

Vibrational Spectroscopy of Trapped and Sympathetically Cooled Molecular Ions

Internship presentation

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- ① Vibrational Spectroscopy
- ② Sympathetic cooling and Laser-trapping
- ③ Post Hartree-Fock methods: CCSDT
- ④ Atom-light interactions and Einstein coefficients (for cooled atoms)

Vibrational Spectroscopy

Degrees of freedom

Molecular structures give rise to internal degrees of freedom, e.g For linear molecules there are total $3N$ degrees of freedom including 3 translational, 2 rotational and $(3N-5)$ vibrational

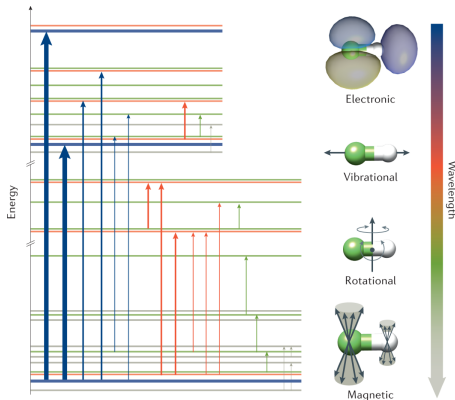


Figure 1: General scheme of the order of transitions for different classes of degrees of freedom

Sympathetic cooling and Laser-trapping

Quadrupole Ion Traps ("AC" Traps)

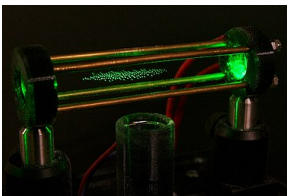


Figure 2: Paul Trap

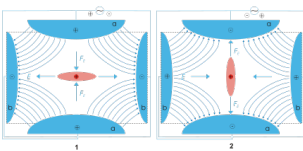


Figure 3: Quadrupole Ion Trap

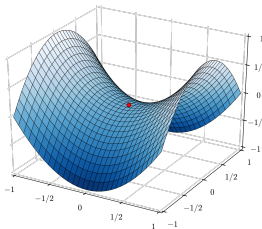


Figure 4: Saddle Point graph analogy

Sympathetic cooling

How?:

- Coulomb interaction between two species (Coolant ions and target ions)
- Laser cooling to reduce kinetic energy

Why?:

- Particles exchanging energy through Coulomb interactions (Maxwell-Boltzmann distribution $f(v_x) = \sqrt{\frac{m}{2\pi k_B T}} \exp\left(-\frac{mv_x^2}{2k_B T}\right)$) leading to **Reduction of Doppler broadening**, $\Delta\lambda_D = \lambda_0 \sqrt{\frac{8k_B T \ln 2}{mc^2}}$.
- Reduce perturbations by the field on target ions
- High-resolution spectroscopy

How good?: reaching $T \sim \mu K$

Ion Coulomb Crystals

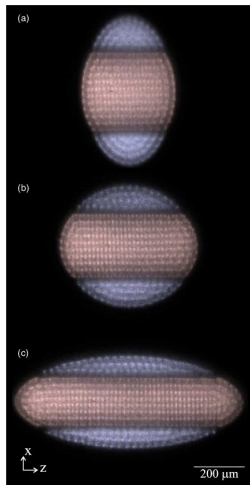


Figure 5: Projection images of two-species ion Coulomb crystals containing 1500 $^{40}\text{Ca}^+$ ions (red) and 2000 $^{44}\text{Ca}^+$ ions (blue) at different settings of the end-cap potential, U_{end} . The camera exposure time is 100 ms and the trap potentials are $U_{\text{rf}} = 540$ V and (a) $U_{\text{end}} = 46.1$ V, (b) $U_{\text{end}} = 30.2$ V and (c) $U_{\text{end}} = 13.8$ V, respectively.

Post Hartree-Fock methods: CCSDT

Why CCSDT?

The Hartree-Fock method:

- The foundational tool in quantum and computational chemistry
- But does not account for electrons' correlation energy

Post HF methods:

- CI (Configuration interaction), (CIS, CISD, CISDT)
- CC (Coupled Cluster)
- DFT (Density Functional Theory)...

Why CCSDT?

CCSDT and its advantages:

- Exponential Ansatz: $|\Psi\rangle = e^T |\Phi_0\rangle$
- Singles (T_1) :

$$T_1 = \sum_{i,a} t_i^a a_a^\dagger a_i$$

Here, i and j denote occupied orbitals, a and b denote virtual orbitals, a_a^\dagger and a_i are creation and annihilation operators, and t_i^a are the single excitation amplitudes. Doubles (T_2) :

$$T_2 = \frac{1}{4} \sum_{i,j,a,b} t_{ij}^{ab} a_a^\dagger a_b^\dagger a_j a_i$$

- Fast convergence.
- Lower computational cost

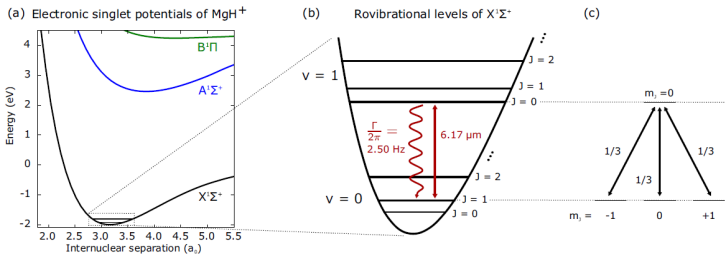
Results for the PES (Potential Energy Surfaces) scans for the MgH^+ molecule

Figure 6: The electronic singlet potentials of MgH^+ . Only the electronic ground state $X^1\Sigma^+$ is considered in the simulation. (b) Rovibrational structure of the electronic ground state $X^1\Sigma^+$ and showing the closed transition of interest $|v=0, J=1\rangle \rightarrow |v=1, J=0\rangle$. The decay rate of $t = 2 \times 2.50 \text{ Hz}$ is the slowest timescale of our experiment and spontaneous emission is negligible. (c) Sublevels of the $|v=0, J=1\rangle$ and $|v=1, J=0\rangle$ states showing the norm squared of the Clebsch-Gordan coefficients.

Atom-light interactions and Einstein coefficients (for cooled atoms)

Toy model of 3 level system

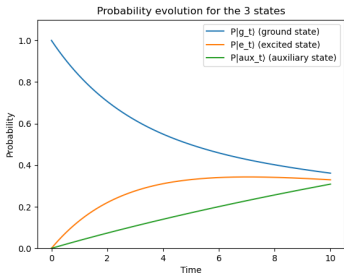


Figure 7: Probability evolution of a 3 state system

$$\begin{aligned} \frac{d}{dt} P_{|g_t\rangle} &= -R_{\text{abs},0} P_{|g_t\rangle} + \left(|\tilde{\xi}|^2 R_{\text{stim},0} + \tilde{D} \Gamma_t \right) P_{|g_t\rangle} \\ \frac{d}{dt} P_{|e_t\rangle} &= -(\Gamma_t + R_{\text{stim},0}) P_{|e_t\rangle} + |\tilde{\xi}|^2 R_{\text{abs},0} P_{|g_t\rangle} \\ \frac{d}{dt} P_{|\text{aux}\rangle} &= \left[(1 - \tilde{D}) \Gamma_t + (1 - |\tilde{\xi}|^2) R_{\text{stim},0} \right] P_{|e_t\rangle} \\ &\quad + (1 - |\tilde{\xi}|^2) R_{\text{abs},0} P_{|g_t\rangle}. \end{aligned}$$

Einstein A coefficients, A_{ij} , are found from the wavefunctions and the permanent dipole moment:

$$A_{ij} = \frac{\omega_{ij}^3 |D_{ij}|}{3g_i c^3 \epsilon_0 \hbar \pi},$$

where $|D_{ij}|$ is the transition dipole moment between the states and g_i is the degeneracy of state i . The corresponding Einstein B coefficients are given by

$$B_{ij} = \frac{c^3}{4h\nu_{ij}^3} A_{ij}$$
$$B_{ji} = \frac{g_i}{g_j} B_{ij},$$

where $\nu_{ij} = \omega_{ij}/2\pi$ is the frequency difference between state i and state j , and $g_i = 2J + 1$ is the degeneracy of the state i with rotational quantum number J .

Expectations

Our current findings for the coefficients agree to about 5% to 10% with the literature

$ v, J\rangle_{ini}$		$\Delta v = 0$		-1		-2		-3		-4
v	J	$\Delta J = -1$	1	-1	1	-1	1	-1	1	-1
0	0									
	1	0.002								
	2	0.022								
	3	0.078								
	4	0.191								
	5	0.381								
	6	0.665								
	7	1.062								
	8	1.589								
1	0		20.565							
	1	7.886	12.773	0.002						
	2	10.141	10.705	0.019						
	3	11.636	9.490	0.068						
	4	12.914	8.586	0.167						
	5	14.123	7.843	0.333						
	6	15.323	7.203	0.581						
	7	16.541	6.637	0.927						
	8	17.793	6.129	1.387						

Figure 8: Einstein A coefficient for transitions within the electronic ground state of MgH⁺

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