# Three-mirror cavity optical behavior for quantum noise reduction

Paul Stevens - 2nd year PhD student
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paul.stevens@ijclab.in2p3.fr

ÉCOLE DOCTORALE PHENIICS
IJCLab Irène Joliot-Curie Laboratoire de Physique des 2 Infinis
université PARIS-SACLAY

# Outline

1. **Context**
2. **Three-mirror cavities optics**
3. **Conclusion**

# Context

Context

# Quantum noise

**<u>Total quantum noise :</u> sum of two sources**

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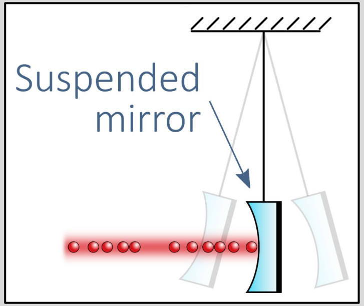
- **Radiation pressure (RP) noise**

Radiation pressure noise
→ Is an amplitude noise

Arises from **impact of laser beam on mirrors**



# Quantum noise

**<u>Total quantum noise :</u> sum of two sources**

- **Radiation pressure (RP) noise**
- **Shot noise**

## Radiation pressure noise
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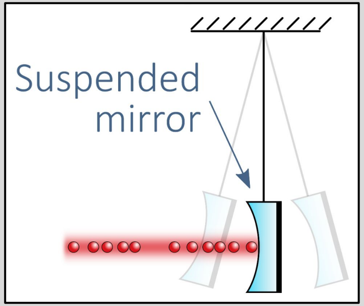
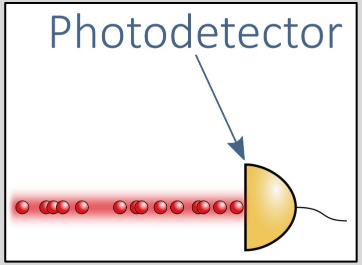



## Shot noise
→ Is a phase noise

Arises from **variation of photon number in the beam** (Poisson statistic)

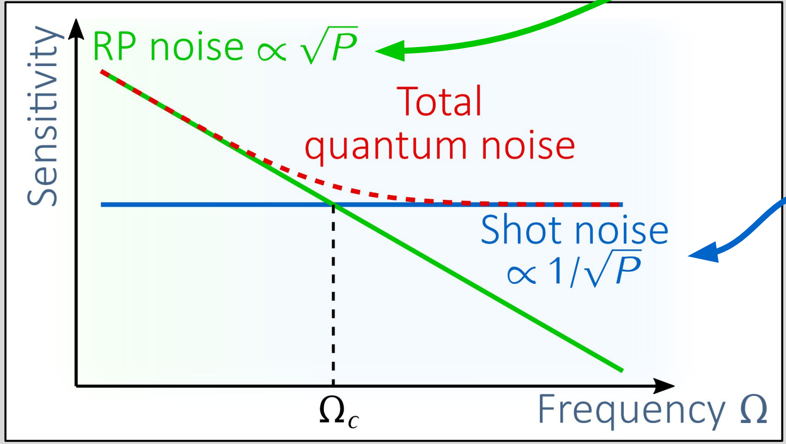


# Quantum noise



**Total quantum noise :** **sum of two sources**

- **Radiation pressure (RP) noise**
- **Shot noise**
- **Cross each other at frequency $\Omega_c$**

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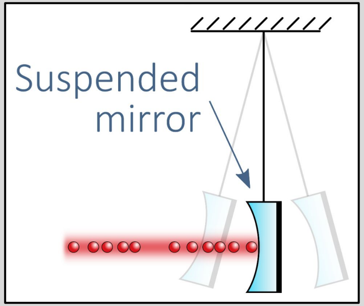


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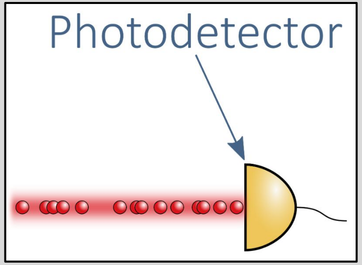
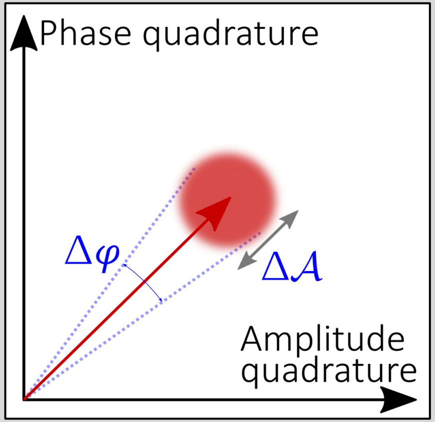


# Squeezed states of light

## Coherent state




Uncertainty disk:
Heisenberg principle
$\Delta A \, \Delta\varphi \geq 1$

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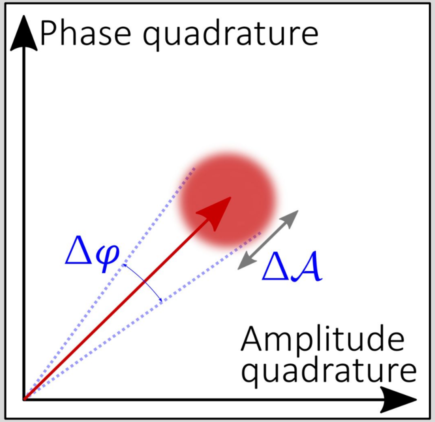


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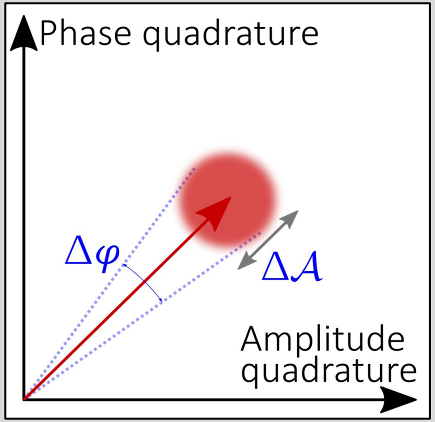


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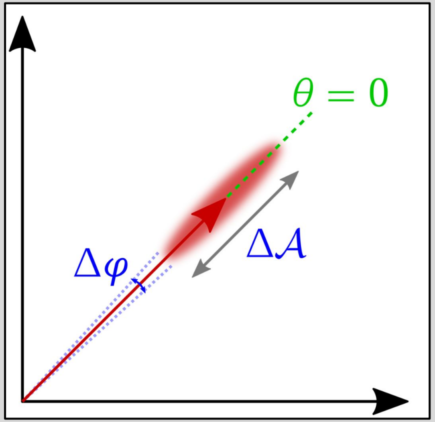

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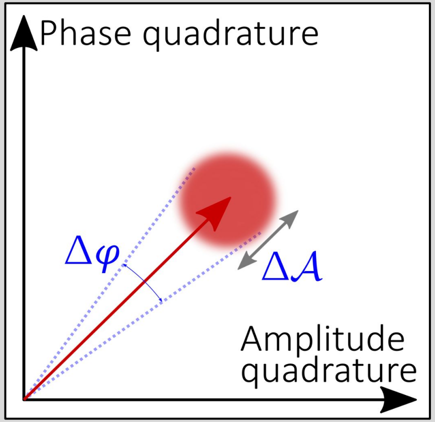


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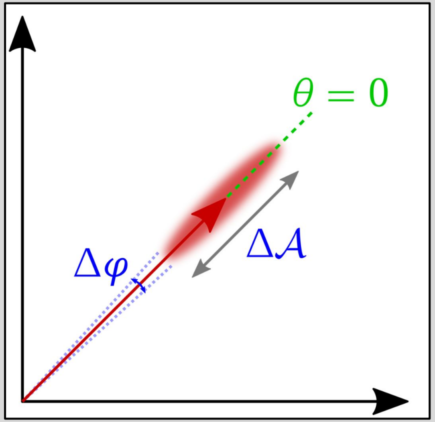
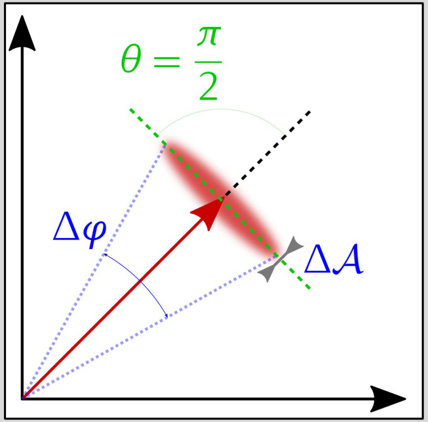



**Amplitude sq. state**



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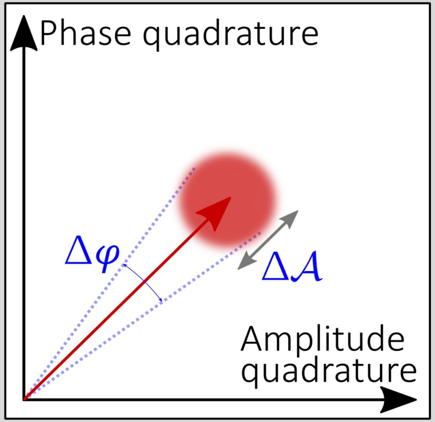


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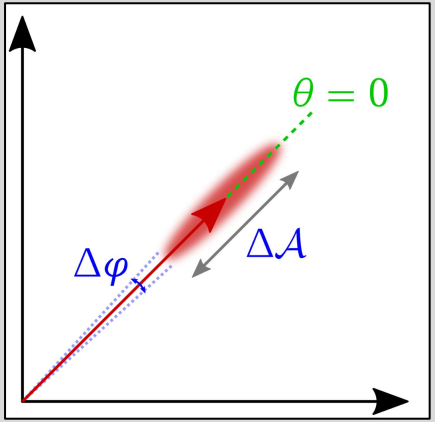
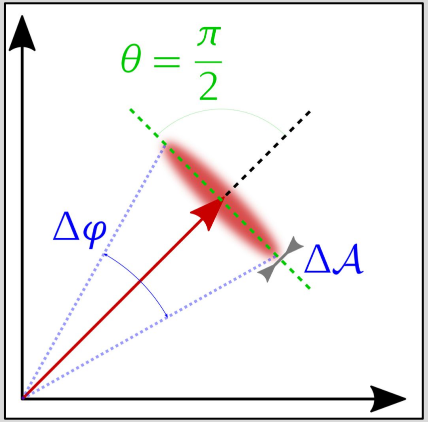


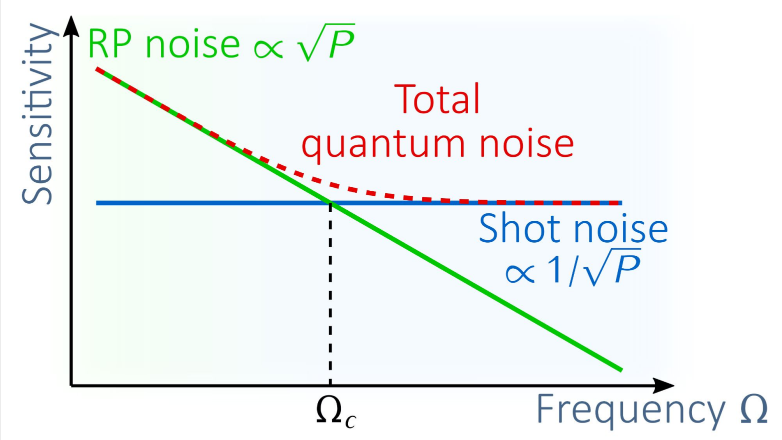

## Amplitude sq. state




- For both cases, the area of uncertainty region is equal to the uncertainty of initial coherent state
- Choose either to reduce phase noise at the prize of increasing amplitude noise or the invert

Context

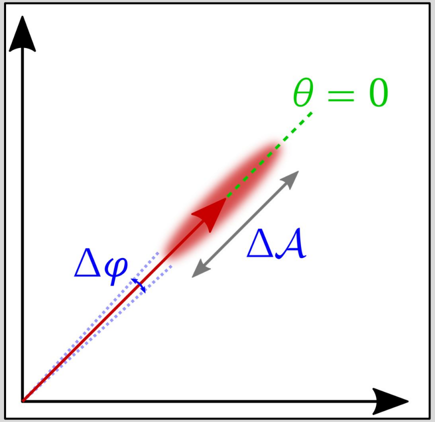
# Squeezed states of light

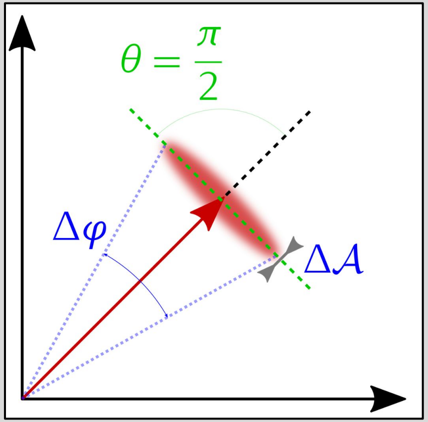



**How the injection of squeezed states into the detector modulate the total quantum noise ?**

Phase sq. state

Amplitude sq. state

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# Squeezed states of light – **phase** squeezing

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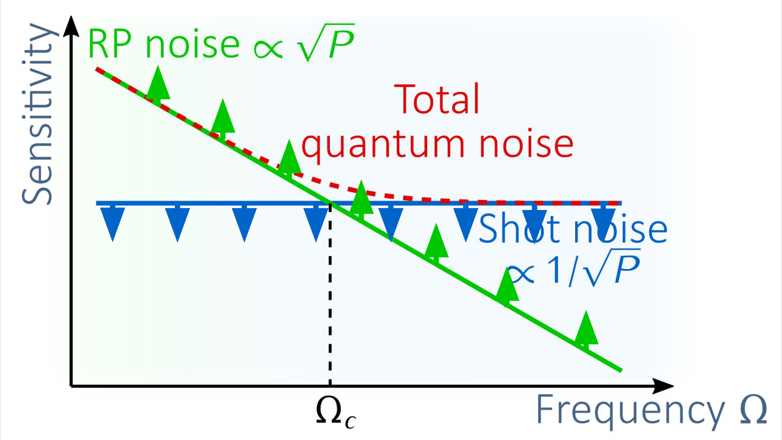
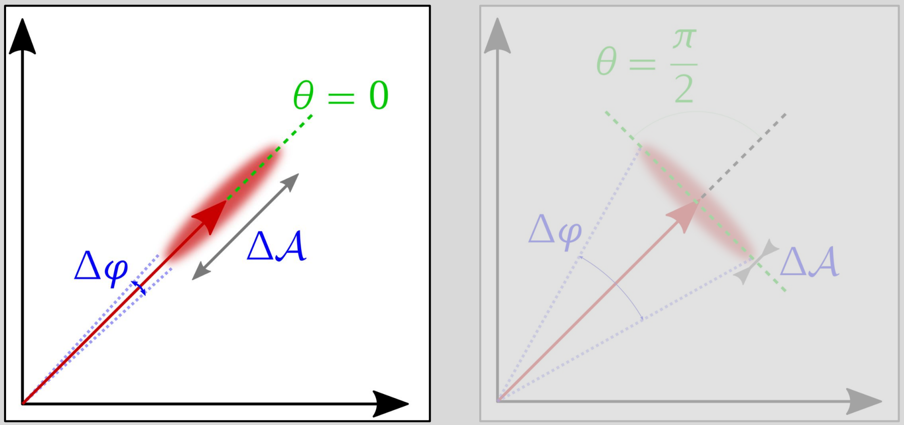



**The total quantum noise <u>increase at low frequencies</u> while <u>decease at high frequencies</u>**

| Phase sq. state | Amplitude sq. state |
|---|---|




- For both cases, the <u>area of uncertainty region is equal to the uncertainty of initial coherent state</u>
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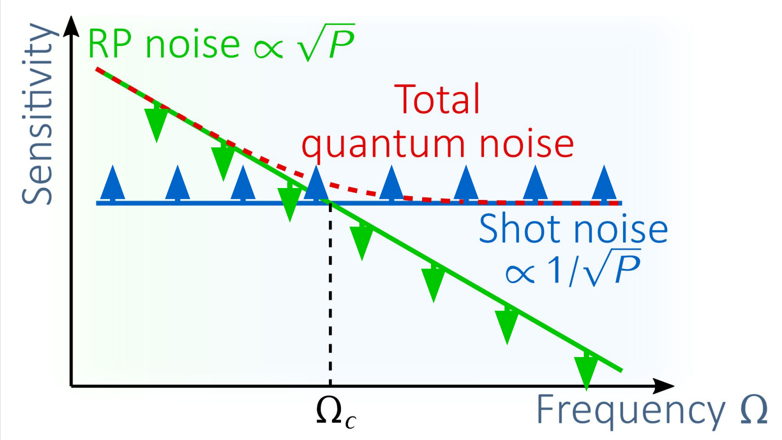
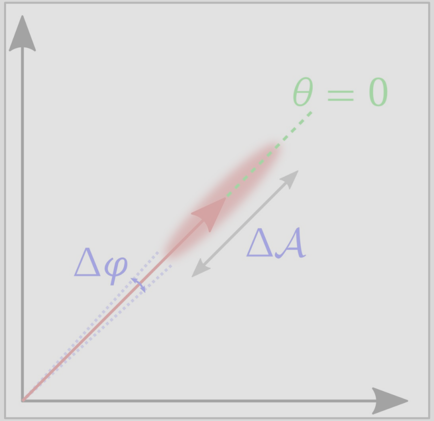
# Squeezed states of light – **amplitude** squeezing

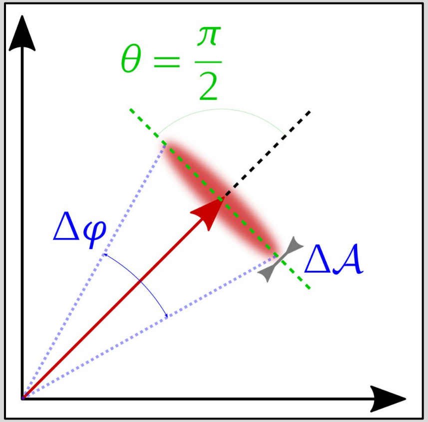



**The total quantum noise decrease at low frequencies while increase at high frequencies**

Phase sq. state




Amplitude sq. state

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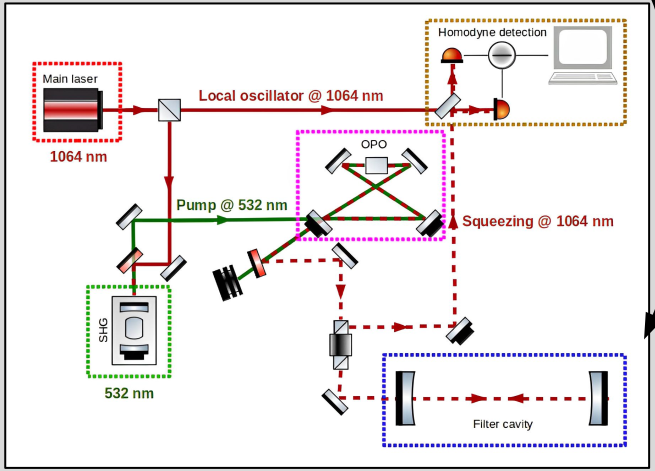
# Frequency dependent squeezing & filter cavity

Context

The squeezed beam produced in the OPO enters into a **« filter cavity »**



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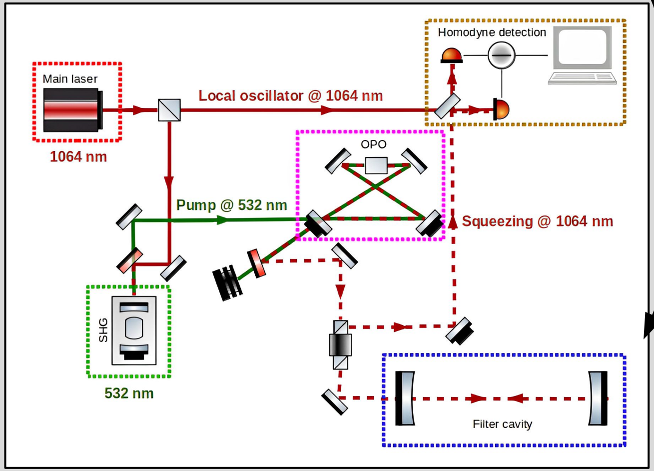


The configuration of the filter cavity define a **frequency $\Omega_t$ around which a transition of squeezing nature (amplitude / phase) occurs**

$$\Omega_t = \frac{\pi c}{\sqrt{2} L F(r_i)}$$

L: cavity length

F: finesse of the cavity (depends on mirrors reflectivities $r_i$)

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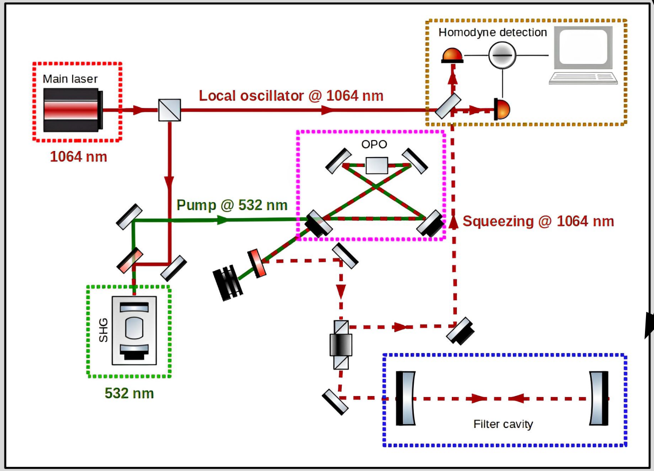


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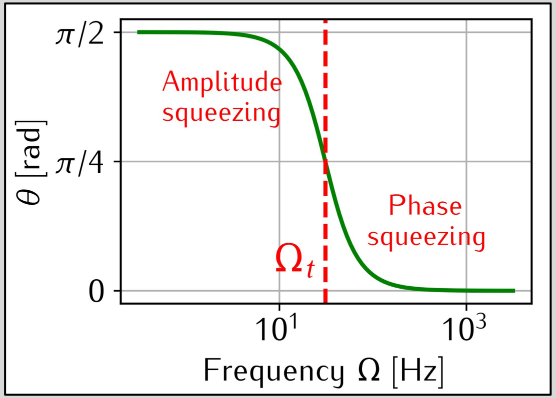


**The squeezing ellipse rotation curve shape is a consequence of filter cavity optical behavior**

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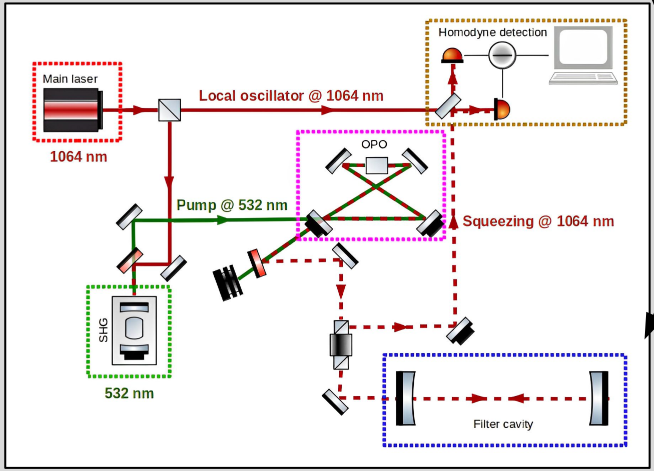


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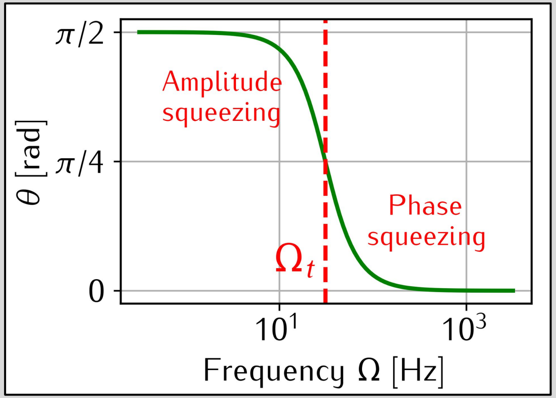
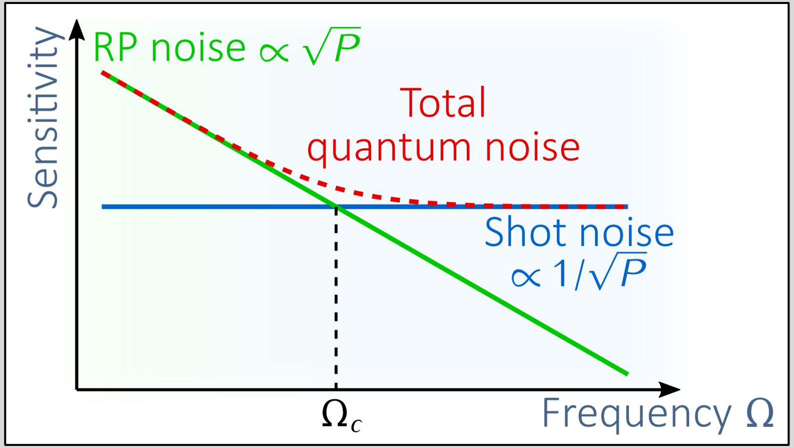



**The squeezing ellipse rotation curve shape is a consequence of filter cavity optical behavior**

Choose filter cavity setting so that $\Omega_t = \Omega_c$

**Resulting situation:**



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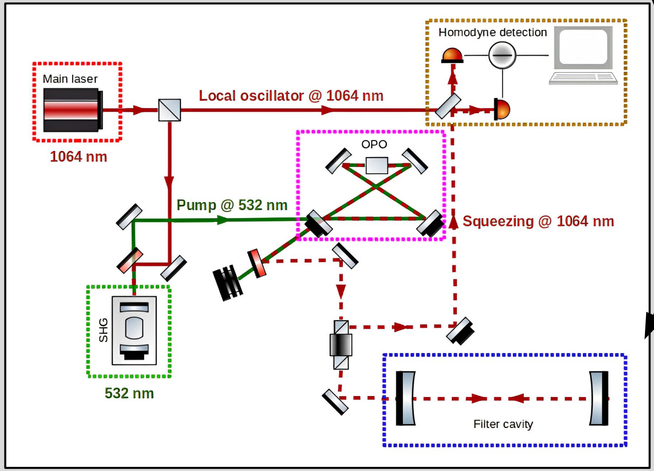


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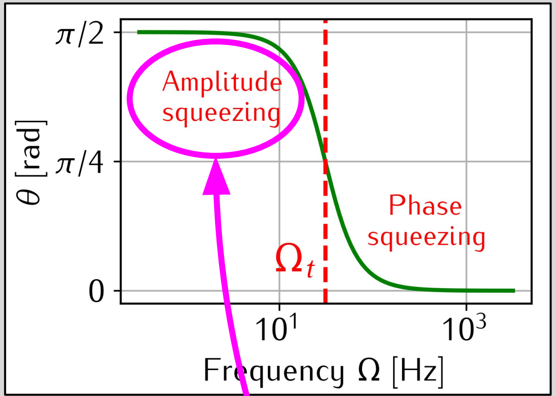
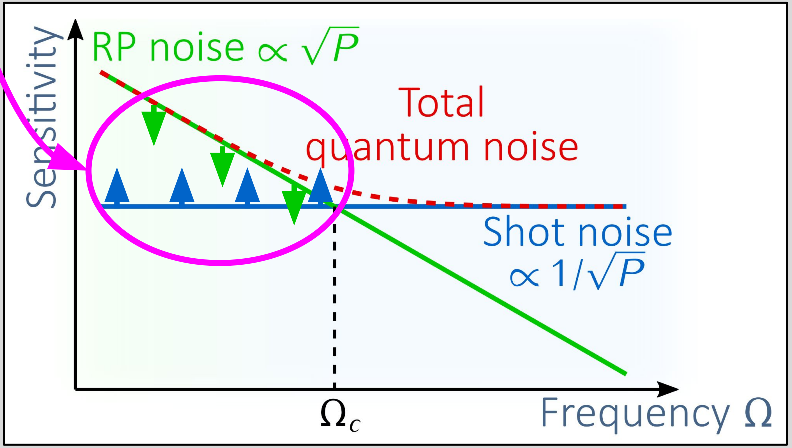



**The squeezing ellipse rotation curve shape is a consequence of filter cavity optical behavior**

Choose filter cavity setting so that $\Omega_t = \Omega_c$

**Resulting situation: below $\Omega_c$ the RP noise is decreased while the shot noise is increased**



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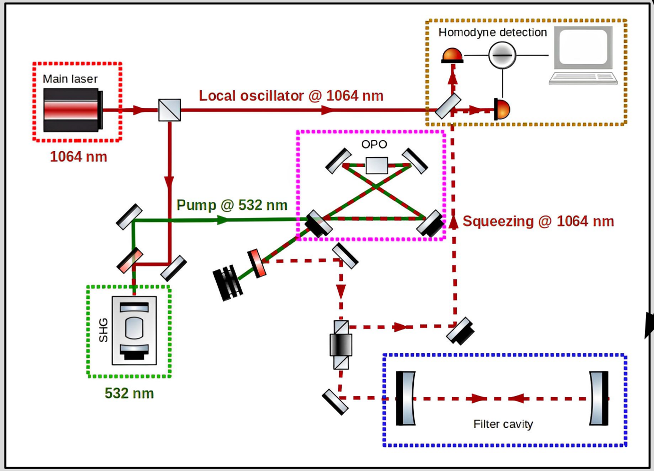


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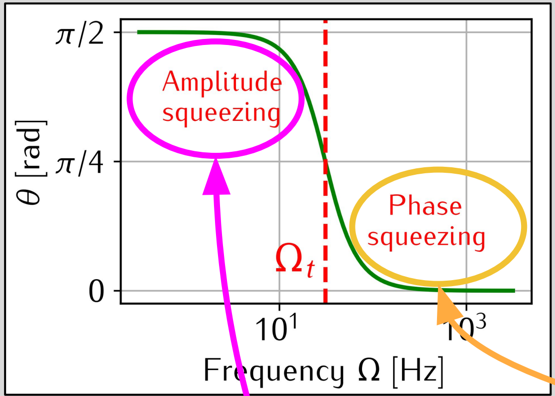
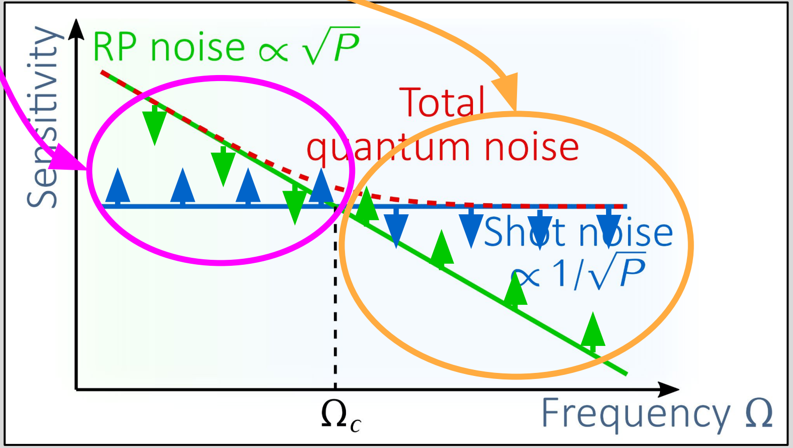



**The squeezing ellipse rotation curve shape is a consequence of filter cavity optical behavior**

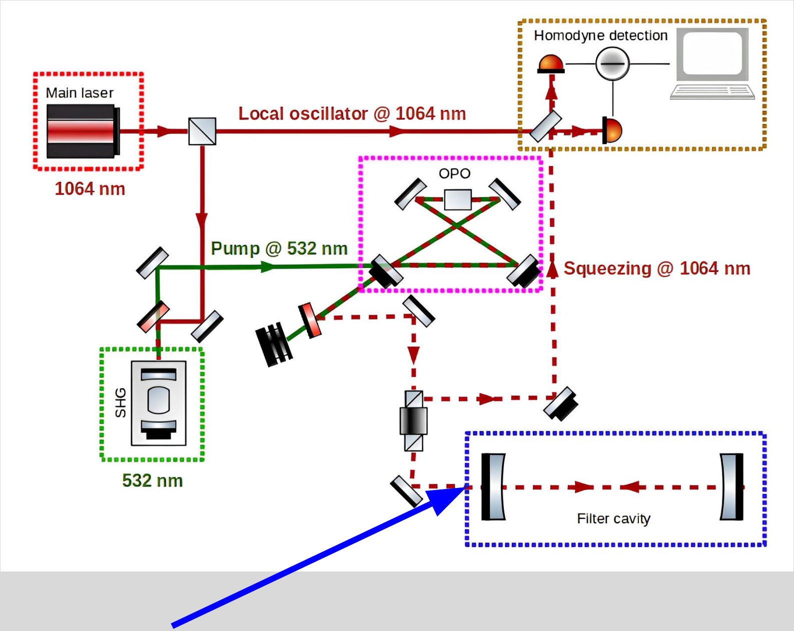
Choose filter cavity setting so that $\Omega_t = \Omega_c$

**Resulting situation: below $\Omega_c$ the RP noise is decreased while the shot noise is increased, above $\Omega_c$ the situation is inverted**

**⇒ QN reduced at all frequencies**



# Frequency dependent squeezing & **three-mirror cavity**




**Fabry-Perot cavity**

# Frequency dependent squeezing & **three-mirror cavity**

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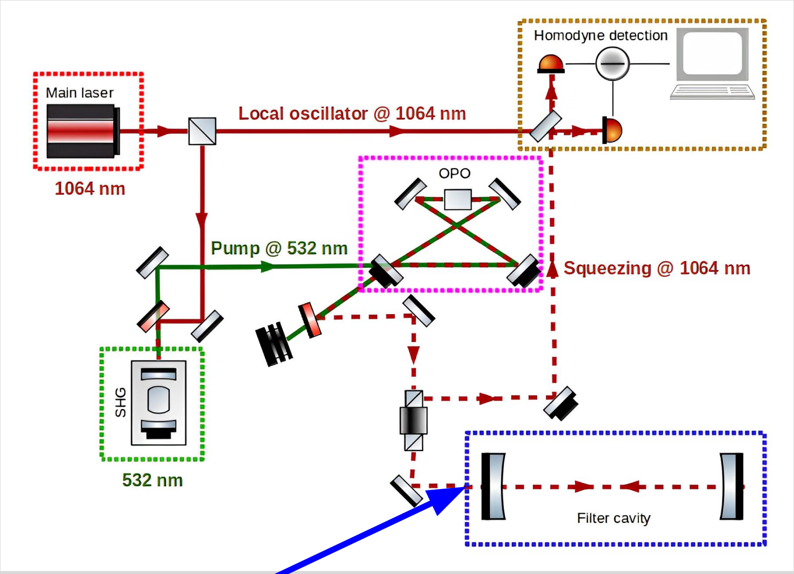


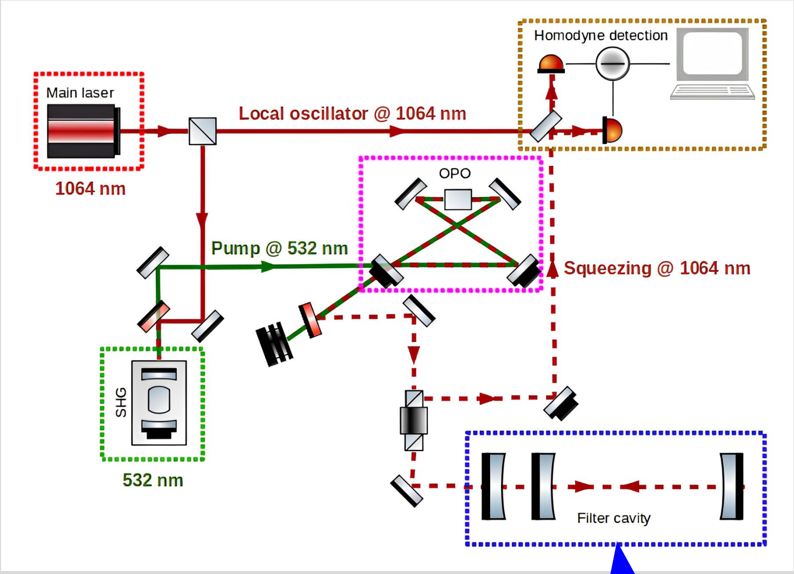


**Fabry-Perot cavity**

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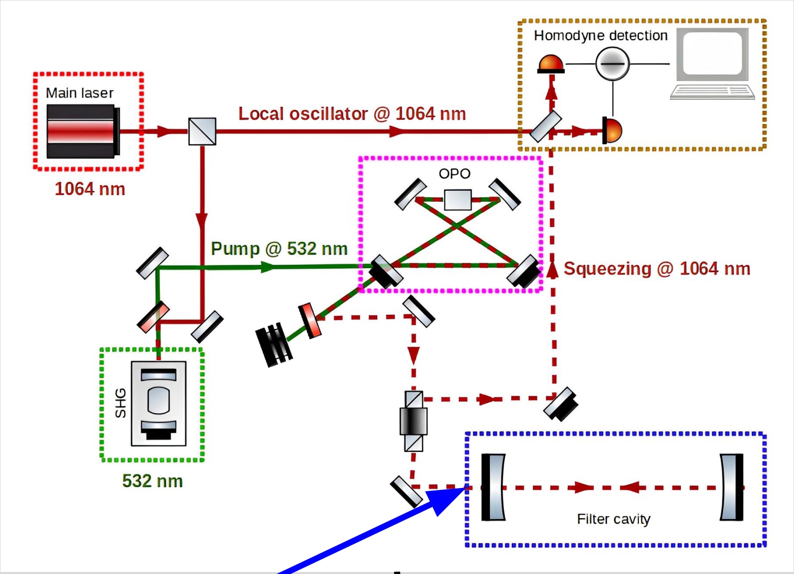


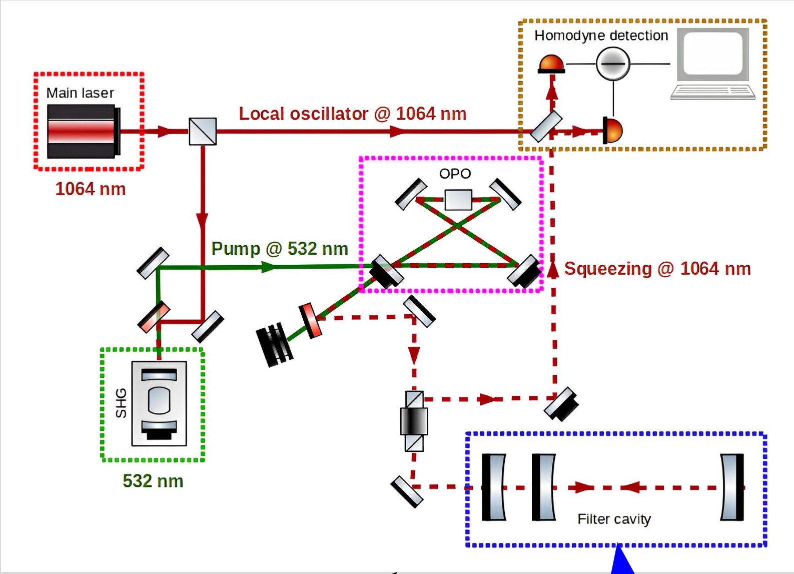


**Fabry-Perot cavity**

**Impact on squeezing properties ?**

**Three-mirror cavity**

Context

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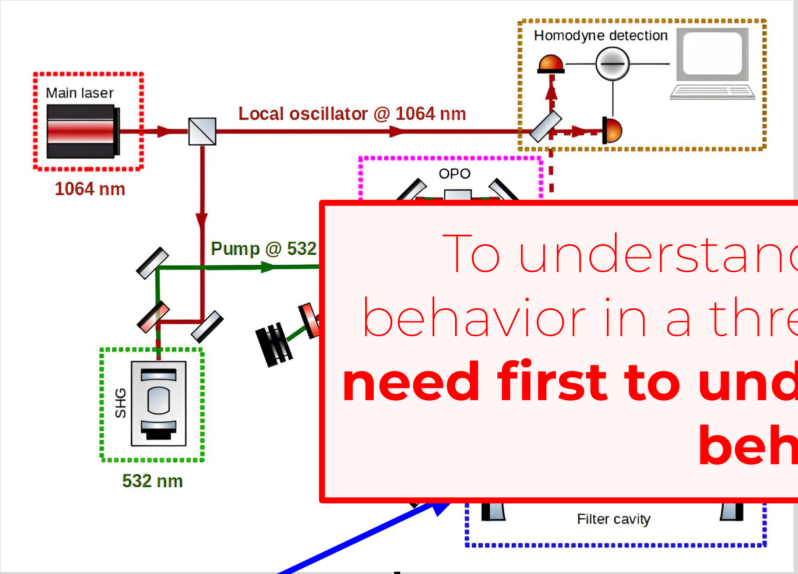
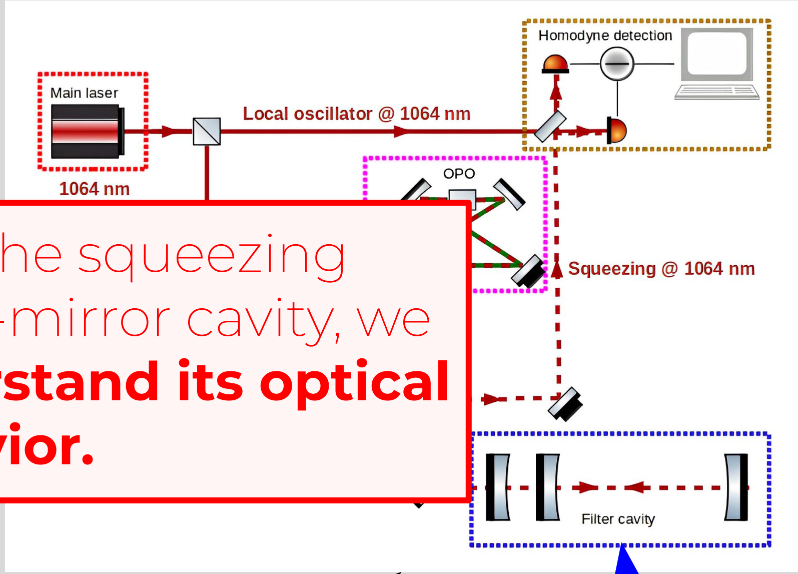



To understand the squeezing behavior in a three-mirror cavity, we **need first to understand its optical behavior.**

**Fabry-Perot cavity**

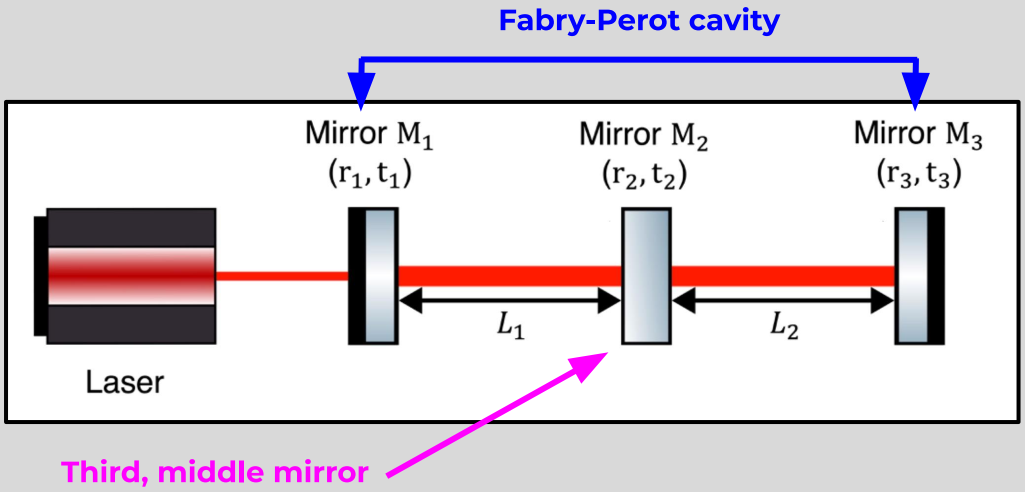
**Impact on squeezing properties ?**

**Three-mirror cavity**

# Three-mirror cavities optics

# Three-mirror cavity

- Simple Fabry-Perot cavity + third, “middle” mirror (two “sub” cavities)
- Three optical resonators
- Despite simple configuration, **non-trivial behavior**



# Modelisation and simulation setup



**To characterize the system: <u>how the global transmissivity and reflectivity of a three-mirror cavity change when we modify the configuration ?</u>**

## 1 - Modelisation

## 2 - Simulations

# Modelisation and simulation setup

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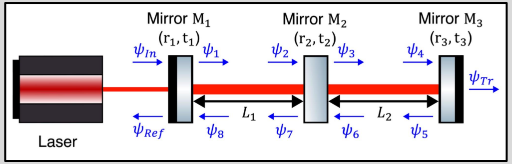


$$\begin{cases} \psi_1 = it_1\psi_{In} + r_1\psi_8 & \psi_2 = \psi_1 e^{-ikL_1} \\ \psi_3 = it_2\psi_2 + r_2\psi_6 & \psi_4 = \psi_3 e^{-ikL_2} \\ \psi_5 = r_3\psi_4 & \psi_6 = \psi_5 e^{-ikL_2} \\ \psi_7 = it_2\psi_6 + r_2\psi_2 & \psi_8 = \psi_7 e^{-ikL_1} \\ \psi_{Ref} = it_1\psi_8 + r_1\psi_{In} & \psi_{Trans} = it_3\psi_4 \end{cases}$$

k: wave-vector; $r_i$ and $t_i$: reflection and transmission coefficients of mirror "i"

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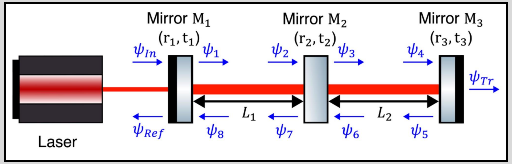


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B - Global reflection and transmission coefficients:

$$r = \frac{\psi_{Ref}}{\psi_{In}} = \frac{r_1 e^{2ik(L_1+L_2)} - r_1 r_2 r_3 e^{2ikL_1} - r_2(r_1^2+t_1^2)e^{2ikL_2} + r_3(r_1^2+t_1^2)(r_2^2+t_2^2)}{e^{2ik(L_1+L_2)} - r_1 r_2 e^{2ikL_2} - r_2 r_3 e^{2ikL_1} + r_1 r_3 (r_2^2+t_2^2)}$$

$$t = \frac{\psi_{Trans}}{\psi_{In}} = \frac{-t_1 t_2 t_3 e^{ik(L_1+L_2)}}{e^{2ik(L_1+L_2)} - r_1 r_2 e^{2ikL_2} - r_2 r_3 e^{2ikL_1} + r_1 r_3 (r_2^2+t_2^2)}$$

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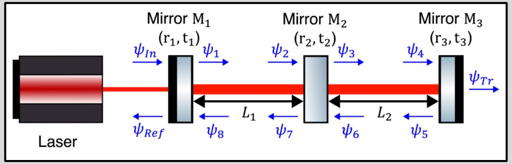


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C - Cavity behavior: complex combination of configuration parameters
➔ **simulations**

## 2 - Simulations

# Modelisation and simulation setup

**To characterize the system: <u>how the global transmissivity and reflectivity of a three-mirror cavity change when we modify the configuration ?</u>**

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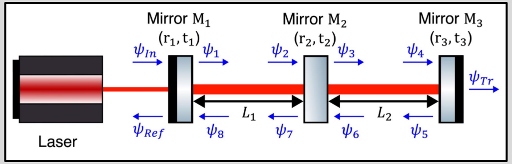


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- Implement global reflection and transmission coefficients in a code

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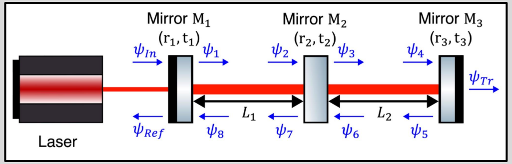


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C - Cavity behavior: complex combination of configuration parameters
→ **simulations**

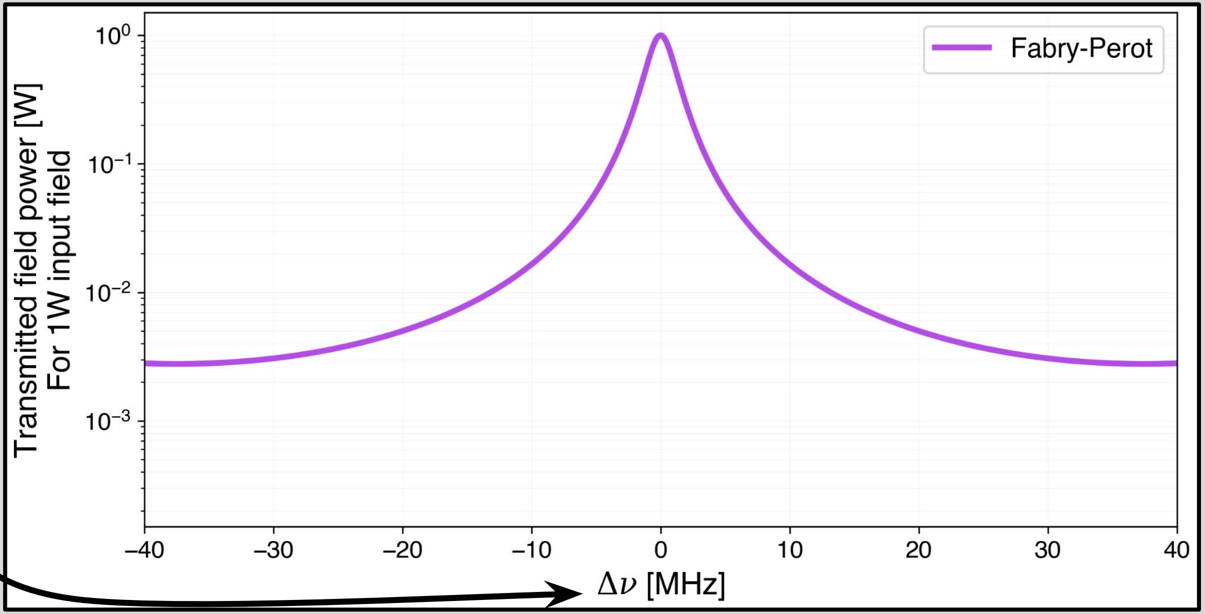
## 2 - Simulations

- Implement global reflection and transmission coefficients in a code
- Parameters to change:
  - **Laser wavelength (wave-vector)**
  - **First, second and third mirrors transmission coefficients**
  - **$L_1$ and $L_2$ distances**

# The doubling of transmission peak

<u>For Fabry-Perot cavity:</u>
**transmission peak for each resonance condition** (cavity length = integer number of half-wavelength)

Scan the input field detuning



# The doubling of transmission peak

Three-mirror cavities optics

For Fabry-Perot cavity:
**transmission peak for each resonance condition** (cavity length = integer number of half-wavelength)

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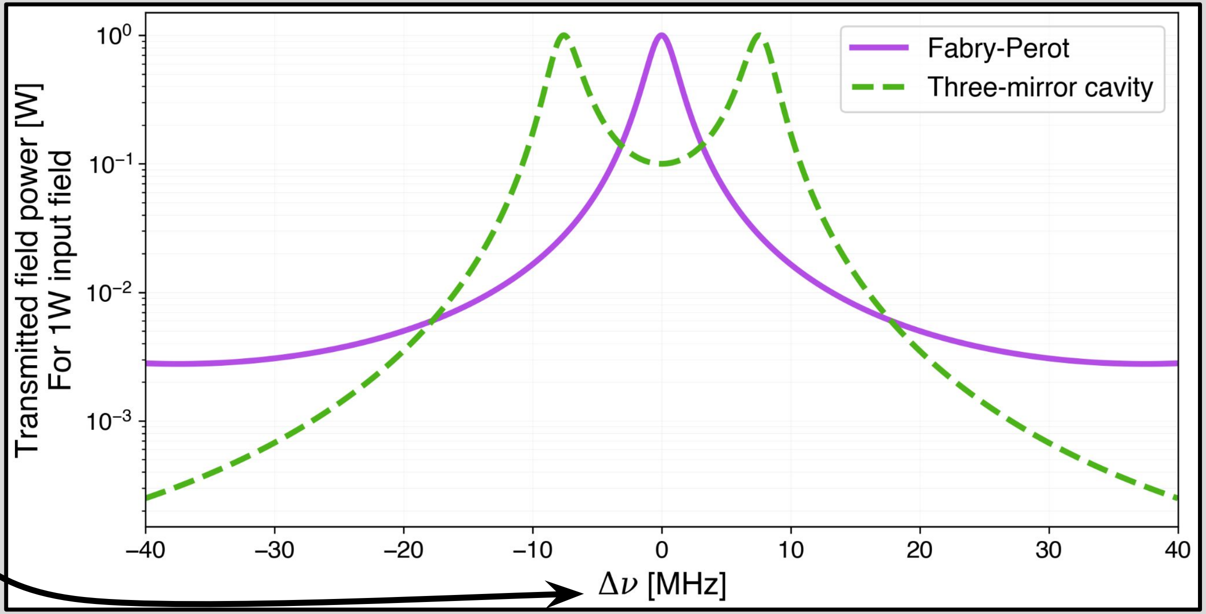


**A <u>three-mirror cavity</u> can show off a <u>doubling of the transmission peak</u>**

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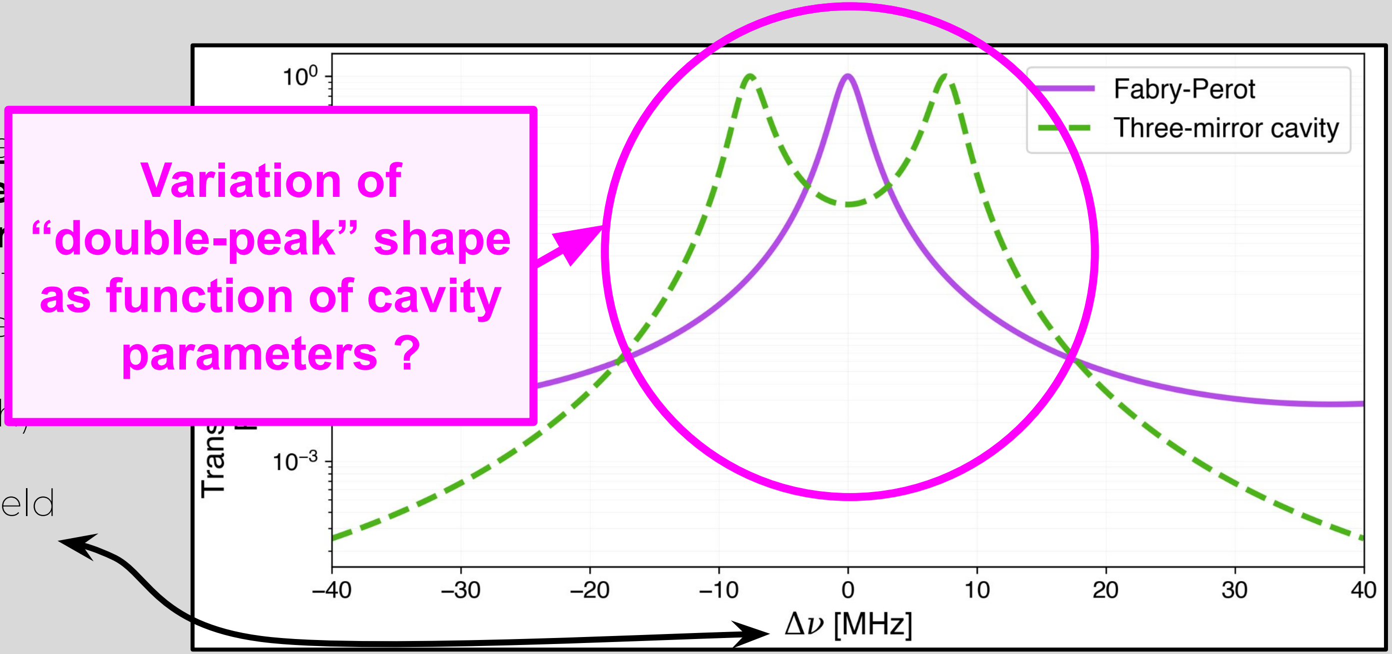
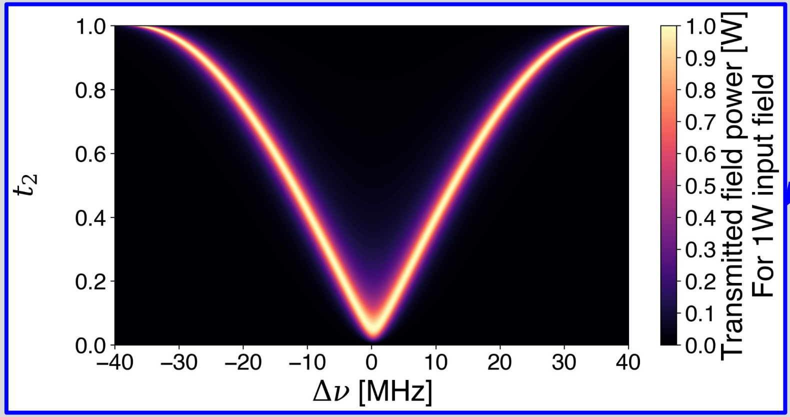



**A <u>three-mirror cavity</u> can show off a <u>doubling of the transmission peak</u>**

# Mirrors transmissivity




**Second mirror** transmissivity → **symmetrical variation of space** between maxima

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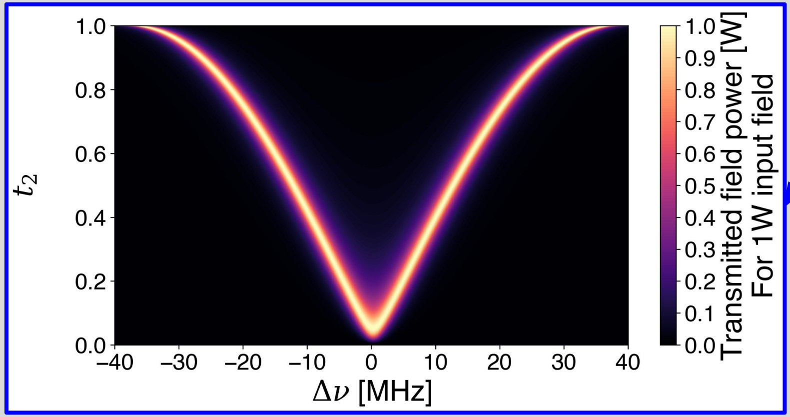


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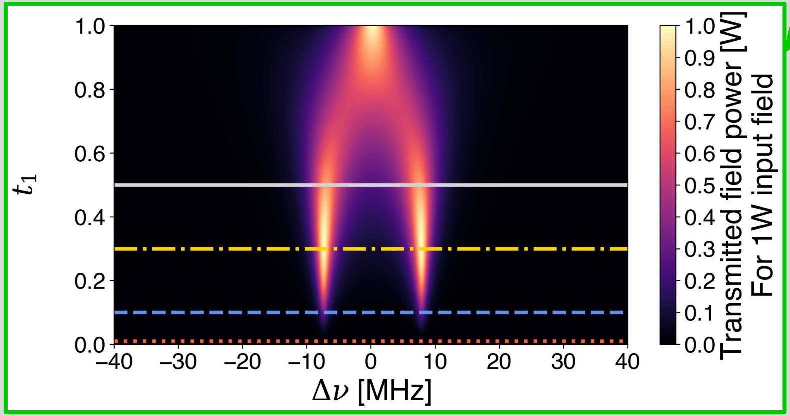
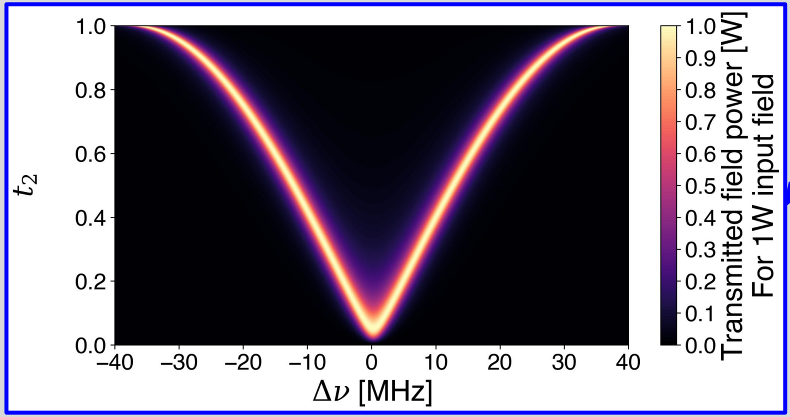



**First (or third) mirror** transmissivity → **sharpen** each maxima

# Mirrors transmissivity




**Second mirror** transmissivity → **symmetrical variation of space** between maxima

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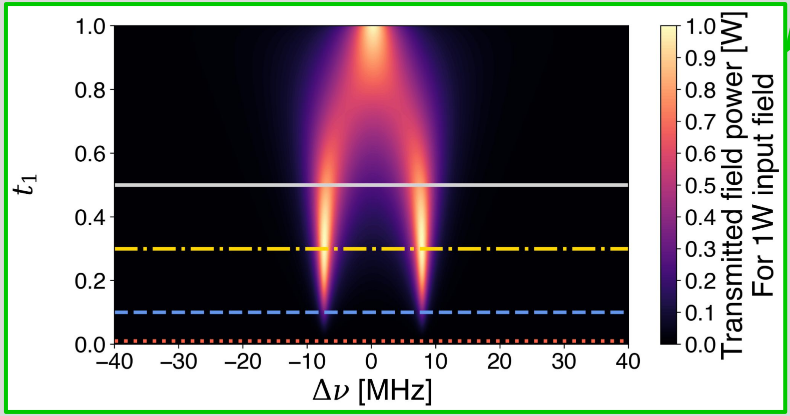
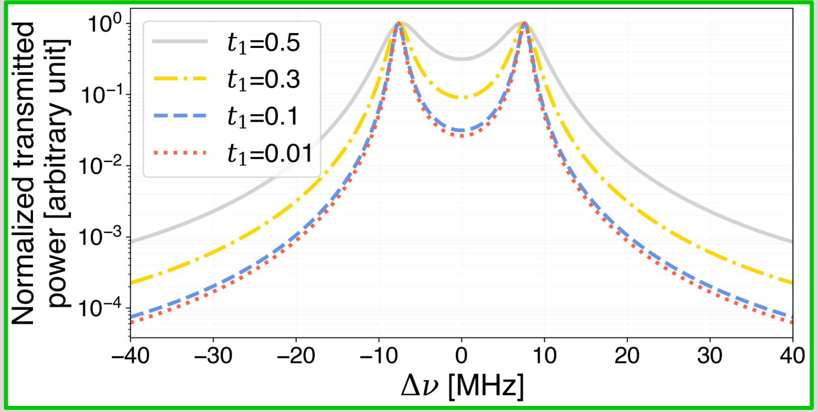
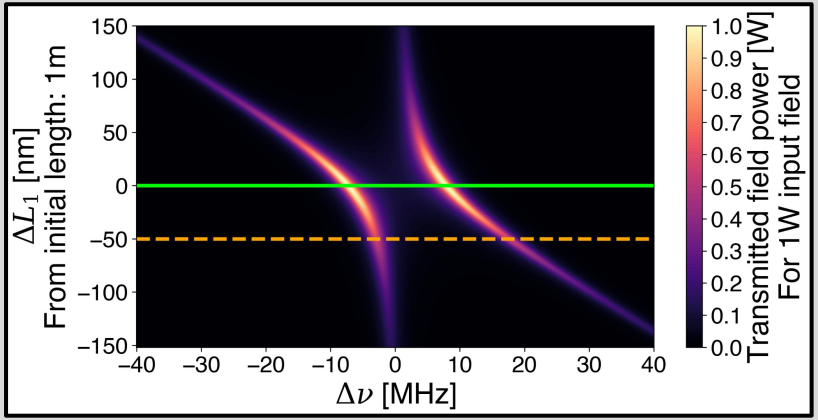
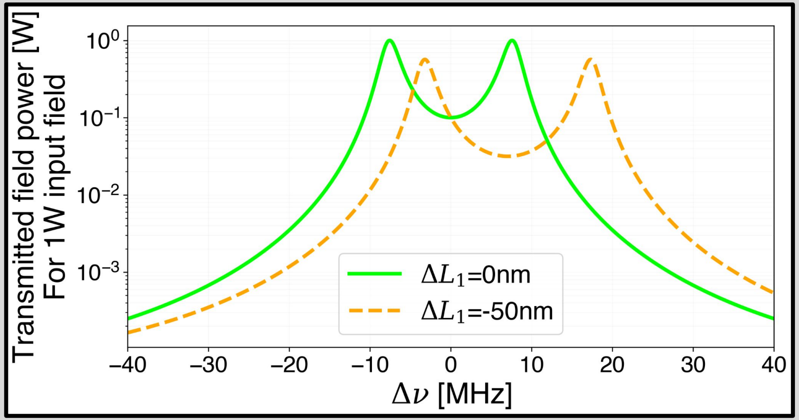


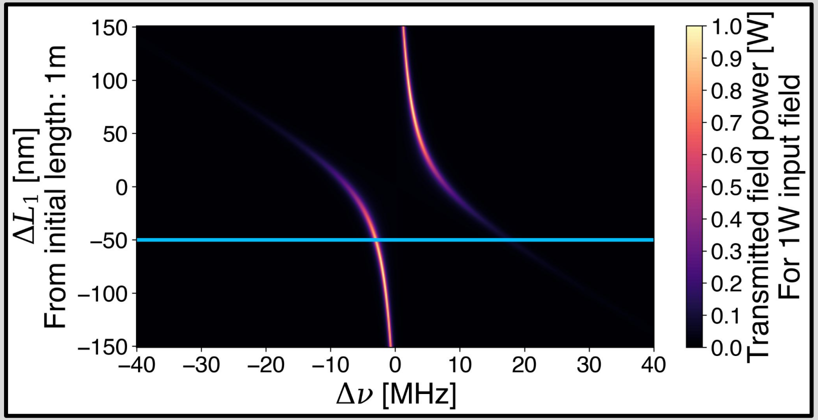
# Microscopic mirrors spacing



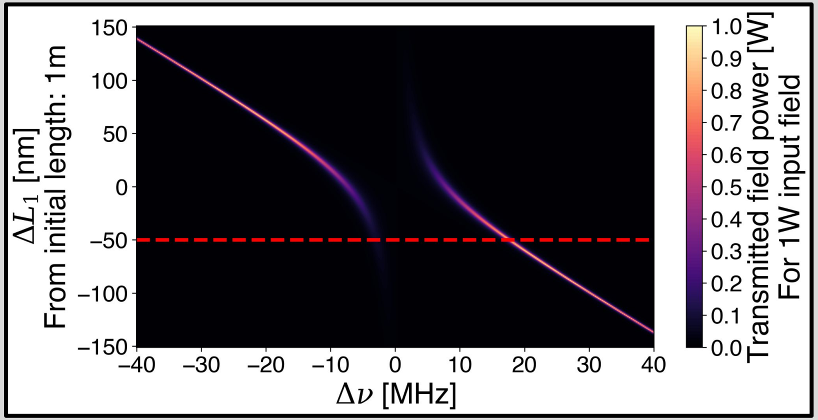



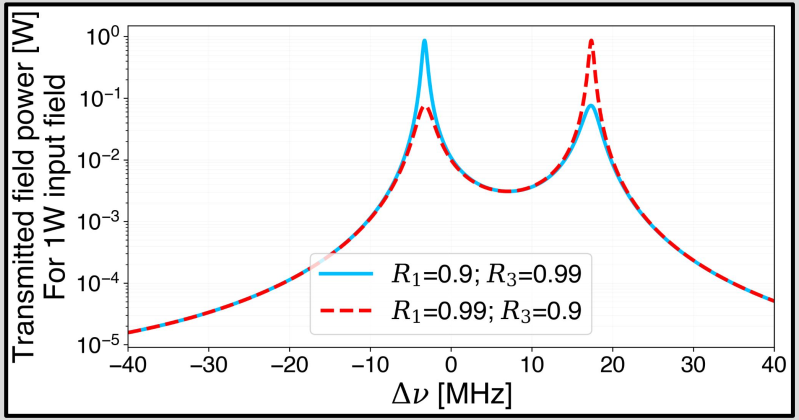

**Asymmetrical variation of maxima spacing (same power in each maxima)**

# Mirrors transmissivity (again)




**Variation of power along resonance lines**



# Conclusion

# Conclusion

- Simulations of three-mirror cavity optics: ✅
  - **Doubling of resonance peak**
  - **Position, height and sharpness of "double-peak" maxima almost completely modulable** by changing the cavity configuration **+ real-time tuning**
    **⇒ Quantum noise reduction for next GW detectors**
  - Full analysis on ArXiv (+ submission to Classical and Quantum Gravity) : Resonant behavior and stability of a linear three-mirror cavity
- Currently:
  - Implementation of a **meter-scale prototype on CALVA platform, IJCLab**
  - **Simulations of squeezing properties** in a three-mirror cavity

# Thank you !

# Backup slides

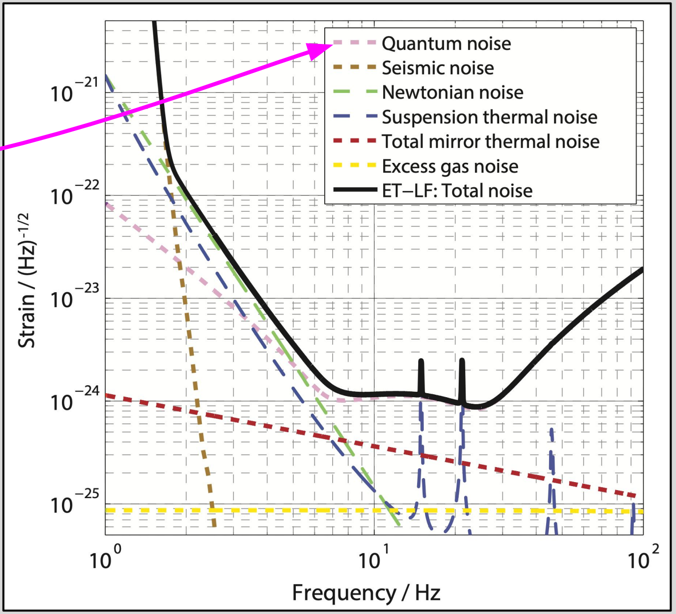
# Frequency dependant squeezing for next generation of GW detectors

Frequency dependant squeezing in current detectors:

⇒ Squeezed beam filtered with a “simple” Fabry-Perot cavity → allow to reduce QN at all frequencies

Future detectors:

⇒ Is it possible to develop a **system for more complex QN shape, Einstein Telescope - Low Frequency (ET-LF) ?**
⇒ Current proposition: two Fabry-Perot cavities in series




Problematic:

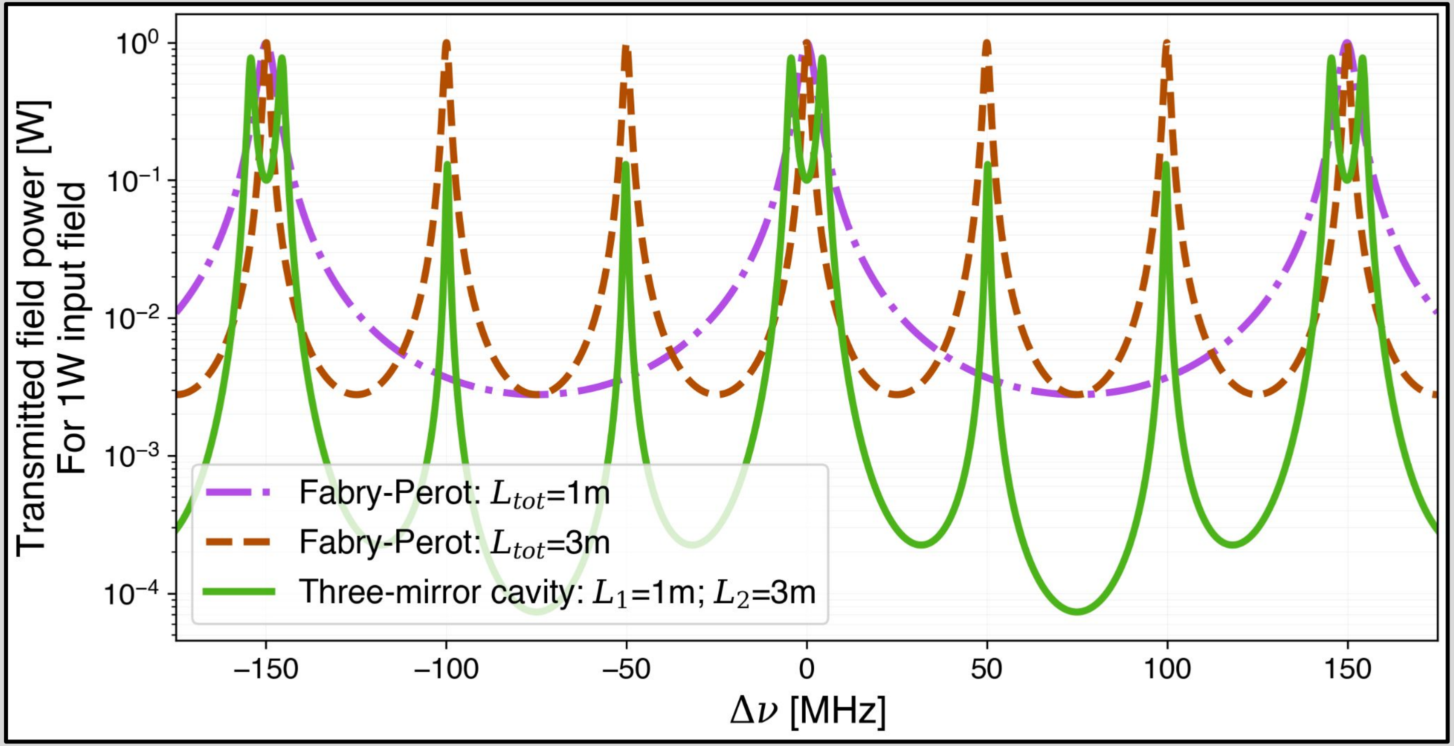
⇒ **Replace the two Fabry-Perot cavities with a three-mirror cavity ?**

To understand the squeezing behavior in a three-mirror cavity, we **need first to understand its optical behavior.**

# Condition for doubling of transmission peak

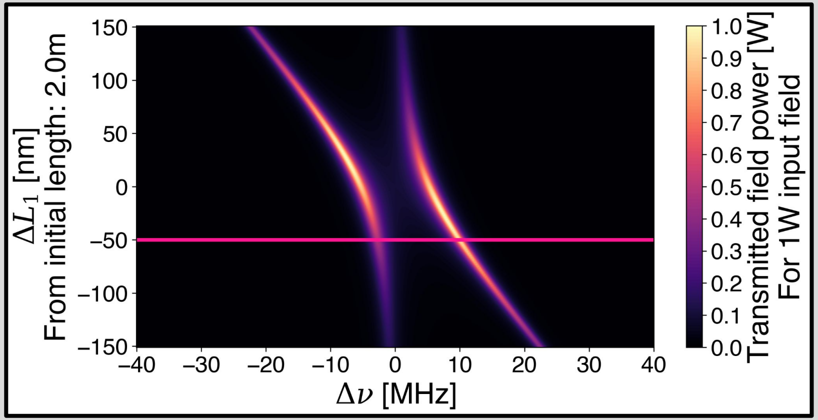


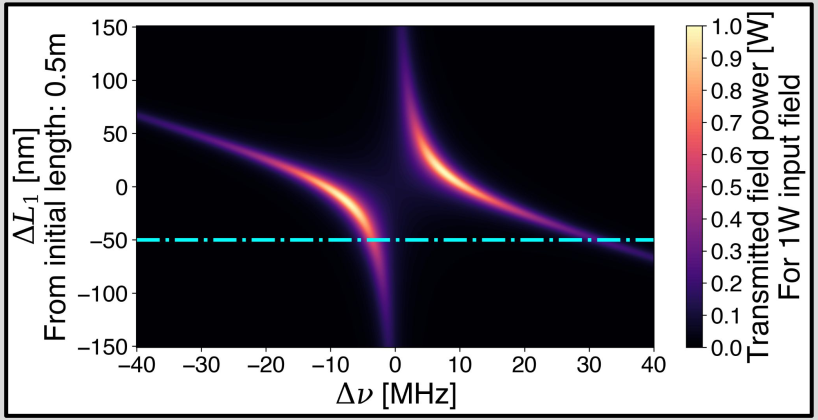
**Each sub-cavity have to be resonant**



# Macroscopic mirrors spacing

Backup slides




**Asymmetrical variation of maxima spacing and power ratio**