A black hole effective theory for strongly interacting matter

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POSTECH
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Holography and dense matter workshop
APC Paris — 12 June 2024

[MJ, Dorin Weissman 2405.17533]

[Romuald Janik, MJ, Hesam Soltanpanahi, Jacob Sonnenschein 2205.06274]
[Romuald Janik, MJ. Jacob Sonnenschein 2106.02642]

National Research Foundation of Korea

Outline

1. Introduction and motivation
   - Connection to gravitational wave production
2. Effective theories from holography
   - Overview of different approaches
   - Application to bubble wall velocity
3. Black holes at large D
   - Membrane-like description of the horizon
4. A generalized black hole effective theory
   - Large-D-inspired approach to low-dimensional theories via holography
5. Conclusion

[See also Mikel's talk]

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Phase transitions at strong coupling

First order transitions and domain walls are fundamental topics in physics

Examples in the context of strongly coupled field theory:

1. QCD
   - (Conjectured) phase transition at finite density probed in heavy-ion collisions
   - Phase transitions in neutron stars and neutron star mergers
2. Strongly coupled extensions of the standard model
   - (First order) phase transitions in the electroweak, hidden sectors, or at higher energies than the standard model
   - Could occur via bubble nucleation in the early universe

Phase transitions in the early universe

Dynamics of a first order phase transition with $T = T_c$

- Bubble nucleation to the low T phase starts when T crosses T_c (thermal fluctuations and quantum tunneling)
- Significant bubble nucleation: rate per volume $\Gamma \sim H^4 \Rightarrow$ approximation to transition temperature $T_*(< T_c)$
- Nucleated bubbles expand and collide
- Gravitational wave production in three stages
  1. Initial collisions of the bubbles
  2. Sound waves created in the collisions (dominant mechanism)
  3. Final turbulent phase
- Gravitational waves might be detectable by LISA or its successors

Domain walls

Static domain walls at $T \approx T_c$

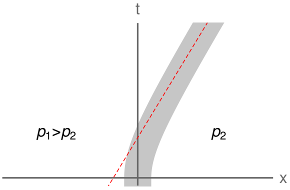
- At a first order phase transition at $T = T_c$, we can have domains of coexisting phases separated by domain walls
- The pressures on both sides are balanced and the domain wall can be static

Going away from $T = T_c$, different situations:

- for nucleated bubbles of a stable phase within an supercooled medium
- at an interface between stable phases away from $T = T_c$
- at an interface between phases at different temperatures

Pressure difference over the domain wall drives its motion

- For strong interactions solving the surface tension, wall profile and velocity are hard questions $\Rightarrow$ use gauge/gravity duality?



Holographic domain walls

Gauge/gravity duality: study phase transitions and domain walls by solving classical higher dimensional gravity

- Finding classical solutions in principle straightforward

However:

- Holographic methods add one coordinate
- Solving Einstein equations numerically somewhat challenging
- Typically would like to study confinement-deconfinement transitions – it turns out to be particularly tricky
  - Confining and deconfining geometries have different structure
  - Leads to a "discontinuity" of the geometry at the wall

Solutions

1. Try to solve the geometry nevertheless (hard)
   [Aharony, Minwalla, Wiseman hep-th/0507219
   Bantilan, Figueras, Mateos 2001.05476]
2. Study simpler (deconfining-deconfining) transitions in gravity
   [See e.g. Bellantuono et al. 1906.00061; Bea et al. 2202.10503]
3. Use effective theory methods (this talk)

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Different simplifications

Solving Einstein equations numerically hard, in particular time evolution. Simplifications:

1. Solve the effective potential V for the order parameter from holography – use the effective potential in field theory computation (ask Niko)
   [Ares, Henriksson, Hindmarsh, Hoyos, Jokela 2109.13784, 2110.14442]
2. Match the results from gravity simulations to a hydrodynamic theory – use the hydro theory to do numerical evoluation (I will discuss this briefly now)
   [Janik, MJ, Sonnenschein 2106.02642; Janik, MJ, Soltanpanahi, Sonnenschein 2205.06274 + WIP]
3. Derive a hydrodynamic theory from gravity in an expansion motivated by dynamics high-dimensional black holes (main topic of this talk)
   [MJ, Weissman 2405.17533]

Generalized hydrodynamic model

One can define the model through a Lagrangian

[Janik, MJ, Sonnenschein 2106.02642]

$$\mathcal{L} = \underbrace{(1-\Gamma(\gamma))p(T)}_{\text{deconfined}} + \underbrace{\Gamma(\gamma)}_{\text{confined}} - \underbrace{\frac{1}{2}a\left((\partial\gamma)^2 + V(\gamma,T)\right)}_{\text{domain wall}}$$

- Directly defined in space-time coordinates (not holographic)
- Perfect fluid hydrodynamics + extra field γ = order parameter
- Equations of motion are

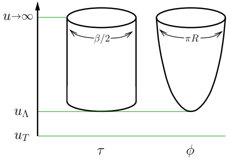
$$\partial_\mu T^{\mu\nu} = 0 \,, \qquad a\,\partial^2\gamma = -\frac{\partial\mathcal{L}}{\partial\gamma}$$

  with $T_{\mu\nu}$ computed from $\mathcal{L}$
- $\Gamma(\gamma)$, $V(\gamma,T)$, and a to be determined by comparing to full gravity solutions in a holographic model (for example)

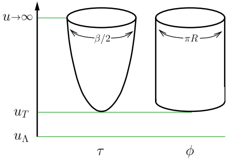
Matching with Witten's model

Use (5 dimensional) Witten model [Witten hep-th/9803131; ...]

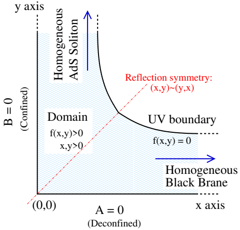
- Two compact coordinates: Euclidean time τ and extra spatial coordinate ϕ
- Confinement-deconfinement transition: cigar-cylinder swap for τ and ϕ geometries [Aharony, Sonnenschein, Yankielowicz hep-th/0604161]
- The domain wall known numerically (highly nontrivial) [Aharony, Minwalla, Wiseman hep-th/0507219]

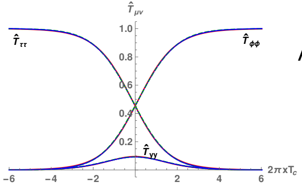


low temperature phase



high temperature phase





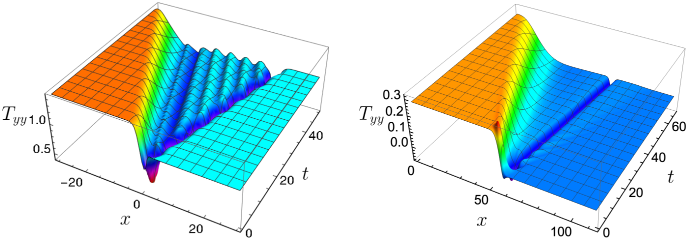
Almost perfect fit to $T_{\mu\nu}$ with hydro model:

$$\begin{aligned}\Gamma(\gamma) &= \gamma^2(3-2\gamma)\\ V(\gamma,T) &= q_*^2\,T^{2+2\Gamma(\gamma)}\gamma^2(1-\gamma)^2\end{aligned}$$

Simulations and the wall speed

Pressure for moving domain wall at $T > T_c$ wall using:

Hydro fitted to Witten's model — Gravity simulation



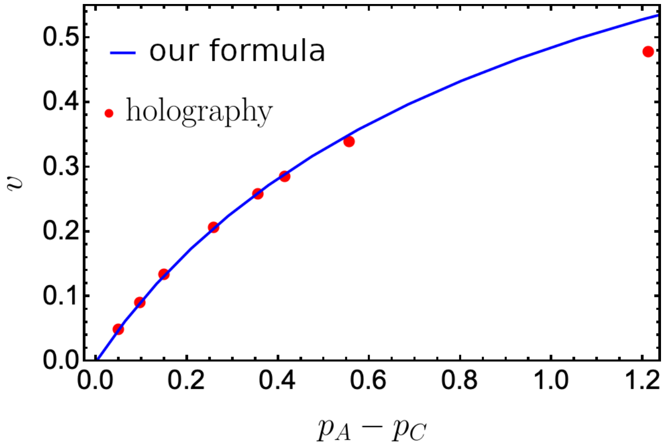
- Full holographic gravity simulation (only) for deconfinement-deconfinement transition
- Domain wall and hydrodynamic waves separate
- Wall velocity determined by the hydrodynamic wave:

$$v_{\text{domain wall}} = \tanh \int_{p_c}^{p_A} \frac{dp}{(\epsilon + p)c_s} \equiv \tanh \int_{T_c}^{T_A} \frac{dT}{T c_s}$$

- Expressed in terms of the equation of state

[Janik, MJ, Soltanpanahi, Sonnenschein 2205.06274]

Velocity formula compared to simulations



$$v_{\text{domain wall}} = \tanh \int_{p_c}^{p_A} \frac{dp}{(\epsilon + p) c_s}$$

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Black hole solutions at large D

Solutions at high-dimensional Einstein gravity

$$S = \frac{1}{16\pi G_D} \int d^D x \sqrt{-\det g}\, [R + \Lambda] \ , \qquad \Lambda = \frac{D(D-1)}{\ell^2}$$

Static (AdS) black hole geometry

$$ds^2 = \frac{\ell^2}{r^2} \left(\frac{dr^2}{f(r)} - f(r) dt^2 + d\vec{x}^2 \right) \ , \quad f(r) = 1 - \left(\frac{r}{r_h} \right)^{D-1}$$

Here $f(r) \approx 1$ unless $r_h - r \sim 1/D \Rightarrow$

- Most of the space $\approx$ empty AdS
- Black hole horizon – membrane with width $\sim 1/D$ – thin at large D
- Fluctuation modes near the horizon decoupled from the rest [Mikel's talk]

Expect that decoupling also works for dynamical black holes. How to describe the dynamics of the membrane?

Effective theory for large D black holes

Consider a moving black hole Ansatz (with $\mathcal{R} = e^{Dr/\ell}$)

$$ds^2 = \frac{\ell^2}{r^2}\left(-f dt^2 - 2\,dt\,dr - 2C_i\,dt\,dx^i + g_{ij}dx^i dx^i\right)$$

$$\begin{aligned} f &= 1 - m(t,x^k)\mathcal{R} + \frac{1}{D}f^{(1)}(\mathcal{R},t,x^k)\,, \\ g_{ij} &= \frac{1}{D}\delta_{ij} + \frac{1}{D^2}\frac{p_i(t,x^k)p_j(t,x^k)}{m(t,x^k)}\mathcal{R}\,, \\ C_i &= \frac{1}{D}p_i(t,x^k)\mathcal{R} + \frac{1}{D^2}C_i^{(1)}(\mathcal{R},t,x^k)\,, \end{aligned}$$

with the mass m and momentum density p^i of the black hole

Solving Einstein equations as a series in $1/D$ gives explicit solutions for $f^{(1)}$, $C^{(1)}$ and [Emparan, Suzuki, Tanabe 1506.06772]

$$\begin{aligned} \partial_t m &= \ell\partial_i\partial^i m - \partial_i p^i\,, \\ \partial_t p_i &= \ell\partial_j\partial^j p_i - \partial_j\left(\frac{p^j p_i}{m}\right) - \partial_i m\,, \end{aligned}$$

- $D-1$ dimensional effective theory
- Non-relativistic hydrodynamics – see scaling of g_{ij} – easy to simulate! [Mikel's talk]

Dimensional reduction

Can the effective theory be useful at lower number of dimensions?

Reduce dimensionally (over flat manifold)

[Goutéraux, Smolic, Smolic, Skenderis, Taylor 1110.2320]

$$S = \frac{1}{16\pi G_D}\int d^D x\,\sqrt{-\det g}\,[R+\Lambda]$$

to

$$S = \frac{1}{16\pi G_5}\int d^5 x\,\sqrt{-\det g}\,\left[R - \frac{4}{3}(\partial\phi)^2 + V(\phi)\right]$$

where

$$V(\phi) = \frac{D^2}{\ell^2}\exp\left(\frac{4}{3}\frac{\sqrt{D-5}}{\sqrt{D-2}}\phi\right) = \frac{D^2}{\ell^2}e^{\frac{4}{3}\phi}\left[1 - \frac{2\phi}{D} + \mathcal{O}\left(\frac{1}{D^2}\right)\right]$$

- The exponential potential $e^{\frac{4}{3}\phi}$ is the critical choice separating confining and deconfining setups [Gürsoy, Kiritsis, Nitti 0707.1349]
- Due to $1/D$ correction deconfinement wins, no domain walls
- Most quasi-normal modes of black hole solution solvable analytically [Betzios, Gürsoy, MJ, Policastro 1708.02252, 1807.01718; Gürsoy, MJ, Policastro, Zinnato 2112.04296]

Can this setup be generalized to theories admitting domain walls?

Outline

Large-D-inspired approach

Starting from the dimensionally reduced action (with $V_0 = D^2/\ell^2$)

$$S = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left[R - \frac{4}{3} (\partial\phi)^2 + V_0 e^{4\phi/3} (1 - 2\phi/D)\right]$$

Large-D-inspired approach

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we consider a generalized potential where δV is arbitrary

[MJ, Weissman 2405.17533]

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[MJ, Weissman 2405.17533]

Potential similar to

- Improved holographic QCD model (first order transition)

$$V(\phi) \sim e^{4\phi/3} \sqrt{\phi}\ , \qquad (\phi \to \infty)$$

[Gürsoy, Kiritsis 0707.1324; Gürsoy, Kiritsis, Nitti 0707.1349]

- Holographic duals of spin models (high order transitions), e.g.

$$V(\phi) \sim e^{4\phi/3} \left(1 + C e^{-\kappa\phi} \right)\ , \qquad (\phi \to \infty)$$

[Gürsoy 1007.4854, 1007.0500]

Generalized effective theory

Plug in (dimensionally reduced) large-D-inspired black hole ansatz

$$ds^2 = e^{2A}\left(-f dt^2 - 2\,dt\,dr - 2C_i\,dt\,dx^i + g_{ij}dx^i dx^i\right)$$

$$\begin{aligned} A &= -\frac{Dr}{3\ell} + \frac{1}{D}A^{(1)}(\mathcal{R}, t, x^k)\,, \\ f &= 1 - m(t, x^k)\mathcal{R} + \frac{1}{D}f^{(1)}(\mathcal{R}, t, x^k)\,, \\ g_{ij} &= \frac{1}{D}\delta_{ij} + \frac{1}{D^2}\frac{p_i(t, x^k)p_j(t, x^k)}{m(t, x^k)}\mathcal{R}\,, \\ C_i &= \frac{1}{D}p_i(t, x^k)\mathcal{R} + \frac{1}{D^2}C_i^{(1)}(\mathcal{R}, t, x^k) \end{aligned}$$

A huge mess ensues . . .

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A huge mess ensues . . . but amazingly, you can still solve the Einstein equations, at leading order in $1/\text{D}$! You get

[MJ, Weissman 2405.17533]

$$\boxed{\begin{aligned} &\partial_t m = \ell\partial_i\partial^i m - \partial_i p^i \\ &\partial_t p_i = \ell\partial_j\partial^j p_i - \partial_j\left(\frac{p^j p_i}{m}\right) + \frac{1}{2}\delta V'\left(-\frac{1}{2}\log m\right)\partial_i m \end{aligned}}$$

- $3+1$ dimensional theory, D just an expansion parameter

Thermodynamics and hydrodynamics

Armed with explicit solutions, it is straightforward to check black hole thermodynamics (for static black holes)

$$T \approx \frac{D}{4\pi\ell}\left[1+\frac{1}{D}\delta V\left(-\frac{1}{2}\log m\right)\right] \approx \frac{D}{4\pi\ell}e^{-4\phi/3}V(\phi)\Big|_{\phi=-\frac{1}{2}\log m}$$

$$s \approx \frac{1}{4G_5}m$$

Fluctuations around a static black hole $\Rightarrow$ hydrodynamic modes

$$\begin{aligned} \omega &= -i\ell q^2 && \text{(shear)} \\ \omega &= \pm\sqrt{-\delta V'\left(-\log m/2\right)/2}\, q - i\ell q^2 && \text{(sound)} \end{aligned}$$

Spinodal instability for $\delta V' > 0$ (as seen both from T and sound modes)

Domain wall solutions

Assuming dependence on only one spatial coordinate

$$
\begin{aligned}
m'' - \frac{(m')^2}{m} &= -\frac{1}{2\ell^2}\int_{m_0}^{m(x)} dm\,\delta V'\left(-\frac{1}{2}\log m\right) \\
\frac{m''}{m} - \frac{(m')^2}{2m^2} &= \frac{1}{\ell^2}\left[\delta V\left(-\frac{1}{2}\log m(x)\right) - \delta V\left(-\frac{1}{2}\log m_0\right)\right]
\end{aligned}
$$

Existence of a static domain wall (m_0 to m_1) implies, consistently with the black hole thermodynamics

$$
0 = \int_{m_0}^{m_1} dm\,\delta V'\left(-\frac{1}{2}\log m\right) \propto -\int s\,dT = \Delta F
$$

$$
0 = \delta V\left(-\frac{1}{2}\log m_0\right) - \delta V\left(-\frac{1}{2}\log m_1\right) \propto \Delta T
$$

Another nice consistency check!

Application to IHQCD: setup

Example: consider domain wall in the improved holographic QCD model (IHQCD)

[Gürsoy, Kiritsis 0707.1324; Gürsoy, Kiritsis, Nitti 0707.1349]

Use the potential fitted to lattice (and other) data

$$V(\phi) = \frac{12}{1+\frac{c_{\rm UV}}{e^\phi}} + V_1 e^\phi + \frac{V_2 e^{2\phi}}{1+c_{\rm IR}e^\phi} + V_{\rm IR}\, e^{-\frac{1}{c_{\rm IR}e^\phi}}\, e^{4\phi/3}\sqrt{\log(1+c_{\rm IR}e^\phi)}$$

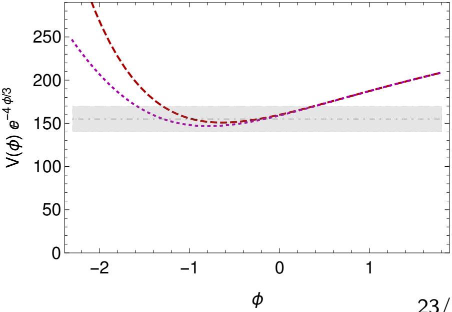
[Jokela, MJ, Remes 1809.07770]

with either $c_{\rm UV} = 0$ (original) or $c_{\rm UV} = 0.075$ (adjusted)

Decompose

$$V(\phi) = V_0 e^{4\phi/3}(1+\delta V(\phi))$$

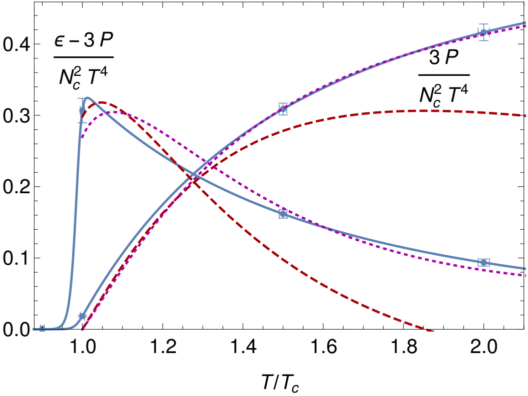
with $V_0 = 155$ and apply the large-D-like expansion



Application to IHQCD: check of thermodynamics

Check fo thermodynamics at $T > T_c$

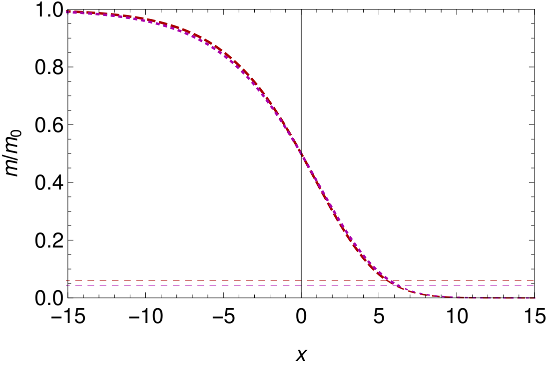
- Note: for T well above T_c, the nonrelativistic approach unlikely to work well



- Unsurprisingly, the original with is passable only around $T \approx T_c$
- The adjusted potential gives better fit

Application to IHQCD: domain walls

Results for the domain wall profile

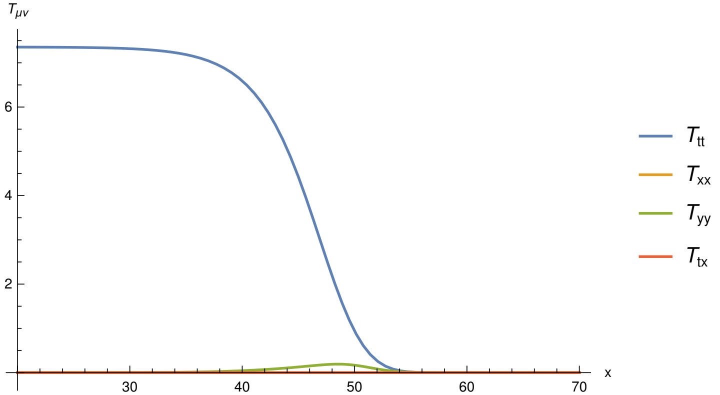


- Despite differences in the thermodynamics for $T > T_c$, profiles nearly identical
- This suggests that description works near $T = T_c$
- Accuracy near $m = 0$ more difficult to check – however only the tail of the solution probes the region where δV becomes sizeable

Application to IHQCD: $T_{\mu\nu}$

(Naive) computation of the $T_{\mu\nu}$ components

[Work in progress]



- Surface tension (bump in T_{yy}) is small
- Uncertainty from fitting procedure larger than the error due to the expansion?
- Result clearly asymmetric

Outline

Conclusion

I discussed two approaches for holographic phase transitions

1. A hydrodynamic setup fitted to the Witten's model
   - Simple formula for domain wall velocity motivated by simulations
2. Large-D-inspired black hole effective theory
   - Generalized theory allows decription of phase transitions
   - Analyzed the domain wall in IHQCD – error due to the $1/D$ expansion small (?)

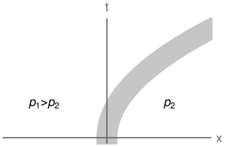
Future directions (for the large-D setup)

- Study time evolution $\Rightarrow$ bubble collisions and gravitational wave production?
- Other setups: black hole – black hole transitions, duals of spin models, ...
- Extend to finite density (charged black holes)
- Compare to exact IHQCD domain wall?
- Study matching with near boundary (near AdS_5) solutions
- Try to extend to higher order?

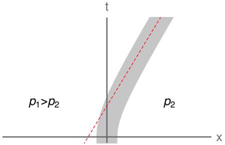
Thank you!

Moving domain walls

- The pressures on both sides are not balanced so one would (naively) expect accelerated motion. . .



- . . . but analysis shows that eventually the acceleration stops and the domain wall approaches a constant velocity

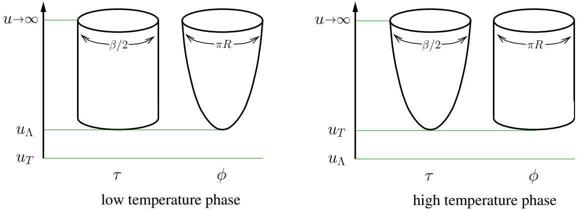


- Common lore: friction balances the net force – challenging to calculate

Our main result: At strong coupling, the domain wall velocity can be understood in a simpler way using essentially only the equation of state

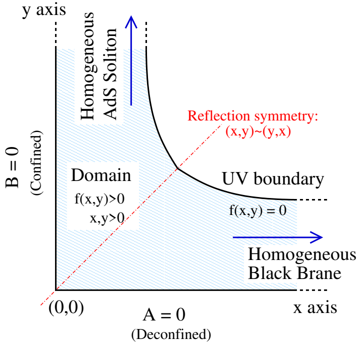
Deconfinement transition in Witten's model

- An example of a holographic theory with a first order confinement/deconfinement phase transition: (a lower dimensional variant of) the Witten model [Witten hep-th/9803131; Brandhuber, Itzhaki, Sonnenschein, Yankielowicz hep-th/9803263]
- On the boundary one compactifies a coordinate (ϕ) on a circle with anti-periodic boundary conditions for the fermions
- At low temperatures the bulk geometry of the ϕ circle closes off into a cigar $\to$ confinement (soliton geometry)
- At high temperatures, the geometry of the Euclidean τ circle closes off into a cigar $\to$ deconfined phase (black brane)
- In between, there is a first order phase transition [Aharony, Sonnenschein, Yankielowicz hep-th/0604161]



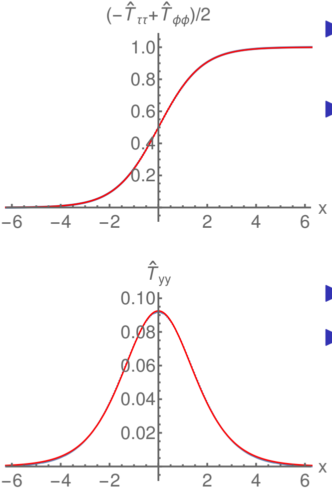
The AMW solution

- AMW constructed numerically a static planar domain wall solution interpolating between confined and deconfined phases
  [Aharony, Minwalla, Wiseman hep-th/0507219]
- The numerical relativity setup is highly nontrivial due to the different topologies of the geometries in the different phases



- We use the solution with 4+1 bulk dimensions, dual to a 3+1 d boundary theory with one compactified coordinate

$T_{\mu\nu}$ of the AMW solution



- $-T_{\tau\tau} + T_{\phi\phi}$ from the numerical solution
- Ridiculously good fit by

$$1 - \gamma(x) = \frac{1}{2}\left(1 + \tanh \frac{q_* x}{2}\right)$$

$\gamma = 0$ deconf. ($\gamma = 1$ conf.) phase

- Only additional feature: bumb in T_{yy}
- Can be fit by

$$\frac{\gamma'^2}{\gamma(1-\gamma)} \propto \frac{c}{\cosh^2 \frac{q_* x}{2}}$$

or

$$\gamma'^2 \propto \frac{\tilde{c}}{\cosh^4 \frac{\tilde{q}_* x}{2}}$$

N.B. tanh fit also works well in nonconformal holographic models
[Attems, Bea, Casalderrey-Solana, Mateos, Zilhão 1905.12544]

Lagrangian for the hydrodynamic model

The hydrodynamic picture can be derived from a simple action (with 2+1 boundary dimensions) [Janik, MJ, Sonnenschein 2106.02642]

$$\mathcal{L} = (1-\Gamma(\gamma))p(T) + \Gamma(\gamma) - \frac{1}{2}a(\gamma)T^{\alpha}\left((\partial\gamma)^2 + T^{\beta}q_*^2\gamma^2(1-\gamma)^2\right)$$

where $3\alpha + \beta = 2 + 2\Gamma$, $p(T)$ is the Lagrangian density for perfect fluid hydrodynamics, and

- **Option A:**
$$\Gamma = \gamma\,, \quad a(x) = \frac{c}{\gamma(x)(1-\gamma(x))}$$
- **Option B:**
$$\Gamma = \gamma^2(3-2\gamma)\,, \quad a(x) = \tilde{c}$$

The order parameter γ is promoted into a dynamical field!

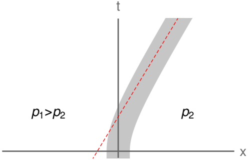
- Coupling of γ to hydrodynamics natural, and will lead to asymmetric effective potential for γ when $T \neq T_c$

Time-dependent solutions in Witten's model hard
[See however Bantilan, Figueras, Mateos 2001.05476]

- Use the hydro model (option B) for simulations

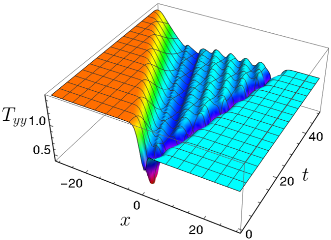
Expectations vs. simulation results

Evolve a domain wall at $T > T_c$: expect that the wall first accelerates and later propagates at constant velocity

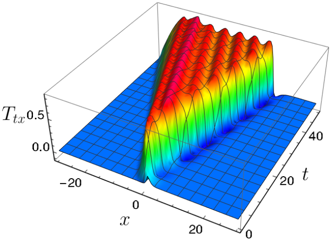


Simulation results in Witten's model:

pressure

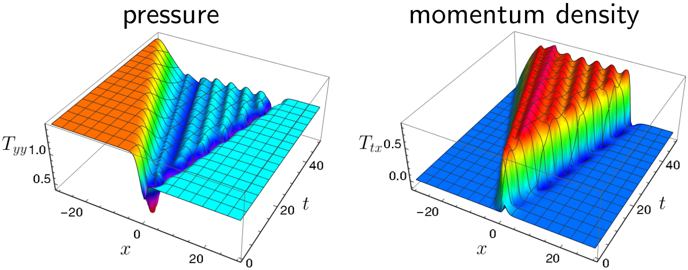


momentum density



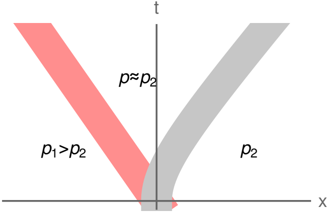
Surprise! Looks rather different ...

What's going on?



- The domain wall moves to the right as expected, with constant speed
- Nontrivial dynamics on the deconfined side (left)
- A hydrodynamic wave moving to the left
- Pressure difference over the hydro wave, no difference over the domain wall
- In the middle, plasma moving to the right following the wall

Schematic picture



The full picture is obtained by gluing together two elements:

1. A domain wall moving at constant speed to the right, with no pressure difference
   - Turns out to be boosted static domain wall!
2. A hydrodynamic wave moving to the left, supporting the pressure difference

Hydrodynamic wave

- The hydrodynamic wave solution should interpolate between static plasma with $p_A > p_c$ and plasma with $p = p_c$ moving with the domain wall velocity
- Since the wave is deep in the deconfined phase, it is described in terms of perfect fluid hydrodynamics only
- Since $v_{\text{fluid}} = v_{\text{domain wall}}$, we may access $v_{\text{domain wall}}$ through a hydrodynamic computation in the deconfined phase!

Nonlinear hydrodynamic solution: "simple wave"

[Landau, Fluid mechanics]

- One assumes that all hydrodynamic quantities are functions of a single variable (e.g. pressure)
- One gets our main result

$$v_{\text{domain wall}} = \tanh \int_{p_c}^{p_A} \frac{dp}{(\epsilon + p)c_s} \equiv \tanh \int_{T_c}^{T_A} \frac{dT}{Tc_s}$$

Hydrodynamic simulations in Witten's model not precise enough to compare to this formula. We will instead use a different holographic model.

Nonconformal holographic model

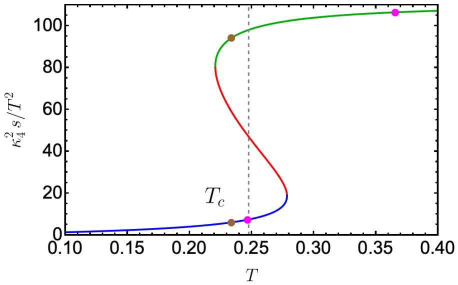
- A gravity+scalar field system in D = 3 + 1 bulk dimensions

$$S = \frac{1}{2\kappa_4^2} \int d^4x \sqrt{-g} \left[R - \frac{1}{2} (\partial\phi)^2 - V(\phi) \right]$$

$$V(\Phi) = -6\cosh\left(\frac{\Phi}{\sqrt{3}}\right) - 0.2\,\Phi^4$$

[Janik, Jankowski, Soltanpanahi 1704.05387;
Bellantuono, Janik, Jankowski, Soltanpanahi 1906.00061]

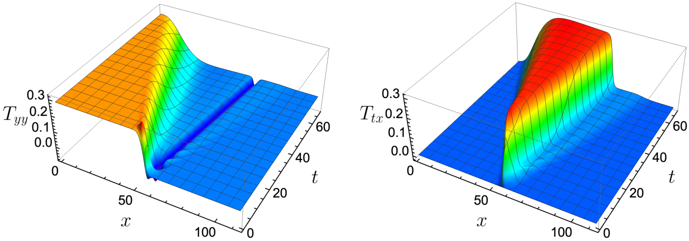
- Two deconfined phases separated by a first order transition



- Horizon in both phases, so much easier to setup a numerical relativity computation

Simulations in the holographic model

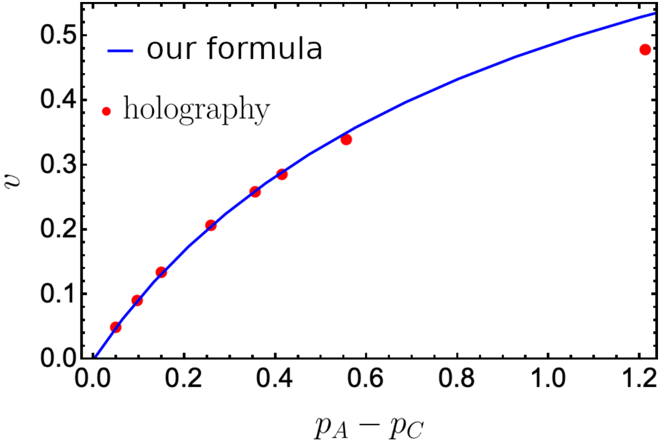
Quite similar results as in Witten's model:



However, some important differences:

- Smooth profiles, due to finite dissipation
- Hydrodynamic evolution also in the low temperature side (instead of empty space)
- Different fluid velocities (not shown) on different sides of the domain wall
- One can show that the low-T phase velocity $v_L \approx v_{\text{domain wall}}$ $\Rightarrow$ apply our formula for the high-T phase

Velocity for the nonconformal model

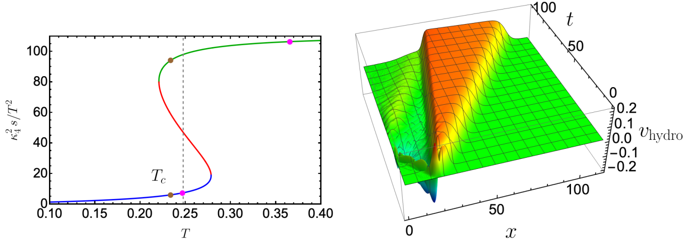


$$v_{\text{domain wall}} = \tanh \int_{p_c}^{p_A} \frac{dp}{(\epsilon + p) c_s}$$

Simulations for bubble nucleation

It is interesting to also study the expansion of small region of the stable low T phase (left) in the supercooled high T phase (right)

- Model for bubble nucleation in the early universe



- Hydrodynamic wave traveling in front of bubble wall ("deflagration")
- Inside the bubble, fluid eventually at rest
- As earlier, pressure difference over the domain wall (not shown) ≈ 0

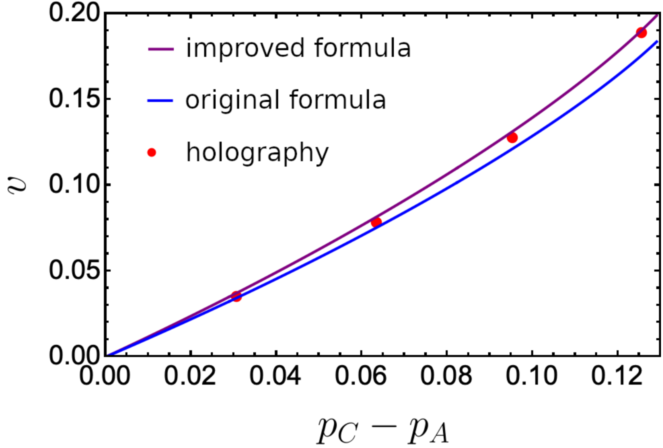
Decorating the velocity formula

For the bubble nucleation case, the fluid inside the bubble is at rest

- Use this to improve the velocity formula: we can now compute v_L (instead of just basically ignoring it)
- Skipping details: a correction factor

$$v_{\rm domain\ \ wall} = \frac{1}{1 - \frac{\epsilon_L + p_H}{\epsilon_H + p_L}} \tanh \int_{p_c}^{p_A} \frac{dp}{(\epsilon + p) c_s}$$

Comparison to the velocity formula



Literature on domain walls and holography

Lots of studies related to phase separation in various models:

- Dynamical, real-time dependent gravity simulations of nonconformal setups (Einstein dilaton gravity)
  - In 4+1 bulk dimensions [Attems, Bea, Casalderrey-Solana, Mateos, Triana, Zilhão 1703.02948; Bea, Casalderrey-Solana, Giannakopoulos, Jansen, Mateos, Sanchez-Garitaonandia, Zilhão 2202.10503; . . .]
  - In 3+1 bulk dimensions [Janik, Jankowski, Soltanpanahi 1704.05387; Bellantuono, Janik, Jankowski, Soltanpanahi 1906.00061]
- Simulations in Witten's model (top-down) [Bantilan, Figueras, Mateos 2001.05476]
- Phase transitions in Witten-Sakai-Sugimoto model (probe flavor branes) [Bigazzi, Caddeo, Cotrone, Paredes 2008.02579; Bigazzi, Caddeo, Cotrone 2104.12817]
- An effective action derived from (bottom-up) holography [Ares, Henriksson, Hindmarsh, Hoyos, Jokela 2109.13784, 2110.14442]

This talk:

1. Hydrodynamic description of static domain walls
2. A hydrodynamic formula for domain wall velocities
3. Comparison to results from holography

Our tools

We study domain walls for the deconfinement transition, and domain wall velocities using

1. (A hydrodynamic description of) the Witten's model
[Witten hep-th/9803131, . . .]
2. A nonconformal holographic model with 3+1 bulk dimensions: Einstein dilaton gravity with nontrivial dilaton potentials
[Janik, Jankowski, Soltanpanahi 1704.05387]
3. Some data from a nonconformal 4+1 dimensional model
[Bea, Casalderrey-Solana, Giannakopoulos, Mateos, Sanchez-Garitaonandia, Zilhão 2104.05708]

A hydrodynamic model for the full $T_{\mu\nu}$

The fit motivates a simple covariant Ansatz:

[Janik, MJ, Sonnenschein 2106.02642]

$$T_{\mu\nu}(x) = \underbrace{\Gamma(x)\eta_{\mu\nu}}_{\text{confined}} + \underbrace{(1-\Gamma(x))\,T^{\text{hydro}}_{\mu\nu}(x)}_{\text{deconfined}} + \underbrace{a(x)\left(\partial_\mu\gamma\partial_\nu\gamma - (\partial\gamma)^2\eta_{\mu\nu} - (1+\Gamma(x))(\partial\gamma)^2 u_\mu u_\nu\right)}_{\text{domain wall}}$$

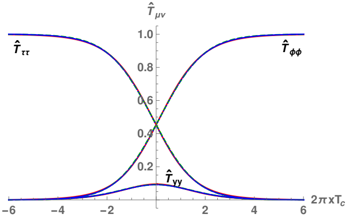
with $T^{\text{hydro}}_{\mu\nu} = p(T)(\eta_{\mu\nu} + 4u_\mu u_\nu)$, u four-velocity at rest, and

- **Option A:**

$$\Gamma = \gamma\,, \quad a(x) = \frac{c}{\gamma(x)(1-\gamma(x))}$$

- **Option B:**

$$\Gamma = \gamma^2(3-2\gamma)\,, \quad a(x) = \tilde{c}$$



Why do we need a hydrodynamic model?

- For the Witten model, a numerical relativity description of time-dependent evolution with two phases is extremely difficult!

  [See however Bantilan, Figueras, Mateos 2001.05476]
- In this case we performed numerical simulations of the evolution of our simplified hydrodynamical model (option B)
- Our initial condition is a tanh profile of the domain wall between the deconfined and confined phases but at $T > T_c$
- Since we are away from T_c, the solution will no longer be static but will start to evolve in time

First ingredient: boosted domain wall

- The most natural solution is the AMW solution boosted to the domain wall velocity
- For this solution, the pressure of the deconfined phase next to domain wall is $p_c = p(T_c)$
- The fluid velocity is equal to the domain wall velocity
- Thanks to its covariance, our hydrodynamic model indeed has such a boosted solution
- (According to numerical checks) it is the only static solution in the rest frame of the wall

How to apply the velocity formula?

Basic idea: since entropy and energy density are low in the low T phase, it is well approximated by empty space
$\Rightarrow$ apply the formula for the high T phase as earlier

"Prove" by going to the rest frame of the domain wall

- By conservation of energy-momentum
[Gyulassy, Kajantie, Kurki-Suonio, McLerran NPB 237, 477 (1984)]

$$\frac{v_H}{v_L} = \frac{\epsilon_L + p_H}{\epsilon_H + p_L} \qquad v_H v_L = \frac{p_H - p_L}{\epsilon_H - \epsilon_L}$$

- We obtain, inserting the simulation result $p_H \approx p_L$,

$$v_H = \frac{\epsilon_L + p_H}{\epsilon_H + p_L} v_L < \frac{\epsilon_L + p_H}{\epsilon_H + p_L} \approx \frac{s_L}{s_H} \ll 1$$

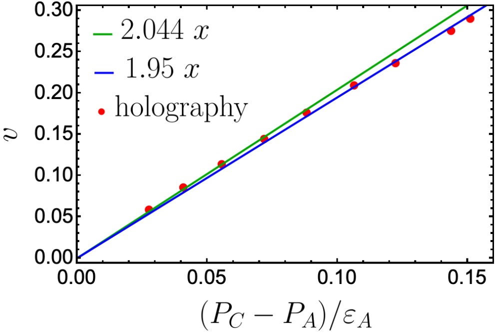
- After boosting back to "lab frame", $v_{\rm domain\ wall} \approx v_H^{\rm lab}$ as earlier

Linear formula of 2104.05708

Recently in similar simulations with a different nonconformal theory (4+1 d bulk), a linear fit was obtained for the velocity:

$$v \approx 1.95 \times \frac{p_c - p_A}{\epsilon_A}$$

[Bea, Casalderrey-Solana, Giannakopoulos, Mateos, Sanchez-Garitaonandia, Zilhão 2104.05708]

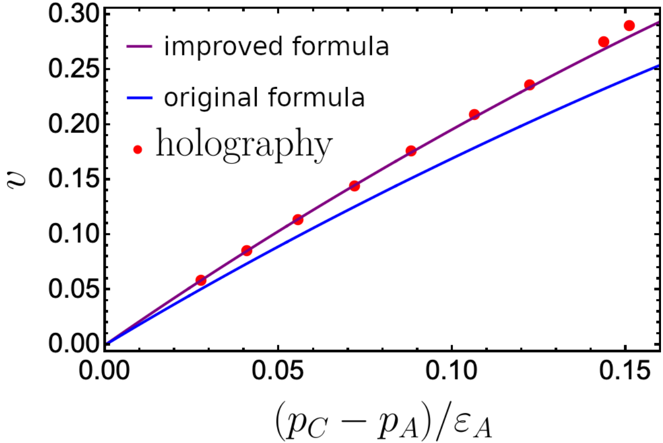


Linearizing our formula at small $p_c - p_A$, we obtain

$$v_{\text{domain wall}}^{\text{linearized}} \approx \frac{\epsilon_H}{\epsilon_H - \epsilon_L} \frac{1}{c_s}_{|T=T_c} \times \frac{p_c - p_A}{\epsilon_A} \approx 2.044 \times \frac{p_c - p_A}{\epsilon_A}$$

- 1.95 a better overall fit, 2.044 perhaps better at low $p_c - p_A$

Also compare the data of 2104.05708 to the full nonlinear formulas



- For the 4+1 d bulk theory, the correction term gives a drastic improvement