Examination of k_t-factorization in a Yukawa theory

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Motivation

High-energy (or k_t) factorization (HEF) in DIS

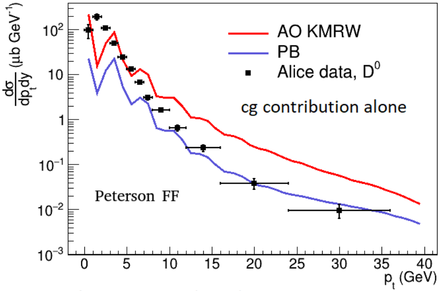
$$F_2(x_{\text{bj}}, Q) = \int_{Q^2/s}^{1} dx \int_0^{k_{\text{max}}^2} d^2k_t F_q(x, k_t; Q) \hat{F}_2(x, Q, k_t) \qquad (1)$$

with (some) unintegrated PDFs (UPDFs) obeying

$$\int_0^{\mu^2} dk_t^2 F(x, k_t; \mu) = f(x; \mu) \qquad (2)$$

1. k_{max} generally not discussed in the litterature.
2. Off-shell cross sections: HEF $\neq$ TMD factorization.
3. If $\mu = Q$, UPDFs constrained on $[0, Q]$ but integration up to k_{max}. UPDFs built **only** from (2) can in principle have any value for $k_t \in [Q, k_{\text{max}}]$, giving any result for the cross section.
4. Overestimation of the D-meson cross section using KaTie. Not the case if $\int_0^\infty dk_t^2 \mathscr{F}(x, k_t; \mu) = f(x; \mu)$.

D-meson cross section



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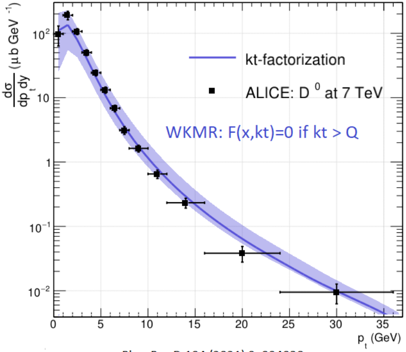

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Default $k_{\max} \gg Q$ and the region $k_t \in [Q, k_{\max}]$ is important.

All these results will be explained by the fact that $k_{\max} = \mu_F$.

Interesting to perform similar calculations for F_2 in a Yukawa theory and compare with the TMD factorization.

The Yukawa theory

Interest: Everything can be calculated perturbatively, including the exact (not factorized) F_2.

$$\mathscr{L} = \sum_j \frac{i}{2}\left[\overline{\psi}_j \gamma^\mu \partial_\mu \psi_j - (\partial_\mu \overline{\psi}_j)\gamma^\mu \psi_j\right] - M_j \overline{\psi}_j \psi_j + \frac{1}{2}(\partial\phi)^2 - \frac{m_s^2}{2}\phi^2 - \lambda\left(\overline{\psi}_1 \psi_2 \phi + \overline{\psi}_2 \psi_1 \phi\right) \quad (3)$$

$j = 1, 2$, $\psi_1 = \psi_N$ represents the target "nucleon" with mass $M_1 = m_p$, $\psi_2 = \psi_q$ is the "quark" field with mass m_q, and ϕ is the "scalar gluon".

- The expression for the exact $\mathscr{O}(\lambda^2)$ F_2 given in *Aslan et al., Phys. Rev. D 107 (2023) 7 074031*

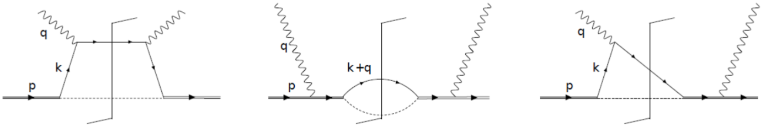


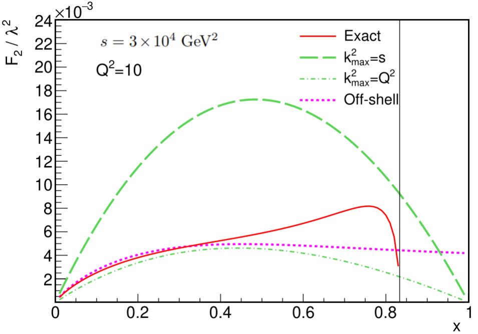
Figure 1. Order $\mathcal{O}(\lambda^2)$ diagrams for the DIS on a quark target in the Yukawa theory.

HEF vs exact result

$$F_2(x,Q) = \int_{Q^2/s}^{1} d\xi \int_0^{k_{\max}} d^2k_t F_q(\xi,k_t;Q)\hat{F}_2(\xi,Q,k_t) \qquad (4)$$

- $\hat{F}_2^{\text{off-shell}} = e_q^2 \frac{\Theta(\Delta(\xi,k_t))}{\sqrt{\Delta(\xi,k_t)}}$: $k_{\max} \sim Q$ (more kinematical constraints in DIS than pp).
- $\hat{F}_2^{\text{on-shell}} = e_q^2 \delta\left(1 - \frac{\xi}{x}\right)$: k_t not restricted.

Using KMR UPDFs or similar.



For sufficiently inclusive observables, coll. fact. is a good proxy for HEF.

TMD factorization

$F_2 = W + Y$. The W term includes the TMD PDF

$$\int_0^\infty d^2k_t f^{(1)}(x,k_t;\mu) \otimes \hat{F}_2^{(0)} + CT = \int_0^{\mu^2} d^2k_t f^{(1)}(x,k_t;\mu) \otimes \hat{F}_2^{(0)} \quad (5)$$

The Y term includes the collinear contribution and a subtraction term to avoid double counting

$$f^{(0)}(x;\mu) \otimes \hat{F}_2^{(1)} - \text{sub.} = a_\lambda (1 - x_{\mathsf{bj}}) \left(\int_0^{\hat{k}_m^2(x_{\mathsf{bj}})} \frac{dk_t^2}{\kappa(x_{\mathsf{bj}}) k_t^2} - \int_0^{\mu^2} \frac{dk_t^2}{k_t^2} \right), \quad (6)$$

and $\hat{k}_m^2(x) = \frac{(1-x)Q^2}{4x}$, $\kappa(x) = \sqrt{1 - \frac{k_t^2}{\hat{k}_m^2(x)}}$, $a_\lambda = \lambda^2/4\pi$.

At small x, the Y term can be written as

$$Y \simeq a_\lambda (1 - x_{\mathsf{bj}}) \int_{\mu^2}^{\hat{k}_m^2(x_{\mathsf{bj}})} \frac{dk_t^2}{\kappa(x_{\mathsf{bj}}) k_t^2} \quad (7)$$

Some comments

- The TMD formula describes well the exact result (improves with larger Q).
- The HEF is the equivalent of W term.
- TMD formalism cuts the k_t integral (W term) at $\mu^2 \sim Q^2$.
- Small $k_t < Q$ in TMD PDFs, large $k_t > Q$ in the hard coefficient.

Back to HEF: Collins' version

- Definition of UPDFs: Start with collinear fact.

$$\sigma(s, 4m^2) = \int_0^1 dx\, f(x;\mu)\hat{X}(xs, k=0, 4m^2) \tag{8}$$

with $\hat{X}(xs, k, 4m^2)$ an off-shell cross section obeying a modified BFKL eq., and $\hat{X}(xs, k=0, 4m^2) = \hat{\sigma}$.

- Then, $X_N(k=0, 4m^2) = C_N(k_t, \mu_F) \otimes I_N(k_t, \mu_F)$
- The resummation factor C_N is universal and can be combined with coll. PDFs.

$$F_N(k_t; \mu, \mu_F) = f_N(\mu) C_N(k_t, \mu_F); \quad C_N(k_t, \mu_F) = \frac{\gamma_N(\alpha_s)}{k_t^2} \left(\frac{k_t^2}{\mu_F^2}\right)^{\gamma_N(\alpha_s)} \tag{9}$$

The resummation factor is solution of the BFKL eq. and obeys

$$\int_0^{\mu_F} dk_t^2 C_N(k_t, \mu_F) = 1 \Longrightarrow \int_0^{\mu_F^2} dk_t^2 F(x, k_t; \mu, \mu_F) = f(x, \mu) \tag{10}$$

Main claim: $k_{\mathsf{max}} = \mu_F$

- Clear, for instance by comparison with TMD fact.
- $X_N(k = 0, 4m^2) = C_N(k_t, \mu_F) \otimes I_N(k_t, \mu_F)$: I_N infrared safe thanks to the subtraction of small $k_t < \mu_F$.

Factorization scale invariance:

Check that F_2, with the on-shell hard coefficient $\propto \delta(1 - x_{\mathsf{bj}})$ and $k_{\mathsf{max}} = \mu_F$, is μ_F independent. For $\mu_F = Q$ and $\mu_F = \sqrt{s} \gg Q$

$$\frac{F_2}{x_{\mathsf{bj}}} = \mathsf{TM}^{-1} \left\{ f_N(\mu) \int_0^{Q^2} dk_t^2 C_N(k_t, Q) \right\} = f(x_{\mathsf{bj}}; \mu) \qquad (11)$$

$$\frac{F_2}{x_{\mathsf{bj}}} = \mathsf{TM}^{-1} \left\{ f_N(\mu) \int_0^{s} dk_t^2 C_N(k_t, \sqrt{s}) \right\} = f(x_{\mathsf{bj}}; \mu) \qquad (12)$$

No integration in regions where the UPDFs are not constrained.

Potential issue with standard calculations

Standard $\Rightarrow \int_0^{\mu^2\sim Q^2} dk_t^2 F(x,k_t;\mu) = f(x,\mu)$. Implies $\mu_F = \mu \sim Q$.
At the same time k_t integrated above Q, say, up to $k^2_{\mathsf{max}} = s = \mu_F^2$.

Inconsistent, leads to an overestimation

$$\frac{F_2}{x_{\mathsf{bj}}} = \mathsf{TM}^{-1}\left\{ f_N(\mu)\int_0^s dk_t^2 C_N(k_t,\alpha_s;\mu)\right\}$$

$$= f(x_{\mathsf{bj}};\mu) + \mathsf{TM}^{-1}\left\{ f_N(\mu)\int_{\mu^2}^s dk_t^2 C_N(k_t,\alpha_s;\mu)\right\} \quad (13)$$

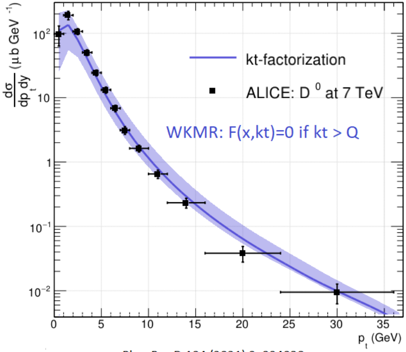
Does not lead to an overestimation, for instance, if:

- $Q \sim \sqrt{s}$
- Large k_t suppressed by the kinematics, e.g, in back-to back jets
- Drell-Yan?

Possible choices: partial answer

Use $\mu_F = \mu \sim Q \Rightarrow k_{\text{max}} \sim Q$.

Works fine with the KMRW UPDFs (but don't neglect flavor-excitation processes!)



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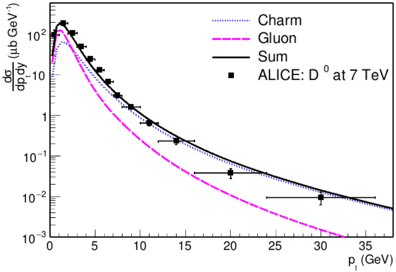
Use $\mu_F \gg Q$ and $k_{\text{max}} \gg Q$,

$\int_0^\infty dk_t^2 \mathscr{F}(x, k_t; \mu; s) = f(x, \mu)$



More on UPDFs

$f(x,\mu) = \int_0^\infty dk_t^2 \mathscr{F}(x,k_t;\mu)$: Why is this quantity not divergent?

In the Yukawa theory, the Feynman rules for bare collinear PDFs are the same as for TMDs integrated over k_t.

$$f(x;\mu) = \int d^2k_t f(x,k_t;\mu) + \text{C.T.} \tag{14}$$

Unusual, but we can write

$$(2\pi\mu)^{2\varepsilon} \int d^{2-2\varepsilon}k_t \ \text{c.t.}(\mu) = \ \text{C.T.} \tag{15}$$

and

$$\mathscr{F}(x,k_t;\mu) = F(x,k_t;\mu) + \ \text{c.t.}(\mu) \tag{16}$$

Illustration with the KMRW UPDFs

Starting with the usual definition

$$F_q(x,k_t;\mu) = \frac{x}{\pi}\frac{\partial}{\partial k_t^2}\left[T_q(k_t,\mu)f_q(x,k_t)\right], \quad k_t \geq \mu_0 \tag{17}$$

$$F_q(x,k_t;\mu^2) = \frac{x}{\mu_0^2\pi}T_q(\mu_0,\mu)f_q(x,\mu_0), \quad k_t < \mu_0, \tag{18}$$

and using the expression for the $\mathscr{O}(\lambda^2)$ collinear PDF, we find

$$F_q(x,k_t;\mu)/x = \frac{a_\lambda}{\pi k_t^2}(1-x), \quad k_t \geq \mu_0 \tag{19}$$

$$F_q(x,k_t;\mu^2)/x = \frac{1}{\mu_0^2\pi}a_\lambda(1-x)\left(\frac{\chi(x)^2}{\Delta(x)^2} + \ln\left(\frac{\mu_0^2}{\Delta(x)^2}\right) - 1\right), \quad k_t < \mu_0, \tag{20}$$

At this order, the Sudakov factor is 1.

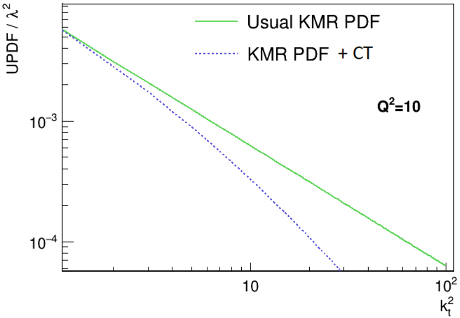
Illustration with the KMRW UPDFs

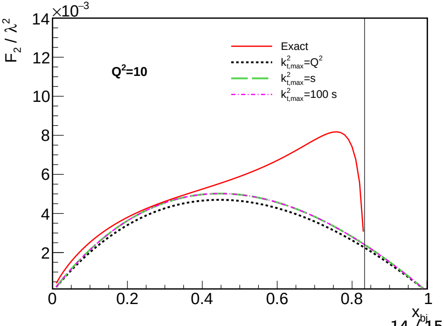
The $\overline{\text{MS}}$ C.T. is

$$a_\lambda(1-x)\left(\frac{1}{\varepsilon}-\gamma_E+\ln 4\pi\right)+\mathscr{O}(\varepsilon)=(2\pi\mu)^{\varepsilon}\int d^{2-2\varepsilon}k_t\,\frac{a_\lambda}{\pi}\frac{1-x}{(k_t^2+\mu^2)}. \tag{21}$$

Including the integrand to the KMRW UPDF leads to the modification

$$\frac{a_\lambda}{\pi k_t^2}(1-x)\to\frac{a_\lambda}{\pi}(1-x)\frac{\mu^2}{k_t^2(k_t^2+\mu^2)} \tag{22}$$





Conclusion

- $k_{\mathsf{max}} = \mu_F$.
- The simpler choice is probably $\mu_F = \mu \sim Q$.
  - Use $\int_0^{\mu^2} dk_t^2 F(x, k_t; \mu) = f(x; \mu)$ with $k_{\mathsf{max}} = \mu$.
- Another option is $\mu_F \sim \infty$.
  - Use $\int_0^{\infty} dk_t^2 \mathscr{F}(x, k_t; \mu) = f(x; \mu)$ with any $k_{\mathsf{max}} > Q$.
  - $\mathscr{F}(x, k_t; \mu)$ gives a negligible contribution in the region $k_t > Q$.
- Avoid $k_{\mathsf{max}} \gg \mu_F$, e.g., $\int_0^{\mu_F^2 = \mu^2 \sim Q^2} dk_t^2 F(x, k_t; \mu) = f(x; \mu)$ and $k_{\mathsf{max}} \gg Q$.