Nuclear equation of state for the study of neutron stars

Orsay Mac Meeting

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A very coincise summary of my PhD

Main Goal:

- Find a reliable probability distribution of nuclear matter parameters that is informed both by nuclear physics and astronomical observations.

First Step:

- Bayesian inferences on nuclear matter parameters with constraints from nuclear experiments -> inclusion of of A_{PV}, α_D

Second Step:

- Employing the results as basis for a second Bayesian inference, this time with neutron stars observables as constraints.

Statistical analysis with Skyrme interaction

1-to1 correspondence with usual Skyrme's parameters!

Nuclear matter parameters

Surface term parameters

Parameters of the model: $n_{sat}, E_{sat}, K_{sat}, E_{sym}, L_{sym}, m_s^*, m_v^*, G_s, G_v, w_0$

Effective masses

Spin-orbit parameter

"hfbcs-qrpa[1]" code to compute observables from parameters

[1]G. Colò, X. Roca-Maza, arXiv:2102.06562v1 [nucl-th]

Observables used

	$B.E.$ [MeV]	R_{ch} [fm]	ΔE_{SO} [MeV]	E^{IS}_{GMR} [MeV]
Pb208	1636.4 ± 2.0	5.50 ± 0.05	2.02 ± 0.50	13.5 ± 0.5
Ca48	416.0 ± 2.0	3.48 ± 0.05	1.72 ± 0.50	
Ca40	342.1 ± 2.0	3.49 ± 0.05		
Ni56	484.0 ± 2.0			
Ni68	590.4 ± 2.0			
Sn100	825.2 ± 2.0			
Sn132	1102.8 ± 2.0	4.65 ± 0.05		
Zr90	783.9 ± 2.0	4.27 ± 0.05		17.7 ± 0.5

	α_D [fm^3]	$\mathfrak{m}(1)$ [MeV fm^2]	A_{PV} (p.p.b.)	E^{IS}_{GQR} [MeV]
Pb208	19.60 ± 0.60	961 ± 22	550 ± 18	10.9 ± 0.5
Ca48	2.07 ± 0.22		2668 ± 113	

2 h. x 10’000’000 points …

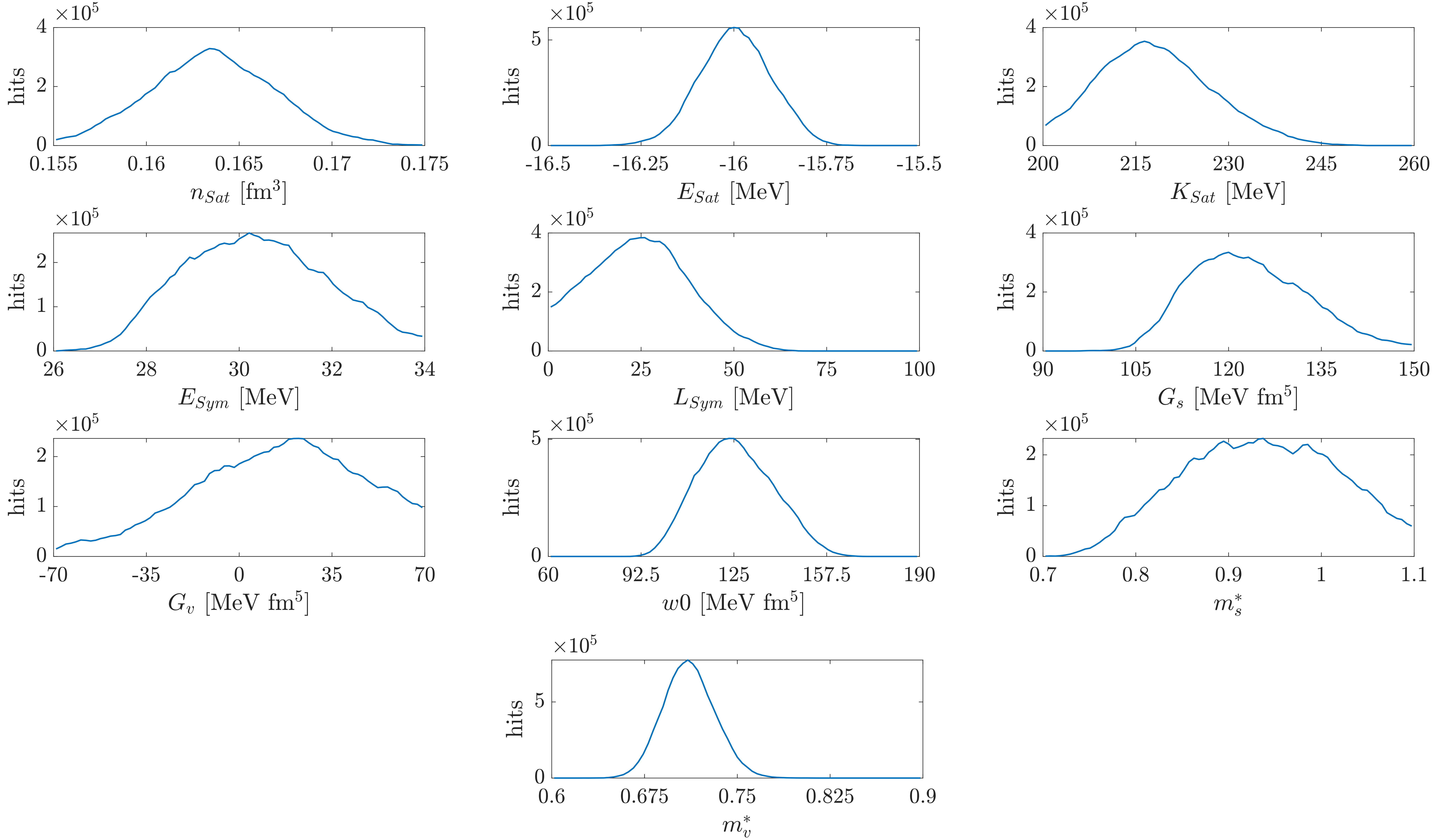
Direct bayesian inference unfeasible!

MADAI package

(Emulator for Bayesian inference)

https://madai.phy.duke.edu/

Marginalized Parameters Posterior Distributions



Posterior observables means and uncertainties

B.E. [MeV]	^{40}Ca	342 ± 1.5 342 ± 2.0
	^{48}Ca	417 ± 1.1 416 ± 2.0
	^{56}Ni	482 ± 1.3 484 ± 2.0
	^{68}Ni	590 ± 1.0 590 ± 2.0
	^{90}Zr	784 ± 1.3 784 ± 2.0
	^{100}Sn	826 ± 1.6 825 ± 2.0
	^{132}Sn	1103 ± 1.6 1103 ± 2.0
	^{208}Pb	1636 ± 1.8 1636 ± 2.0

$\mathbf{R}_{ch}$ [fm]	^{40}Ca	3.49 ± 0.01 3.48 ± 0.05
	^{48}Ca	3.50 ± 0.02 3.48 ± 0.05
	^{90}Zr	4.26 ± 0.02 4.27 ± 0.05
	^{132}Sn	4.69 ± 0.02 4.65 ± 0.05
	^{208}Pb	5.47 ± 0.03 5.50 ± 0.05
$\mathbf{\Delta E}_{SO}$ [MeV]	^{48}Ca ($\nu 2p$)	1.91 ± 0.18 1.72 ± 0.50
	^{208}Pb ($\pi 2f$)	2.33 ± 0.15 1.96 ± 0.50

m(1) IV [MeV]	^{208}Pb	957 ± 22 961 ± 22
E^{IS}_{GQR} [MeV]	^{208}Pb	10.8 ± 0.4 10.9 ± 0.5
E^{IS}_{GMR} [MeV]	^{90}Zr	17.8 ± 0.4 17.7 ± 0.5
	^{208}Pb	13.5 ± 0.3 13.5 ± 0.5
α_D [fm^3]	^{48}Ca	2.3 ± 0.1 2.1 ± 0.2
	^{208}Pb	19.5 ± 0.5 19.6 ± 0.6
A_{PV} [p.p.b.]	^{48}Ca	2571 ± 47 2668 ± 113
	^{208}Pb	587 ± 5 550 ± 18

$$\sigma_c = \sqrt{\sigma_{exp}^2 + \sigma_{inf}^2} \longrightarrow |x_{exp} - x_{inf}| \in [1,2)\,\sigma_c$$

Neutron stars from nuclear Equation of state

Caen group's code

Input:
Nuclear matter parameters

Meta-Model nEoS →

Output:
Neutron star equation of state
+ properties

Bayesian analysis

Parameters:
nuclear matter parameters

Constraints:
ab-initio calculations; neutron stars

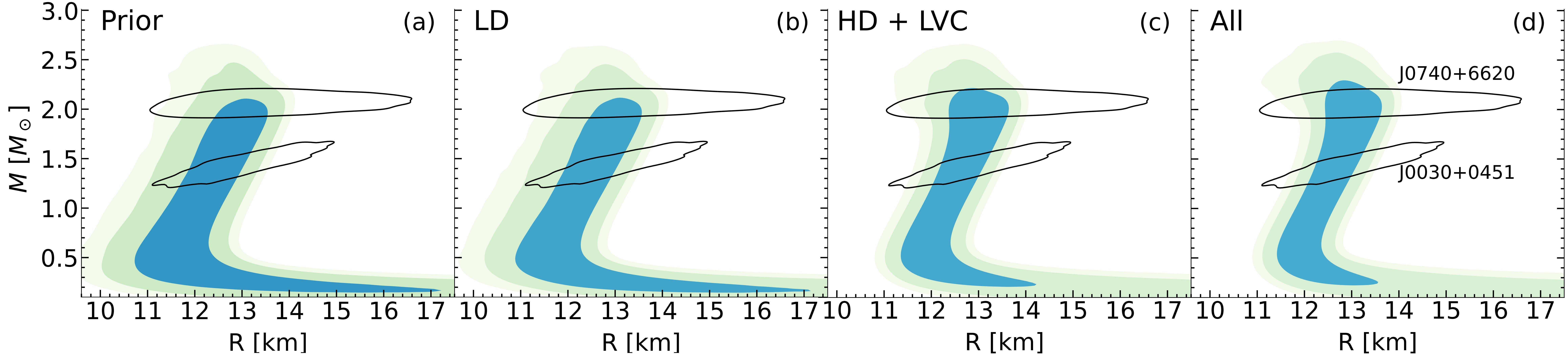
Maximum mass M observed,
Ligo-Virgo-Collaboration
tidal deformability results,
NICER mission simultaneous
mass-radius measurements

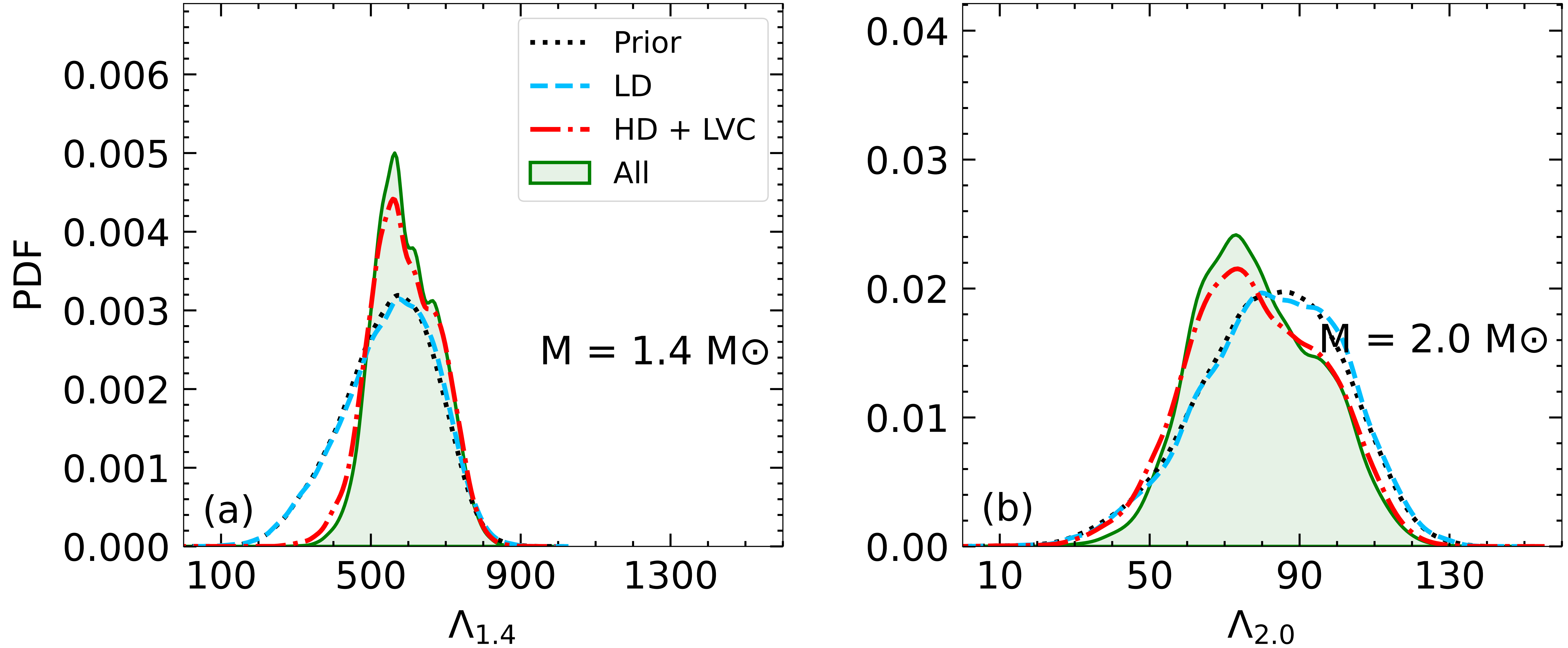
Previous **posterior distribution**
=
new **prior distribution**

Final posterior distribution informed by both nuclear and astrophysics

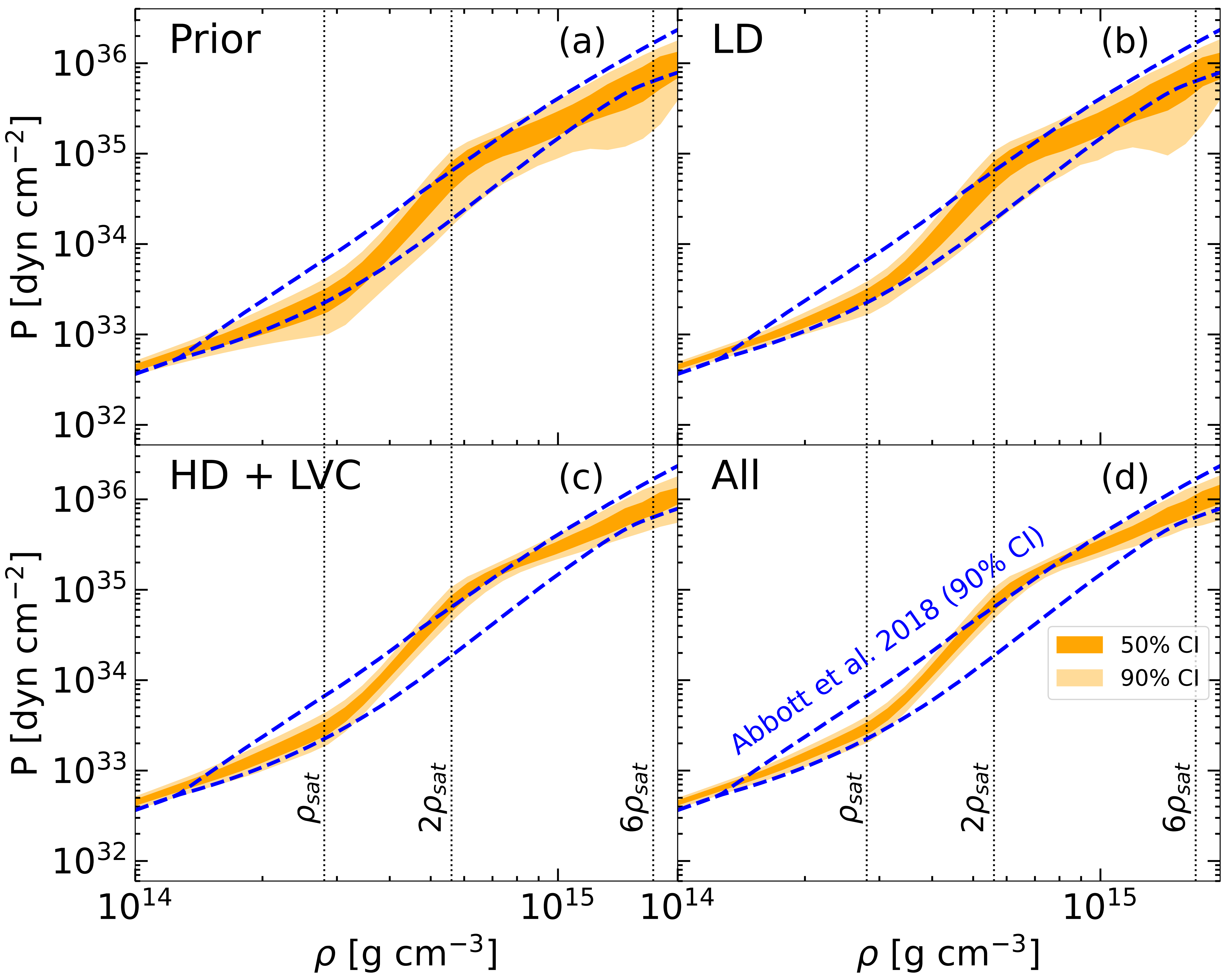
Marginalized Parameters Posterior Distributions

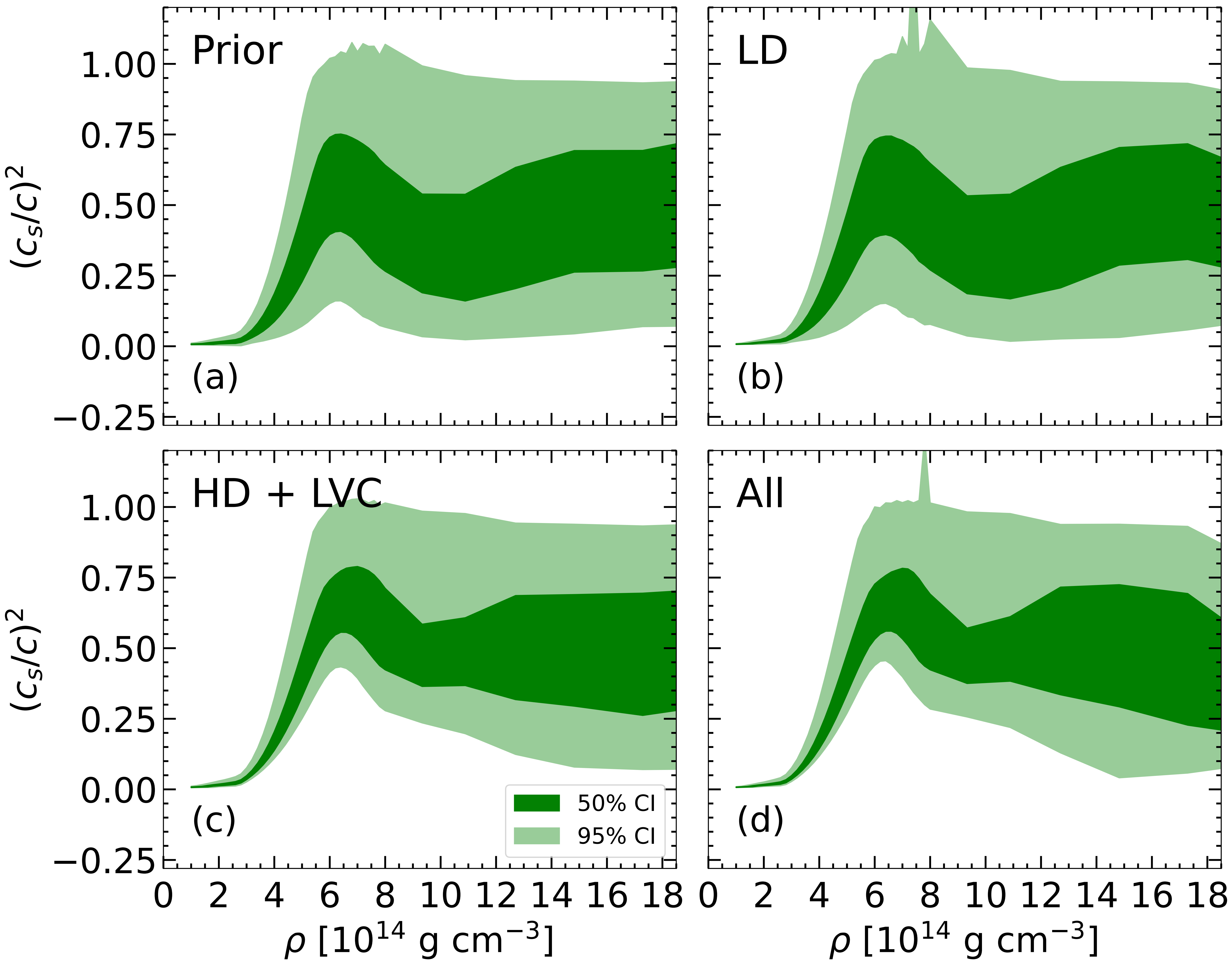
Mass-Radius relation and $\Lambda_{1.4M_{\odot}}$, $\Lambda_{2.0M_{\odot}}$ posterior distributions





Equation of state and Sound Speed posterior distributions





What's next? Neutron star crust

From CLDM to ETF

Skyrme EDF approximated within extended Thomas Fermi theory at second order in $\hbar^2$

$$E_{WS} = \int_{V_{WS}} \mathcal{E}_{ETF}^{Sky}(n(\mathbf{r}), n_p(\mathbf{r}))d^3\mathbf{r}$$

$$n(r) = n_{r-cl}(r) + n_{r-gas}(r) = \frac{n_0}{1+\exp\left(\frac{r-R}{a}\right)} + \frac{n_g}{1+\exp\left(-\frac{r-R}{a}\right)}$$

$$n_p(r) = n_{p,\,r-cl}(r) = \frac{n_{0,\,p}}{1+\exp\left(\frac{r-R_p}{a_p}\right)}$$

Parametrized density profiles for improved computational efficiency