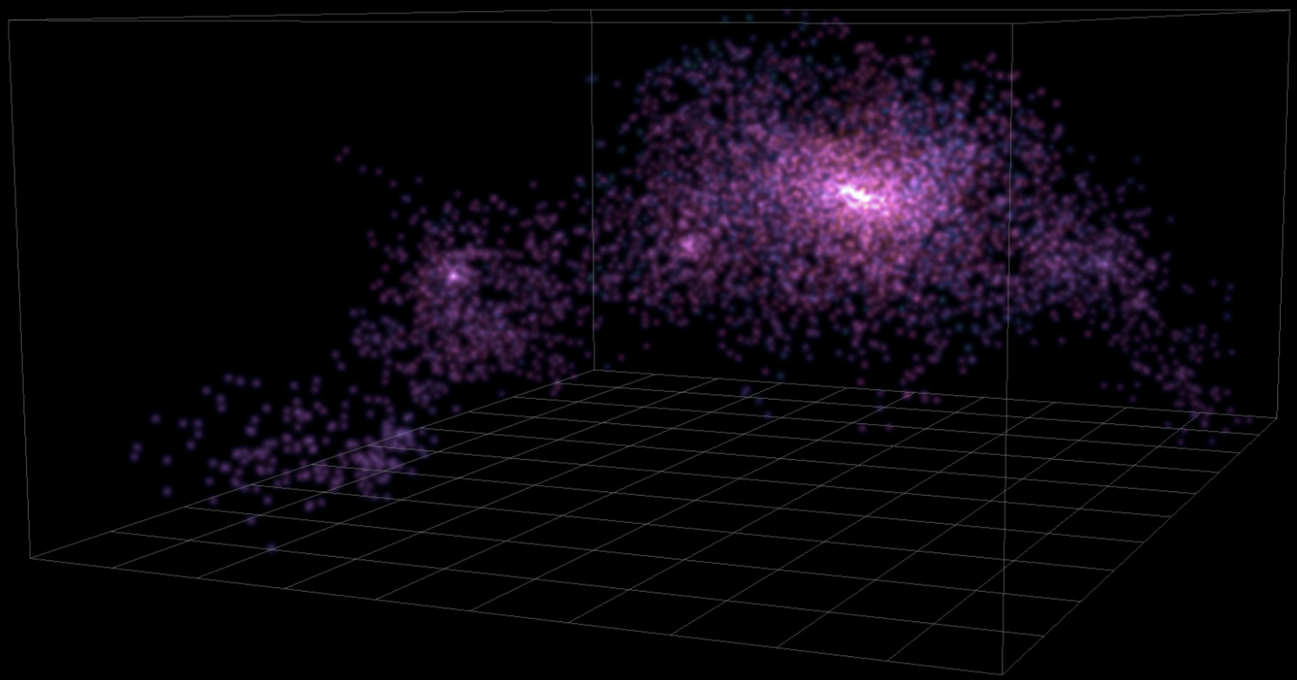
LUTH Meeting Recap: Foundations of the mass function



LSST Group Meeting – April 30th, 2024 – LAPP, Annecy

Vincent Reverdy & Narei Lorenzo Martinez

LUTH Meeting

Meeting with the theoretical/numerical cosmology team at LUTH on April 10th

- Laboratory Universe and Theories (LUTH) at the Paris-Meudon Observatory (OBSPM)
- Discussions during the entire day
- ⇒ Yann Rasera: expert in cosmological simulations
- ⇒ Inigo Saez Casares: PhD student on cosmological emulators
- + during lunch with Pier-Stefano Corasaniti: expert in cosmostatistics

2 main topics discussed

- History and foundations of the mass function
- Cosmological emulators to reproduce the mass function (+ demo)

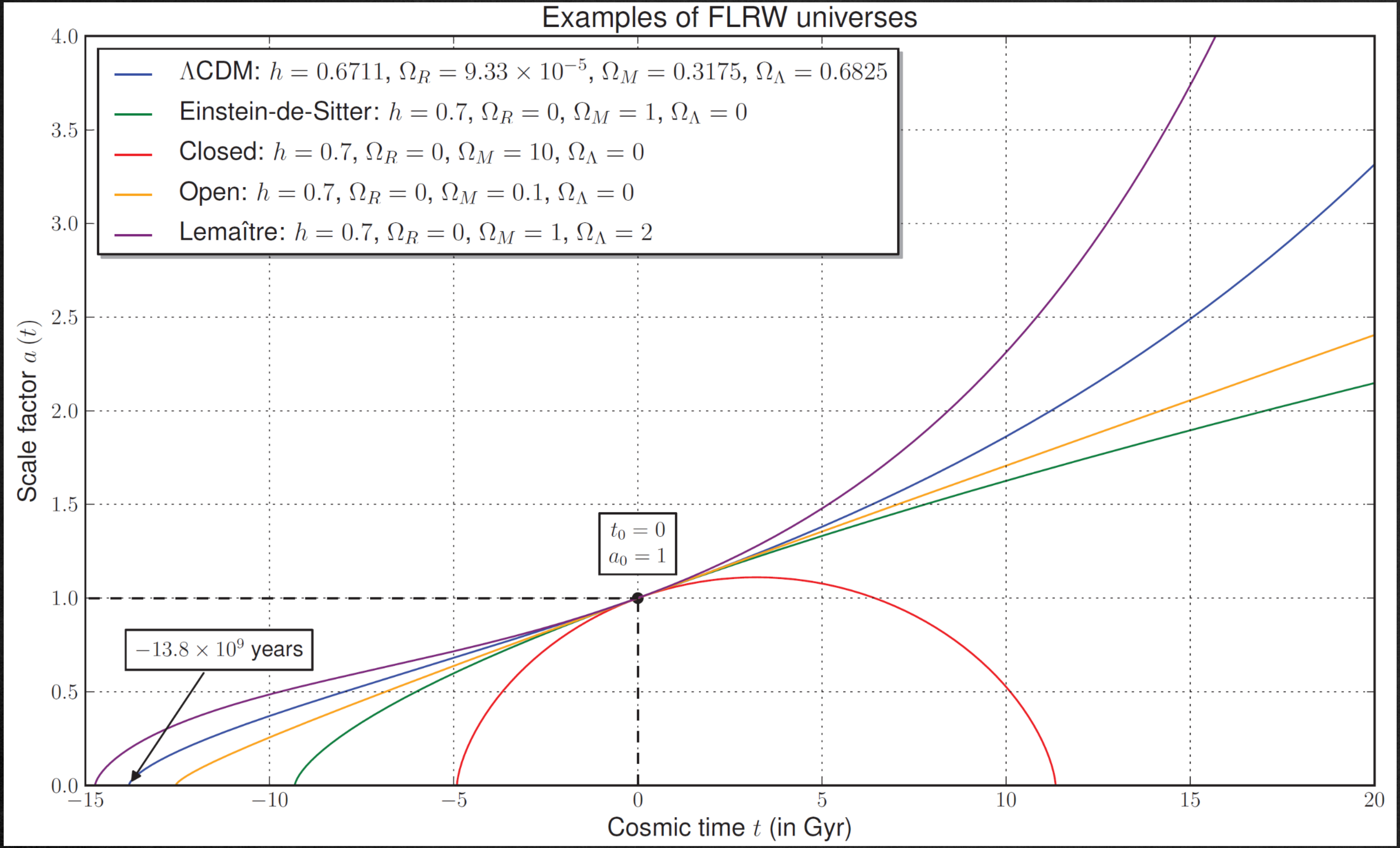
⇒ We consider a spherical perturbation of radius r

⇒ Birkhoff's theorem: it can be treated as a local Universe

⇒ Spherical FLRW metric dominated by matter

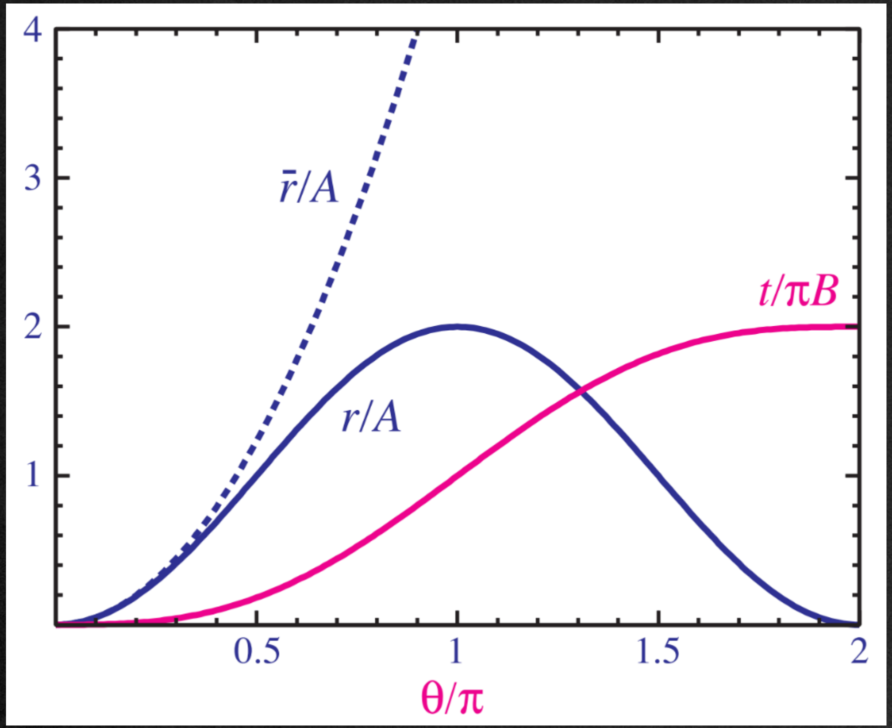
"Closed Einstein-de-Sitter" metric

$$\frac{1}{a}\frac{da}{dt} = H_0\sqrt{\Omega_M a^{-3} + (1-\Omega_M)a^{-2}}$$


Examples of FLRW universes
ΛCDM: $h = 0.6711$, $\Omega_R = 9.33 \times 10^{-5}$, $\Omega_M = 0.3175$, $\Omega_\Lambda = 0.6825$
Einstein-de-Sitter: $h = 0.7$, $\Omega_R = 0$, $\Omega_M = 1$, $\Omega_\Lambda = 0$
Closed: $h = 0.7$, $\Omega_R = 0$, $\Omega_M = 10$, $\Omega_\Lambda = 0$
Open: $h = 0.7$, $\Omega_R = 0$, $\Omega_M = 0.1$, $\Omega_\Lambda = 0$
Lemaître: $h = 0.7$, $\Omega_R = 0$, $\Omega_M = 1$, $\Omega_\Lambda = 2$
$t_0 = 0$
$a_0 = 1$
-13.8×10^9 years
Scale factor $a(t)$
Cosmic time t (in Gyr)
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
−15
−10
−5
0
5
10
15
20

⇒ Scenario: will expand first and then collapse

$$\rho = \frac{M}{\frac{4}{3}\pi r^3} \qquad M = \frac{4}{3}\pi r_0^3 \Omega_{M\,0}\rho_c = \frac{4}{3}\pi r_0^3 \Omega_{M\,0}\frac{3H_0^2}{8\pi G} = \frac{r_0^3 \Omega_{M\,0} H_0^2}{2G}$$



$$t_{turn-around} = \frac{\pi}{2H_0} \times \frac{\Omega_{M\,0}}{\left(\Omega_{M\,0} - 1\right)^{\frac{3}{2}}}$$

$$r_{turn-around} = r_0 \times \frac{\Omega_{M\,0}}{\Omega_{M\,0} - 1}$$

$$t_{collapse} = 2t_{turn-around}$$

⇒ Perturbative approach for the density

$$\rho = \bar{\rho}(1+\delta) \qquad \delta = \frac{\rho - \bar{\rho}}{\bar{\rho}} \qquad \Delta = \frac{\rho}{\bar{\rho}} = 1 + \delta$$

$$\delta_{turn-around} = \frac{3}{20}(6\pi)^{2/3} \approx 1.06$$

$$\delta_{collapse} = \frac{3}{20}(12\pi)^{2/3} \approx 1.686$$

⇒ In practice it does not collapse back to $r = 0$

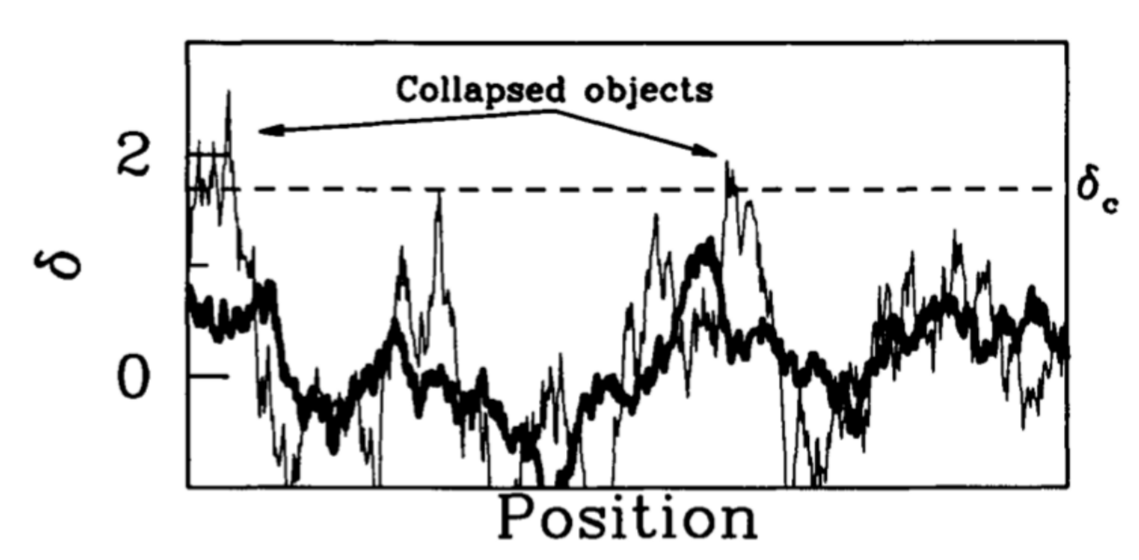
⇒ Virialization takes place (virial equilibrium)

$$E = U + K \qquad\qquad U = -2K$$

$$r_{vir} = \frac{1}{2} r_{turn-around}$$

$$\Delta_{vir} = 1 + \delta_{vir} = 18\pi^2 \approx 177.65 \approx 178$$

⇒ Regions where the local density contrast is such that $\delta > \delta_c \approx 1.686$ collapse and evolve in virialized halos



$$\sigma^2(M) = \frac{1}{(2\pi)^3} \int dk \left|\widetilde{W}(kR)\right|^2 P_m^L(k)$$

$$\frac{dn}{dM} = \sqrt{\frac{2}{\pi}} \times \frac{\bar{\rho}}{M} \frac{\delta_c}{\sigma^2} \frac{d\sigma}{dM} e^{-\frac{\delta_c^2}{2\sigma^2}}$$

⇒ Generalization of the mass function through a multiplicity function $f(\sigma)$

$$\frac{dn}{dM} = f(\sigma)\,\frac{\bar{\rho}}{M}\,\frac{d\ln\sigma^{-1}}{dM}$$

ID	mdefs	z-dep.	Reference
press74	fof	delta_c	Press& Schechter 1974
sheth99	fof	delta_c	Sheth & Tormen 1999
jenkins01	fof	No	Jenkins et al. 2001
reed03	fof	delta_c	Reed et al. 2003
warren06	fof	No	Warren et al. 2006
reed07	fof	delta_c	Reed et al. 2007
tinker08	Any SO	Yes	Tinker et al. 2008
crocce10	fof	No	Crocce et al. 2010
bhattacharya11	fof	Yes	Bhattacharya et al. 2011
courtin11	fof	No	Courtin et al. 2011
angulo12	fof	No	Angulo et al. 2012
watson13	fof, any SO	Yes (SO)	Watson et al. 2013
bocquet16	200m,200c,500c	Yes	Bocquet et al. 2016
despali16	Any SO	Yes	Despali et al. 2016
comparat17	vir	No	Comparat et al. 2017
diemer20	sp-apr-*	No	Diemer 2020b
seppi20	vir	Yes	Seppi et al. 2020

⇒ NFW (Navarro–Frenk–White) profile: fitted on simulations!

$$\rho(r) = \frac{\rho_S}{\frac{r}{r_S}\left(1+\frac{r}{r_S}\right)^2}$$

$$M = \int_0^{R_{max}} 4\pi r^2 \rho(r) dr = 4\pi \rho_S r_S^3 \left[\ln\left(\frac{r_S+R_{max}}{r_s}\right) - \frac{R_{max}}{r_S+R_{max}}\right]$$

$\lim_{R_{max}\to+\infty} M = +\infty$: Divergence ⇒ Where to stop?

$R_{vir} = c r_S$: with c the concentration parameter

$R_\Delta \to M_\Delta$: Radius such that $\bar{\rho} = \Delta \times \rho_C$

$R_{200} \to M_{200}$: Radius such that $\bar{\rho} = 200\rho_C$

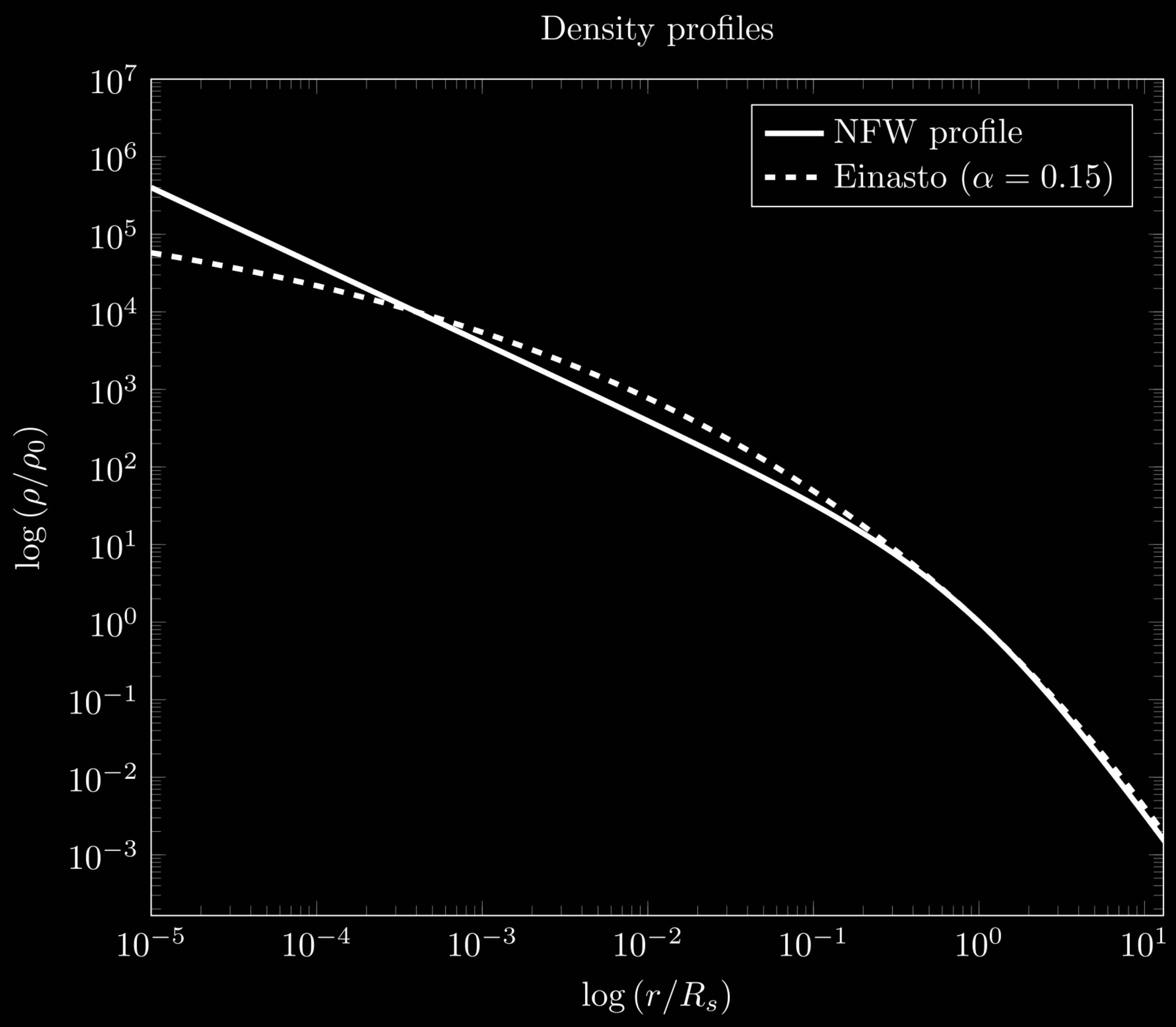
$R_{500} \to M_{500}$: Radius such that $\bar{\rho} = 500\rho_C$

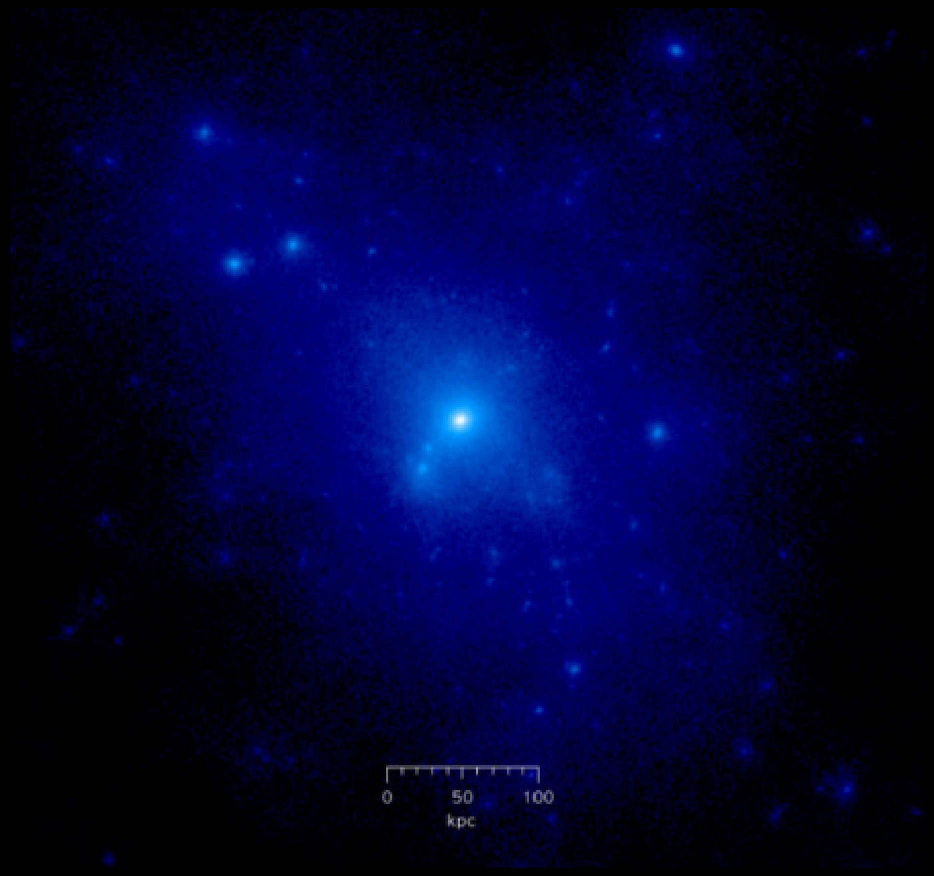
$R_{1000} \to M_{1000}$: Radius such that $\bar{\rho} = 1000\rho_C$

$\Delta = \frac{\rho}{\rho_c}$: Overdensity

$\rho_c = \frac{3H^2}{8\pi G}$: Critical density

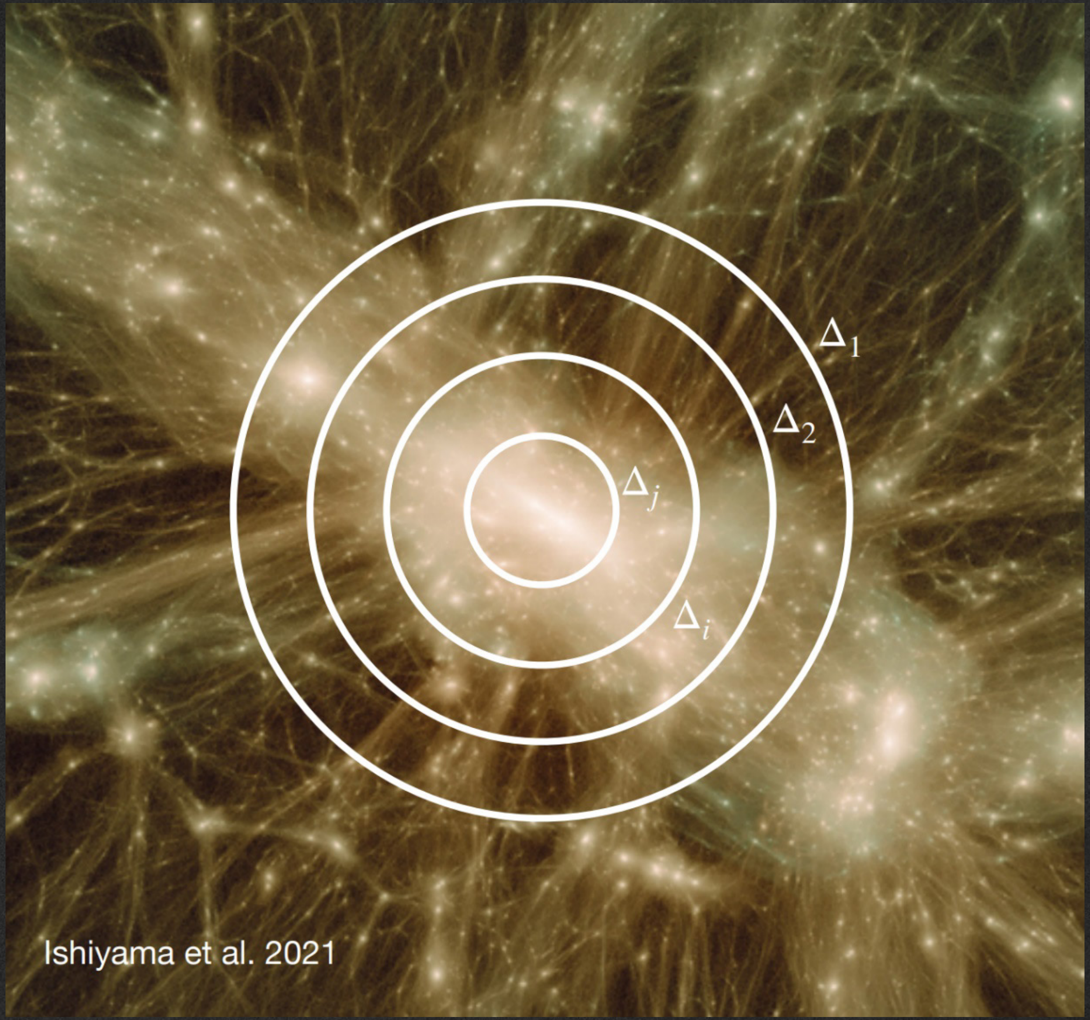
Halo model: visualization



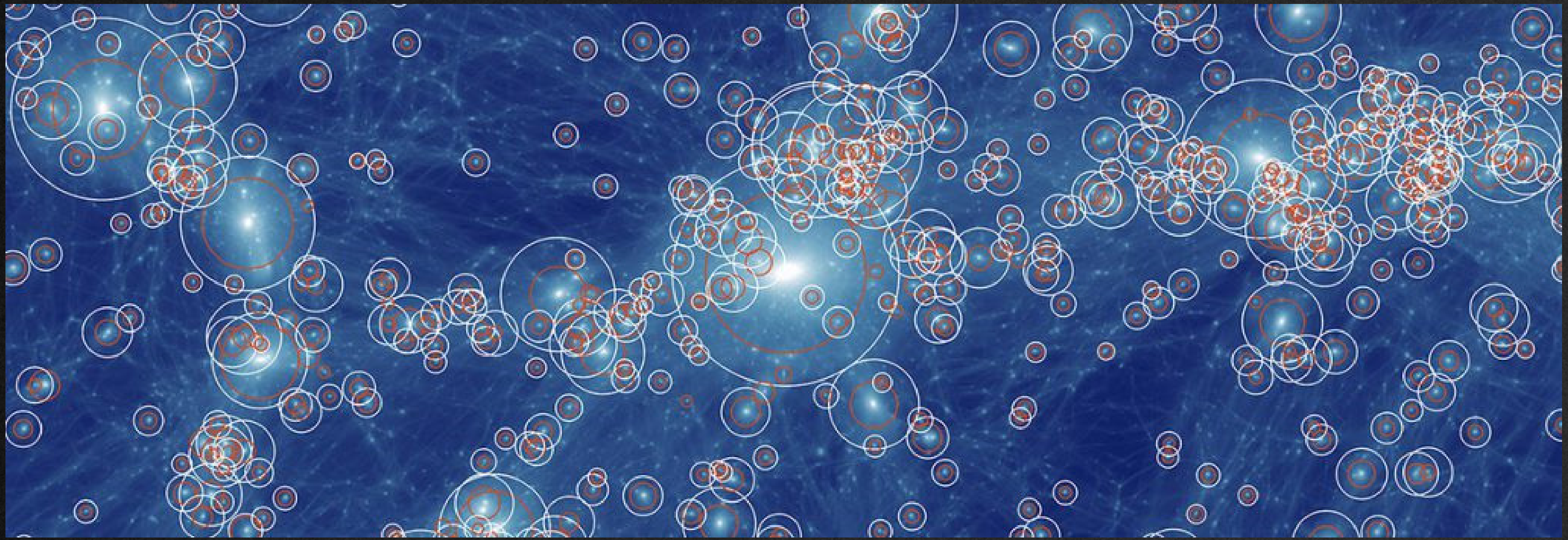


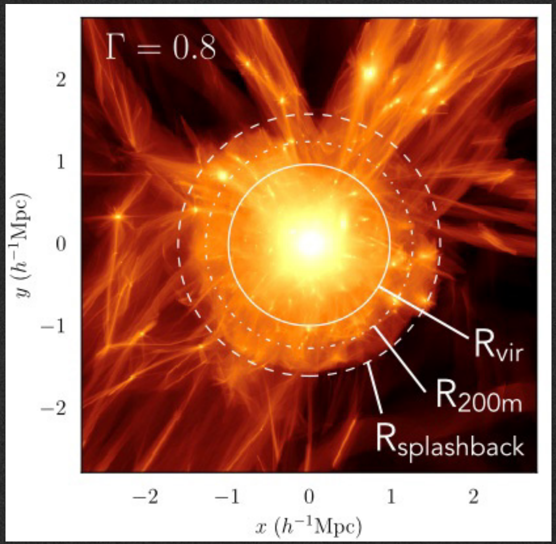
Halo model: sparsity

$$s_{\Delta_i,\Delta_j} = \frac{M_{\Delta_i}}{M_{\Delta_j}}$$

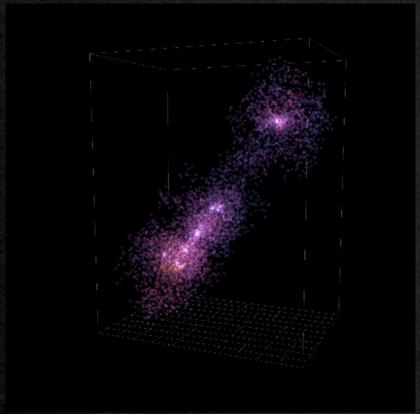


⇒ Spherical Overdensity (SOD) [parameter: Δ]





⇒ Friend-of-Friend (FoF) [linking length: b]



$$[b = 0.2] \sim [\Delta = 200] \sim [\Delta_{vir} = 178]$$

Key questions

$\Rightarrow$ Mass functions obtained through simulation fit depends on mass definition (choice of algorithm)

$\Rightarrow$ Mass function universality?