Determination of α_S from the Z-boson q_T distribution at hadron colliders

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Results from:

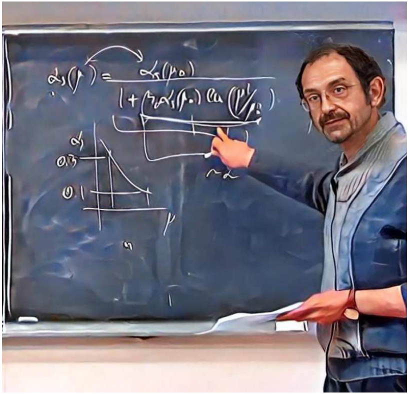
S. Camarda, G. F., M. Schott, e-Print:2203.05394, EPJC 84 (2024)

ATLAS Coll., e-Print:2309.12986

REF 2024

IPhT – Saclay – 16/10/2024

Stefano Catani (1958-2024)



Wonderful person, outstanding physicist

The idea: α_S from semi-inclusive processes

QCD COHERENT BRANCHING AND SEMI-INCLUSIVE PROCESSES AT LARGE x*

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$$\alpha_s^{(\mathrm{MC})} = \alpha_s^{(\overline{\mathrm{MS}})}\left(1 + K\frac{\alpha_s^{(\overline{\mathrm{MS}})}}{2\pi}\right),$$

$$\begin{aligned}\Lambda_{\mathrm{MC}} &= \Lambda_{\overline{\mathrm{MS}}}\exp(K/4\pi\beta_0)\\ &\simeq 1.569\,\Lambda_{\overline{\mathrm{MS}}} \quad \text{for } N_f = 5.\end{aligned}$$

In this paper we have studied [...] the next-to-leading logarithmic terms in **semi-inclusive hard processes such as the DIS and DY processes** *at large x. Since the Monte Carlo algorithm with these improvements is accurate to next-to-leading order in the large-x region,* **it can be used to determine the fundamental QCD scale $\Lambda_{\overline{MS}}$**

The idea: α_S from semi-inclusive processes

Advantages:

- **higher sensitivity to α_S** w.r.t. *inclusive* observables;
- calculable at **higher theoretical accuracy** w.r.t. *exclusive* observables.

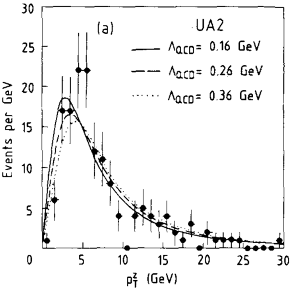
Challenges:

- sensitivity to **infrared (Sudakov) logs**;
- sensitivity **non perturbative QCD** effects.

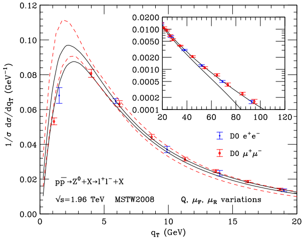
Classical semi-inclusive obs. at hadron colliders:
high invariant-mass Drell–Yan lepton pair
at small transverse-momentum ($\mathbf{q_T}$).

α_S from Z-boson q_T distribution

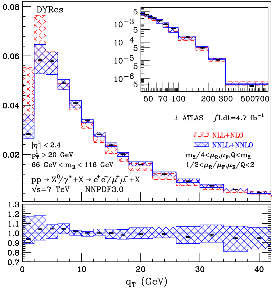
Sp$\bar{p}$S ($\sqrt{s} = 0.63$ TeV) **Tevatron ($\sqrt{s} = 1.96$ TeV)** **LHC ($\sqrt{s} = 7 - 8$ TeV)**



[UA2 Coll.('92)]
compared with
[Altarelli et al.('84)]

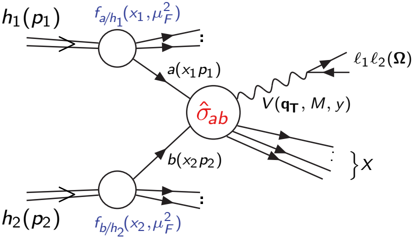


[D0 Coll.('08,'10)]
compared with
[Catani et al.('10)]



[ATLAS Coll.('14)]
compared with
[Catani et al.('15)]

Drell–Yan $\mathbf{q_T}$ distribution



$$\mathbf{h_1(p_1) + h_2(p_2) \;\rightarrow\; V + X \;\rightarrow\; \ell_1 + \ell_2 + X}$$

where $\quad V = Z^0/\gamma^*, W^\pm$

QCD factorization formula:

$$\frac{d\sigma}{dq_T^2} = \sum_{a,b}\int_0^1 dx_1 \int_0^1 dx_2 \; f_{a/h_1}(x_1,\mu_F^2)\; f_{b/h_2}(x_2,\mu_F^2)\; \frac{d\hat{\sigma}_{ab}}{dq_T^2}(\alpha_S(\mu_R^2),\mu_R^2,\mu_F^2).$$

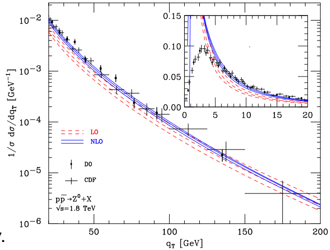
Fixed-order perturbative expansion reliable only for $q_T \sim M$. When $q_T \ll M$:

$$\int_0^{q_T^2} d\bar{q}_T^2 \,\frac{d\hat{\sigma}_{q\bar{q}}}{d\bar{q}_T^2} \sim 1 + \alpha_S\left[c_{12}L_{q_T}^2 + c_{11}L_{q_T} + \cdots\right]$$

$$+\alpha_S^2\left[c_{24}L_{q_T}^4 + \cdots + c_{21}L_{q_T} + \cdots\right] + \mathcal{O}(\alpha_S^3)$$

with $\alpha_S^n L_{q_T}^m \equiv \alpha_S^n \log^m(M^2/q_T^2) \gtrsim 1$.

Resummation of logarithmic corrections mandatory.



$\mathbf{q_T}$ resummation in QCD [Catani,de Florian,Grazzini('01)] [Bozzi,Catani,de Florian,Grazzini('03,'06)]

$$\boxed{\frac{d\hat{\sigma}}{dq_T^2} = \frac{d\hat{\sigma}^{(res)}}{dq_T^2} + \frac{d\hat{\sigma}^{(fin)}}{dq_T^2};}$$

In the impact parameter space: $q_T \ll M \Leftrightarrow Mb \gg 1$, $\log M/q_T \gg 1 \Leftrightarrow \log Mb \gg 1$

$$\frac{d\hat{\sigma}^{(res)}}{dq_T^2} = \frac{M^2}{\hat{s}} \int \frac{d^2\mathbf{b}}{4\pi} e^{i\mathbf{b}\cdot\mathbf{q_T}}\, \mathcal{W}(b, M),$$

In the Mellin space (with respect to $z = M^2/\hat{s}$) we have:

$$\boxed{\mathcal{W}_N(b,M) = \mathcal{H}_N(\alpha_S) \times \exp\left\{\mathcal{G}_N(\alpha_S, L)\right\}}$$

with $L \equiv \log(M^2 b^2)$ and $\alpha_S L \sim 1$

$$\mathcal{G}(\alpha_S, L) = L\, g^{(1)}(\alpha_S L) + g^{(2)}(\alpha_S L) + \frac{\alpha_S}{\pi} g^{(3)}(\alpha_S L) + \cdots \qquad \mathcal{H}(\alpha_S) = \hat{\sigma}^{(0)} \left(1 + \frac{\alpha_S}{\pi}\mathcal{H}^{(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 \mathcal{H}^{(2)} + \cdots\right)$$

LL ($\sim\alpha_S^n L^{n+1}$): $g^{(1)}$, $(\hat{\sigma}^{(0)})$; NLL ($\sim\alpha_S^n L^n$): $g^{(2)}$, $\mathcal{H}^{(1)}$; $\cdots$ N^{k}LL ($\sim\alpha_S^n L^{n+k-1}$): $g^{(k+1)}$, $\mathcal{H}^{(k)}$;

Resummed result at small q_T *matched* with corresponding fixed "finite" part at large q_T: *uniform accuracy* for $q_T \ll M$ and $q_T \sim M$.

- Resummed effects exponentiated in a universal of Sudakov form factor, process-dependence factorized in the hard-virtual factor $H_c^F(\alpha_S)$ via all-order formula [Catani,Cieri,de Florian,G.F.,Grazzini('14)].
- Resummation performed at partonic cross section level: (collinear) PDF evaluated at $\mu_F \sim M$, $f_N(b_0^2/b^2) = \exp\left\{-\int_{b_0^2/b^2}^{\mu_F^2} \frac{dq^2}{q^2}\,\gamma_N(\alpha_S(q^2))\right\} f_N(\mu_F^2)$: no PDF extrapolation in the non perturbative region, study of μ_R and μ_F dependence as in fixed-order calculations.
- No need for NP models: Landau singularity of α_S regularized using a *Minimal Prescription* without power-suppressed corrections [Laenen et al.('00)],[Catani et al.('96)].
- Introduction of resummation scale $Q \sim M$: variations give an estimate of the uncertainty from uncalculated logarithmic corrections.

$$\ln\left(M^2 b^2\right) \rightarrow \ln\left(Q^2 b^2\right) + \ln\left(M^2/Q^2\right)$$

- Perturbative unitarity constraint: recover *exactly* the total cross-section (upon integration on q_T)

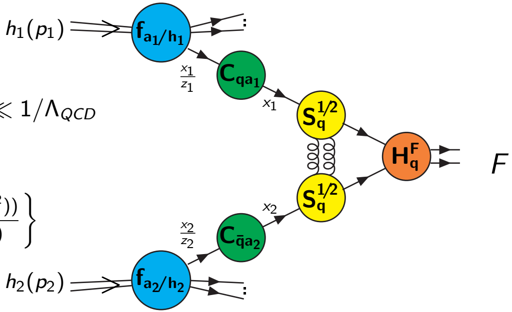
$$\ln\left(Q^2 b^2\right) \rightarrow \widetilde{L} \equiv \ln\left(Q^2 b^2 + 1\right) \quad \Rightarrow \quad \exp\left\{\alpha_S^n \widetilde{L}^k\right\}\Big|_{b=0} = 1 \;\Rightarrow\; \int_0^\infty dq_T^2 \left(\frac{d\hat{\sigma}}{dq_T^2}\right) = \hat{\sigma}^{(tot)};$$

- General procedure to treat the q_T recoil [Catani,de Florian,G.F.,Grazzini('15)]:

$$\frac{d\hat{\sigma}^{(0)}}{d\mathbf{\Omega}} = \hat{\sigma}^{(0)}(M^2)\, F(\mathbf{q_T}; M^2, \mathbf{\Omega}) \;\; with \;\; F(\mathbf{q_T}; M^2, \mathbf{\Omega}) = F(\mathbf{0}; M^2, \mathbf{\Omega}) + \mathcal{O}(\mathbf{q_T^2}/M^2)$$

Connection with CSS and TMD formalisms

[Collins,Soper,Sterman('85)]



$$M \gg \Lambda_{QCD}\ , \quad b \gg 1/M\ , \quad b \ll 1/\Lambda_{QCD}$$

$$C(\alpha_S(b_0^2/b^2)) = C(\alpha_S(M^2))$$
$$\times\ \exp\left\{ -\int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2}\ \beta(\alpha_S(q^2))\ \frac{d\ln C(\alpha_S(q^2))}{d\ln \alpha_S(q^2)} \right\}$$

$$\frac{d\sigma_F^{(res)}}{dq_T^2} = \frac{M^2}{s}\ \sigma^{(0)}_{q\bar{q},F}\ H_q^F(\alpha_S(M^2)) \sum_{a_1,a_2} \int \frac{d^2\mathbf{b}}{2\pi}\ e^{i\mathbf{b}\cdot\mathbf{q_T}}\ S_q(M,b)$$
$$\times \int_{x_1}^1 \frac{dz_1}{z_1}\ C_{qa_1}(z_1;\alpha_S(b_0^2/b^2))\ f_{a_1/h_1}(x_1/z_1, b_0^2/b^2) \int_{x_2}^1 \frac{dz_2}{z_2}\ C_{\bar{q}\,a_2}(z_2;\alpha_S(b_0^2/b^2))\ f_{a_2/h_2}(x_2/z_2, b_0^2/b^2)$$

$$\tilde{F}_{q_f/h}(x,b,M) = \sum_a \int_x^1 \frac{dz}{z}\ \sqrt{S_q(M,b)}\, C_{q_f\,a}(z;\alpha_S(b_0^2/b^2))\ f_{a/h}(x/z, b_0^2/b^2)$$

$\mathbf{q_T}$ resummation: perturbative accuracy

- Formalism implemented in numerically efficient and publicly available code:

  DYTurbo: computes resummed and fixed-order fiducial cross section and related distributions it retains full kinematics of the vector boson and of its leptonic decay products [Camarda,Boonekamp,Bozzi,Catani,Cieri,Cuth,G.F.,de Florian,Glazov, Grazzini,Vincter,Schott('20)]

  https://dyturbo.hepforge.org.

- We have explicitly included in DYTurbo up to:
  - N^{4}LL logarithmic contributions to all orders (i.e. up to $\exp(\sim\alpha_S^n L^{n-3})$);
  - Approximated N^{4}LO corrections (i.e. up to $\mathcal{O}(\alpha_S^4)$) at small q_T;
  - NLO corrections (i.e. up to $\mathcal{O}(\alpha_S^2)$) at large q_T;
- Matching with NNLO corrections (i.e. up to $\mathcal{O}(\alpha_S^3)$) at large q_T from results in [Boughezal et al.('16)], [Gehrmann-De Ridder et al.('16)], [MCFM ('23)];
- Results up to N^{3}LO (i.e. up to $\mathcal{O}(\alpha_S^3)$) recovered for the total cross section (from unitarity).

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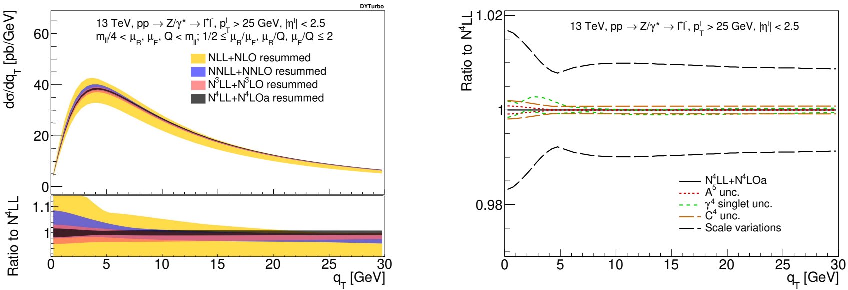
Z/γ^* production at N^{4}LL+N^{4}LOa resummed

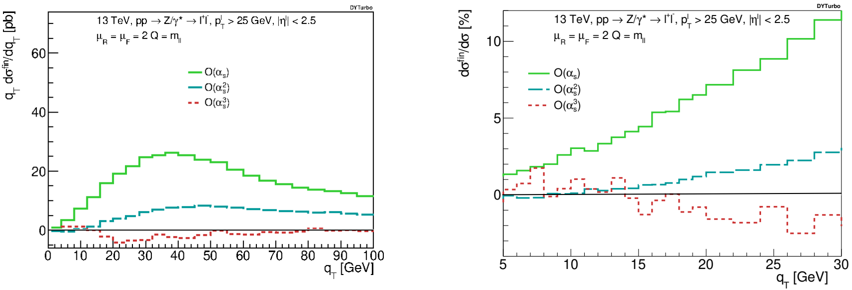
[Camarda,Cieri,G.F.('23)]



DYTurbo results. Left: Resummed NLL, NNLL, N^{3}LL and N^{4}LLa bands for Z/γ^* (left). Right: Uncertainties from approximations of the perturbative coefficients at N4LL+N4LOa compared to scale variations.

Z/γ^* production: finite part

[Camarda,Cieri,G.F.('21)]



Finite part at $\mathcal{O}(\alpha_S)$, $\mathcal{O}(\alpha_S^2)$ and $\mathcal{O}(\alpha_S^3)$ (left) and ratio wrt matched results (right).

Combining QED and QCD q_T resummation

[Cieri,G.F.,Sborlini('18)]

We start considering QED contributions to the q_T spectrum in the case of colourless and **neutral** high mass systems, e.g. on-shell Z boson production

$$h_1 + h_2 \ \rightarrow \ Z^0 + X$$

In the impact parameter and Mellin spaces resummed partonic cross section reads:

$$\mathcal{W}_N(b,M) = \hat{\sigma}^{(0)}\, \mathcal{H}'_N(\alpha_S, \alpha) \times \exp\left\{\mathcal{G}'_N(\alpha_S, \alpha, L)\right\}$$

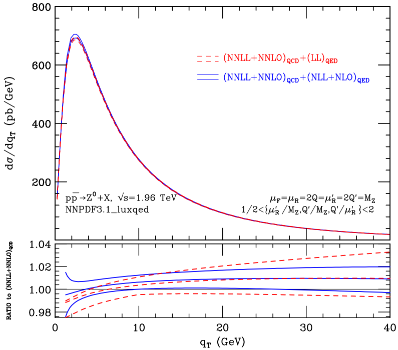
$$\mathcal{G}'(\alpha_S, \alpha, L) = \mathcal{G}(\alpha_S, L) + \boxed{L\, g'^{(1)}(\alpha L) + g'^{(2)}(\alpha L) + \textstyle\sum_{n=3}^{\infty} \left(\frac{\alpha}{\pi}\right)^{n-2} g'^{(n)}(\alpha L)}$$

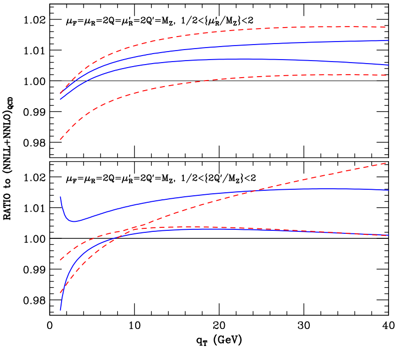
$$+ \boxed{g'^{(1,1)}(\alpha_S L, \alpha L) + \sum_{\substack{n,m=1\\ n+m\neq 2}}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^{n-1} \left(\frac{\alpha}{\pi}\right)^{m-1} g_N'^{(n,m)}(\alpha_S L, \alpha L)}$$

$$\mathcal{H}'(\alpha_S, \alpha) \quad = \quad \mathcal{H}(\alpha_S) + \boxed{\frac{\alpha}{\pi}\mathcal{H}'^{(1)} + \textstyle\sum_{n=2}^{\infty} \left(\frac{\alpha}{\pi}\right)^{n} \mathcal{H}_N'^{(n)}} + \boxed{\textstyle\sum_{n,m=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^{n} \left(\frac{\alpha}{\pi}\right)^{m} \mathcal{H}_N'^{F\,(n,m)}}$$

LL QED ($\sim\alpha^n L^{n+1}$): $g'^{(1)}$; NLL QED ($\sim\alpha^n L^n$): $g'^{(2)}$, $\mathcal{H}'^{(1)}$;
LL mixed QCD-QED ($\sim\alpha_S^n \alpha^n L^{2n}$): $g'^{(1,1)}$;

Combined QED and QCD q_T resummation for Z production at the Tevatron [Cieri,G.F.,Sborlini ('18)]

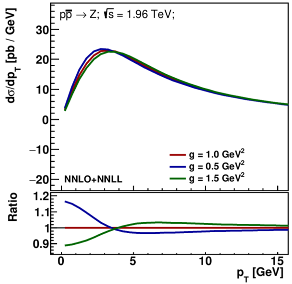


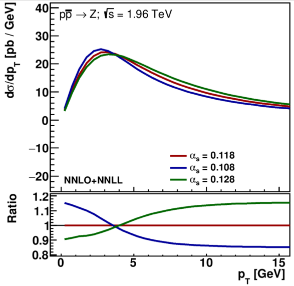


Z qT spectrum at the LHC. NNLL+NNLO QCD combined with the LL (red dashed) and NLL+NLO (blue solid) QED with corresponding QED uncertainty bands.

Ratio of the resummation (upper panel) and renormalization (lower panel) QED scale-dependent results with respect to the central value NNLL+NNLO QCD result.

Non perturbative effects





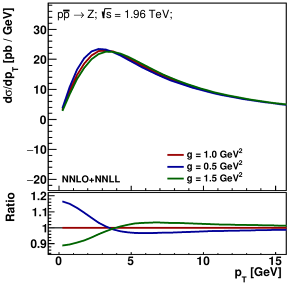
- Up to now discussed result in a complete perturbative framework (except for PDFs).
- Non perturbative *intrinsic* k_T effects parametrized by a NP form factor:

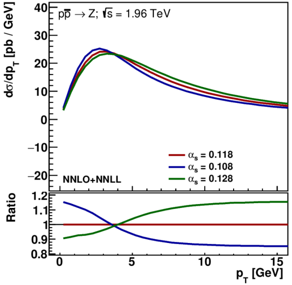
$$\exp\{\mathcal{G}_N(\alpha_S,\widetilde{L})\} \quad \to \quad \exp\{\mathcal{G}_N(\alpha_S,\widetilde{L})\}\, S_{NP}$$

e.g. $S_{NP} = \exp\{-gb^2\}$ with $g \sim 0.5\, GeV^2$:

- NP effects increase the hardness of the q_T spectrum at small values of q_T. Non trivial interplay of perturbative and NP effects.
- However possible to disentangle the effects: scale of the NP effects is $\langle q_T \rangle \sim 1\, GeV$ ($g \sim 0.5 GeV^2$), scale of "soft gluon" recoil is $\langle q_T \rangle \sim 10\, GeV$.

Non perturbative effects





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Non perturbative effects

$$S_{\mathrm{NP}}(b) = \exp\left[-g_j(b) - g_K(b)\log\frac{m_{\ell\ell}^2}{Q_0^2}\right]$$

$$g_j(b) = \frac{g\,b^2}{\sqrt{1+\lambda\,b^2}} + \mathrm{sign}(q)\left(1-\exp\left[-|q|\,b^4\right]\right)$$

$$g_K(b) = g_0\left(1-\exp\left[-\frac{C_F\alpha_s(b_0/b_*)b^2}{\pi g_0\,b_{\mathrm{lim}}^2}\right]\right),$$

S_{NP} parameterization from [Collins,Rogers('15)].

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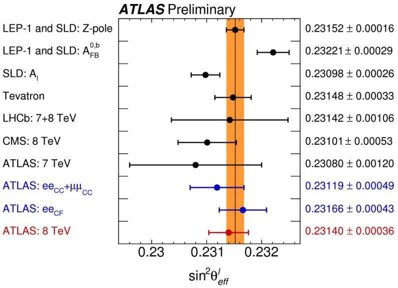
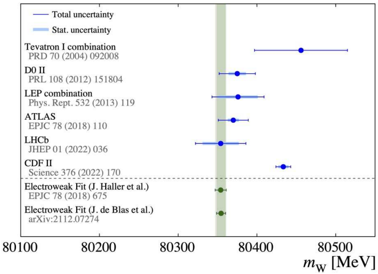
Modelling Z (and W) production for $\sin^2\theta^l_{eff}$ and M_W determinations

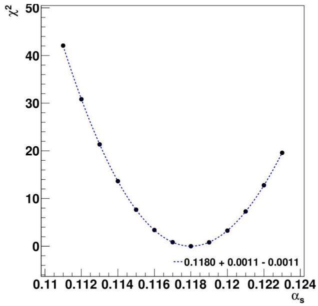


Comparison of the measurements of the $\sin^2\theta^l_{eff}$.



Measured values of M_W compared with the prediction of from the global electroweak fit

Methodology for α_S determination

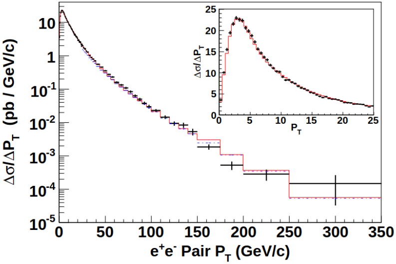


- DYTurbo interfaced to xFitter.
- Defined χ^2 with experimental (β_{exp}) and PDFs (β_{th}) uncertainties (equivalent to including the new dataset in the PDF using profiling/reweighting).
- The non-perturbative form factor is S_{NP} left free in the fit.

$$\chi^2(\beta_{\rm exp}, \beta_{\rm th}) = \sum_{i=1}^{N_{\rm data}} \frac{\left(\sigma_i^{\rm exp} + \sum_j \Gamma_{ij}^{\rm exp} \beta_{j,{\rm exp}} - \sigma_i^{\rm th} - \sum_k \Gamma_{ik}^{\rm th} \beta_{k,{\rm th}}\right)^2}{\Delta_i^2} + \sum_j \beta_{j,{\rm exp}}^2 + \sum_k \beta_{k,{\rm th}}^2 \,.$$

Z-boson q_T measurement at CDF

The CDF measurement of $Z/\gamma^* \to e^+e^-$ ($\sqrt{s} = 1.96\,TeV$ with $\int \mathcal{L} = 2.1 fb^{-1}$) [CDF Coll.('10)] is ideal for $\alpha_S(m_Z)$ determination.



[CDF Coll.('10)]

- Measurement in full-lepton phase space with small extrapolation using angular coefficients method ⇒ allows fast analytic predictions with DYTurbo.
- $p\bar{p}$ collisions: small contribution from heavy-flavour in initial state (0.4% $b\bar{b} \to Z$, 1.3% $c\bar{c} \to Z$). Quark mass effects negligible.
- Low pile-up and good electron resolution. Fine q_T bins ($0.5\,GeV$) with relatively small bin-to-bin correlations.

Theory uncertainties

	PDF fit	Hessian profiling
$\alpha_S(m_Z)$	0.1188 ± 0.0008	0.1184 ± 0.0006
g [GeV^2]	0.69 ± 0.05	0.71 ± 0.05
Dataset	χ^2/points	χ^2/points
NC DIS H1-ZEUS e^+p	955/905	
CC DIS H1-ZEUS e^+p	46/39	
NC DIS H1-ZEUS e^-p	219/159	
CC DIS H1-ZEUS e^-p	53/42	
H1-ZEUS correlated χ^2	91	
CDF Z p_T	41/55	40/55
Total	1405 / 1184	

- Bias from α_S-PDFs correlations [Forte,Kassabov('20)] $\to$ PDFs refitted.
- NNPDF4.0 PDF at NNLO. Other sets considered: CT18, CT18Z, MSHT20, HERAPDF2.0, ABMP16 (NNLO) and MSHT20an3lo. The midpoint value is the nominal result and the PDF envelope as an additional uncertainty.
- Uncertainty from missing higher orders: $m_{ll}/2 < \{\mu_R, \mu_F, Q\} < 2m_{ll}$ with $0.5 < \{\mu_R/\mu_F, \mu_R/Q, \mu_F/Q\} < 2$.
- NP effects: $S_{NP} = \exp\{-gb^2\}$: b_*-pr. $b_* = b/\sqrt{1 + b^2/b_{lim}^2}$ ($b_{lim} = 2 - 3$ GeV^{-1}) and *minimal* pr. ($b_{lim} \to \infty$); quartic term $\exp(-qb^4)$ and S_{NP} [Collins,Rogers('15)].
- Uncerainty from finite component at $\mathcal{O}(\alpha_S^3)$.
- Check with D0 data and fit boundaries.

Theory uncertainties

	$\alpha_S(m_Z)$	g [GeV^2]	χ^2/dof
NNPDF4.0	0.1192 ± 0.0008	0.66 ± 0.05	41/53
CT18	0.1189 ± 0.0010	0.67 ± 0.05	40/53
CT18Z	0.1198 ± 0.0009	0.62 ± 0.05	41/53
MSHT20	0.1185 ± 0.0009	0.72 ± 0.05	40/53
HERAPDF2.0	0.1188 ± 0.0008	0.69 ± 0.05	40/53
ABMP16	0.1185 ± 0.0007	0.62 ± 0.05	42/53
MSHT20an3lo (N^{4}LL)	0.1184 ± 0.0009	0.73 ± 0.05	40/53
PDF fit	0.1184 ± 0.0006	0.71 ± 0.05	1405/1184

- Bias from α_S-PDFs correlations [Forte,Kassabov('20)] $\rightarrow$ PDFs refitted.
- NNPDF4.0 PDF at NNLO. Other sets considered: CT18, CT18Z, MSHT20, HERAPDF2.0, ABMP16 (NNLO) and MSHT20an3lo. The midpoint value is the nominal result and the PDF envelope as an additional uncertainty.
- Uncertainty from missing higher orders: $m_{ll}/2 < \{\mu_R, \mu_F, Q\} < 2m_{ll}$ with $0.5 < \{\mu_R/\mu_F, \mu_R/Q, \mu_F/Q\} < 2$.
- NP effects: $S_{NP} = \exp\{-gb^2\}$: b_*-pr. $b_* = b/\sqrt{1 + b^2/b_{lim}^2}$ ($b_{lim} = 2 - 3$ GeV^{-1}) and *minimal* pr. ($b_{lim} \to \infty$); quartic term $\exp(-qb^4)$ and S_{NP} [Collins,Rogers('15)].
- Uncerainty from finite component at $\mathcal{O}(\alpha_S^3)$.
- Check with D0 data and fit boundaries.

Theory uncertainties

$\mu_R/m_{\ell\ell}$	$\mu_F/m_{\ell\ell}$	$Q/m_{\ell\ell}$	$\alpha_S(m_Z)$	g [GeV^2]	χ^2/dof
1	1	1	0.1192 ± 0.0008	0.66 ± 0.05	41/53
1	1	2	0.1183 ± 0.0007	0.77 ± 0.05	40/53
1	1	0.5	0.1196 ± 0.0008	0.57 ± 0.05	42/53
1	2	1	0.1194 ± 0.0008	0.66 ± 0.05	41/53
1	2	2	0.1183 ± 0.0007	0.77 ± 0.05	41/53
1	0.5	1	0.1193 ± 0.0008	0.68 ± 0.05	42/53
1	0.5	0.5	0.1196 ± 0.0008	0.59 ± 0.05	42/53
2	1	1	0.1193 ± 0.0008	0.67 ± 0.05	42/53
2	1	2	0.1194 ± 0.0008	0.70 ± 0.05	41/53
2	2	1	0.1192 ± 0.0008	0.65 ± 0.05	42/53
2	2	2	0.1192 ± 0.0008	0.67 ± 0.05	41/53
0.5	1	1	0.1184 ± 0.0007	0.75 ± 0.05	42/53
0.5	1	0.5	0.1192 ± 0.0007	0.64 ± 0.05	41/53
0.5	0.5	1	0.1183 ± 0.0007	0.75 ± 0.05	42/53
0.5	0.5	0.5	0.1192 ± 0.0007	0.64 ± 0.05	42/53

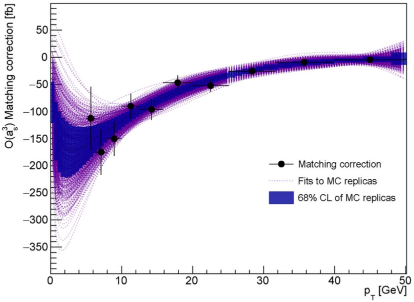
- Bias from α_S-PDFs correlations [Forte,Kassabov('20)] $\rightarrow$ PDFs refitted.
- NNPDF4.0 PDF at NNLO. Other sets considered: CT18, CT18Z, MSHT20, HERAPDF2.0, ABMP16 (NNLO) and MSHT20an3lo. The midpoint value is the nominal result and the PDF envelope as an additional uncertainty.
- Uncertainty from missing higher orders: $m_{ll}/2 < \{\mu_R, \mu_F, Q\} < 2m_{ll}$ with $0.5 < \{\mu_R/\mu_F, \mu_R/Q, \mu_F/Q\} < 2$.
- NP effects: $S_{NP} = \exp\{-gb^2\}$: b_*-pr. $b_* = b/\sqrt{1 + b^2/b^2_{lim}}$ ($b_{lim} = 2 - 3$ GeV^{-1}) and *minimal* pr. ($b_{lim} \to \infty$); quartic term $\exp\left(-qb^4\right)$ and S_{NP} [Collins,Rogers('15)].
- Uncerainty from finite component at $\mathcal{O}(\alpha_S^3)$.
- Check with D0 data and fit boundaries.

Theory uncertainties

	$\alpha_S(m_Z)$	g [GeV^2]
$b_{\rm lim} = 2$ GeV^{-1}	0.1187 ± 0.0007	0.83 ± 0.05
$b_{\rm lim} \to \infty$	0.1199 ± 0.0008	0.42 ± 0.05
g_k	0.1186 ± 0.0008	0.65 ± 0.05
$q = 0.1$ GeV^4	0.1197 ± 0.0008	0.51 ± 0.05
VFN PDF evolution	0.1190 ± 0.0007	0.71 ± 0.05

- Bias from α_S-PDFs correlations [Forte,Kassabov('20)] $\to$ PDFs refitted.
- NNPDF4.0 PDF at NNLO. Other sets considered: CT18, CT18Z, MSHT20, HERAPDF2.0, ABMP16 (NNLO) and MSHT20an3lo. The midpoint value is the nominal result and the PDF envelope as an additional uncertainty.
- Uncertainty from missing higher orders: $m_{ll}/2 < \{\mu_R, \mu_F, Q\} < 2m_{ll}$ with $0.5 < \{\mu_R/\mu_F, \mu_R/Q, \mu_F/Q\} < 2$.
- NP effects: $S_{NP} = \exp\{-gb^2\}$: b_*-pr. $b_* = b/\sqrt{1 + b^2/b_{lim}^2}$ ($b_{lim} = 2 - 3$ GeV^{-1}) and *minimal* pr. ($b_{lim} \to \infty$); quartic term $\exp(-qb^4)$ and S_{NP} [Collins,Rogers('15)].
- Uncerainty from finite component at $\mathcal{O}(\alpha_S^3)$.
- Check with D0 data and fit boundaries.

Theory uncertainties



- Bias from α_S-PDFs correlations [Forte,Kassabov('20)] $\rightarrow$ PDFs refitted.
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- Uncerainty from finite component at $\mathcal{O}(\alpha_S^3)$.
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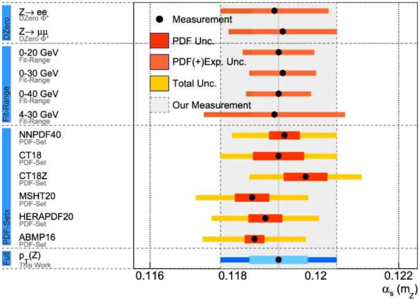
Theory uncertainties



- Bias from α_S-PDFs correlations [Forte,Kassabov('20)] $\rightarrow$ PDFs refitted.
- NNPDF4.0 PDF at NNLO. Other sets considered: CT18, CT18Z, MSHT20, HERAPDF2.0, ABMP16 (NNLO) and MSHT20an3lo. The midpoint value is the nominal result and the PDF envelope as an additional uncertainty.
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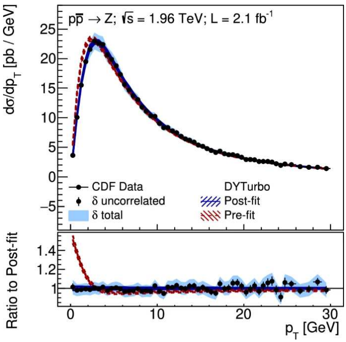
- Uncerainty from finite component at $\mathcal{O}(\alpha_S^3)$.
- Check with D0 data and fit boundaries.

Fit results

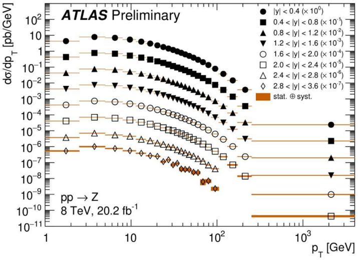


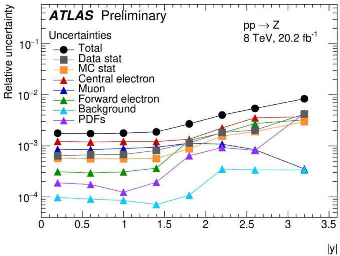
Statistical uncertainty		±0.7	
Experimental systematic uncertainty		±0.1	
PDF uncertainty (NNPDF4.0)		±0.4	
PDF uncertainty (envelope of PDFs)		±0.7	
Scale variations uncertainties	+0.4		− 0.9
Matching at $\mathcal{O}(\alpha_S^3)$		±0.1	
Non-perturbative model		±0.7	
Flavour model	0		− 0.3
QED ISR		< ±0.1	
Lower limit of fit range		±0.2	
Total	+1.3		− 1.6

Simultaneous fit of $\alpha_S(m_Z)$ and g at N^{3}LL+$\mathcal{O}(\alpha_S^3)$ (N^{3}LL+N^{3}LO):

$$\alpha_S(m_Z) = 0.1191^{+0.0013}_{-0.0016}$$

Z-boson q_T measurement at ATLAS



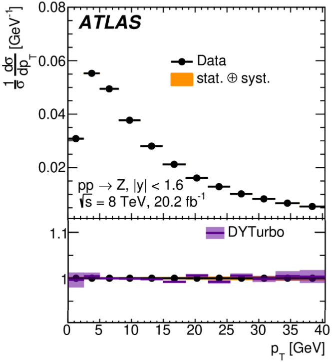


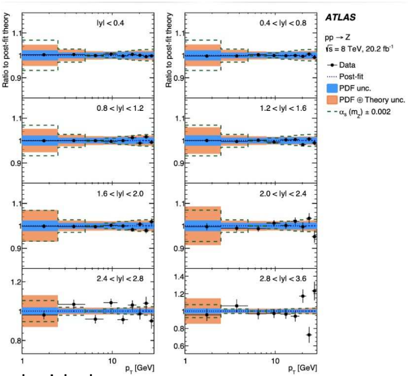
First measurement at the LHC of full-lepton phase space cross sections. Double differential in pT and rapidity.

Permille level precision in the central region, subpercent uncertainties up to $|y| < 3.6$. Dominant uncertainties from lepton calibration. Very small (negligible) theory uncertainties.

	UA1/UA2	LEP	Tevatron 1.96 TeV	LHC 8 TeV	LHC 13 TeV
Z→ ℓℓ events	200	500 K	300 K	15 M	150 M

Z-boson q_T measurement at ATLAS vs theory

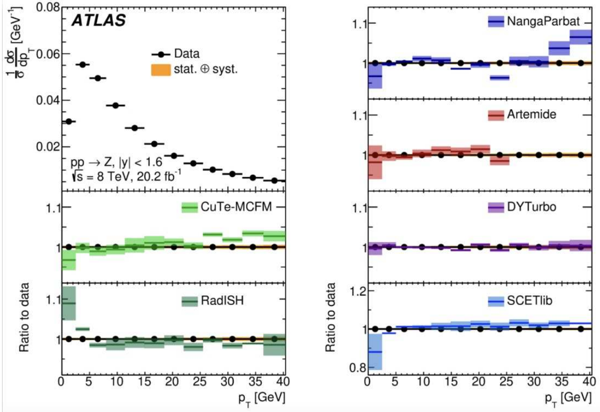




DYTurbo predictions at N^{4}LLa accuracy compared with data [ATLAS Coll.('23)].

Time performance of $\mathcal{O}(seconds)$: (with exception of V+jet term with fiducial lepton cuts).

Z-boson q_T measurement at ATLAS vs theory



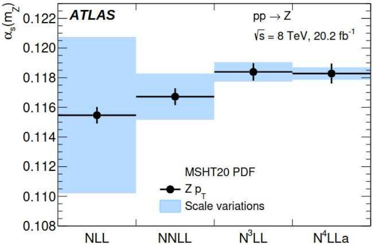
Theory predictions compared with data [ATLAS Coll.('23)].

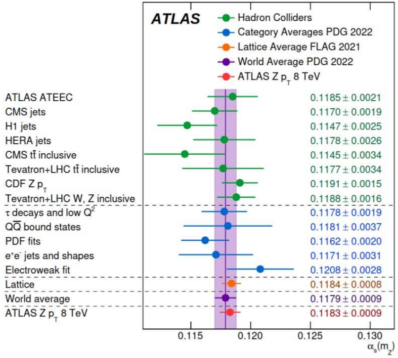
Exp. and theory uncertainties

Table 1: Summary of the uncertainties in the determination of $\alpha_s(m_Z)$, in units of 10^{-3}.

Experimental uncertainty	± 0.44	
PDF uncertainty	± 0.51	
Scale variation uncertainties	± 0.42	
Matching to fixed order	0	-0.08
Non-perturbative model	$+0.12$	-0.20
Flavour model	$+0.40$	-0.29
QED ISR	± 0.14	
N^4LL approximation	± 0.04	
Total	$+0.91$	-0.88

Fit results

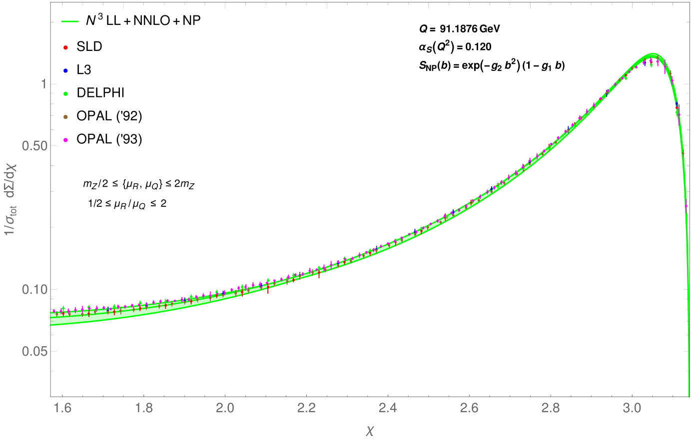




Simultaneous fit of $\alpha_S(m_Z)$ and NP parameters at N^{4}LL+$\mathcal{O}(\alpha_S^3)$:

$$\boldsymbol{\alpha_S(m_Z) = 0.11828^{+0.00084}_{-0.00088}}$$

α_S from Energy-Energy-Correlation in e^+e^-



Comparison with data of the resummed EEC spectrum at N^3LL+NLO with non perturbative k_T *dependent* effects parameterized by a NP form factor $S_{NP} = \exp\{-g_2 b^2\}(1 - g_1 b)$ [G.F.,Aglietti('24)].

Conclusions

- Novel methodology for determination of $\alpha_S(m_Z)$ based on Z-boson small-q_T distribution.
- Based on N^{4}LL+$\mathcal{O}(\alpha_S^3)$ resummed QCD predictions.
- Extraction from CDF Tevatron and ATLAS LHC data.
- Result in agreement with the world average.
- Precise collider determination: less than 1% relative uncertainty.
- Crucial development of `DYTurbo` program to compute fast and accurate theoretical predictions: https://dyturbo.hepforge.org