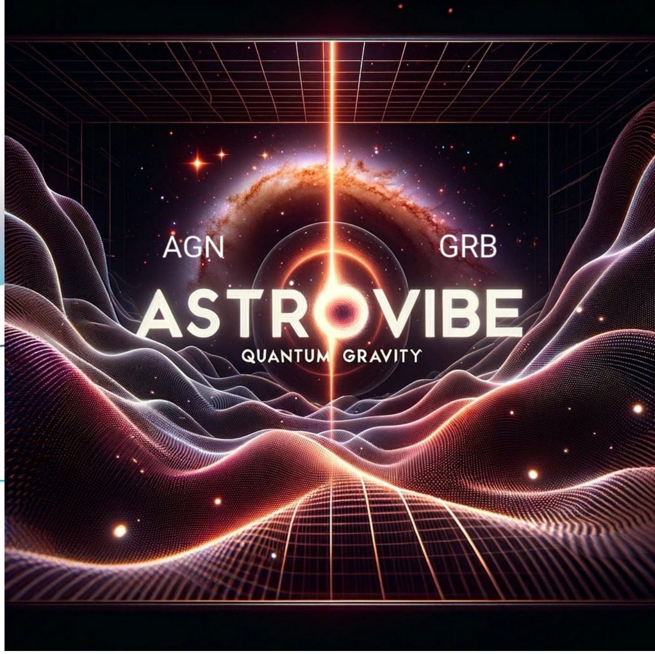


LIVelihood: A code for Lorentz Invariance Violation searches with time lag, and beyond ?

Sami Caroff (LAPP)
16 July 2024

Based on Bolmont et al 2022 ApJ 930 75

What are we looking for ?

Time lag method

- Lorentz Invariance Violation (or Deformation) is expected by some candidates theory for Quantum Gravity
- Concept of a minimal length is a potential manifestation of quantum characteristics of space-time, anticipated to be of the order of Planck length ($1.6\ 10^{-35}$ m)
- Integrated successfully in special relativistic framework (Doubly Special Relativity http://arXiv.org/abs/hep-th/0112090v2), but as well in classical quantum mechanics (Generalized Uncertainty Principle https://arxiv.org/abs/2308.13788)
- In both cases, leads to a possibly modified dispersion relation, leading to detectable effects
- Variability of speed of light versus energy is one of them (with an energy scale of the order of Planck Energy $\sim 10^{19}$ GeV)

Modified dispersion relation

QG pertubation term

$$E^2 = \frac{p^2 c^2}{a^2(t)} (1 + h_{QG}(E, E_{QG}))$$

Cosmological scale factor

Taylor series expansion

$$E^2 = \frac{p^2 c^2}{a^2(t)} \left(1 + \sum_1^n \left(\frac{E}{E_{QG,n}}\right)^n\right)$$

Speed for a given order

E(t) because of scale factor (redshift)

$$v(E,t) = \frac{\partial E}{\partial p} \simeq \frac{c}{a(t)} \left(1 + \frac{n+1}{2} \left(\frac{E(t)}{E_{QG,n}}\right)^n\right) \qquad \left(\frac{E(t)}{E_{QG,n}}\right)^n \simeq \left(\frac{pc}{a(t) E_{QG,n}}\right)^n$$

Time lag

$$\left(\frac{E(t)}{E_{QG,n}}\right)^n \simeq \left(\frac{pc}{a(t)E_{QG,n}}\right)^n$$

Time of flight

$$t(E,z) = \frac{1}{c}\int v(E,t')dt' = \int_0^z \frac{1}{H(z')}dz' + \frac{n+1}{2}\left(\frac{E_0}{E_{QG}}\right)^n \int_0^z \frac{(1+z')^n}{H(z')}dz'$$

Energy independent term

Energy dependent term

Lag between E_h and E_l events

$$\Delta t(E_h, E_l, z) = \frac{n+1}{2}\frac{E_h^n - E_l^n}{E_{QG}^n}\int_0^z \frac{(1+z')^n}{H(z')}dz' = \frac{n+1}{2}\frac{\Delta E_n}{H_0 E_{QG}^n}\kappa(z)$$

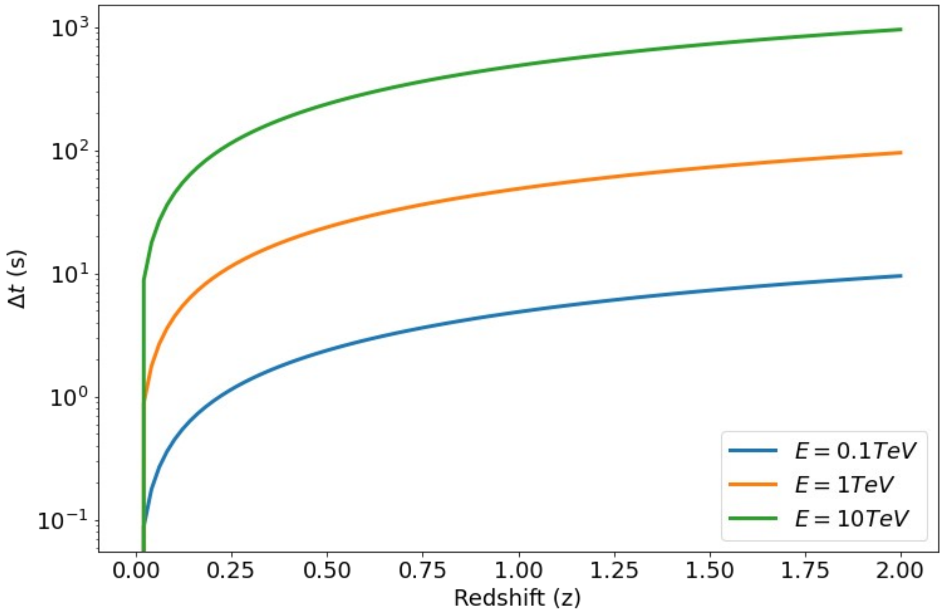
Lag between E_h and E_l events

Lambda independent of both redshift and energy (only n dependent)

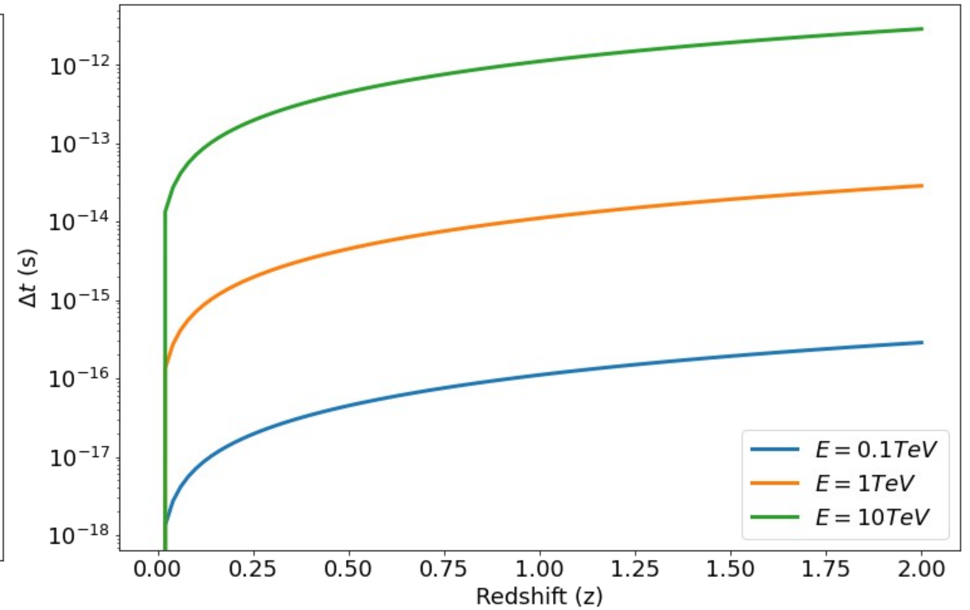
$$\lambda_n \equiv \frac{\Delta t_n}{\Delta E_n\ \kappa_n(z)} = \pm\frac{n+1}{2\mathrm{H}_0\ E_{QG}^n},$$

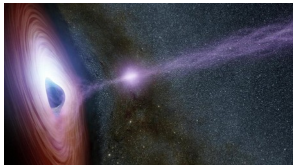
Time lag

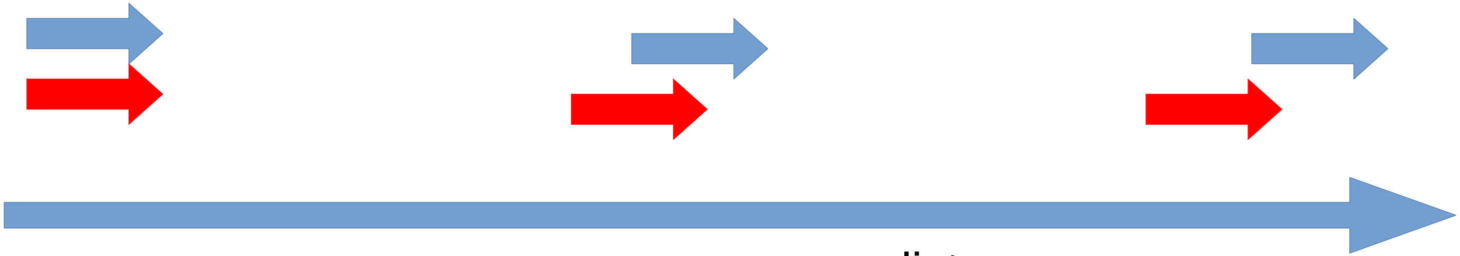
n=1



n=2





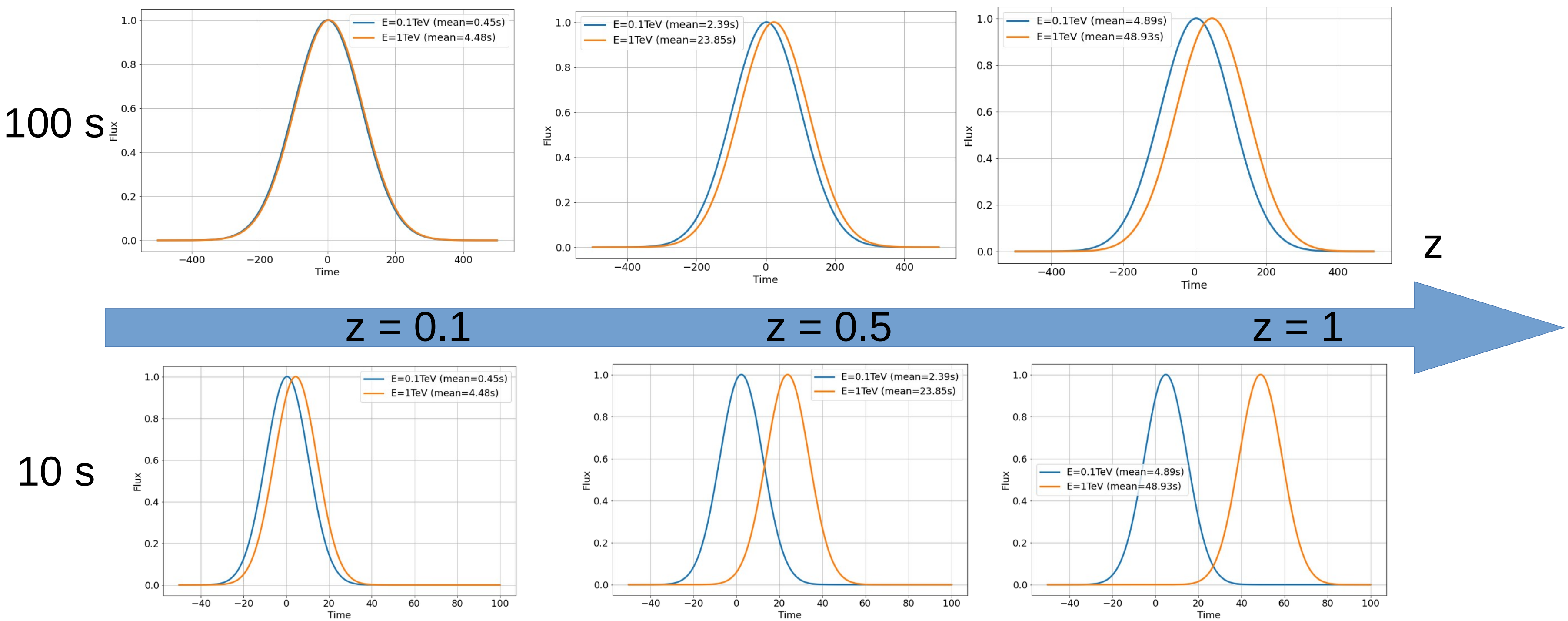


distance

Impact on flaring sources

$$\frac{dP}{dtdE}(t,E) = \phi_E(E)\phi_t(t)$$

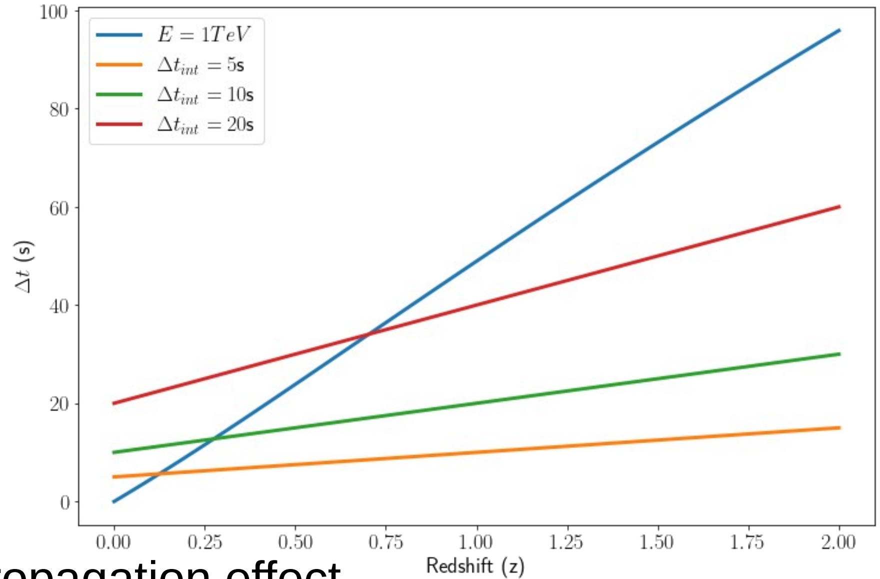
$$\frac{dP}{dtdE}(t+\boxed{\lambda_n \kappa_n(z) E^n}, E) = \phi_E(E)\phi_t(t+\boxed{\lambda_n \kappa_n(z) E^n})$$



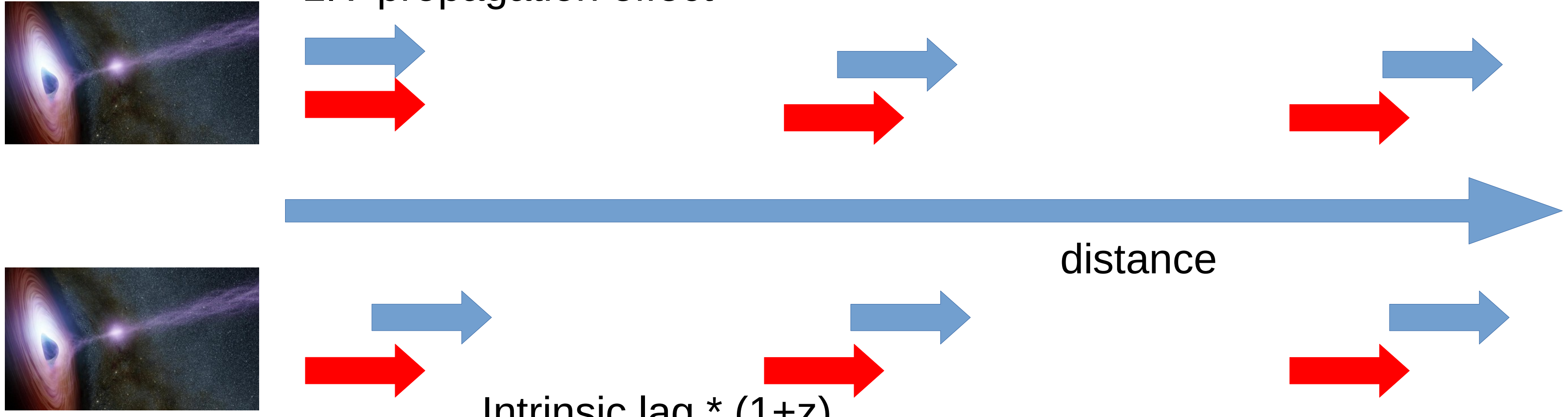
Intrinsic time delays ?

Intrinsic delay

$$\Delta t_{total}(z) = \Delta t_{LIV}(z) + \Delta t_{int}(1+z) \qquad \Delta\tau_{total}(z) = \Delta\tau_{LIV}\kappa(z) + \Delta\tau_{int}(1+z)^2$$



LIV propagation effect

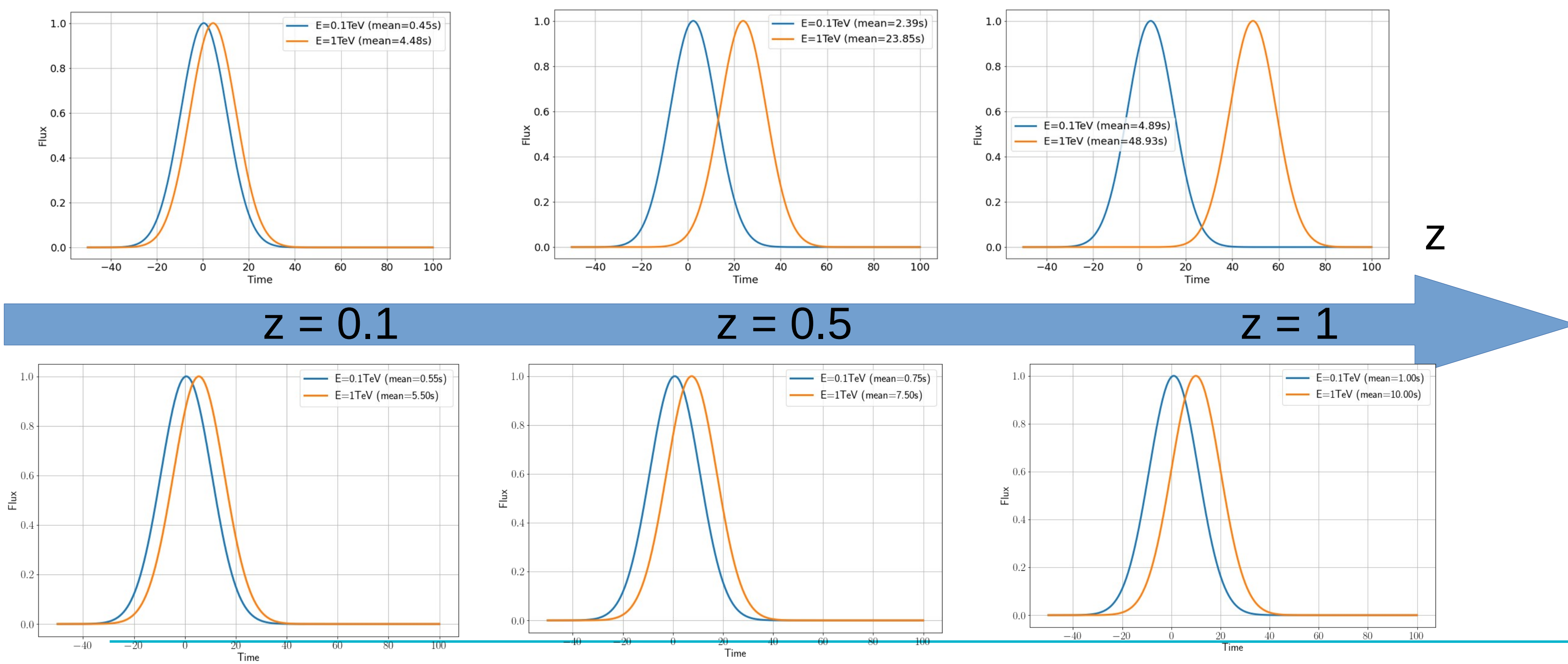


Intrinsic lag * (1+z)

Impact on light curve

LIV $$\frac{dP}{dtdE}(t + \boxed{\lambda_n \kappa_n(z) E^n}, E) = \phi_E(E)\phi_t(t + \boxed{\lambda_n \kappa_n(z) E^n})$$

Intrinsic (linear) $$\frac{dP}{dtdE}(t + \boxed{\lambda(1+z)^2 E}, E) = \phi_E(E)\phi_t(t + \boxed{\lambda(1+z)^2 E})$$



Some more thoughts on intrinsic time delay

- More detailed presentation will be given this afternoon, but here some arguments :
  - Mostly two approaches tested so far :
    - Source population studies
    - Intrinsic lag simulation (AGN – cf Hélène presentation this afternoon)

  - Current status in analysis is that intrinsic lag are **under control** :
    - We do not see any lag in most of the sources used (lag_int << experimental precision)
    - Combination of many sources type (AGN, GRB, Pulsar) at many redshift, permits to avoid the possible case of lag_int = -lag_LIV

  - But, when experimental precision will became bigger with lag_int, we will have to deal with it…
  - … and we need to be ready for that

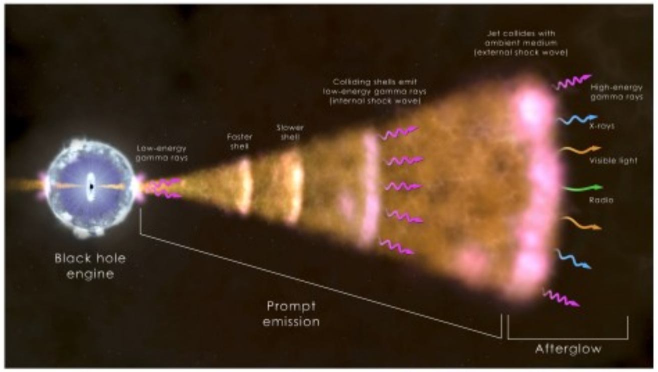
Astrophysical sources combination

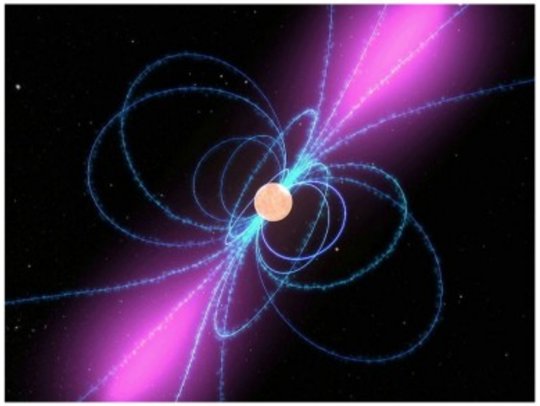
- Need distant, variable, energetic and important statistics

- Flaring AGNs → distant sources, short variability (~10 mins)
- Pulsar → local sources but high variability, can accumulate data to improve measurement
- GRBs → Fast variability (prompt emission ~mins) and distance

- LIV implies same effect to all these sources independently of the acceleration processes

- Why combination of different sources :
  - Source intrinsic effects
  - different redshifts (redshift dependence of LIV time lag)
  - Increase the statistics





- Combination of all available data from the three leading Imaging Atmospheric Cherenkov Array
- Preparing the CTAO era (combination of sources and observatories North and South)
- Joined analysis of different type of sources (AGNs, pulsars, GRB)
- Signed MoU between experiments to share data for LIV study
- First step done, production of a common software for LIV analysis : LIVelihood

LIVelihood code general description

- Code made for time-lag study and combination of different data of experiments

- Code based on ROOT C++

- Several functionnalities
  - Unbinned likelihood taking into account IRFs
  - Simulations of lightcurve with or without injected lag
  - Systematics uncertainties treatment with profiled likelihood
  - Different model of redshift distance term implemented (J&P and DSR)
  - Able to run on both lightcurves (GRBs and AGNs) and phasograms (pulsars)

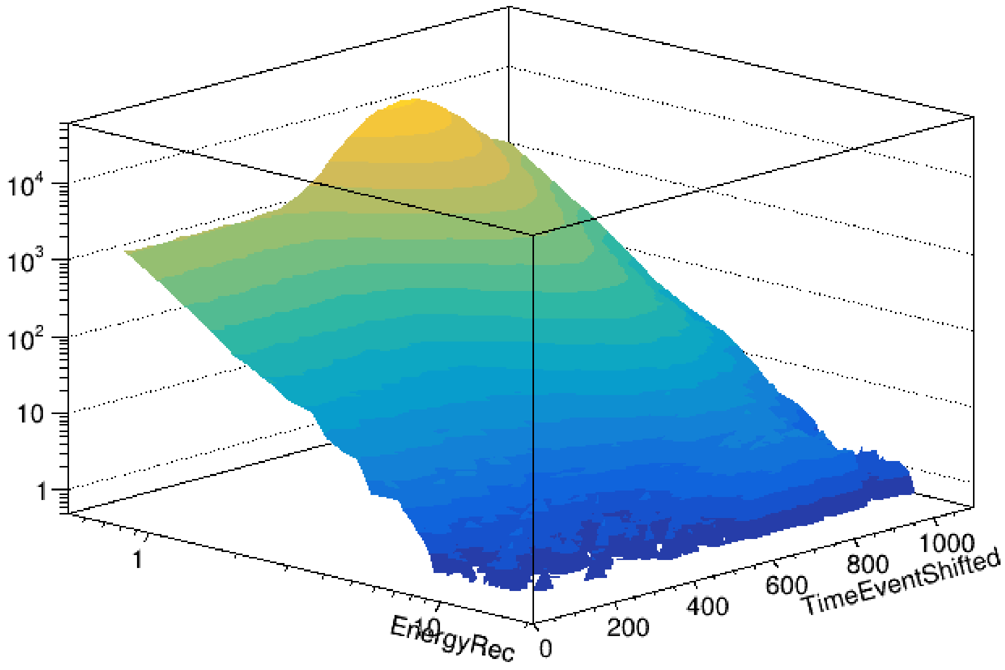
- Made by design to be easily used for diverse experiments, designed for HESS+MAGIC+VERITAS+LST but easily tunable for other IACTs or other experiments (Fermi)

- Support the DL3 CTAO data format (~without event type)

- Definition of a new experiment needs some ingredients :
  - Systematics (Energy scale, Signal/Background uncertainty, Spectral Index)
  - IRFs (Migration Matrix, Effective Area)

Simulations

- Light curve simulation code included in the LIVelihood code
- Able to run simulations for light curve and phasograms (pulsar)
- Parameters needed :
  - Light curve shape (or phasogram shape)
  - PL index of signal and bakground
  - IRFs (Acceptance, Energy Migration) can handle time dependent IRFs
  - Redshift
  - Injected Time lag per energy
  - N_signal and N_background



Likelihood

$$F_s(E_t, t; \lambda_n) = \frac{\underbrace{\Gamma_s(E_t)}_{\text{Spectra}} \underbrace{C_s\left(t - D(E_t, \lambda_n, z)\right)}_{\text{Light curve}}}{N_s}, \qquad (7)$$

Lag corrected by redshift

$$D(E_t, \lambda_n, z) = \boxed{\lambda_n} \times \kappa_n(z) \times E_t^n, \qquad (8)$$

- Simple likelihood with a « perfect » instrument (no Emig and flat acceptance)
- Lag corrected by redshift used as a free parameter
- Can be shared as free parameter between sources at different redshift

Likelihood

Background/Signal proportion

IRFs

Light curve * spectra

$$\frac{dP}{dE_m dt} = \boxed{w_s} \frac{\int \boxed{A(E_t,\vec{\varepsilon})M(E_t,E_m)} \times \boxed{F_s(E_t,t;\lambda_n)} dE_t}{\boxed{N'_s}} + \sum_k \boxed{w_{b,k}} \frac{\int \boxed{A(E_t,\vec{\varepsilon})M(E_t,E_m)} \times \boxed{F_{b,k}(E_t,t)} dE_t}{\boxed{N'_{b,k}}}, \tag{12}$$

Normalisation

$$L_S(\lambda_n) = -\sum_{\mathrm{i}} \log\left(\frac{dP}{dE_m dt}(E_{m,i},t_i);\lambda_n\right). \tag{13}$$

- Likelihood takes into account IRFs and background
- Several types of background can be added (so far, quiescent state + gamma-like hadrons)
- Should be generic enough for every type of gamma-ray observatory
- Very time consuming (1D + 3D integral for each event !), using precomputation and tabulation of likelihood, and interpolation for the minimization
- Can handle time dependent IRFs

Combination and systematics

$$L_{comb}(\lambda_n) = \sum_{\text{all sources}} L_S(\lambda_n).$$

- Combination done by summing log-likelihood over sources and/or observation nights
- Systematic error terms added to each source log-likelihood

$$L(\lambda_n, \vec{\theta}) = L_{\mathrm{S}}(\lambda_n, \vec{\theta}) + L_{\mathrm{template}}(\vec{\theta}_{\mathrm{C}}) + L_\gamma(\theta_\gamma) + L_{\mathrm{B}}(\vec{\theta}_{\mathrm{B}}) + L_{\mathrm{ES}}(\theta_{\mathrm{ES}}) + L_{\mathrm{z}}(\theta_{\mathrm{z}}), \quad (15)$$

a. $\vec{\theta}_{\mathrm{C}}$, the parameters of the light curve analytic parameterization,

b. θ_γ, the power law index of signal events spectrum,

c. $\vec{\theta}_{\mathrm{B}}$, the ratio of signal and of background event numbers to the total number of events,

d. θ_{ES}, the energy scale,

e. θ_{z}, the distance or redshift.

Some parameters depend of the instrument used for the observation

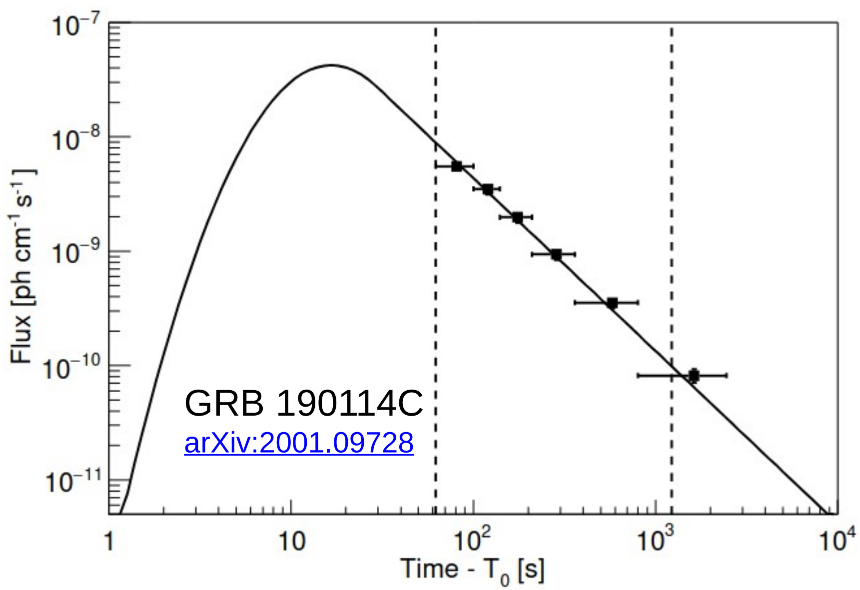
Simulated sources

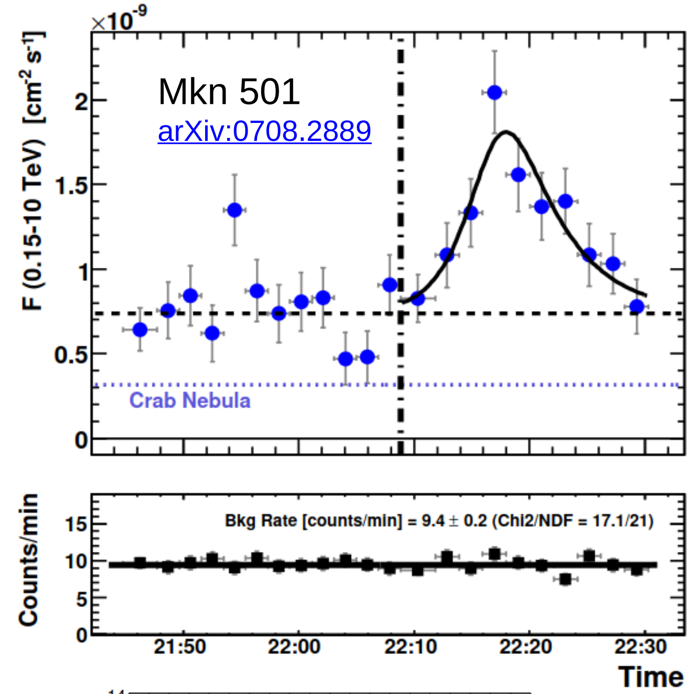
- Tested on 6 different sources already published by consortium collaborations :
  - Different types AGNs, GRB and pulsar phasograms
  - Some of them observed by two different observatories and combined (Crab pulsar)
- Simulation done using the instruments IRFs for these particular observations
- Light curve simulation code included in the LIVelihood code
  - Again modular code, IRFs can be tuned to study other observatories and sources

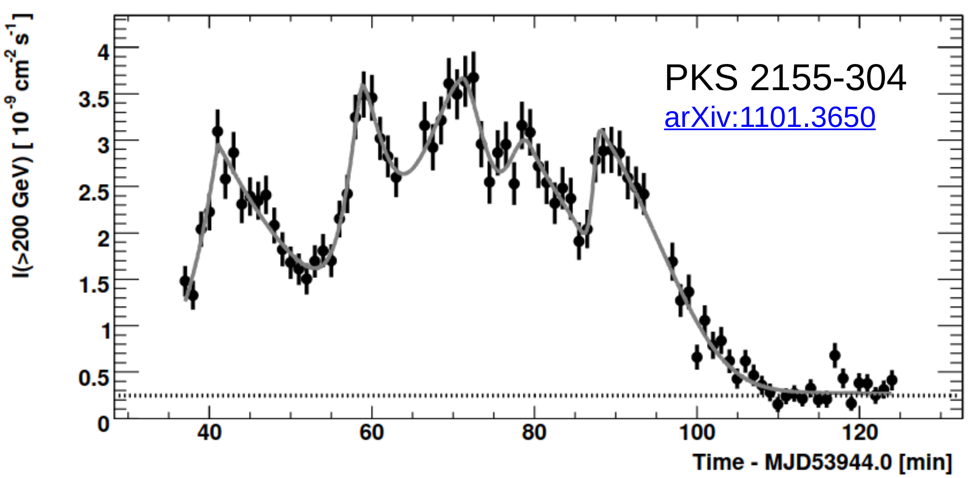
Table 3. Simulation settings for the individual sources.

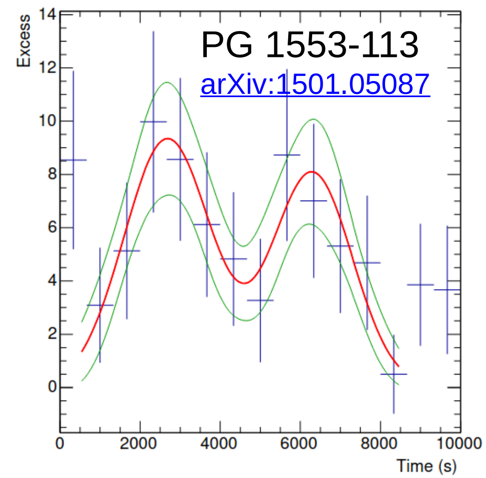
Source	Energy Range (TeV)	Time Range[a]	Spectral index Γ_s, Γ_b	Lightcurve shape	Number of events likelihood[b], template[c]	Background proportion hadronic, baseline
GRB 190114C	0.3 - 2	60 - 1200 s	5.43, -	curved power law	726, -	0.055, 0.
PG 1553+113	0.4 - 0.8	0 - 8000 s	4.8, 4.8	double Gauss	72, 82	0.29, 0.15
Mrk 501	0.25 - 11	0 - 1531 s	2.2, 2.2	single Gauss	1800, -	0.39, 0.
PKS 2155-304	0.28 - 4	0 - 4000 s	3.46, 3.32	5 asymmetric Gauss	2965, 561	0., 0.02
Crab (M)	0.4 - 7	0.36 - 0.45	2.81, 2.47	single Gauss + Baseline	14869, -	0., 0.961
Crab (V)	0.2 - 10	0.37 - 0.43	3.25, 2.47	single Gauss + Baseline	22764, -	0., 0.964
Vela	0.06 - 0.15	0.50 - 0.60	3.9, 1.75	asymmetric Lorentzian	330820, -	0., 0.998

Simulated sources

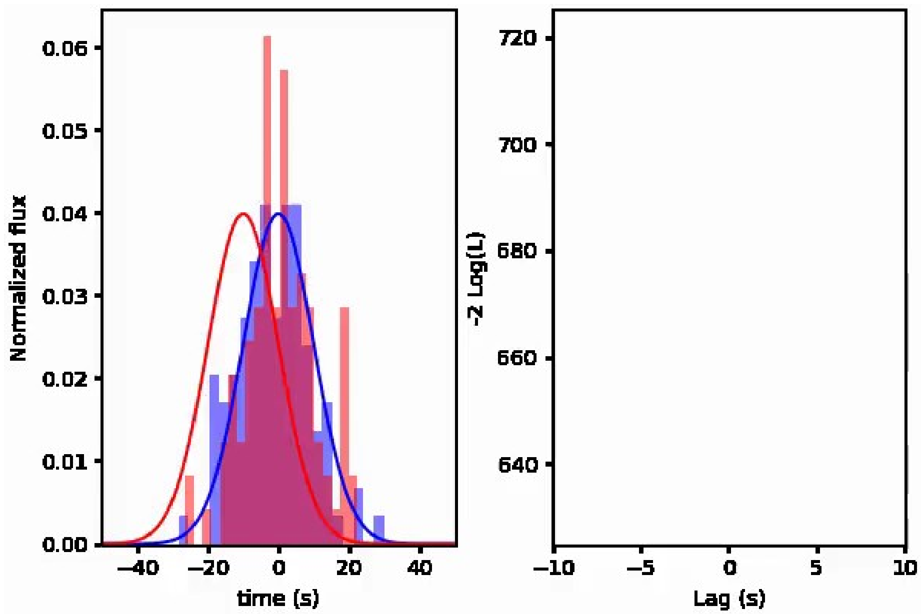


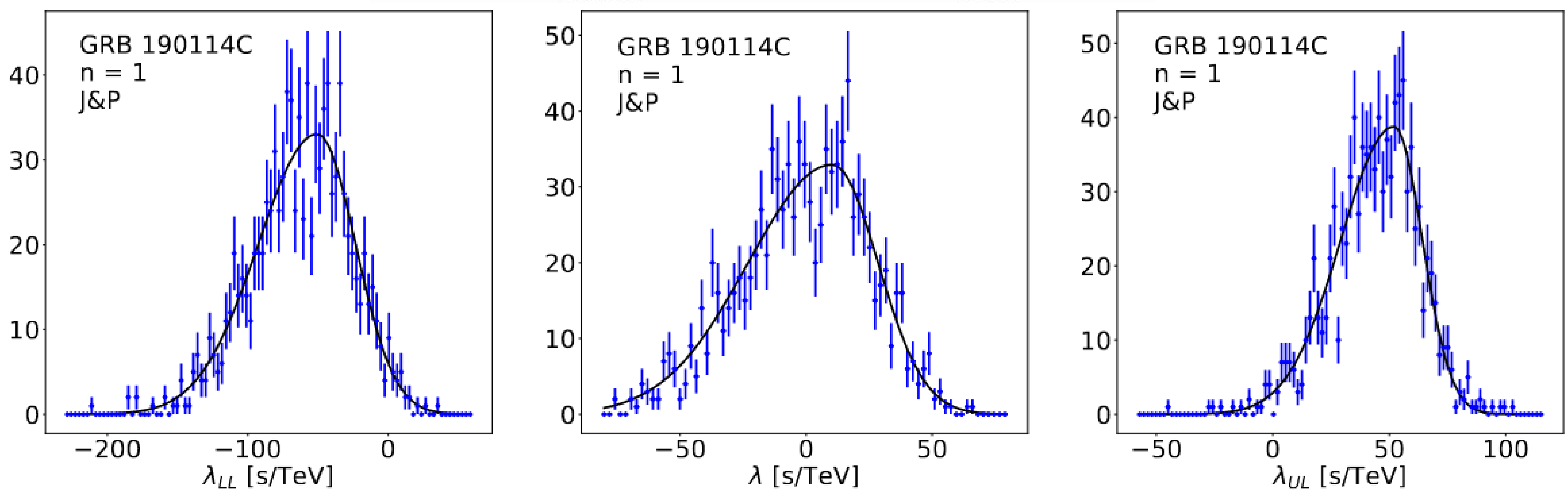






Code validation





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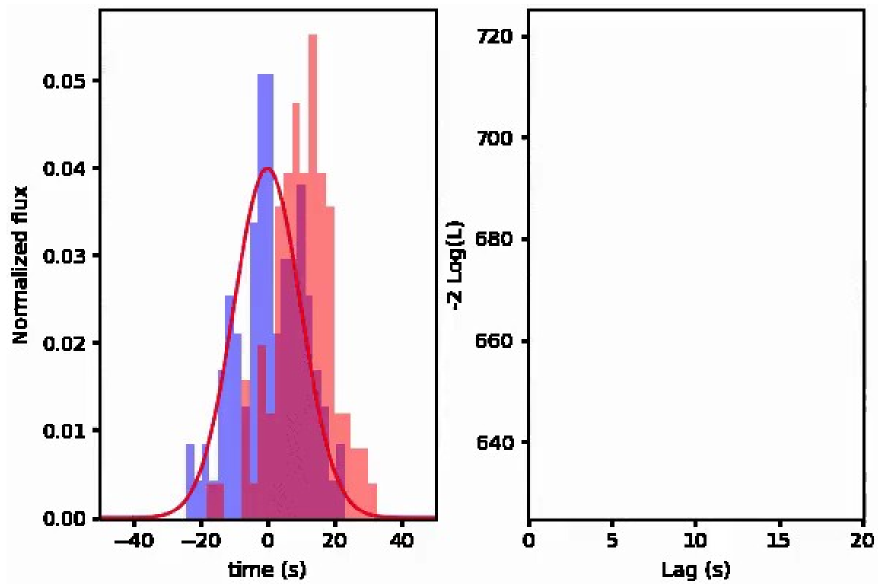
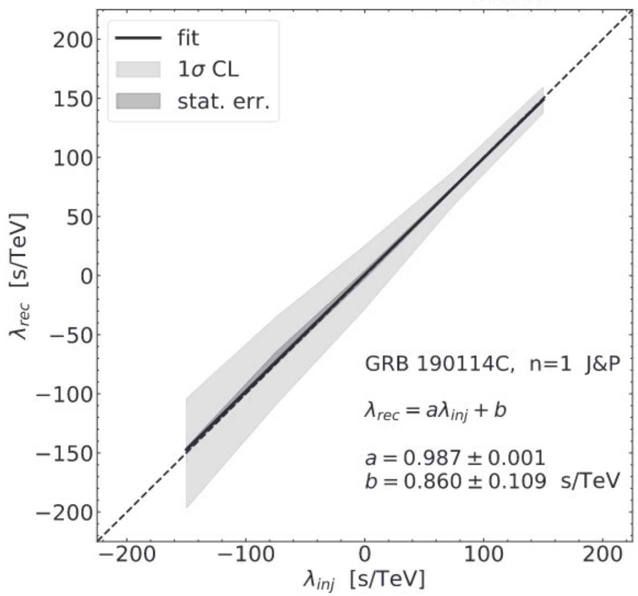
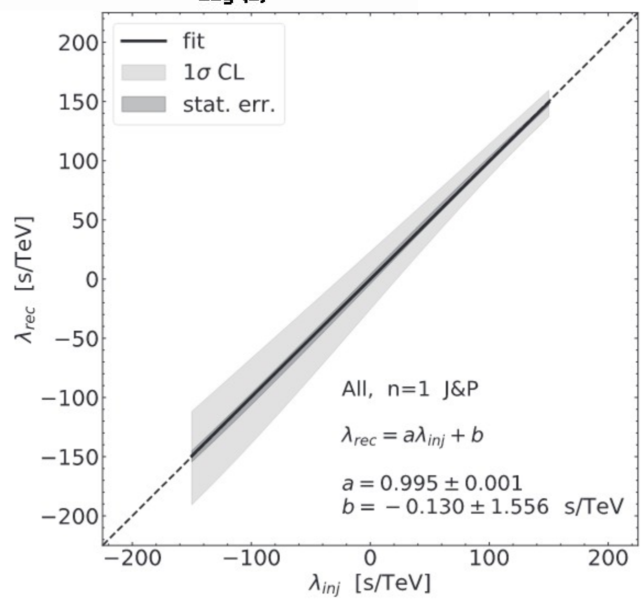
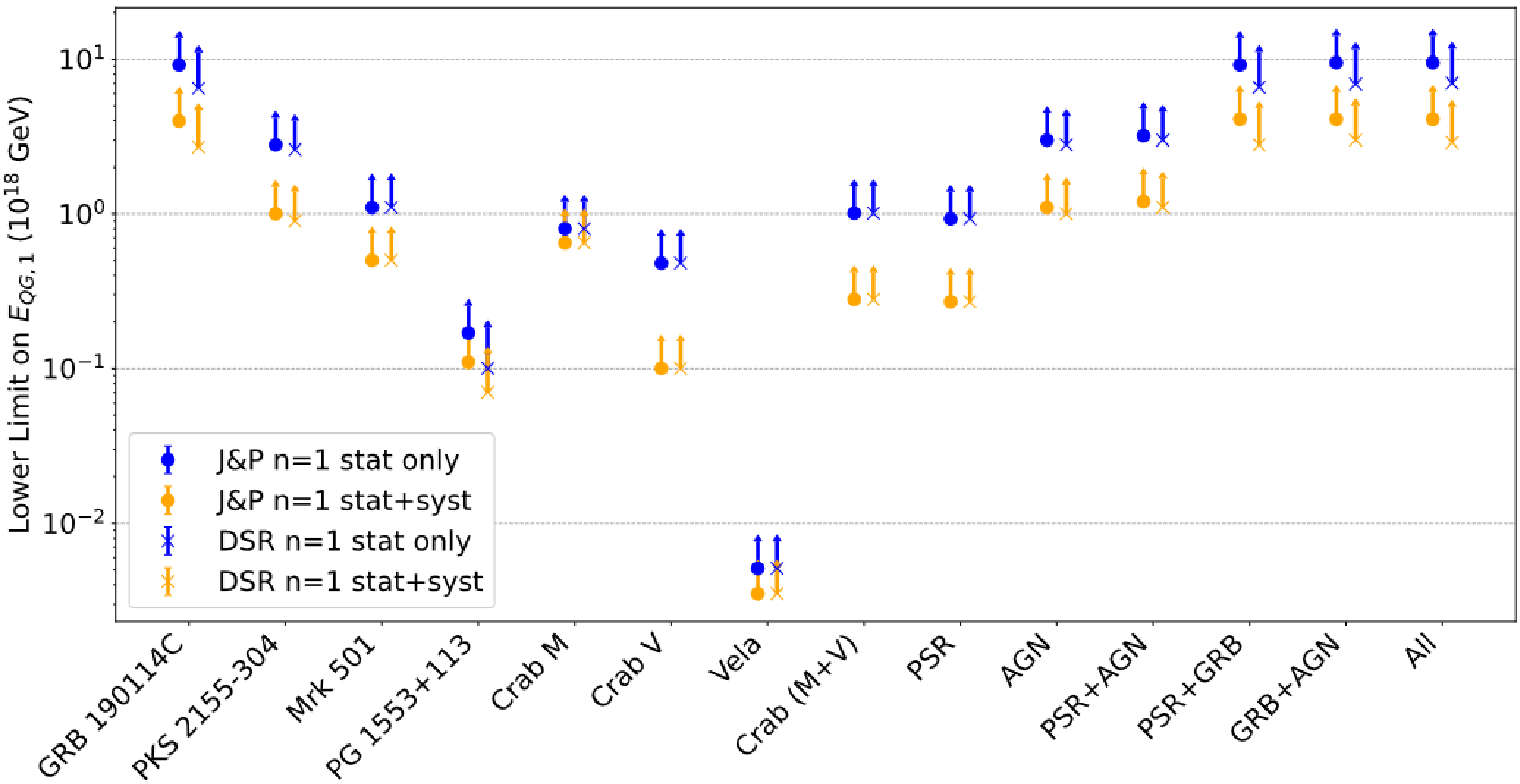






Limits



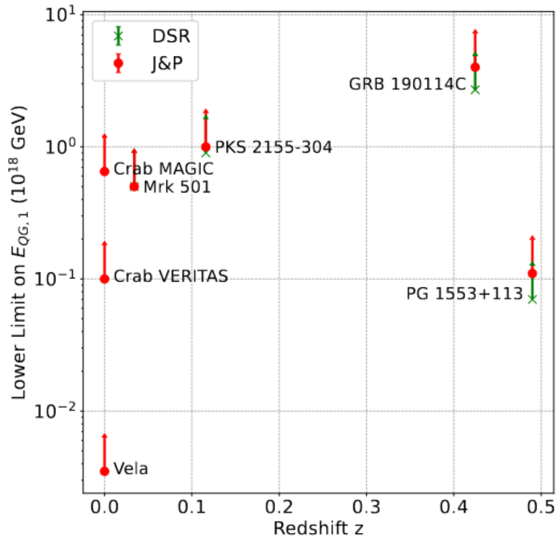
Lower Limit on $E_{QG,1}$ (10^{18} GeV)
10^{1}
10^{0}
10^{-1}
10^{-2}
J&P n=1 stat only
J&P n=1 stat+syst
DSR n=1 stat only
DSR n=1 stat+syst
GRB 190114C
PKS 2155-304
Mrk 501
PG 1553+113
Crab M
Crab V
Vela
Crab (M+V)
PSR
AGN
PSR+AGN
PSR+GRB
GRB+AGN
All

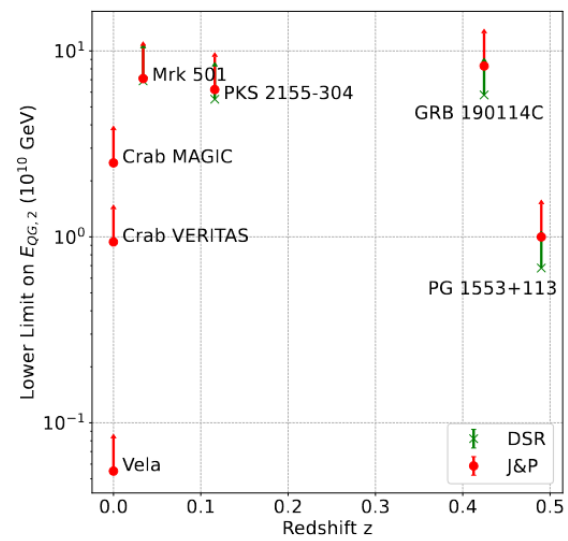
Why LIVelihood is interesting for AGN/GRB physics?

- More detail this afternoon but
- Made by design to track delay versus energy, whatever the reason of those delays
- In principle, any varying event distribution versus energy can be compared to data with this code
- So far mostly made for LIV studies, but :
  - DL3 format implemented (so we can work with CTAO or all IACTs data)
  - Simulation can handle simulation of time delays due to intrinsic effects
  - Model can be changed to some other scenarios than LIV
  - Started to explore the possibility to constrain (or even measure) the redshift distance function → can be done as well in a context of intrinsic lag
- More this afternoon…

Redshift variability of the limit

- Redshift variability of the limit is ruled by three processes :
  - **Distance** (reduce events by D_L^2)
  - **EBL absorption** (high energy events absorbed)
  - **Delay increase with redshift**

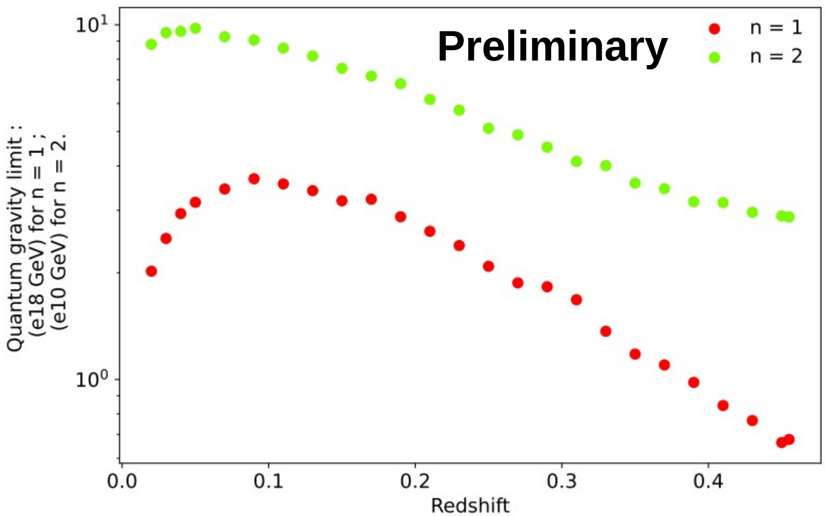


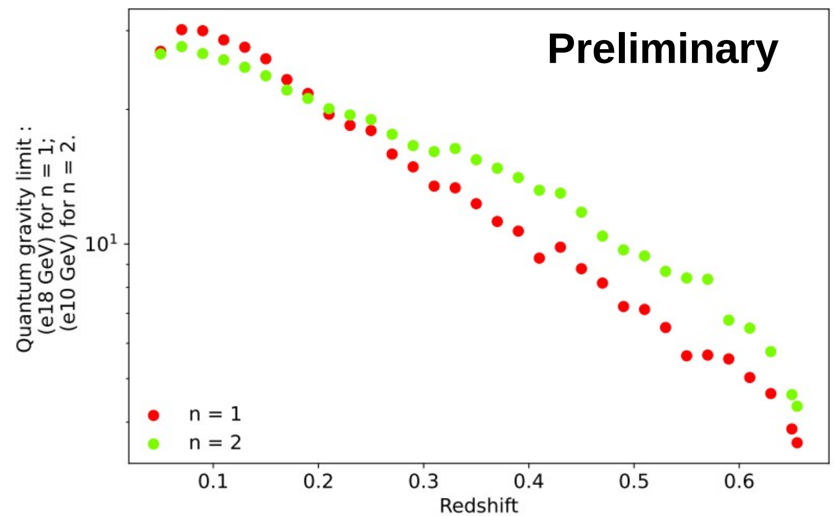


PKS 2155-137

M1 thesis : Amélie Nigou

GRB 190114C





Backup