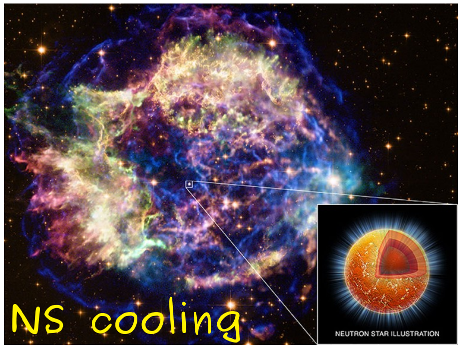
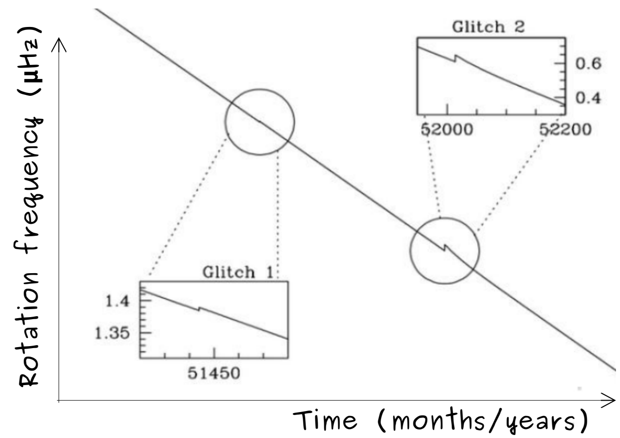
Macroscopic superfluidity in neutron stars

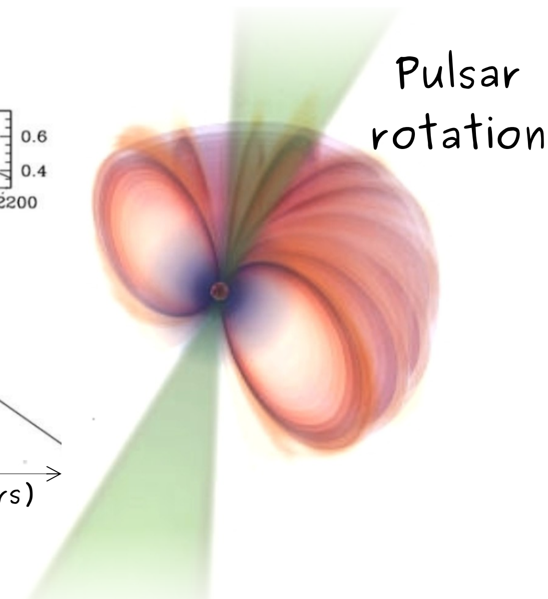


Superfluidity affects neutrino emission and heat capacity

Superfluidity allows for more and different oscillation modes
(like second sound in Helium-II)

Pulsar glitches (and maybe timing noise) are related to the presence of superfluid neutrons
Review: arXiv:2301.12769 (2022)





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Thursday, May 30, 2024 – MAC Meeting, Paris Orsay-Saclay

Pairing channels

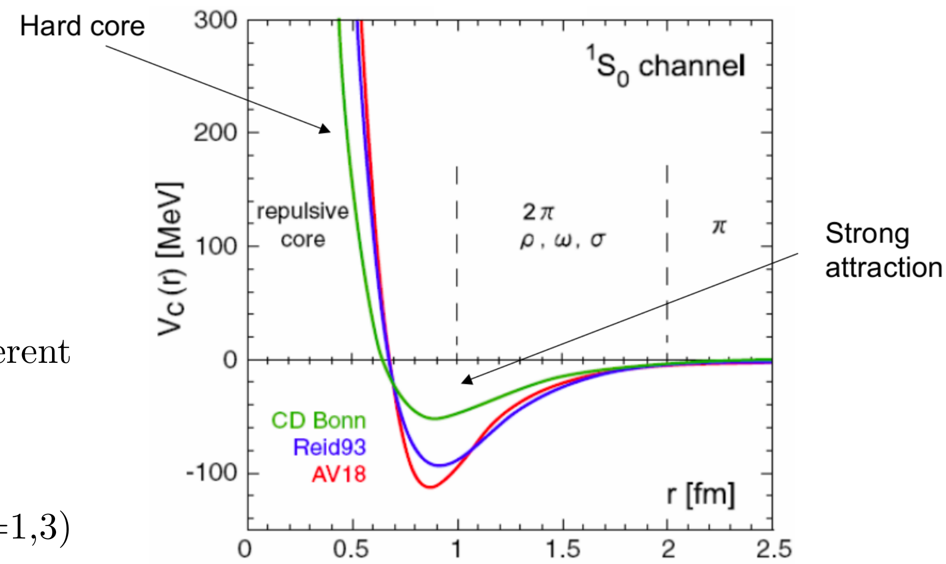
Total angular momentum operator: $\mathbf{J} = \mathbf{L} + \mathbf{S}$
Notation: $^{2S+1}L_J$ (L=0,1,2,3... → S,P,D,F...)

$^1S_0$ isotropic pairing: Δ = "energy gap" ~ 0.57 T_c

$^3S_1$–$^3D_1$ binds the deuteron: but in NS **n** and **p** have very different Fermi surfaces → **no n-p pairing**

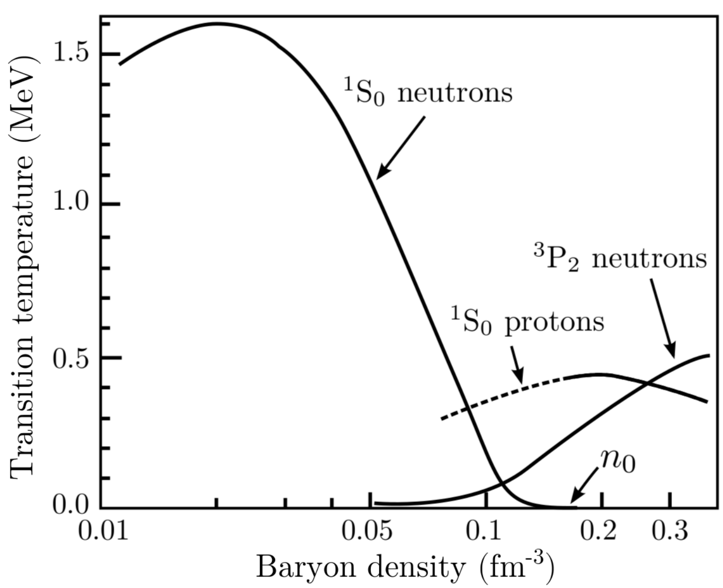
$^3PF_2$ partial–wave channel (Δ has contributions from both L=1,3) preferred at larger Fermi momenta, where $^1S_0$ is repulsive. Uncertain gap, usually treated as free **parameter** in cooling simulations.

NN potential for S=0, L=0, J=0

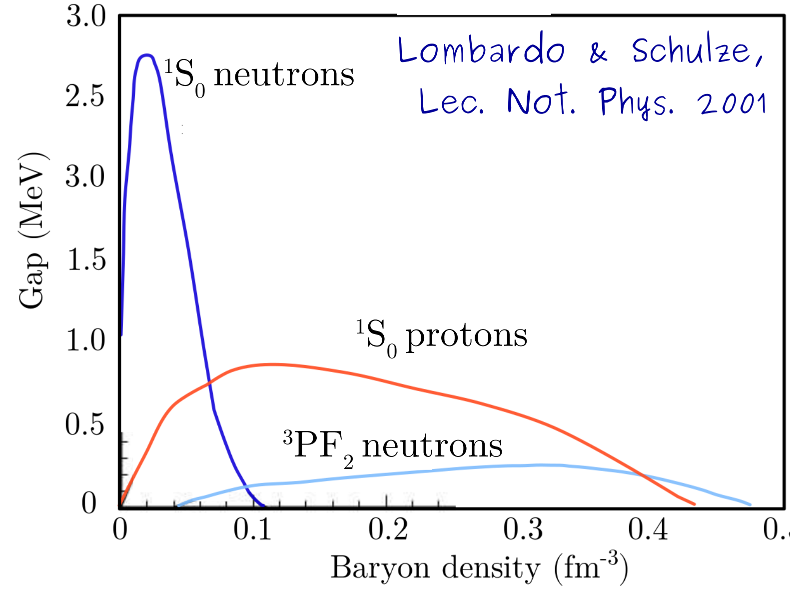


Ishii et al, PRL 2007

Transition temperature

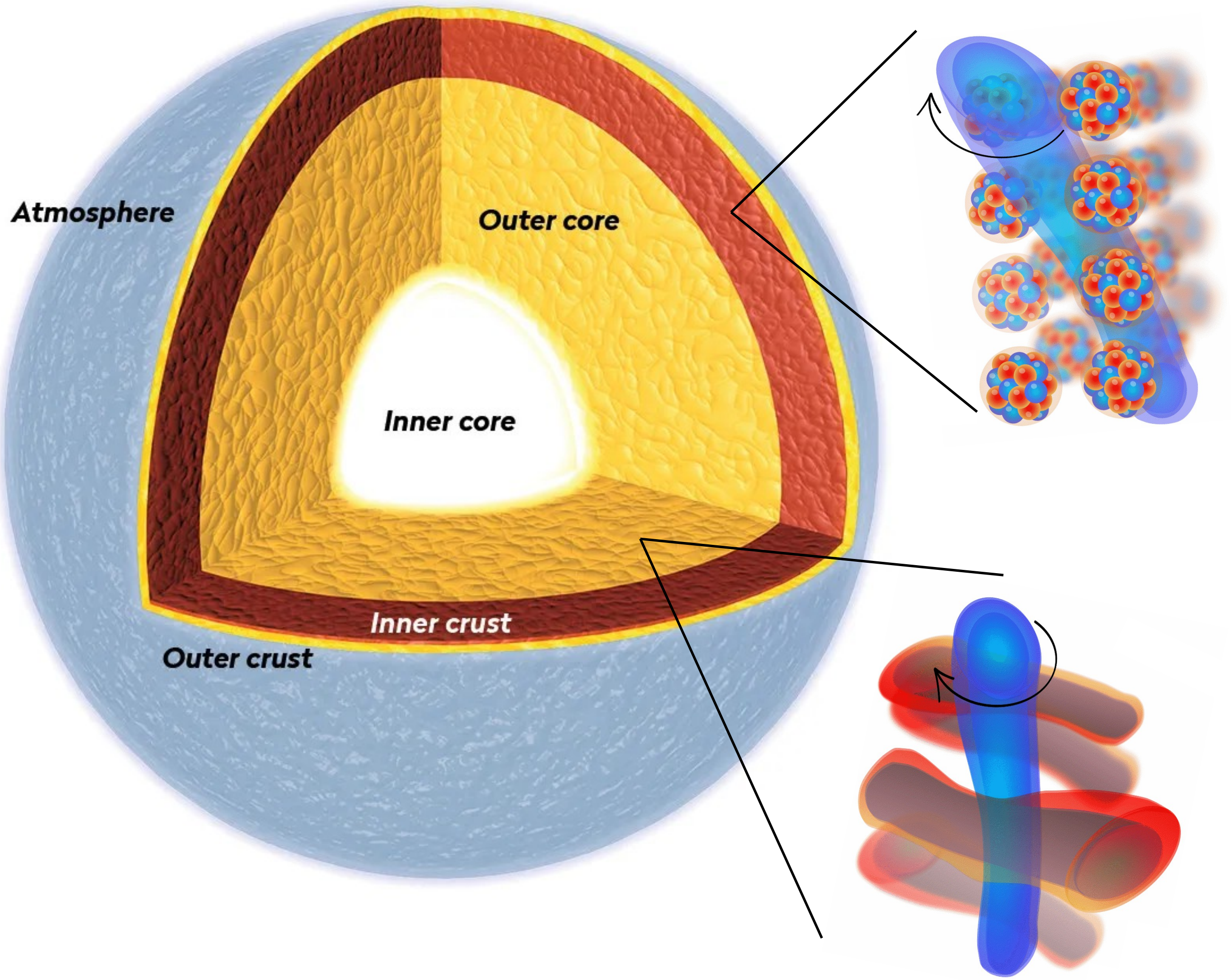


Pairing gap Δ ~ 1 MeV ~ 10^{10} K



Continuum description?

Not a simple fluid but many layers with different inhomogeneities, defects, currents



Inner crust

(Visco-)elastic lattice

Neutron "scalar" superfluid

Excitations (entropy/heat)

Ideal gas of electrons

B field

Frame of the lattice: 3 currents

Outer core

Neutron superfluid

Proton superconductor

Excitations (entropy/heat)

Ideal gas of electrons

B field in fluxtubes (type II?)

Different phenomena may be well described with a smaller number of "fluids"

e.g. cooling, glitches, oscillation modes, "mountains", B evolution...

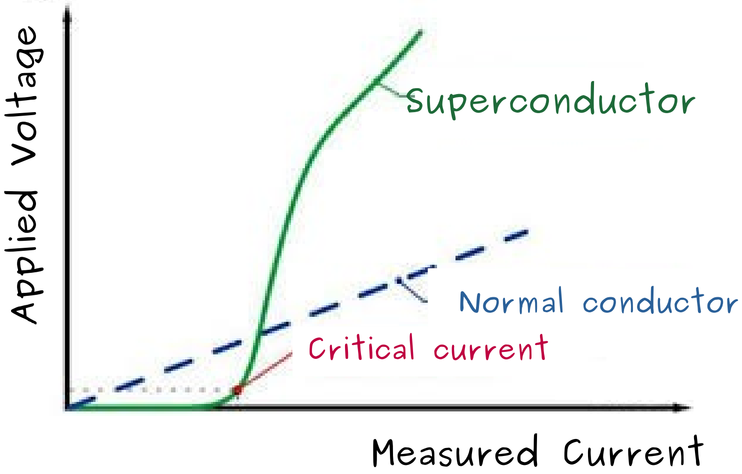
...it is however true that the "mageto-thermo-rotational" evolution is coupled

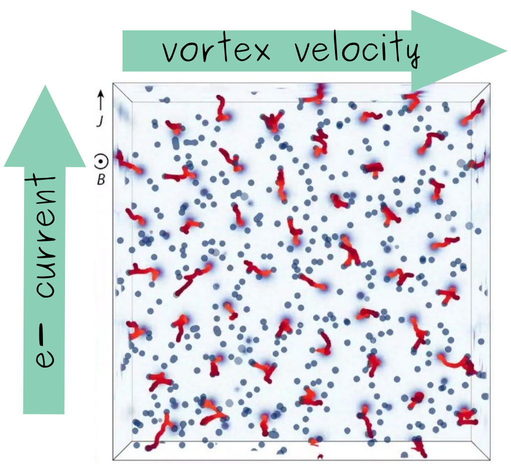
Role of vortices in the continuum description?

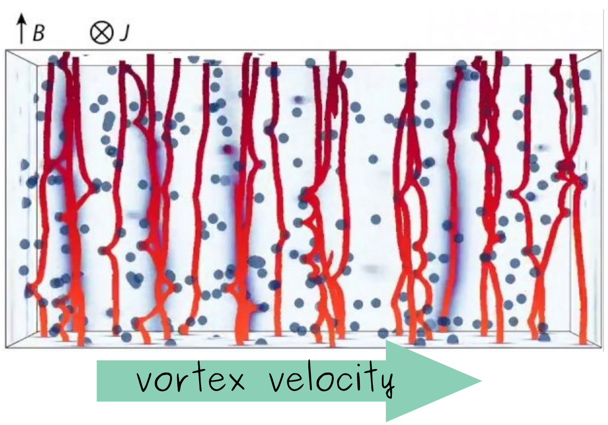
Many coexisting species and independent currents, but **what is the role of the topological defects?**
Hint: consider laboratory type-II superconductors

Vortex-defect interaction associated with a **critical current** in the V-I relation

The slope is the resistance:
Flat region → no dissipation







Willa et al. (2018)

Nutron Stars – Lab Superconductor

Crust's frame of reference – Frame of the lattice
Delocalized neutrons – Electron sea
Vorticity – magnetic field
Vortices – Flux-tubes
Neutron current – electric current
Magnus force – Lorentz force

Conservative regime:
Pinned vortices – pinned flux-tubes

Dissipative regime:
Moving vortices – Moving flux-tubes

Superfluid hydrodynamics

Relativistic formulation: Gavassino, MA
ArXiv: 2012.10288, 2001.08951
Review: arXiv:2301.12769 (2022)

Vortex core scale:
~ 10 fm in a NS

Microscopic models needed:

Helium: stochastic GPE, mean field...

NS: phenomenological Ginzburg-Landau+GPE, TDLDA, HFB...

Conserved baryon charge:
Baryons $\sim n_0\,(10\ \mathrm{fm})^3 \sim 10^3$

Inter-vortex scale:
$\sim 10^{-3}$ cm in a NS

Mescoscopic models needed:

Helium: Vortex filament model
(K. Schwarz's 80s papers)

NS: the same but needs extension to non-homogeneous environment

Conserved baryon charge:
Baryons $\sim n_0\,(10^{-3}\ \mathrm{cm})^3 \sim 10^{29}$

Fluid element:
from ~*mm* in a NS

Macroscopic hydro:

Helium: HVBK hydro
(Hall & Vinen 1956)

NS: extensions/decorations of HVBK hydro (more species, GR)

Conserved baryon charge:
Not so relevant, need to include many vortices in the fluid element

Relativistic formulation: Gavassino, MA
ArXiv: 2012.10288, 2001.08951
Review: arXiv:2301.12769 (2022)

Superfluid hydrodynamics

Vortex core scale:
~ 10 fm in a NS

Inter-vortex scale:
~ 10^{-3} cm in a NS

Fluid element:
from ~*mm* in a NS

We can not take into account each vortex (~10^{16} in a pulsar) → HVBK hydrodynamics

2 Euler-like equations + **entrainment** + **mutual friction**

$$\partial_t \rho_x + \nabla_i(\rho_x v_x^i) = 0$$

$$(\partial_t + v_x^j \nabla_j)(v_i^x + \varepsilon_x w_i^{yx}) + \nabla_i(\tilde{\mu}_x + \Phi) + \varepsilon_x w_{yx}^j \nabla_i v_j^x = f_i^x/\rho_x$$

The **dynamics of vortices** in a fluid element gives the form and strength of **mutual friction**

$x,y=n$ → superfluid neutrons $x,y=p$ → normal component (electrons, excitations, protons, lattice...)

Superfluid hydrodynamics

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2 Euler-like equations + **entrainment** + **mutual friction**

$$(\partial_t + v_{\rm x}^j \nabla_j)(v_i^{\rm x} + \varepsilon_{\rm x} w_i^{\rm yx}) + \nabla_i(\tilde{\mu}_{\rm x} + \Phi) + \varepsilon_{\rm x} w_{\rm yx}^j \nabla_i v_j^{\rm x} = f_i^{\rm x}/\rho_{\rm x}$$

$$p_x^i = m_x \left[(1-\epsilon_x) v_x^i + \epsilon_x v_y^i \right]$$

Hydrodynamic canonical momenta

Uniform matter, external force applied to a single species (e.g. the neutrons)

$$\mathbf{F}_n^{ext} = n_n \dot{\mathbf{p}}_n = \rho_n \dot{\mathbf{v}}_n$$

No entrainment

$$\mathbf{F}_n^{ext} = n_n \dot{\mathbf{p}}_n = \rho_{nn}^* \dot{\mathbf{v}}_n + \rho_{pn}^* \dot{\mathbf{v}}_p$$

With entrainment

Entrainment is a non-dissipative coupling (but you can not see this at this level, entropy is needed)

Superfluid hydrodynamics

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Vortex core scale:
~ 10 fm in a NS

Inter-vortex scale:
~ 10^{-3} cm in a NS

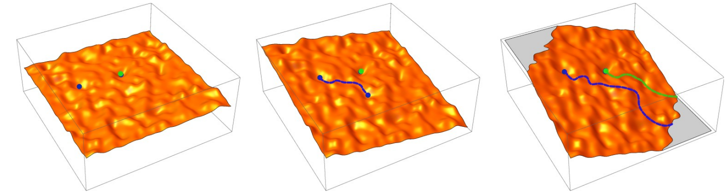
Fluid element:
from ~*mm* in a NS

We can not take into account each vortex (~10^{16} in a pulsar) → HVBK-like hydrodynamics

2 Euler-like equations + **entrainment** + **mutual friction**

$$\rho_n D_t \mathbf{v}_n + \ldots = \mathbf{F}_{MF}$$
$$\rho_p D_t \mathbf{v}_p + \ldots = -\mathbf{F}_{MF}$$

The **dynamics of vortices** in a fluid element gives the form and strength of the macroscopic **"mutual friction"**

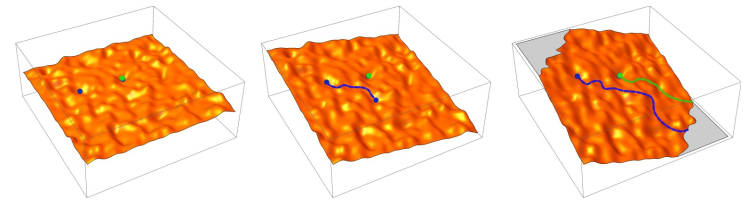


$x,y=n$ → superfluid neutrons $x,y=p$ → normal component (electrons, excitations, protons, lattice...)

Vortex motion → Mutual friction (with pinning)

Antonelli & Haskell, arXiv:2007.11720 (2020)

It's a "kinetic approach" but with point vortices instead of particles



- Fix a background "lag" (background current of superfluid neutrons)
- Assign random position of a vortex in the pinning landscape and solve the trajectory

disordered pinning force field

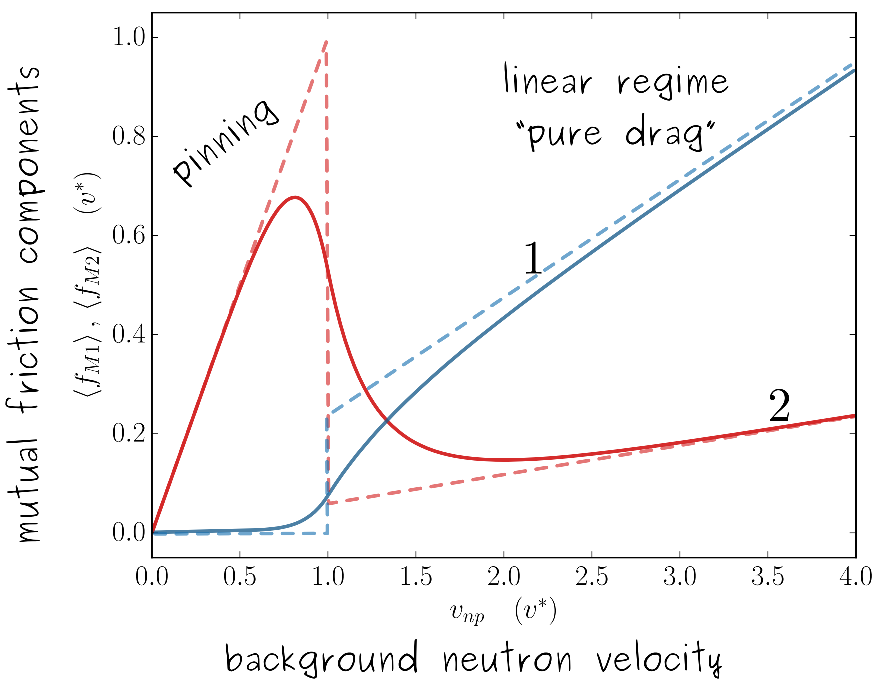
$$\hat{\boldsymbol{\kappa}} \times (\dot{\boldsymbol{x}}(t) - \boldsymbol{v}_{np}) - \mathcal{R}\, \dot{\boldsymbol{x}}(t) + \boldsymbol{f} = 0$$

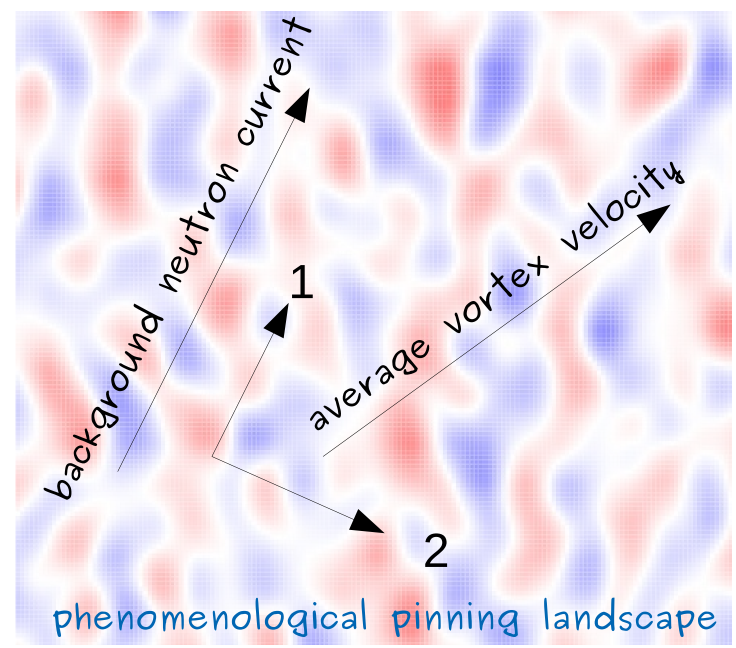
- Repeat many times and find the average vortex velocity for the given "lag"
- The mutual friction is given by

$$\boldsymbol{F}_n = -\kappa\, n_v\, \hat{\boldsymbol{\kappa}} \times (\langle \dot{\boldsymbol{x}} \rangle - \boldsymbol{v}_{np})$$

average vortex velocity

assigned "lag"



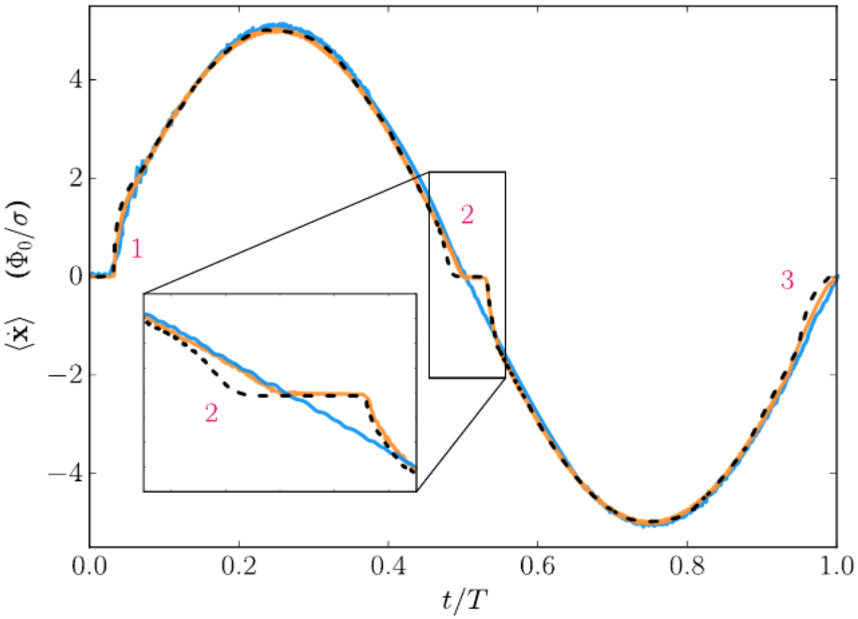


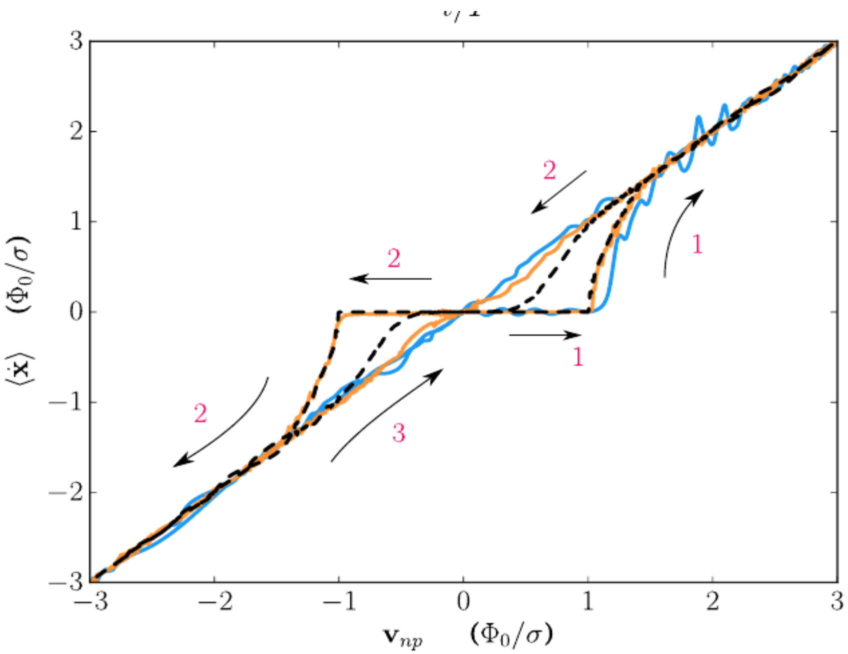
Beyond hydrodynamics: hysteresis

Rate-dependent hysteresis: lag between an input and an output that disappears if the input is varied more slowly. If the input is reduced to zero, the output continues to respond for a finite time.

Instantaneous drop to null lag → vortex velocity drops to zero immediately (if NO pinning forces)
Instantaneous drop to null lag → vortex velocity relaxes to zero (with pinning forces)

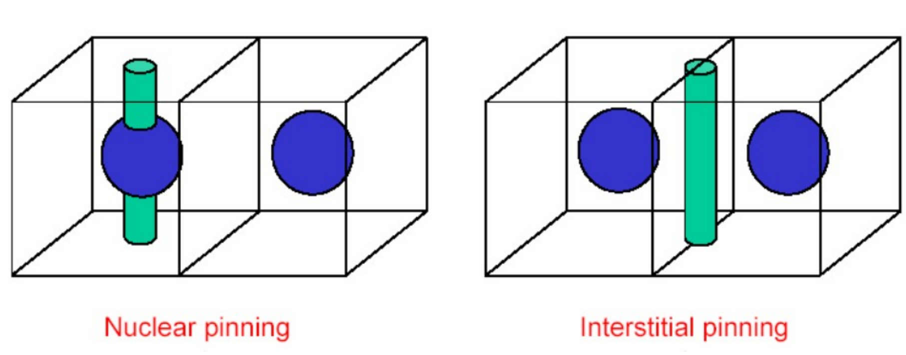
Rate-independent hysteresis → vortex-vortex interactions



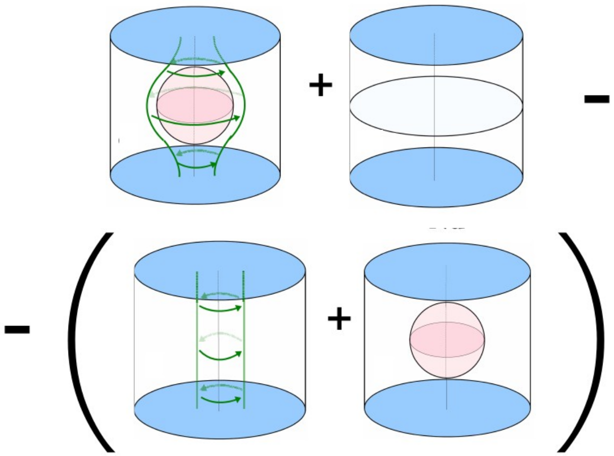


Antonelli & Haskell 2020 arXiv:2007.11720

Pinning energy (vortex – single nucleus)



Donati & Pizzochero, Phys Lett B, 640 (2006)
Semiclassical approach: static LDA calculation (local Fermi momentum is a function of the neutron number density)



Klausner et al, arXiv:2303.18151 (2023)
Hartree-Fock-Bogolyubov

Energy contributions to pinning:

→ negative condensation energy of the order of Δ^2 / E_F

→ kinetic energy of the irrotational vortex-induced flow

→ Fermi energy E_F of neutrons

→ nuclear cluster energy (Woods-Saxon potential)

Uncertain pairing gap Δ: modifies the strength and location of the pinning energies (in-medium effects!)
Maximum pinning energies < 3.5 MeV
Significant pinning occurs only in a restricted range: $0.07\ n_0 < n_B < 0.2\ n_0$

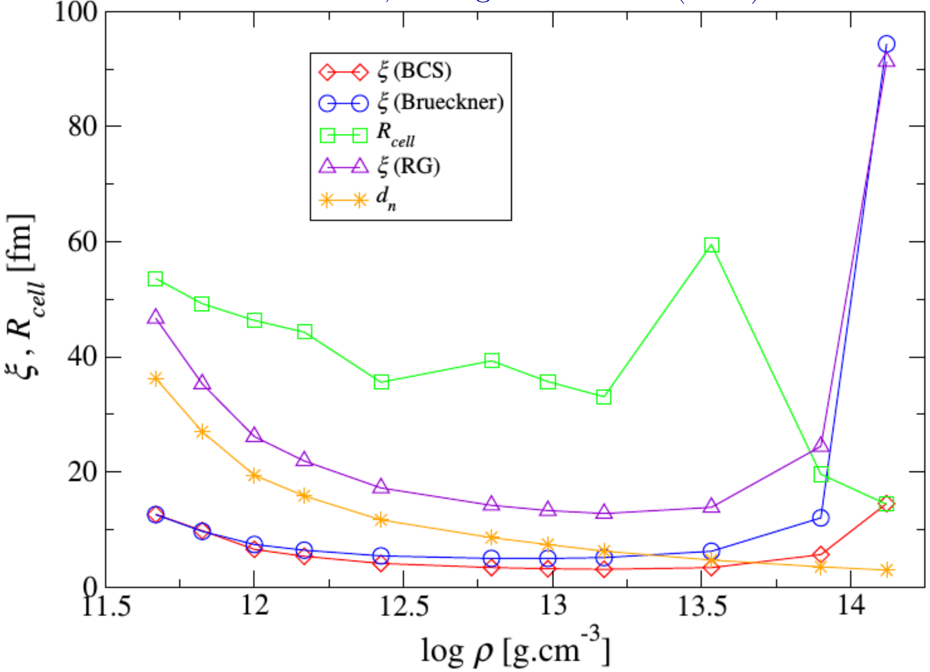
Improvements: TDLDA, Wlazłowski+ (2016), Hartree-Fock-Bogolyubov Klausner+ (2023)

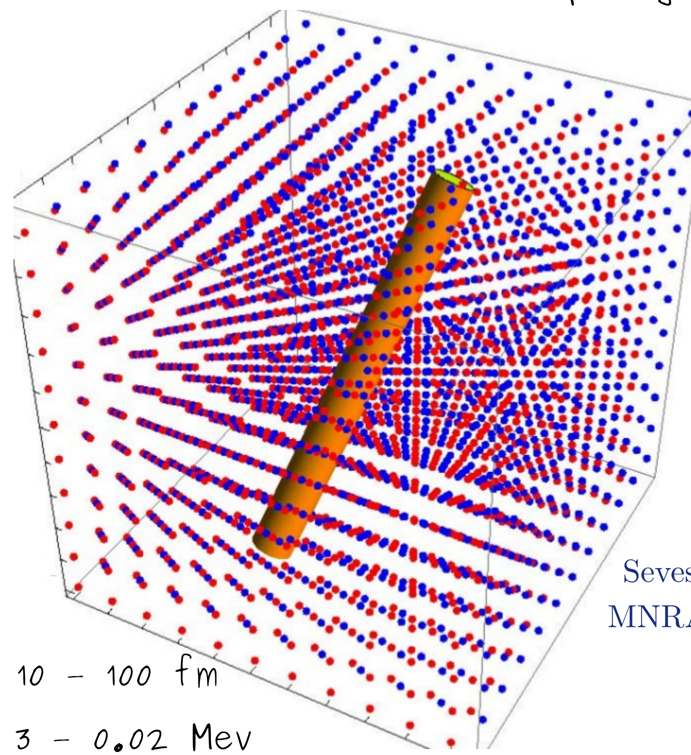
Pinning forces (inner crust)

Lattice spacing: 50-10 fm

Lattice spacing: 50–10 fm

Chamel & Haensel, Living Rev.Rel. 11 (2008)





Seveso et al MNRAS 2016

ξ = 10 – 100 fm

E_{pin} = 3 – 0.02 Mev

Coherence length ξ ~ vortex core radius

Strong pinning when ξ < lattice spacing

Pinning to **single defects** VS "**collective pinning**":

Rigid (straight) vortices are "less pinned"

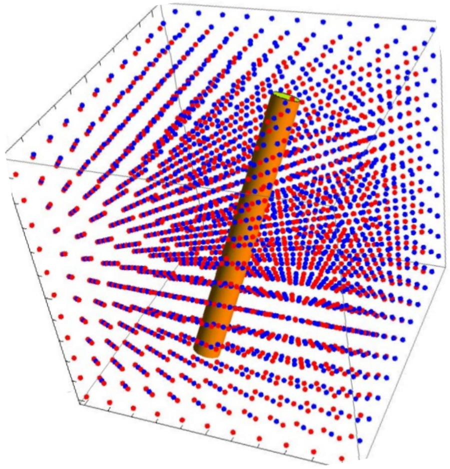
Coherence length ξ estimates: Mendell, ApJ 38 1991

Inner crust:

Problem: how to calculate the "vortex-lattice" interaction from the "vortex-nucleus" interaction ?

IDEA: consider a segment of vortex line (the length L is given by the tension) and average over translations and rotations of the total pinning force divided by L

Stationary test (glitch amplitude)



We can **test** theoretical single-vortex pinning forces with glitches of large amplitude:

- Choose the EOS
- Choose the pinning forces (function of the baryon density)
- Solve the hydrostatic equilibrium in slow rotation ("Hartle") and calculate:

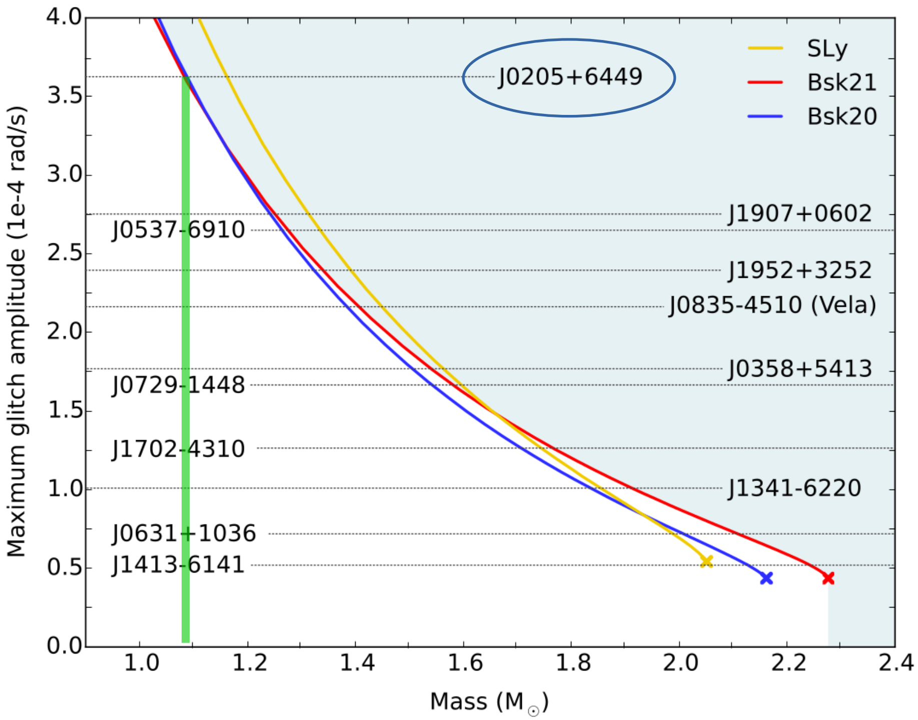
$$\Delta\Omega_{\rm abs} = \frac{\pi^2}{I\kappa}\int_0^{R_d} dr\, r^3\, e^{\Lambda(r)}\,\frac{\mathcal{E}(r)+P(r)}{m_n\, n_B(r)\, c^2}\, f_P(r)$$

→ Generous theoretical **upper bound** based only on angular momentum balance
→ No superfluid fraction and no entrainment
→ No need to solve the internal dynamics
→ No dependence on the assumed vortex configuration

Compare with bounds on the **minimum** mass:

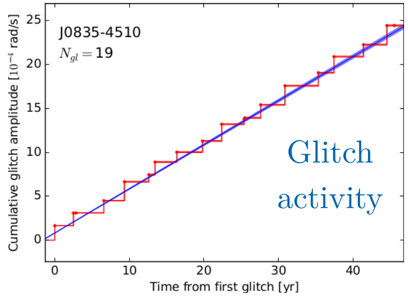
Observed: M=1.174 M⊙ Martinez+2015 arXiv:1509.08805

CCS simulations: M≈1.15 M⊙ Suwa+2017 arXiv:1808.02328



Review: Antonelli, Montoli, Pizzochero arXiv:2301.12769 (2022)

Stationary test (glitch activity)

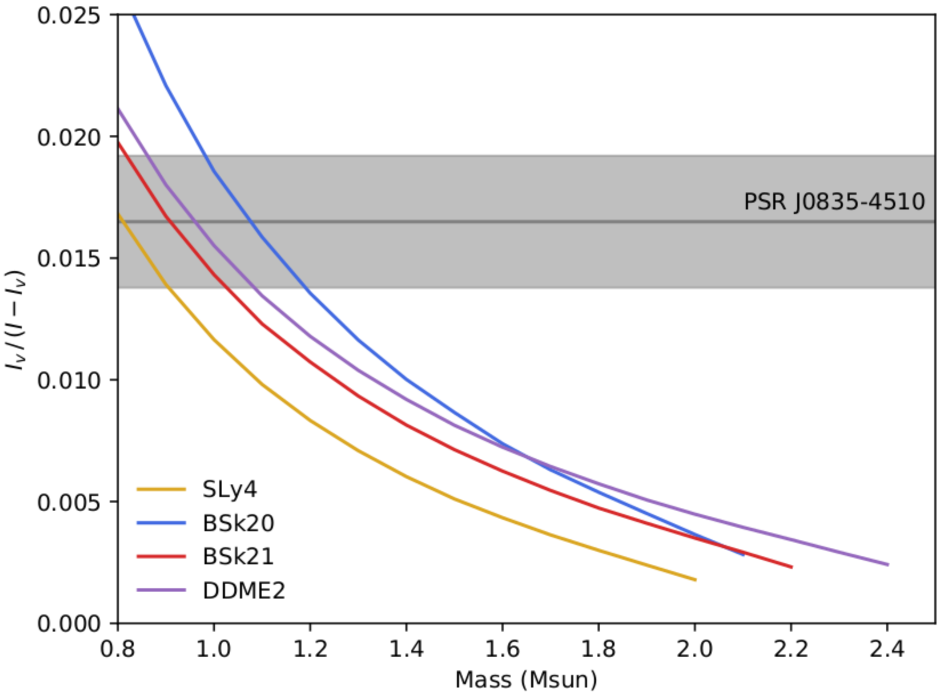


Result: the *superfluid in the crust is not enough* to explain Vela's activity when strong entrainment is accounted for Andersson+2012 arXiv:1207.0633, Chamel 2013 arXiv:1210.8177

Revised argument: Montoli+2020 arXiv:2012.01539

$$I_v = \frac{8\pi}{3}\int_0^{R_d} dr\, r^4 \, \frac{\rho_n(r)}{1-\epsilon_n(r)} \qquad \frac{\mathcal{A}}{|\dot{\Omega}|} < \frac{I_v}{I - I_v}$$

Note: no need to solve the internal dynamics ("stationary")



"Heteroscedastic" linear regression: uncertainties larger by a factor ~10 for the Vela (Montoli+2020 arXiv:2012.01539)

$$\mathcal{A}_a = \frac{\sum_i \Delta\Omega_i}{\sum_i \Delta t_i} = \frac{\sum_i \Delta\Omega_i}{t_{N_{gl}-1} - t_0} \qquad \mathrm{Var}(\mathcal{A}_a) = \frac{1}{(N_{gl}-1)(t_{N_{gl}-1} - t_0)} \sum_i \frac{(\Delta\Omega_i - \mathcal{A}_a \Delta t_i)^2}{\Delta t_i}$$

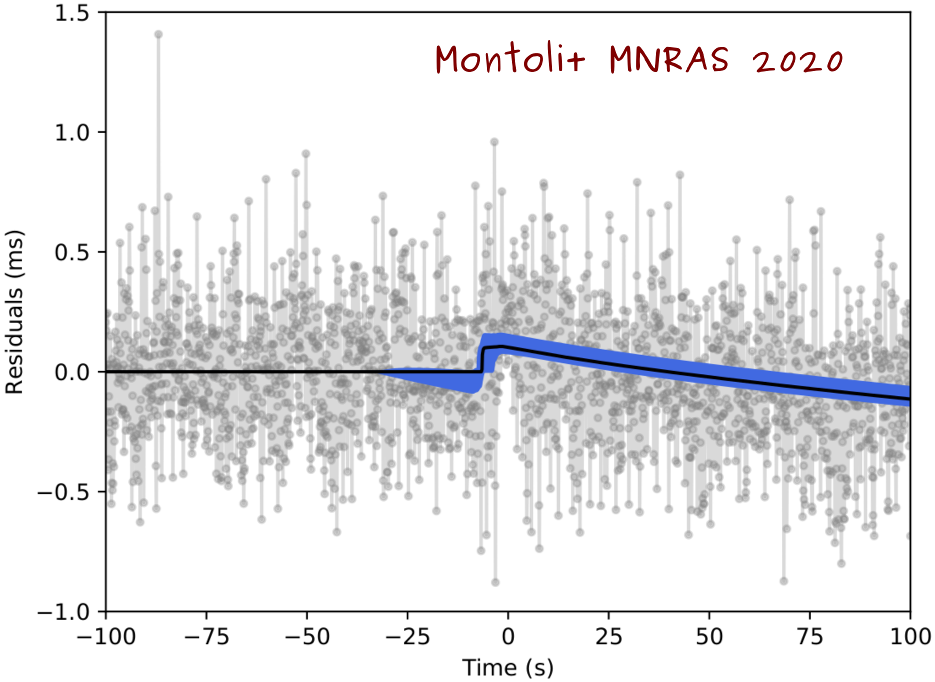
Unbiased estimators for cumulated data

Compare with bounds on the minimum mass of a NS:

Observed: M=1.174 M⊙ Martinez+2015 arXiv:1509.08805 CCS simulations: M≈1.15 M⊙ Suwa+2017 arXiv:1808.02328

Review: Antonelli, Montoli, Pizzochero arXiv:2301.12769

Bayesian fit of Vela 2016



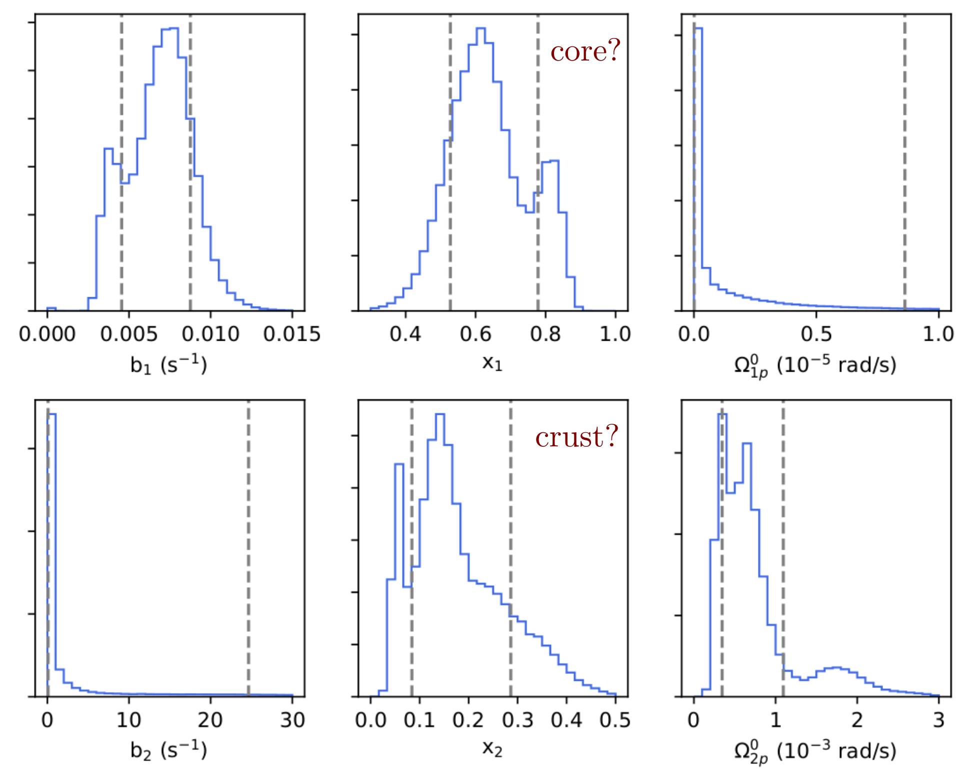
Normal $x_p \dot{\Omega}_p = -\mathcal{T}_1 - \mathcal{T}_2 - |\dot{\Omega}_\infty|$

Super (core?) $x_1 \dot{\Omega}_1 = \mathcal{T}_1$

Super (crust?) $x_2 \dot{\Omega}_2 = \mathcal{T}_2\,,$

$$\mathcal{T}_i = -x_i b_i (\Omega_i - \Omega_p) \qquad i = 1, 2$$

Posteriors for the **physical** parameters



Fit of the TOA residuals of Palfreyman+2018 with a 3-component model:

→ Estimated **moment of inertia fractions** ($x_2+x_1<1$):

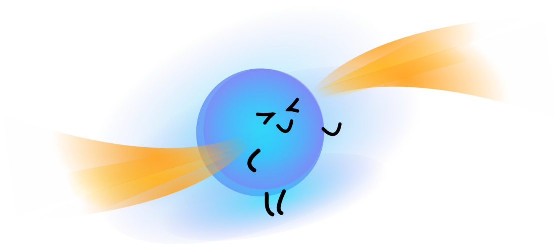
“active” $x_2 \sim 0.1 - 0.3$, “passive” $x_1 \sim 0.5 - 0.7$

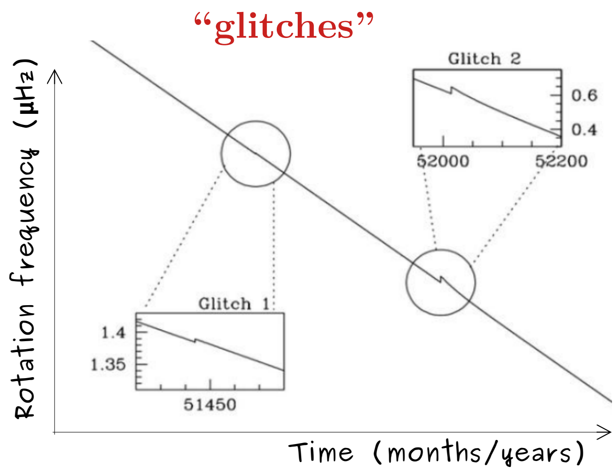
→ Confirmed **overshoot** found by Ashton+2019, Pizzochero+2020

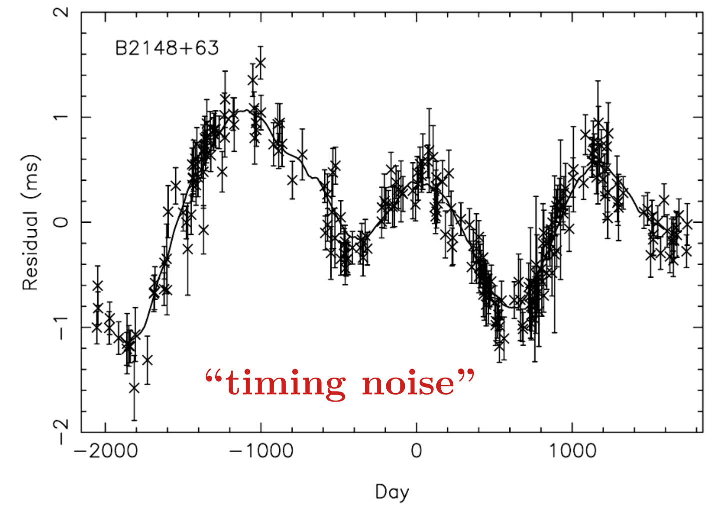
→ Posteriors for **friction parameters** in agreement with the revised estimates of friction by Graber+2018

→ **First “clue”** that the superfluid in the **outer core** is pinned before the glitch, regardless of entrainment strength in the inner crust

Pulsar "timing noise"







Antonelli, Basu, Haskell, MNRAS (2023) - arXiv:2206.10416

→ we import physical ideas from "glitch theory" to model "timing noise"

→ understanding the properties of timing noise is important also for Virgo/Ligo

NS emitting *continuous* GW is a "gravitational pulsar"

→ need to keep track of pulsar timing (including glitches and noise) for "targeted" and "narrow-band" searches of continuous GW @Ligo/Virgo

Antonelli, Basu, Haskell
Arxiv:*2206.10416*

Stochastic process for timing noise

fluctuating internal torque — fluctuating external torque

Non-superfluid component (coupled to magnetosphere)

$$x_p \dot{\Omega}_p = -\mathcal{T} - \eta_t^{\mathcal{T}} + \mathcal{T}_\infty + \eta_t^\infty$$

Superfluid component (not directly observable)

$$x_1 \dot{\Omega}_1 = \mathcal{T} + \eta_t^{\mathcal{T}}$$

fluctuating internal torque

Independent standard Wiener processes

$$\eta_t^\infty = \sigma_\infty \dot{W}_t^\infty$$

$$\eta_t^{\mathcal{T}} = \sigma_{\mathcal{T}} \dot{W}_t^{\mathcal{T}}$$

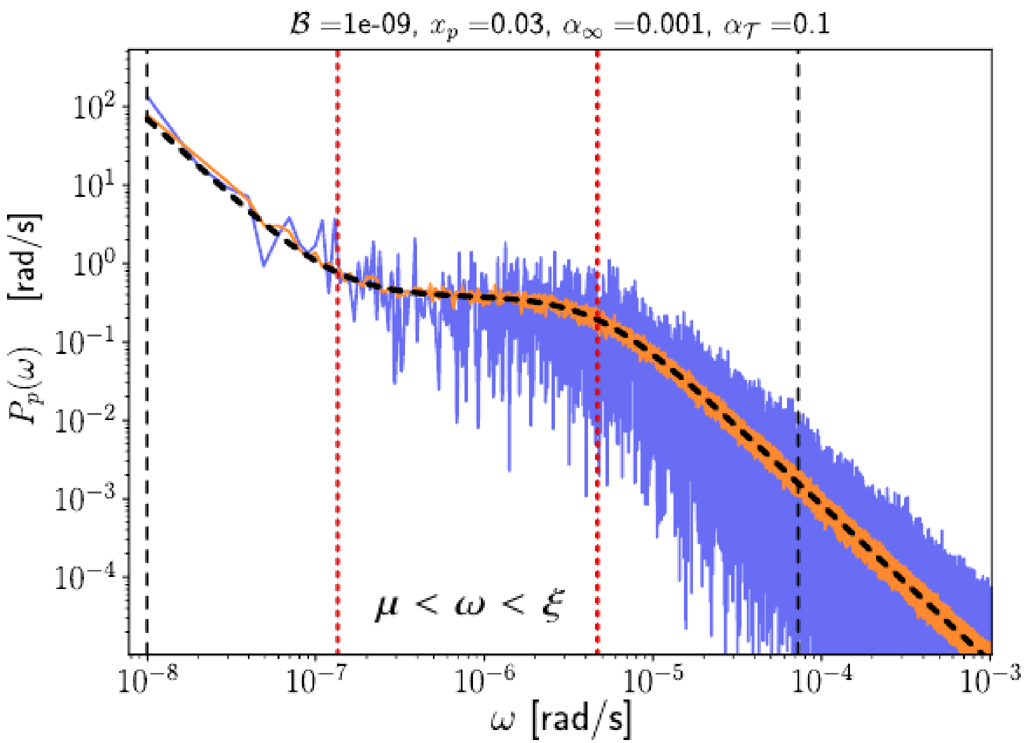
$$\langle \dot{W}_t^i \dot{W}_s^j \rangle = \delta_{ij} \delta(t-s)$$

Only **external** fluctuations: flat region in the PSD shrinks
→ pure Wiener process + "inertial" corrections

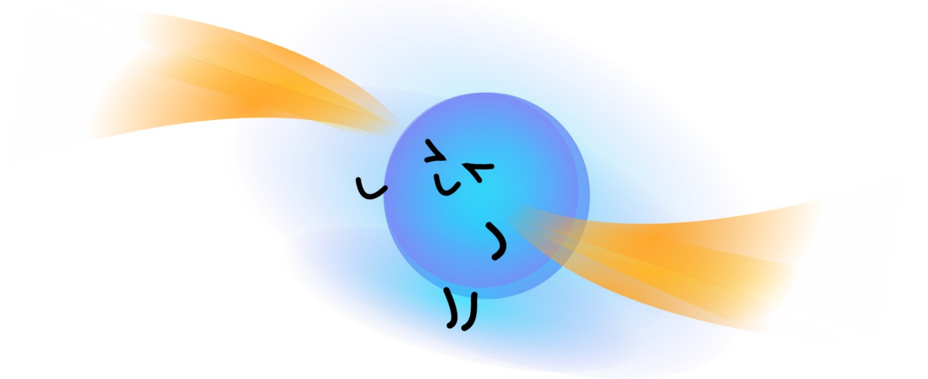
$$P_p(\omega) = \frac{\alpha_\infty^2 \, \Omega \, \dot{\Omega}}{x_p^2 \, \omega^2} \cdot \frac{\omega^2 + x_p^2 \tau^{-2}}{\omega^2 + \tau^{-2}} \approx \frac{\alpha_\infty^2 \Omega \dot{\Omega}}{\omega^2}$$

Only **internal** fluctuations: flat region extends to the origin
→ typical Lorentzian PSD (Ornstein-Uhlenbeck process)

$$P_p(\omega) = \frac{\alpha_{\mathcal{T}}^2 x_1^2 \dot{\Omega}^2}{2 x_p^2 \mathcal{B} \, \Omega} \cdot \frac{1}{\omega^2 + \tau^{-2}}$$



Final considerations



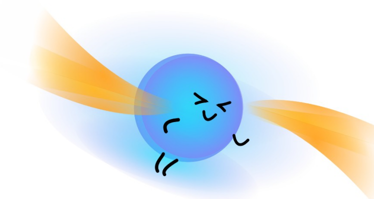
Glitches provide us with some interesting theoretical challenges:

...thank you spinning pulsar!

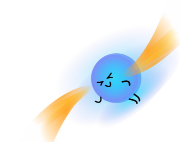
→ vortex dynamics in non-homogeneous environments
→ collective avalanche dynamics
→ how to describe pinning at the microscopic scale?
→ physics in the core: superfluid-superconductor mixture

Cross contamination between different fields, especially condensed matter, is necessary.
Some open questions:

→ role of starquakes? (can we really have quakes in a NS?)
→ role of entrainment (strong/weak? affected by disorder?)
→ better understanding of dissipation at micro/meso scale
→ collective aspects of vortex dynamics (Viscoelasticity? Hysteresis? Collective pinning?)



The most important thing: more and better observations
Improved timing techniques (and more observation time) → test current models



(some) References

Stationary “activity” test (morally: “effective moment of inertia of pinned region” > “observed activity”)
Link+1999 arXiv:9909146, Andersson+2012 arXiv:1207.0633, Chamel 2013 arXiv:1210.8177, Montoli+2020b arXiv:2012.01539
Stationary “glitch size” test (morally: “glitch from maximal critical lag” > “observed glitch amplitude”)
Antonelli+2018 arXiv:1710.05879
Beyond the stationary tests (need to solve temporal evolution $\rightarrow$ model dependent):
Ho+2015 arXiv:1703.00932 (activity+cooling: need to integrate the thermal evolution), Pizzochero+2017 arXiv:1611.10223, Montoli+2020 arXiv1809.07834 (glitch size+activity: need to integrate the superfluid reservoir evolution)

General Relativity corrections: Sourie+2017 arXiv:1607.08213 (2 rigid components), Antonelli+2018 arXiv:1710.05879 (fluid, corrections for the glitch size), Gavassino+2019 arXiv:2001.08951, arXiv:2012.10288 (fluid, corrections for the spin-up timescale)

Revisiting the starquake paradigm: Giliberti+2019 arXiv:1902.06345, arXiv:1809.08542 (continuously stratified), Reconret+2021 arXiv:2106.12604 (“starquake is not enough” argument), Bransgrove+2020 arXiv:2001.08658 (quake and null pulses in Vela 2016)

Glitch overshoot: virtually any model with more degrees of freedom than Baym+ Nature 224 1969:
Fluid models that are in the same class of Alpar+ ApJ 273 1984: Haskell+2012 arXiv:1107.5295, Antonelli+2017 arXiv:1603.02838, Graber+2018 arXiv:1804.02706 (revised friction parameters)
Models with 3 rigid components: Pizzochero+2020 arXiv:1910.00066 (mathematical condition for the overshoot)

Vela 2016 (Palfreyman+ Nature 556 2018)
Ashton+2019 arXiv:1907.01124 (agnostic Bayesian fit), Montoli+2020a arXiv:2005.01594 (theory of 3-component model, Bayesian fit of physical parameters), Sourie+2020 arXiv:2001.09668 (pinning in the core), Gügercinoğlu+2020 arXiv:2003.08724 (fit with the vortex creep model)

Beyond hydrodynamics: we are still “stealing” from Anderson&Itoh Nature 256 1975 (short but very dense paper!)
Melatos+2018 arXiv:1809.03064 (correlations in glitches), Khomenko+2018 arXiv:1801.01413, Antonelli+2020 arXiv:2007.11720 (pinning/depinning transition, hysteresis loop), Howitt+2020 arXiv:2008.00365 (point vortex simulations), Haskell+2020 arXiv:2007.02748 (quantum turbulence), Carlin+2021 arXiv:2105.13588

Entrainment coupling: crust and core

In the inner crust (lattice of ions & S-wave superfluid):

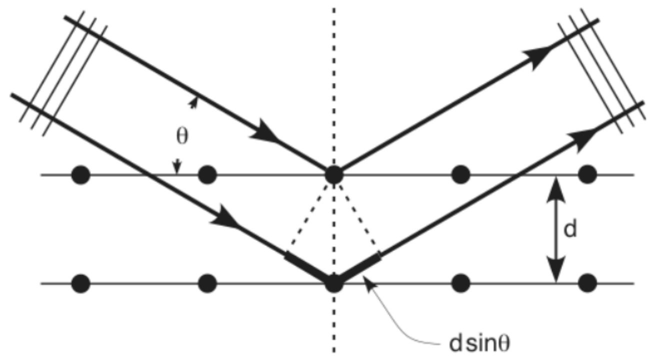
Chamel, PRC 2012

Bragg scattering by crustal lattice entrains the "free" neutrons. Non-local effect: $m^* > 1$

→ Consequence: the crustal superfluid is entrained by the normal component: **reduced mobility** of "free" neutrons is a potential problem for pulsar glitch theory.

Chamel PRL 2013, Montoli, Antonelli et al, Universe 2020

$$d \sin\theta = N\pi/k$$



In the core (S-wave superconductor & P-wave superfluid):

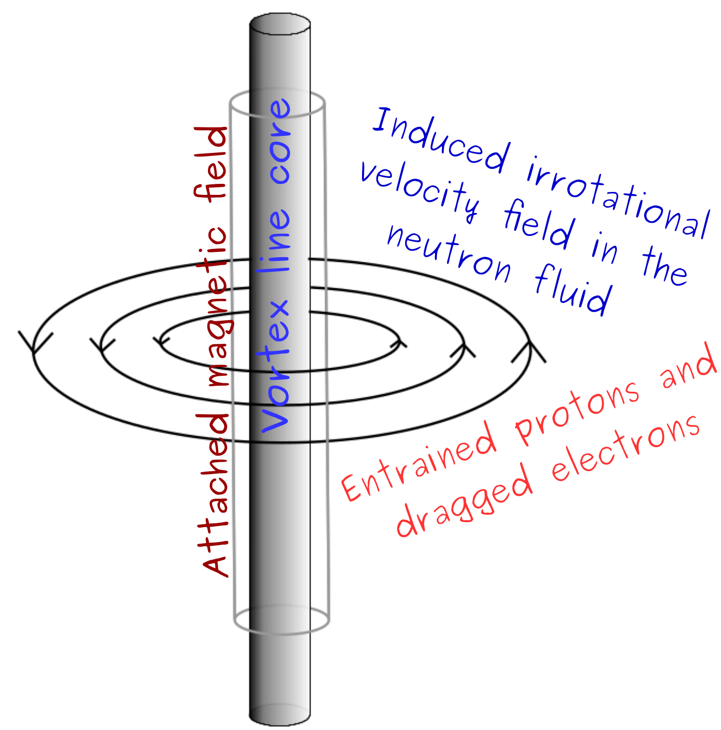
Chamel & Haensel PRC 2006

Entrainment is due to the strong interaction between protons and neutrons. Local effect: $m^* < 1$

–Consequence #1: Scattering of electrons off vortex cores: the core is coupled to the crust on the timescale of a second

Alpar et al, ApJ 1984

–Consequence #2: Dipole–dipole interaction with flux–tubes (core pinning?)



Pinning – Length scales

Core $\rightarrow$ "Abrikosov lattice" spacing between flux-tubes ~ 1000 fm

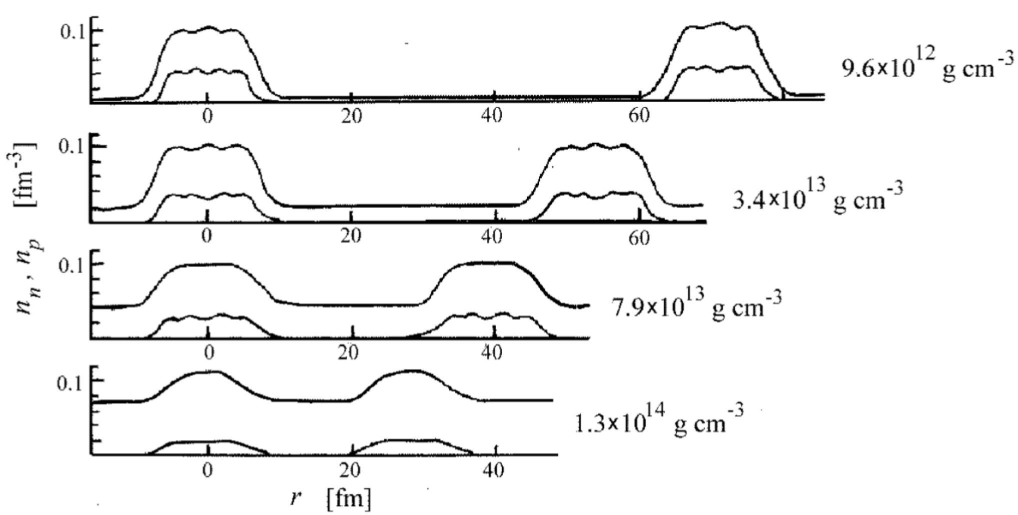
Crust $\rightarrow$ crustal lattice spacing ~ 100 – 20 fm

Vortex-*nucleus* interaction $\rightarrow$ coherence length ~ 10 – 100 fm

Vortex dynamics and vortex-*lattice* interaction $\rightarrow$ "mesoscale" (inter-vortex spacing)

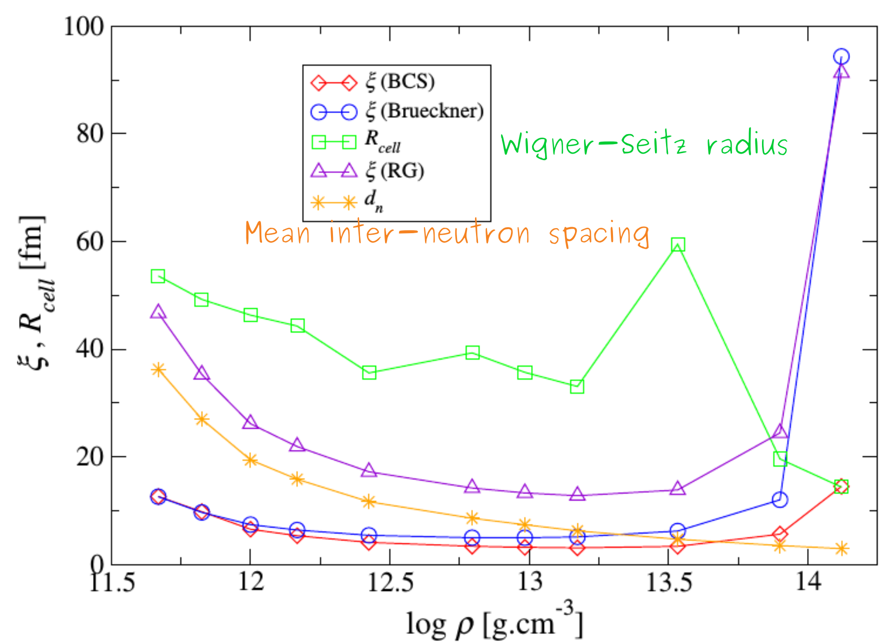
Inter-vortex spacing

$$l_v = \frac{\sqrt{\kappa P}}{2\pi} \approx 7 \times 10^{-3} \sqrt{P} \text{ cm}$$



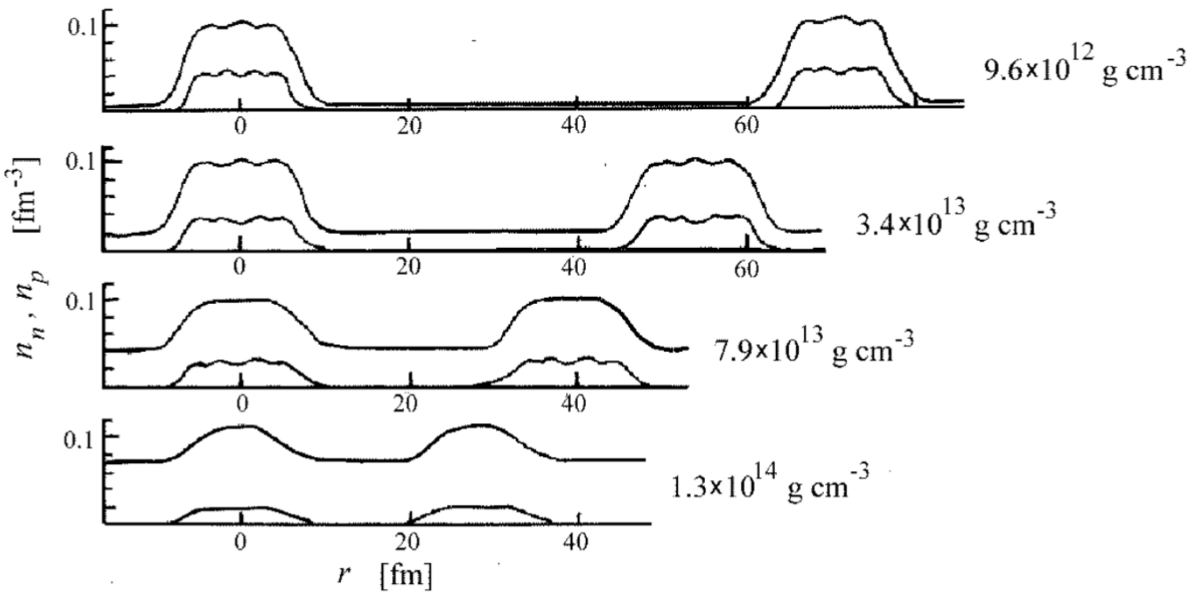
Negele & Vautherin (1973)

Neutron star matter at sub-nuclear densities



Chamel & Haensel, Living Rev.Rel. 11 (2008)

Inner crust structure



Density profiles of neutron and protons, at several average densities, along a line joining the centers of two adjacent unit cells (HF calculation of the GS in the **inner crust** with effective NN interaction, **no pairing correlations**)

Negele & Vautherin, *Neutron star matter at sub-nuclear densities* (1973)

Include **pairing correlations**: Baldo et al, *The role of superfluidity in the structure of the neutron star inner crust* (2005)

Band theory of solids: Carter et al, *Entrainment Coefficient and Effective Mass for Conduction Neutrons in Neutron Star Crust* (2006)

Heteroscedastic linear regression

Montoli+2020 arXiv:2012.01539

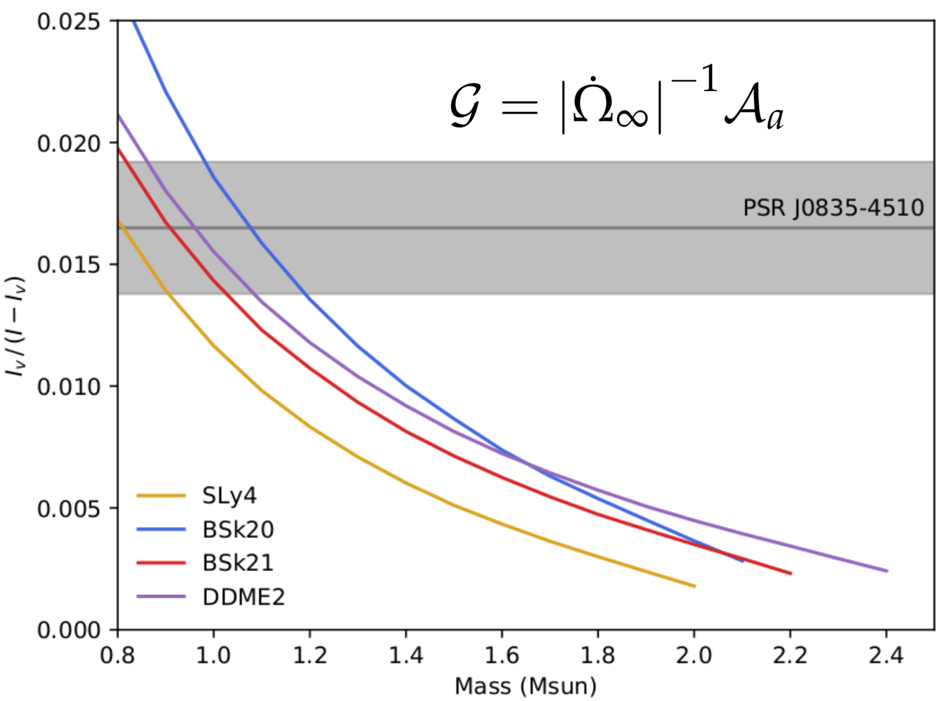
$$\Delta\Omega_i = \mathcal{A}_a \, \Delta t_i + \varepsilon_i \qquad \Omega_i = \mathcal{A}_a \, t_i + \sum_{j=1}^{i} \varepsilon_j$$

Usual regression: deviations i.i.d.

Cumulated data: deviations are not i.i.d.

Important: careful inclusion of the "intercept" may lower the estimated uncertainty.

Pulsar	$\mathcal{G}_{\mathbf{hom}}(\%)$	$\mathcal{G}_{\mathbf{het}}(\%)$
0534+2200	0.0079 ± 0.0007	0.008 ± 0.006
0537-6910	0.874 ± 0.003	0.85 ± 0.15
0631+1036	1.77 ± 0.18	2.03 ± 1.95
0835-4510	1.62 ± 0.02	1.6 ± 0.2
1341-6220	1.52 ± 0.10	1.9 ± 0.6
1740-3015	1.22 ± 0.04	1.3 ± 0.7



$$\mathcal{A}_a = \frac{\sum_i \Delta\Omega_i}{\sum_i \Delta t_i} = \frac{\sum_i \Delta\Omega_i}{t_{N_{\rm gl}-1} - t_0} \qquad \mathrm{Var}(\mathcal{A}_a) = \frac{1}{(N_{\rm gl}-1)(t_{N_{\rm gl}-1} - t_0)} \sum_i \frac{(\Delta\Omega_i - \mathcal{A}_a \Delta t_i)^2}{\Delta t_i}$$

Unbiased estimators for cumulated data