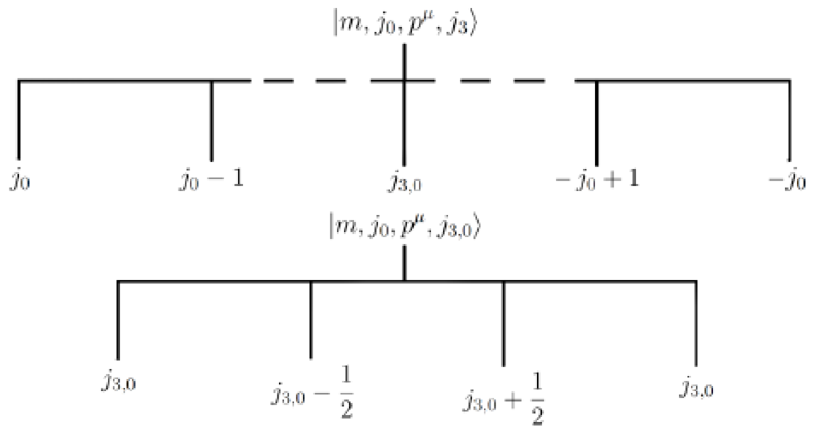


FIGURE 5 – General schematic of massive supermultiplet states

In the case where we start with $j_0 = 0$, we have a scalar state $|\Omega\rangle$ and four states with $j_{3,0} \in \{0, -\frac{1}{2}, \frac{1}{2}, 0\}$.The chiral fermion is here again described by a WEYL spinor with two components and the rest of the states are a scalar and a pseudo-scalar. Two of such supermultiplets are needed to create a DIRAC fermion out of two WEYL spinors. The corresponding four spin zero states can be described as two complex sfermions (superpartners of the left and right chiral components of the DIRAC fermion).

D) Superspace

Let's do a bit of formalism before in order to define what really is our superspace. We know from the usual group theory that every continuous group G defines a manifold $\mathcal{M}_G$ via its parameters $\{r_a\}$:

$$\begin{aligned} \Lambda : G &\longrightarrow \mathcal{M}_G \\ \{g = e^{ir_a T^a}\} &\longrightarrow \{r_a\} \end{aligned} \tag{4.1}$$

where $\dim(G) = \dim(\mathcal{M}_G)$. To identify our structure, let's define a coset G/H where $g \in G$ is identified with $g \cdot h$, $\forall\, h \in H \subset G$. For example, we can obtain the set of translations $\{a^\mu = x^\mu\}$ simply by considering the coset Poincaré/Lorentz $= \{\omega^{\mu\nu}, a^\mu\}/\{\omega^{\mu\nu}\}$ where the parameters are defined in the corresponding section. This space is in fact MINKOWSKI space. We thus define $N = 1$ superspace to be the coset :

$$\text{SuperPoincaré/Lorentz} = \{\omega^{\mu\nu}, a^\mu, \theta^\alpha, \bar{\theta}_{\dot{\alpha}}\}/\{\omega^{\mu\nu}\} \tag{4.2}$$

meaning that we go from an element in the group that was parametrized by :

$$e^{i(\omega^{\mu\nu} M_{\mu\nu} + a^\mu P_\mu + \theta^\alpha Q_\alpha + \bar{Q}_{\dot{\alpha}} \bar{\theta}^{\dot{\alpha}})} \tag{4.3}$$

to :

$$e^{i(a^\mu P_\mu + \theta^\alpha Q_\alpha + \bar{Q}_{\dot{\alpha}} \bar{\theta}^{\dot{\alpha}})} \tag{4.4}$$

D) Superspace

$$\mathcal{X}(0,\epsilon,\bar{\epsilon})\mathcal{X}(x,\theta,\bar{\theta}) = e^{i(\epsilon^\alpha Q_\alpha + \bar{Q}_{\dot{\alpha}}\bar{\epsilon}^{\dot{\alpha}})} e^{x^\mu P_\mu + i(\theta^\alpha Q_\alpha + \bar{Q}_{\dot{\alpha}}\bar{\theta}^{\dot{\alpha}})}$$
$$= e^{x^\mu \partial_\mu + i\left((\theta+\epsilon)Q + \bar{Q}(\bar{\theta}+\bar{\epsilon})\right) + \frac{1}{2}\left[\theta Q + \bar{Q}\bar{\theta}, \epsilon Q + \bar{Q}\bar{\epsilon}\right]} \tag{4.7}$$
$$= e^{x^\mu + i\left((\theta+\epsilon)Q + \bar{Q}(\bar{\theta}+\bar{\epsilon})\right) + i\left(\epsilon\sigma^\mu\bar{\theta} - \theta\sigma^\mu\bar{\epsilon}\right)} \qquad \text{(left)}$$

$$\mathcal{X}(x,\theta,\bar{\theta})\mathcal{X}(0,\epsilon,\bar{\epsilon}) = e^{x^\mu P_\mu + i(\theta^\alpha Q_\alpha + \bar{Q}_{\dot{\alpha}}\bar{\theta}^{\dot{\alpha}})} e^{i(\epsilon^\alpha Q_\alpha + \bar{Q}_{\dot{\alpha}}\bar{\epsilon}^{\dot{\alpha}})}$$
$$= e^{x^\mu \partial_\mu + i\left((\theta+\epsilon)Q + \bar{Q}(\bar{\theta}+\bar{\epsilon})\right) - \frac{1}{2}\left[\theta Q + \bar{Q}\bar{\theta}, \epsilon Q + \bar{Q}\bar{\epsilon}\right]} \tag{4.8}$$
$$= e^{x^\mu + i\left((\theta+\epsilon)Q + \bar{Q}(\bar{\theta}+\bar{\epsilon})\right) - i\left(\epsilon\sigma^\mu\bar{\theta} - \theta\sigma^\mu\bar{\epsilon}\right)} \qquad \text{(right)}$$

This allows us to identify the supersymmetric transformations for our supercoordinates depending on the side of the action :

$$\begin{aligned}
\delta_{(left)}x^\mu &= i(\epsilon\sigma^\mu\bar{\theta} - \theta\sigma^\mu\bar{\epsilon}) \\
\delta_{(right)}x^\mu &= -i(\epsilon\sigma^\mu\bar{\theta} - \theta\sigma^\mu\bar{\epsilon}) \\
\delta_{(left)}\theta^\alpha &= \delta_{(right)}\theta^\alpha = \epsilon^\alpha \\
\delta_{(left)}\bar{\theta}_{\dot{\alpha}} &= \delta_{(right)}\bar{\theta}_{\dot{\alpha}} = \bar{\epsilon}_{\dot{\alpha}}
\end{aligned} \tag{4.9}$$

such that :

$$x^\mu \longrightarrow x^\mu + \delta x^\mu, \qquad \theta^\alpha \longrightarrow \theta^\alpha + \delta\theta^\alpha, \qquad \bar{\theta}_{\dot{\alpha}} \longrightarrow \bar{\theta}_{\dot{\alpha}} + \delta\bar{\theta}_{\dot{\alpha}} \tag{4.10}$$

D) Superspace

1. $\delta f(x) = \sqrt{2}\epsilon\zeta(x) + \sqrt{2}\bar{\epsilon}\bar{\chi}(x)$
2. $\delta m(x) = \frac{i}{\sqrt{2}}\partial_\mu\zeta(x)\sigma^\mu\bar{\epsilon} + \bar{\epsilon}\bar{\lambda}$
3. $\delta n(x) = -\frac{i}{\sqrt{2}}\epsilon\sigma^\mu\partial_\mu\bar{\chi}(x) + \epsilon\xi(x)$
4. $\delta d(x) = \frac{i}{2}\partial\mu\xi(x)\sigma^\mu\bar{\epsilon} - \frac{i}{2}\epsilon\sigma^\mu\partial_\mu\bar{\lambda}(x)$
5. $\delta A_\mu(x) = -\frac{i}{\sqrt{2}}\epsilon\partial_\mu\zeta(x) - i\sqrt{2}\epsilon\sigma_{\mu\nu}\partial^\nu\zeta(x) + \frac{i}{\sqrt{2}}\bar{\epsilon}\partial_\mu\bar{\chi}(x) - i\sqrt{2}\bar{\epsilon}\bar{\sigma}_{\nu\mu}\partial^\nu\bar{\chi}(x) - \bar{\epsilon}\bar{\sigma}_\mu\xi(x) - \bar{\lambda}\bar{\sigma}_\mu\epsilon$
6. $\sqrt{2}\delta\zeta(x) = 2\epsilon m(x) + \sigma^\mu\bar{\epsilon}\left(A_\mu(x) - i\partial_\mu f(x)\right)$
7. $\delta\xi(x) = -i\sigma^\mu\bar{\epsilon}\partial_\mu n(x) + \frac{i}{2}\epsilon\partial A - \frac{i}{2}\sigma^{\mu\nu}\epsilon F_{\mu\nu} + \epsilon d(x)$
8. $\sqrt{2}\delta\bar{\chi}(x) = 2n(x)\bar{\epsilon} - \bar{\sigma}^\mu\epsilon\left(A_\mu(x) + i\partial_\mu f(x)\right)$
9. $\delta\bar{\lambda}(x) = -i\bar{\sigma}^\mu\epsilon\partial_\mu m(x) - \frac{i}{2}\bar{\epsilon}\partial A + \frac{i}{2}\bar{\sigma}^{\mu\nu}\bar{\epsilon}F_{\mu\nu} + \bar{\epsilon}d(x)$

D) Superspace

1. $\delta\phi = \sqrt{2}\epsilon\psi \longrightarrow \sqrt{2}\epsilon\psi$
2. $\delta\psi = -\sqrt{2}\epsilon F - i\sqrt{2}\sigma^{\mu}\bar{\epsilon}\partial_{\mu}\phi \longrightarrow -\sqrt{2}\epsilon F - i\sqrt{2}\sigma^{\mu}\bar{\epsilon}D_{\mu}\phi$
3. $\delta F = -i\sqrt{2}\partial_{\mu}\psi\sigma^{\mu}\bar{\epsilon} \longrightarrow -i\sqrt{2}D_{\mu}\psi\sigma^{\mu}\bar{\epsilon} - 2ig\bar{\epsilon}\bar{\lambda}\phi$

Degrees of freedom	ϕ (boson)	ψ (fermion)	F (boson)
on-shell	2	2	0
off-shell	2	4	2

1. $\delta v_{\mu} = i\left(\epsilon\sigma_{\mu}\bar{\lambda} - \lambda\sigma_{\mu}\bar{\epsilon}\right) \longrightarrow i\left(\epsilon\sigma_{\mu}\bar{\lambda} - \lambda\sigma_{\mu}\bar{\epsilon}\right)$
2. $\delta\lambda = \sigma^{\mu\nu}\epsilon F_{\mu\nu} + i\epsilon D \longrightarrow \sigma^{\mu\nu}\epsilon F_{\mu\nu} + i\epsilon D$
3. $\delta D = \partial_{\mu}\lambda\sigma^{\mu}\bar{\epsilon} + \epsilon\sigma^{\mu}\partial_{\mu}\bar{\lambda} \longrightarrow D_{\mu}\lambda\sigma^{\mu}\bar{\epsilon} + \epsilon\sigma^{\mu}D_{\mu}\bar{\lambda}$

Degrees of freedom	v_{μ} (boson)	λ (fermion)	D (boson)
on-shell	2	2	0
off-shell	3	4	1

E) Superfields

For the chiral superfield :

on shell : - 2 real degrees of freedom for the complex scalar

- 2 polarization states for the fermion

off shell : - 4 real degrees of freedom for the complex fermion

$$
\begin{aligned}
V(x,\theta,\bar\theta) &= \Phi(y,\theta) + \Phi^\dagger(y,\theta) \\
&= \left(\phi(x) + \sqrt{2}\theta\psi(x) - \theta\theta F(x) - i\theta\sigma^\mu\bar\theta\partial_\mu\phi(x) + \frac{i}{\sqrt{2}}\theta\theta\partial_\mu\psi(x)\sigma^\mu\bar\theta - \frac{1}{4}\theta\theta\bar\theta\bar\theta\Box\phi(x)\right) \\
&\quad + \left(\phi^\dagger(x) + \sqrt{2}\bar\theta\bar\psi(x) - \bar\theta\bar\theta F^\dagger(x) + i\theta\sigma^\mu\bar\theta\partial_\mu\phi^\dagger(x) - \frac{i}{\sqrt{2}}\bar\theta\bar\theta\theta\sigma^\mu\partial_\mu\bar\psi(x) - \frac{1}{4}\theta\theta\bar\theta\bar\theta\Box\phi^\dagger(x)\right) \\
&= 2\mathfrak{Re}\left(\phi(x)\right) + \sqrt{2}\theta\psi(x) + \sqrt{2}\bar\theta\bar\psi(x) - \theta\theta F(x) - \bar\theta\bar\theta F^\dagger(x) - 2\theta\sigma^\mu\bar\theta\partial_\mu\mathfrak{Im}\left(\phi(x)\right) \\
&\quad + \frac{i}{\sqrt{2}}\theta\theta\bar\theta\bar\sigma^\mu\partial_\mu\psi(x) - \frac{i}{\sqrt{2}}\bar\theta\bar\theta\theta\sigma^\mu\partial_\mu\bar\psi(x) - \frac{1}{2}\theta\theta\bar\theta\bar\theta\partial^\mu\partial_\mu\mathfrak{Re}\left(\phi(x)\right)
\end{aligned}
\tag{4.47}
$$

F) Lagrangian

Let's define a gauge group $G = U(1)_q$ under which a set of matter superfields Φ^a transforms as :

$$\Phi^a \longrightarrow e^{-2i\Lambda q_a}\Phi^a, \qquad \Phi_a^\dagger \longrightarrow e^{2i\Lambda^\dagger q_a}\Phi_a^\dagger \tag{5.38}$$

q_a are defined as real charges and Λ is a complex function of $(x, \theta, \bar{\theta})$ specifying the local gauge transformation. We impose that Λ has vanishing chiral derivatives, basically turning it into a chiral superfield, which ensures that the chiral nature of Φ^α and $\Phi_\alpha^\dagger$ is preserved. We see that a chiral superfield is mapped into a chiral superfield and that the Φ^α carry a representation of $U(1)_q$. The factor 2 in the exponential comes from the fact that the imaginary part of the scalar component of the Λ superfield is half the usual gauge transformation function in an abelian gauge theory. Because $\Lambda \neq \Lambda^\dagger$, the kinertic D-term is not gauge invariant by itself :

$$\Phi^\dagger\Phi \longrightarrow e^{2i\Lambda^\dagger q_a}\Phi_a^\dagger e^{-2i\Lambda q_a}\Phi^a = e^{2i(\Lambda^\dagger - \Lambda)q_a}\Phi_a^\dagger\Phi^a \tag{5.39}$$

so we see that we must introduce a compensating gauge superfield to restore the invariance under gauge transformation. This is rectified by introducing a gauge vector superfield V with gauge transformation :

$$V \longrightarrow V + i\Lambda - \Lambda^\dagger \tag{5.40}$$

and by modifying the kinetic term :

$$\left[\Phi^\dagger\Phi\right]_D \longrightarrow \left[\Phi^\dagger e^{2qV}\Phi\right]_D \tag{5.41}$$

F) Lagrangian

$$[T_a, T_b] = if_{ab}{}^c T_c, \qquad \mathrm{tr}\,\{T_a T_b\} = \tau_{\mathcal{R}} \delta_{ab} \tag{5.53}$$

where $\tau_{\mathcal{R}}$ is the representation constant of the representation $\mathcal{R}$. Concerning the vector superfield, we assume that it is given by $V = V^a T_a$. If the representation is unitary, meaning $T_a = T_a^\dagger$, we directly have a real vector superfield. In order to define the gauge transformation, it is convenient to use :

$$\Lambda = \Lambda^a T_a \tag{5.54}$$

with Λ^a being a chiral superfield. The gauge transformation, with g the coupling constant, is :

$$e^{2gV} \longrightarrow e^{-2ig\Lambda} e^{2gV} e^{2ig\Lambda^\dagger}, \qquad e^{-2gV} \longrightarrow e^{-2ig\Lambda^\dagger} e^{-2gV} e^{2ig\Lambda} \tag{5.55}$$

To fix the ideas, we choose to work in the WESS-ZUMINO gauge of the superfield V^a where triple products of V or those with a higher number of V factors vanish. In this gauge, any vector superfield does not carry a dependence in θ-terms meaning that the θ component of Λ^a must be real. Using the BAKER-CAMPBELL-HAUSDORFF formula, that tells us :

$$\delta V = i\left[gV, \Lambda + \Lambda^\dagger\right] - i \cdot \coth\left(\left[gV, \Lambda - \Lambda^\dagger\right]\right) \tag{5.56}$$

G) Breaking

$$\begin{aligned}\langle\Omega| H |\Omega\rangle &= \langle\Omega| \left(\frac{1}{4}\{Q_1, \bar{Q}_{\dot{1}}\} + \frac{1}{4}\{Q_2, \bar{Q}_{\dot{2}}\}\right) |\Omega\rangle \\ &= \frac{1}{4}\sum_{\alpha,\dot{\alpha}=1}^{2}\sum_{n}\left(|\langle\Omega| Q_\alpha |n\rangle|^2 + \left|\langle\Omega| \bar{Q}_{\dot{\alpha}} |n\rangle\right|^2\right)\end{aligned} \tag{7.3}$$

For any state $|\Omega\rangle$, this tells us that $\langle\Omega| H |\Omega\rangle \geq 0$ and it also tells us that states with vanishing energy density are supersymmetric ground states of the theory. A supersymmetric vacuum state is defined by the condition that it remains invariant under any supersymmetry transformation, which is equivalent to say that a supersymmetric vacuum is annihilated by the supersymmetry generators Q and $\bar{Q}$. Such states are called ground states because the expectation value of H may never be negative : they are supersymmetric because $\langle 0| H |0\rangle = 0$ implies $Q|0\rangle = \bar{Q}|0\rangle = 0$. Ground states of zero energy preserve supersymmetry, while those of positive energy break it spontaneously.

H) MSSM

To access supersymmetry breaking, we study the derivatives of the scalar potential to deduce the minimum values :

$$\frac{\partial V_{neutral}}{\partial H_1^0} = 0 = H_1^{0\dagger}|\mu|^2 + \frac{{g_1}^2 + {g_2}^2}{4} H_1^{0\dagger} \left(H_1^{0\dagger} H_1^0 - H_2^{0\dagger} H_2^0 \right) \quad (8.25)$$

$$\frac{\partial V_{neutral}}{\partial H_2^0} = 0 = H_2^{0\dagger}|\mu|^2 - \frac{{g_1}^2 + {g_2}^2}{4} H_2^{0\dagger} \left(H_1^{0\dagger} H_1^0 - H_2^{0\dagger} H_2^0 \right) \quad (8.26)$$

If we assume that $\mu \neq 0$, the only solution to be able to satisfy both equations if to take H_1^0 and H_2^0 to have vanishing vacuum expectation values :

$$\left\langle H_1^0 \right\rangle = \left\langle H_2^0 \right\rangle = 0 \quad (8.27)$$

This is a zero minimum and thus supersymmetry in the electroweak sector is not broken at all. The potential created by the Higgs bosons alone is not sufficient to ensure a supersymmetry breaking in this sector, meaning that the model cannot compete with experimental results. This means that we have to add terms to softly break supersymmetry in the Lagrangian, this is what we said when introducing $\mathcal{L}_{SOFT}$. To deal with this problem, we introduce a soft breaking term in the Lagrangian that we denote :

$${m_1}^2|H_1|^2 + {m_2}^2|H_2|^2 + b\mu H_1 H_2 + h.c \quad (8.28)$$

H) MSSM

We can then find the expression of the selectron mass matrix :

$$\begin{pmatrix}\tilde{e}_L^\dagger & \tilde{e}_R^\dagger\end{pmatrix} M^2 \begin{pmatrix}\tilde{e}_L \\ \tilde{e}_R\end{pmatrix} = \begin{pmatrix}\tilde{e}_L^\dagger & \tilde{e}_R^\dagger\end{pmatrix} \begin{pmatrix} {m_e}^2 + {m_L}^2 + \cos(2\beta)(\sin^2(\theta_W) - \frac{1}{2}){M_Z}^2 & m_e(A_e^* + \mu\tan(\beta)) \\ m_e(A_e + \mu^*\tan(\beta)) & {m_e}^2 + {m_L}^2 - \cos(2\beta)\sin^2(\theta_W){M_Z}^2 \end{pmatrix} \begin{pmatrix}\tilde{e}_L \\ \tilde{e}_R\end{pmatrix} \quad (8.83)$$

Notation	Interpretation	Parameters
$\tan(\beta)$	ratio of the vacuum expectation values of the two Higgs doublets	1
M_{A^0}	mass of the pseudoscalar Higgs boson	1
μ	higgsino mass parameter	1
$m_1,\ m_2,\ m_3$	bino, wino and gluino mass parameters	3
$m_{\tilde{q}},\ m_{\tilde{u}_R},\ m_{\tilde{d}_R}$	first and second generation squark masses	3
$m_{\tilde{q}_3},\ m_{\tilde{t}_R},\ m_{\tilde{b}_R}$	third generation squark masses	3
$m_{\tilde{l}},\ m_{\tilde{e}_R}$	first and second generation slepton masses	2
$m_{\tilde{l}_3},\ m_{\tilde{\tau}_R}$	third generation slepton masses	2
$A_t,\ A_b,\ A_\tau$	third generation trilinear couplings	3

H) MSSM

1. **neutralinos** : what we call neutralinos are the neutral fermions of the theory, namely $\tilde{B}$, $\tilde{W}_3$ (we saw that the gauge partner of the sparticle is the boson responsible for the Z^0 boson and the photon), and the two neutral components of the higgsinos. The mass matrix will be a 4×4 matrix involving couplings between the different particles :

$$-\frac{1}{2}\begin{pmatrix}\tilde{B} & \tilde{W}_3 & \tilde{H}_1^0 & \tilde{H}_2^0\end{pmatrix} M_{\tilde{\chi}^0} \begin{pmatrix}\tilde{B} \\ \tilde{W}_3 \\ \tilde{H}_1^0 \\ \tilde{H}_2^0\end{pmatrix} \tag{8.111}$$

with :

$$M_{\tilde{\chi}^0} = \begin{pmatrix} m_1 & 0 & -iM_W\tan(\theta_W)\cos(\beta) & iM_W\tan(\theta_W)\sin(\beta) \\ 0 & m_2 & iM_W\cos(\beta) & -iM_W\sin(\beta) \\ -iM_W\tan(\theta_W)\cos(\beta) & iM_W\cos(\beta) & 0 & \mu \\ iM_W\tan(\theta_W)\sin(\beta) & -iM_W\sin(\beta) & \mu & 0 \end{pmatrix} \tag{8.112}$$