Laboratoire Kastler Brossel
Physique quantique et applications

ENS · Sorbonne Université · Collège de France · CNRS

GW detectors — Paris — 17/06/2024

Basics on Quantum Noise and Squeezing

Serge Reynaud
Laboratoire Kastler Brossel (LKB)
CNRS, Sorbonne Université, ENS-PSL, Collège de France

Thanks to the many colleagues with whom I have collaborated or discussed on the topics in this talk

Quantum noise and squeezing

Quantum noise arises from the basic principles of quantum theory. It is a limitation in all high-sensitivity measurements, when other sources of noise are mastered

When the quantum noise level is attained, there still exist ways to reduce the impact of quantum noise on sensitivity by "squeezing" its effect on the observable of interest

Gravitational wave detection has always been a deeply rooted motivation for developing ideas and techniques to attain the quantum noise level and go beyond it if possible

Today, GW detection is an amazing application of squeezing with an eminent place among the many applications proposed along the years

VIRGO
LIGO
KAGRA
GEO ...

Quantum noise and squeezing

As for many quantum phenomena, there are different manners of discussing qualitatively the origin and effects of quantum noise and squeezing

First, a laser beam is a flow of photons. Shot noise is due to the statistics of photon counting and it is the simplest way to understand quantum noise in intensity measurements

Then, a deeper understanding requires to describe quantum fluctuations of the various field observables involved in a more general measurement

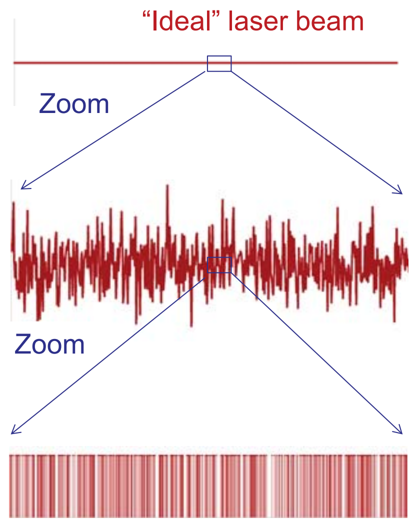
In the talk, I give a simple presentation of this understanding and discuss the principles of its application to the domain of gravitational wave detection with optical interferometers

Standard photon noise

An "ideal" laser beam is described in classical optics as having a constant intensity

But a laser beam is a flow of photons. Shot noise arises from discreteness of light quanta, and is visible when looking closely at time varying intensity

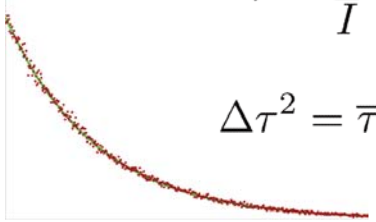
The flow of discrete quanta could be seen with perfect time resolution and perfect intensity measurement



Standard photon noise

Standard shot noise can be described by a flow with independent random delays τ drawn in an exponential distribution; mean delay is the inverse of mean intensity

$$\overline{\tau} = \frac{1}{\overline{I}}$$



$$\Delta\tau^2 = \overline{\tau}^2$$

The number N of photons in any time interval T obeys a Poisson law with the variance equal to the mean number of photons

$$\Delta N_T^2 = \overline{N}_T = \overline{I}T$$

Histogram of a large number of independent random delays and simulated time-varying flow

The intensity noise spectrum is constant (white photon noise)

$$S_I(\omega) = \overline{I}$$

Quantum fluctuations of the field

A deeper and more general understanding requires to describe the quantum fluctuations of the field

$$E(t) = \int \mathcal{E}_\omega \left(a_\omega e^{-\imath\omega t} + a_\omega^\dagger e^{\imath\omega t} \right) \mathrm{d}\omega$$

$$\left[a_\omega, a_{\omega'}^\dagger \right] = 2\pi\delta\left(\omega - \omega'\right)$$

Here $\mathcal{E}_\omega \to \mathcal{E}_{\omega_0}$

The field is an infinite set of modes, each one equivalent to an harmonic oscillator with two “cosine” and “sine” quadratures analogous to position and momentum of a material harmonic …

$$a_1(\Omega) = a_{\omega_0-\Omega}^\dagger + a_{\omega_0+\Omega} \ , \ a_2(\Omega) = \imath \left(a_{\omega_0-\Omega}^\dagger - a_{\omega_0+\Omega} \right)$$

$$\left[a_1(\Omega), a_2(\Omega') \right] = 2\imath \times 2\pi\delta\left(\Omega - \Omega'\right)$$

Quantum fluctuations of the field

For each mode, the two quadrature observables are conjugated; notations without frequencies are focused onto one mode and disregard the mode counting Dirac function $2\pi\ \delta(\Omega\text{-}\Omega')$

$$[a_1, a_2] = 2\imath$$

The variances and covariances of the two observables are constrained by generalized Heisenberg inequalities

$$\Delta a_1^2 = \left\langle (\delta a_1)^2 \right\rangle$$

$$\Delta a_1^2 \Delta a_2^2 - \Gamma_{a_1 a_2}^2 \geq 1$$

$$\Delta a_2^2 = \left\langle (\delta a_2)^2 \right\rangle$$

$$\delta a_j \equiv a_j - \overline{a}_j$$

$$\Gamma_{a_1 a_2} = \langle \delta a_1 \cdot \delta a_2 \rangle$$

The dot indicates a symmetrized product

Quantum fluctuations of the field

Minimal uncertainty states are defined by equality in the Heisenberg inequality

$$\Delta a_1^2 \Delta a_2^2 - \Gamma_{a_1 a_2}^2 = 1$$

This definition contains the vacuum as the particular case with null mean field, minimal uncertainties and symmetry in the exchange of the two quadratures a_1 and a_2

$$\overline{a}_1 = \overline{a}_2 = 0 \ , \ \Delta a_1 = \Delta a_2 = 1 \ , \ \Gamma_{a_1 a_2} = 0$$

Vacuum is defined by an equation

$$(a_1 + \imath a_2) \left| \Psi_{\text{vac}} \right\rangle = 0$$

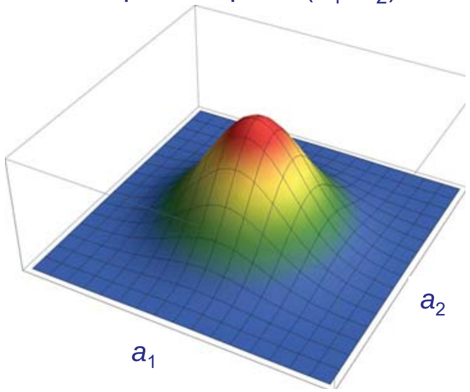
which is solved by a Gaussian function in the representation a_1 or a_2 as each quadrature is a derivative with respect to the other one

Wigner representation

The density matrix is a function of two parameters in a_1-space (or a_2-space with Fourier transform from one space to the other). The Wigner function is a mixed representation in phase space (a_1, a_2)

Variances and covariances in this distribution match quantum (co)variances defined with symmetrized ordering for non-commuting observables

The Wigner function is positive for all minimal uncertainty states (this is not true for general states)



Wigner representation of vacuum fluctuations in phase space

Quantum fluctuations of the field

The general minimal uncertainty state has one component "squeezed" and the conjugated component "anti-squeezed"

Rotated quadrature components can be defined

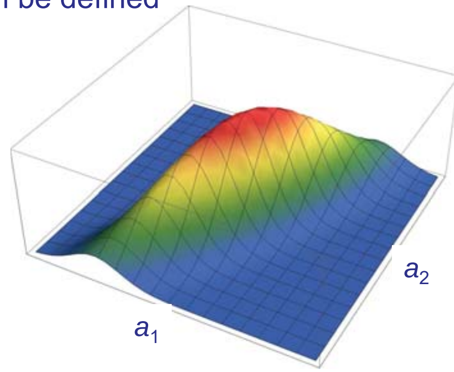
$$a_1^\theta = a_1 \cos\frac{\theta}{2} + a_2 \sin\frac{\theta}{2}$$

$$a_2^\theta = a_2 \cos\frac{\theta}{2} - a_1 \sin\frac{\theta}{2}$$

for which variances are given by a squeezing or anti-squeezing factor

$$\Delta a_1^\theta = e^{-\zeta} \; , \; \Delta a_2^\theta = e^{\zeta}$$

$$\Gamma_{a_1^\theta a_2^\theta} = 0$$

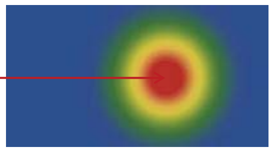


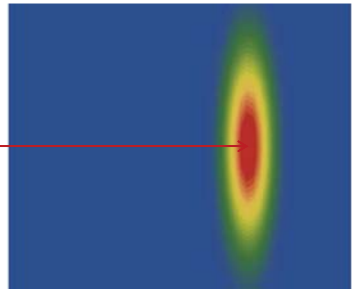
Wigner representation of squeezed vacuum fluctuations

Quantum fluctuations of the field

Minimal uncertainty states can be centered on non null mean fields

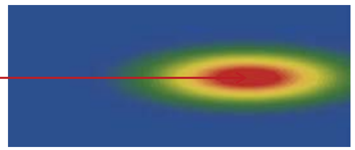
Displacing vacuum state defines the "coherent states"





Wigner representations for coherent, amplitude-squeezed or phase-squeezed states

Displacing squeezed states produces "amplitude-squeezed" or "phase-squeezed" states depending on the relative phase of the mean field and the axis of uncertainty ellipse



Interpretation of photon noise

Intensity measurements directly probe amplitude fluctuations

At a high-intensity field with mean value aligned along quadrature a_1, the intensity noise spectrum is proportional to mean intensity and noise spectrum of quadrature a_1

$$S_I(\Omega) = \bar{I}\, S_{a_1}(\Omega)$$

Observation of standard photon noise (white noise spectrum for intensity) implies that the fluctuations of the amplitude a_1 are vacuum fluctuations over the frequency interval on which the measurement is done

Squeezing photon noise is possible but this implies to degrade phase noise

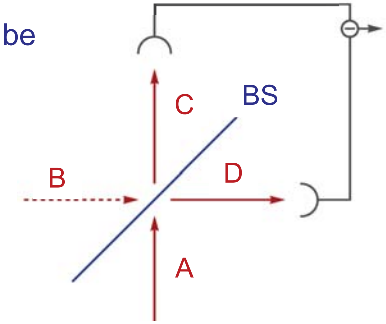
$$S_{a_2}(\Omega) \geq \frac{1}{S_{a_1}(\Omega)}$$

Measuring quadrature fluctuations

The fluctuations of any quadrature can be measured after a beam splitter

$$c = \frac{a+b}{\sqrt{2}} \quad , \quad d = \frac{b-a}{\sqrt{2}}$$

$$c^\dagger c - d^\dagger d = a^\dagger b + b^\dagger a$$



For a high-intensity laser in the port A with mean field along a_1, the noise spectrum on the difference of intensities after the beam splitter is proportional to mean laser intensity and noise spectrum of the quadrature b_1 of the field in the second input port B

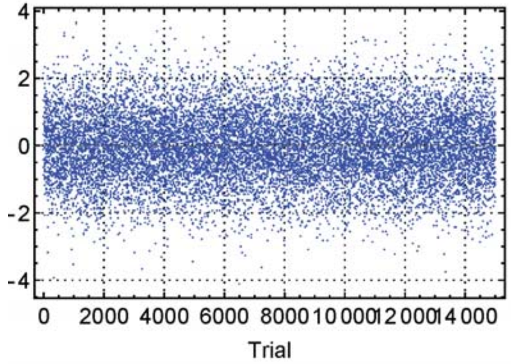
$$I_{CD} = I_C - I_D \quad , \quad S_{I_{CD}}(\Omega) = \overline{I}_A \, S_{b_1}(\Omega)$$

Measuring vacuum fluctuations

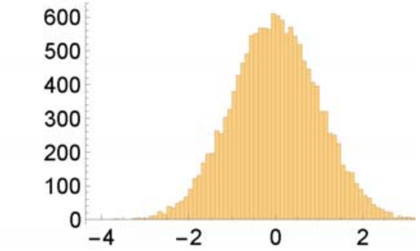
The figure shows experimental homodyne measurement of vacuum fluctuations of one quadrature of the field, with the measurement repeated a large number of times

Thanks to Claude Fabre and Adrien Dufour (LKB)

Values found in 16000 repeated measurements

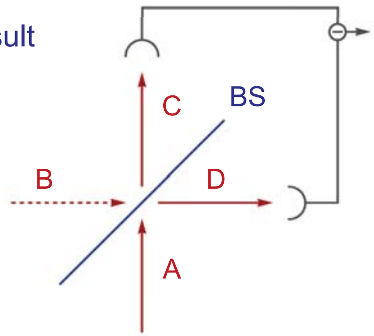


Histogram showing the Gaussian distribution of values



Regularized statistics of beam splitting

Let me stress the significance of this result in terms of statistics of beam splitting



When vacuum fluctuations enter the port B, the noise spectrum is

$$S_{I_{CD}} = \overline{I}_A$$

Each photon hitting the beam splitter from input port A goes randomly in ports C or D independently of what other photons do

When fluctuations of the quadrature b_1 are squeezed, a more regular splitting statistics is produced

$$S_{I_{CD}}(\Omega) = \overline{I}_A \, S_{b_1}(\Omega)$$

Producing squeezing

Minimum uncertainty states are generated from any one of these states (say the vacuum) by linear canonical transformations (linear transformations preserving the canonical commutator)

$$U^\dagger a U = a\cosh(\zeta) - a^\dagger \sinh(\zeta) e^{\imath\theta}$$

$$U^\dagger a^\dagger U = a^\dagger \cosh(\zeta) - a \sinh(\zeta) e^{-\imath\theta}$$

$$U^\dagger a_1^\theta U = a_1^\theta e^{-\zeta} \quad , \quad U^\dagger a_2^\theta U = a_2^\theta e^{\zeta}$$

$$U = \exp\left(\frac{\zeta}{2}\left(e^{-\imath\theta} a^2 - e^{\imath\theta} a^{\dagger\, 2}\right)\right)$$

Transformations are unitary squeezing operators which are mapped to classical transformations of Wigner distribution

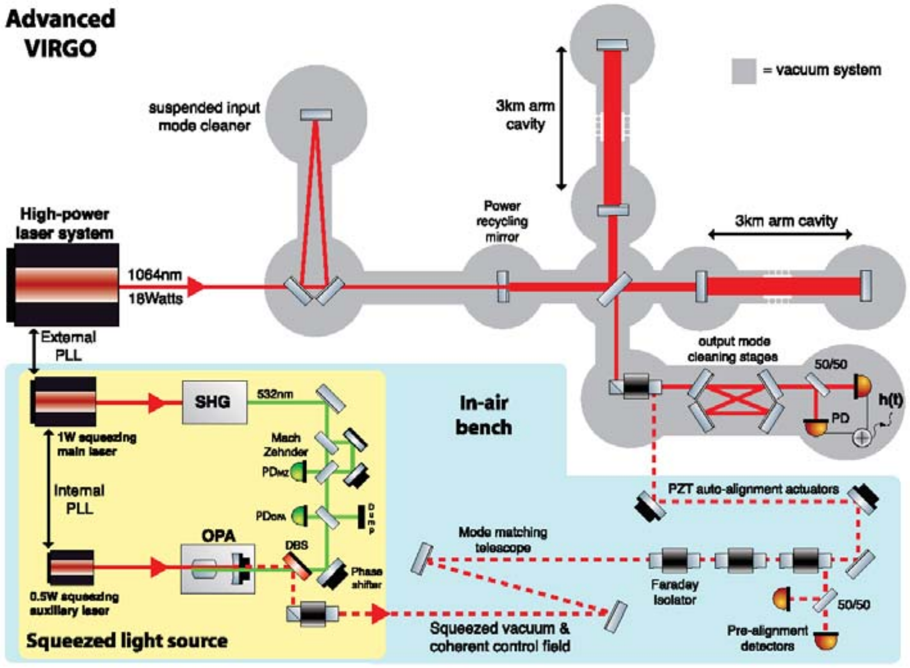
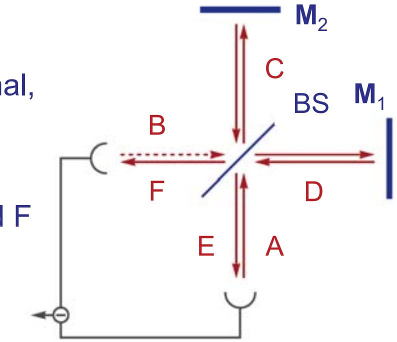


Fig. 1 in Acernese and the Virgo Collaboration, PRL **123** 231108 (2019)

Using squeezing in interferometers

For discussing quantum noise in the interferometric detection of a phase signal, we consider a configuration with high-intensity laser light entering port A, and intensity difference in output ports E and F measured (balanced for a null phase)



When vacuum fluctuations enter the port B, the noise spectrum on the intensity difference is standard splitting noise

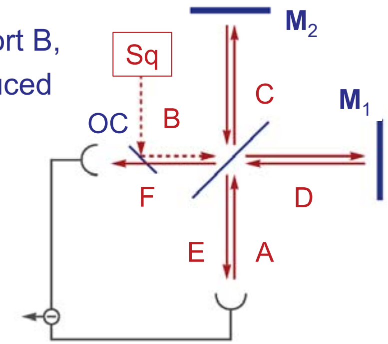
$$S_{I_{EF}} = \bar{I}_A$$

The phase signal is measured as a length ℓ with photon (splitting) noise obtained as inversely proportional to the intensity

$$S_{\ell_{PN}} = \frac{1}{4k_0^2 \bar{I}_A}$$

Using squeezing in interferometers

When squeezed fluctuations enter the port B, a more regular splitting statistics is produced for photons entering the port A (interferometer seen as a beam splitter)



The photon noise on the length measurement can thus be reduced

$$S_{\ell_{PN}}(\Omega) = \frac{S_{b_2}(\Omega)}{k_0^2 \bar{a}_1^2} \qquad \bar{a}_1 = \sqrt{4\bar{I}_A}$$

Using a squeezing factor improves the sensitivity as if the mean laser intensity was increased by the same factor

C.M. Caves, Phys. Rev. D **23** 1698 (1981)

Using squeezing in interferometers

Illustrating the interest of squeezing for improving the SNR for the detection of a modulated phase signal

(simulation by B. Hage, Albert Einstein Institute)

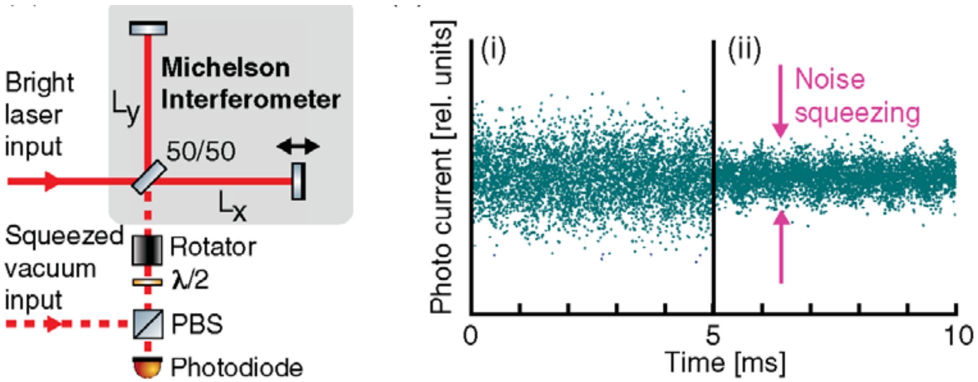


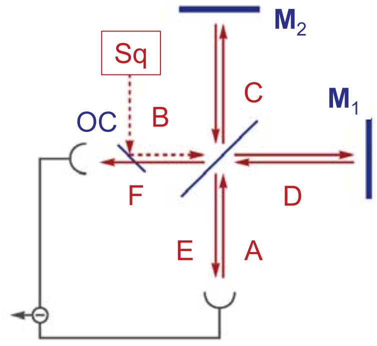
Figure 1 in Barsotti et al, Rep. Progr. Phys. **82** 016905 (2019)

Radiation pressure noise

Radiation pressure fluctuations induce random motions of the mirrors

$$\delta\ell_{\mathrm{RP}}(\Omega) = \frac{2\hbar k_0\, I_{CD}(\Omega)}{-M\Omega^2}$$

$$I_{CD} = I_C - I_D$$



The radiation pressure noise is given by the difference of intensities on the two mirrors

$$S_{I_{CD}}(\Omega) = \overline{I}_A\, S_{b_1}(\Omega)$$

It is determined by fluctuations of the quadrature conjugated to that determining photon noise

$$S_{\ell_{\mathrm{RP}}} = \frac{\hbar^2 k_0^2 \overline{a}_1^2}{M^2\Omega^4} S_{b_1}$$

Acernese and the Virgo Collaboration, PRL **125** 131101 (2020)

The standard quantum limit

When considering the effects of photon noise and radiation pressure as uncorrelated, one computes an optimal sensitivity called the standard quantum limit (SQL)

$$S_{\ell_{\mathrm{PN}}} = \frac{S_{b_2}}{k_0^2 \overline{a}_1^2} \qquad S_{\ell_{\mathrm{RP}}} = \frac{\hbar^2 k_0^2 \overline{a}_1^2}{M^2\Omega^4} S_{b_1} \geq \frac{\hbar^2}{M^2\Omega^4}\frac{1}{S_{\ell_{\mathrm{PN}}}}$$

$$S_{\ell_{\mathrm{PN}}} + S_{\ell_{\mathrm{RP}}} \geq S_{\mathrm{SQL}} \quad , \quad S_{\mathrm{SQL}} = \frac{2\hbar}{M\Omega^2}$$

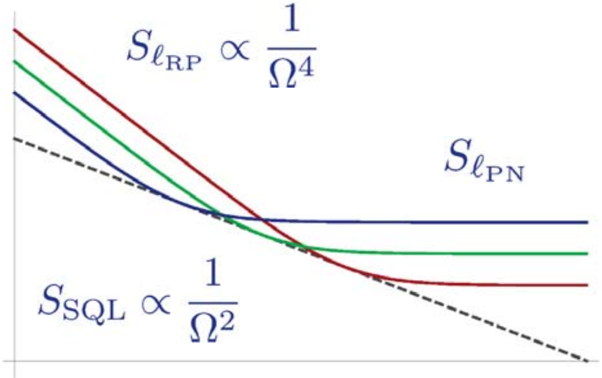
The standard quantum limit has for a long time been considered as a general ultimate accuracy for any position sensing of a free mass.

$$\Delta\ell \geq \sqrt{\frac{\hbar\tau}{M}}$$

C.M. Caves, Phys. Rev. D **23** 1698 (1981)

The standard quantum limit

Representation of the variation with frequency for various values of the ratio $\dfrac{S_{b_2}}{\overline{a}_1^2}$



But the standard quantum limit is not an ultimate accuracy for position sensing because the effects of photon noise and radiation pressure may be correlated.

M.T. Jaekel, S. Reynaud, Eur. Phys. Lett. **13** 301 (1990)

Beyond the standard quantum limit

The effects of photon noise and radiation pressure are due to the two quadrature observables and should be added in a coherent linear superposition

$$\delta\ell_{\mathrm{PN}}(\Omega) + \delta\ell_{\mathrm{RP}}(\Omega) = \frac{b_2(\Omega)}{k_0\overline{a}_1} - \frac{\hbar k_0 \overline{a}_1}{M\Omega^2} b_1(\Omega)$$

If the other sources of noise are mastered, it is in principle possible to squeeze the fluctuations of the quadrature observable which determines the noise on the measurement of the phase (its frequency-dependence has to be taken into account)

M.T. Jaekel, S. Reynaud, Eur. Phys. Lett. **13** 301 (1990)

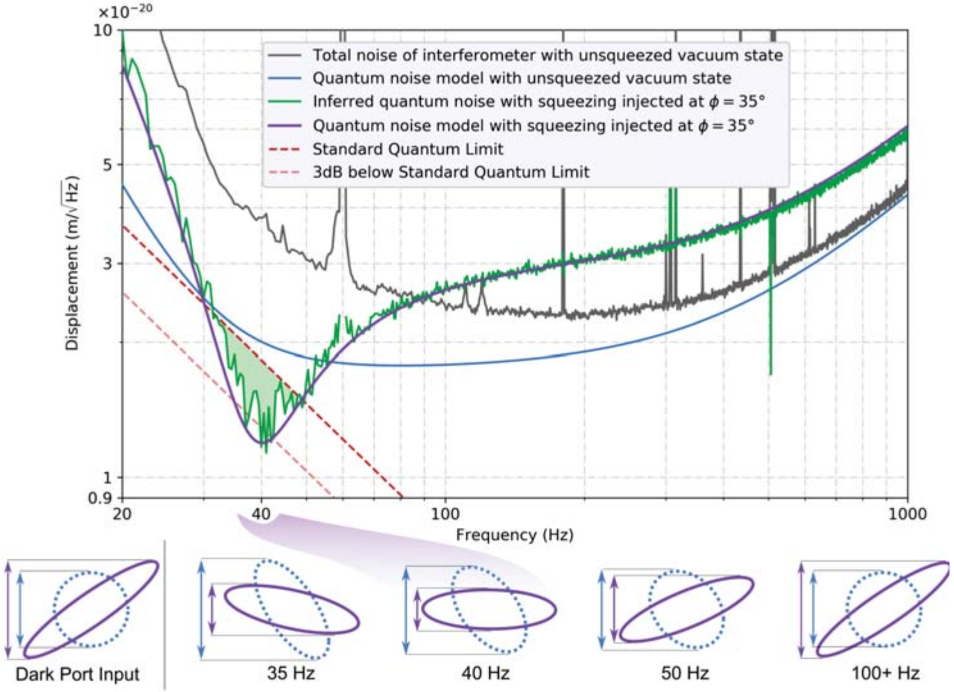


Figure 2 in Yu and the Ligo Collaboration, Nature **583** 43 (2020)

Beyond the standard quantum limit

As radiation pressure acts directly on the motions of mirrors, correlations of photon noise and radiation pressure imply that motions of mirrors and field fluctuations are correlated

It would be great to be able to prove experimentally that the motions of the large-macroscopic-mass mirrors in the interferometric GW detectors are quantum !

Macroscopic quantum mechanics in GW observatories …
R. Schnabel, M. Korobko, AVS Quantum Sci. **4** 014701 (2022)

serge.reynaud@lkb.upmc.fr

Thanks

Introduction à la réduction du bruit quantique
S. Reynaud, Annales de Physique **15** 63-162 (1990)

Quantum Fluctuations in Optical Systems
S. Reynaud, A. Heidmann, E. Giacobino, C. Fabre,
Progress in Optics **30** 1-85 (1992)

Quantum noise of optical fields: a semi-classical approach
C. Fabre, S. Reynaud,
Les Houches Summer School **53** 675-711 (1992)

Quantum-mechanical noise in an interferometer
C.M. Caves, Phys. Rev. D **23** 1698 (1981)

Quantum limits in interferometric measurements
M.T. Jaekel, S. Reynaud, Eur. Phys. Lett. **13** 301 (1990)

Squeezed states of light: an (incomplete) survey of experimental progress and prospects
H. J. Kimble, Physics Reports **219** 227 (1992)

30 years of squeezed light generation
U.L Andersen, T. Gehring, C. Marquardt, G. Leuchs,
Physica Scripta **91** 053001 (2016)

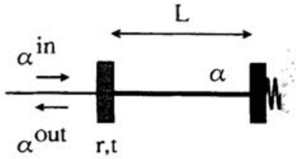
Squeezed states of light and their applications in laser interferometers
R. Schnabel, Physics Reports **684** 1–51 (2017)

Squeezed vacuum states of light for GW detectors
. L. Barsotti, J. Harms, R. Schnabel,
Reports on Progress in Physics **82** 016905 (2019)

Opto-mechanical field squeezing

Radiation pressure can be used to squeeze the field in the cavity

Motion of the mirrors induced by radiation pressure squeezes the cavity field and the output field



C. Fabre, M. Pinard, S. Bourzeix, A. Heidmann, E. Giacobino, S. Reynaud, Phys. Rev. A **49** 1337 (1994)

This field squeezing mechanism has been observed in opto-mechanical experiments

H. Safavi-Naeini, S. Gröblacher, J. T. Hill, J. Chan, M. Aspelmeyer, O. Painter, Nature **500** 185 (2013)

T. P. Purdy, P.-L. Yu, R.W. Peterson, N. S. Kampel, C. A. Regal, Phys. Rev. X **3** 031012 (2013)

W. Hvidtfelt, P. Nielsen, Y. Tsaturyan, C. Bo Møller, E. S. Polzik, A. Schliesser, Proc. Nat. Acad. Sci. **114** 62 (2016)

Cavity optomechanics

M. Aspelmeyer, T. J. Kippenberg, F. Marquardt Rev. Mod. Phys. **86** 1391 (2014)

Recent advances toward mesoscopic quantum optomechanics

M. Croquette, S. Deléglise, T. Kawasaki, K. Komori, M. Kuribayashi, A. Lartaux-Vollard, N. Matsumoto, Y. Michimura, M. Andia, N. Aritomi et al., AVS Quantum Sci. **5** 014403 (2023)

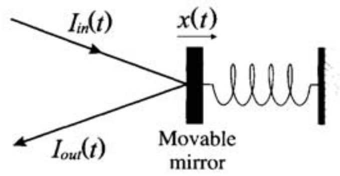
Suppressing Recoil Heating in Levitated Optomechanics Using Squeezed Light

C. Gonzalez-Ballestero, J.A. Zielinska, M. Rossi, A. Militaru, M. Frimmer, L. Novotny, P. Maurer, O. Romero-Isart, PRX Quantum **4** 030331 (2023)

Opto-mechanical field squeezing

One may even squeeze the field without the help of cavity feedback

This mechanism is simpler in principle but much more difficult in practice



A. Heidmann, S. Reynaud, Phys. Rev. A **50** 4237 (1994)

It has been observed in experiments with levitated nano-particles

L. Magrini, V. A. Camarena-Chávez, C. Bach, A. Johnson, M. Aspelmeyer, Phys. Rev. Lett. **129** 053601 (2022)

A. Militaru, M. Rossi, F. Tebbenjohanns, O. Romero-Isart ,M. Frimmer, L. Novotny, Phys. Rev. Lett. **129** 053602 (2022)

R. Berkowitz, Physics July 25, 2022, physics.aps.org/articles/v15/114