Fermi gases and dilute neutron matter with low-momentum interactions

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Outline

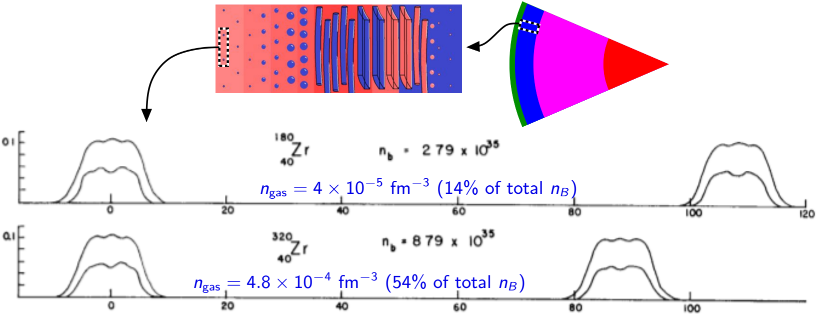
- Dilute neutron matter and ultracold atoms
- Low-momentum interactions for cold atoms
- Hartree-Fock-Bogoliubov with perturbative corrections
- Results for cold atoms
- Results for neutron matter
- Induced 3-body force
- Conclusions and outlook

More details:

M.U. and S. Ramanan, PRA 103, 063306 (2021),
V. Palaniappan, S. Ramanan, and M.U., PRC 107, 025804 (2023).

What is “dilute” neutron matter?

- Upper layers of the inner crust (close to neutron-drip density $\sim 2.5 \times 10^{-4}$ fm^{-3})



[Negele and Vautherin, NPA 207 (1973); similar results by Baldo et al., PRC 76 (2007)]

- In spite of its “low” density (still $\rho \gtrsim 10^{11}$ g/cm^3), the neutron gas is relevant because it occupies a much larger volume than the clusters

- Deeper in the crust: $n_{\rm gas}$ increases up to $\sim n_0/2 = 0.08$ fm^{-3}

Comparison with ultracold trapped Fermi gases

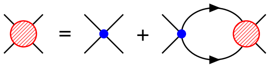
	neutron gas	trapped Fermi gas (e.g. ^{6}Li)
n	$4 \times 10^{-5} \dots 0.08$ fm^{-3}	$\sim 1\ \mu$m^{-3}
$k_F = (3\pi^2 n)^{1/3}$	$0.1 \dots 1.3$ fm^{-1}	$\sim 1\ \mu$m^{-1}
scattering length a	-18 fm	adjustable (Feshbach resonance)
effective range r_{eff}	2.5 fm	~ 1 nm
$1/k_F a$	$-0.5 \dots -0.07$	unitary limit: 0 BCS-BEC crossover: $-1 \dots 1$
$k_F r_{\text{eff}}$	$0.25 \dots 3$	10^{-3}

- r_{eff} can be neglected in cold atoms but not in neutron matter
- the neutron gas is close to the crossover regime but not in the unitary limit

Standard regularization procedure for a contact interaction

- Scattering length a for coupling constant $g < 0$ and cutoff Λ $\quad \left(\epsilon_k = \frac{k^2}{2m}\right)$

$$\frac{4\pi a}{m} = g + g\int^{\Lambda} \frac{d^3k}{(2\pi)^3}\,\frac{1}{-2\epsilon_k}\,\frac{4\pi a}{m}$$



- Express g in terms of a, e.g. in the gap equation $\quad \left(E_k = \sqrt{(\epsilon_k - \mu)^2 + \Delta^2}\right)$

$$\Delta = -g\int^{\Lambda} \frac{d^3k}{(2\pi)^3}\,\frac{\Delta}{2E_k} \quad \Leftrightarrow \quad \Delta = -\frac{4\pi a}{m}\int^{\Lambda} \frac{d^3k}{(2\pi)^3}\left(\frac{\Delta}{2E_k} - \frac{\Delta}{2\epsilon_k}\right)$$

$\Rightarrow$ now the cutoff can be removed

- Coupling constant vanishes for $\Lambda \to \infty$: $\quad \dfrac{1}{g} = \dfrac{m}{4\pi a} - \dfrac{m\Lambda}{2\pi^2}$
- Keeping Λ finite would induce a finite effective range: $r_{\text{eff}} = \dfrac{4}{\pi\Lambda}$
- For cold atoms one usually takes the limit $\Lambda \to \infty$

$V_{\text{low-}k}$-like interaction for cold atoms

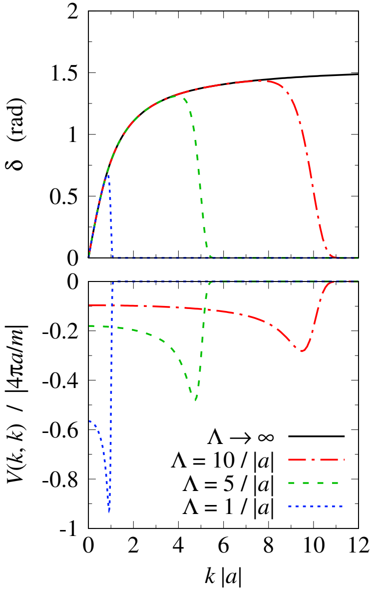
- In nuclear physics: "soft" $V_{\text{low-}k}$ or SRG interactions reproduce exactly the low-momentum scattering phase shifts of the full *NN* interaction below the cutoff

- We can construct a separable *s*-wave interaction [Tabakin 1969] that gives scattering phase shifts with $r_{\text{eff}} = 0$

$$\delta(k) = R\Big(\frac{k}{\Lambda}\Big)\ \text{arccot}\Big(-\frac{1}{ka}\Big)$$

  here: smooth regulator $R(x) = e^{-x^{20}}$

- Physical results are cutoff independent but a given theoretical method might only work (or work better) in a limited range of cutoffs

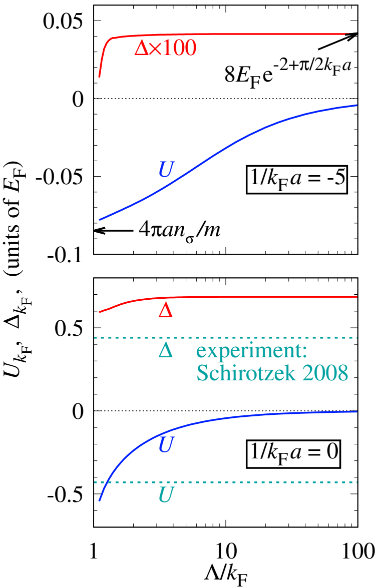


Hartree-Fock-Bogoliubov (HFB)

- In nuclear physics: hard core of "realistic" potentials requires explicit inclusion of short-range correlations, and nuclei are not bound in HF(B) approximation
- Soft interactions ($V_{\text{low-}k}$, SRG) much better suited for perturbative methods
- HFB with perturbative corrections can give good results for open-shell nuclei [e.g., Tichai et al. 2019]
  $\rightarrow$ try this method for cold atoms
- Momentum dependent gap Δ_k and mean field U_k:

$$\Delta_k = -\int \frac{d^3p}{(2\pi)^3} V(k,p)\, u_p v_p$$

$$U_k = \int \frac{d^3p}{(2\pi)^3} V\left(\frac{\mathbf{p}-\mathbf{k}}{2}, \frac{\mathbf{p}-\mathbf{k}}{2}\right) v_p^2$$



Bogoliubov Many-Body Perturbation Theory (BMBPT)

- Express $\hat{K} = \hat{H} - \mu\hat{N}$ in terms of quasiparticle operators

$$\beta_{\mathbf{k}\uparrow} = u_k\, a_{\mathbf{k}\uparrow} - v_k\, a^\dagger_{-\mathbf{k}\downarrow}\,, \qquad \beta_{\mathbf{k}\downarrow} = u_k\, a_{\mathbf{k}\downarrow} + v_k\, a^\dagger_{-\mathbf{k}\uparrow}$$

$$\hat{K} = \mathcal{E}_{\mathrm{HFB}} + \sum_{\mathbf{k}\sigma} E_k\, \beta^\dagger_{\mathbf{k}\sigma}\beta_{\mathbf{k}\sigma} + {:}\hat{V}{:}$$

$${:}\hat{V}{:} = V_{04}\,\beta\beta\beta\beta + V_{13}\,\beta^\dagger\beta\beta\beta + V_{22}\,\beta^\dagger\beta^\dagger\beta\beta + V_{31}\,\beta^\dagger\beta^\dagger\beta^\dagger\beta + V_{40}\,\beta^\dagger\beta^\dagger\beta^\dagger\beta^\dagger$$

- BMBPT: treat ${:}\hat{V}{:}$ as a perturbation

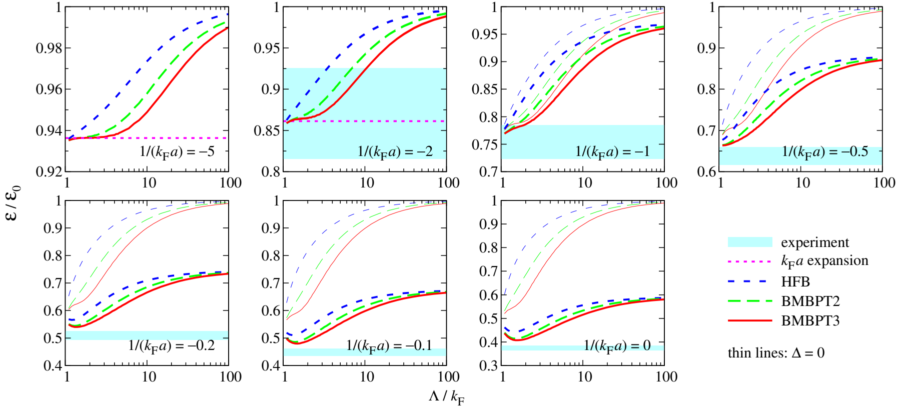
- Example: leading correction to ground-state energy is second order

$$\mathcal{E}_2 = -\frac{1}{4!}\sum_{ijkl}\frac{|\langle ijkl|\hat{V}_{40}|\mathrm{HFB}\rangle|^2}{E_i + E_j + E_k + E_l} \qquad \text{with} \qquad |ijkl\rangle = \beta^\dagger_i\beta^\dagger_j\beta^\dagger_k\beta^\dagger_l|\mathrm{HFB}\rangle$$

- Very large number of terms at higher orders $\longrightarrow$ Mathematica

- Summation over intermediate quasiparticle states $\longrightarrow$ Monte-Carlo integration

HF(B)+(B)MBPT results for ultracold atoms



[exp.: Horikoshi et al. (2017), $k_F a$ expansion: Wellenhofer et al. (2021)]

- Approximate cutoff independence reached in a region of small cutoffs ($\Lambda \lesssim 3k_F$) at weak coupling
- Inclusion of pairing (thick vs thin lines) very important at stronger coupling
- For $\Lambda \simeq 1.5 - 2k_F$, results are close to experimental ones

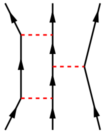
Discussion

- The Fermi momentum k_F is a natural scale for the cutoff Λ
- BMBPT3 weakens cutoff dependence but is not enough to remove it

What is missing?

- Higher orders of BMBP
- Resummation of certain classes of diagrams:
  - (Q)RPA to account for Bogoliubov-Anderson mode
  - Screening in the gap equation
- Induced three-body force (3BF) and higher-body forces:

  even if there is no 3BF in the limit $\Lambda \to \infty$, at finite Λ one has to compensate for the missing contributions of loop momenta above Λ in diagrams like this one



Differences between ultracold atoms and neutron matter

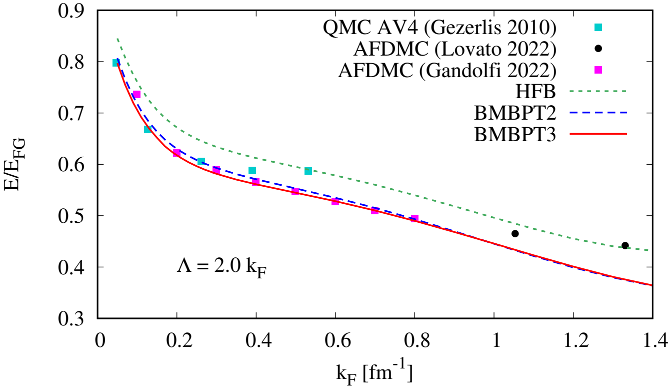
The nn interaction is more complicated:

- Finite range of the nn interaction is never negligible (even at the lowest relevant densities)
- Not only s-wave, but also higher partial waves: we include $\ell \leq 6$
- Coupling between different ℓ due to tensor force
- We use $V_{\text{low-}k}$ and SRG matrix elements generated from AV18 or chiral interactions (both give almost identical results)
- Although it is relatively weak in pure neutron matter, the bare 3BF (neglected here) could play a role at higher densities

  (note: contact nnn interaction forbidden by Pauli principle)

HFB+BMBPT results for neutron matter ($V_{\text{low-}k}$)

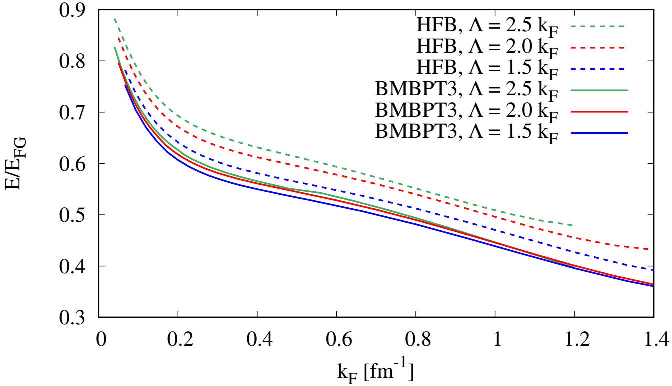
- As in the cold atoms case, we use density dependent cutoffs $\Lambda \simeq 1.5 - 3k_F$



- With $V_{\text{low-}k}$ ($\Lambda = 2k_F$), BMBPT seems to converge rapidly
- Good agreement with QMC results at low densities
- Energies too low at high densities: missing (bare) 3BF?

Cutoff dependence of neutron matter results ($V_{\text{low-}k}$)

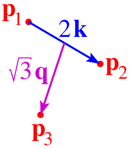
- Physical results should be independent of the ratio Λ/k_F



- Varying Λ/k_F in a reasonable range, we see that the BMBPT results show much less cutoff dependence than the HFB results
- The residual cutoff dependence indicates the necessity of including higher orders of BMBPT or induced many-body forces

Similarity Renormalization Group: induced 3-body force

- Unlike $V_{\text{low-}k}$, the SRG allows us to compute the induced 3BF
- In the 3-body space, we use the basis of hyperspherical harmonics



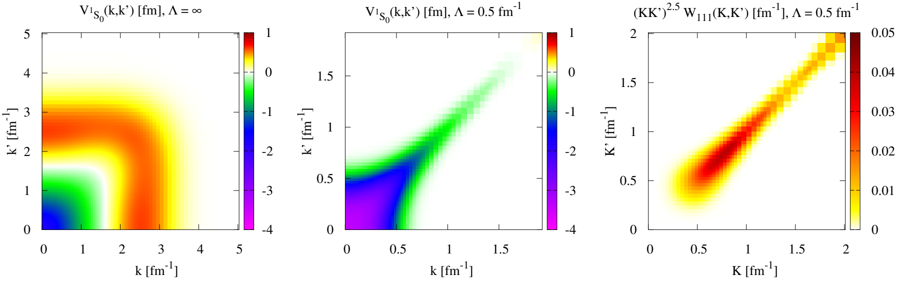
$$K = \sqrt{k^2 + q^2}$$

$$\alpha = \arccos \frac{k}{K}$$

$$\mathcal{Y}_{L\ell_1 m_1 \ell_2 m_2} = Y_{\ell_1 m_1}(\hat{\mathbf{k}}) Y_{\ell_2 m_2}(\hat{\mathbf{q}}) \mathcal{P}_L^{\ell_2 \ell_1}(\alpha)$$

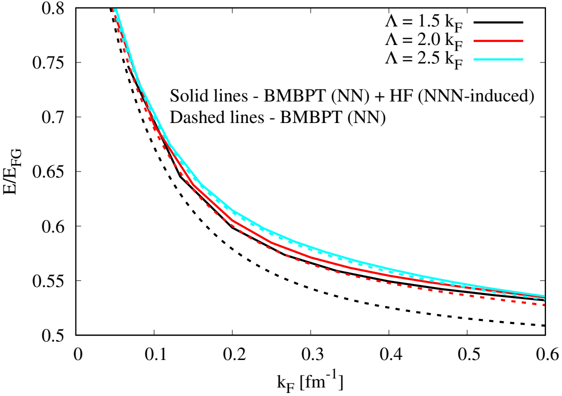
$\rightarrow$ needs less memory than the Jacobi partial wave basis [Hebeler 2012]

- So far, only 3BF induced by $^1S_0$ 2-body interaction (bare $V_{^1S_0}$: chiral)



Cutoff dependence with induced 3-body force (SRG)

- Since induced 3BF is weak, we include it perturbatively (HF)



- BMBPT3 with SRG has stronger cutoff dependence than with $V_{\text{low-}k}$ (reason currently under investigation)

- Cutoff dependence almost cancelled by the contribution of the induced 3BF

Conclusions

- The HFB+BMBPT scheme with low-momentum interactions can be applied to uniform systems
- In infinite matter, it is natural to scale the cutoff Λ with k_F
- In cold atoms: low-momentum interactions give a HF field and hence better results already at the mean-field (HFB) level
- In neutron matter: BMBPT seems to converge at small cutoffs
- Contribution of induced 3BF is small

Outlook

- In progress: perturbative corrections to U_k and Δ_k within the diagrammatic (Nambu-Gorkov) formalism
- Contribution of bare 3BF (2-pion exchange) in neutron matter
- Role of collective modes and screening
- Inclusion of protons (neutron-star core)