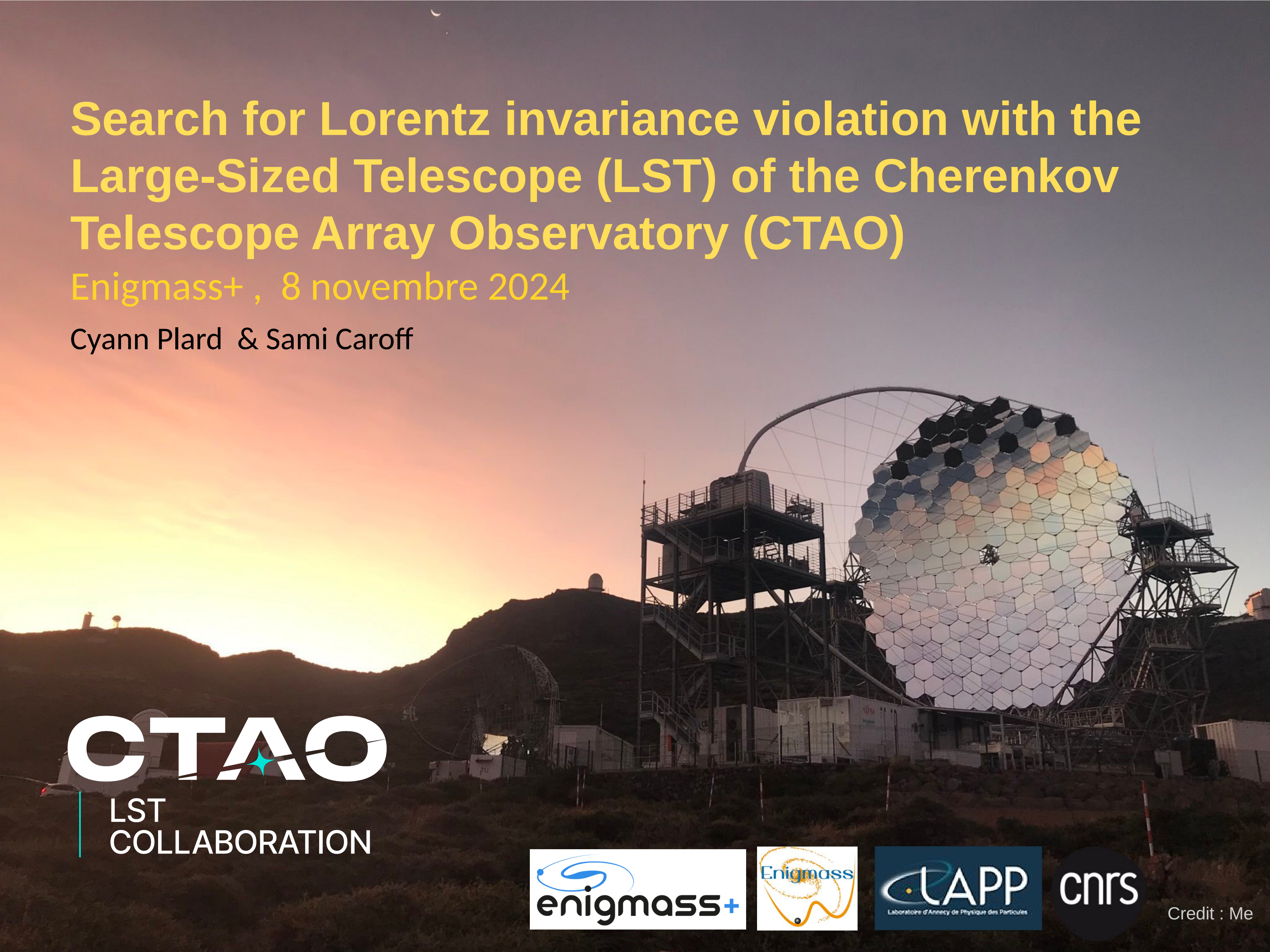
Search for Lorentz invariance violation with the Large-Sized Telescope (LST) of the Cherenkov Telescope Array Observatory (CTAO)

Enigmass+ , 8 novembre 2024

Cyann Plard & Sami Caroff



CTAO

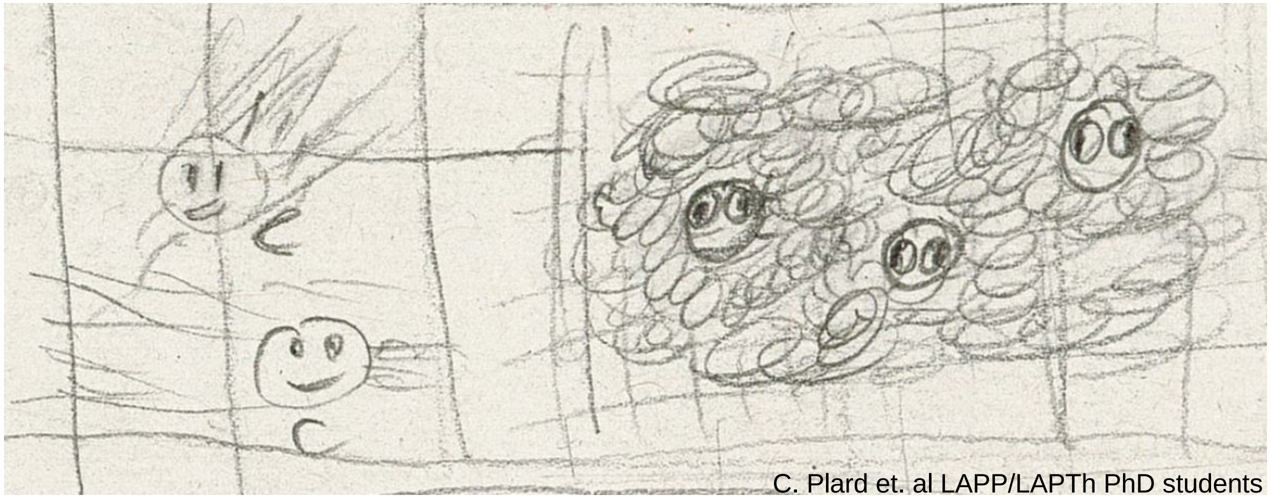
LST COLLABORATION

Credit : Me

Context

- Unification of general relativity and quantum field theory: quantum gravity
  - → effects of QG expected to appear at Planck scale $E_P \sim 10^{19}\mathrm{GeV}$

- Some quantum gravity models allow a violation of Lorentz invariance
  - → may be observable using high energetic gamma-rays

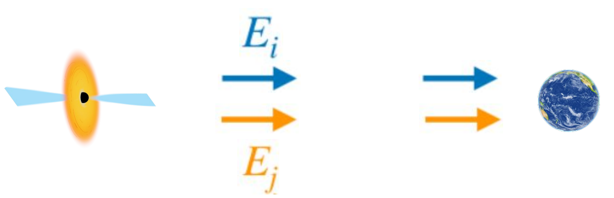

C. Plard et. al LAPP/LAPTh PhD students

Lorentz invariance violation (LIV)

Lorentz invariance: speed of light c in vacuum is energy-independent

$$E^2 = p^2c^2$$

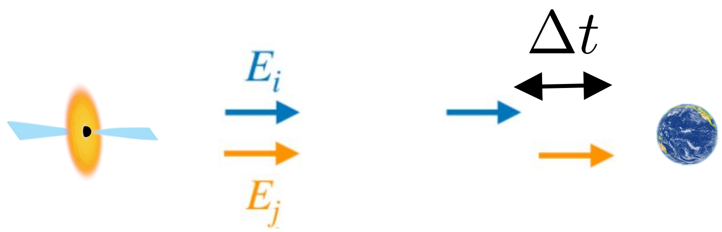
$$v_\gamma = c$$

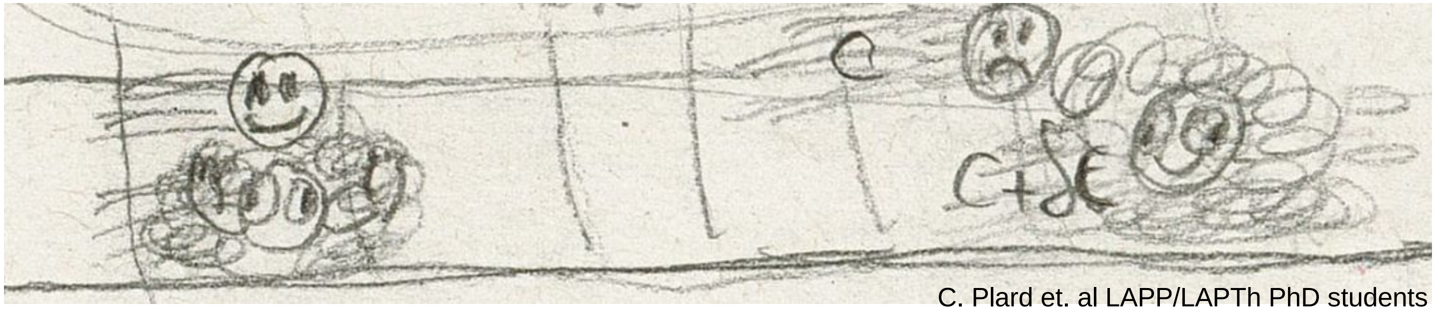


$\longrightarrow$

$$E^2 = p^2c^2 \times \left[1 \pm \sum_{n=1}^{\infty} \left(\frac{E}{E_{QG,n}}\right)^n\right]$$

$$v_\gamma(E_\gamma^n, E_{QG,n}) \neq c$$





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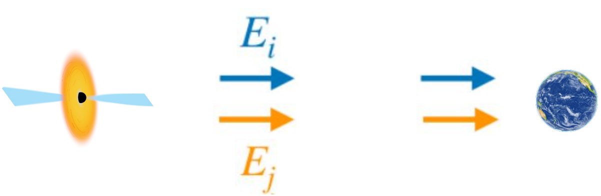

→

$$E^2 = p^2 c^2 \times \left[1 \pm \sum_{n=1}^{\infty} \left(\frac{E}{E_{QG,n}} \right)^n \right]$$

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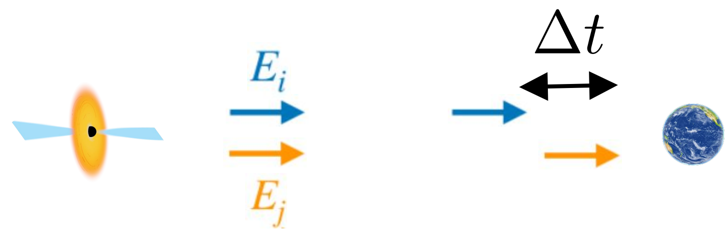


$$\lambda_n = \frac{\Delta t}{\Delta E^n \kappa_n(z)} = \pm \frac{n+1}{2 H_0 E_{QG,n}^n}$$

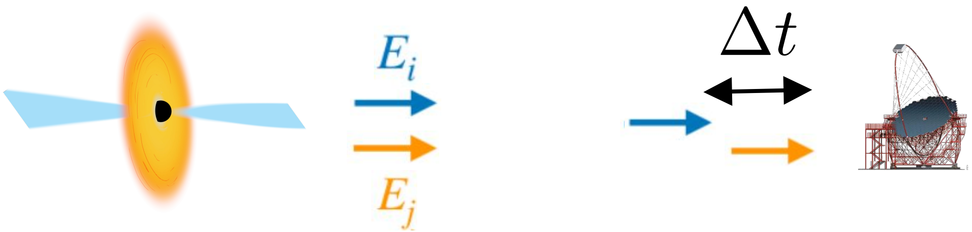
→

Search for a limit on $E_{QG,n}$ at the first order $n = 1$

Sources

- Large range of energy
- Cosmological distance

$$\Delta t_{LIV} = \pm \frac{n+1}{2} \frac{\Delta E^n}{E^n_{QG,n}} \times \kappa_n(z)$$



- Large range of energy
- Cosmological distance
- Highly variable and active source

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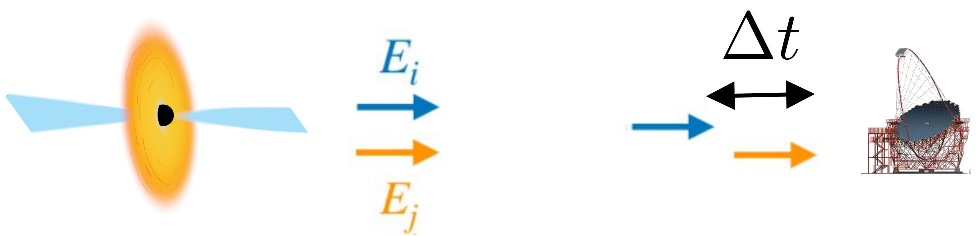

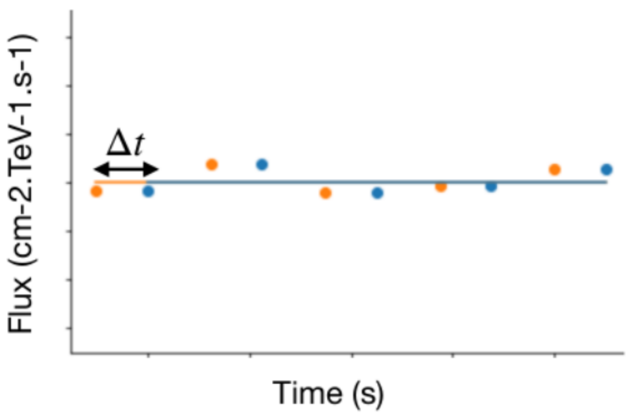

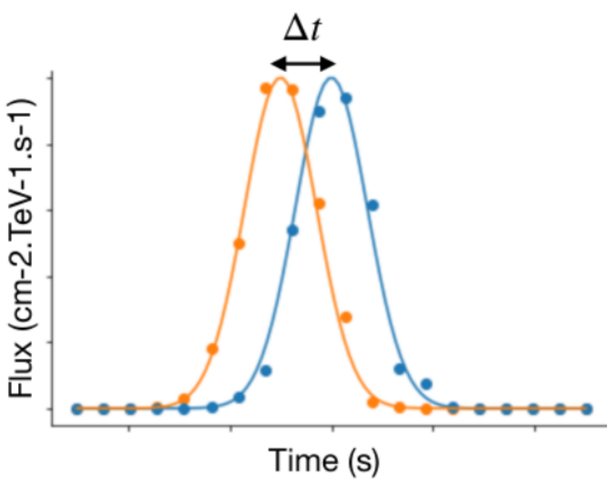

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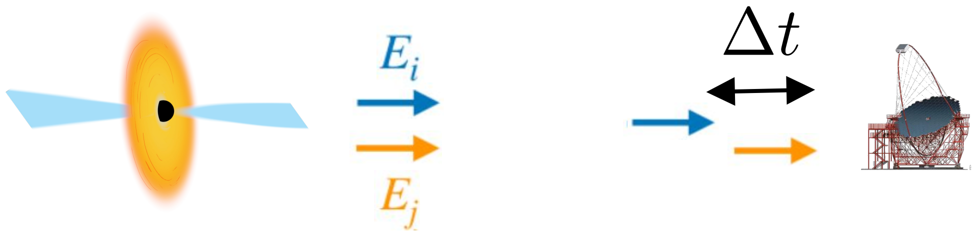
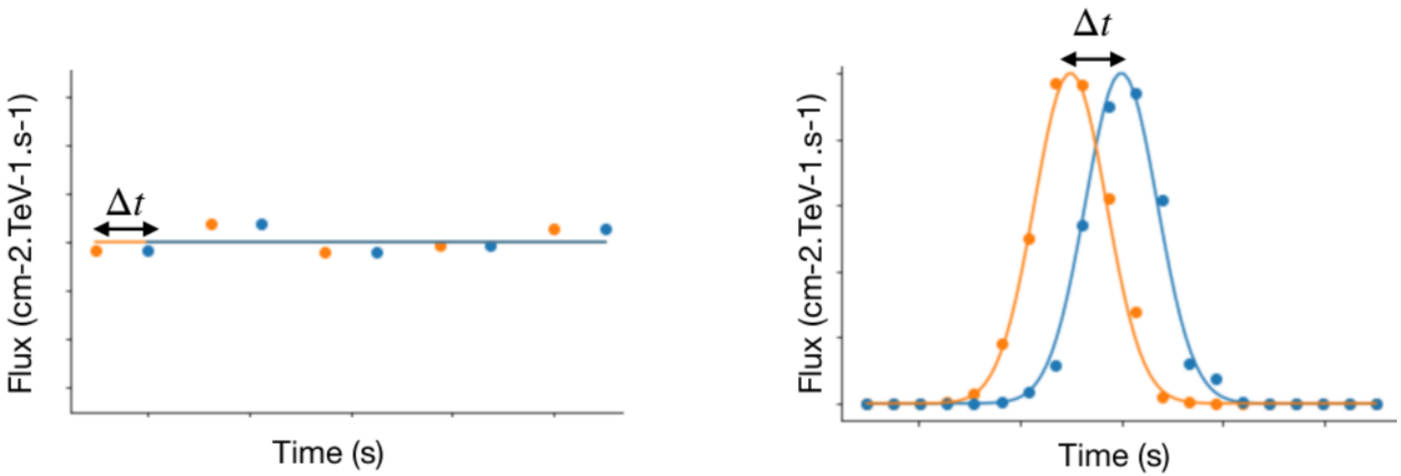


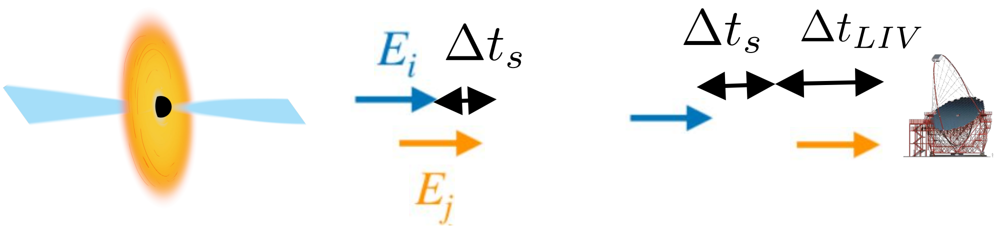
→ blazars, gamma-ray bursts, pulsars



No guarantee that photons are emitted at the same time (and region)

→ Intrinsic source delay:

$$\Delta t = \Delta t_{LIV} + (1+z)\Delta t_{source}$$



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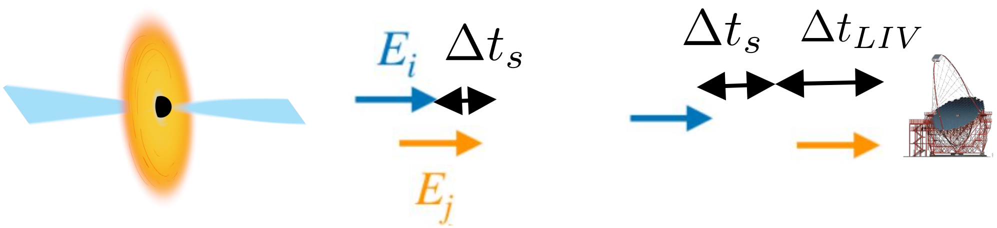

Δt_{source}	Δt_{LIV}
Redshift-independent	Redshift-dependent
Sources and flares -dependent	Sources and flares -independent

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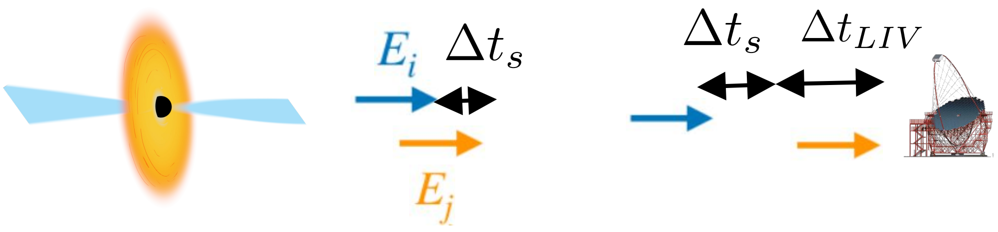


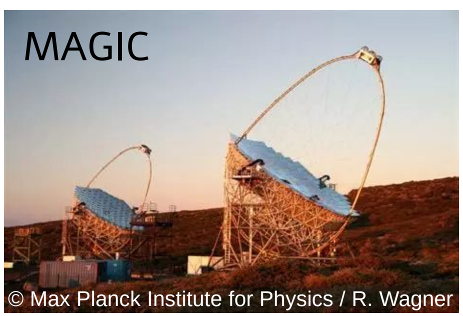
Δt_{source}	Δt_{LIV}
Redshift-independent	Redshift-dependent
Sources and flares -dependent	Sources and flares -independent

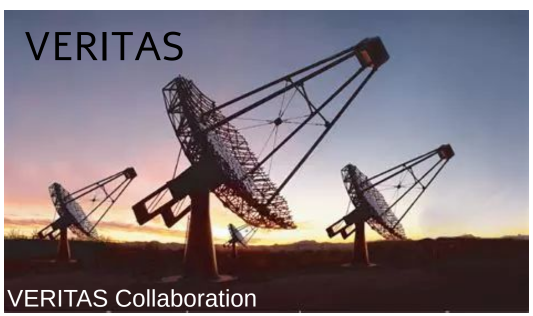
→ Combination of different flares and different (types of) sources

Combination of sources: γLIV WG

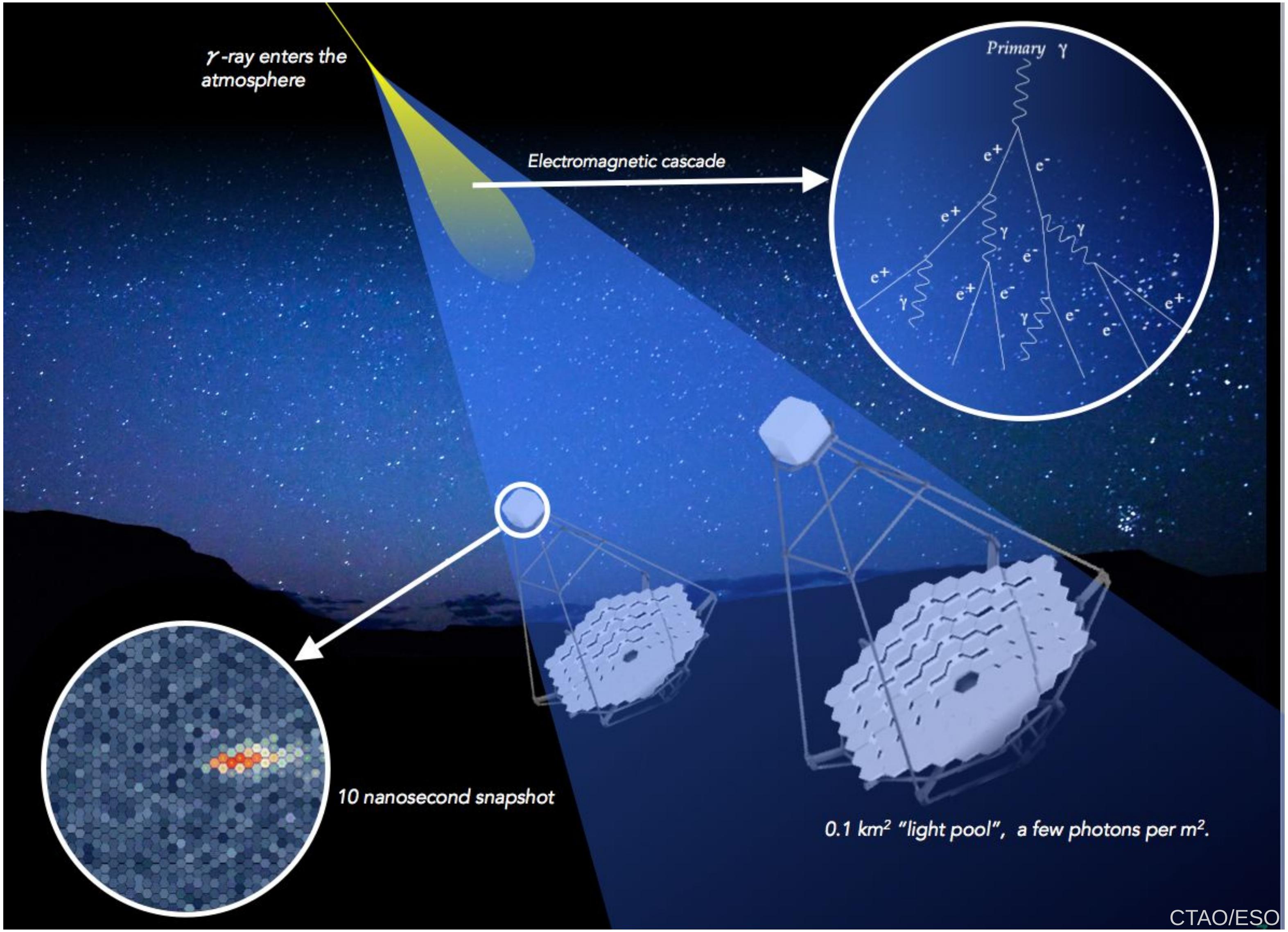
- Limits already extracted from **isolated sources** of Cherenkov telescopes observations but **data were never combined**
- *Gamma-ray LIV Working Group (γLIV WG)* between H.E.S.S., MAGIC, VERITAS and LST to **combine** observations from various experiments
- Combination of **simulations** (without LST) already performed: *Bolmont et. al., 2022, ApJ 930 75*
- This work: prototype of a global and consistent analysis on all **blazar** data of **LST-1** to combine them with the *γLIV WG* data in the future

→ beginning of a **new way** to study LIV: **population study**



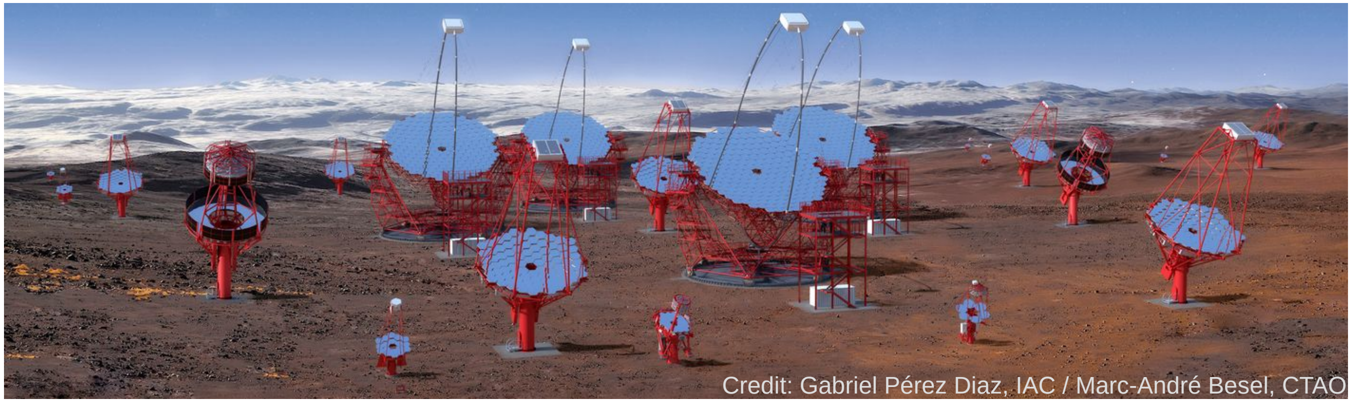


Imaging atmospheric Cherenkov telescopes



Cherenkov telescope array observatory

Tens of telescopes split into 2 geographic sites : North (La Palma, Spain) and South (Chile)



LST-1 inaugurated in 2019

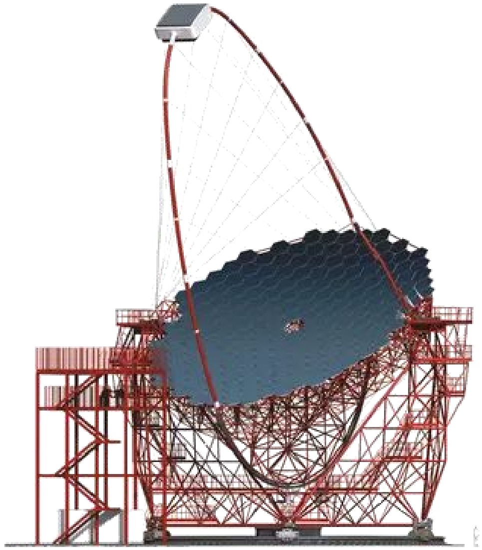
LST-4 construction site

Analysis: step 1

Several blazars data of LST-1 from January 2021 to June 2023

	M87	Mrk421	Mrk501	1ES 1959+650	BL Lac	PG 1553+113	TON0599
Redshift z	0.0043	0.03002	0.034	0.047	0.069	0.433	0.72
Observation time (hours)	9	72	66	13	44	24	9

237 hours of observations

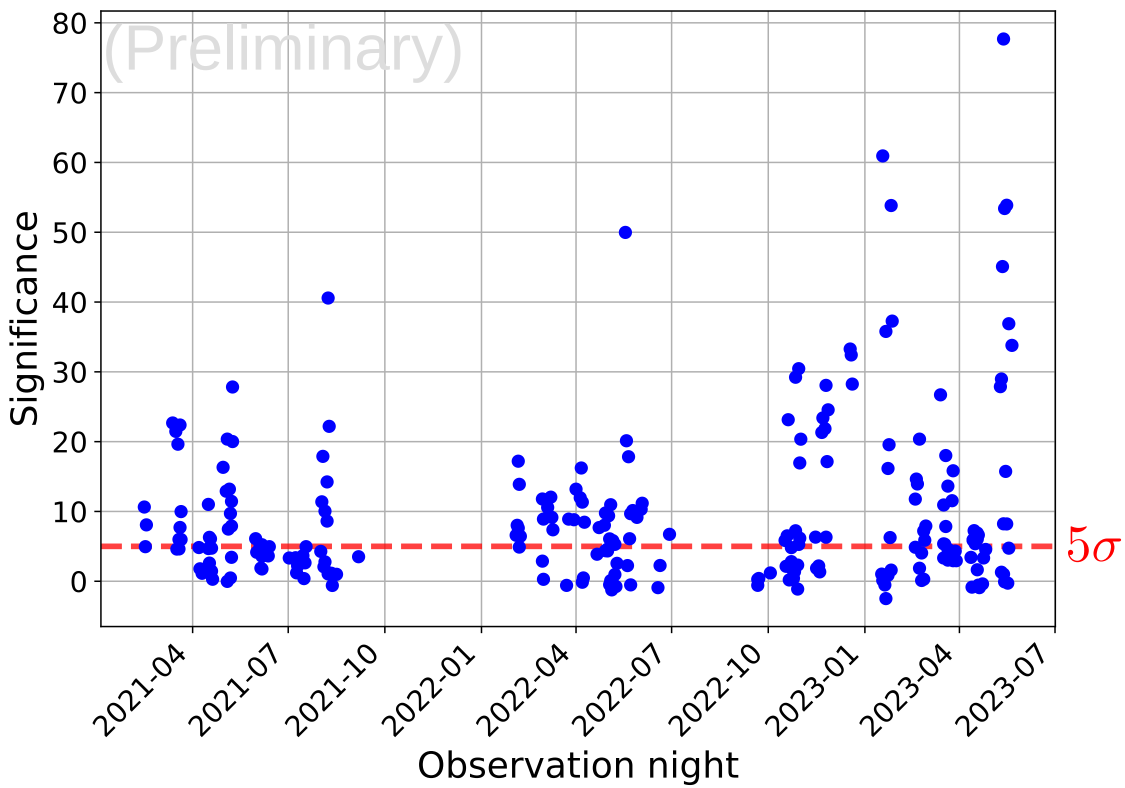


Several blazars data of LST-1 from January 2021 to June 2023

Variability test on each **significant** observation night

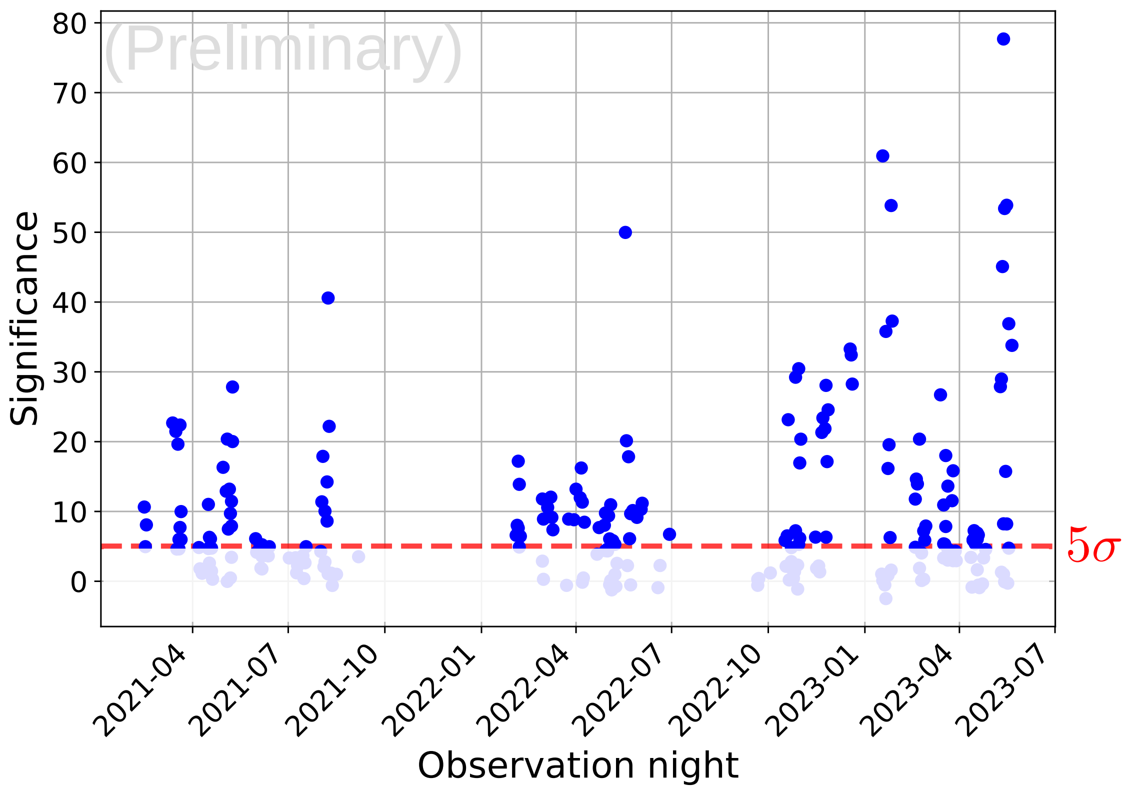
Step 2: intra-night variability

Several blazars data of LST-1 from January 2021 to June 2023



Step 2: intra-night variability

Several blazars data of LST-1 from January 2021 to June 2023



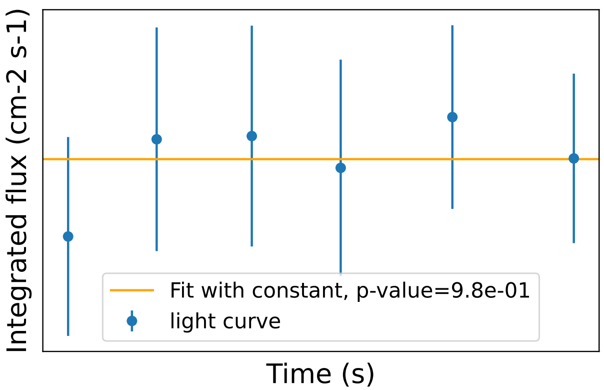
Several blazars data of LST-1 from January 2021 to June 2023

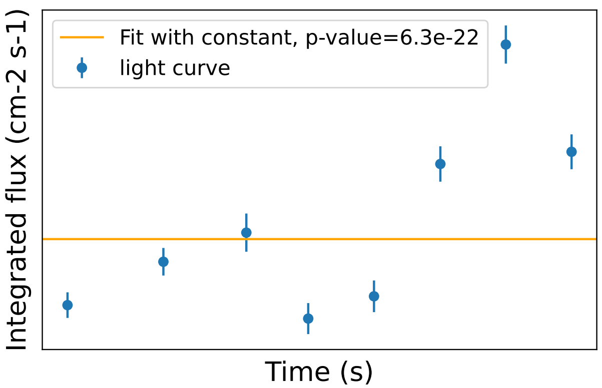
Variability test on each significant observation night

Non-variable sample

Variable sample

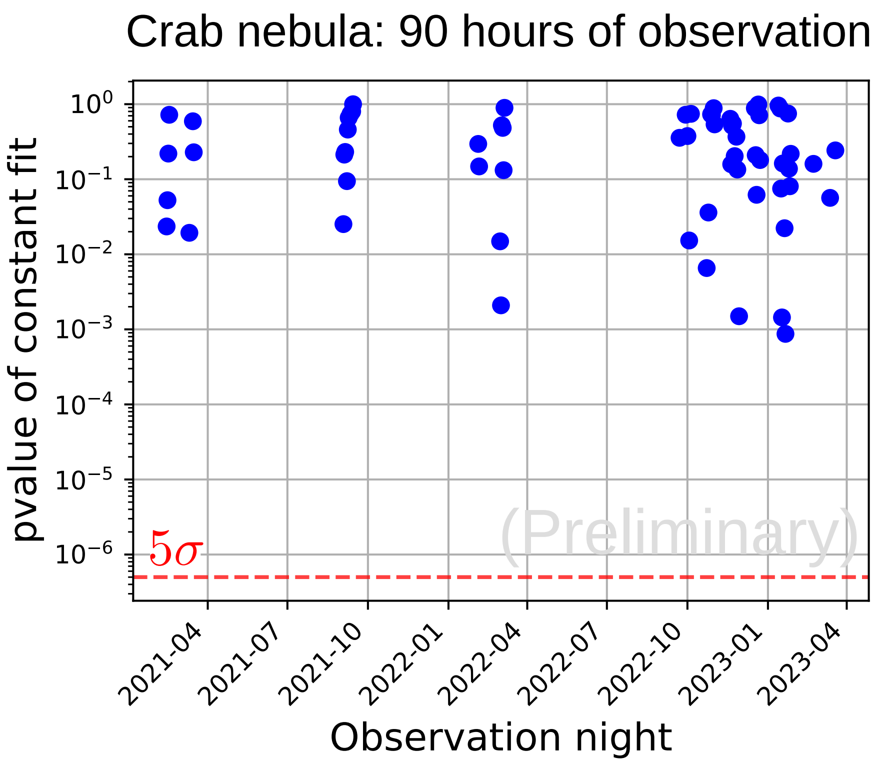
Constant fit
pvalue < 3×10^{-7}
(5σ)





Step 2: sanity check with Crab nebula

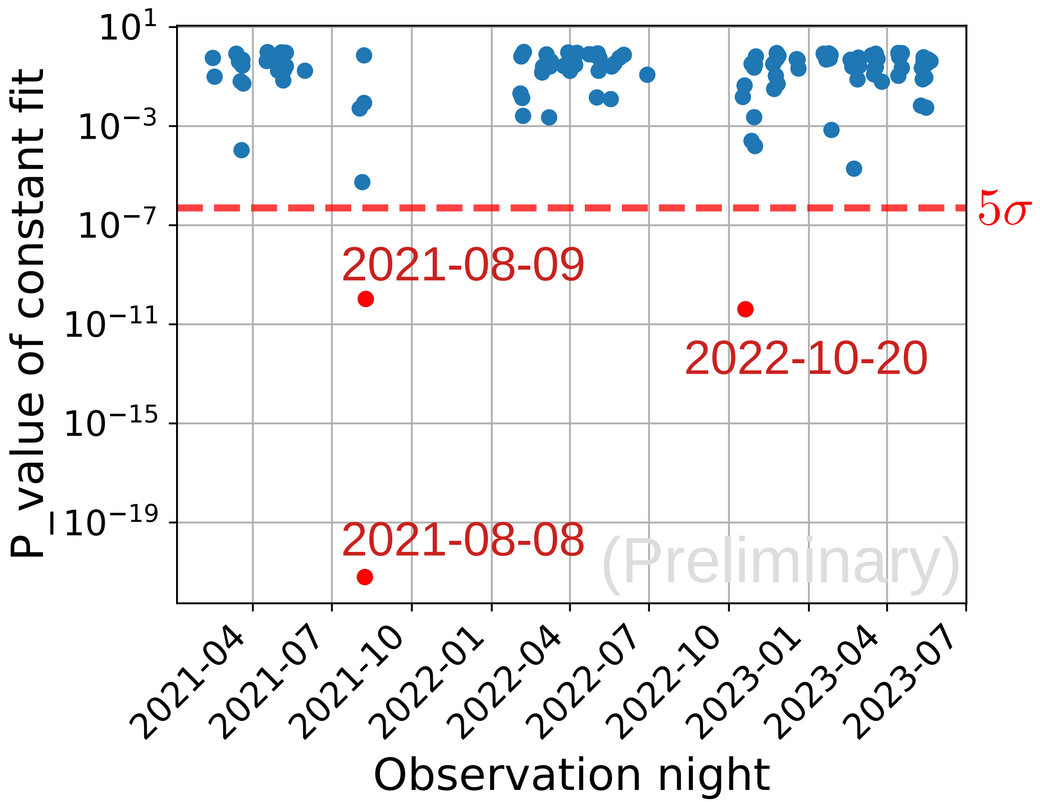
Crab nebula is a **stable source** ↔ **expecting 0** night with intra-night variability (p-value >> 10^{-7})



✅

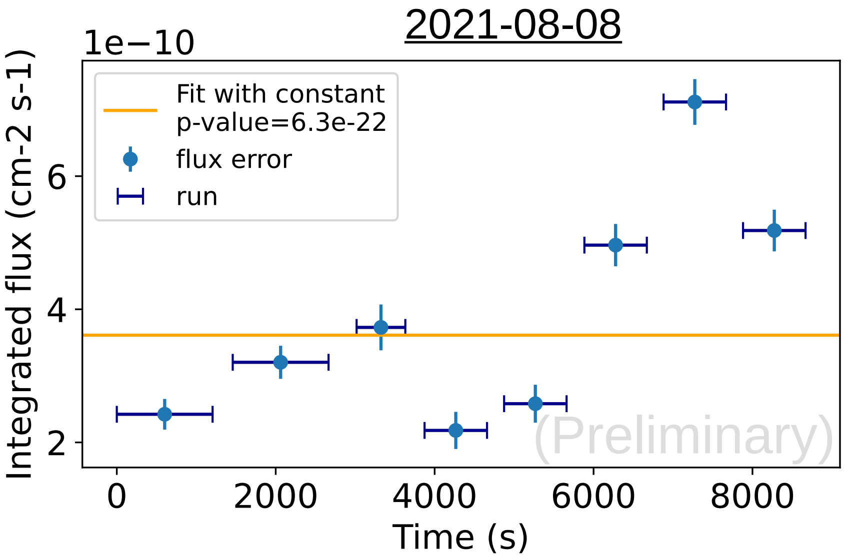
Step 2: intra-night variability

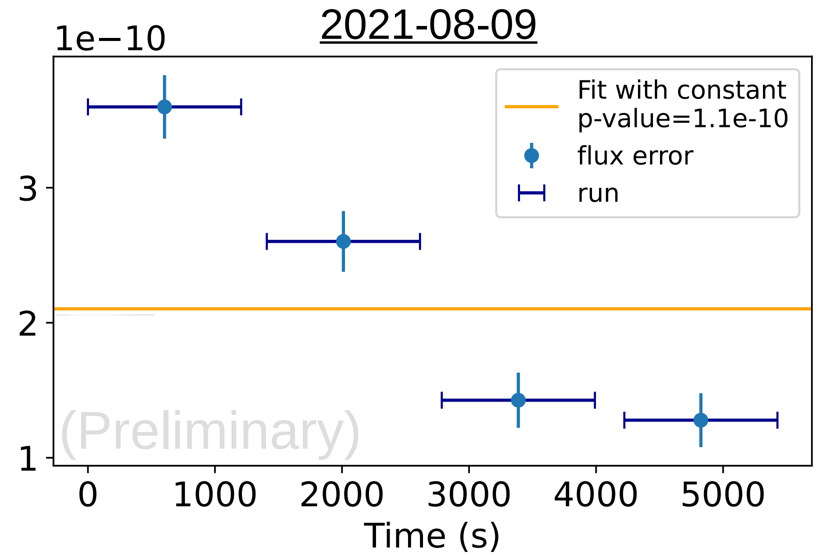
Several blazars data of LST-1 from January 2021 to June 2023

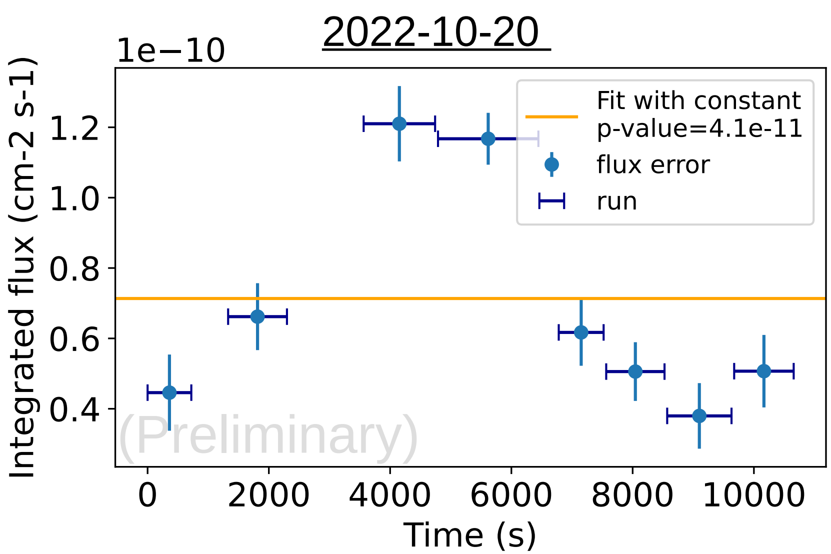


Found 1 source showing intra-night variability: **BL Lacertae** with 3 nights (~7h)

Variable lightcurves of BL Lac







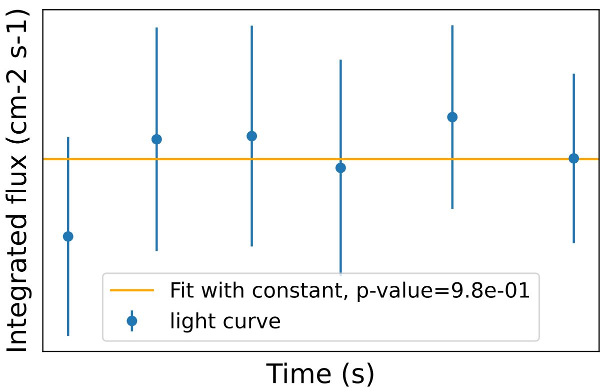
Analysis: step 3

Several blazars data of LST-1 from January 2021 to June 2023

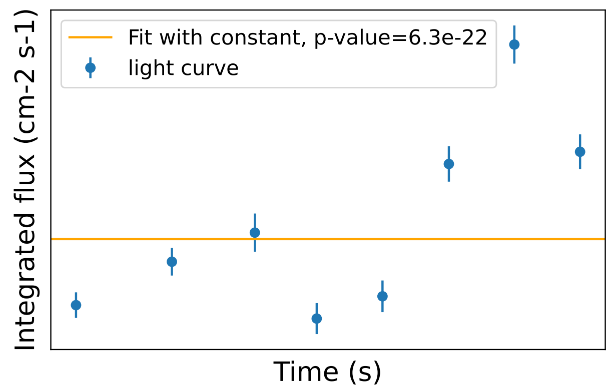
Variability test on each significant observation night

Constant fit
pvalue < 5σ

Non-variable sample



Variable sample



- is implemented in the LST-1 real time analysis
- will be available to the CTAO community as a catalog of blazars fast variability

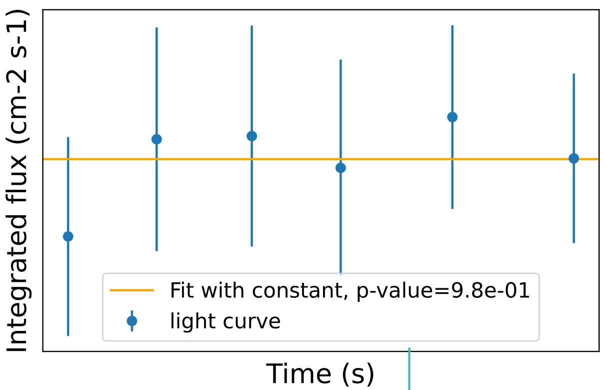
Analysis: step 4

Several blazars data of LST-1 from January 2021 to June 2023

Variability test on each significant observation night

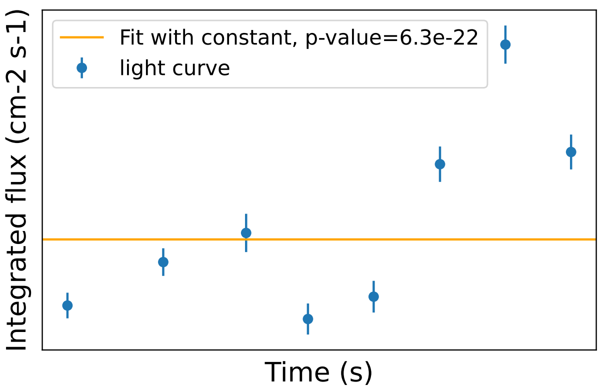
Constant fit
pvalue < 5σ

Non-variable sample



Optimization of analysis cut

Variable sample



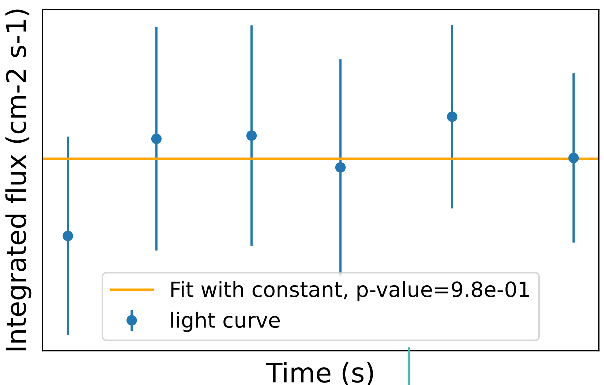
Analysis: step 5

Several blazars data of LST-1 from January 2021 to June 2023

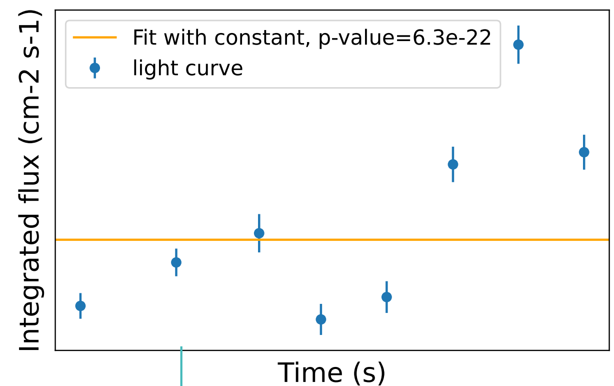
Variability test on each significant observation night

Constant fit
pvalue < 5σ

Non-variable sample



Variable sample



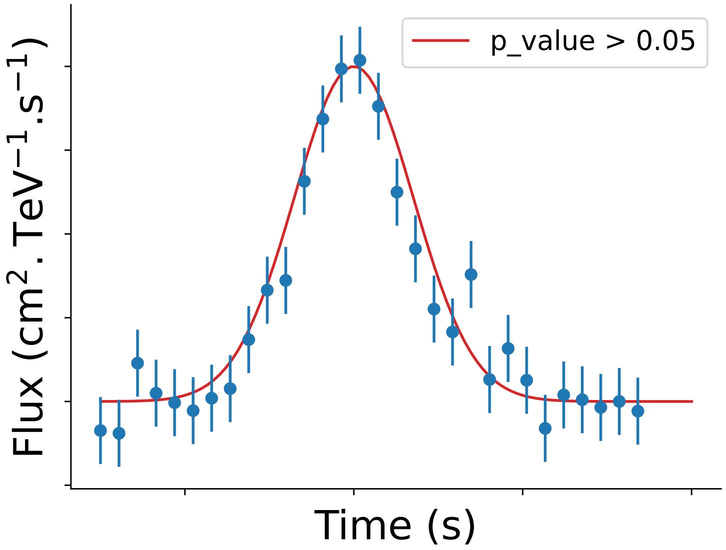
Optimization of analysis cut

Search for a variability pattern and extract sample properties (energetic and temporal distributions)

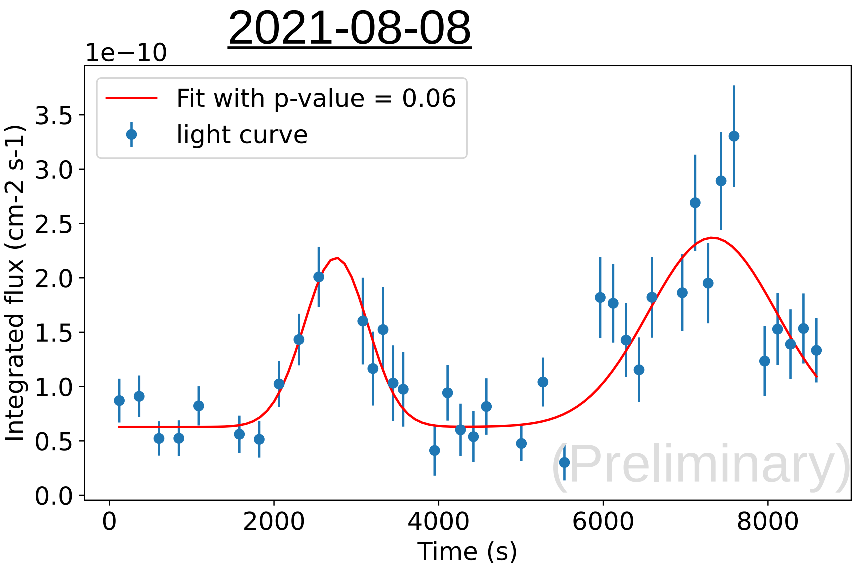
Step 5: variability pattern

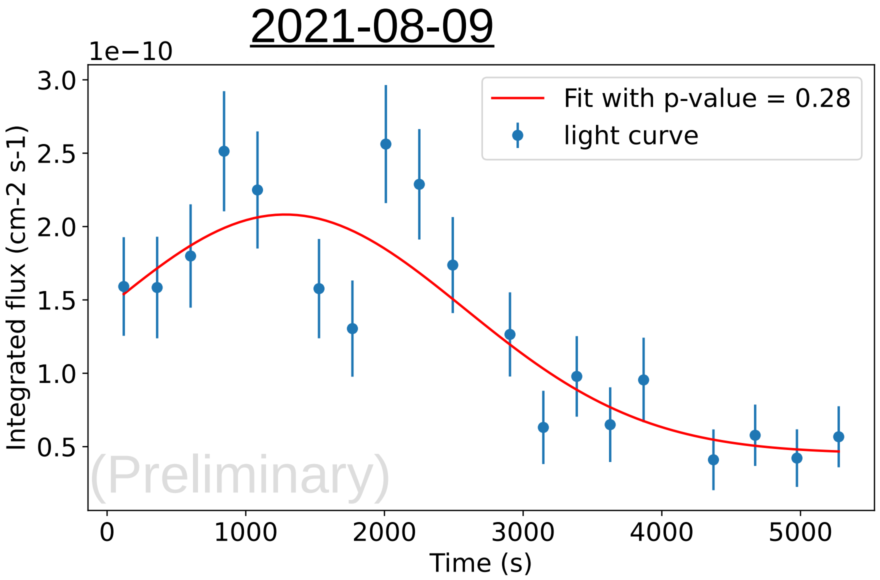
Search for a variability pattern and extract sample properties (energetic and temporal distributions)

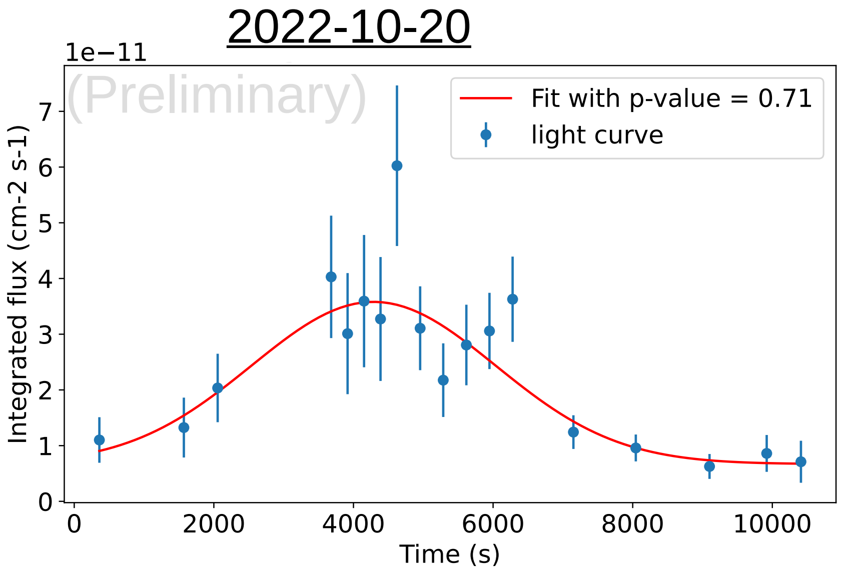
- Find a model to describe the lightcurve variability: non-rejected if p-value > 0.05 (2σ)



- $A_1 e^{\frac{(t-\mu_1)^2}{2\sigma_1^2}} + A_2 e^{\frac{(t-\mu_2)^2}{2\sigma_2^2}} + C_0$
- $A e^{\frac{(t-\mu)^2}{2\sigma^2}} + C$







Step 5: variability pattern

Search for a variability pattern and extract sample properties (energetic and temporal distributions)

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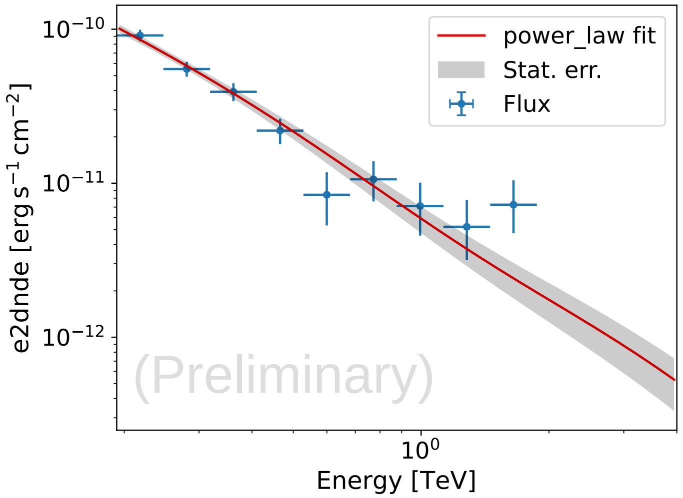
Search for a variability pattern and extract sample properties (energetic and temporal distributions)

- Find a model to describe the lightcurve variability: non-rejected if p-value > 0.05 (2σ)
- Find a model to extract the parameters of the energetic distribution

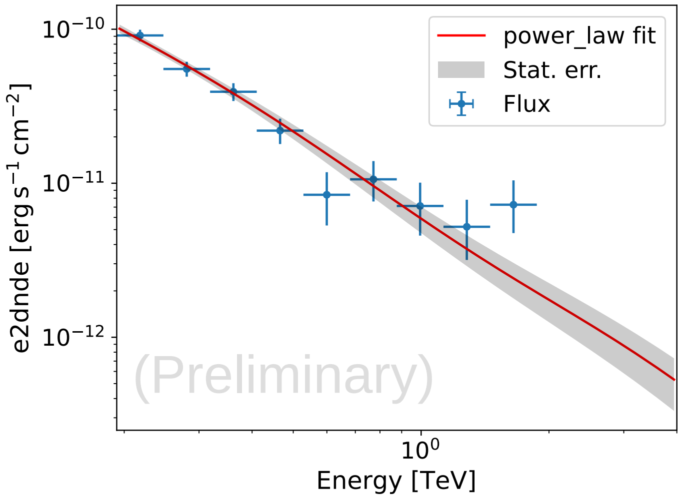


$$A_0\left(\frac{E}{E_0}\right)^{-\alpha}$$

Step 5: variability pattern

Search for a variability pattern and extract sample properties (energetic and temporal distributions)

- Find a model to describe the lightcurve variability: non-rejected if p-value > 0.05 (2σ)
- Find a model to extract the parameters of the energetic distribution
- Check: no significant time-variation of energetic distribution ✅



$$A_0\left(\frac{E}{E_0}\right)^{-\alpha}$$

time-independent index ?

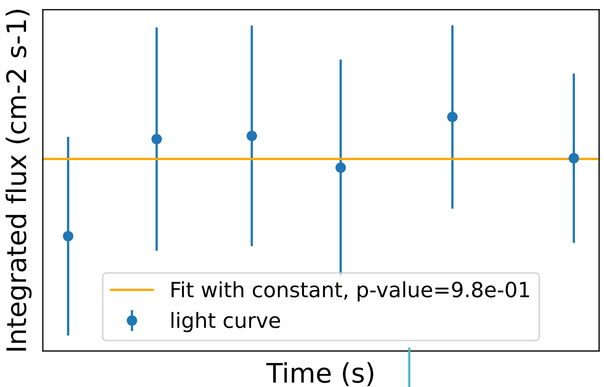
Analysis: step 5

Several blazars data of LST-1 from January 2021 to June 2023

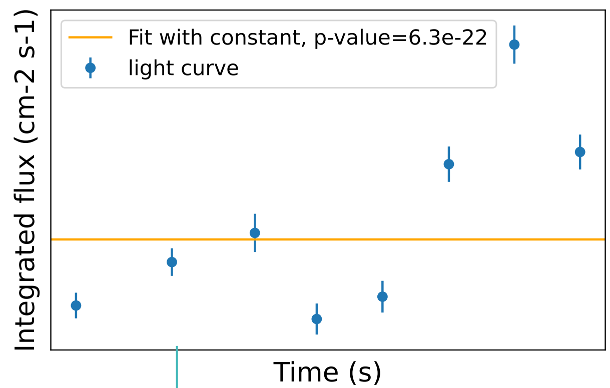
Variability test on each significant observation night

Constant fit
pvalue < 5σ

Non-variable sample



Variable sample



Optimization of analysis cut

Search for a variability pattern and extract sample properties (energetic and temporal distributions)

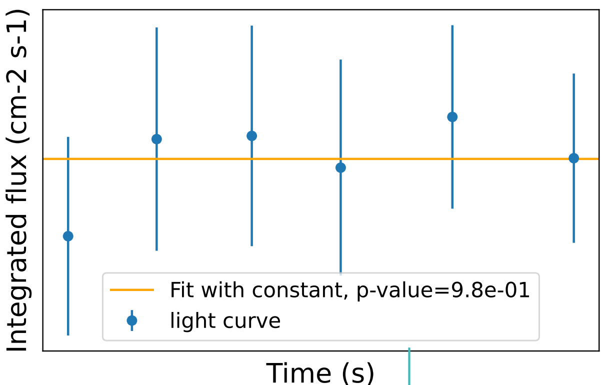
Analysis: step 6

Several blazars data of LST-1 from January 2021 to June 2023

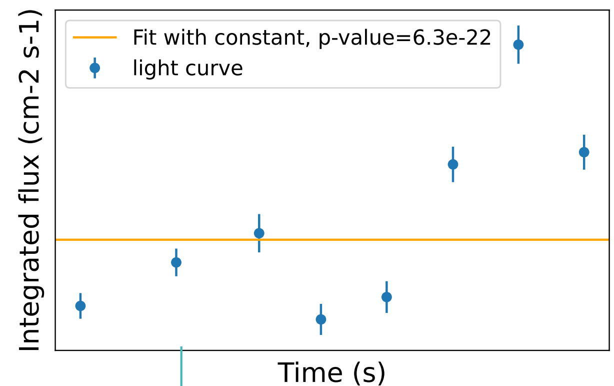
Variability test on each significant observation night

Constant fit
pvalue < 5σ

Non-variable sample



Variable sample



Optimization of analysis cut

Search for a variability pattern and extract sample properties (energetic and temporal distributions)

LIV analysis

LIV analysis

↓

LIVelihood software

- Created in the context of the γLIV working group
- Uses the **likelihood method**:
  λ_n is a free parameter that minimizes the likelihood function

for one source (or night): $$\mathcal{L}_S(\lambda_n) = -\sum_{\text{event } \mathbf{i}} \log\left(\frac{dP(E_{R,\mathbf{i}}, t_{\mathbf{i}}; \lambda_n)}{dE_R dt}\right)$$

for combination: $$\mathcal{L}_{\text{comb}}(\lambda_n) = \sum_{\text{source } S} \mathcal{L}_S(\lambda_n)$$

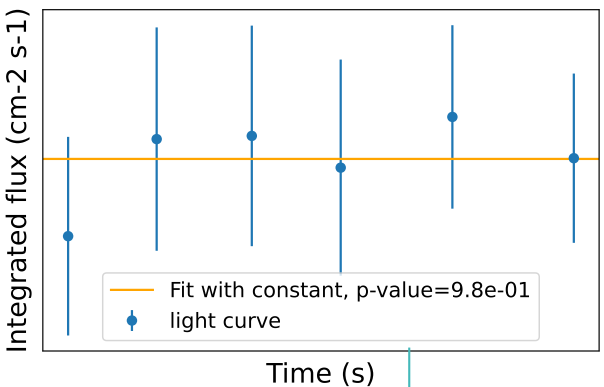
Analysis: step 7

Several blazars data of LST-1 from January 2021 to June 2023

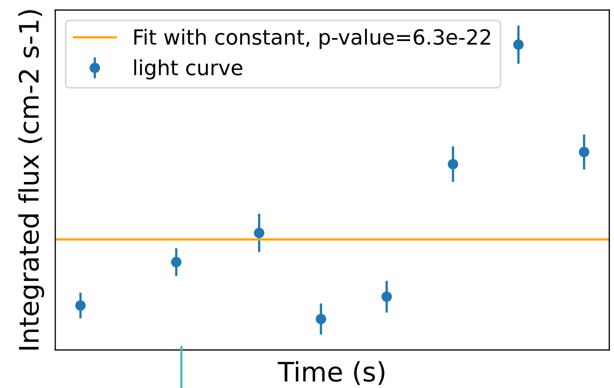
Variability test on each significant observation night

Constant fit
pvalue < 5σ

Non-variable sample



Variable sample



Optimization of analysis cut

Search for a variability pattern and extract sample properties (energetic and temporal distributions)

LIV analysis

Analysis: step 7

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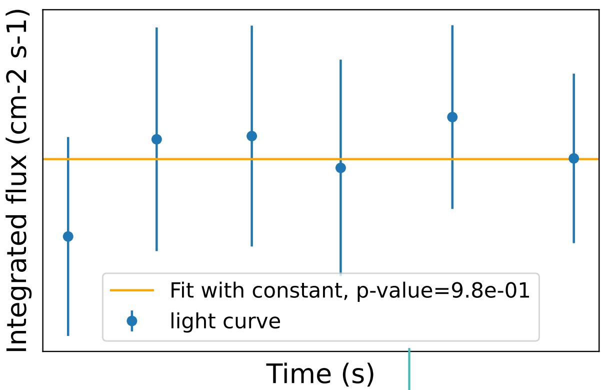
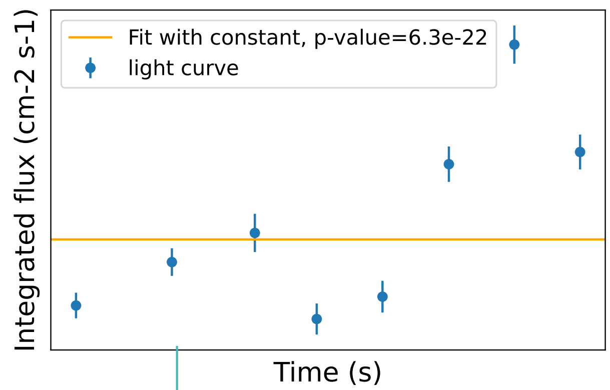
Non-variable sample



Variable sample



Optimization of analysis cut

Search for a variability pattern and extract sample properties (energetic and temporal distributions)

LIV analysis

Sample of simulations for calibration

Step 7: Likelihood for calibration

Lag λ_n : free parameter, can be shared between sources with different redshifts

For one night: $$\mathcal{L}(\lambda_n) = -\sum_{\text{event i}} \log\left(\frac{dP(E_{R,\text{i}}, t_\text{i}; \lambda_n)}{dE_R dt}\right)$$

with $$\frac{dP}{dE_R dt} = W_\mathbf{s} \frac{\int \mathrm{E_{ff}A}(E_T, t)\mathrm{MM}(E_T, E_R) \times \mathrm{F_\mathbf{s}}(E_T, t; \lambda_n) dE_T}{N'_\mathbf{s}}$$

$$+ \sum_{\mathbf{k}=\{\mathbf{b},\ \mathbf{h}\}} W_\mathbf{k} \frac{\int \mathrm{E_{ff}A}(E_T, t)\mathrm{MM}(E_T, E_R) \times \mathrm{F_\mathbf{k}}(E_T) dE_T}{N'_\mathbf{k}}$$

Step 7: Likelihood for calibration

Lag λ_n : free parameter, can be shared between sources with different redshifts

For one night: $$\mathcal{L}(\lambda_n) = -\sum_{\text{event i}} \log\left(\frac{dP(E_{R,\text{i}}, t_\text{i}; \lambda_n)}{dE_R dt}\right)$$

with $$\frac{dP}{dE_R dt} = W_\mathbf{s} \frac{\int \mathrm{E_{ff}A}(E_T, t)\mathrm{MM}(E_T, E_R) \times \mathrm{F}_\mathbf{s}(E_T, t; \lambda_n) dE_T}{N'_\mathbf{s}}$$

Signal

$$+ \sum_{\mathbf{k}=\{\mathbf{b},\ \mathbf{h}\}} W_\mathbf{k} \frac{\int \mathrm{E_{ff}A}(E_T, t)\mathrm{MM}(E_T, E_R) \times \mathrm{F}_\mathbf{k}(E_T) dE_T}{N'_\mathbf{k}}$$

Backgrounds k: baseline and hadrons

Step 7: Likelihood for calibration

Lag λ_n : free parameter, can be shared between sources with different redshifts

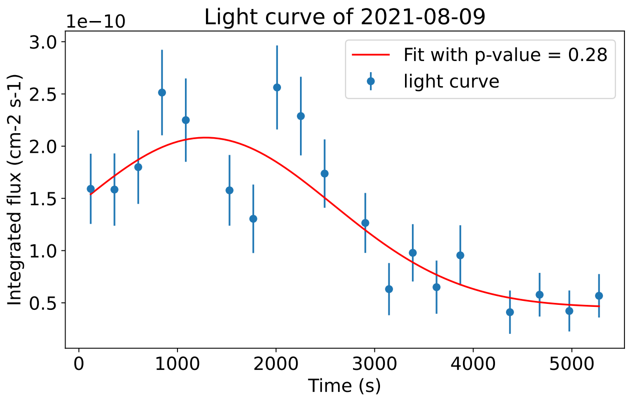
For one night:
$$\mathcal{L}(\lambda_n) = -\sum_{\text{event i}} \log\left(\frac{dP(E_{R,\mathrm{i}}, t_\mathrm{i}; \lambda_n)}{dE_R dt}\right)$$

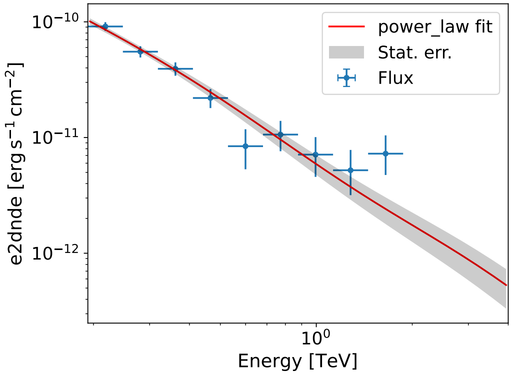
with
$$\frac{dP}{dE_R dt} = W_\mathbf{s} \frac{\int \mathrm{E_{ff}A}(E_T, t)\mathrm{MM}(E_T, E_R) \times \mathrm{F}_\mathbf{s}(E_T, t; \lambda_n) dE_T}{N'_\mathbf{s}}$$

Instrumental response functions

$$+ \sum_{\mathbf{k}=\{\mathbf{b},\ \mathbf{h}\}} W_\mathbf{k} \frac{\int \mathrm{E_{ff}A}(E_T, t)\mathrm{MM}(E_T, E_R) \times \mathrm{F}_\mathbf{k}(E_T) dE_T}{N'_\mathbf{k}}$$

Step 7: Likelihood for calibration





with $$\frac{dP}{dE_R dt} = W_{\mathbf{s}} \frac{\int \mathrm{E_{ff}A}(E_T, t)\mathrm{MM}(E_T, E_R) \times \mathrm{F}_{\mathbf{s}}(E_T, t; \lambda_n) dE_T}{N'_{\mathbf{s}}}$$

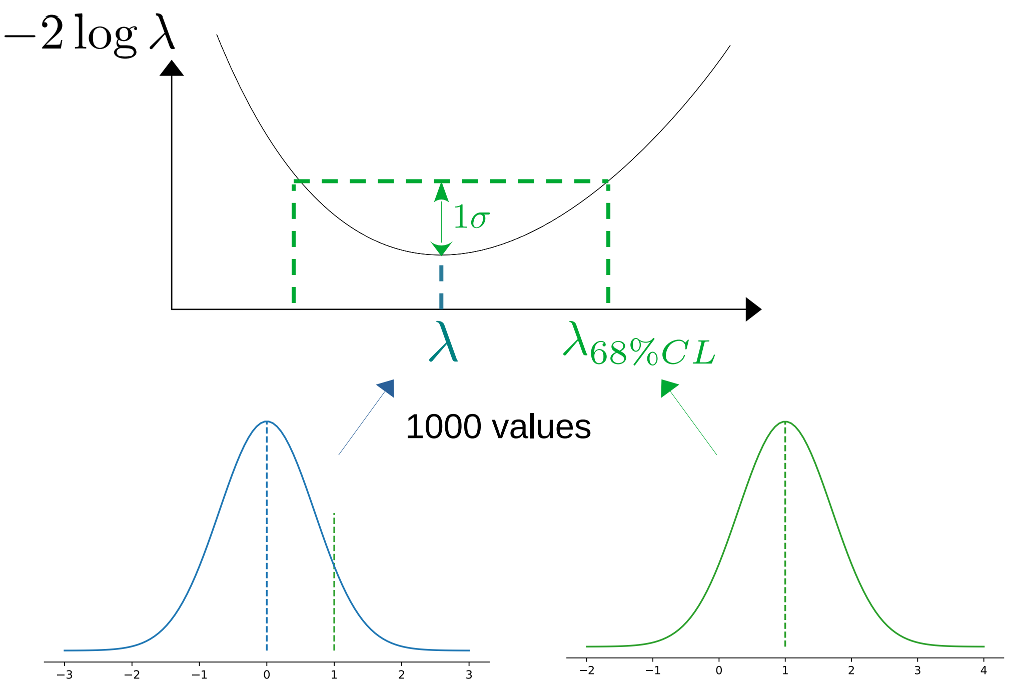
Lightcurve x spectra

$$+ \sum_{\mathbf{k}=\{\mathbf{b},\ \mathbf{h}\}} W_{\mathbf{k}} \frac{\int \mathrm{E_{ff}A}(E_T, t)\mathrm{MM}(E_T, E_R) \times \mathrm{F}_{\mathbf{k}}(E_T) dE_T}{N'_{\mathbf{k}}}$$

Step 7: Calibration

Simulations sample for calibration

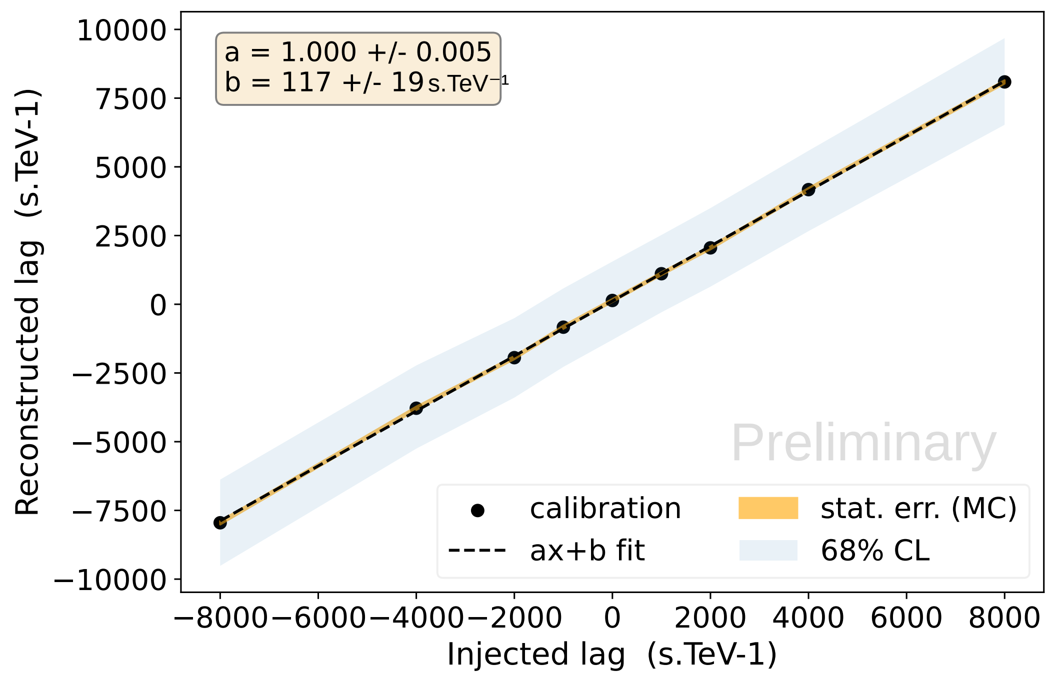
- Perform 1000 dataset simulations



Simulations sample for calibration

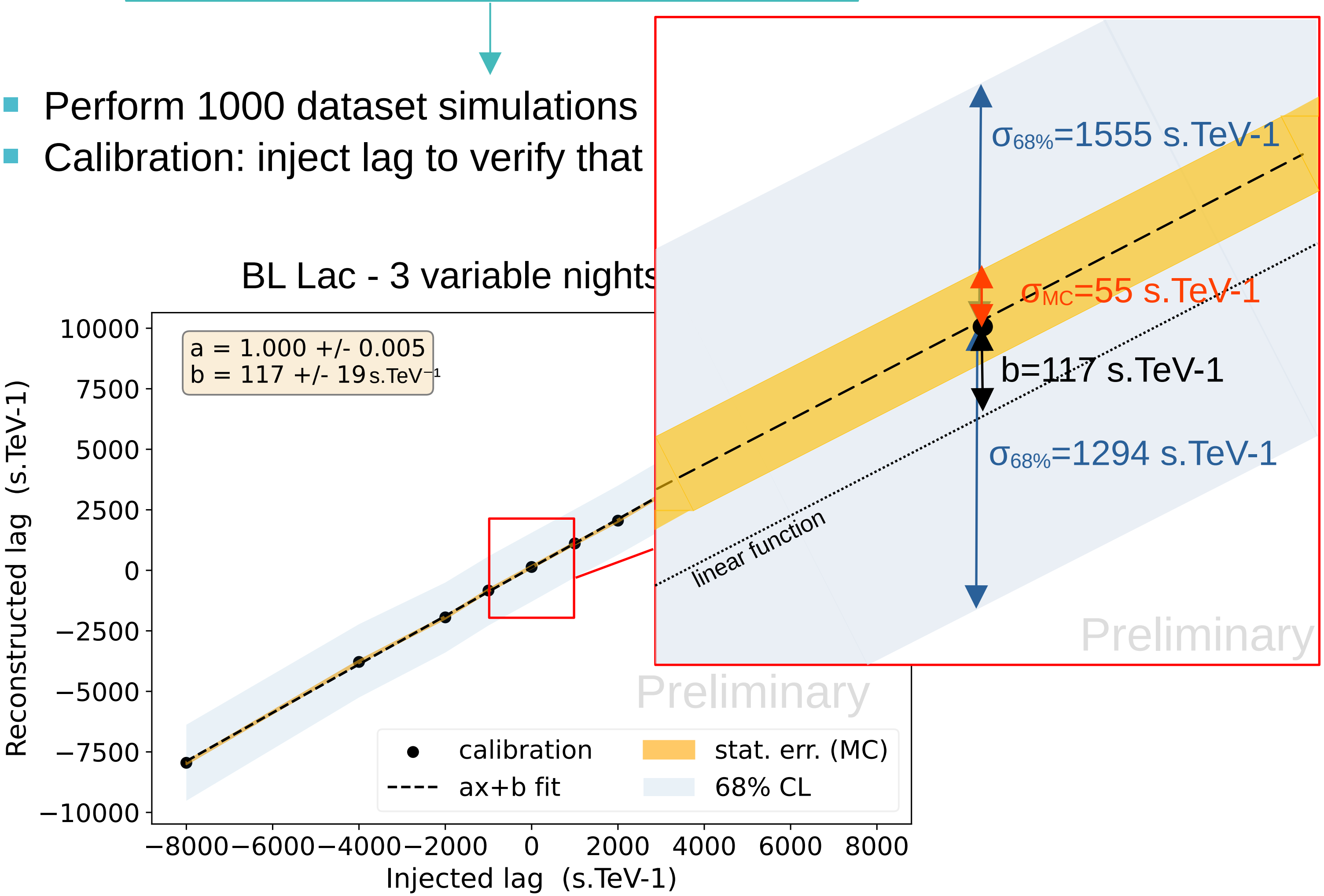
- Perform 1000 dataset simulations
- Calibration: inject lag to verify that LIVelihood reconstructs it well

BL Lac - 3 variable nights combined



Simulations sample for calibration

- Perform 1000 dataset simulations
- Calibration: inject lag to verify that



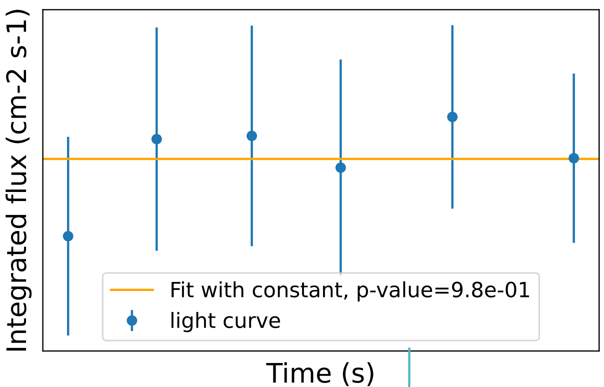
Analysis: step 7

Several blazars data of LST-1 from January 2021 to June 2023

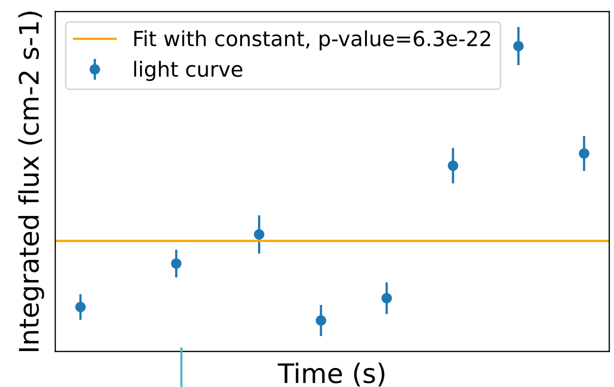
Variability test on each significant observation night

Constant fit pvalue < 5σ

Non-variable sample



Variable sample



Optimization of analysis cut

Search for a variability pattern and extract sample properties (energetic and temporal distributions)

LIV analysis

Sample of simulations for calibration

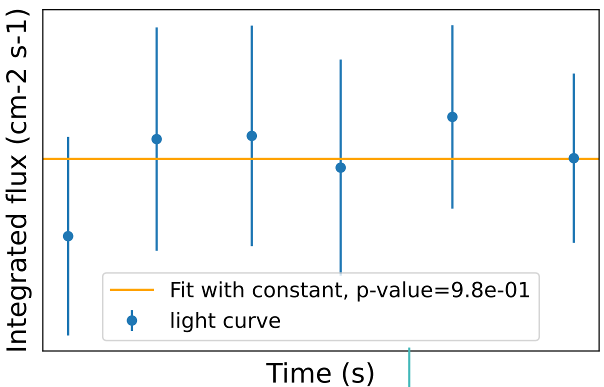
Several blazars data of LST-1 from January 2021 to June 2023

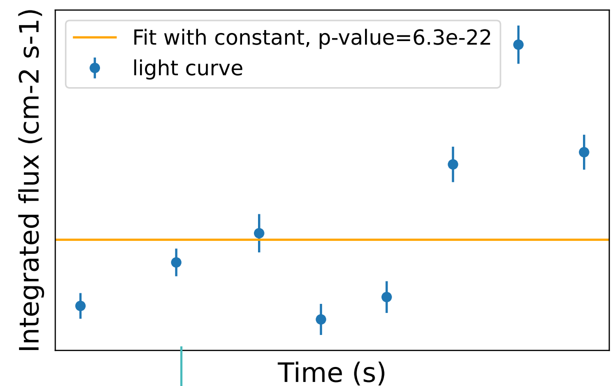
Variability test on each significant observation night

Non-variable sample

Constant fit
pvalue < 5σ

Variable sample





Optimization of analysis cut

Search for a variability pattern and extract sample properties (energetic and temporal distributions)

LIV analysis

Sample of simulations for calibration

Application on real data

Step 8: Likelihood for real data

Lag λ_n : free parameter, can be shared between sources with different redshifts

For one night: $$\mathcal{L}(\lambda_n) = -\sum_{\text{event i}} \log\left(\frac{dP(E_{R,\mathrm{i}}, t_\mathrm{i}; \lambda_n)}{dE_R dt}\right)$$

with $$\frac{dP}{dE_R dt} = W_\mathbf{s} \frac{\int \mathrm{EffA}(E_T, t)\mathrm{MM}(E_T, E_R) \times \mathrm{F}_\mathbf{s}(E_T, t; \lambda_n) dE_T}{N'_\mathbf{s}}$$

signal

$$+ W_\mathbf{b} \frac{\int \mathrm{EffA}(E_T, t)\mathrm{MM}(E_T, E_R) \times \mathrm{F}_\mathbf{b}(E_T) dE_T}{N'_\mathbf{b}}$$

baseline background

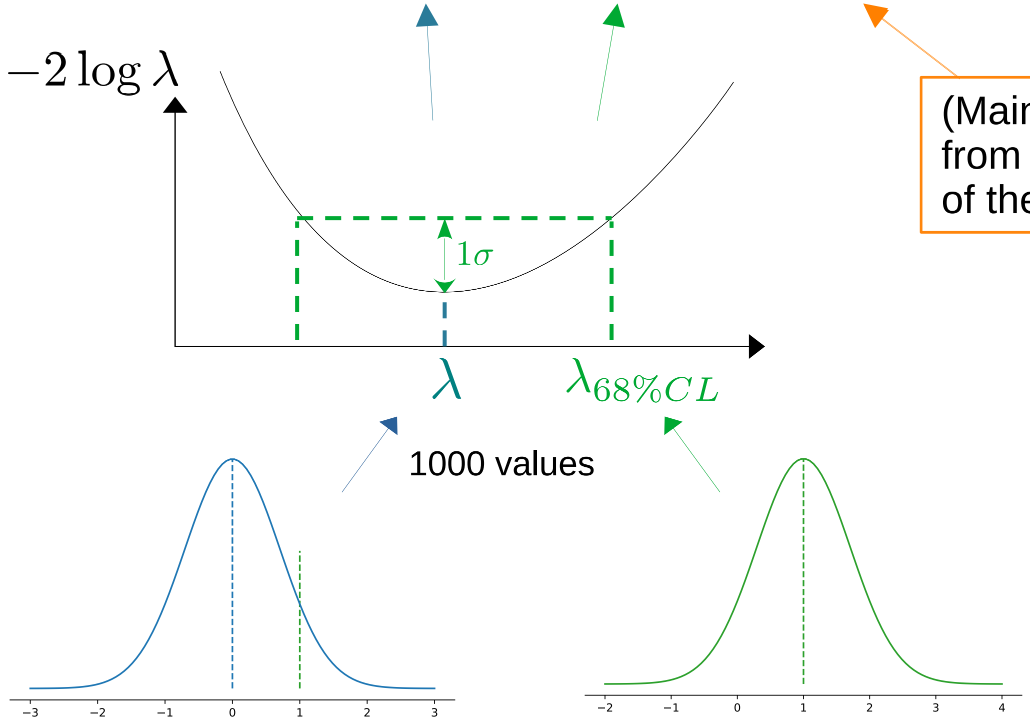
$$+ W_\mathbf{h} \frac{dN_\mathbf{bkg}}{dE_R} \times \frac{1}{T} \times \frac{1}{N'_\mathbf{h}}$$

hadronic background

Step 8: LIV analysis on real data

Application on real data

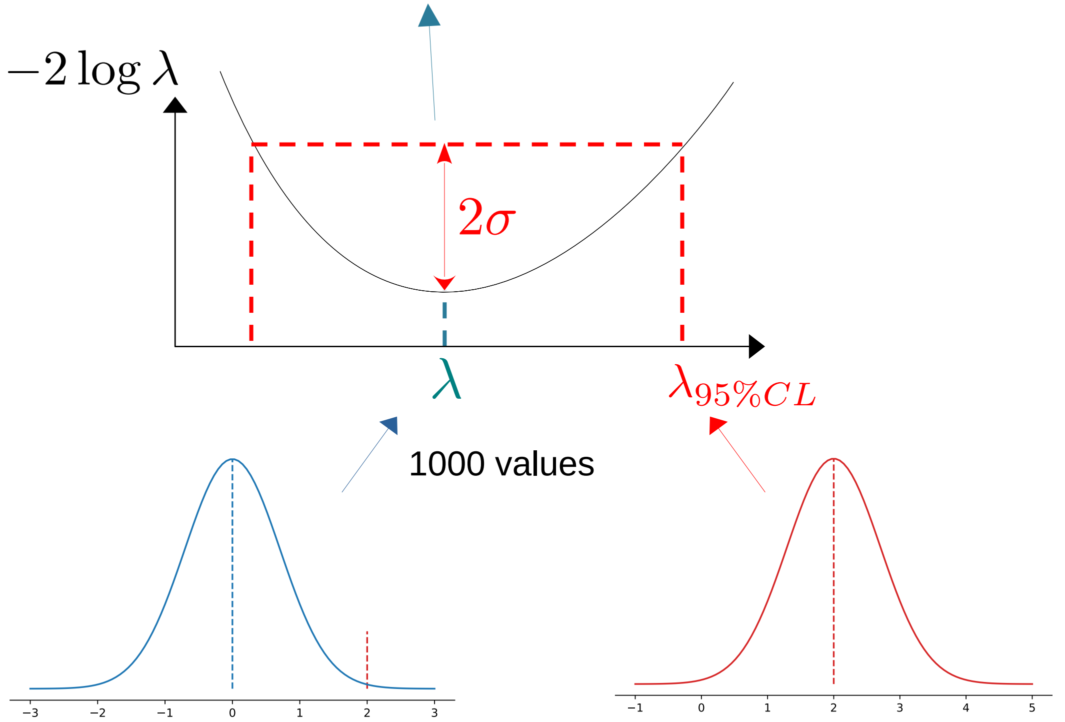
Time delay: $\lambda_1 = (3768 \; {}^{+1475}_{-1466} \; {}^{+3433}_{-3414}) \, s.TeV^{-1}$



(Main) systematic error arising from the statistical uncertainty of the light curve template

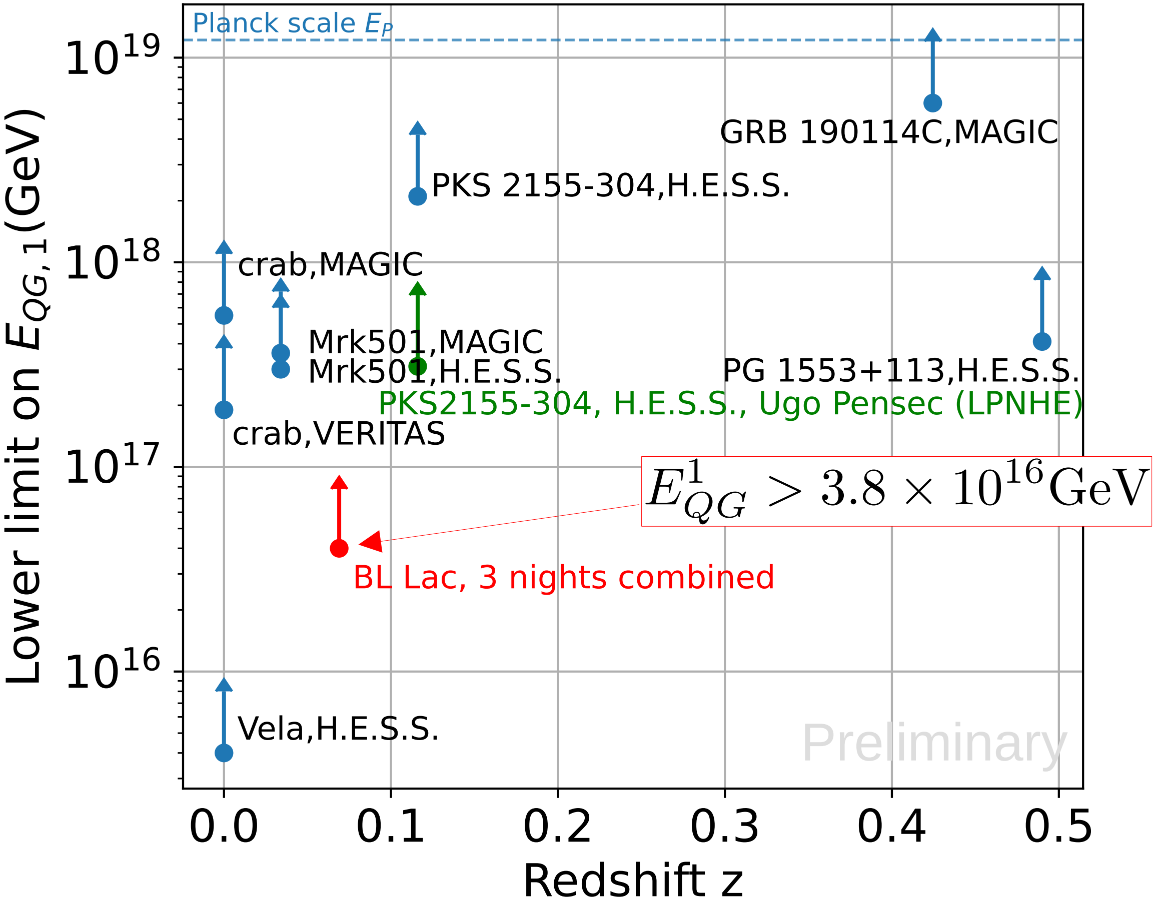
Application on real data

Time delay: $\lambda_1 = (3768 \; {}^{+1475}_{-1466} \; {}^{+3433}_{-3414})\,\mathrm{s.TeV^{-1}}$



Use $\lambda_{1,95\%CL} = \pm \frac{n+1}{2H_0 E^1_{QG,lim}}$ to extract: $E^1_{QG} > 3.8 \times 10^{16}\,\mathrm{GeV}$

Comparison with other limits



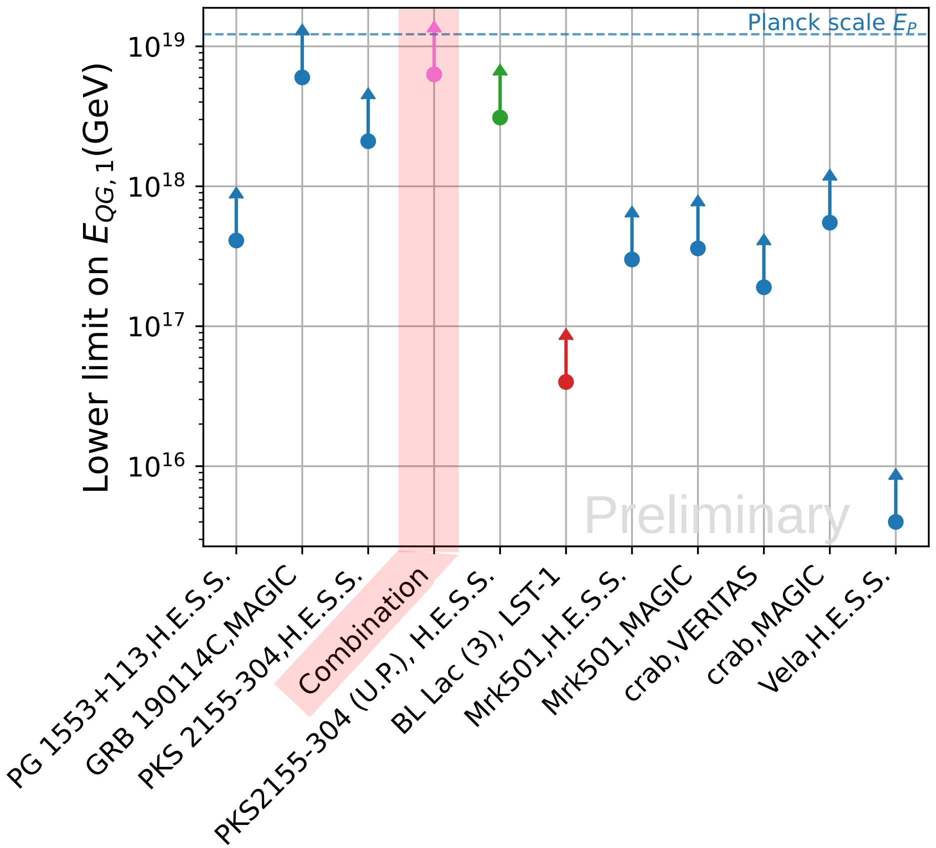
Bolmont et al., 2022

Combination with other experiments

LIVelihood is ready for real data combination, including LST-1 data !

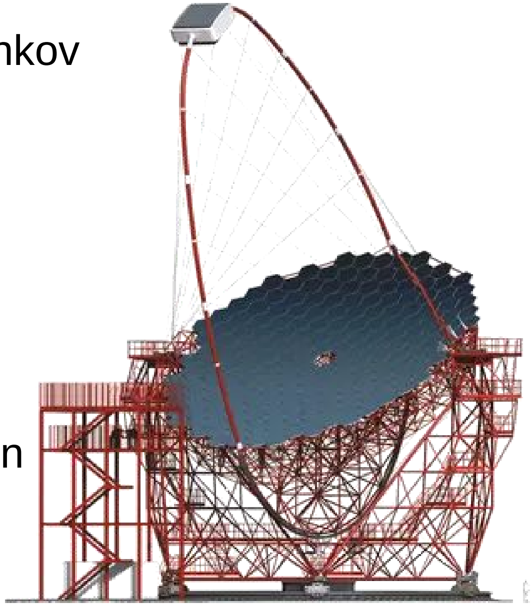
$$\mathcal{L}_{\mathrm{comb}} = \mathcal{L}_{H.E.S.S.} + \mathcal{L}_{MAGIC} + \mathcal{L}_{VERITAS} + \mathcal{L}_{LST-1}$$

First test of combination with H.E.S.S. data on PKS 2155-304 flare by Ugo Pensec, LPNHE



Conclusion

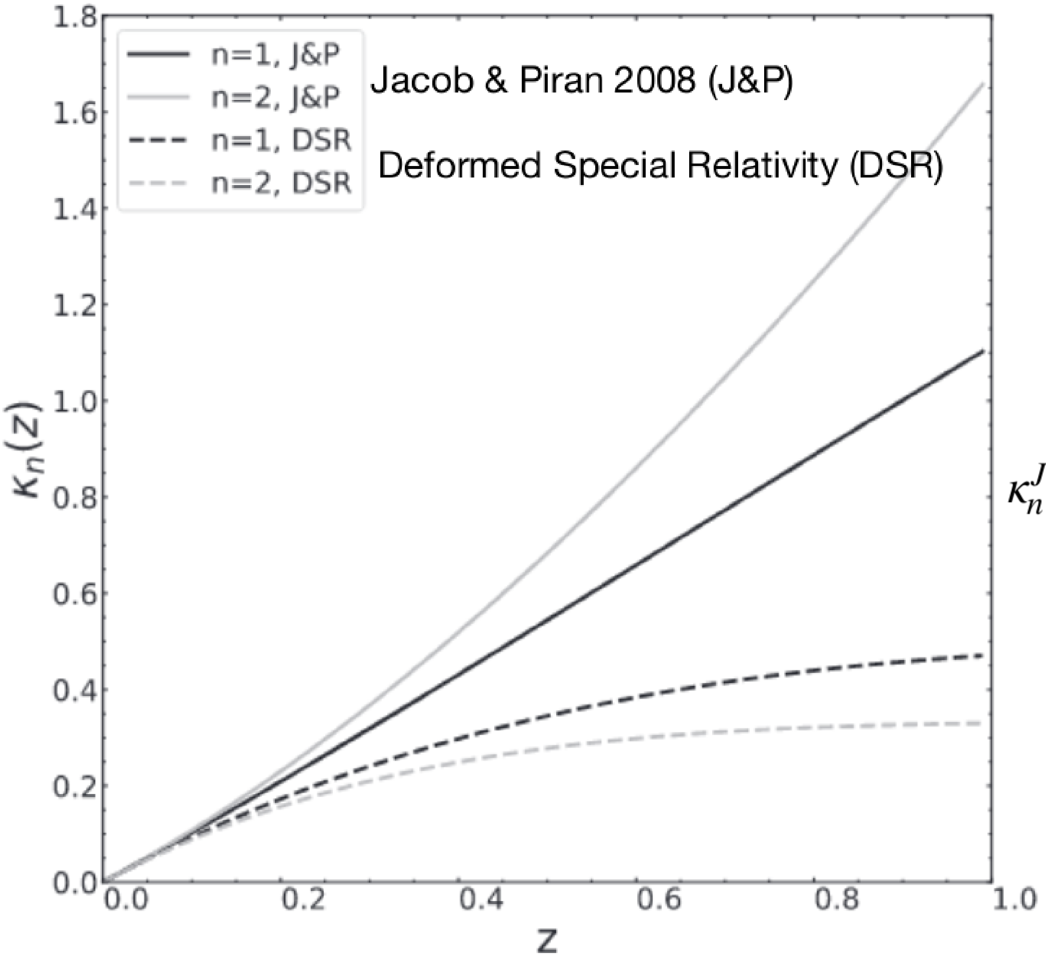
- **Systematic** analysis of several blazars from LST-1 data until June 2023, searching for variability in the lightcurve of a given night
- Found **3** nights showing **intra-night variability** (BL Lac) and combined them to extract a limit on E_{QG}^{1} using **real data**
- **First analysis** performing a **combination of flares** with Cherenkov telescope data



Ongoing work:

- Working on **extended** dataset and **most recent** data
- **VarCat improvement** using the Crab nebula: more efficient on **furthest** sources and tune the **threshold** for intra-night variability detection
- **Combination of LST-1 analysis with the γLIV WG data**

H.E.S.S. MAGIC VERITAS CTAO LST COLLABORATION



Bolmont et al 2022

$$\kappa_n^{J\&P}(z) = \frac{1}{z_0}\int_0^z \frac{(1+z')^n}{\sqrt{\Omega_m(1+z')^3+\Omega_\Lambda}}dz'$$

Intrinsic source delay

- **only** Mrk 501 **MAGIC** flare of july 2005: (4±1)min between E < 0.25TeV & E>1.2TeV [Albert et. al., 2077]
- ongoing work to study it via **simulations** (Levy thesis 2022)
- **Perenne et. al. 2019**: analytical solution outlining the electron spectrum evolution
  - synchrotron losses **>** inverse-Compton losses
  - 2 regimes depending on the mechanism **driving** most energetic electrons when the lightcurve reaches its **peak**:
    - **radiative** cooling effect

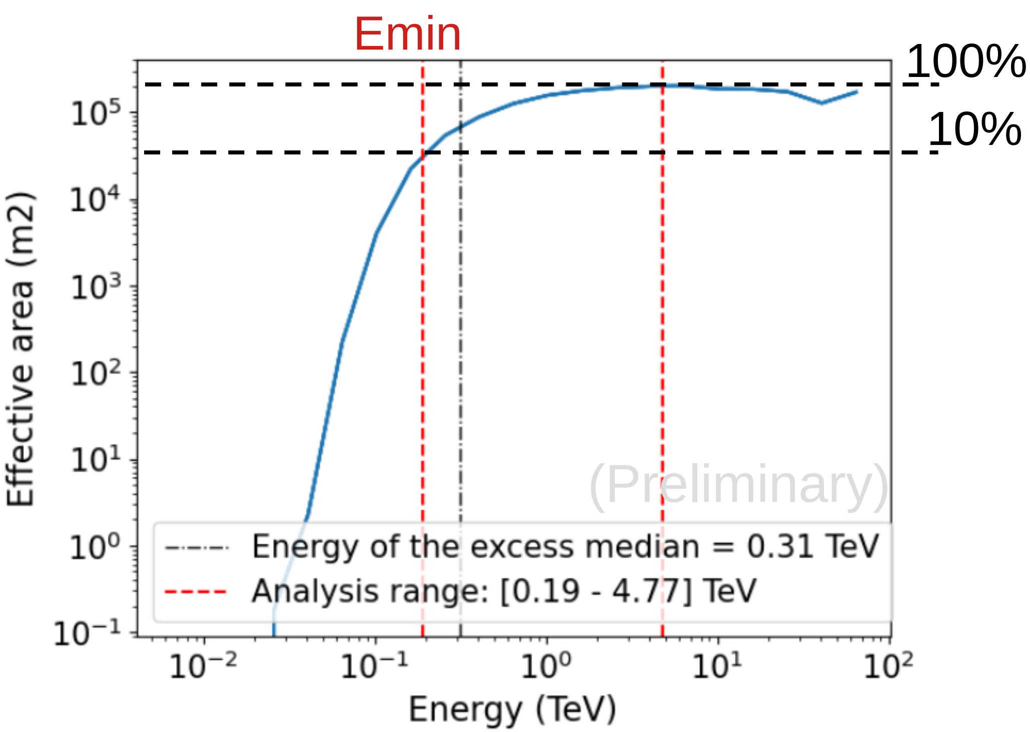
      → *cooling-driven regime*
    - **acceleration** emitting VHE electrons goes on while flare starts to decay because of **other mechanisms** (ex: diminution of ambiant magnetic field)

      → *acceleration-driven mechanisms*

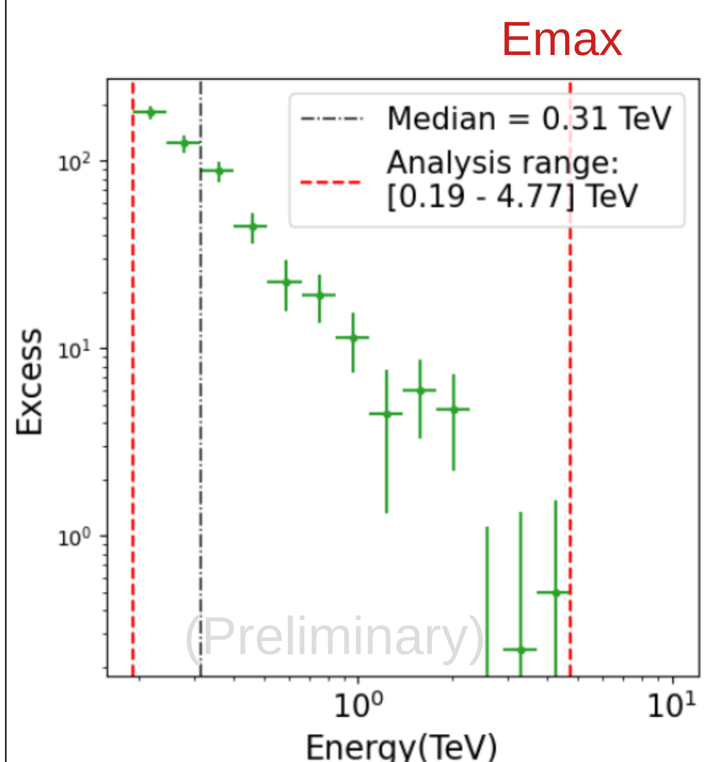
Night-wise energy ranges:

2021-08-08	2021-08-09	2022-10-20
[0.25 ; 4.8] TeV	[0.19 ; 4.8] TeV	[0.40 ; 7.7] TeV

1. Apply a safe mask threshold of 10% of effective area for each run and keep most conservative values for the whole night



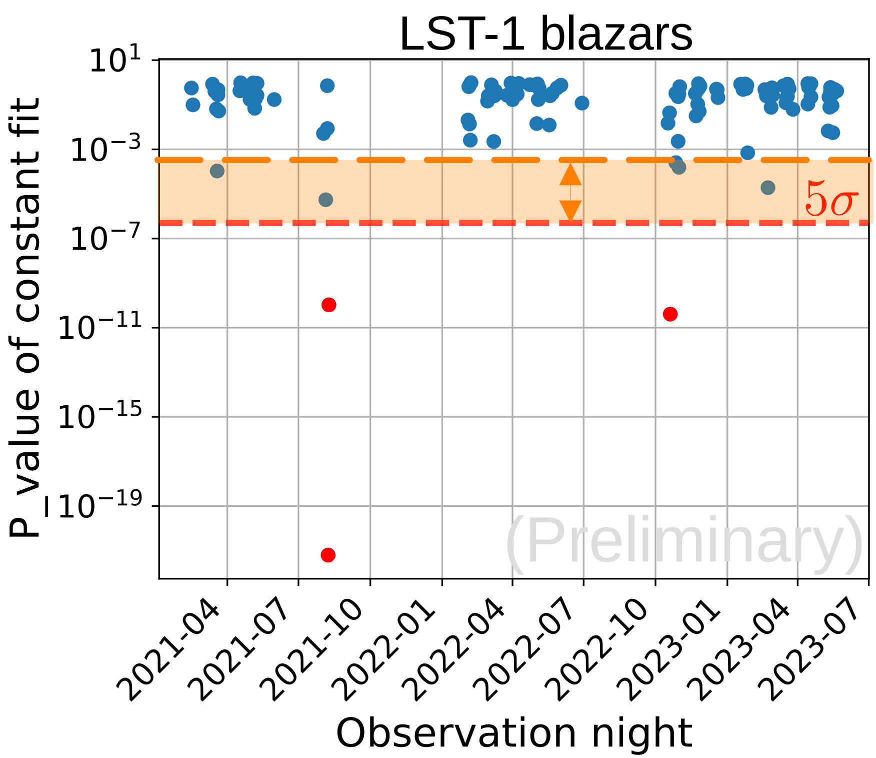
2. Keep the extrema non-zero energy bins of excess

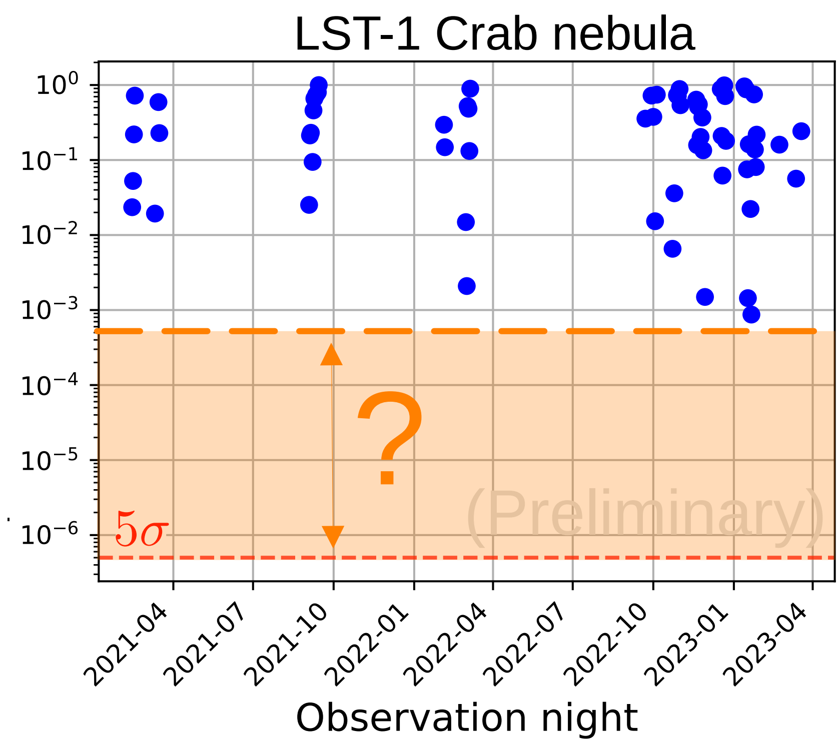


Step 2: using the Crab nebula

Crab nebula is a **stable source** ↔ **expecting 0** night with intra-night variability ✅
(p-value >> 10^{-7})

Possibility to tune the p-value using Crab → more nights with intra-night variability ?



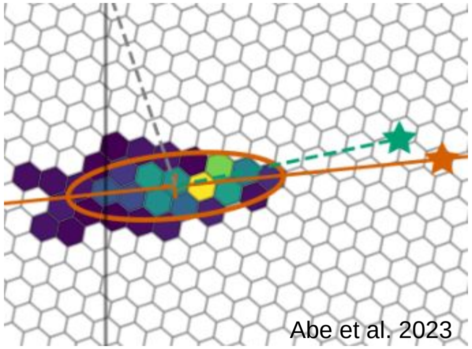


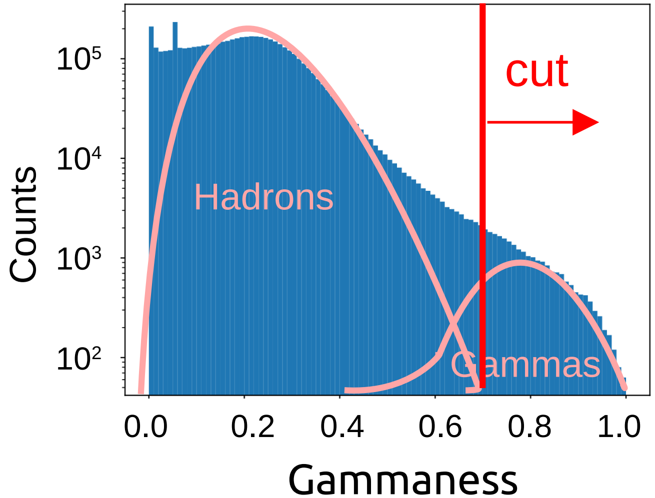
Step 3: cut optimization

Non-variable sample

↓

Optimization of gammaness cut





Reconstruction of event properties

- direction
- energy
- type of particle (gamma, hadron, ...)

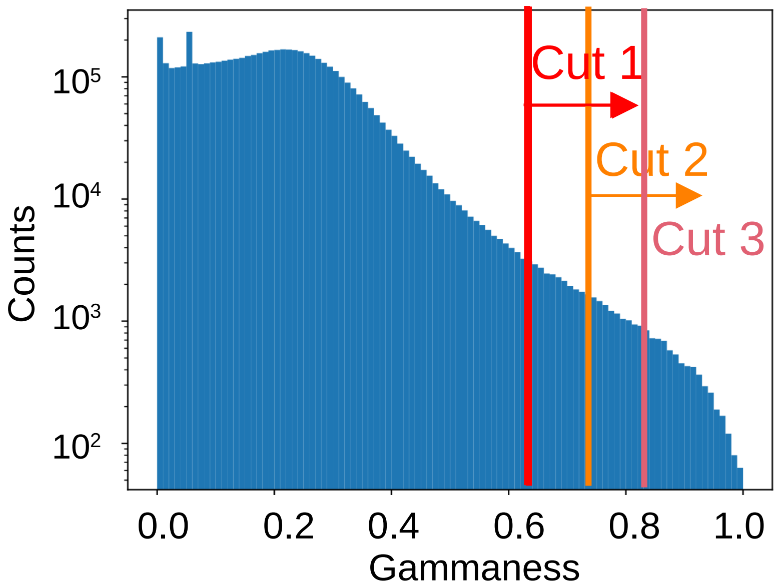
score of how likely an event is expected to be a gamma rather than background

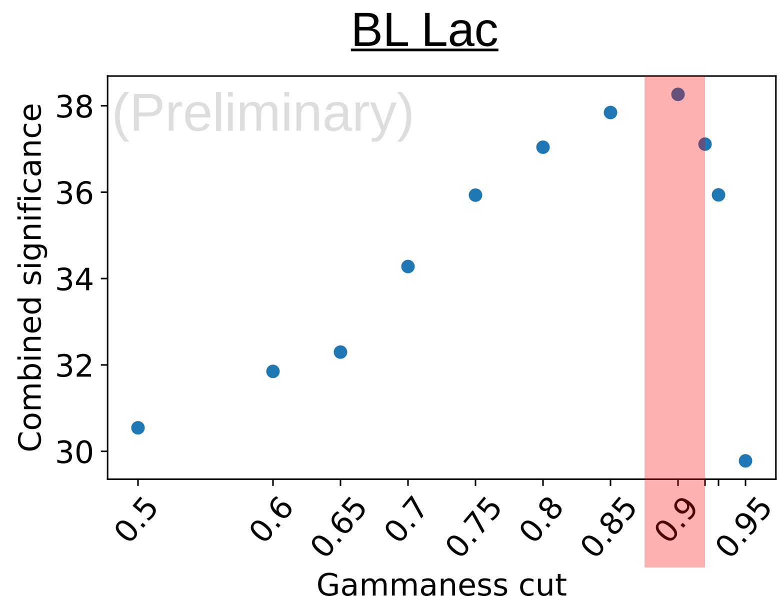
Step 3: cut optimization

Non-variable sample

↓

Select different gammaness cuts and search for the significance

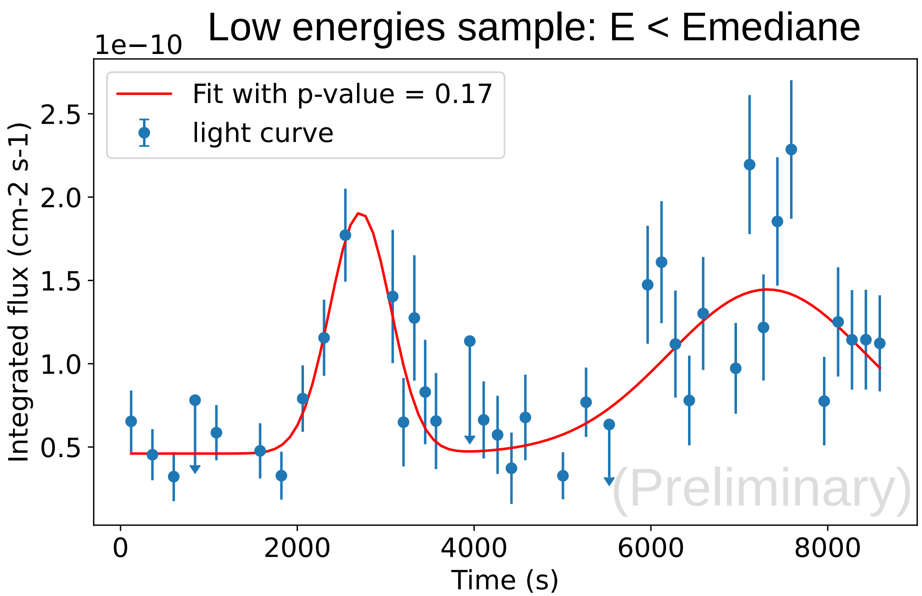


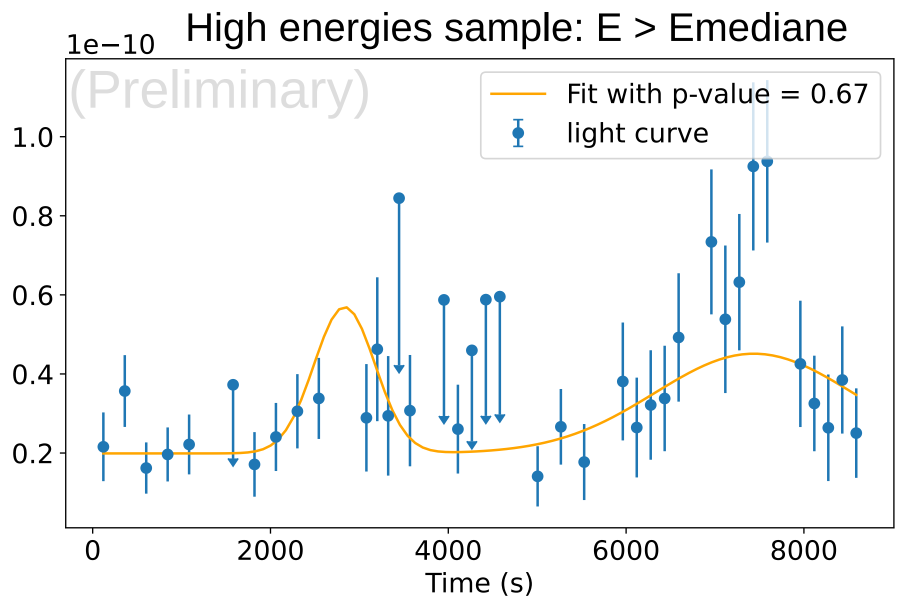


Given a source s and its sample of non-variable nights N_s: $S_{\mathrm{s,\ cut}} = \sqrt{\sum_{n \in N_s} S_n^2}$

Step 5: Band comparison method

Example of 2021-08-08 night





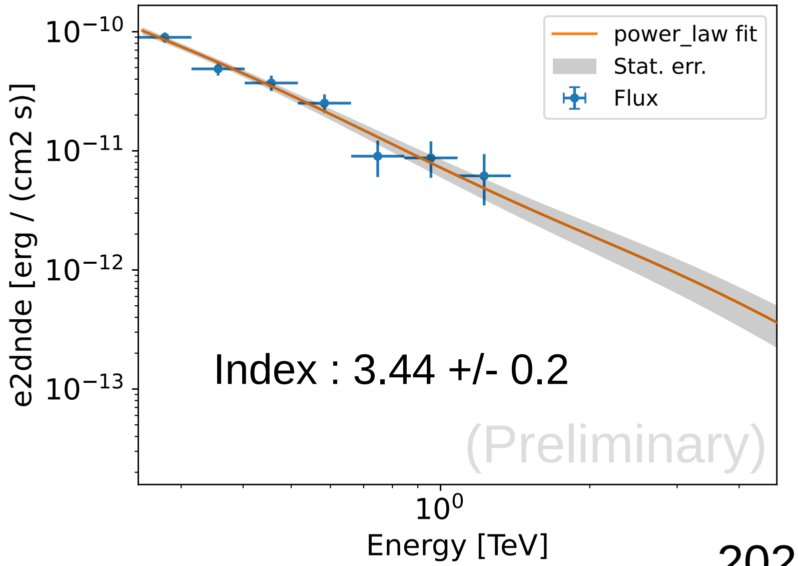
$$G_{LE}(t) = A_1 e^{\frac{(t-\mu_1)^2}{2{\sigma_1}^2}} + A_2 e^{\frac{(t-\mu_2)^2}{2{\sigma_2}^2}} + C_0 \quad G_{HE}(t) = (G_{LE}(t - \Delta t) - C_0) \times A + C$$

(Preliminary)	Energy range (TeV)	Median (TeV)	Delay Δt (s)	Delay significance (σ)
2021-08-08	[0.25 ; 4.8]	0.40	79±69	1.1
2021-08-09	[0.19 ; 4.8]	0.31	-136±397	0.3
2022-10-20	[0.40 ; 7.7]	0.69	-953±526	1.8

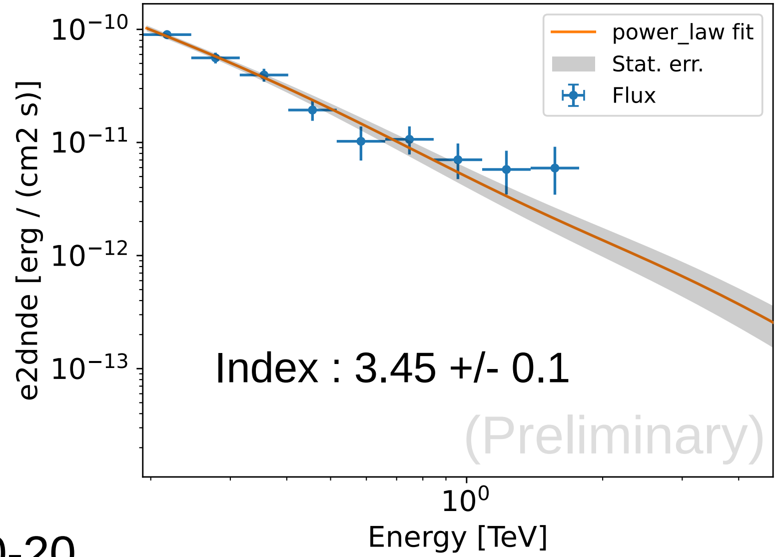
IAPP

Step 5: Spectra fit

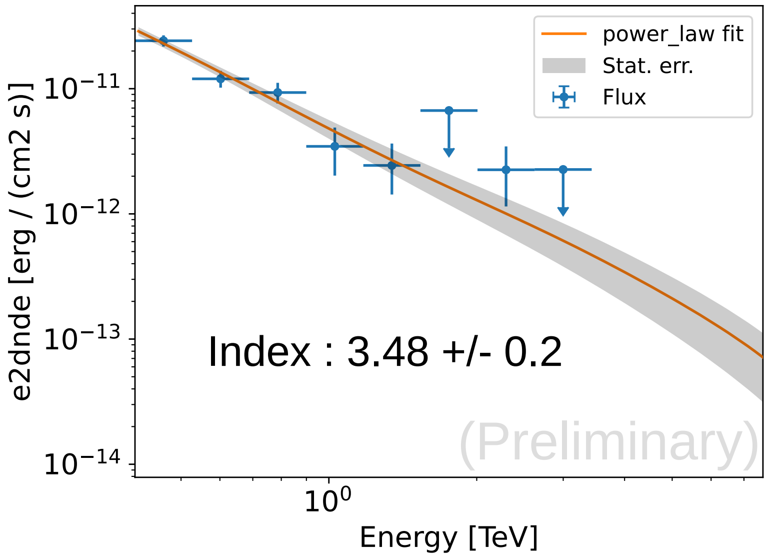
2021-08-08



2021-08-09

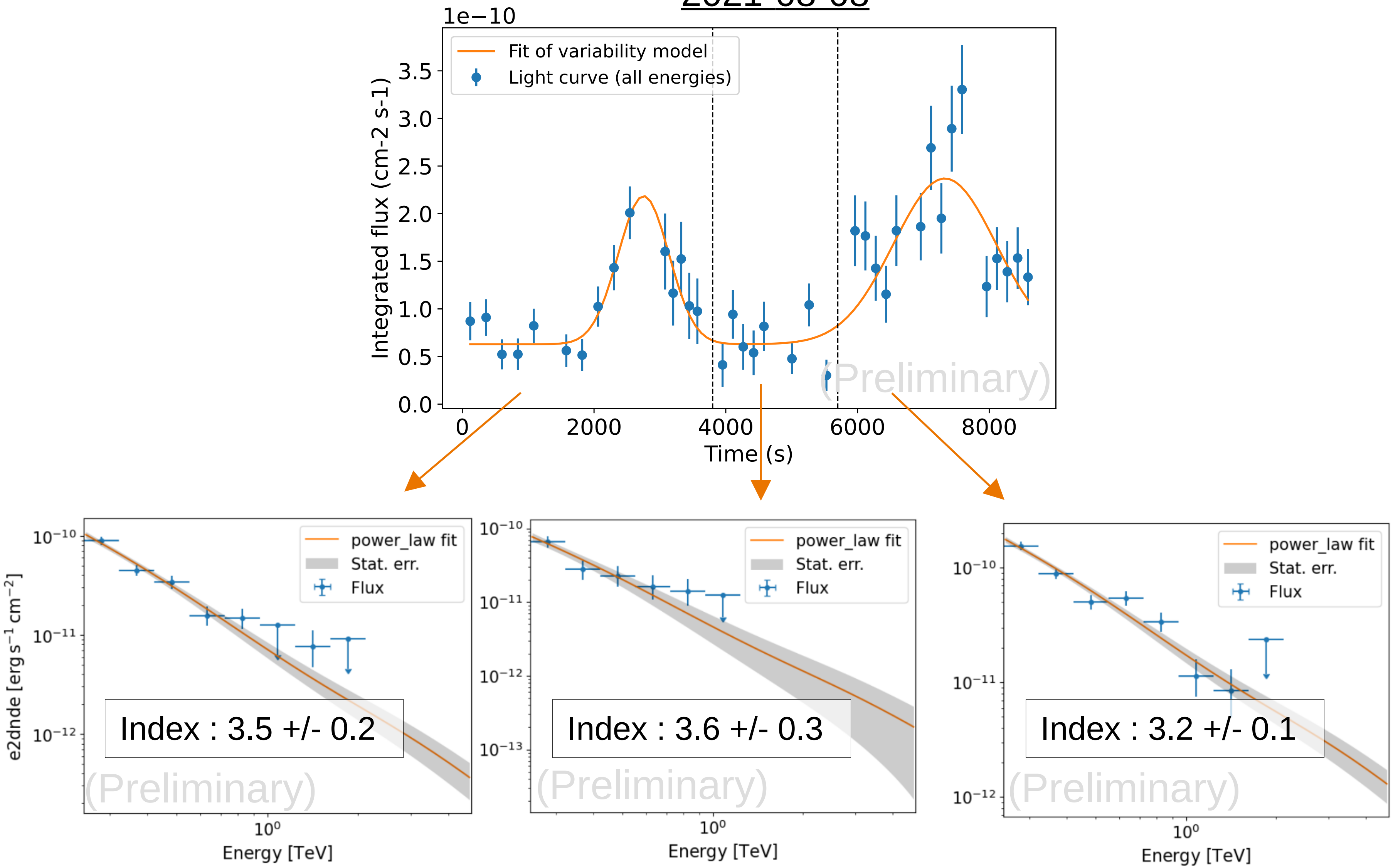


2022-10-20

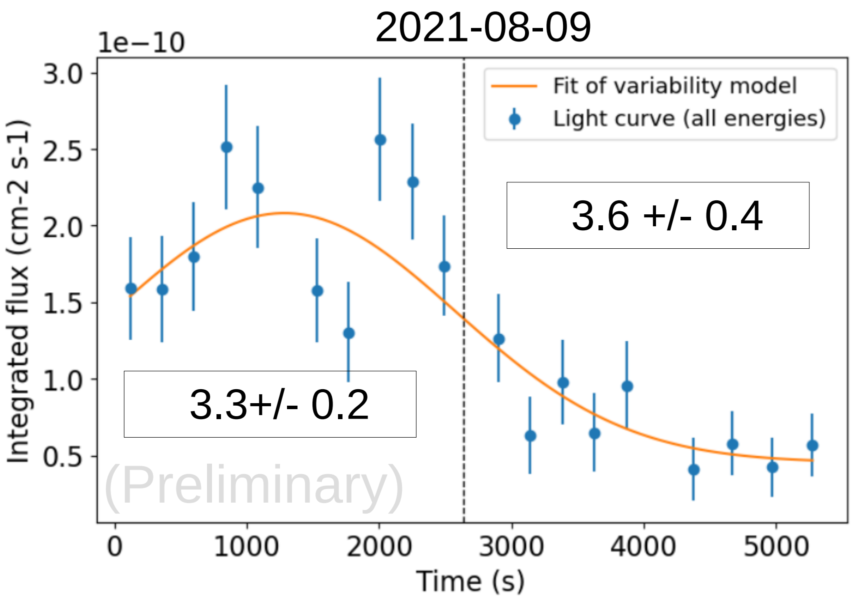


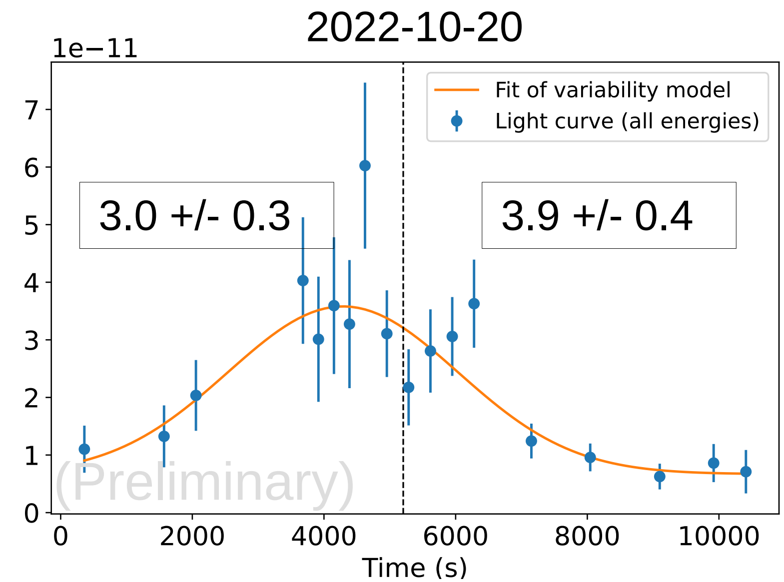
Step 5: Time-independency of spectra

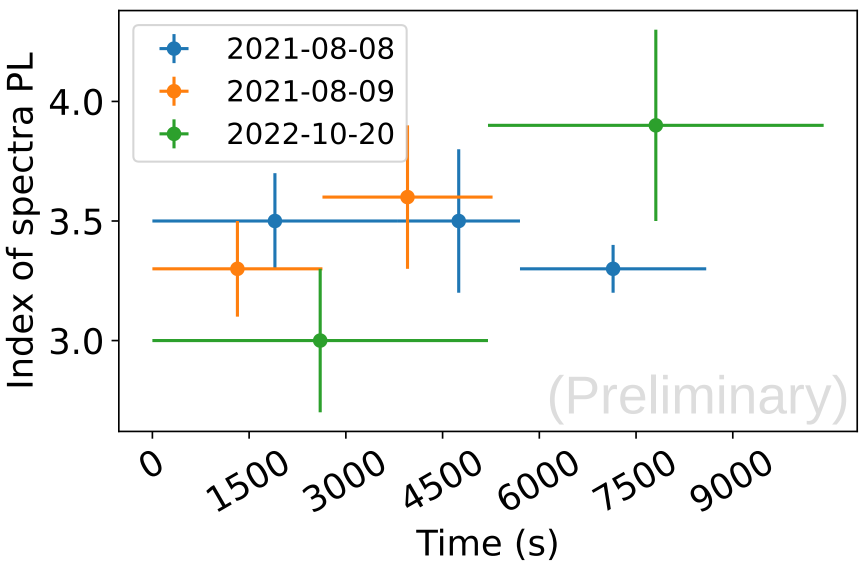
2021-08-08



Step 5: Time-independency of spectra







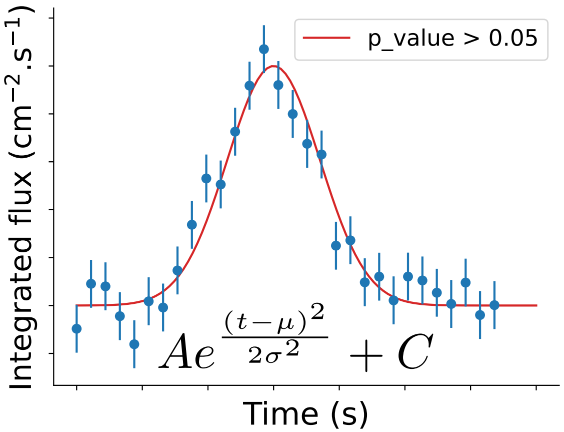
for 2 given time bins j and k:

$$S = \frac{|\mathrm{index}_j - \mathrm{index}_k|}{\sqrt{\sigma_j^2 + \sigma_k^2}}$$

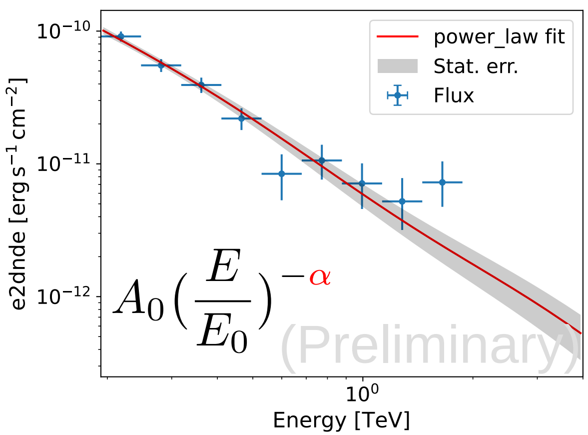
2021-08-08	2021-08-09	2022-10-20
1st - 2nd: 0.3σ 2nd - 3rd: 1.1σ 1st - 3rd: 1.0σ	0.9σ	2.2σ

(Preliminary)

Search for a variability pattern and extract sample properties (energetic and temporal distributions)



Preliminary	Lightcurve model (Gaussians)						
	C (cm-2.s-1)	A_1 (cm-2.s-1)	μ_1 (s)	σ_1 (s)	A_2 (cm-2.s-1)	μ_2 (s)	σ_2 (s)
2021-08-08	6.3e-11	1.6e-10	2752	386	1.8e-10	7326	778
2021-08-09	4.5e-11	1.7e-10	1284	1293			
2022-10-20	6.7e-12	2.9e-11	4288	1750			



Preliminary	Spectra model (power law)
	Index α
2021-08-08	3.44±0.2
2021-08-09	3.45±0.1
2022-10-20	3.48±0.2

Step 6: Likelihood for LIV analysis on real data

$$\frac{dP}{dE_R dt} = W_{\mathbf{s}} \frac{\int \mathrm{E_{ff}A}(E_T,t)\mathrm{MM}(E_T,E_R) \times \mathrm{F}_{\mathbf{s}}(E_T,t;\lambda_n) dE_T}{N'_{\mathbf{s}}}$$
$$+ W_{\mathbf{b}} \frac{\int \mathrm{E_{ff}A}(E_T,t)\mathrm{MM}(E_T,E_R) \times \mathrm{F}_{\mathbf{b}}(E_T) dE_T}{N'_{\mathbf{b}}}$$
$$+ W_{\mathbf{h}} \frac{\int \mathrm{E_{ff}A}(E_T,t)\mathrm{MM}(E_T,E_R) \times \mathrm{F}_{\mathbf{h}}(E_T) dE_T}{N'_{\mathbf{h}}}$$

	Spectral model (power law)	Lightcurve model (Gaussians)							
	index α	C (cm-2.s-1) (baseline background)	A_1 (cm-2.s-1)	μ_1 (s)	σ_1 (s)	A_2 (cm-2.s-1)	μ_2 (s)	σ_2 (s)
2021-08-08	3.44	6.3e-11	1.6e-10	2752	386	1.8e-10	7326	778
2021-08-09	3.45	4.5e-11	1.7e-10	1284	1293			
2022-10-20	3.48	6.7e-12	2.9e-11	4288	1750			
hadronic background	2.7 (Aguilar et al, 2015)	C						

Ongoing: systematics

$$\mathcal{L}(\lambda_n,\vec{\theta}) = \mathcal{L}_S(\lambda_n,\vec{\theta})+$$

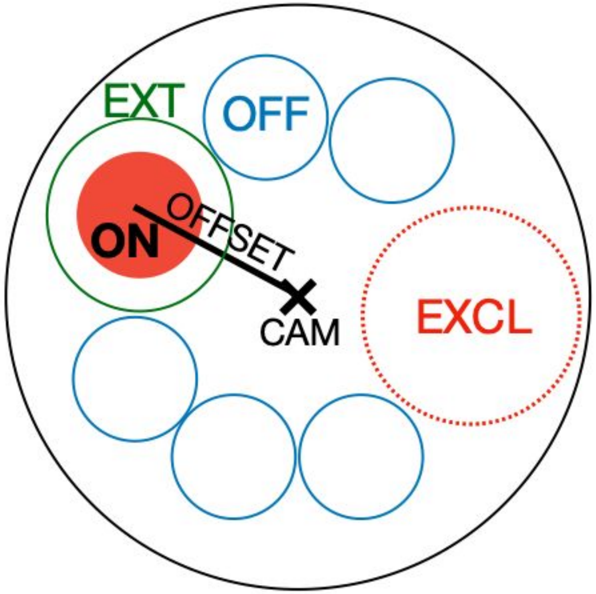
$$+\mathcal{L}_{\rm template}(\vec{\theta}_C) + \mathcal{L}_{\gamma}(\vec{\theta}_{\gamma}) + \mathcal{L}_B(\vec{\theta}_B) + \mathcal{L}_{ES}(\vec{\theta}_{ES}) + \mathcal{L}_z(\vec{\theta}_z)$$

- $\vec{\theta}_C$: parameters of lightcurve analytic parametrization → dominant (Bolmont et al. 2022)
- $\vec{\theta}_{\gamma}$: power-law index of signal events spectrum
- $\vec{\theta}_B$: $\frac{\text{signal}}{\text{total events}}$ & $\frac{\text{hadrons}}{\text{total background}}$
- $\vec{\theta}_{ES}$: energy scale → can increase when n=2
- $\vec{\theta}_z$: distance (redshift) → ~ 0

Reflected region background technique

Hypothesis: radial symmetry of background in the field-of-view

- X CAM: camera pointing direction
- OFFSET: regions dispersion radius
- ON: source (gammas) + background
- EXT: exclusion of potential remaining source events
- EXCL: exclusion of a potential other source
- OFF: background

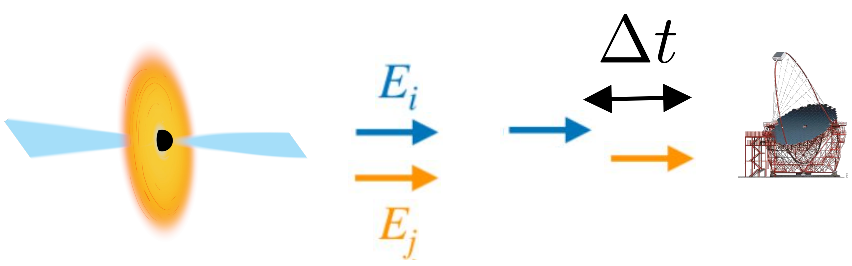


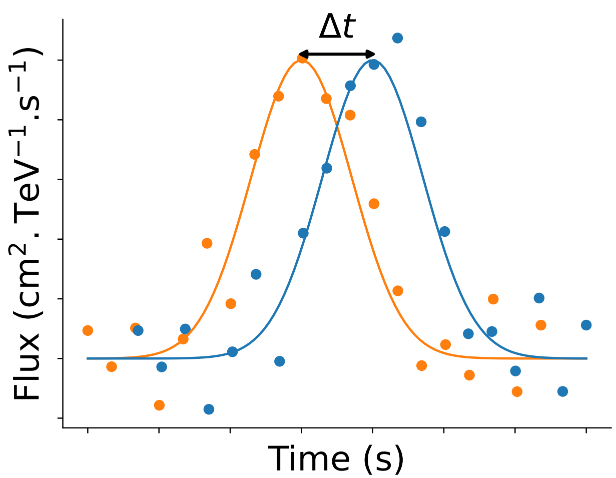
$$N_s = N_{ON} - \alpha N_{OFF} = N_{ON} - \frac{1}{n}\sum_n N_{n,OFF}$$

$$S = \sqrt{2}\left\{N_{on}\ln\left[\frac{1+\alpha}{\alpha}\left(\frac{N_{on}}{N_{on}+N_{off}}\right)\right] + N_{off}\ln\left[(1+\alpha)\left(\frac{N_{off}}{N_{on}+N_{off}}\right)\right]\right\}^{1/2}$$

$$S = \frac{N_S}{\hat{\sigma}(N_S)} = \frac{N_{on} - \alpha N_{off}}{\sqrt{\alpha(N_{on}+N_{off})}}$$

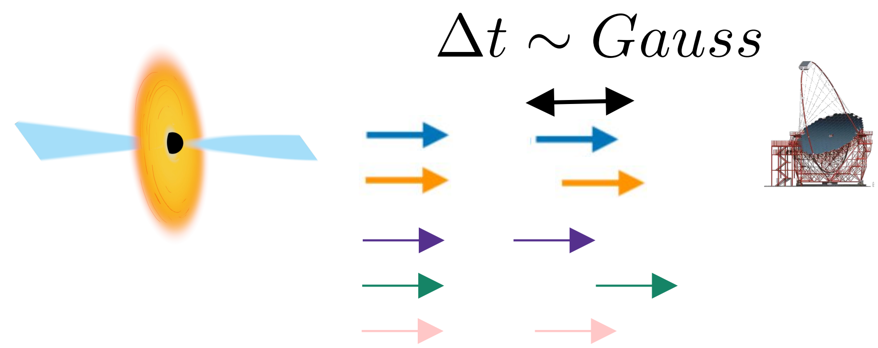
Deterministic LIV

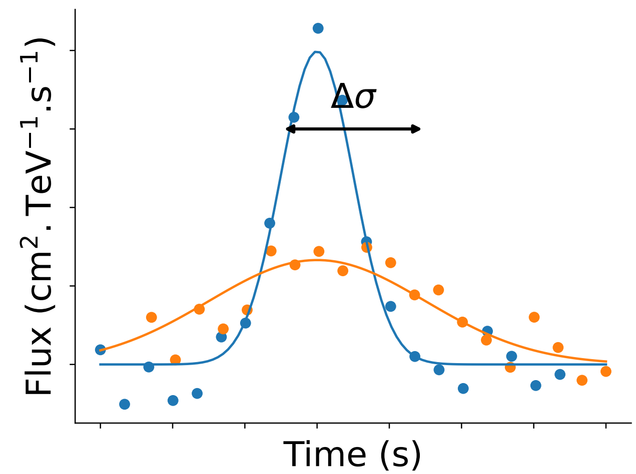




$$\lambda_{\mathrm{determ},n} = \frac{\Delta t_n}{\Delta E_n \kappa_n(z)} = \pm \frac{n+1}{2H_0 E_{QG,n}^n}$$

Stochastic LIV

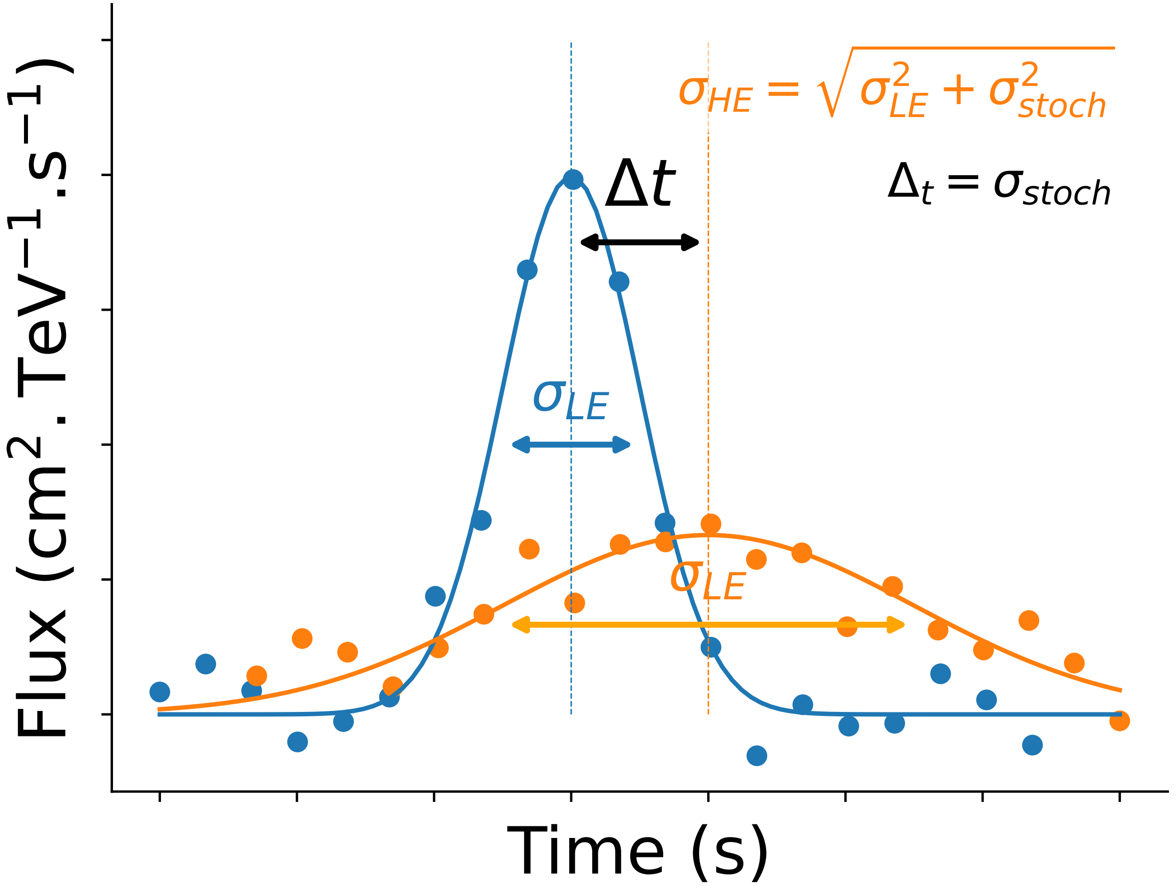




$$\lambda_{\mathrm{stoch},n} = \frac{\Delta \sigma_n}{\Delta E_n \kappa_n(z)} = \pm \frac{n+1}{2H_0 E_{QG,n}^n}$$

different $E_{QG,determ}$ & $E_{QG,stoch}$ for deterministic and stochastic LIV effects separately

but same E_{QG} when constraining them together



Lag λ_n : free parameter, can be shared between sources with different redshifts

For one night: $\mathcal{L}(\lambda_{\mathrm{stoch},n}) = -\sum_{\mathrm{event\ i}} \log\left(\frac{dP(E_{R,\mathrm{i}}, t_\mathrm{i}; \lambda_{\mathrm{stoch},n})}{dE_R dt}\right)$

$$W_{\mathbf{s}} \frac{\int \mathrm{E_{ff}A}(E_T, t)\mathrm{MM}(E_T, E_R) \times \mathrm{F}_{\mathbf{s}}(E_T, t; \lambda_{\mathrm{stoch},n}) dE_T}{N'_{\mathbf{s}}} + \sum_{k=\{b,h\}} W_k \frac{dP_k}{dE_R dt}$$

Stochastic LIV: likelihood (2)

Lag λ_n : free parameter, can be shared between sources with different redshifts

For one night: $$\mathcal{L}(\lambda_{\mathrm{stoch},n}) = -\sum_{\mathrm{event\ i}} \log\left(\frac{dP(E_{R,\mathrm{i}}, t_\mathrm{i}; \lambda_{\mathrm{stoch},n})}{dE_R dt}\right)$$

$$W_\mathbf{s} \frac{\int \mathrm{E_{ff}A}(E_T, t)\mathrm{MM}(E_T, E_R) \times \mathrm{F}_\mathbf{s}(E_T, t; \lambda_{\mathrm{stoch},n}) dE_T}{N'_\mathbf{s}} + \sum_{k=\{b,h\}} W_k \frac{dP_k}{dE_R dt}$$

$$\mathrm{F}_\mathbf{s}(E_T, t; \lambda_{\mathrm{stoch},n}) = f_t(t) * g_{\mathrm{LIV}}(t, E, z) \times f_E(E)$$

lightcurve $\mathrm{Gauss}(\mu, \sigma)$

$\mathrm{Gauss}(0, \sigma_{\mathrm{LIV}}(E))$

spectra $A_0\left(\frac{E}{E_0}\right)^{-\alpha}$

Lag λ_n : free parameter, can be shared between sources with different redshifts

For one night: $\mathcal{L}(\lambda_{\mathrm{stoch},n}) = -\sum_{\text{event i}} \log\left(\frac{dP(E_{R,\mathrm{i}}, t_\mathrm{i}; \lambda_{\mathrm{stoch},n})}{dE_R dt}\right)$

$$W_\mathbf{s} \frac{\int \mathrm{EffA}(E_T, t)\mathrm{MM}(E_T, E_R) \times \mathrm{F}_\mathbf{s}(E_T, t; \lambda_{\mathrm{stoch},n}) dE_T}{N'_\mathbf{s}} + \sum_{k=\{b,h\}} W_k \frac{dP_k}{dE_R dt}$$

$$\mathrm{F}_\mathbf{s}(E_T, t; \lambda_{\mathrm{stoch},n}) = f_t(t) * g_{\mathrm{LIV}}(t, E, z) \times f_E(E)$$

$$f_t * g_{\mathrm{LIV}} = \mathrm{Gauss}\left(\mu, \sqrt{\sigma^2 + \sigma^2_{\mathrm{LIV}}}\right)$$