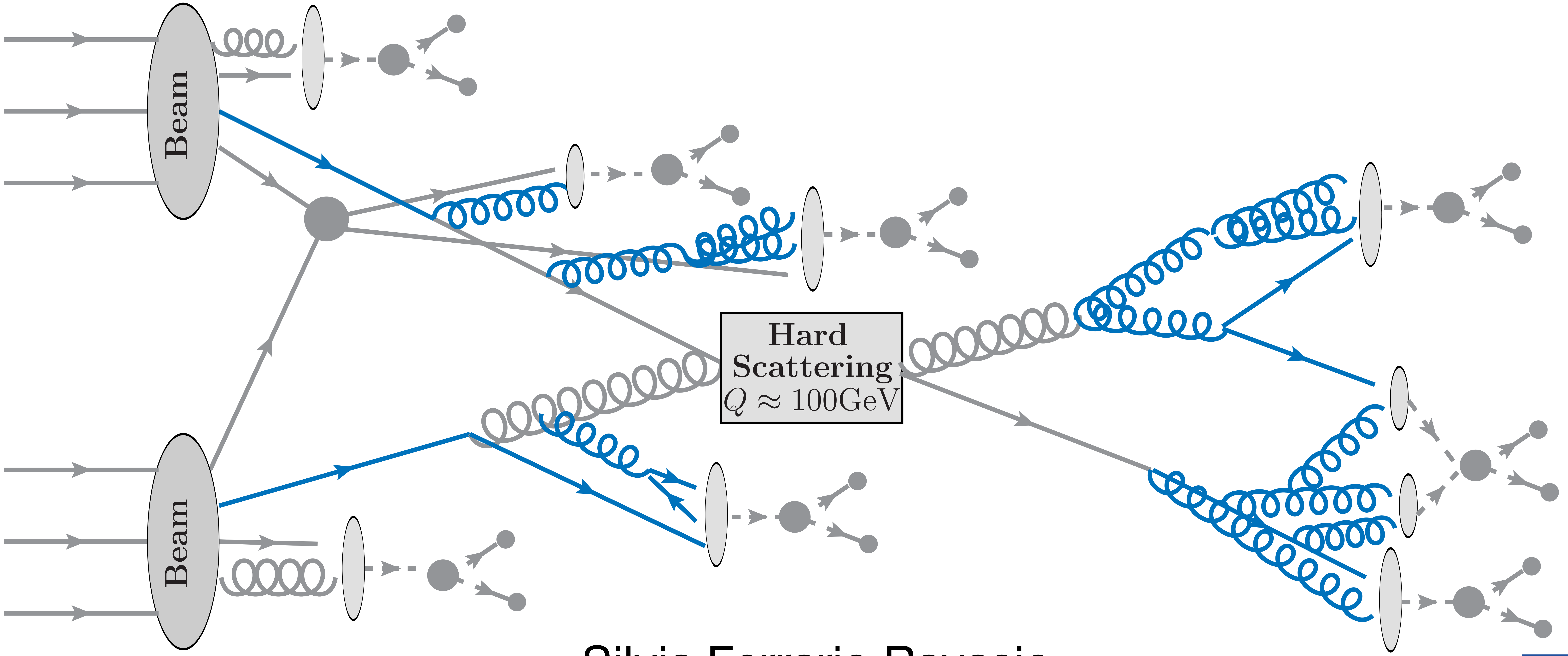
Parton Showers with higher logarithmic accuracy



Silvia Ferrario Ravasio

Resummation, Evolution, Factorization 2024

17th October 2024, nstitut de Physique Théorique, Saclay, France

REF 2024 is the 11th edition in the series of workshops on **Resummation, Evolution and Factorization**.

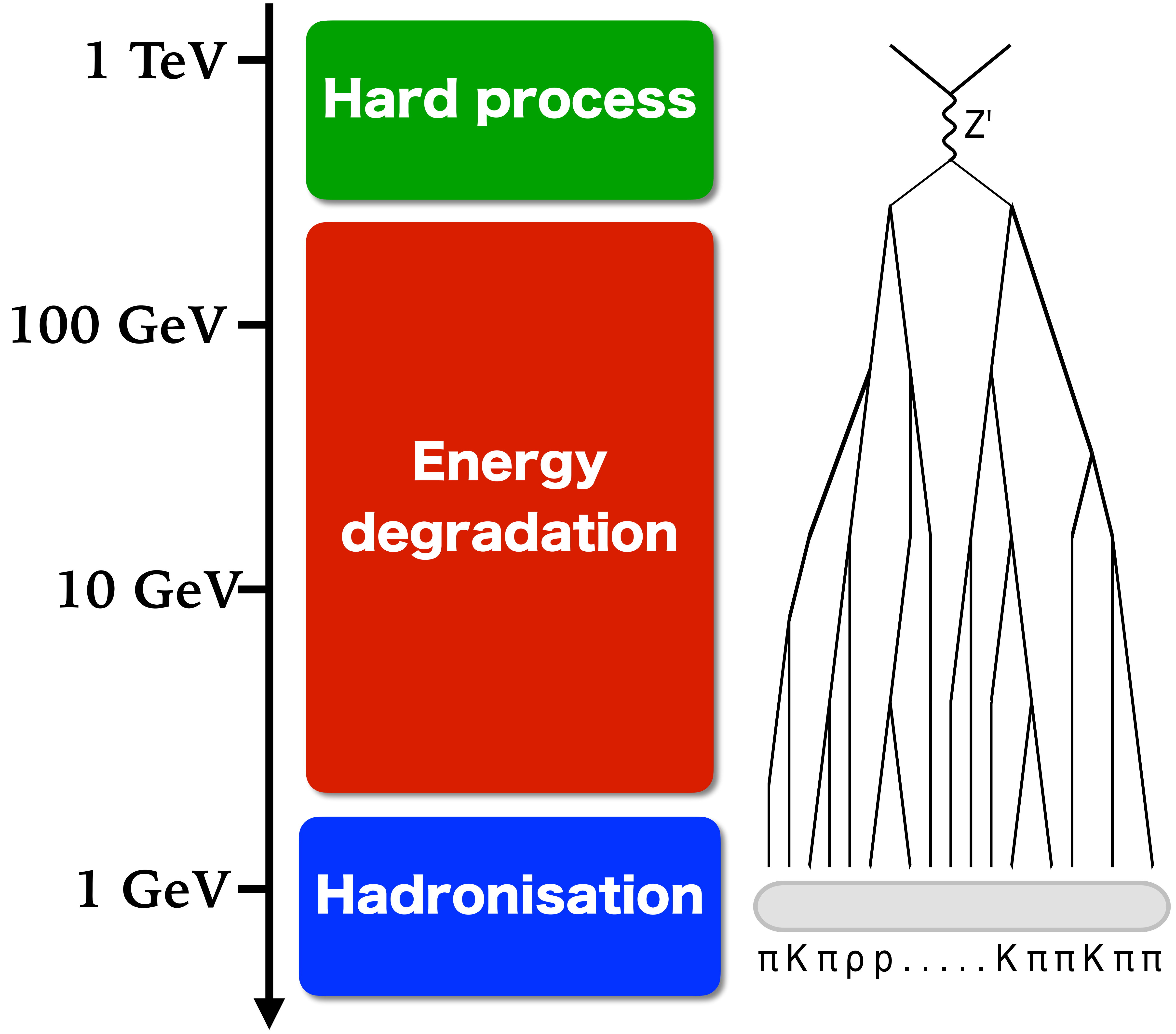
The workshop brings together specialists in different areas, from effective field theory to lattice to QCD factorization methods. **The main focus will be on transverse momentum dependent distributions (TMDs) and their connection with Monte Carlo event generators**, as well as on the experimental measurements aimed at extracting information on TMDs at present and future colliders. The interplay between the factorization theorems, resummation of large logarithms, and the corresponding evolution equations are crucial for higher precision calculations, necessary not only for understanding the data recorded by past and present facilities, such as the LHC, HERA and Belle, but especially for future experiments, such as HL-LHC, EIC, and FCC.

REF 2024 is the 11th edition in the series of workshops on **Resummation, Evolution and Factorization**.

The workshop brings together specialists in different areas, from effective field theory to lattice to QCD factorization methods. **The main focus will be** on transverse momentum dependent distributions (TMDs) and their connection with **Monte Carlo event generators**, as well as on the experimental measurements aimed at extracting information on TMDs at present and future colliders. The interplay between the factorization theorems, **resummation of large logarithms,** and the corresponding **evolution equations** are crucial for higher precision calculations, necessary not only for understanding the data recorded by past and present facilities, such as the LHC, HERA, **LEP** and Belle, but especially for future experiments, such as HL-LHC, EIC, and **FCC(-ee)**.

Shower Monte Carlo event generators

SHOWER MONTE CARLO EVENT GENERATORS = **default tool** for interpreting collider data

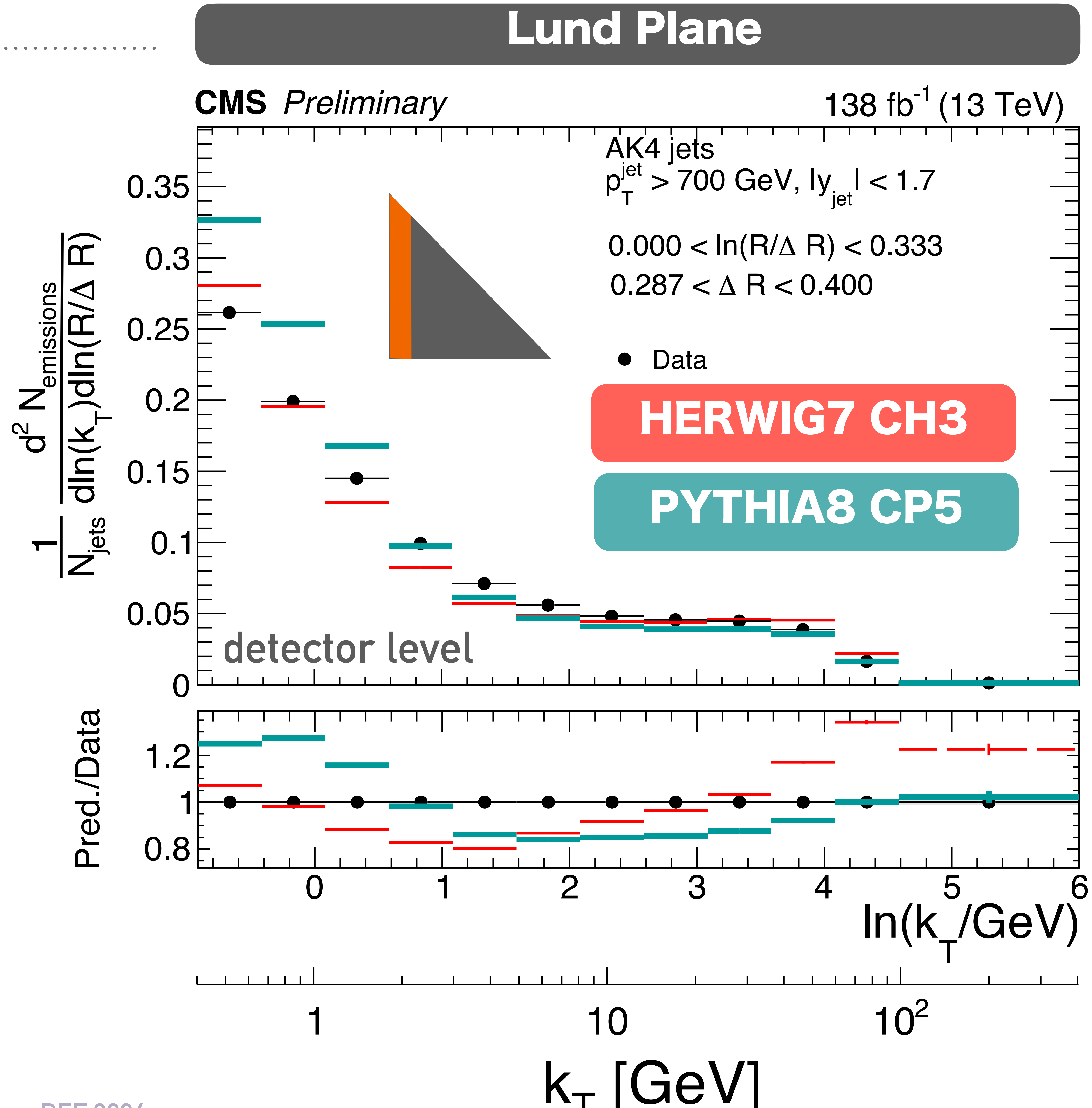


Parton Showers

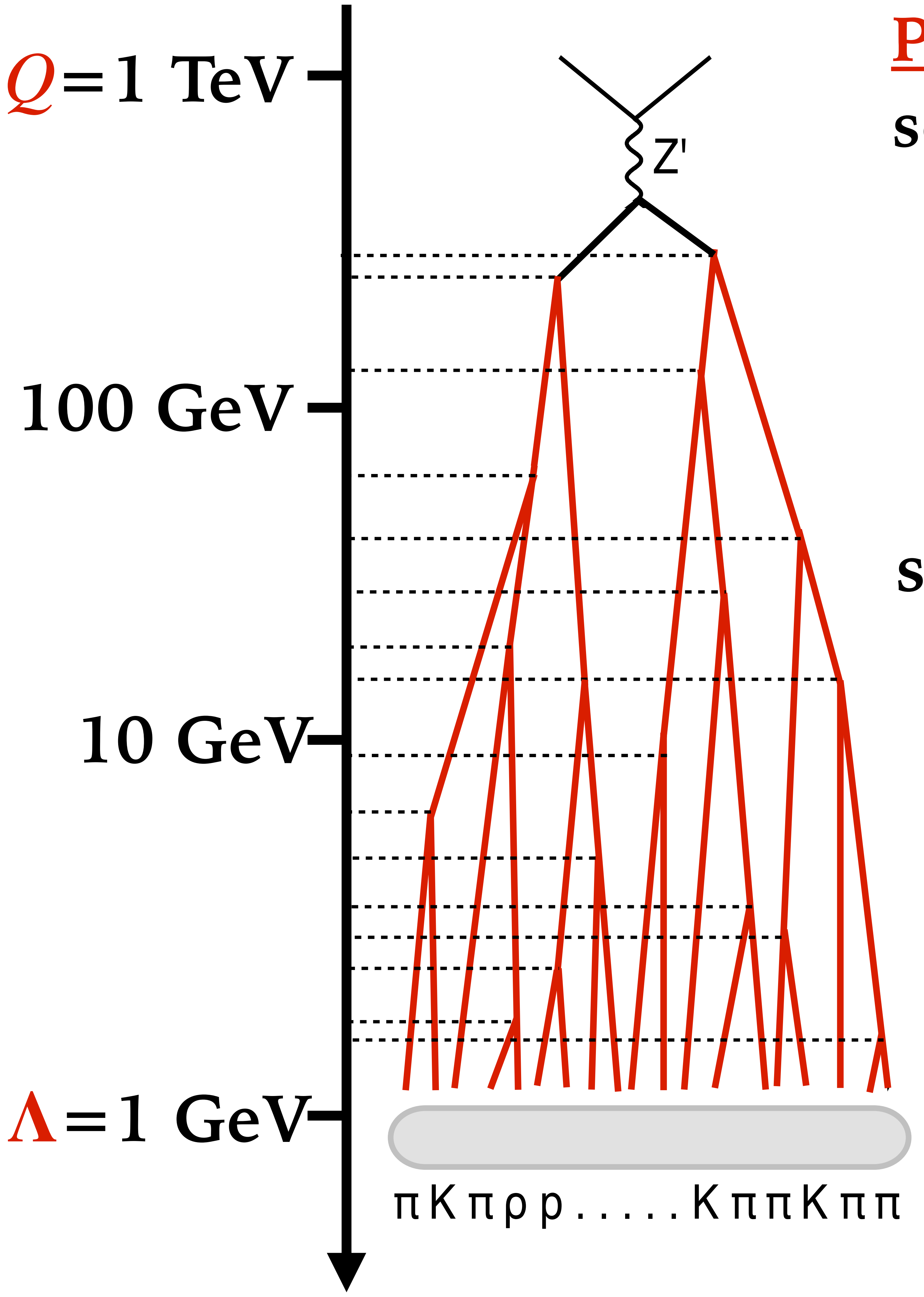
- Energy degradation of hard particles produced during the collision

Are current showers good enough?

- showers do an amazing job on many observables for **LHC**
- various places see **10–30% discrepancies** between showers and data
- A lot of work is required to meet the **percent precision target**!



Logarithmically-accurate Parton Showers



PARTON SHOWERS = energy degradation via an iterated sequence of **softer and softer** emissions

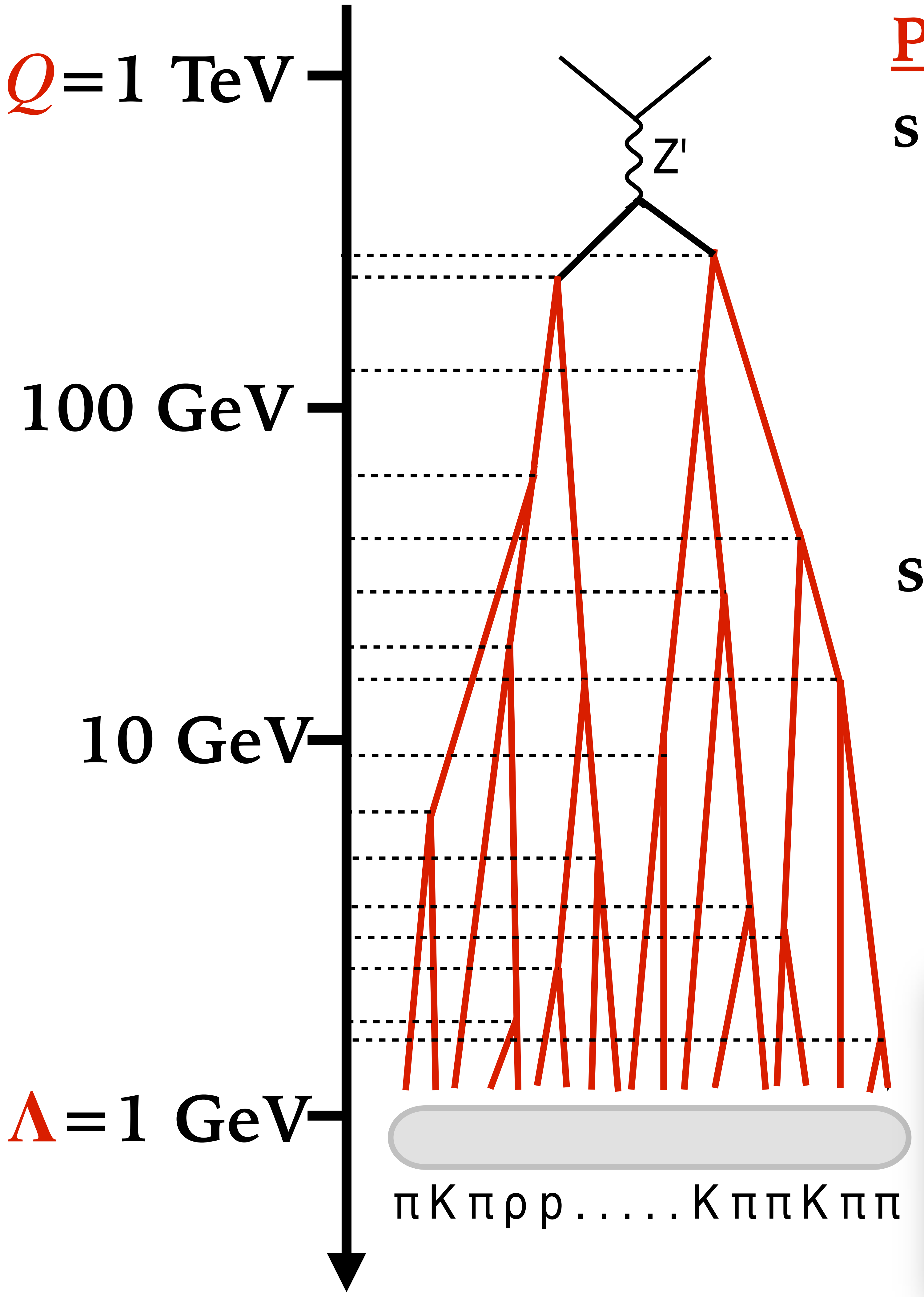
$$L = \ln \frac{Q}{\Lambda} \gg 1$$

simple algorithm to include the **dominant radiative corrections** at all orders for **any observable**!

$$\Sigma(O < e^{-L}) = \exp\left(-L g_{\mathrm{LL}}(\beta_0 \alpha_s L) + \ldots\right)$$

LL = leading logs

Logarithmically-accurate Parton Showers



PARTON SHOWERS = energy degradation via an iterated sequence of **softer and softer** emissions

$$L = \ln \frac{Q}{\Lambda} \gg 1$$

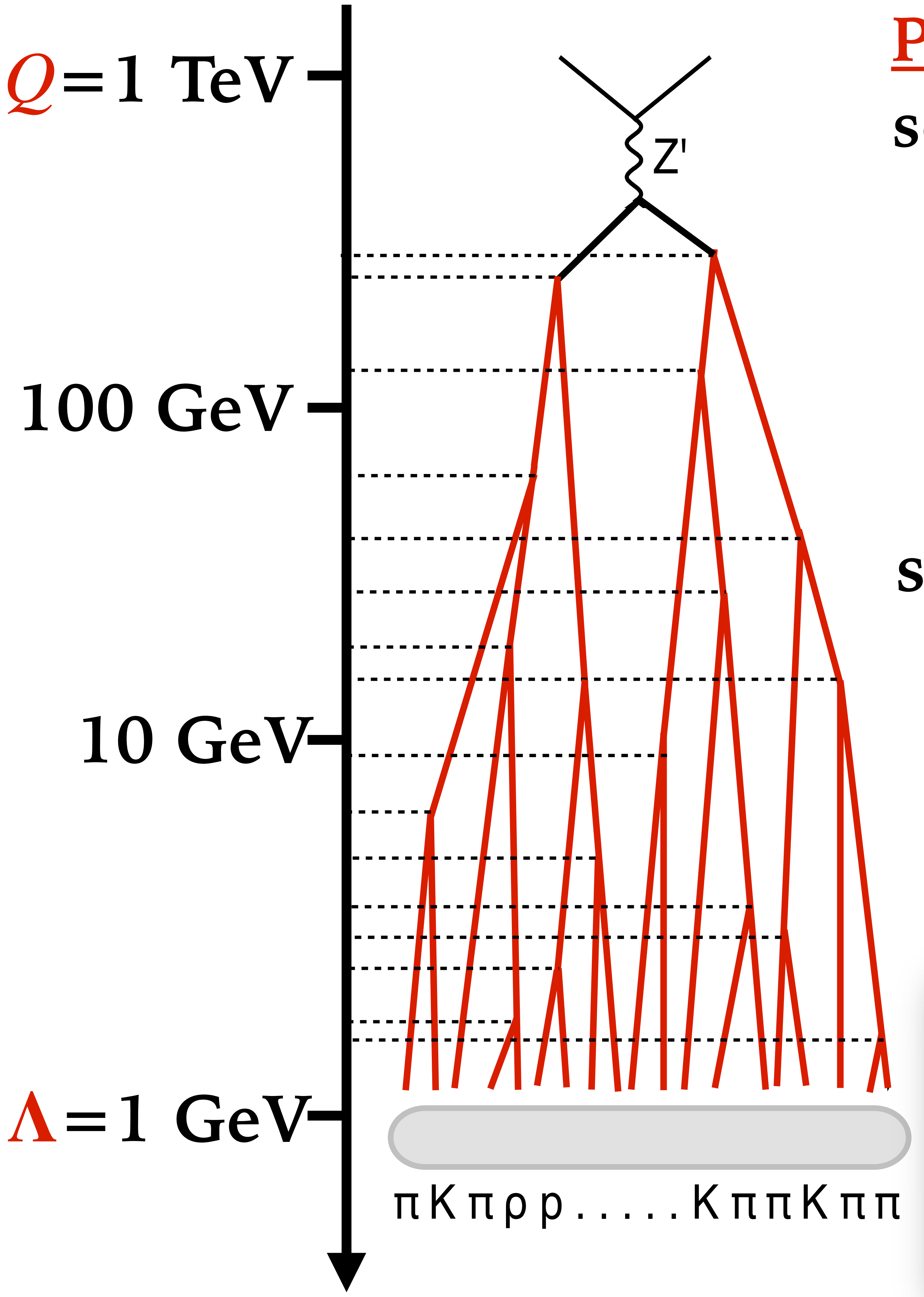
simple algorithm to include the **dominant radiative corrections** at all orders for **any observable**!

??

$$\Sigma(O < e^{-L}) = \exp\left(-L g_{\mathrm{LL}}(\beta_0 \alpha_s L) + g_{\mathrm{NLL}}(\beta_0 \alpha_s L) + \dots\right)$$

For $Q \sim 50 - 10000\,\mathrm{GeV}$, $\beta_0 \alpha_s L \sim 0.3 - 0.5$:
Next-to-Leading Logarithms needed for quantitative predictions!

Logarithmically-accurate Parton Showers



Parton showers = energy degradation via an iterated sequence of **softer and softer** emissions

$$L = \ln \frac{Q}{\Lambda} \gg 1$$

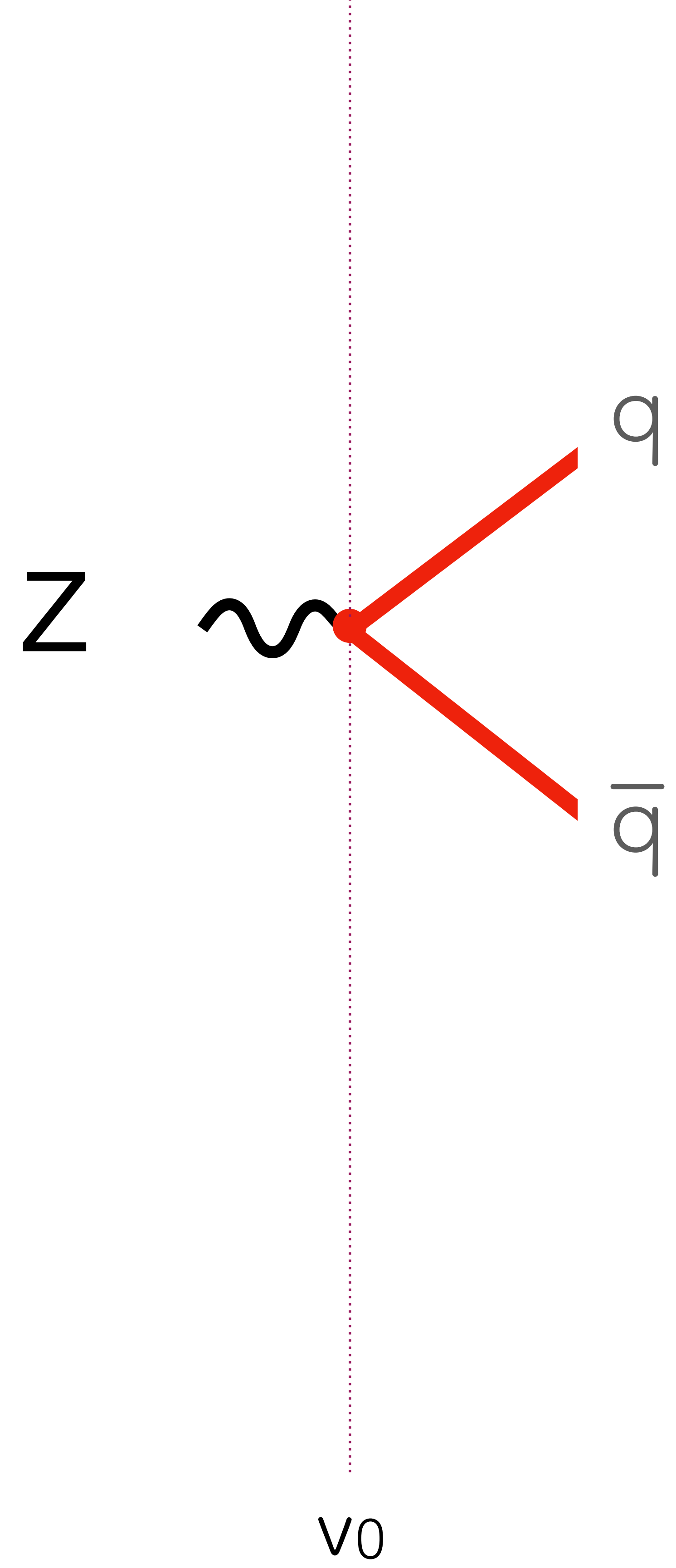
simple algorithm to include the **dominant radiative corrections** at all orders for **any observable**!

$$\Sigma(O < e^{-L}) = \exp\left(-Lg_{\mathrm{LL}}(\beta_0\alpha_s L) + \overset{??}{g_{\mathrm{NLL}}(\beta_0\alpha_s L)} + \overset{??}{\alpha_s g_{\mathrm{NNLL}}(\beta_0\alpha_s L)} + \ldots\right)$$

For $Q \sim 50 - 10000\,\mathrm{GeV}$, $\beta_0\alpha_s L \sim 0.3 - 0.5$:
Next-to-Next-to-Leading Logarithms needed for **%-level**

Parton Showers in a nutshell

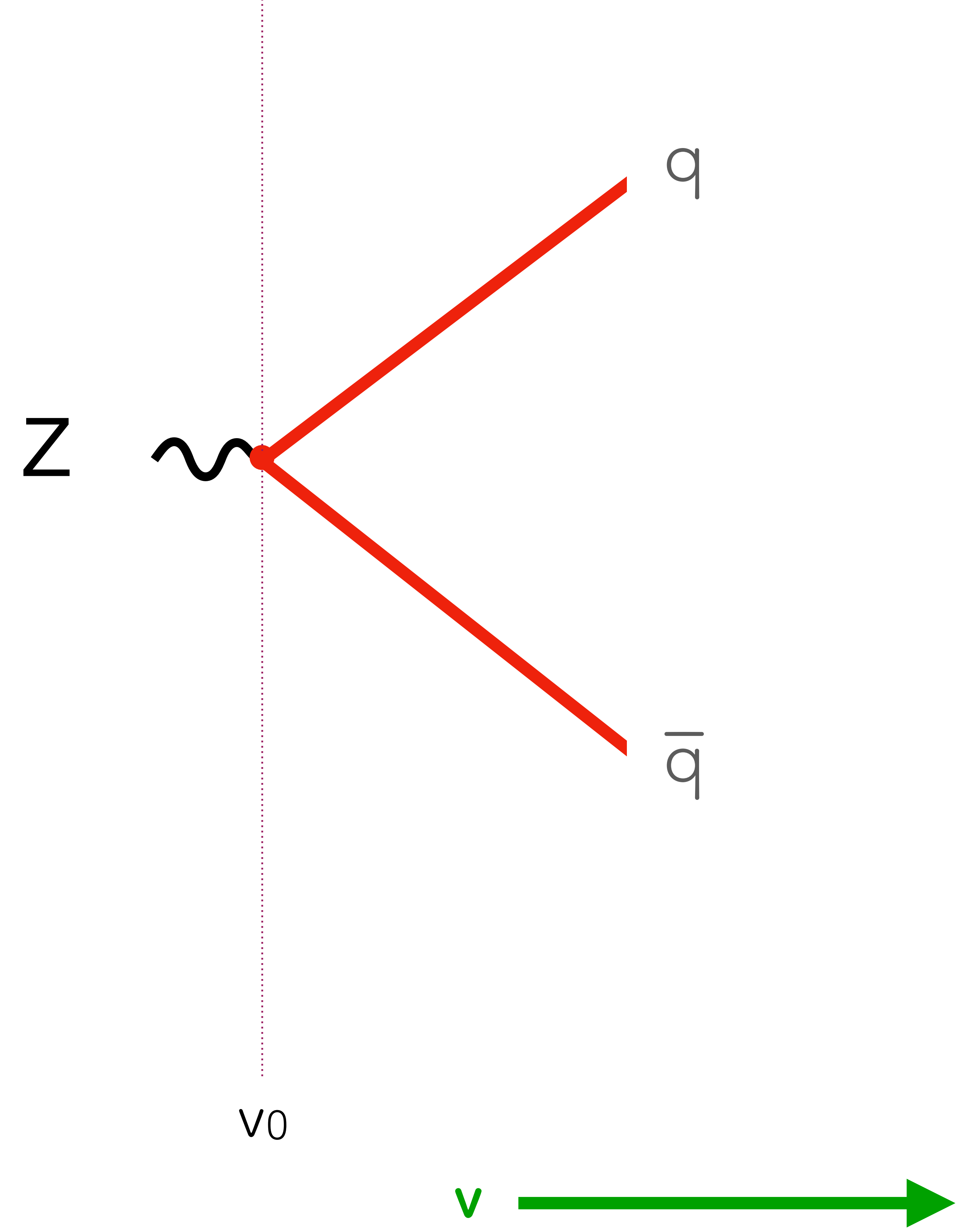
Dipole showers [Gustafson, Pettersson, '88] are the most used shower paradigm



Start with $q\bar{q}$ state produced at a hard scale v_0.

Parton Showers in a nutshell

Dipole showers [Gustafson, Pettersson, '88] are the most used shower paradigm



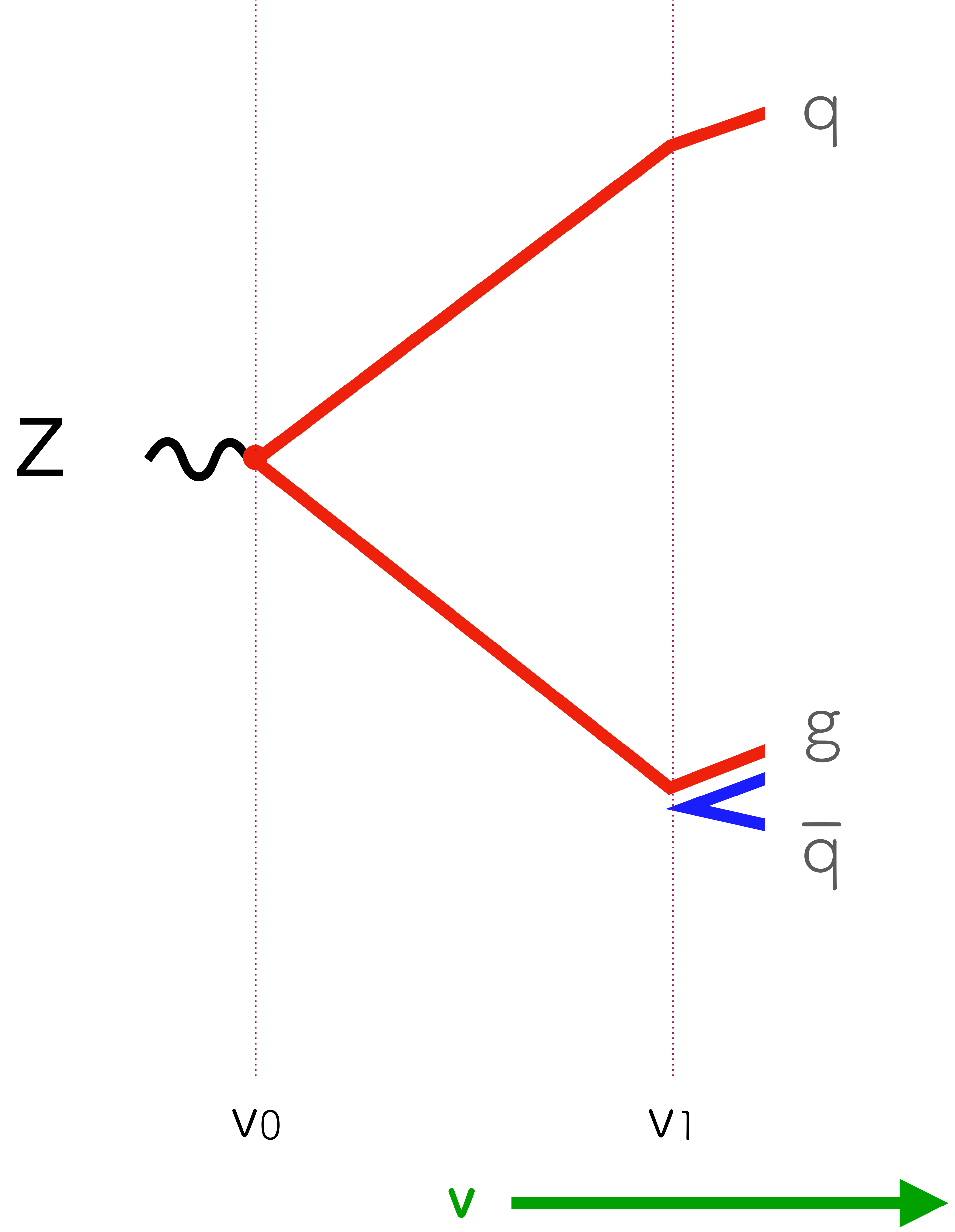
Start with $q\bar{q}$ state produced at a hard scale v_0.

Throw a random number to determine down to what **scale** state persists unchanged

$$\Delta(v_0, v) = \exp\left(-\int_{v}^{v_0} dP_{q\bar{q}}(\Phi)\right)$$

Parton Showers in a nutshell

Dipole showers [Gustafson, Pettersson, '88] are the most used shower paradigm



Start with $q\bar{q}$ state produced at a hard scale v_0.

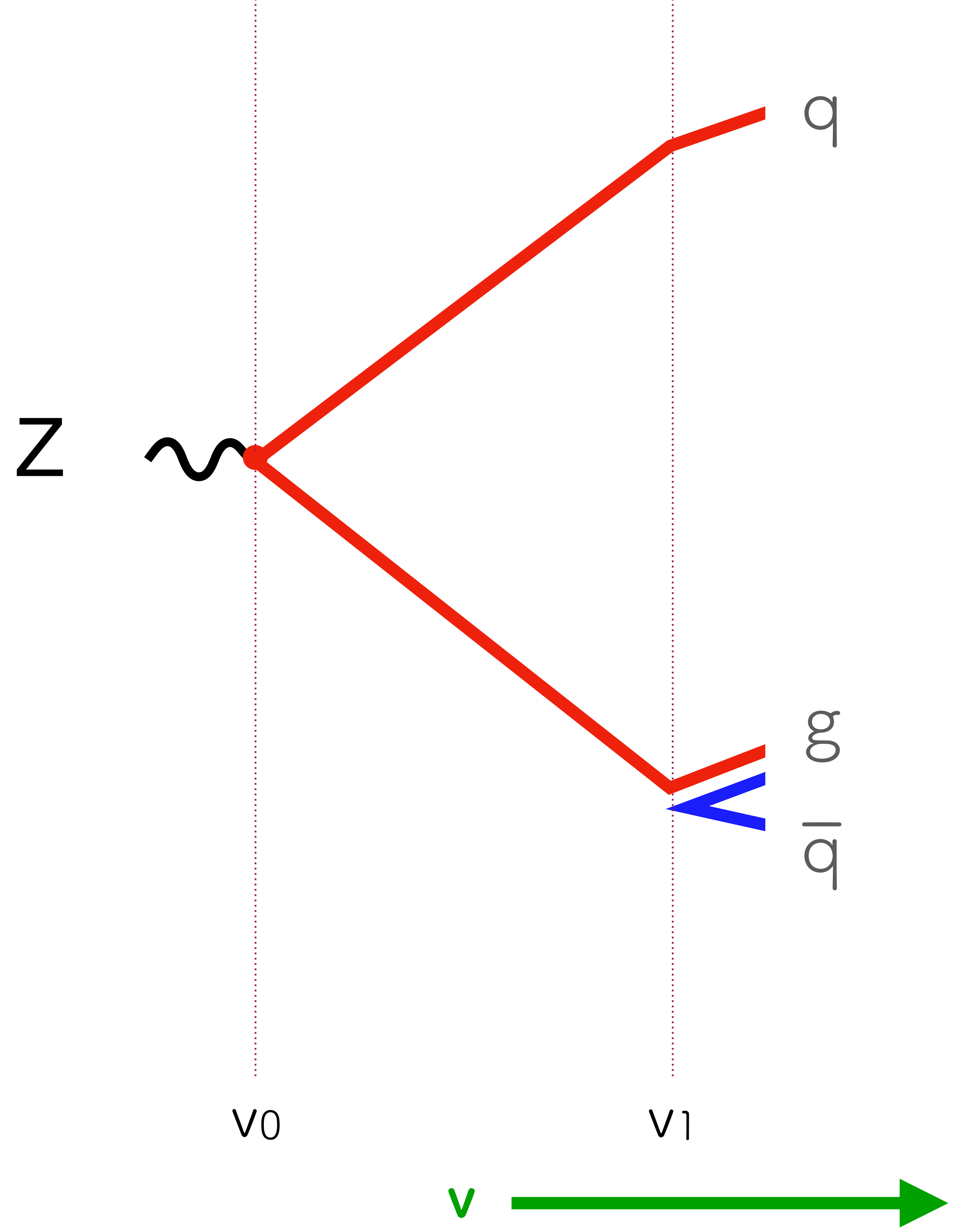
Throw a random number to determine down to what **scale** state persists unchanged

At some point, **state splits** (2→3, i.e. emits gluon) at a scale $v_1 < v_0$. The kinematic (rapidity and azimuth) of the gluon is chosen according to

$$dP_{q\bar{q}}(\Phi(v_1)) \qquad \Phi = \{v, \eta, \varphi\}$$

Parton Showers in a nutshell

Dipole showers [Gustafson, Pettersson, '88] are the most used shower paradigm



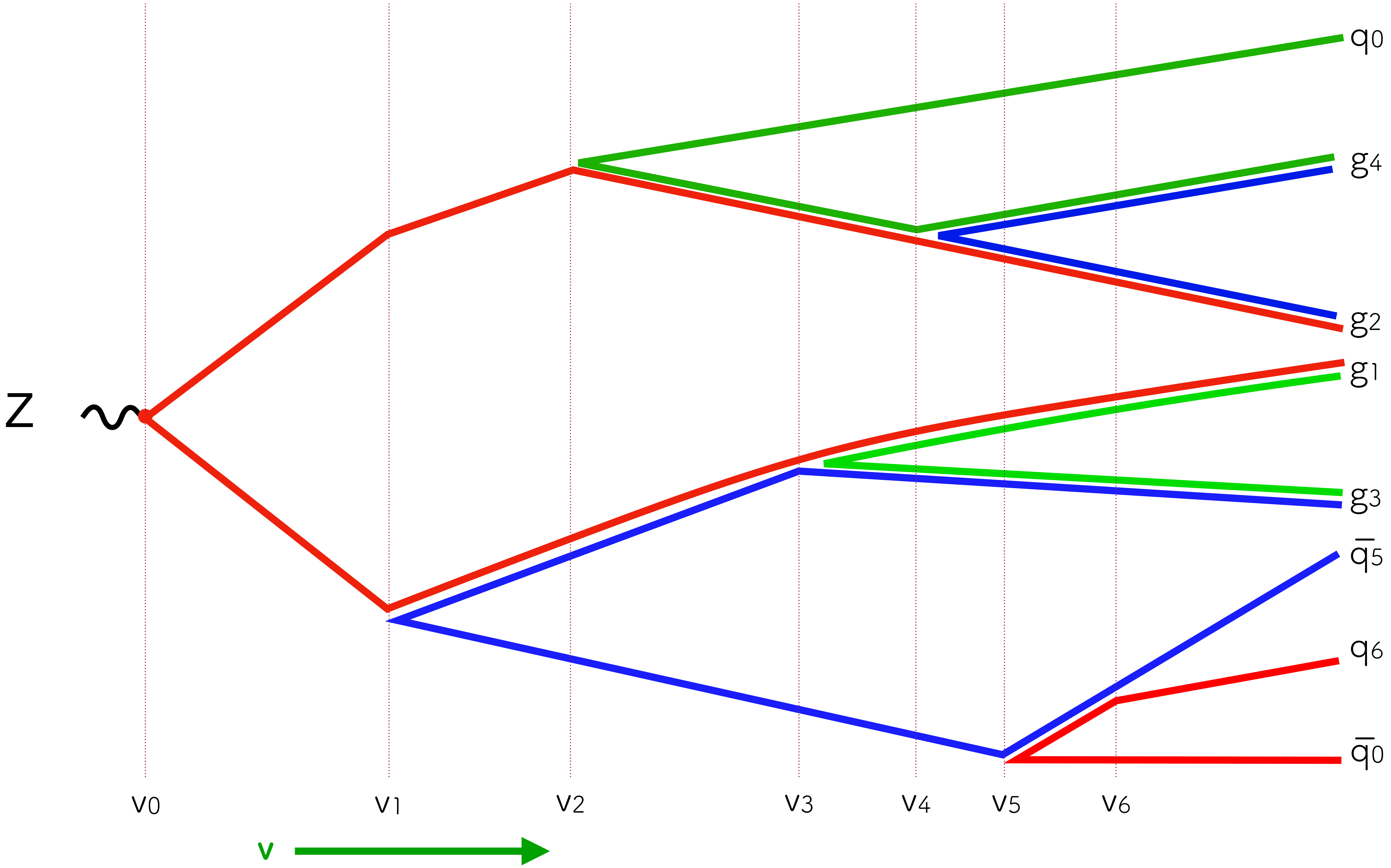
Start with $q\bar{q}$ state produced at a **hard scale** v_0.

Throw a random number to determine down to what **scale** state persists unchanged

At some point, **state splits** (2→3, i.e. emits gluon) at a scale $v_1 < v_0$.

The gluon is part of two dipoles (qg), $(g\bar{q})$.

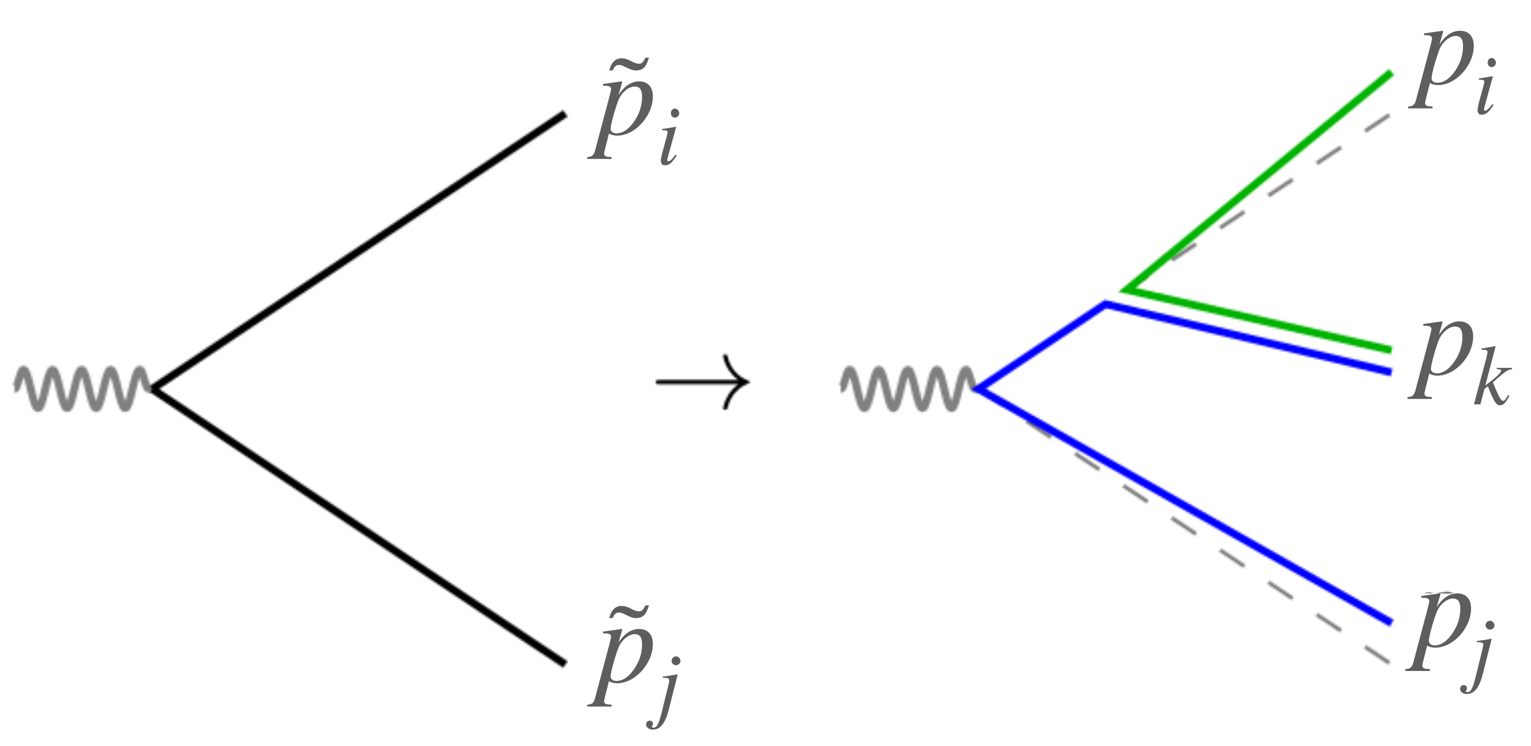
Iterate the above procedure for both dipoles independently, using v_1 as starting scale.



self-similar evolution continues until it reaches a non-perturbative scale

Dissecting the parton shower emission probability

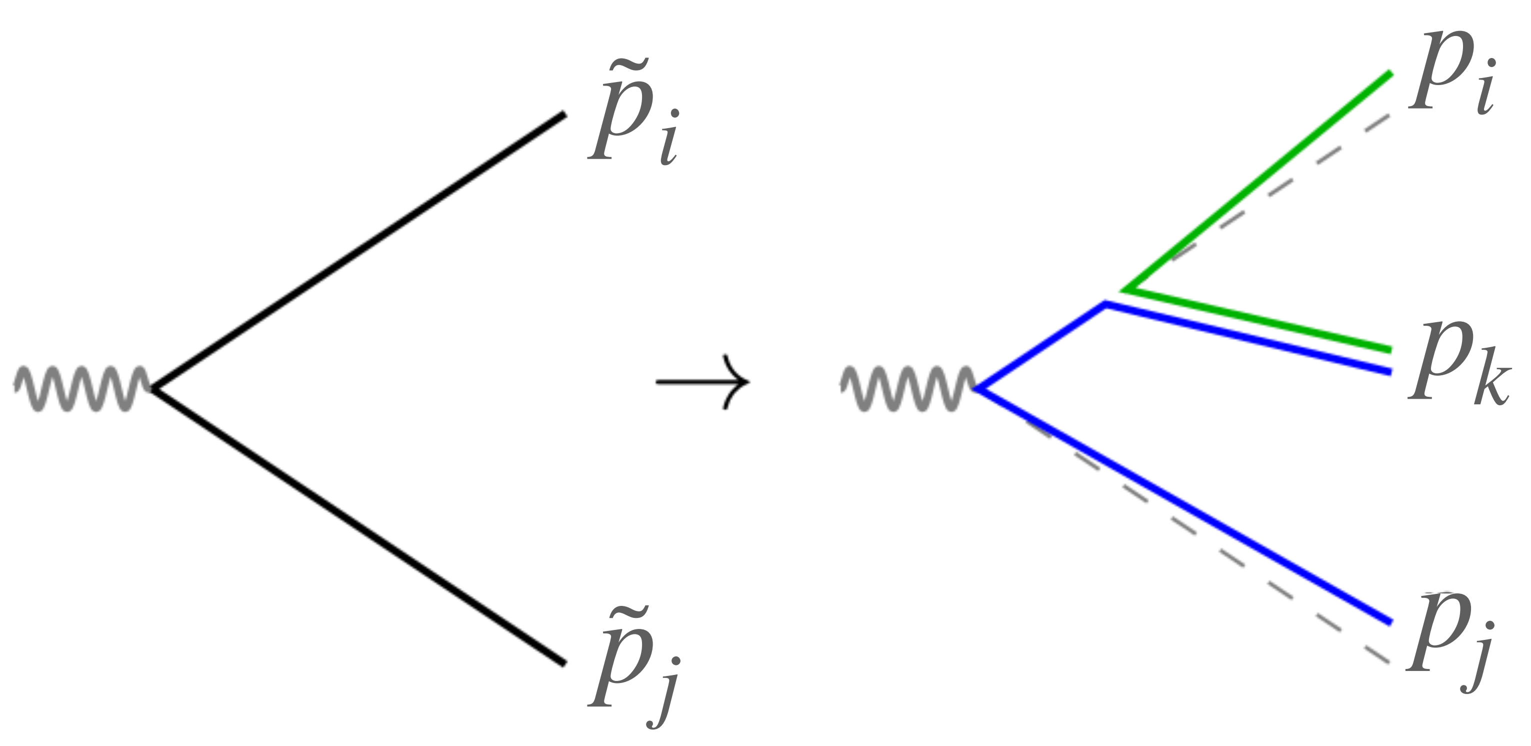
Starting from a $e^+e^- \to Z^* \to q\bar{q}$ system, what is the splitting probability?



$$\mathrm{d}\mathscr{P}_{\tilde{i}\tilde{j}\to ijk} \sim \frac{\mathrm{d}v^2}{v^2}\mathrm{d}\bar{\eta}\,\frac{\mathrm{d}\varphi}{2\pi}\;P_{\tilde{i},\tilde{j}\to i,j,k}(v,\bar{\eta},\varphi)$$

Dissecting the parton shower emission probability

Starting from a $e^+e^- \to Z^* \to q\bar{q}$ system, what is the splitting probability?

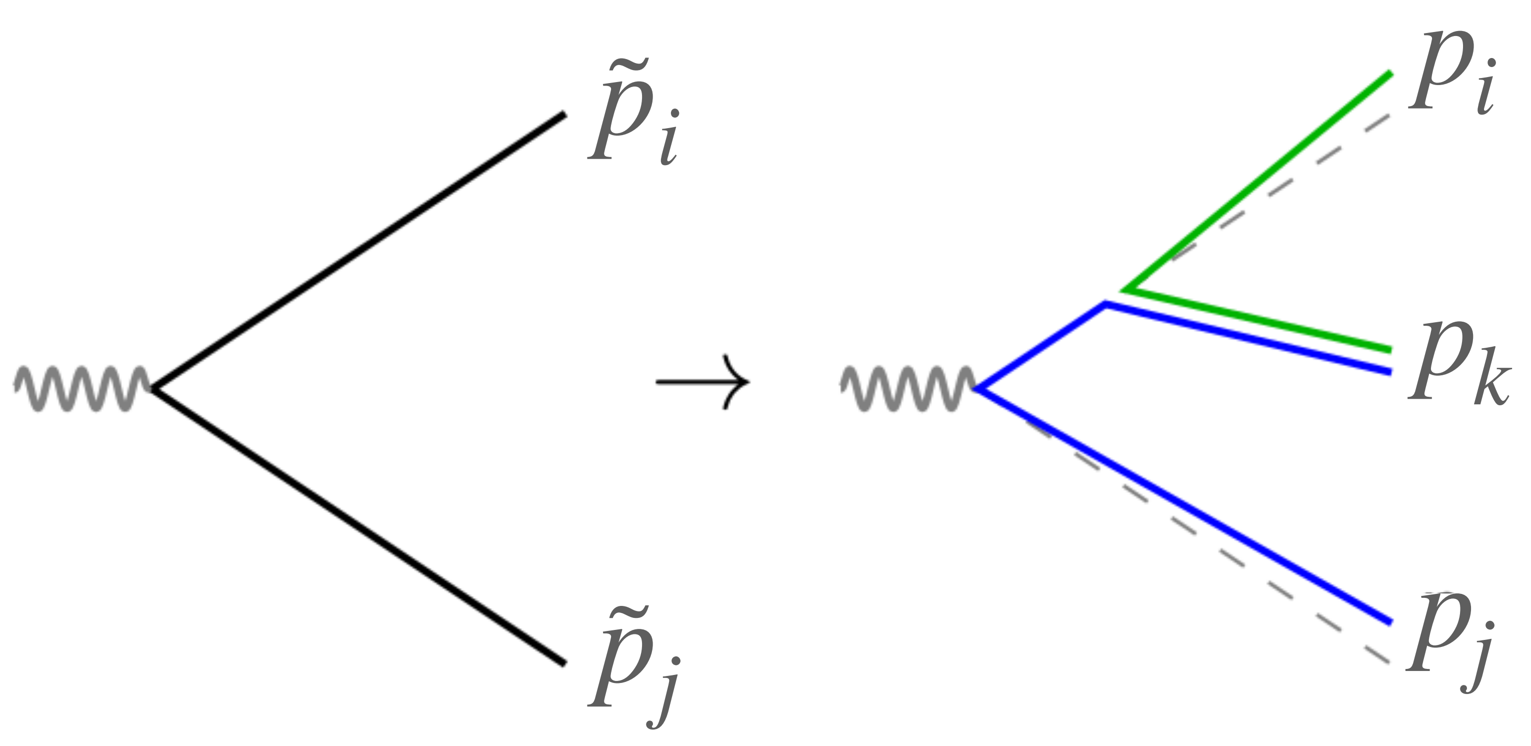


$$\mathrm{d}\mathcal{P}_{\tilde{i}\tilde{j}\to ijk} \sim \frac{\mathrm{d}v^2}{v^2}\mathrm{d}\bar{\eta}\,\frac{\mathrm{d}\varphi}{2\pi}\,P_{\tilde{i},\tilde{j}\to i,j,k}(v,\bar{\eta},\varphi)$$

Matrix element for emitting a parton k from a parton i (or j)

Dissecting the parton shower emission probability

Starting from a $e^+e^- \to Z^* \to q\bar{q}$ system, what is the splitting probability?



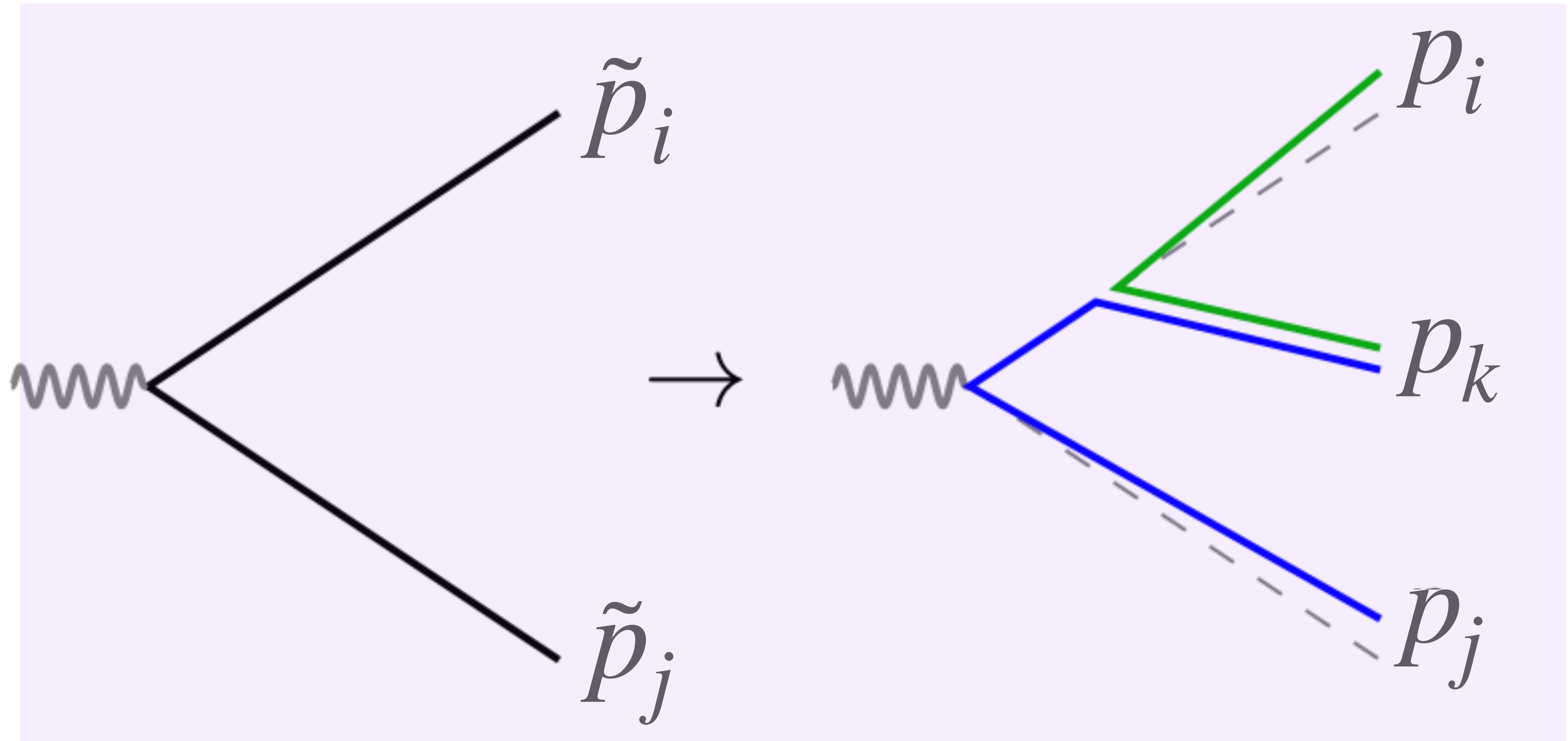
$$\mathrm{d}\mathcal{P}_{\tilde{i}\tilde{j}\to ijk} \sim \frac{\mathrm{d}v^2}{v^2}\mathrm{d}\bar{\eta}\,\frac{\mathrm{d}\varphi}{2\pi}\,P_{\tilde{i},\tilde{j}\to i,j,k}(v,\bar{\eta},\varphi)$$

Evolution variable: emissions are ordered $Q > v_1 > v_2 > \ldots > \Lambda$

Matrix element for emitting a parton k from a parton i (or j)

Dissecting the parton shower emission probability

Starting from a $e^+e^- \to Z^* \to q\bar{q}$ system, what is the splitting probability?



Kinematic mapping: how to reshuffle the momenta of i and j after the emission takes place

$$\mathrm{d}\mathscr{P}_{\tilde{i}\tilde{j}\to ijk} \sim \frac{\mathrm{d}v^2}{v^2}\mathrm{d}\bar{\eta}\,\frac{\mathrm{d}\varphi}{2\pi}\, P_{\tilde{i},\tilde{j}\to i,j,k}(v,\bar{\eta},\varphi)$$

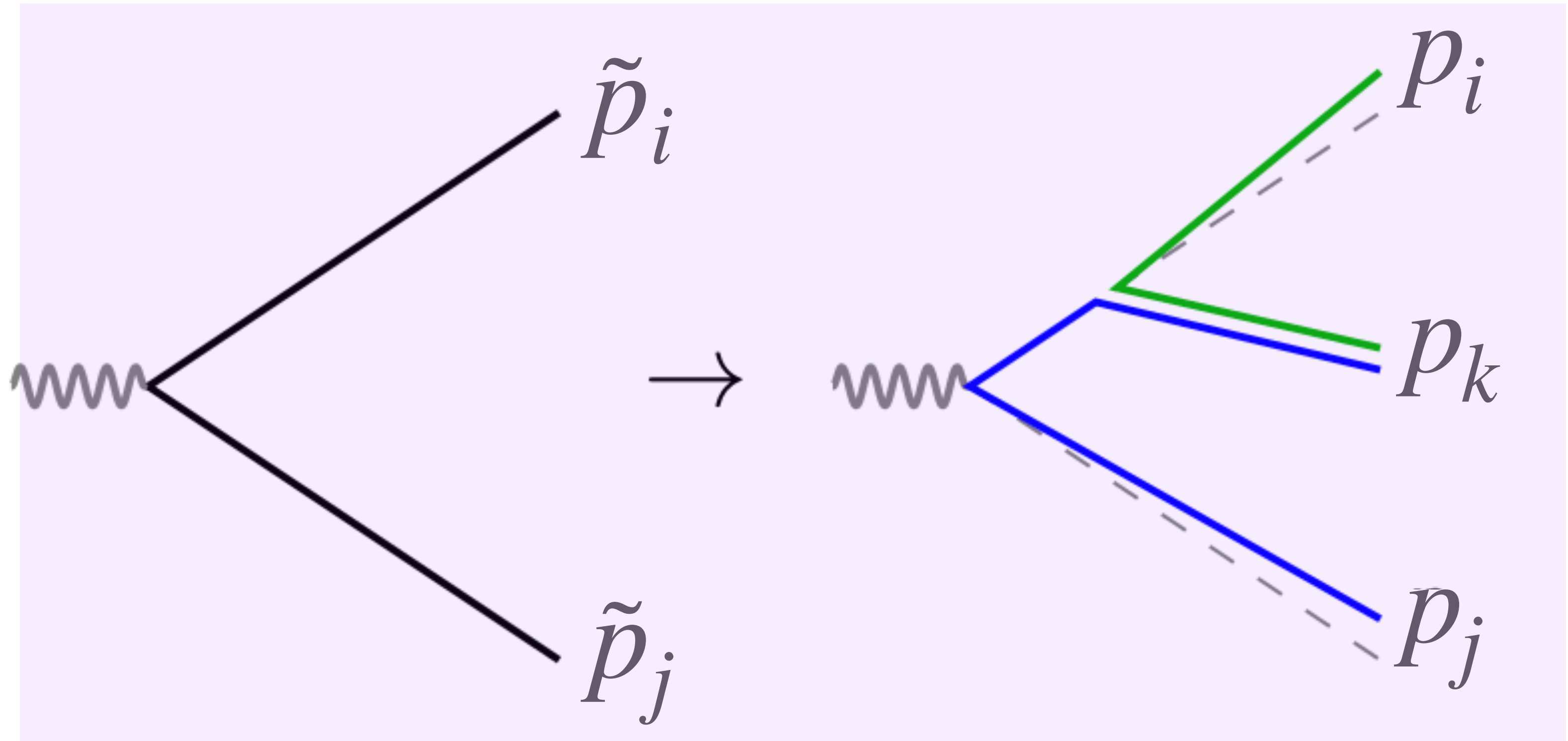
Evolution variable: emissions are ordered $Q > v_1 > v_2 > \ldots > \Lambda$

Matrix element for emitting a parton k from a parton i (or j)

Dissecting the parton shower emission probability

Starting from a $e^+e^- \to Z^* \to q\bar{q}$ system, what is the splitting probability?

Their inteplay determines the shower logarithmic accuracy



Kinematic mapping: how to reshuffle the momenta of i and j after the emission takes place

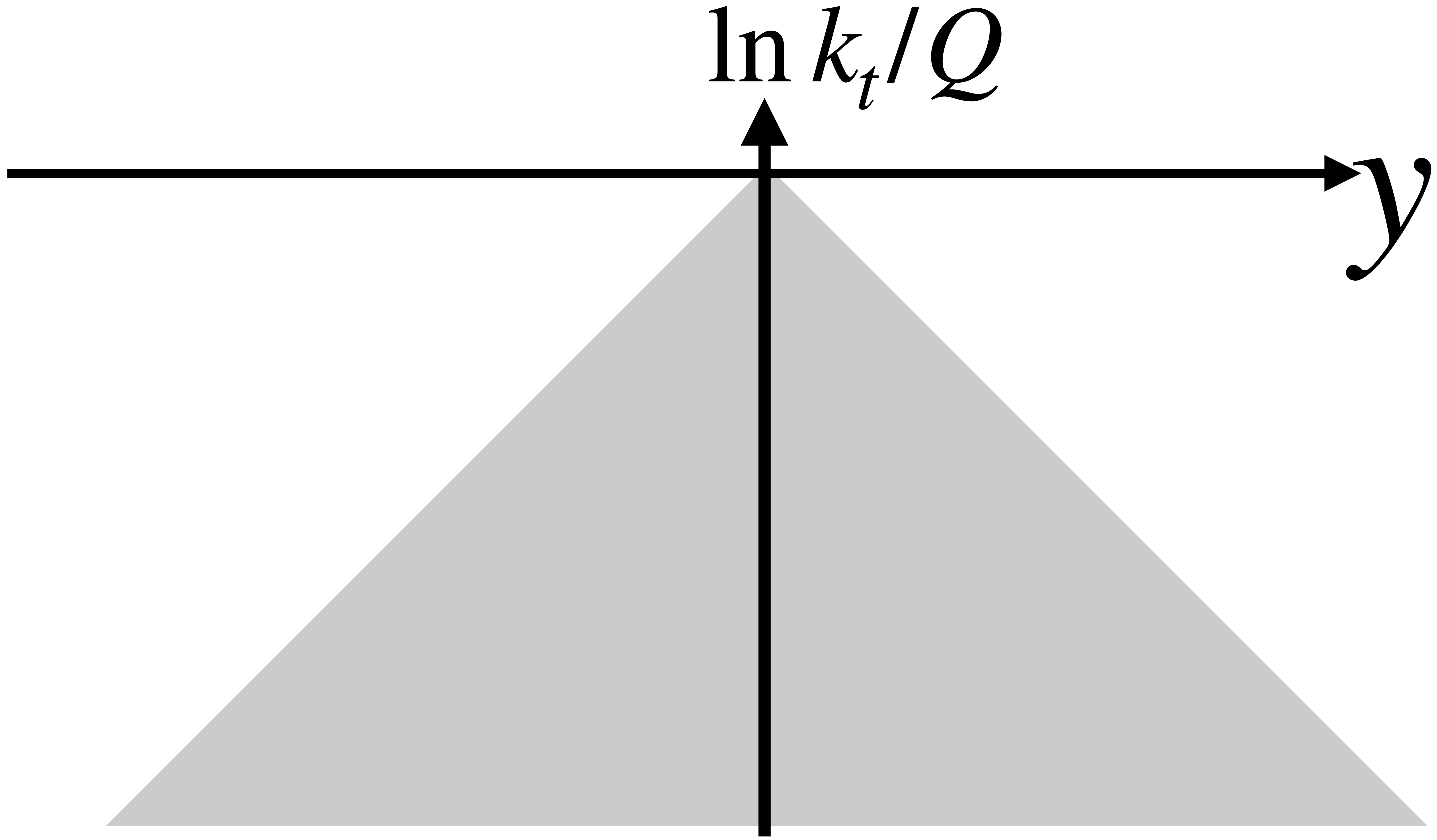
$$\mathrm{d}\mathscr{P}_{\tilde{i}\tilde{j}\to ijk} \sim \frac{\mathrm{d}v^2}{v^2}\mathrm{d}\bar{\eta}\,\frac{\mathrm{d}\varphi}{2\pi}\,P_{\tilde{i},\tilde{j}\to i,j,k}(v,\bar{\eta},\varphi)$$

Evolution variable: emissions are ordered $Q > v_1 > v_2 > \ldots > \Lambda$

Matrix element for emitting a parton k from a parton i (or j)

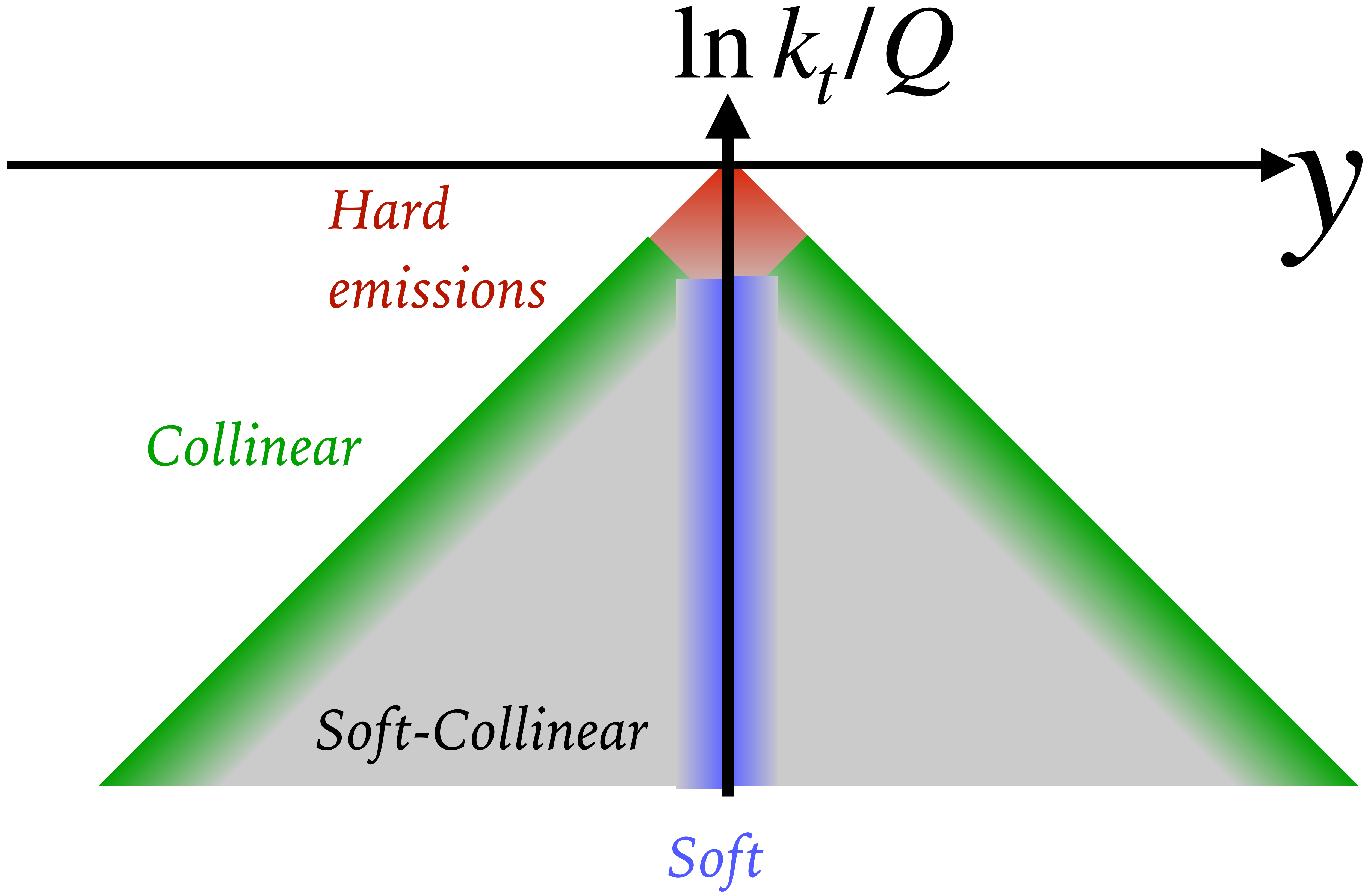
How to build a logarithmically-accurate parton shower?

- **<u>The Lund plane:</u>** diagnostic tools for resummation and parton showers



How to build a logarithmically-accurate parton shower?

- **The Lund plane:** diagnostic tools for resummation and parton showers



How to build a **LL** parton shower?

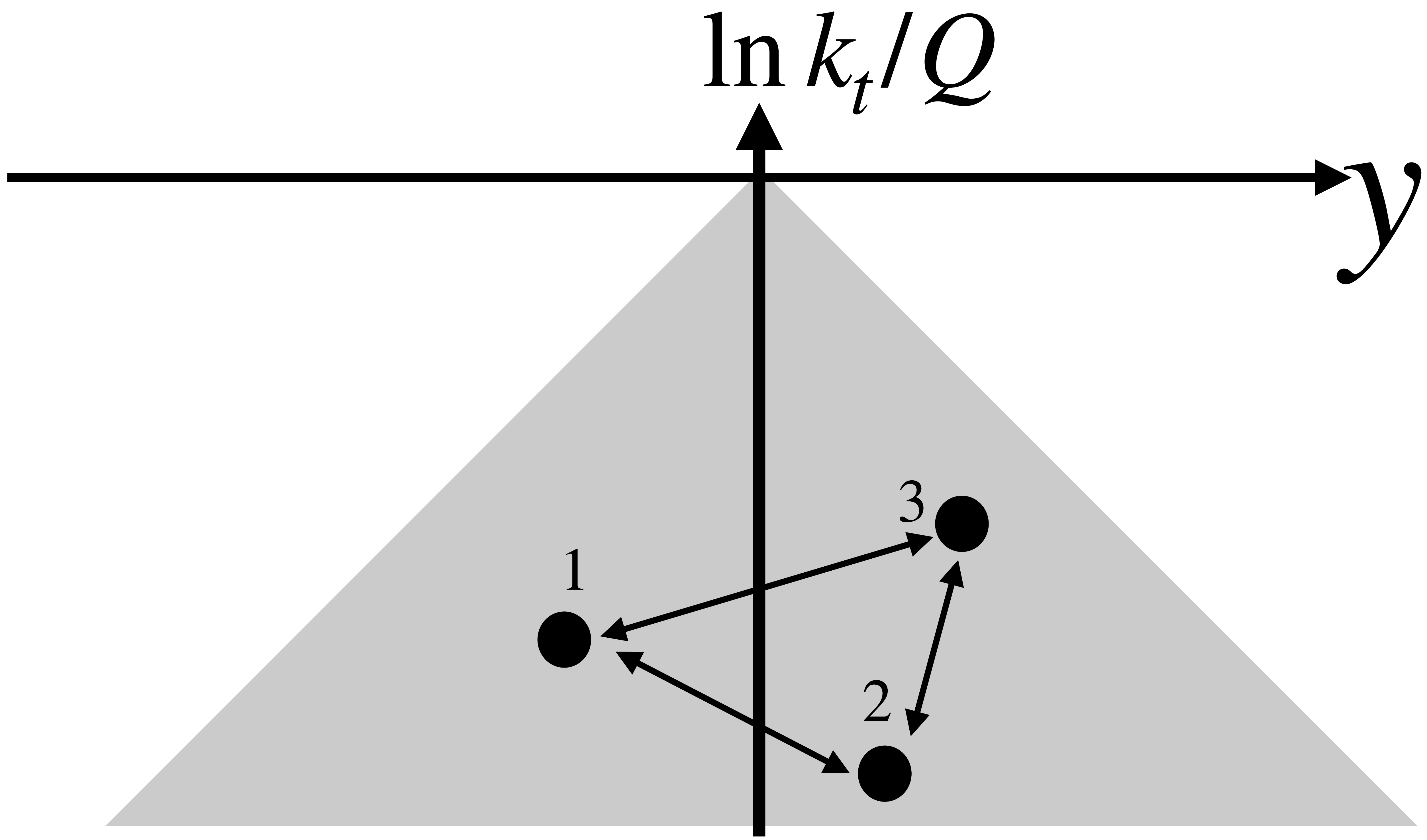
- **The Lund plane:** diagnostic tools for resummation and parton showers

- At **Leading Logarithmic accuracy** we only care about **soft-collinear emissions** very separated between each others

$$dP_i = \frac{\alpha_s(k_t)}{\pi}\frac{2C_F}{z}dz\,d\ln k_t$$

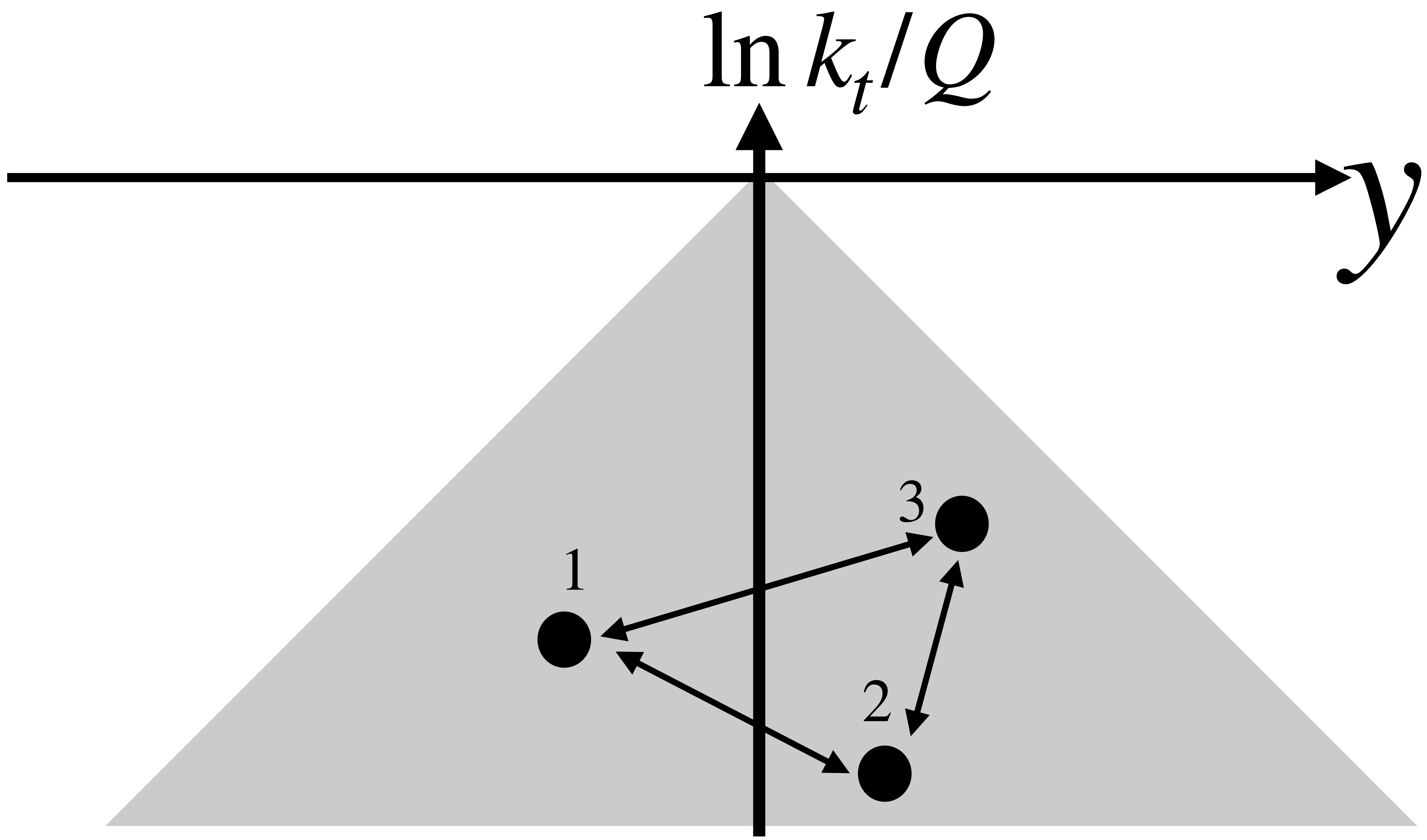
One-loop QCD coupling constant at $\mu_R = k_t$

LO soft splitting function



How to build a **LL** parton shower?

- **The Lund plane:** diagnostic tools for resummation and parton showers
- At **Leading Logarithmic accuracy** we only care about **soft-collinear emissions** very separated between each others



$$dP_i = \frac{\alpha_s(k_t)}{\pi}\frac{2C_F}{z}dz\,d\ln k_t$$

One-loop QCD coupling constant at $\mu_R = k_t$

LO soft splitting function

This tells us what **matrix element** should we use to generate a new emission

How to build a **LL** parton shower?

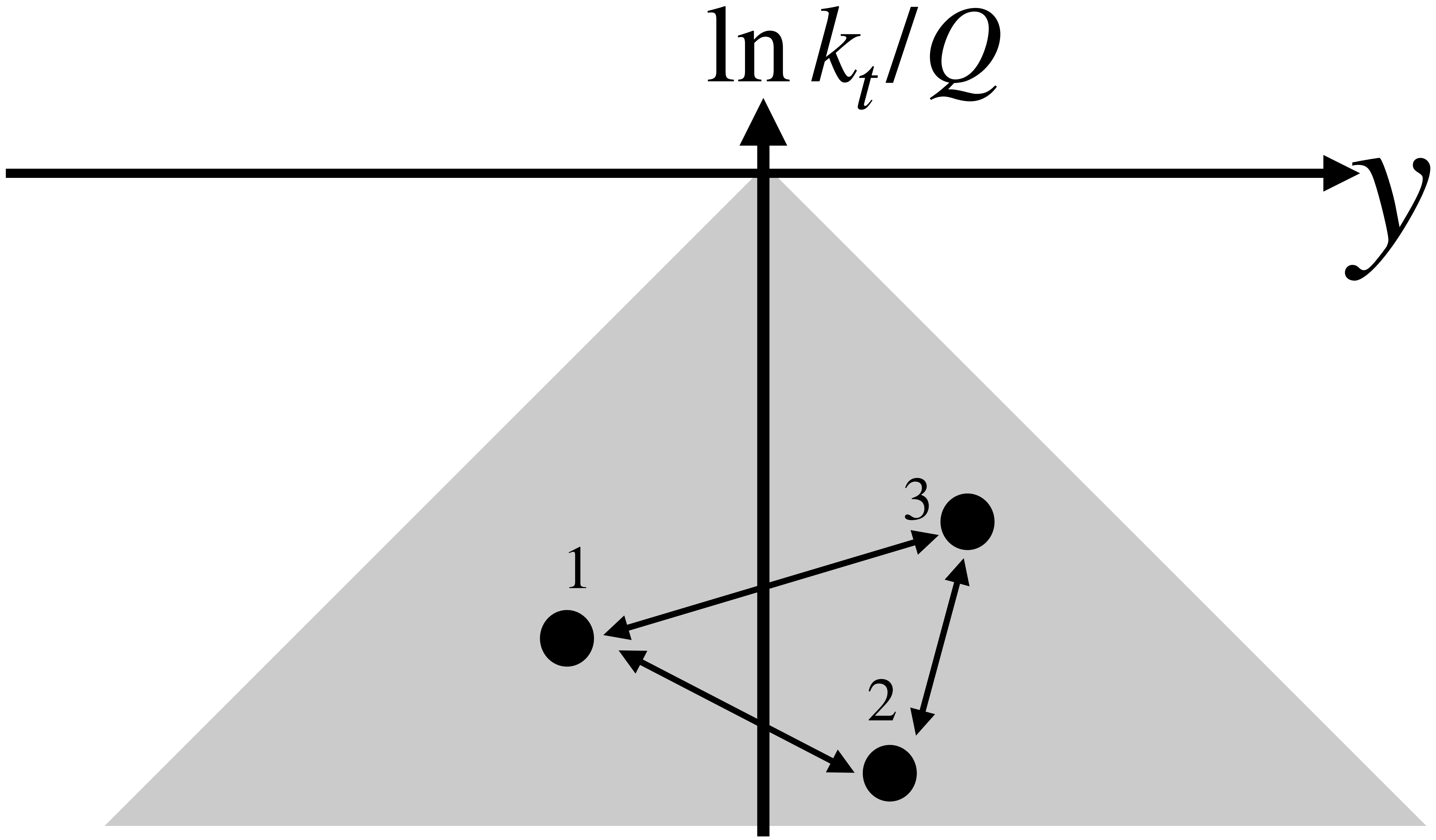
- **The Lund plane:** diagnostic tools for resummation and parton showers

- At **Leading Logarithmic accuracy** we only care about **soft-collinear emissions** very separated between each others

$$dP_i = \frac{\alpha_s(k_t)}{\pi} \frac{2C_F}{z} dz\, d\ln k_t$$

One-loop QCD coupling constant at $\mu_R = k_t$

LO soft splitting function



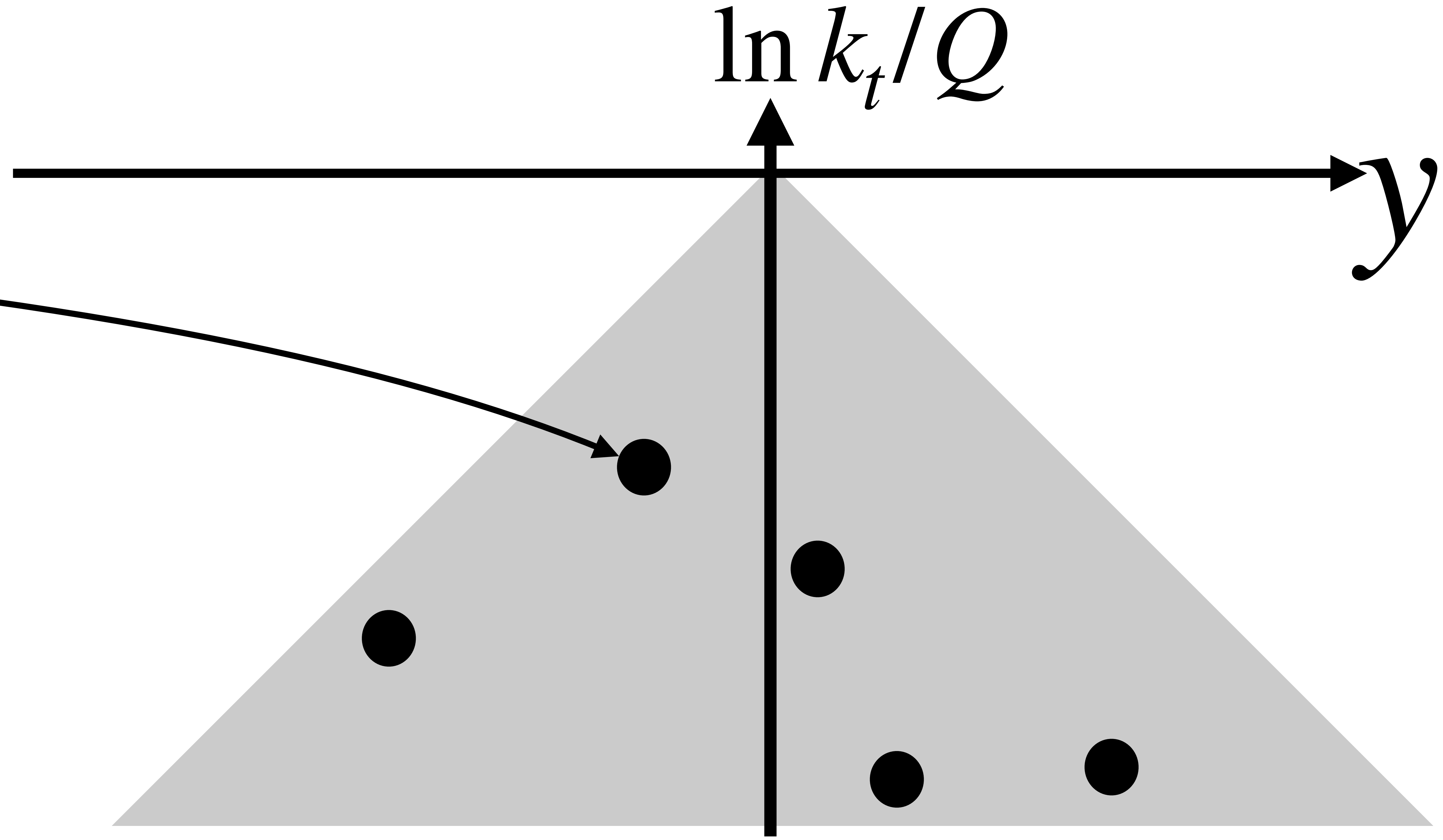
This constraints the **kinematic mapping** $\Phi_n \to \Phi_{n+1}$ and the **ordering variable** choice: **emissions well separated in rapidity and transverse momentum are independent from each others**

How to build a **NLL** parton shower?

At **NLL accuracy:**

- The rate for **soft-collinear emissions** must be correct at NLO

$$dP_i = \frac{\alpha_s(k_t)}{\pi}\left(1+\frac{\alpha_s(k_t)}{2\pi}K_1\right)\frac{2C_F}{z}dz\,d\ln k_t$$



How to build a **NLL** parton shower?

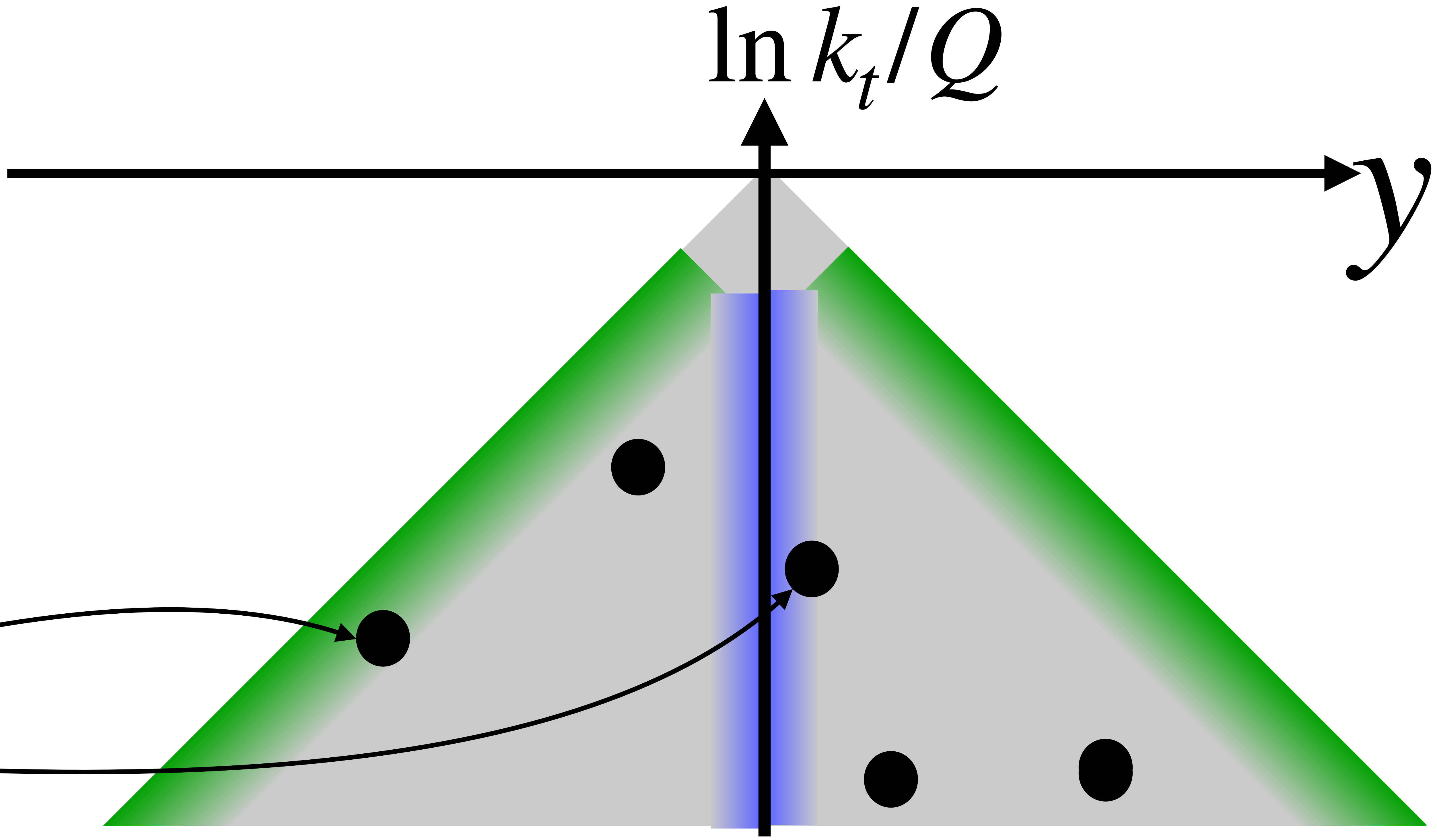
At **NLL accuracy:**

- The rate for **soft-collinear emissions** must be correct at NLO

$$dP_i = \frac{\alpha_s(k_t)}{\pi}\left(1+\frac{\alpha_s(k_t)}{2\pi}K_1\right)\frac{2C_F}{z}dz\, d\ln k_t$$

- We need to include soft and collinear contributions at LO

$$dP_i = \frac{\alpha_s(k_t)}{\pi}P(z)\, dz\, d\ln k_t$$



How to build a **NLL** parton shower?

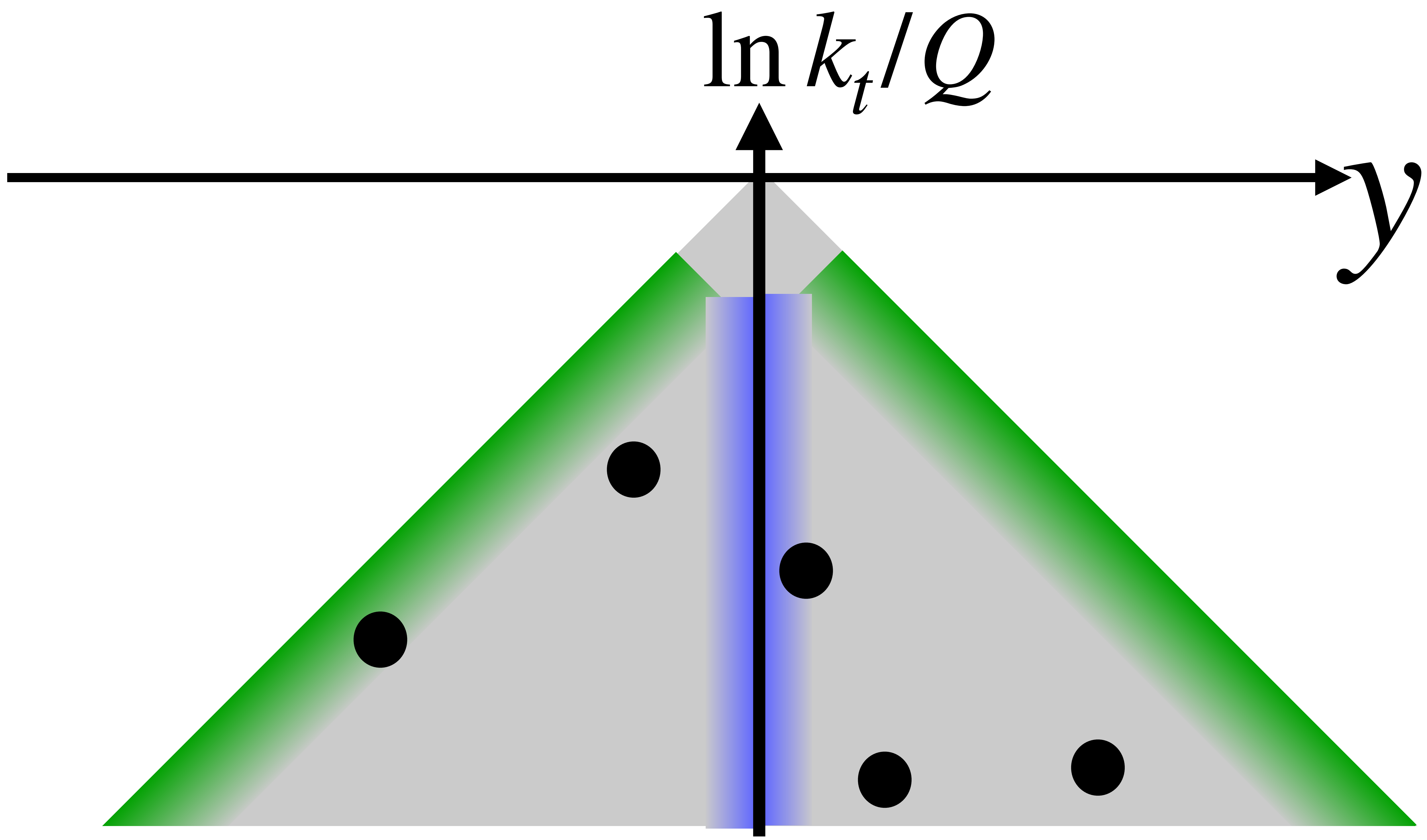
At **NLL accuracy:**

- The rate for **soft-collinear emissions** must be correct at NLO

$$dP_i = \frac{\alpha_s(k_t)}{\pi}\left(1+\frac{\alpha_s(k_t)}{2\pi}K_1\right)\frac{2C_F}{z}dz\,d\ln k_t$$

- We need to include soft and collinear contributions at LO

$$dP_i = \frac{\alpha_s(k_t)}{\pi}P(z)\,dz\,d\ln k_t$$



This tells us what **matrix element** should we use to generate a new emission

Catani, Marchesini, Webber '91

How to build a **NLL** parton shower?

At **NLL accuracy:**

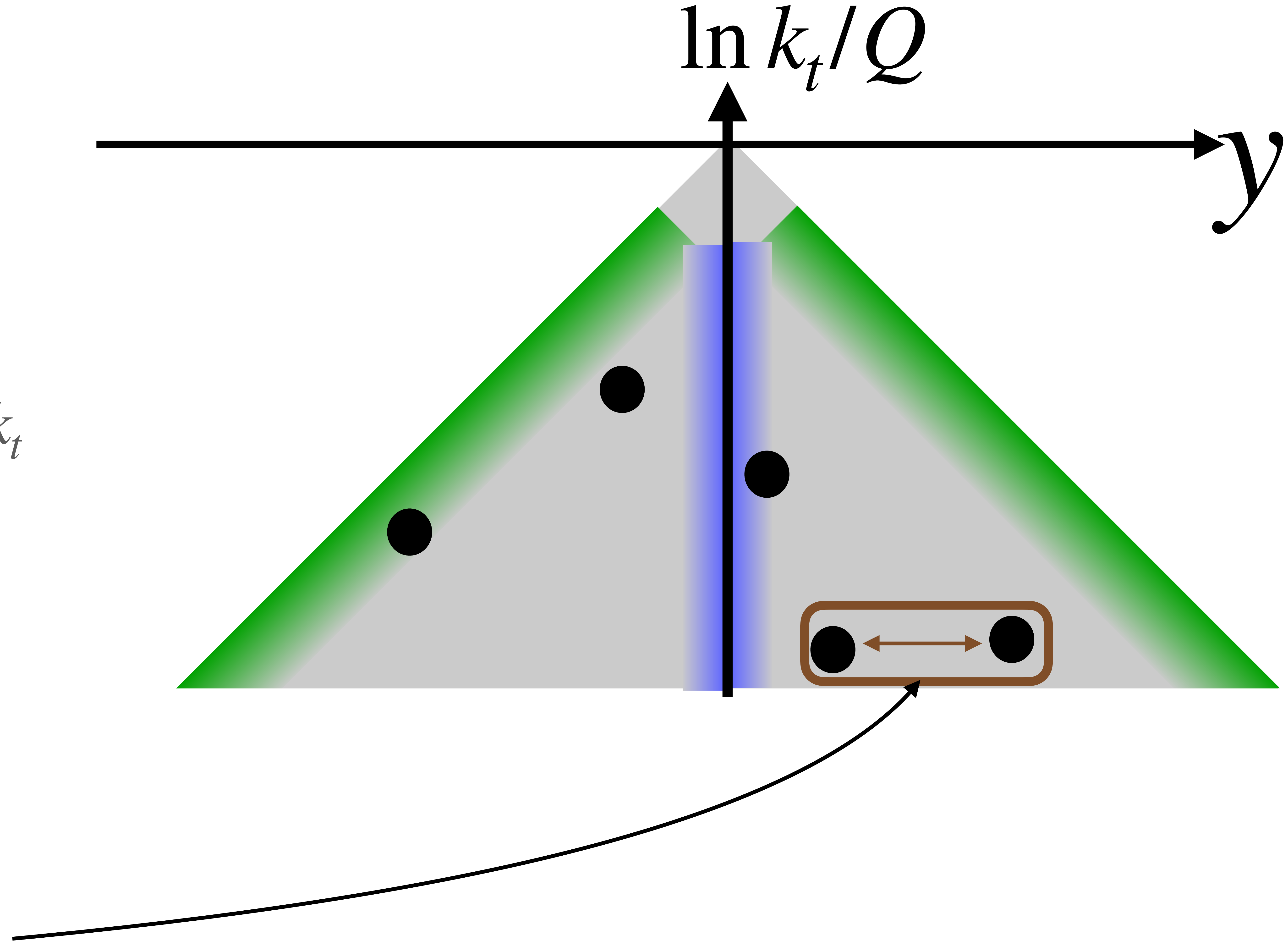
- The rate for **soft-collinear emissions** must be correct at NLO

$$dP_i = \frac{\alpha_s(k_t)}{\pi}\left(1+\frac{\alpha_s(k_t)}{2\pi}K_1\right)\frac{2C_F}{z}dz\,d\ln k_t$$

- We need to include soft and collinear contributions at LO

$$dP_i = \frac{\alpha_s(k_t)}{\pi}P(z)\,dz\,d\ln k_t$$

- Emissions separated in **just one direction** in the Lund plane enter at this order



How to build a **NLL** parton shower?

At **NLL accuracy:**

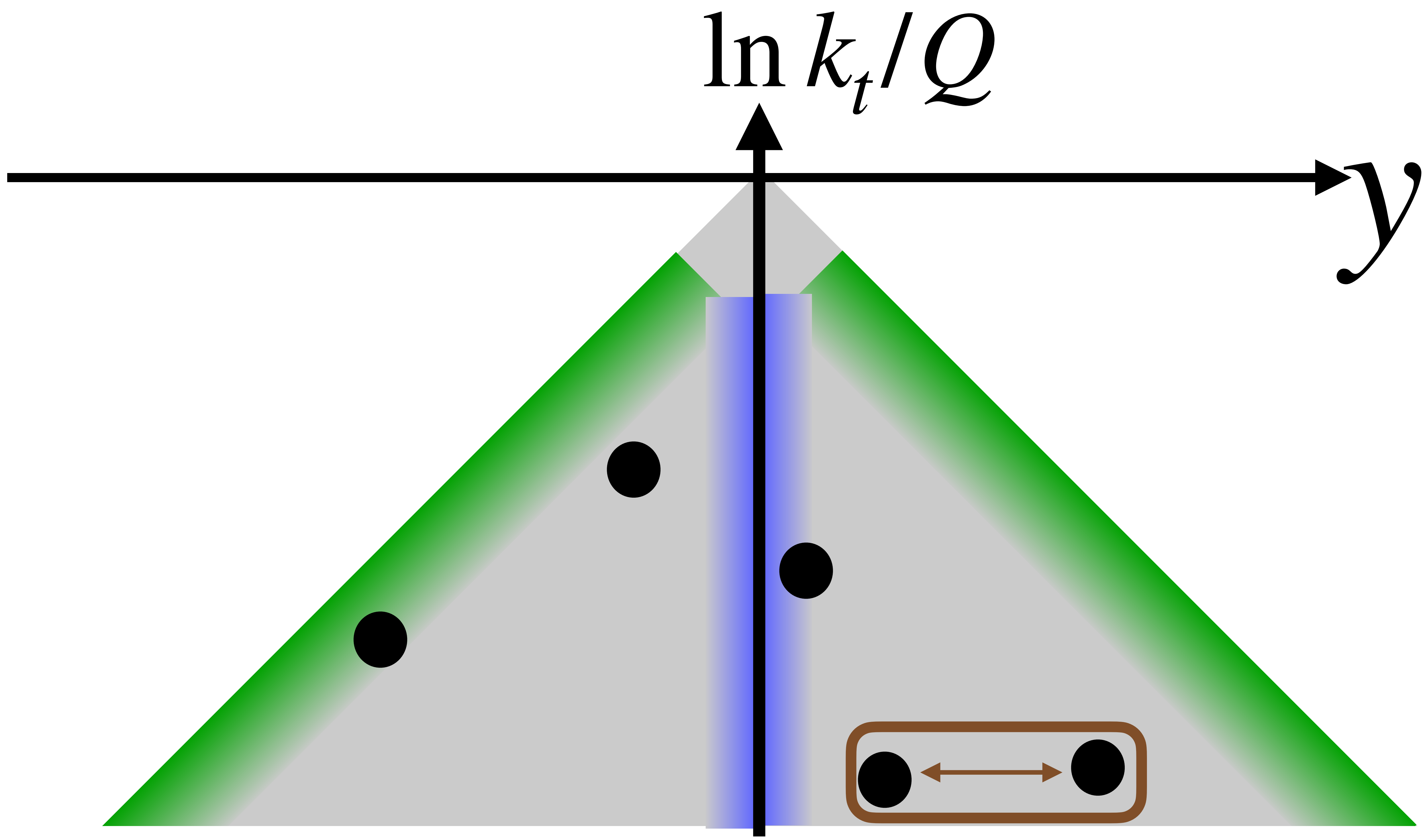
- The rate for **soft-collinear emissions** must be correct at NLO

$$dP_i = \frac{\alpha_s(k_t)}{\pi}\left(1+\frac{\alpha_s(k_t)}{2\pi}K_1\right)\frac{2C_F}{z}dz\, d\ln k_t$$

- We need to include soft and collinear contributions at LO

$$dP_i = \frac{\alpha_s(k_t)}{\pi}P(z)\, dz\, d\ln k_t$$

- Emissions separated in **just one direction** in the Lund plane enter at this order



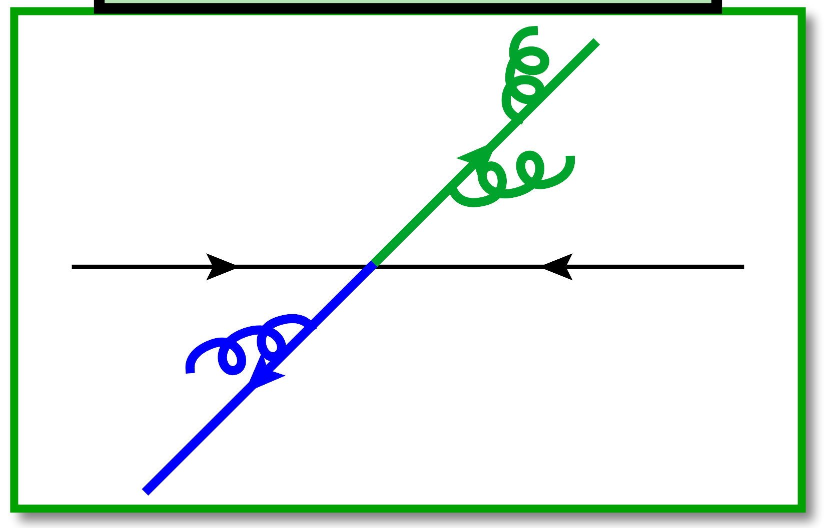
Constraints **kinematic mapping $\Phi_n \to \Phi_{n+1}$** and **ordering variable**: **emissions well separated in rapidity are independent from each other, even if they have similar transverse momentum**

Dasgupta, Dreyer, Hamilton, Monni, Salam, 1805.09327 ;+ Soyez, 2002.11114

Status of **NLL PanScales** showers

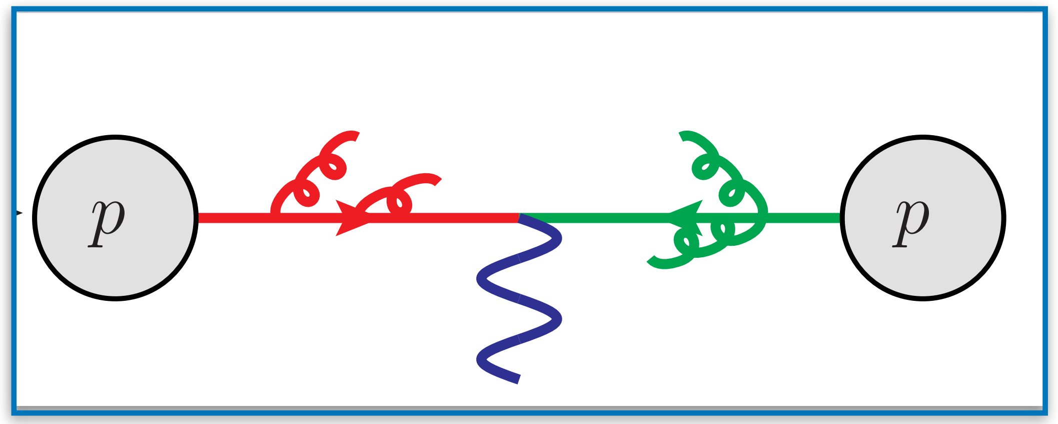
➤ This enabled the **PanScales** to devise the **first** showers with **general** NLL accuracy for

$e^+e^- \to j_1 j_2$



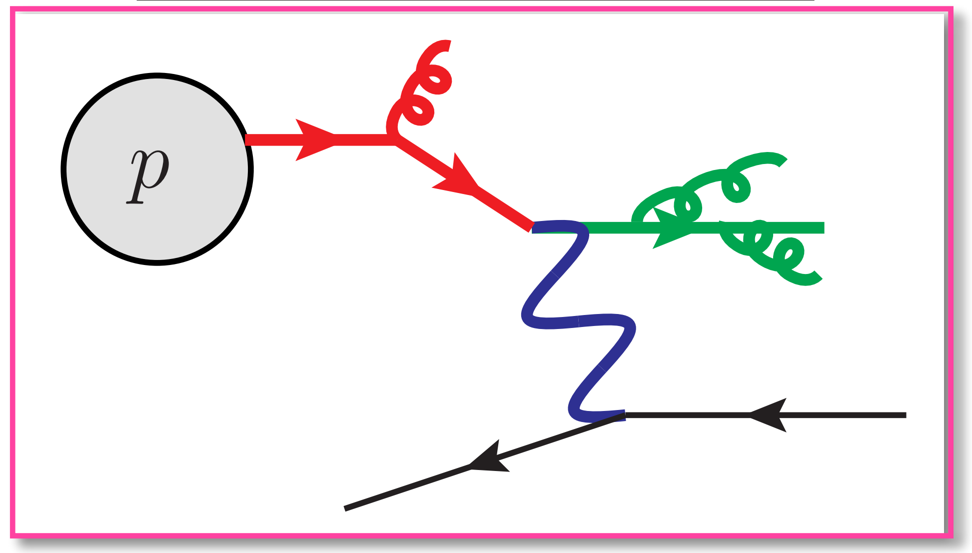
Dasgupta, Dreyer, Hamilton, Monni, Salam, Soyez, 2002.11114

$pp \to$ *colour singlet*



van Beekveld, **SFR**, Soto-Ontoso, Salam, Soyez, Verheyen, 2205.02237, + Hamilton 2207.09467

DIS & VBF



van Beekveld, **SFR**, 2305.08645

…with **subleading colour** (2011.10054) and **spin correlations** (2103.16526, 2111.01161)

- **Herwig7 angular-ordered shower** for the same processes is NLL but only for **global event shapes** (Bewick, **SFR**, Richardson, Seymour,1904.11866, 2107.04051)
- **Deductor** has been proven to be NLL at least for $e^+e^- \to j_1 j_2$ (Nagy, Soper 2011.04777)
- **Alaric** is NLL at leading colour for $e^+e^- \to j_1 j_2$ (2208.06057), recently extended to generic pp collisions (2404.14360) — expected to retain NLL accuracy for $pp \to$ colour singlet
- **FHP** proposal for $e^+e^- \to j_1 j_2$ (2003.06400), currently under implementation in Herwig7
- **Apollo**: NLL shower for $e^+e^- \to j_1 j_2$, hybrid between Vincia and Alaric, with two NLO matchings available (Preuss, 2403.19452)

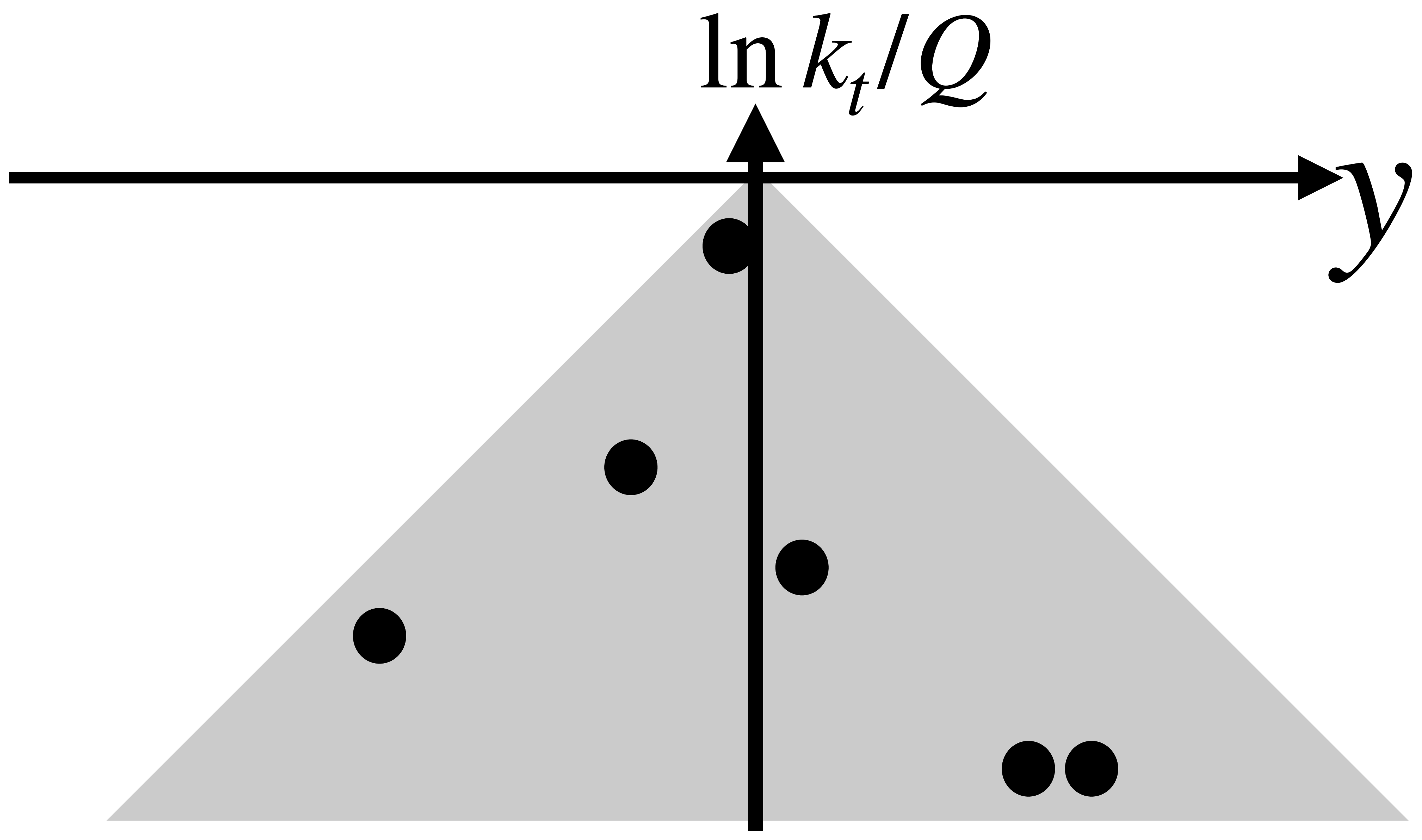
How to go **beyond NLL** in a parton shower?

[**SFR**, Hamilton, Karlberg, Salam, Scyboz, Soyez 2307.11142]

Focus on **soft emissions**

NLL

- ✓ **Soft-collinear** emsns at **NLO**
- ✓ **Soft** (large angle) emsns at **LO**
- ✓ Correct rate for **pair of emsns** separated only in **one Lund coordinate**



How to go **beyond NLL** in a parton shower?

[**SFR**, Hamilton, Karlberg, Salam, Scyboz, Soyez 2307.11142]

Focus on soft emissions

NLL

✓ **Soft-collinear** emsns at **NLO**

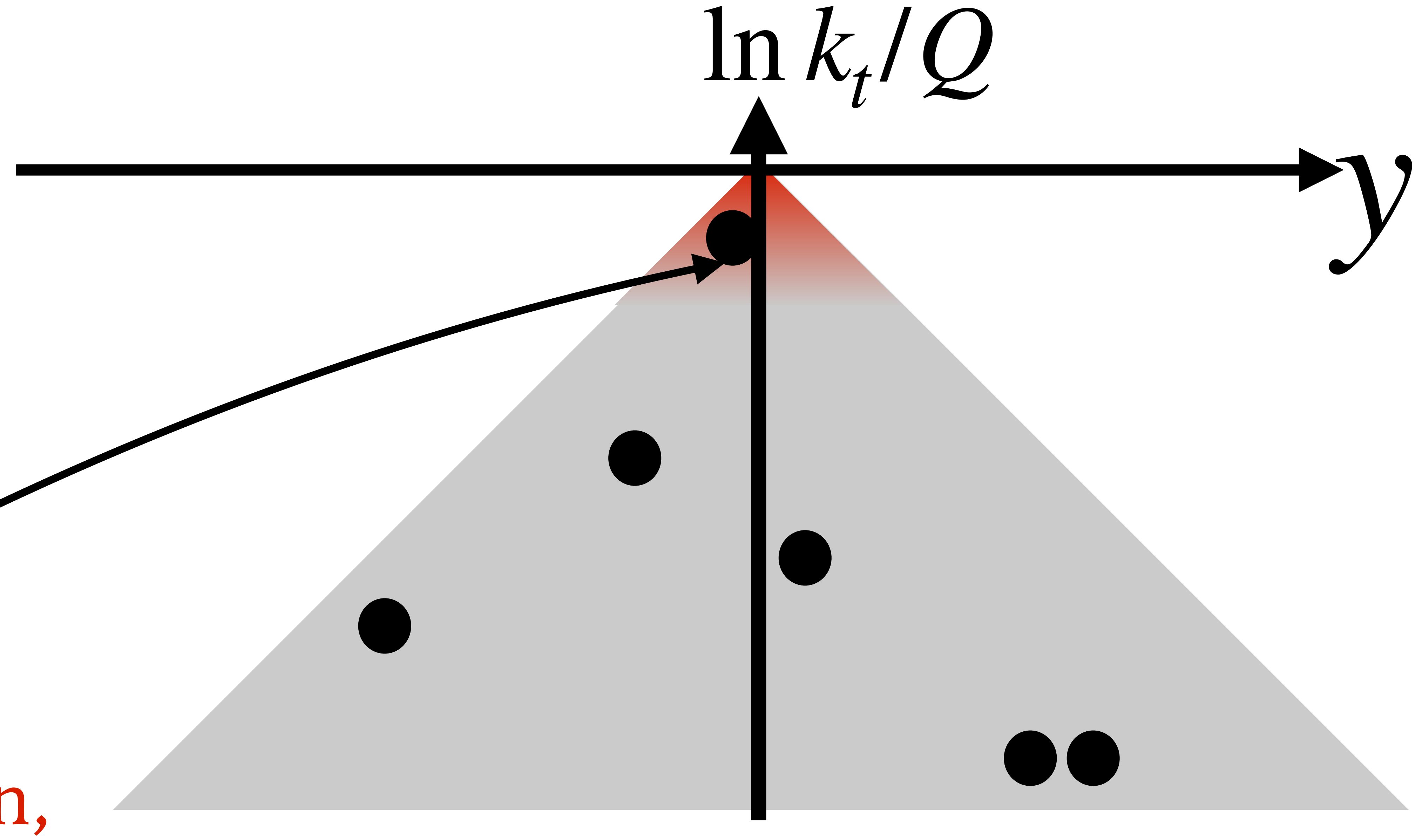
✓ **Soft** (large angle) emsns at **LO**

✓ Correct rate for **pair of emsns** separated only in **one Lund coordinate**

NNLL

✓ **Hard** emissions at **LO**

[Hamilton, Karlberg, Scyboz, Salam, Verheyen, 2301.09645]



How to go **beyond NLL** in a parton shower?

[**SFR**, Hamilton, Karlberg, Salam, Scyboz, Soyez 2307.11142]

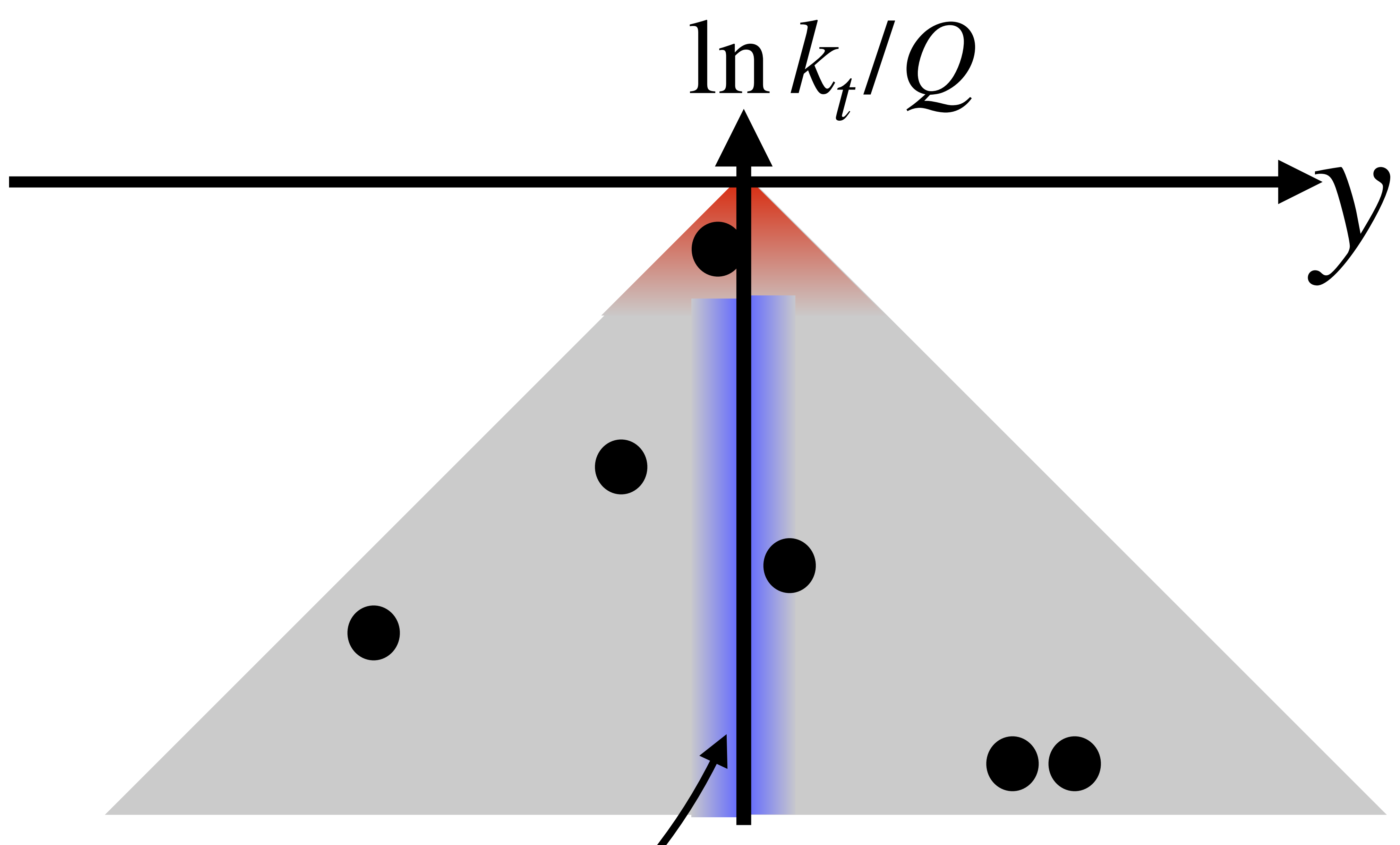
Focus on **soft emissions**

NLL

- ✓ **Soft-collinear** emsns at **NLO**
- ✓ **Soft** (large angle) emsns at **LO**
- ✓ Correct rate for **pair of emsns** separated only in **one Lund coordinate**

NNLL

- ✓ **Hard** emissions at **LO**
- ✓ **Soft** (large angle) emsns at **NLO**



How to go **beyond NLL** in a parton shower?

[**SFR**, Hamilton, Karlberg, Salam, Scyboz, Soyez 2307.11142]

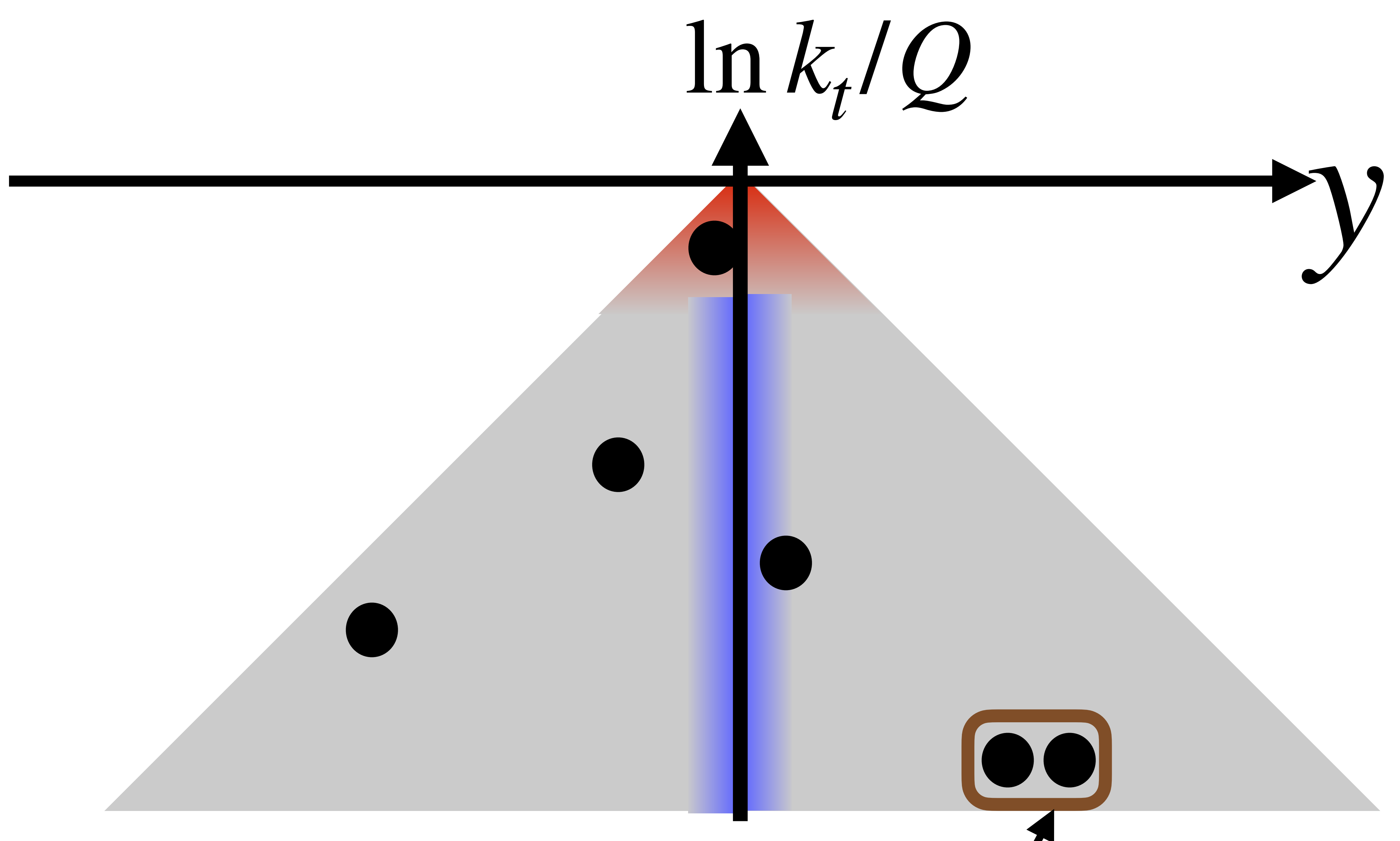
Focus on soft emissions

NLL

- ✓ Soft-collinear emsns at NLO
- ✓ Soft (large angle) emsns at LO
- ✓ Correct rate for **pair of emsns** separated only in **one Lund coordinate**

NNLL

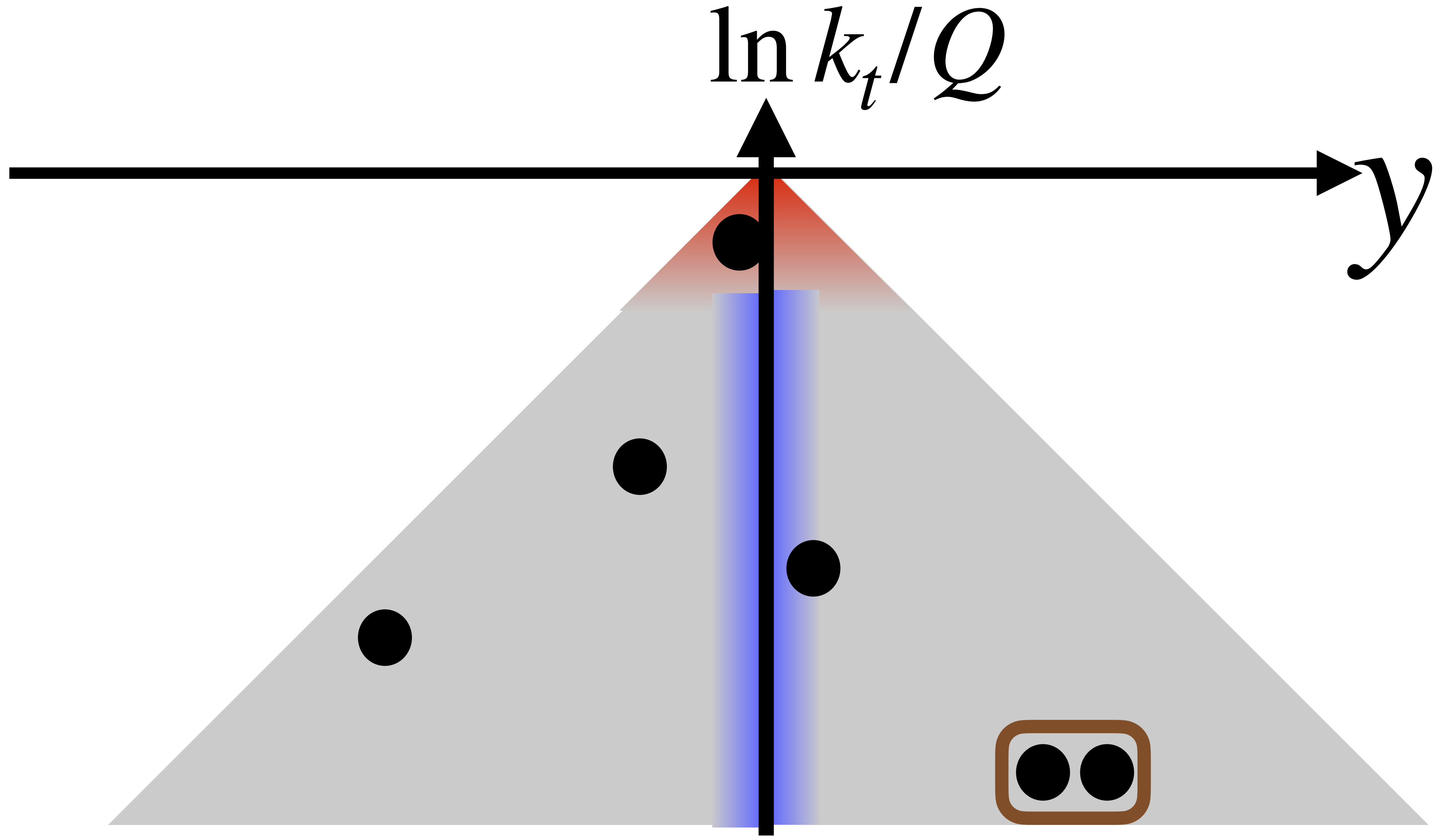
- ✓ Hard emissions at LO
- ✓ Soft (large angle) emsns at NLO
- ✓ Correct rate for **pair of emsns close in the Lund plane**



How to go **beyond NLL** in a parton shower?

[**SFR**, Hamilton, Karlberg, Salam, Scyboz, Soyez 2307.11142]

Focus on **soft emissions**



NLL

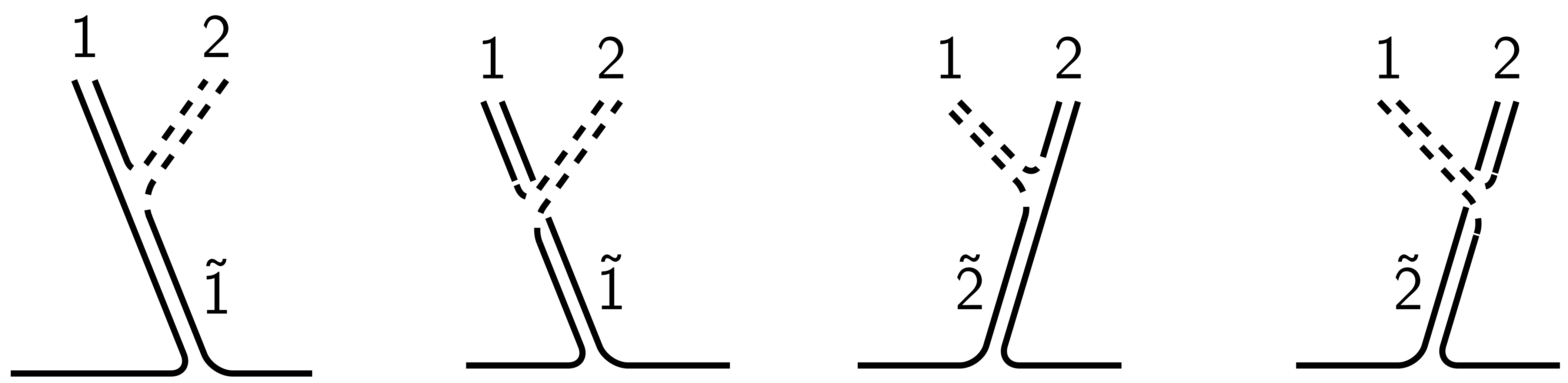
- ✓ **Soft-collinear** emsns at **NLO**
- ✓ Soft (large angle) emsns at LO
- ✓ Correct rate for **pair of emsns** separated only in **one Lund coordinate**

NNLL

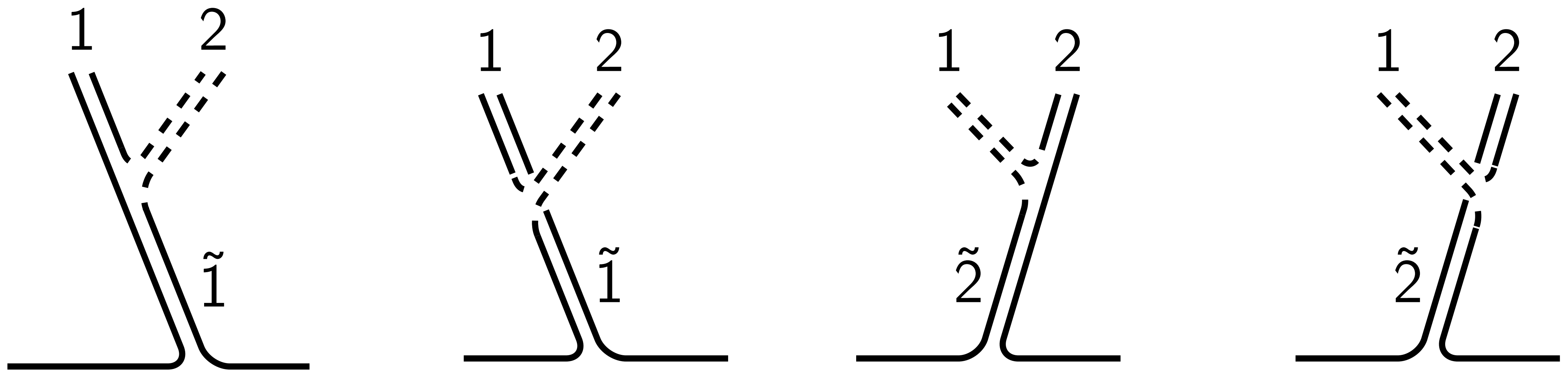
- ✓ **Hard** emissions at **LO**
- ✓ **Soft** (large angle) emsns at **NLO**
- ✓ Correct rate for **pair of emsns close in the Lund plane**
- ✓ …

- ➤ **NNDL** for [subjet] multiplicities, i.e. $\alpha_s^n L^{2n}$, $\alpha_s^n L^{2n-1}$, $\boldsymbol{\alpha_s^n L^{2n-2}}$
- ➤ **Next-to-Single-Log (NSL)** for non-global logarithms, e.g. energy in a slice, all terms $\alpha_s^n L^n$ and $\boldsymbol{\alpha_s^n L^{n-1}}$

Correct rate for **pairs** or soft emissions = Real corrections

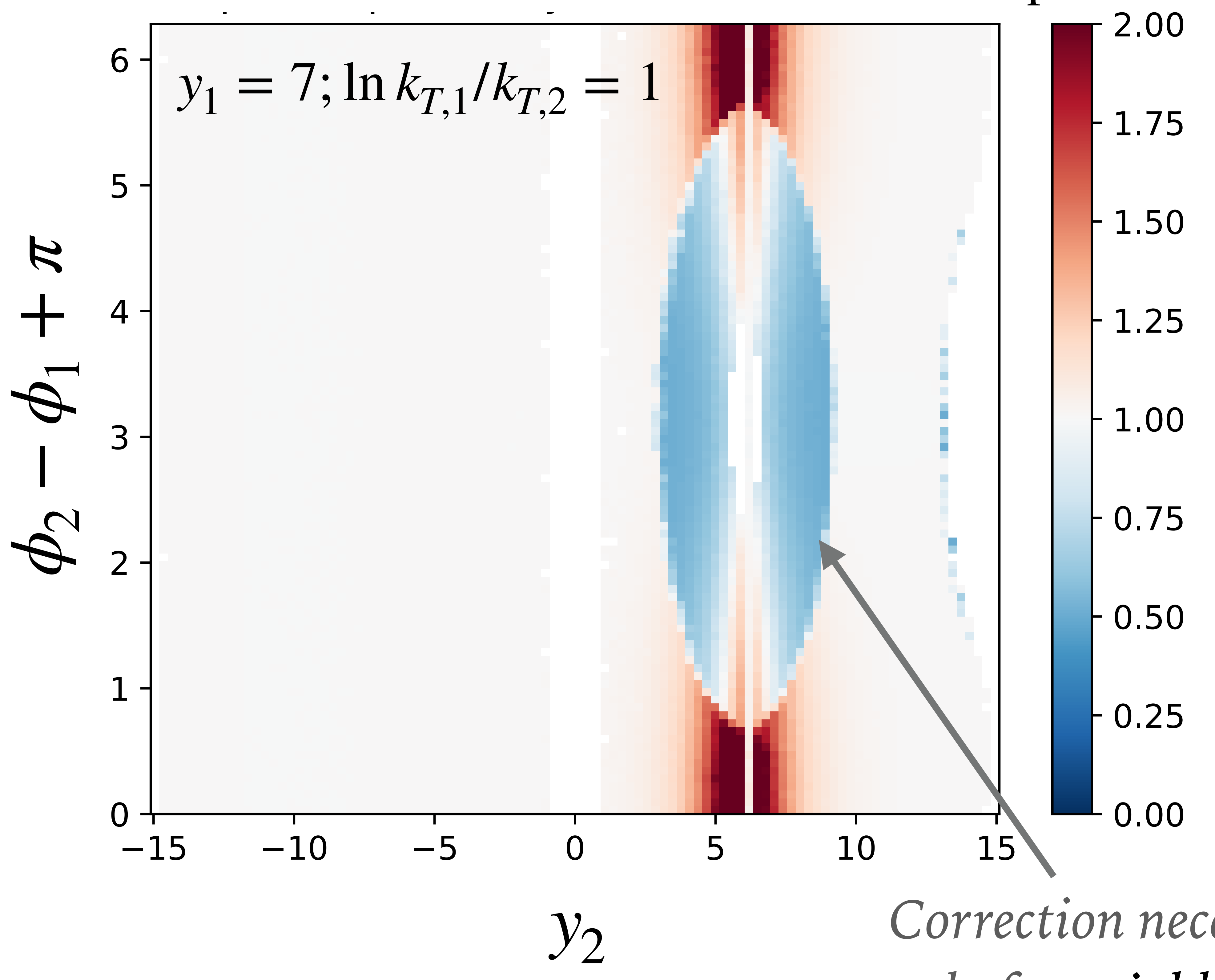


Correct rate for **pairs** or soft emissions = Real corrections



- a given two-emission configuration can come from several shower histories
- **accept a given emission with exact double-soft $M^{(DS)}_{exact}$ divided by shower's effective double-soft matrix element** summed over the histories h that could have produced that configuration

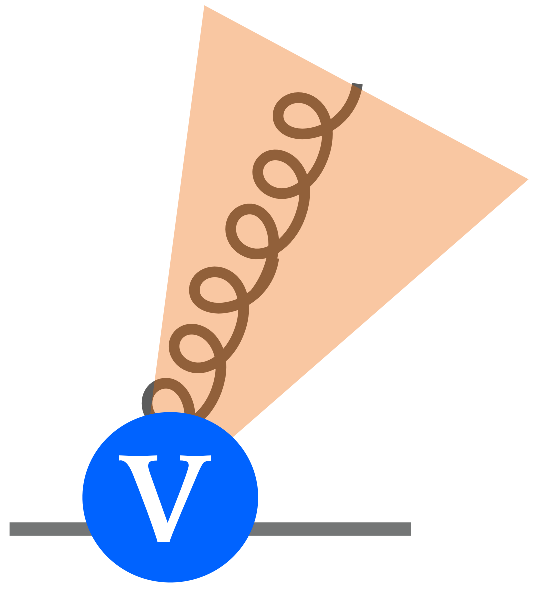
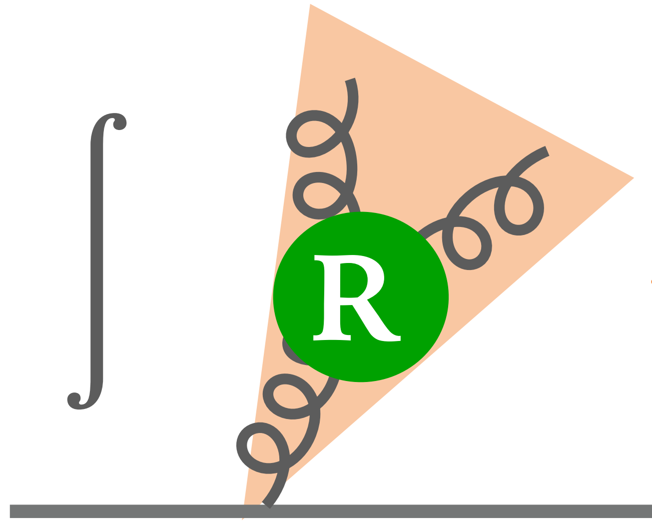
Double-soft acceptance P_{accept}



Correction necessary only for ***neighbouring emsn*** *as the shower is already NLL*

$$P_{accept} = \frac{M^{(DS)}_{exact}}{\sum_h M^{(DS)}_{h,PS}}$$

NLO corrections to a single soft emission: standard behaviour

➤ For a soft emission  $+ \int$ 

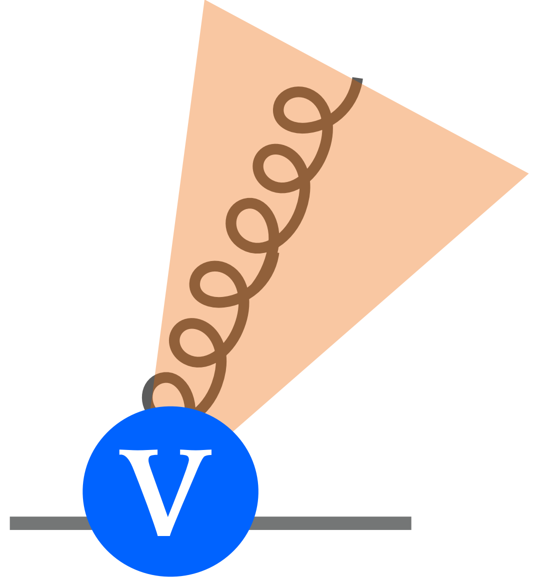
$y, p_\perp$ *fixed*

$$= \boxed{\frac{\alpha_s}{2\pi} K_1}$$

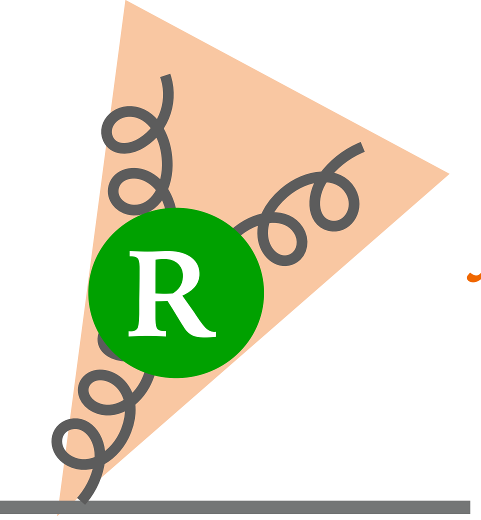
➤ If this happens also in a **parton shower** simulation, we have the emission rate correct at $\mathcal{O}(\alpha_s^2)$

NLO corrections to a single soft emission: standard behaviour

➤ For a soft emission



$+ \int$

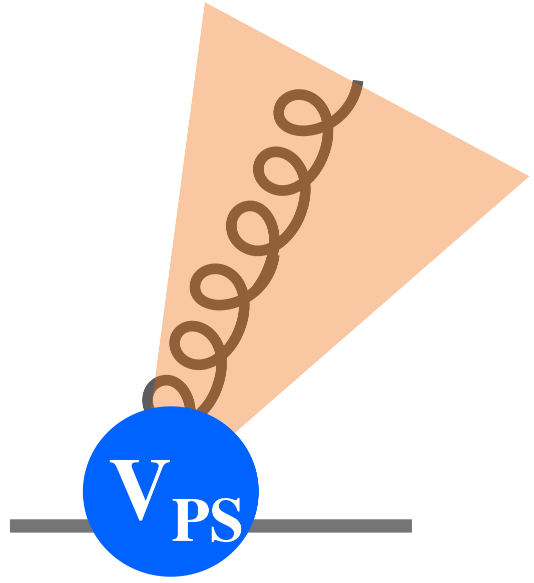


$y, p_\perp$ *fixed*

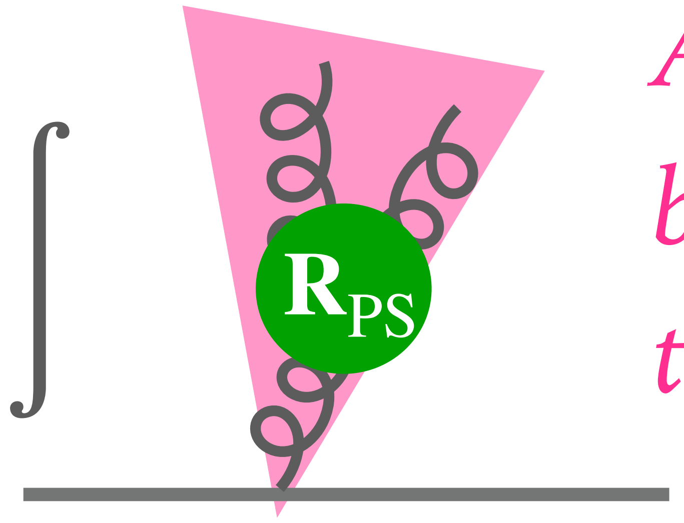
$$= \frac{\alpha_s}{2\pi} K_1$$

➤ If this happens also in a **parton shower** simulation, we have the emission rate correct at $\mathcal{O}(\alpha_s^2)$

➤ In a parton shower, **virtual corrections** are obtained by unitarity (=no emission probability)



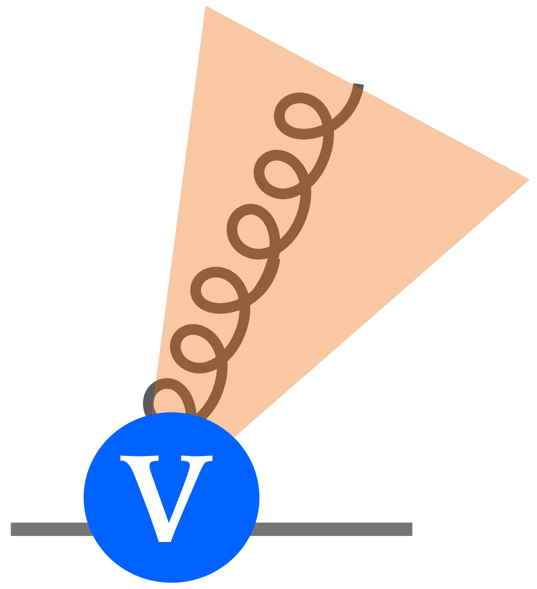
$\equiv - \int$



At fixed "shower variables", but the rapidity and $p_\perp$ of the jet can vary

NLO corrections to a single soft emission: standard behaviour

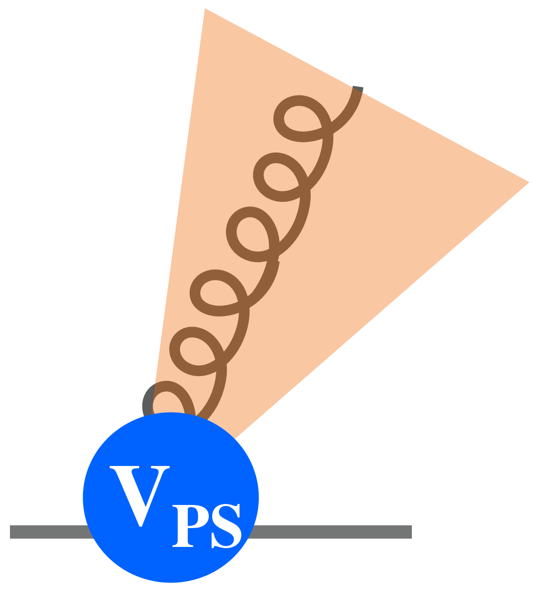
- For a soft emission



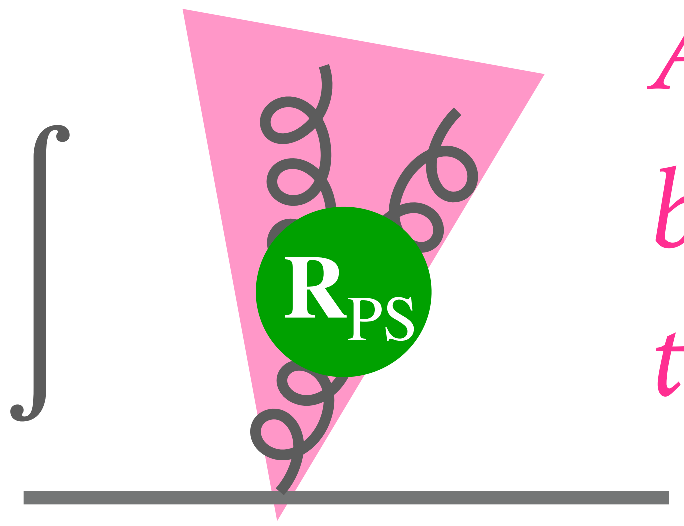
$+ \int$

$$= \frac{\alpha_s}{2\pi} K_1$$

- If this happens also in a **parton shower** simulation, we have the emission rate correct at $\mathcal{O}(\alpha_s^2)$
- In a parton shower, **virtual corrections** are obtained by unitarity (=no emission probability)



$\equiv - \int$



At fixed "shower variables", but the rapidity and $p_\perp$ of the jet can vary

- Catani, Marchesini and Webber defined the "CMW" scheme for the coupling in the shower [*Nucl.Phys.B* 349 (1991) 635-654]

$$\alpha_s^{\mathrm{CMW}} = \alpha_s \left(1 + \frac{\alpha_s}{2\pi} K_1 \right)$$

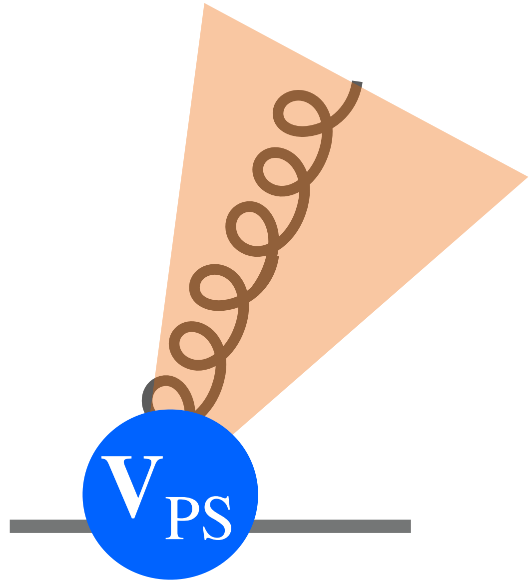
Additional virtual correction added directly to the splitting function

Ensures **"on average"**

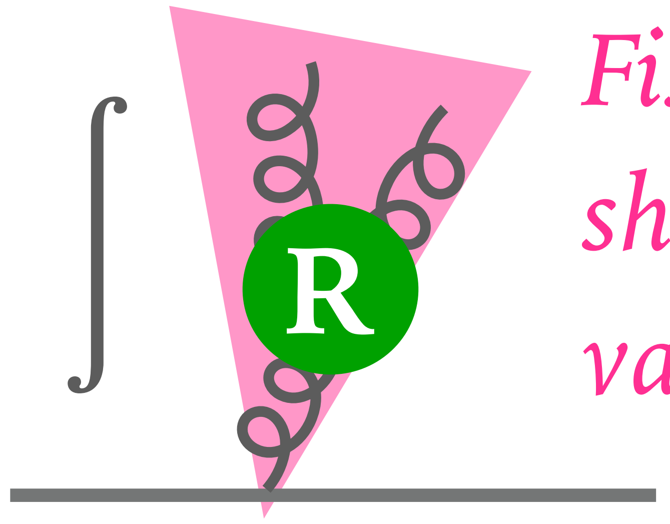
$$\mathbf{V_{PS}} + \int \mathbf{R_{PS}} = \frac{\alpha_s}{2\pi} K_1$$

Revisiting **virtual** corrections to a single soft emission

- With our double soft acceptance we have $\mathbf{R}_{\mathbf{PS}} = \mathbf{R}$. This yields



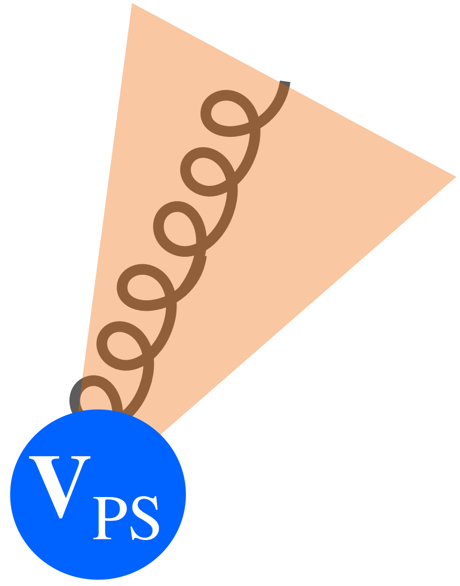
$$= \frac{\alpha_s}{2\pi} K_1 \; - \int$$



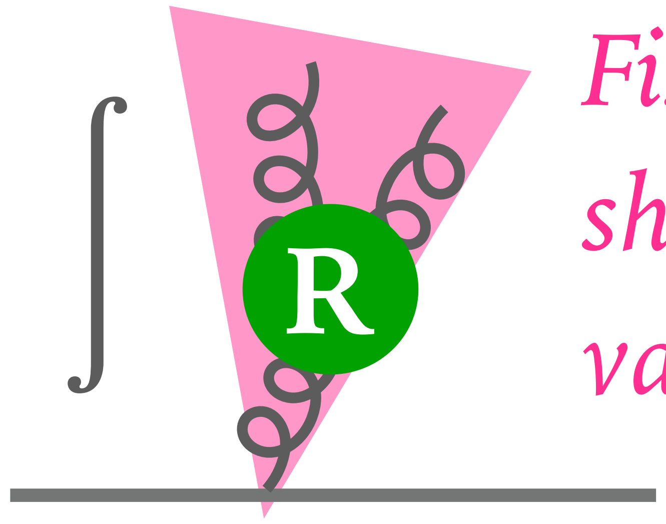
Fixed shower variables

Revisiting **virtual** corrections to a single soft emission

- With our double soft acceptance we have $\mathbf{R}_{\mathbf{PS}} = \mathbf{R}$. This yields



$= \frac{\alpha_s}{2\pi} K_1 - \int$

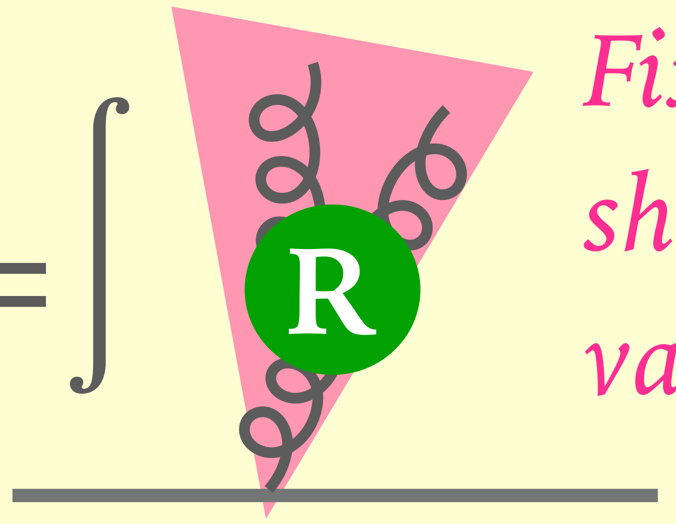


Fixed shower variables

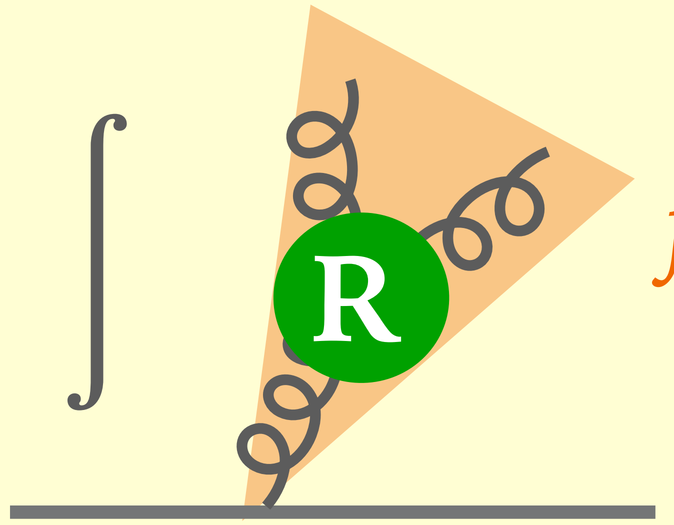
- We modify the CMW scheme

$$K_1 \to K_1 + \Delta K_1(\Phi_{\mathrm{PS}}^{(1)})$$

$\frac{\alpha_s}{2\pi} \Delta K_1(\Phi_{\mathrm{PS}}^{(1)}) = \int$



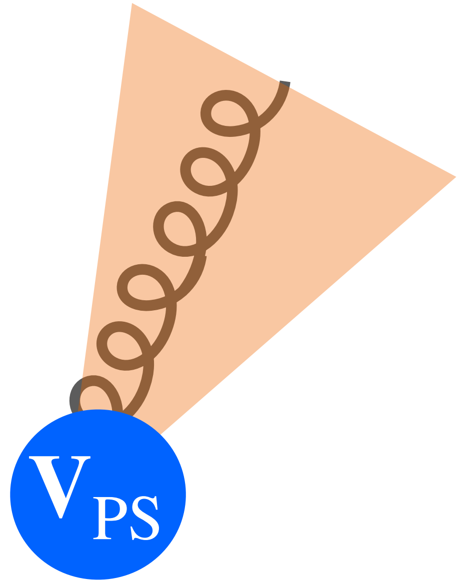
Fixed shower variables $- \int$



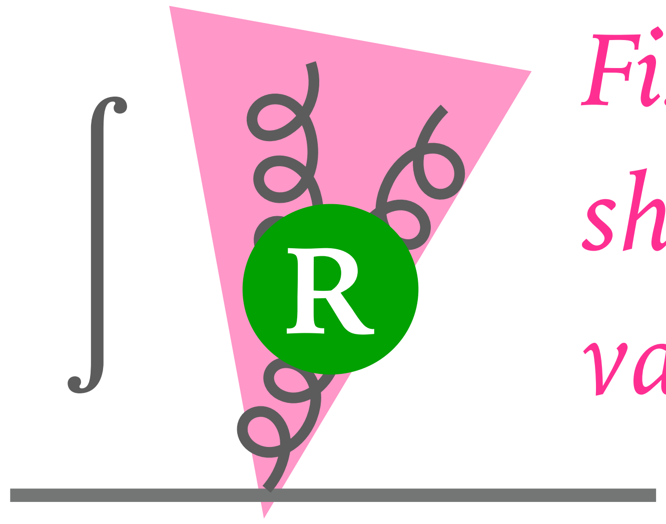
$y, p_\perp$ fixed

Revisiting **virtual** corrections to a single soft emission

- With our double soft acceptance we have $\mathbf{R_{PS}} = \mathbf{R}$. This yields



$$= \frac{\alpha_s}{2\pi} K_1 - \int$$

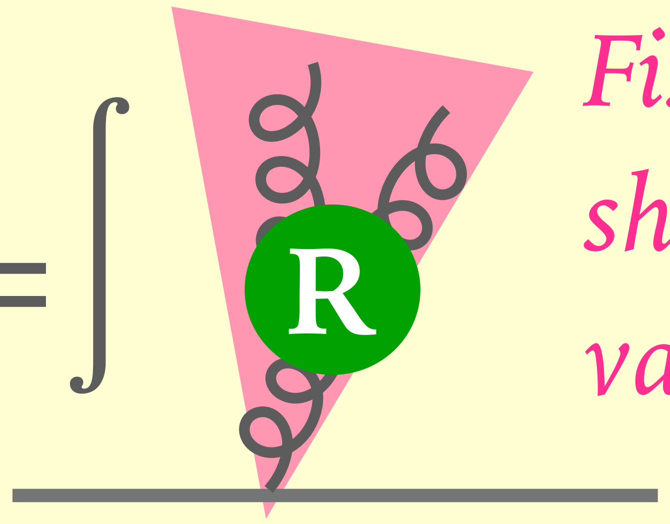


Fixed shower variables

- We modify the CMW scheme

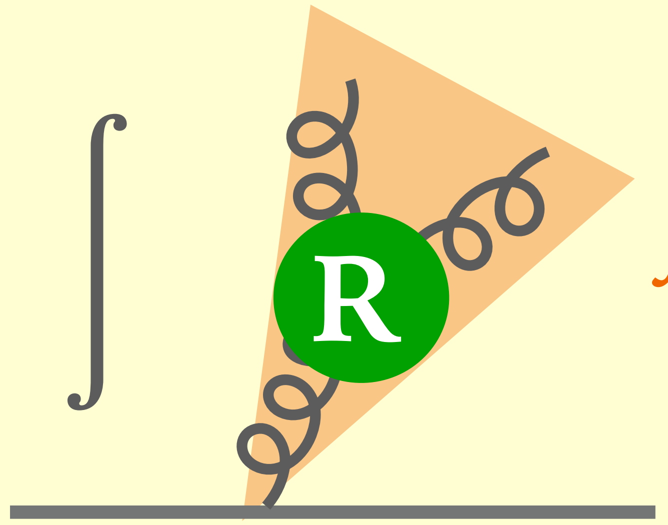
$$K_1 \to K_1 + \Delta K_1(\Phi_{PS}^{(1)})$$

$$\frac{\alpha_s}{2\pi} \Delta K_1(\Phi_{PS}^{(1)}) = \int$$



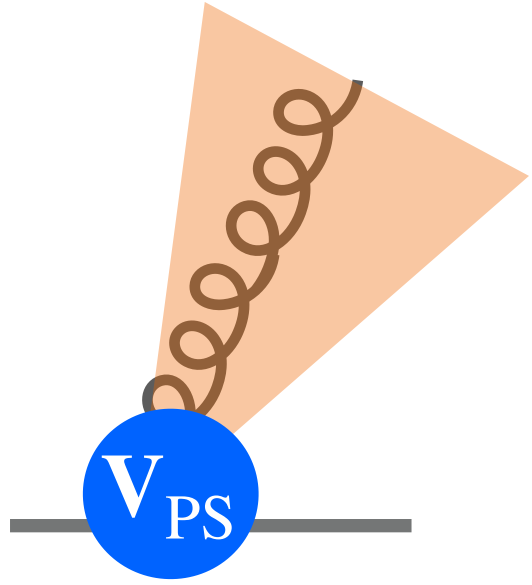
Fixed shower variables

$$- \int$$

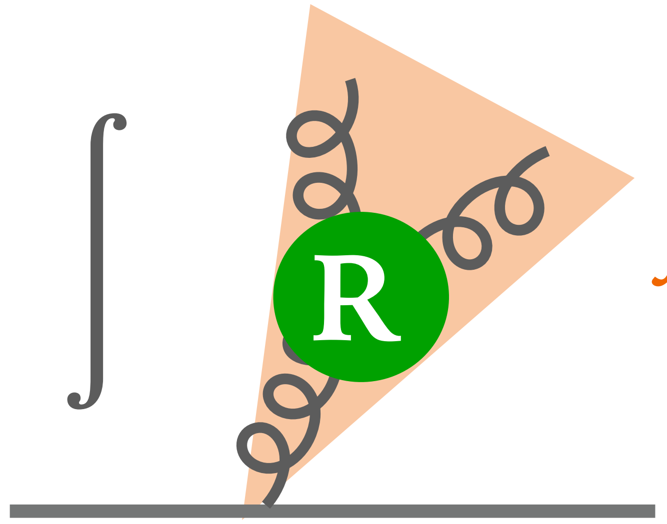


$y, p_\perp$ *fixed*

- …so to have

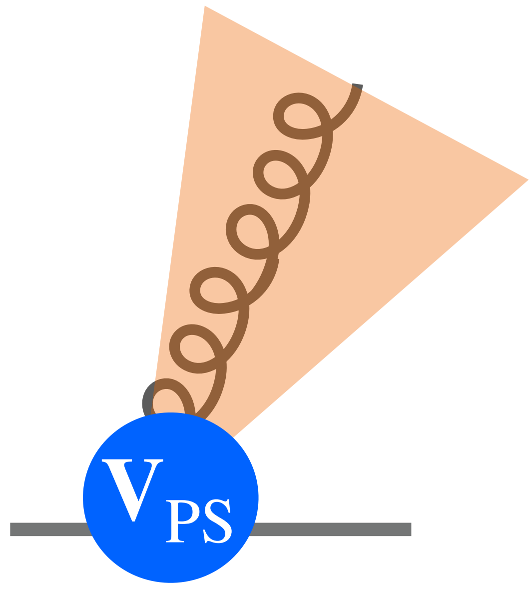


$$= \frac{\alpha_s}{2\pi} K_1 - \int$$

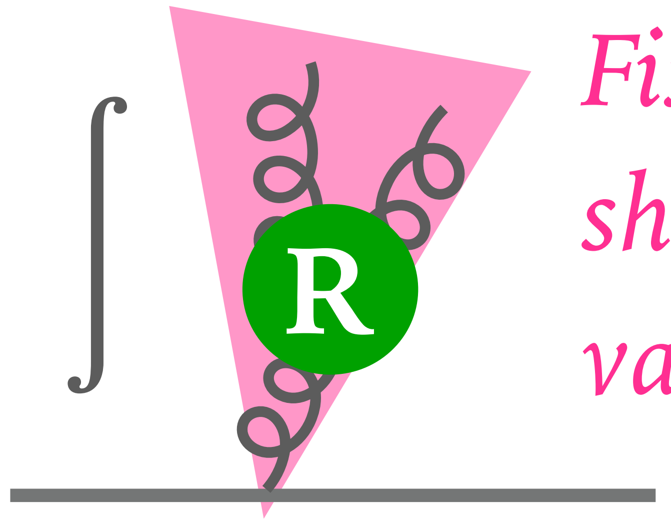


$y, p_\perp$ *fixed*

Virtual corrections to a single soft emission

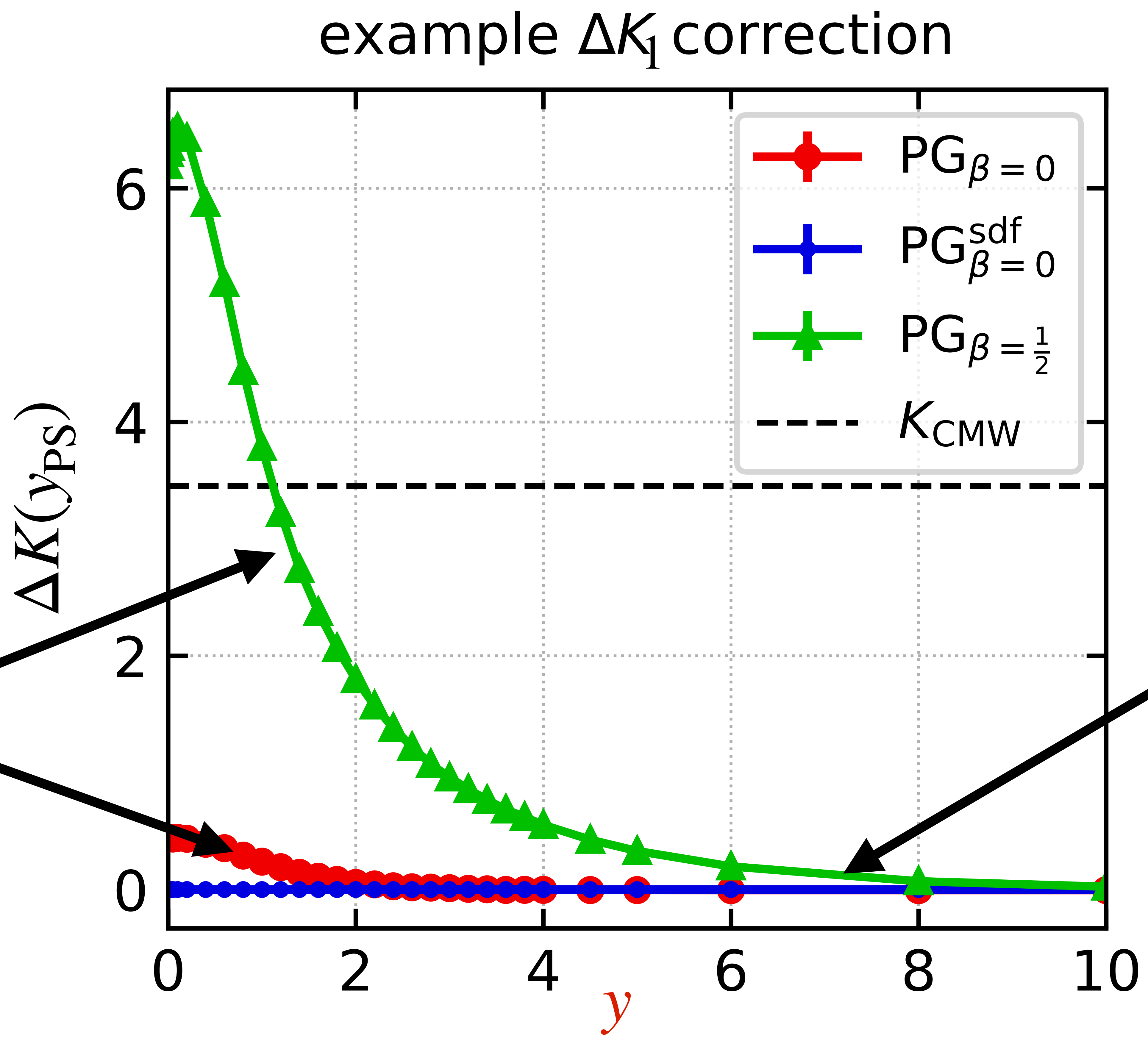


$$= \frac{\alpha_s}{2\pi}\left(K_1 + \Delta K_1(\Phi^{(1)}_{\mathrm{PS}})\right) - \int$$



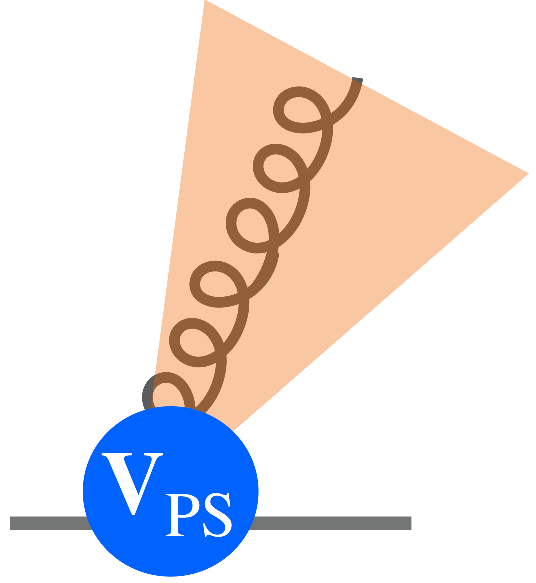
Fixed shower variables

Soft **large-angle** emissions can require a "large" ΔK_1

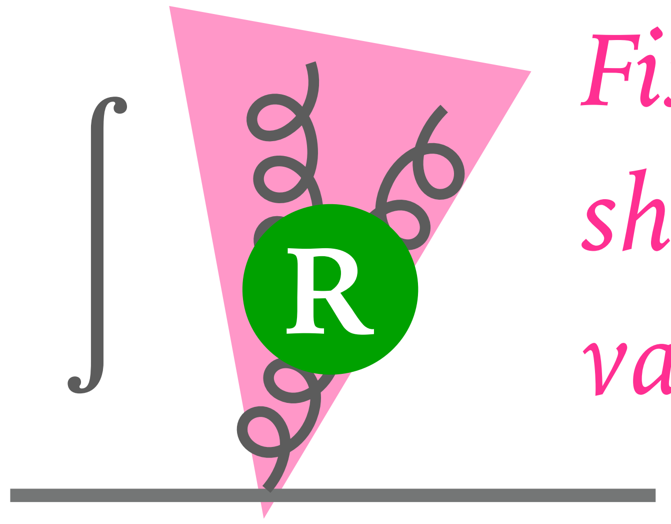


Soft-collinear emissions are already OK (because the shower is NLL)

Virtual corrections to a single soft emission



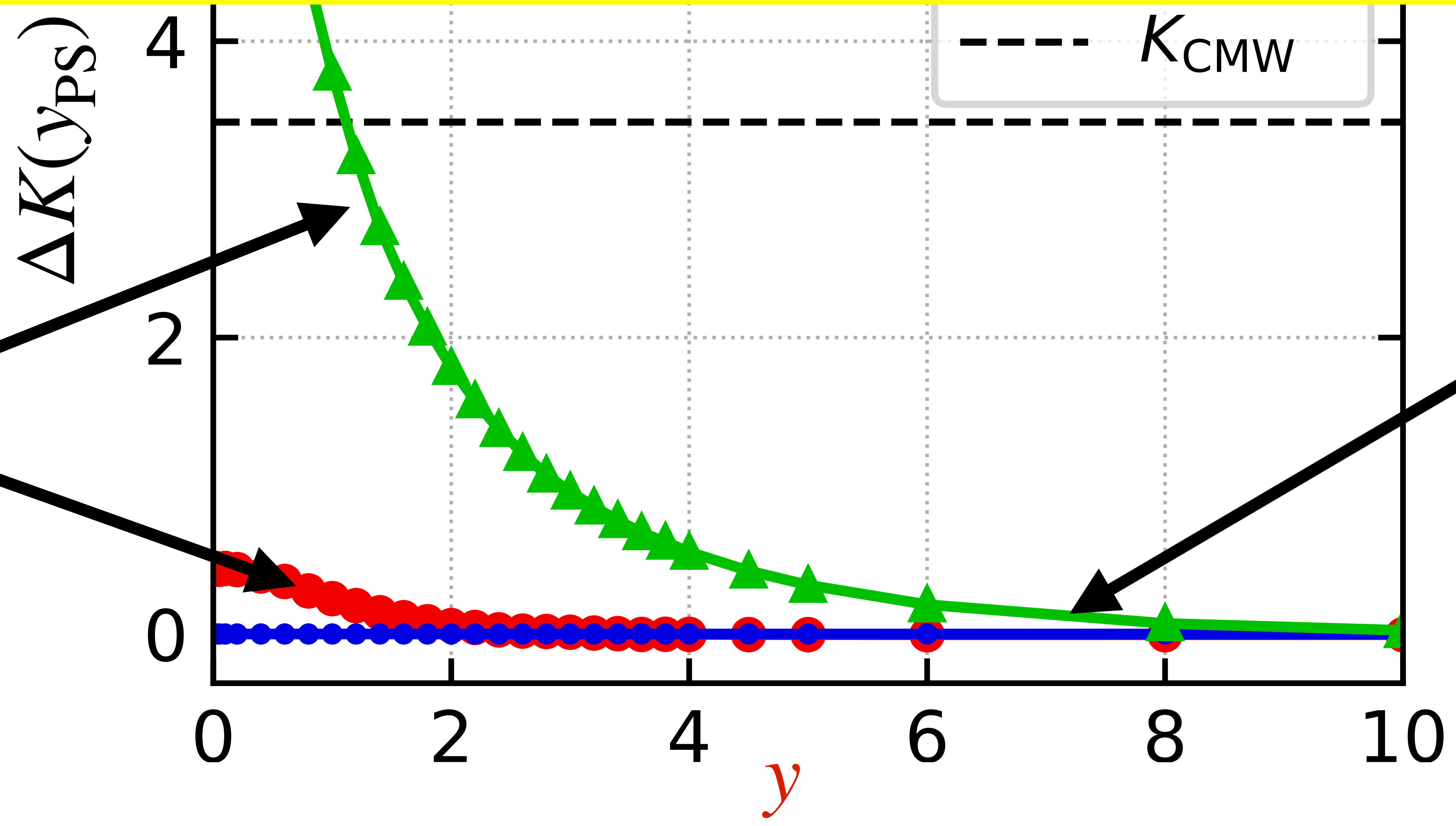
$$= \frac{\alpha_s}{2\pi}\left(K_1 + \Delta K_1(\Phi^{(1)}_{\mathrm{PS}})\right) - \int$$



Fixed shower variables

Augmenting the order of the splitting function used is not sufficient to achieve superior logarithmic accuracy!

Soft **large-angle** emissions can require a "large" ΔK_1

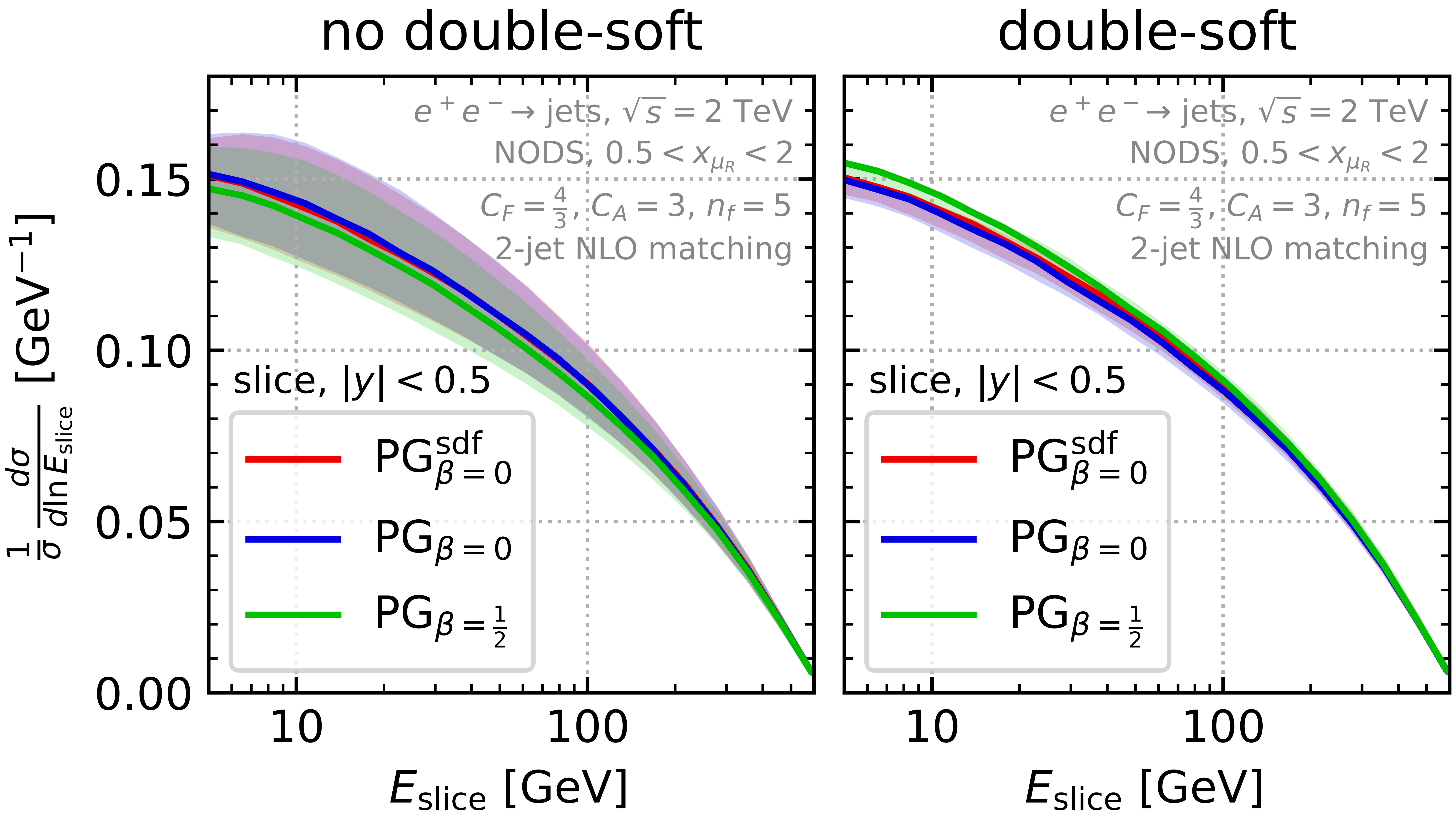


Soft-collinear emissions are already OK (because the shower is NLL)

NSL Pheno outlook

S.F.R., Hamilton, Karlberg, Salam, Scyboz, Soyez
2307.11142

- Energy flow in slice between two 1 TeV jets
- **Double-soft reduces uncertainty band**



Uncertainty here is estimated varying the renormalisation scale

$$\alpha_s^{\mathrm{CMW}}(k_t; x_R) = \alpha_s(x_R k_t)\left(1 + \frac{\alpha_s(x_R k_t)}{2\pi}(K_1 + \Delta K_1(\Phi)) + 2\alpha_s(x_R k_t) b_0 (1-z) \ln x_R\right)$$

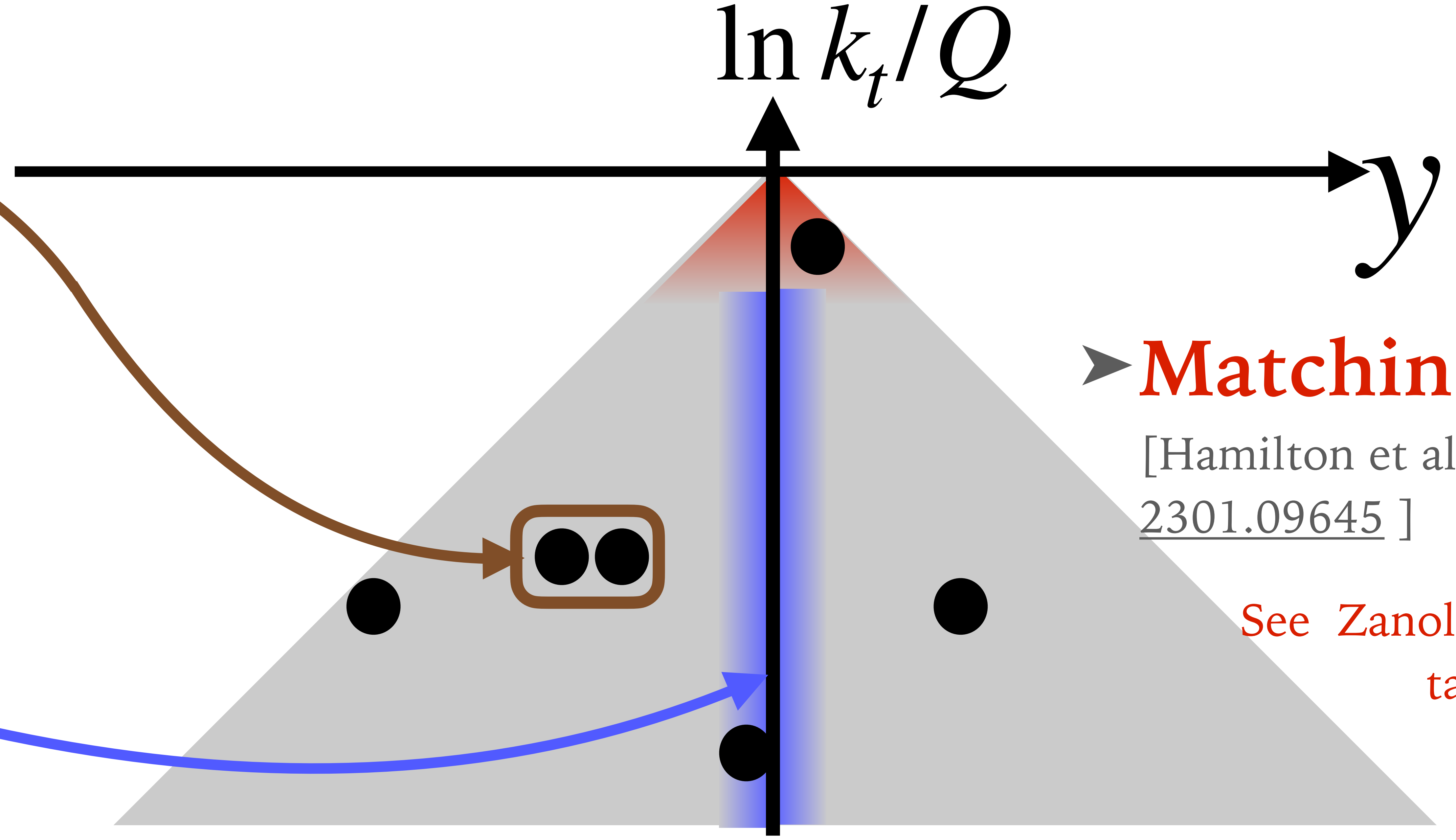
Building a NNLL shower

SFR et al, 2307.11142

- **Double-soft "reweighting"** for neighbouring soft-collinear emsns
- NLO corrections for soft, large-angle emissons

$$\alpha_s^{\text{eff}}(k_t) = \alpha_s(k_t)\left(1 + \frac{\alpha_s(k_t)}{2\pi}(K_1 + \Delta K_1)\right)$$

Catani, Marchesini, Webber, '91



- **Matching**

[Hamilton et al. 2301.09645]

See Zanoli's talk

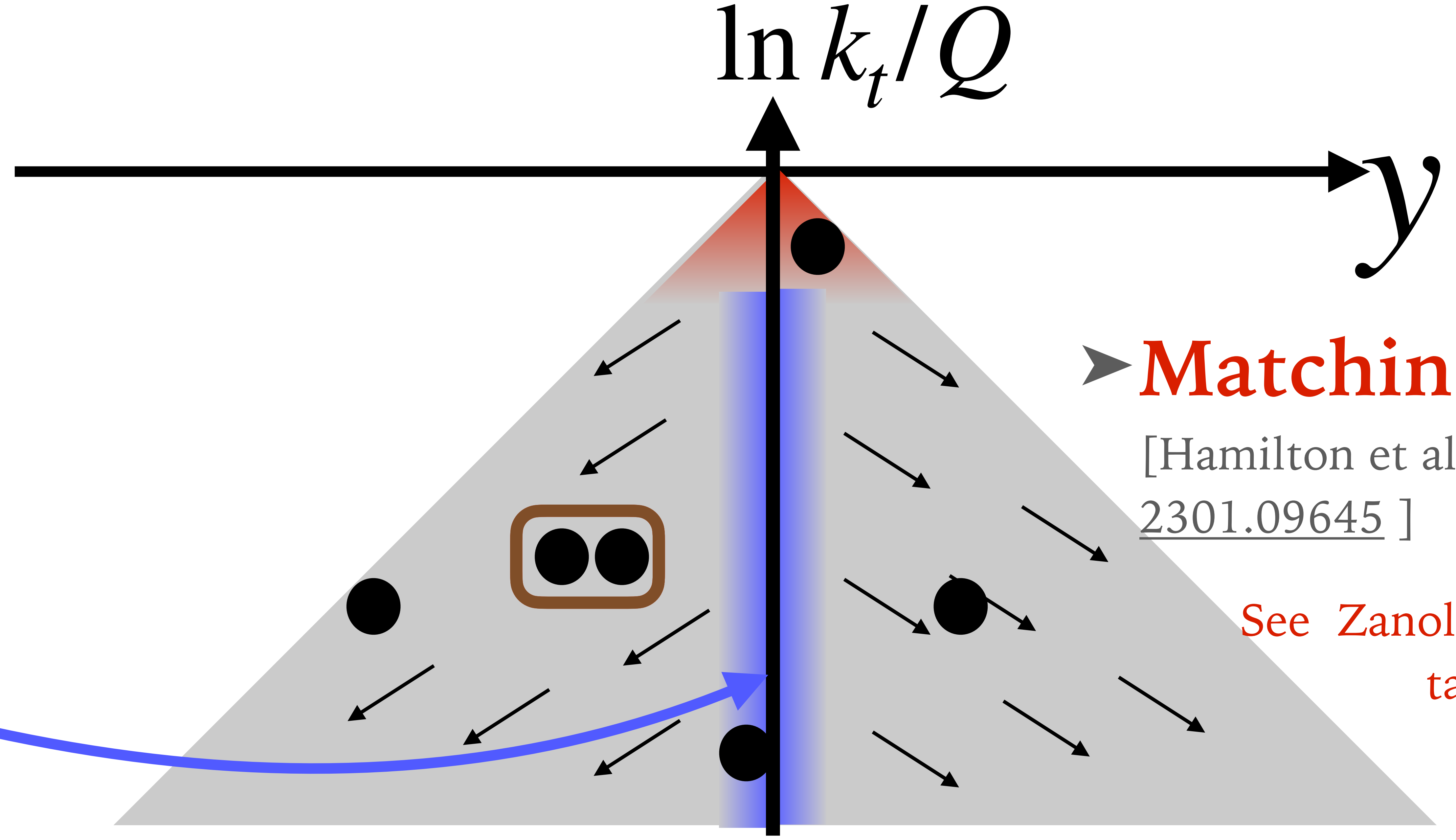
Building a NNLL shower

SFR et al, 2307.11142

- **Double-soft "reweighting"** for neighbouring soft-collinear emsns
- NLO corrections for soft, large-angle emissons

$$\alpha_s^{\text{eff}}(k_t) = \alpha_s(k_t)\left(1 + \frac{\alpha_s(k_t)}{2\pi}(K_1 + \Delta K_1)\right)$$

Catani, Marchesini, Webber, '91



- **Matching**
  [Hamilton et al. 2301.09645]
  See Zanoli's talk

Drift in rapidity *of an emission when it further branches*

$$\int 2C_F d\eta \Delta K_1(\eta) \propto \langle \Delta y \rangle$$

Building a NNLL shower

SFR et al, 2307.11142

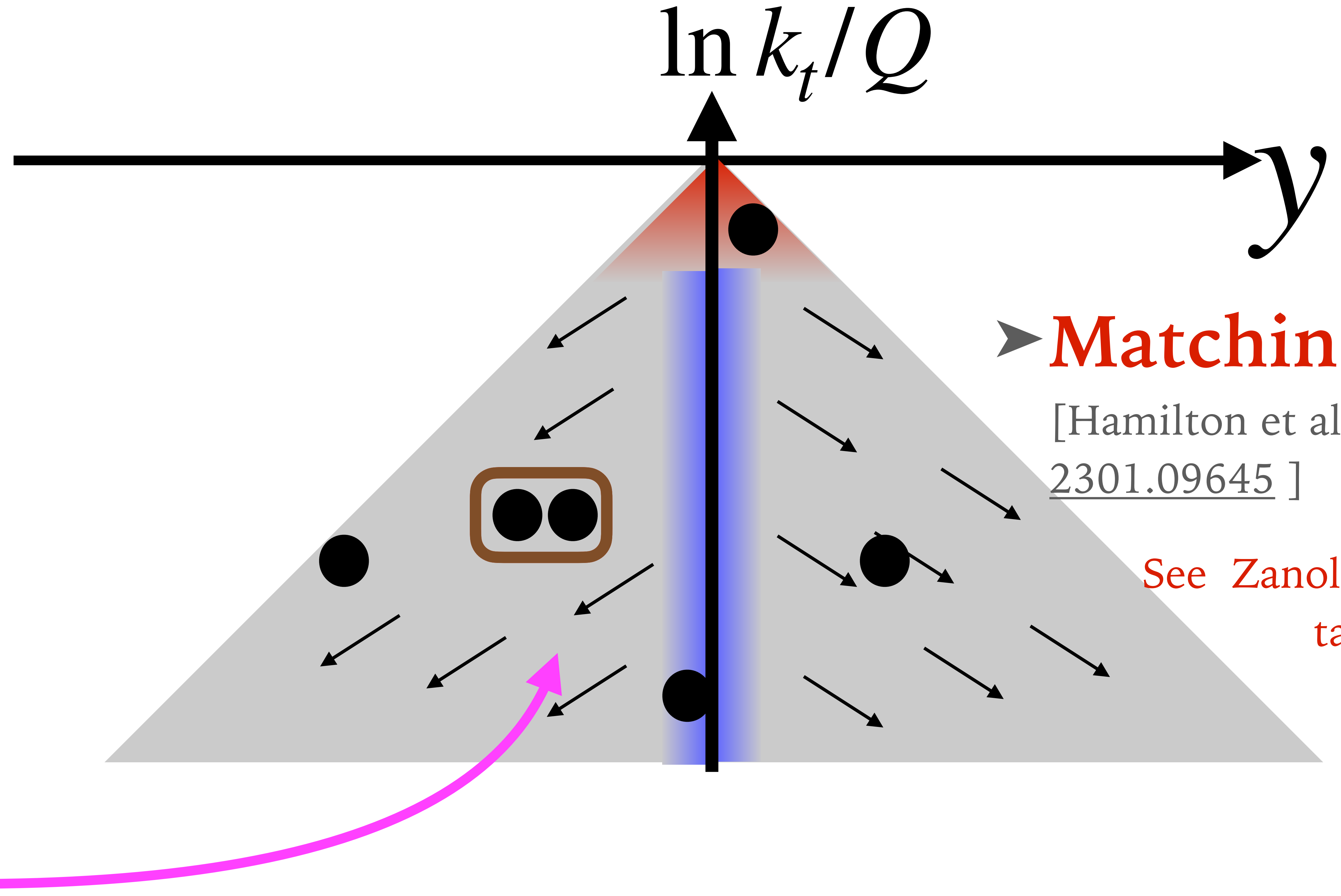
- **Double-soft "reweighting"** for neighbouring soft-collinear emsns
- NLO corrections for soft, large-angle emissons

$$\alpha_s^{\text{eff}}(k_t) = \alpha_s(k_t)\left(1 + \frac{\alpha_s(k_t)}{2\pi}(K_1 + \Delta K_1)\right)$$

- <u>**NNLO corrections**</u> for **soft-collinear emsns**

$$\alpha_s^{\text{eff}}(k_t) = \alpha_s(k_t)\left(\ldots + \frac{\alpha_s^2(k_t)}{4\pi^2}(K_2 + \Delta K_2)\right)$$

Banfi, El-Menoufi, Monni, 1807.11487



- **Matching** [Hamilton et al. 2301.09645]

See Zanoli's talk

<u>**Drift in $\ln k_t$**</u> *of an emission when it further branches*

$$\Delta K_2 \propto \beta_0 \langle \Delta \ln k_t \rangle$$

Building a NNLL shower

SFR et al, 2307.11142

- **Double-soft "reweighting"** for neighbouring soft-collinear emsns
- NLO corrections for soft, large-angle emissons

$$\alpha_s^{\text{eff}}(k_t) = \alpha_s(k_t)\left(1 + \frac{\alpha_s(k_t)}{2\pi}(K_1 + \Delta K_1)\right)$$

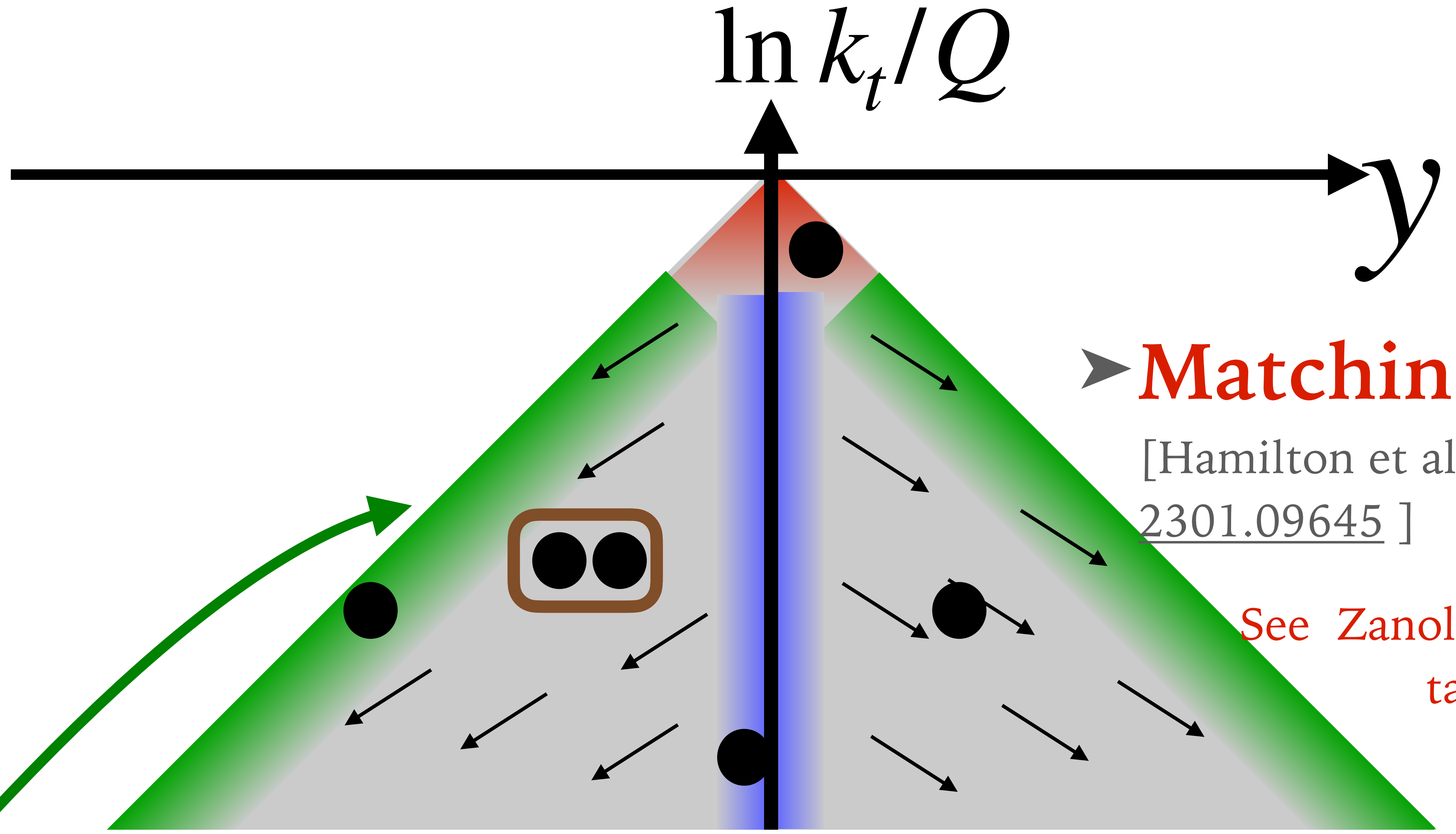
- **NNLO corrections** for **soft-collinear emsns**

$$\alpha_s^{\text{eff}}(k_t) = \alpha_s(k_t)\left(\ldots + \frac{\alpha_s^2(k_t)}{4\pi^2}(K_2 + \Delta K_2)\right)$$

- **NLO corrections** for **collinear emsns**

$$d\mathcal{P}_{\text{coll}} \propto P(z)\left(1 + \frac{\alpha_s}{2\pi}\left(B_2(z) + \Delta B_2(z)\right)\right)$$

Dasgupta, El-Menoufi 2109.07496,
+van Beekveld, Helliwell, Monni 2307.15734,
++Karlberg 2402.05170



- **Matching** [Hamilton et al. 2301.09645]

See Zanoli's talk

Drift in $\ln z = \ln k_t + y$ *of an emission when it further branches*

$$\int P(z)dz\Delta B_2(z) \propto -\langle \Delta \ln z \rangle$$

At this accuracy, it is sufficient to get the integral right, not the functional form of $\Delta B_2(z)$

Building a NNLL shower

➤ Double-soft "reweighting" for

$\ln k_t/Q$

A new standard for the logarithmic accuracy of parton showers

Melissa van Beekveld,[1] Mrinal Dasgupta,[2] Basem Kamal El-Menoufi,[3] Silvia Ferrario Ravasio,[4] Keith Hamilton,[5] Jack Helliwell,[6] Alexander Karlberg,[4] Pier Francesco Monni,[4] Gavin P. Salam,[6,7] Ludovic Scyboz,[3] Alba Soto-Ontoso,[4] and Gregory Soyez[8]

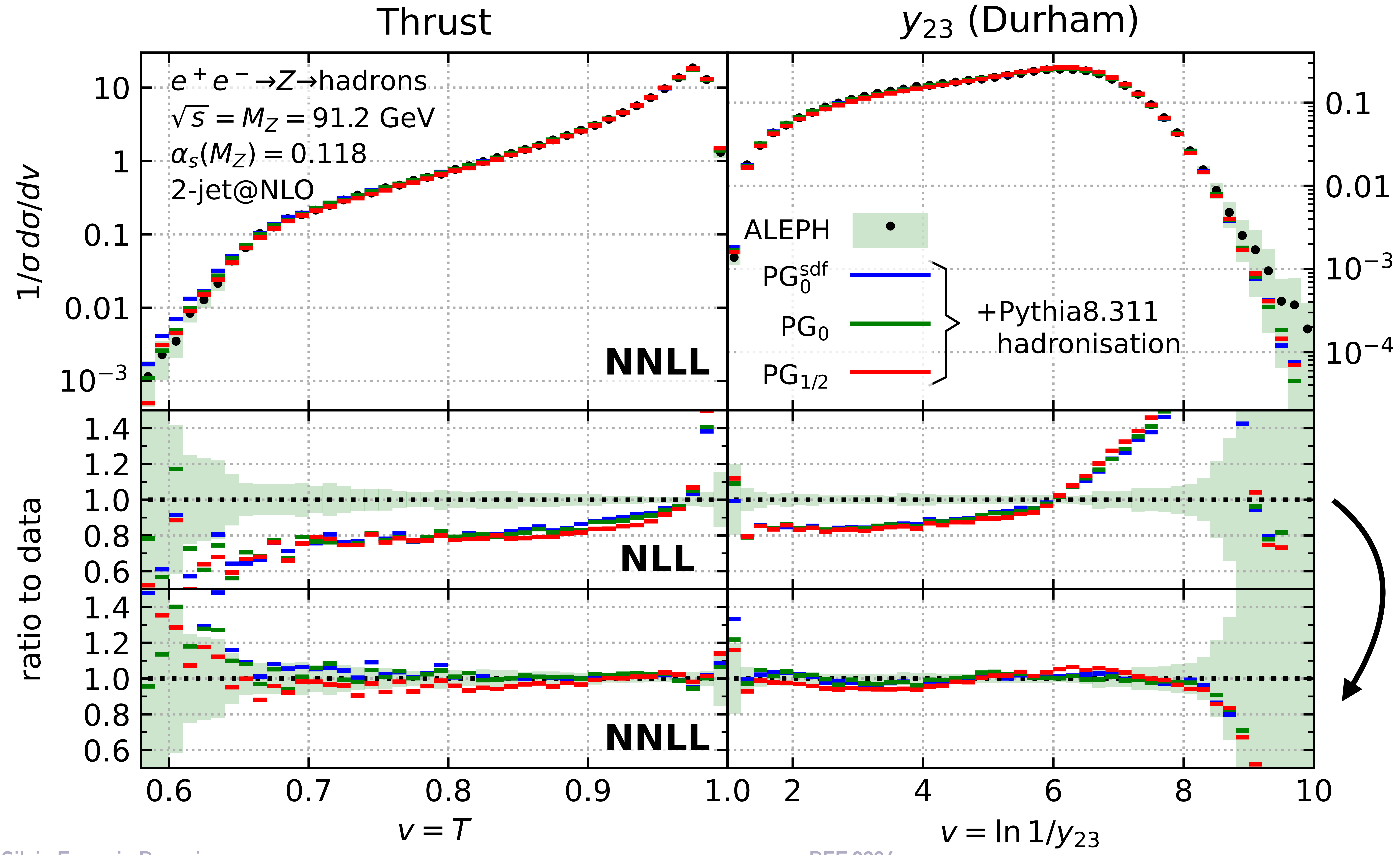
We report on a major milestone in the construction of logarithmically accurate final-state parton showers, achieving next-to-next-to-leading-logarithmic (NNLL) accuracy for the wide class of observables known as event shapes. The key to this advance lies in the identification of the relation between critical NNLL analytic resummation ingredients and their parton-shower counterparts. Our analytic discussion is supplemented with numerical tests of the logarithmic accuracy of three shower variants for more than a dozen distinct event-shape observables in two final states. The NNLL terms are phenomenologically sizeable, as illustrated in comparisons to data.

2406.02661

Dasgupta, El-Menoufi 2109.07496,
+van Beekveld, Helliwell, Monni 2307.15734,
++Karlberg 2402.05170

...to get the integral
...the functional form of $\Delta B_2(z)$

NNLL showers vs NLL showers: pheno outlook



The PanScales collaboration, 2406.02661

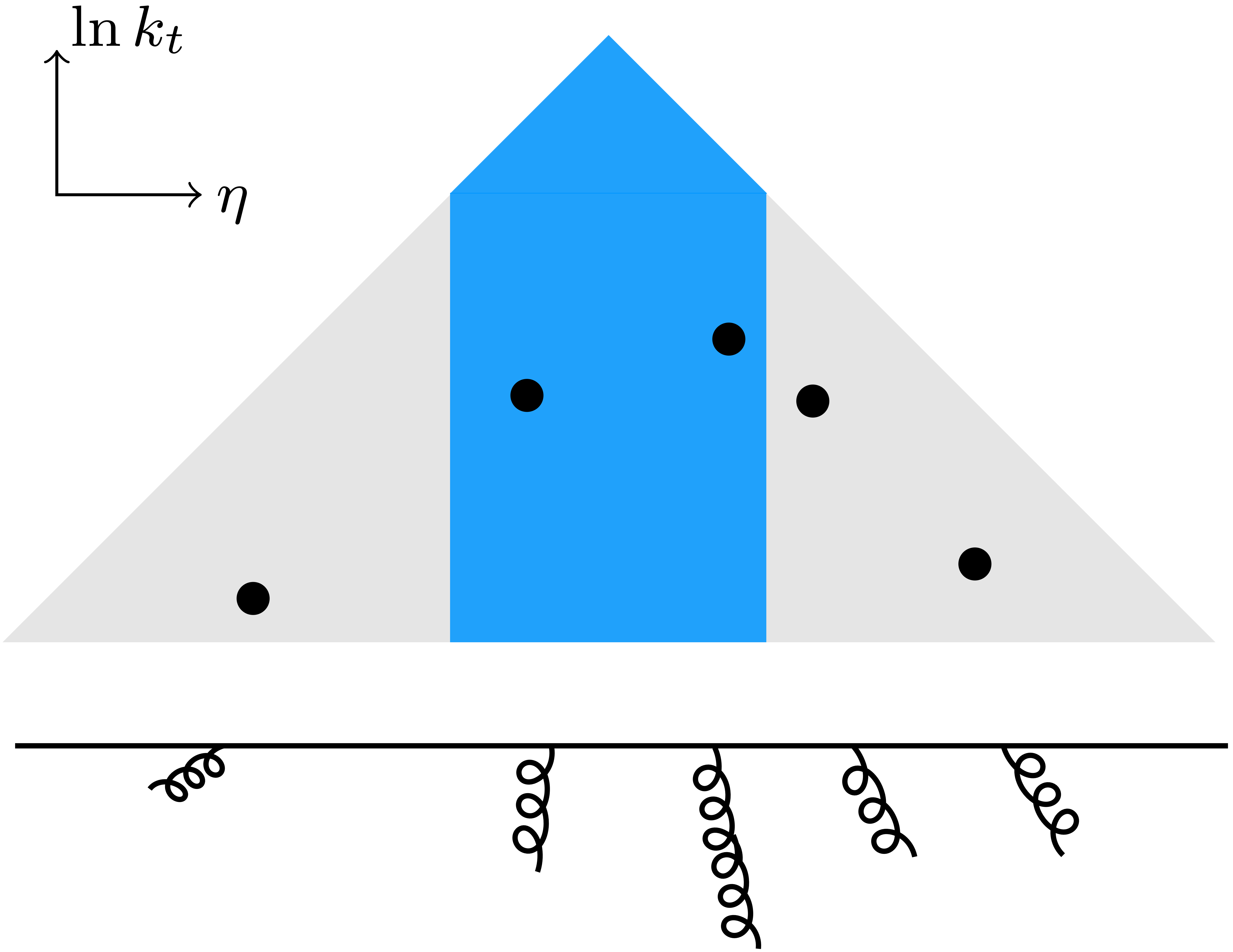
Agreement to **data substantially better** when using **NNLL** showers

Conclusions

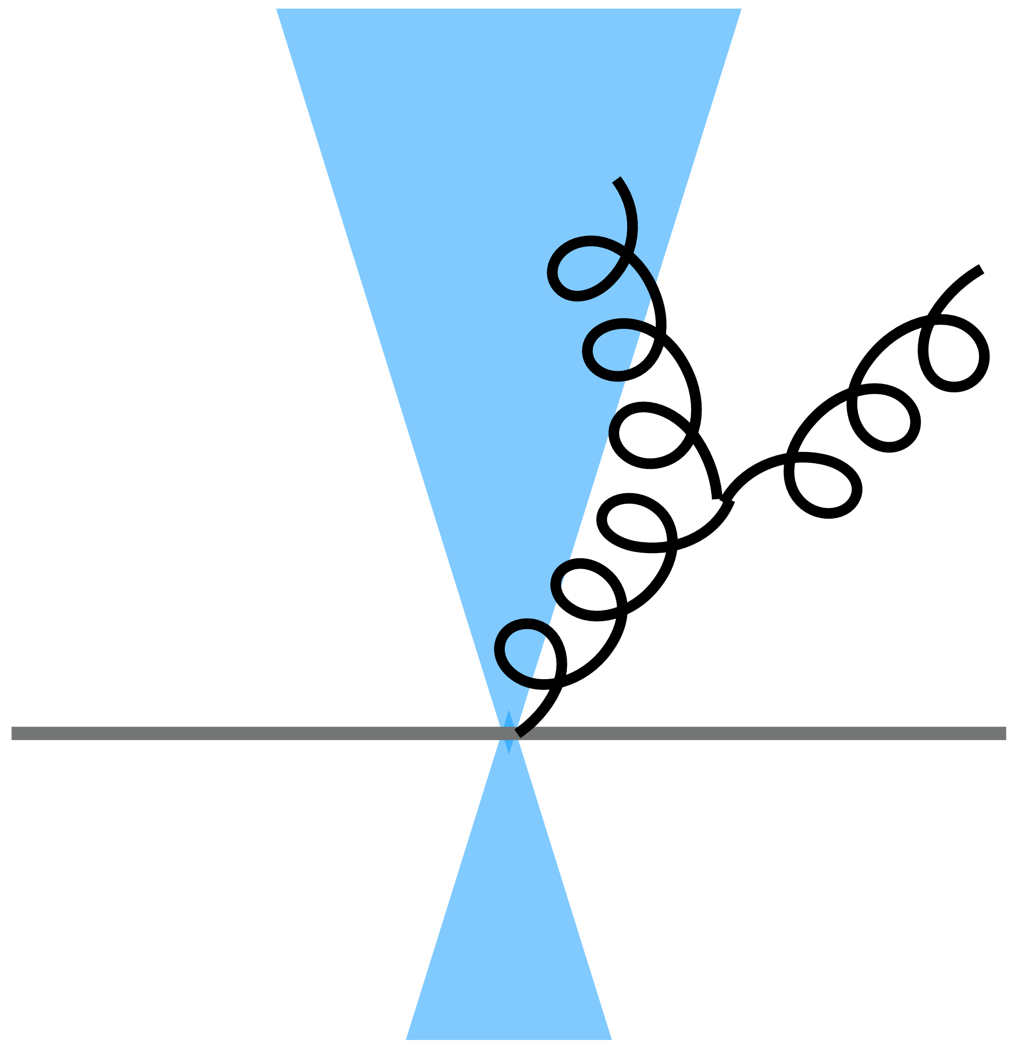
- **PanScales is first validated NLL shower**
  - All processes with **two colour legs** have been rigorously tested to be NLL for both global and non-global event shapes
  - benefits of **LL → NLL** include **reduced uncertainties** (**reliable** estimate)
  - NLO matching in place for some simple processes
- **Higher log accuracy is one of the next frontiers**
  - Double-soft (+ virtual) corrections: **NSL** accuracy for **non-global** event shapes, **NNDL** accuracy for subjet **multiplicites**.
  - **NNLL** accuracy for **global event shapes** in $e^+e^- \to j_1 j_2$
- Public code
  - https://gitlab.com/panscales/panscales-0.X

The PanScales collaboration, 2312.13275

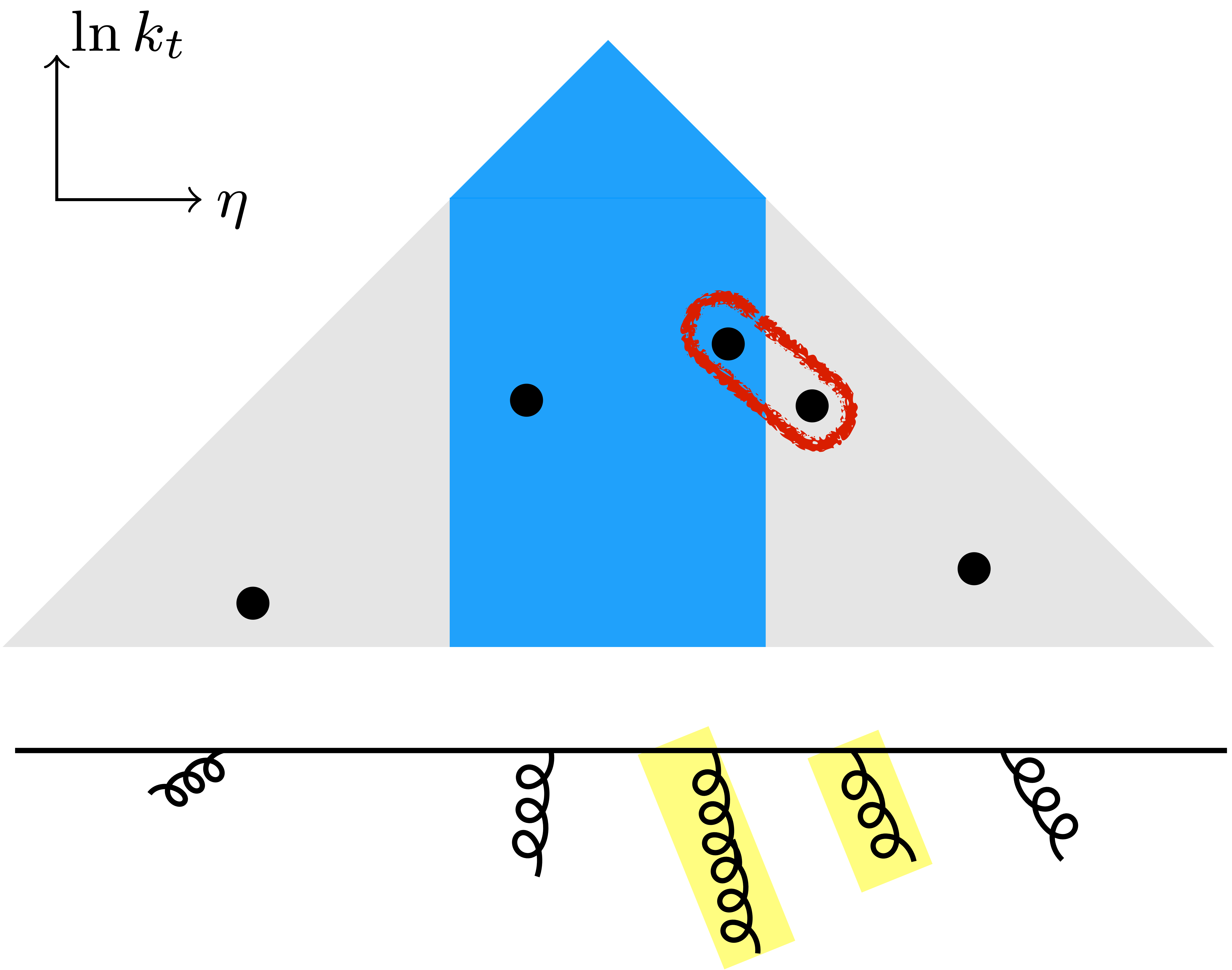
NSL for the energy flow in a rapidity slice



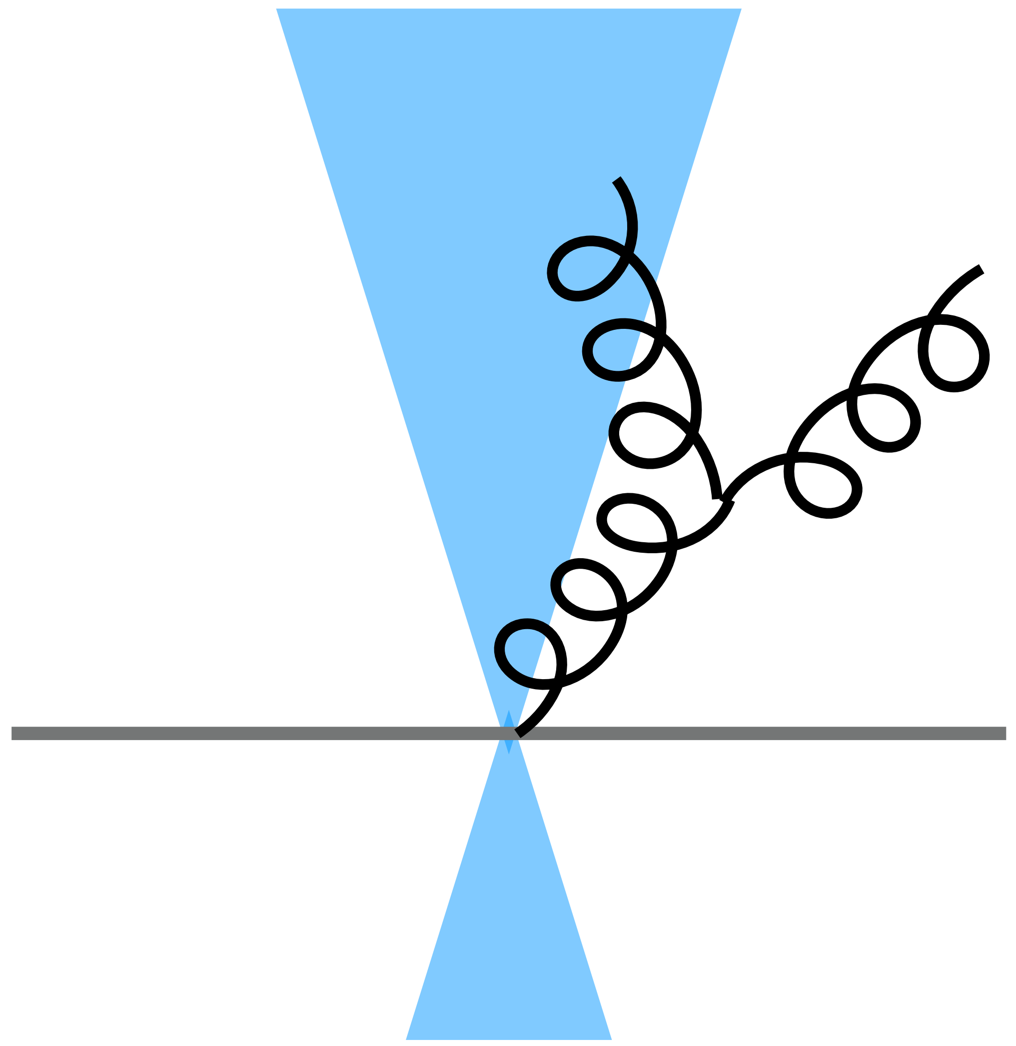
Non-global
observable



NSL for the energy flow in a rapidity slice

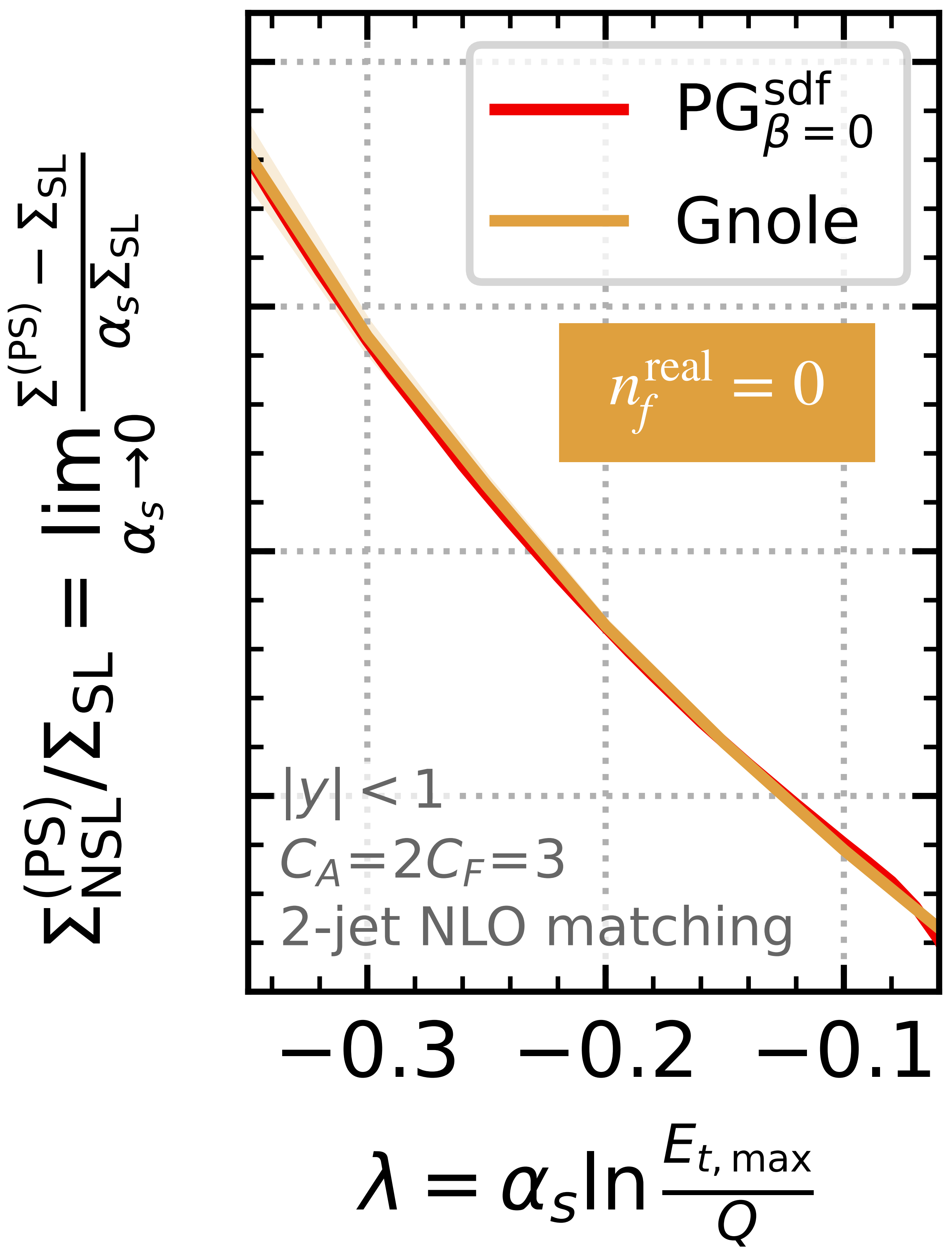


Non-global observable



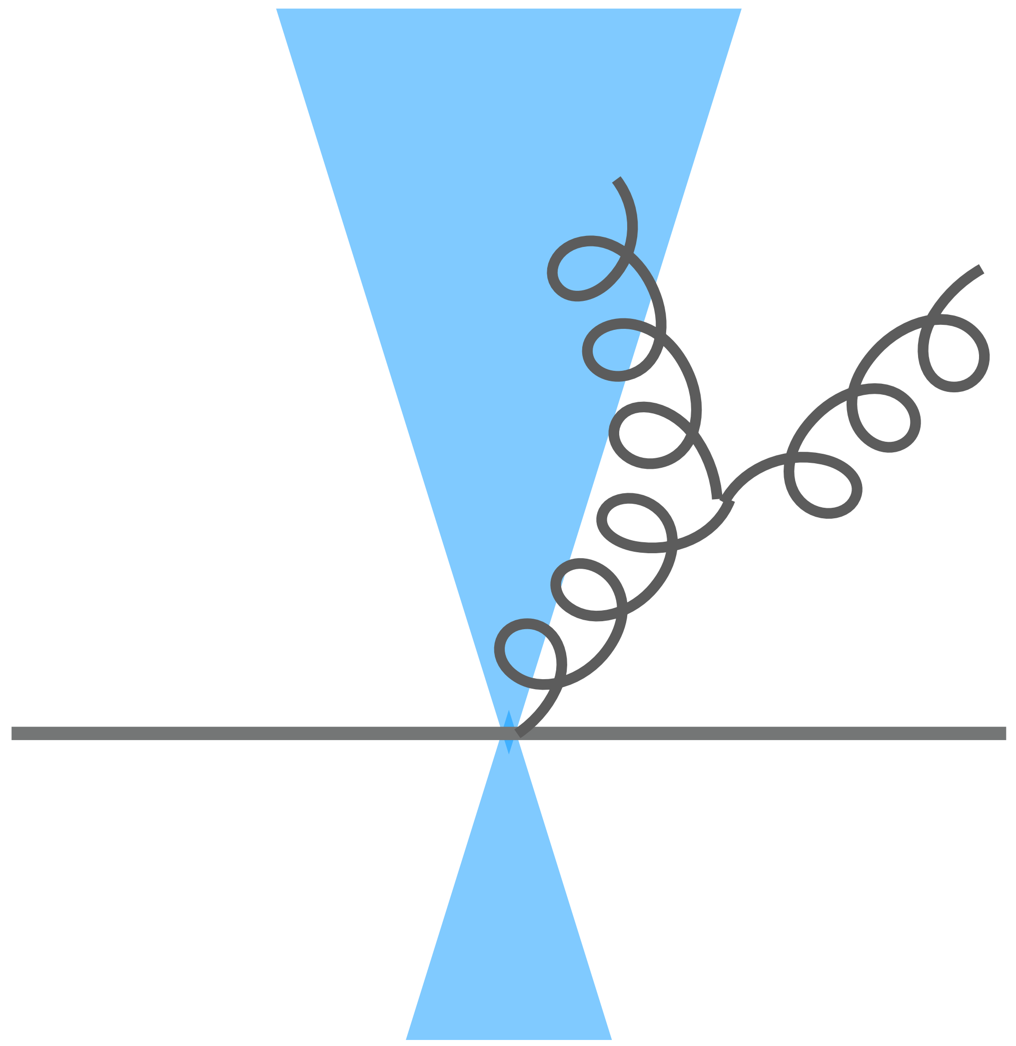
- **NSL** ($\alpha_s^n L^{n-1}$) analytic reference from Banfi, Dreyer, Monni, 2104.06416, 2111.02413 **("Gnole")**
  [NB: see also Becher, Schalch, Xu, 2307.02283]

NSL for the energy flow in a rapidity slice



S.F.R., Hamilton, Karlberg, Salam, Scyboz, Soyez 2307.11142

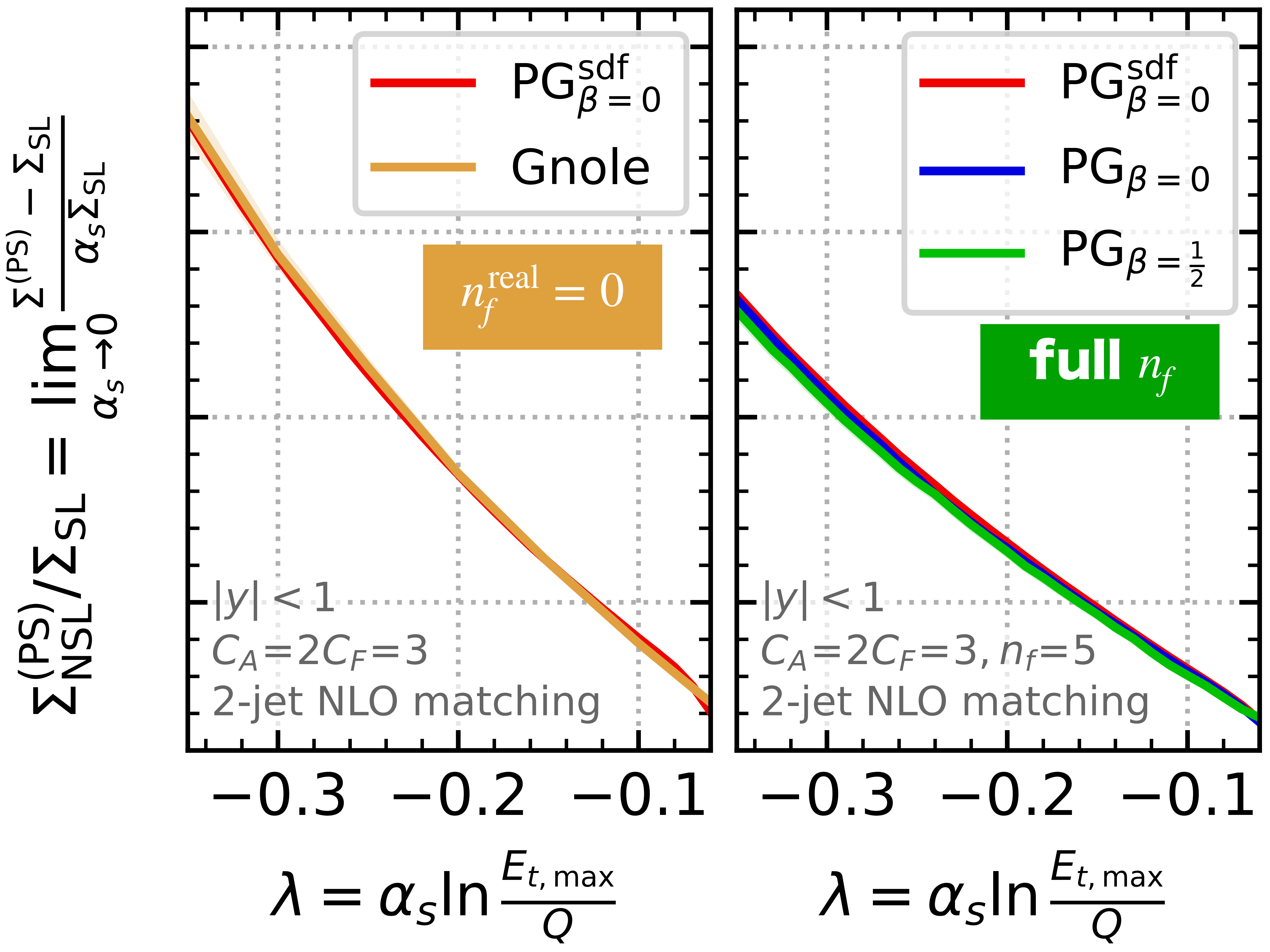
Non-global observable



➤ **NSL** ($\alpha_s^n L^{n-1}$) analytic reference from Banfi, Dreyer, Monni, 2104.06416, 2111.02413 **("Gnole")**

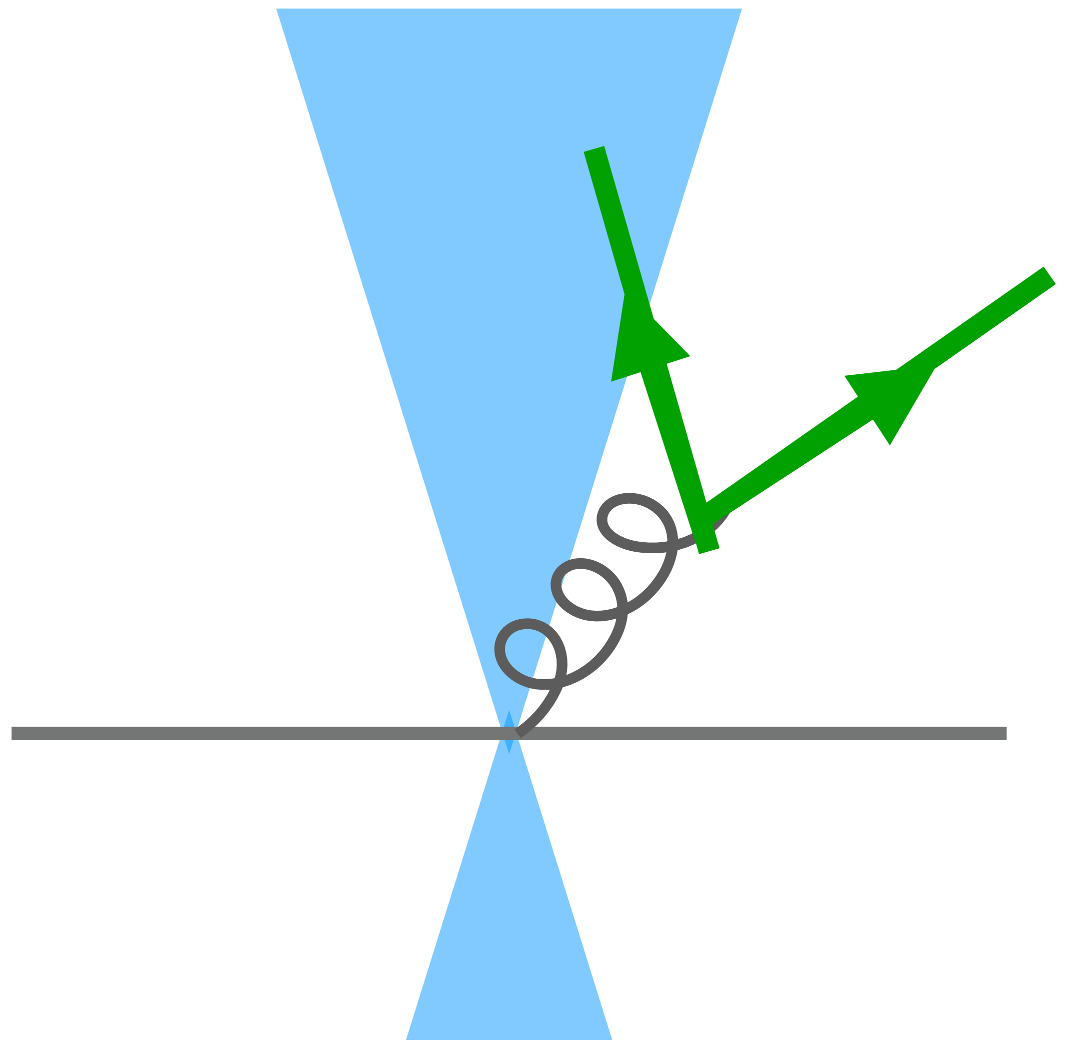
[NB: see also Becher, Schalch, Xu, 2307.02283]

NSL for the energy flow in a rapidity slice



S.F.R., Hamilton, Karlberg, Salam, Scyboz, Soyez 2307.11142

Non-global
observable

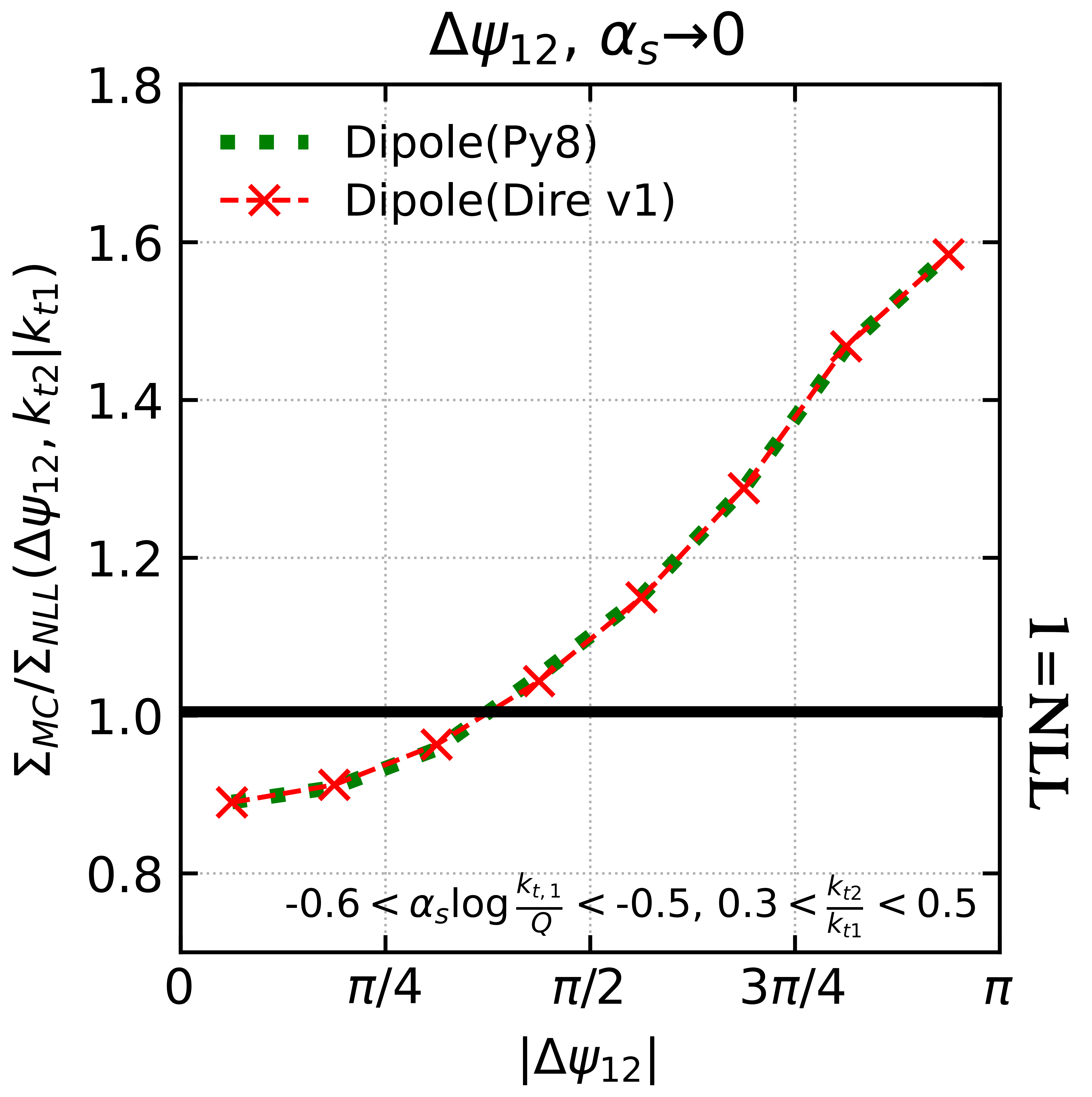


- **NSL** ($\alpha_s^n L^{n-1}$) analytic reference from Banfi, Dreyer, Monni, 2104.06416, 2111.02413 **(“Gnole”)**
  [NB: see also Becher, Schalch, Xu, 2307.02283]
- First large-N_c **full-n_f** results for NSL non-global logs

What is available in Shower Monte Carlo generators?

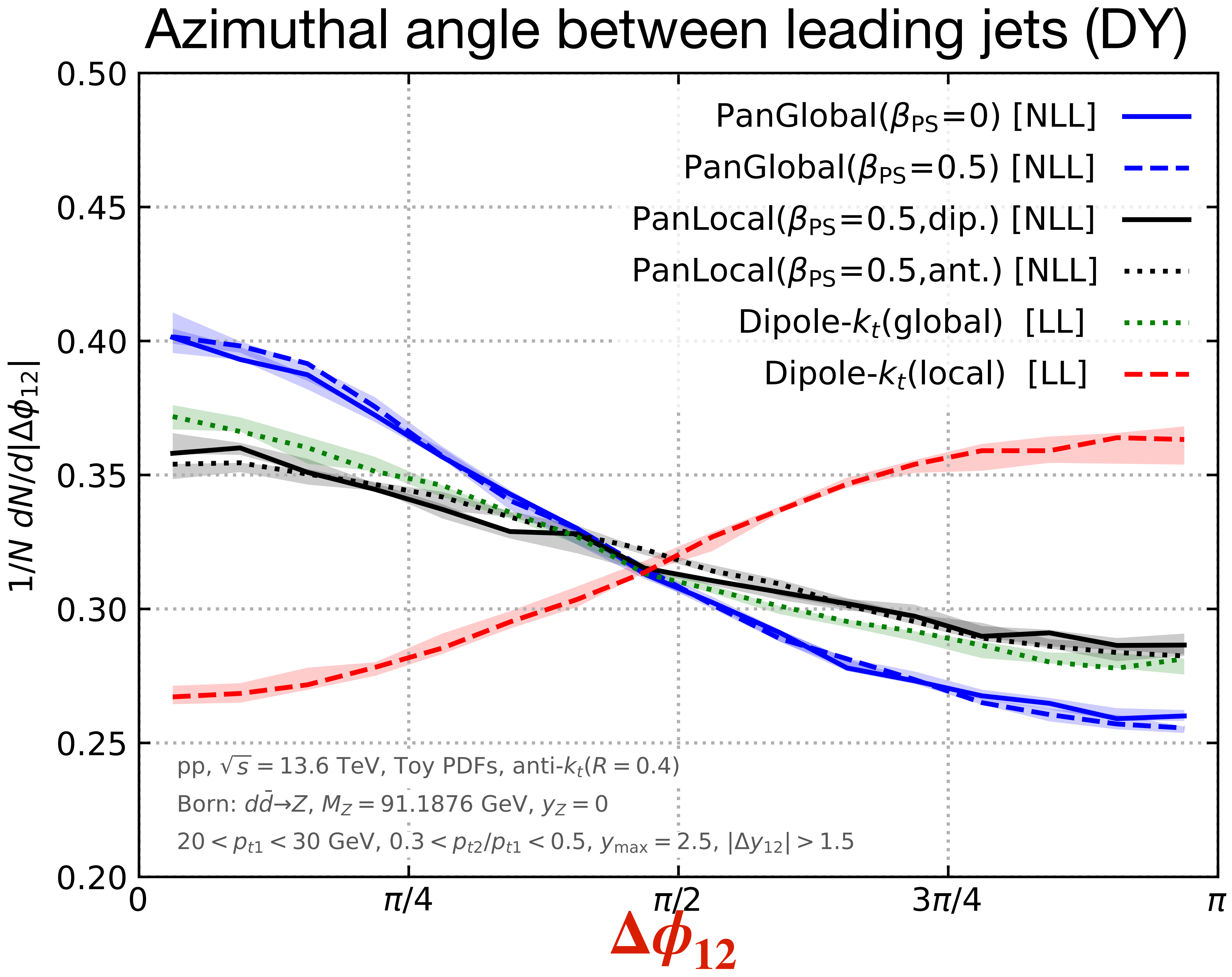
- Showers routinely used to interpret LHC (and LEP) data are **not NLL**!

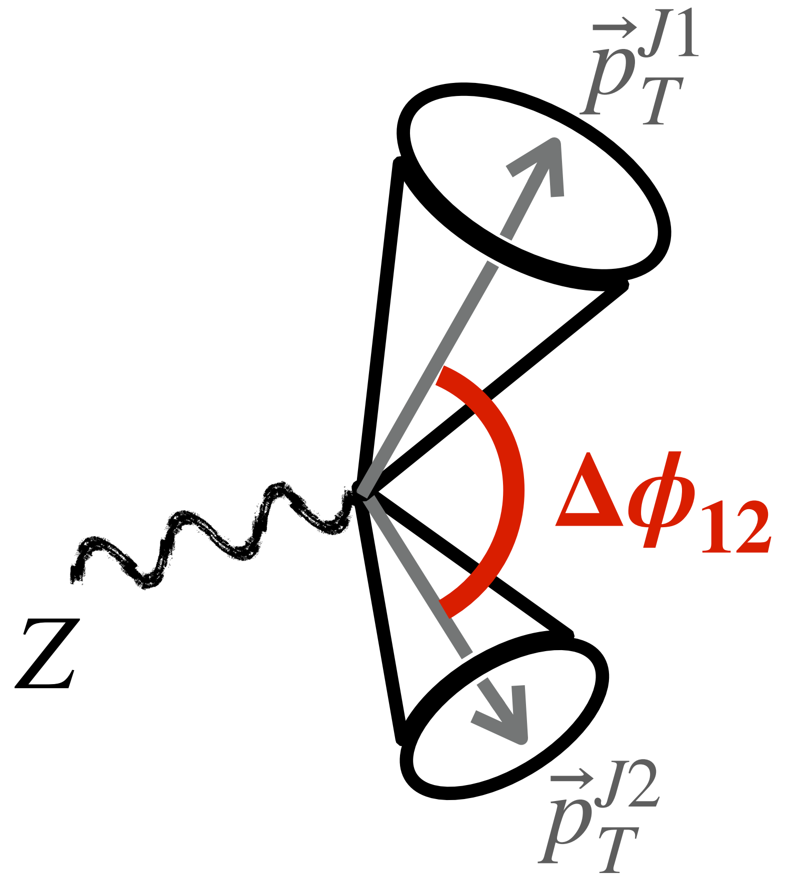
Dasgupta et al. 2002.11114



Exploratory phenomenology for Drell-Yan at the LHC

$m_{\ell\ell} = 91.2\,\mathrm{GeV}$

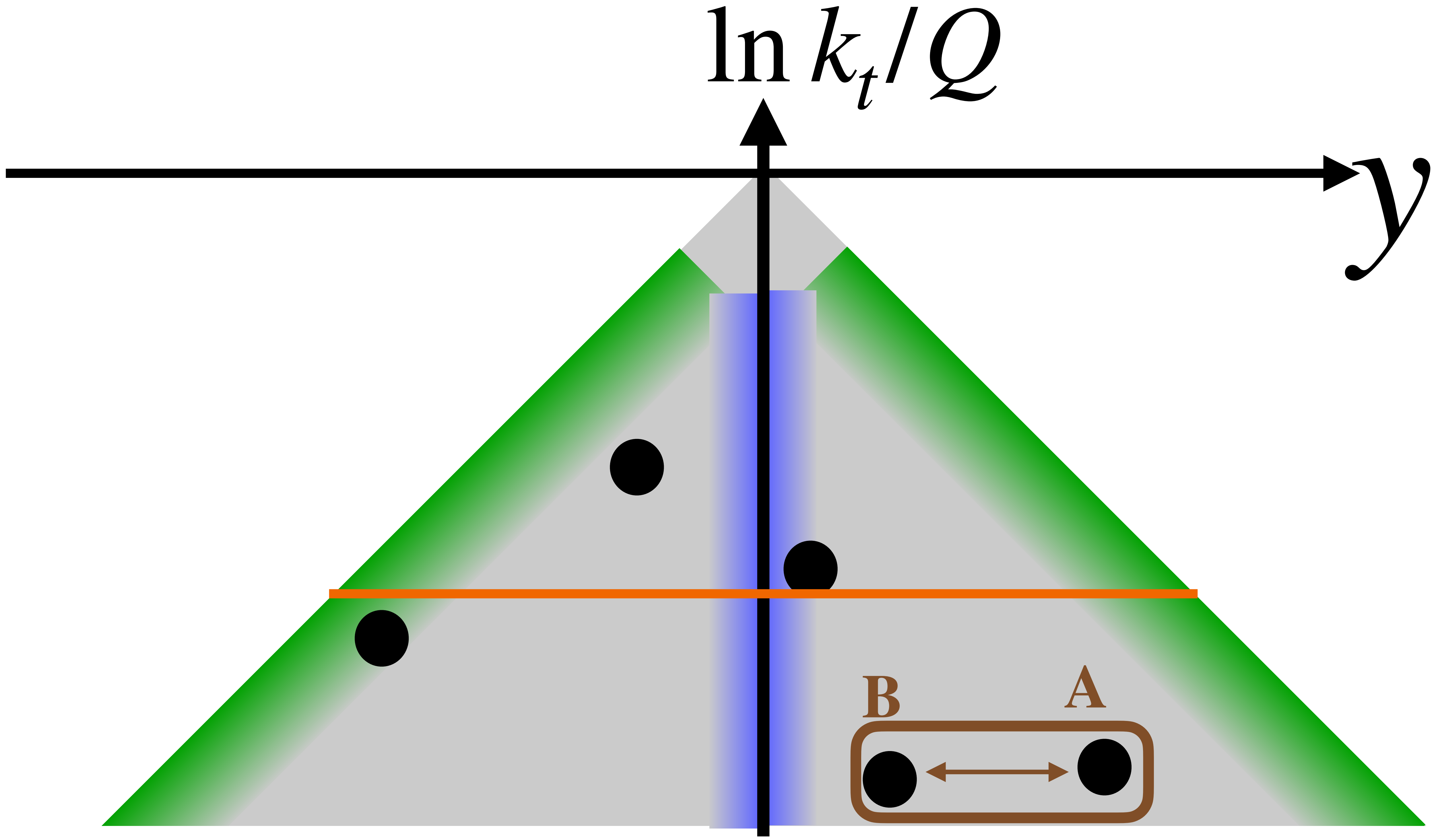




PanScales for $pp \to$ colour singlet: 2207.09467, van Beekveld, **SFR**, Hamilton, Salam Soto Ontoso, Soyez, Verheyen:

How to build a **NLL** parton shower?

- Standard showers implement **local transverse momentum k_t conservation** and **transverse momentum ordering:** emission **A** will change substantially after emission **B**!

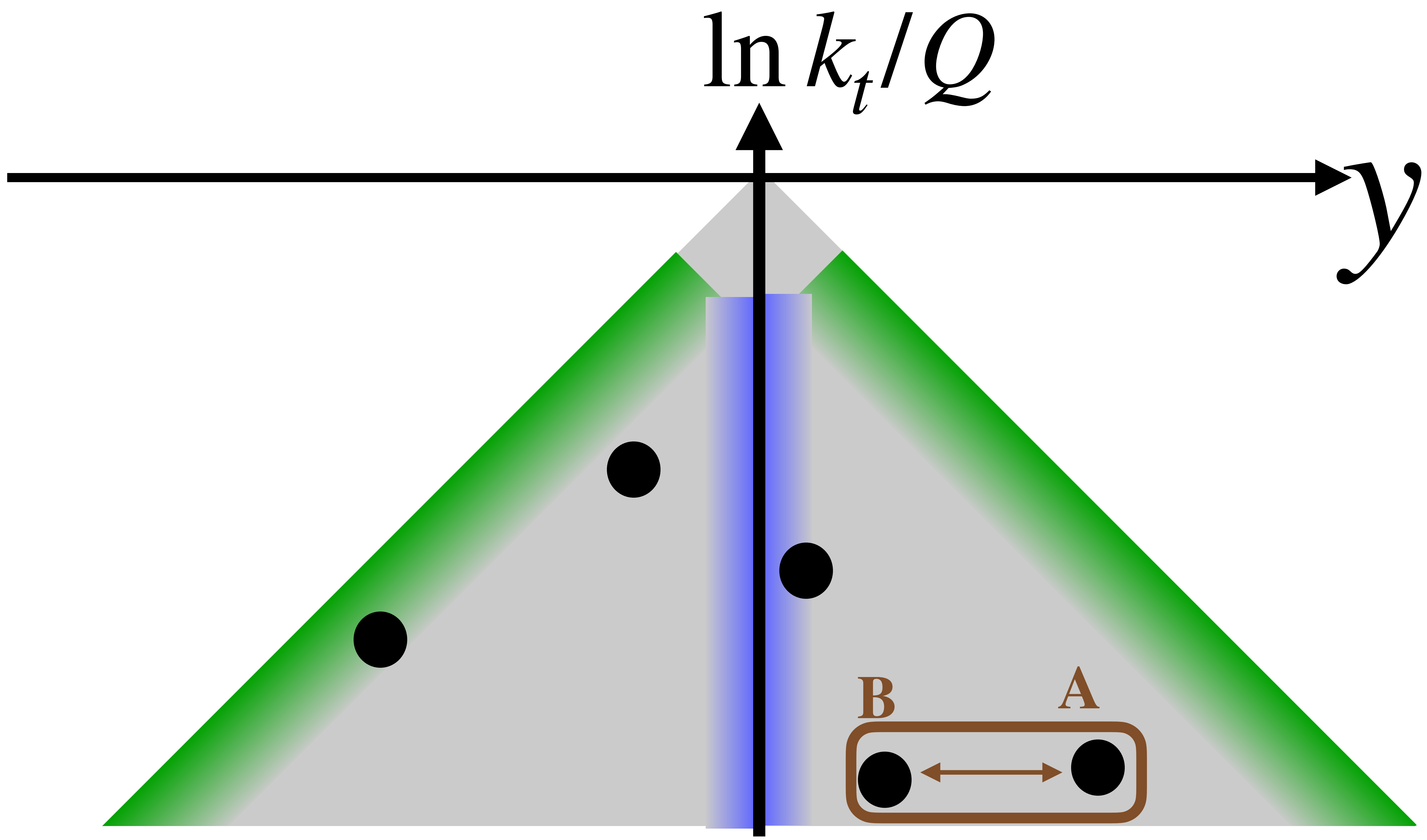


Constraints **kinematic mapping $\Phi_n \to \Phi_{n+1}$** and **ordering variable**: **emissions well separated in rapidity are independent from each other, even if they have similar transverse momentum**

How to build a NLL parton shower?

- Standard showers implement local transverse momentum k_t conservation and transverse momentum ordering: emission **A** will change substantially after emission **B**!

Global k_t conservation
PanScales
FHP 2003.06400 ,
Alaric 2208.06057,
Apollo 2403.19452



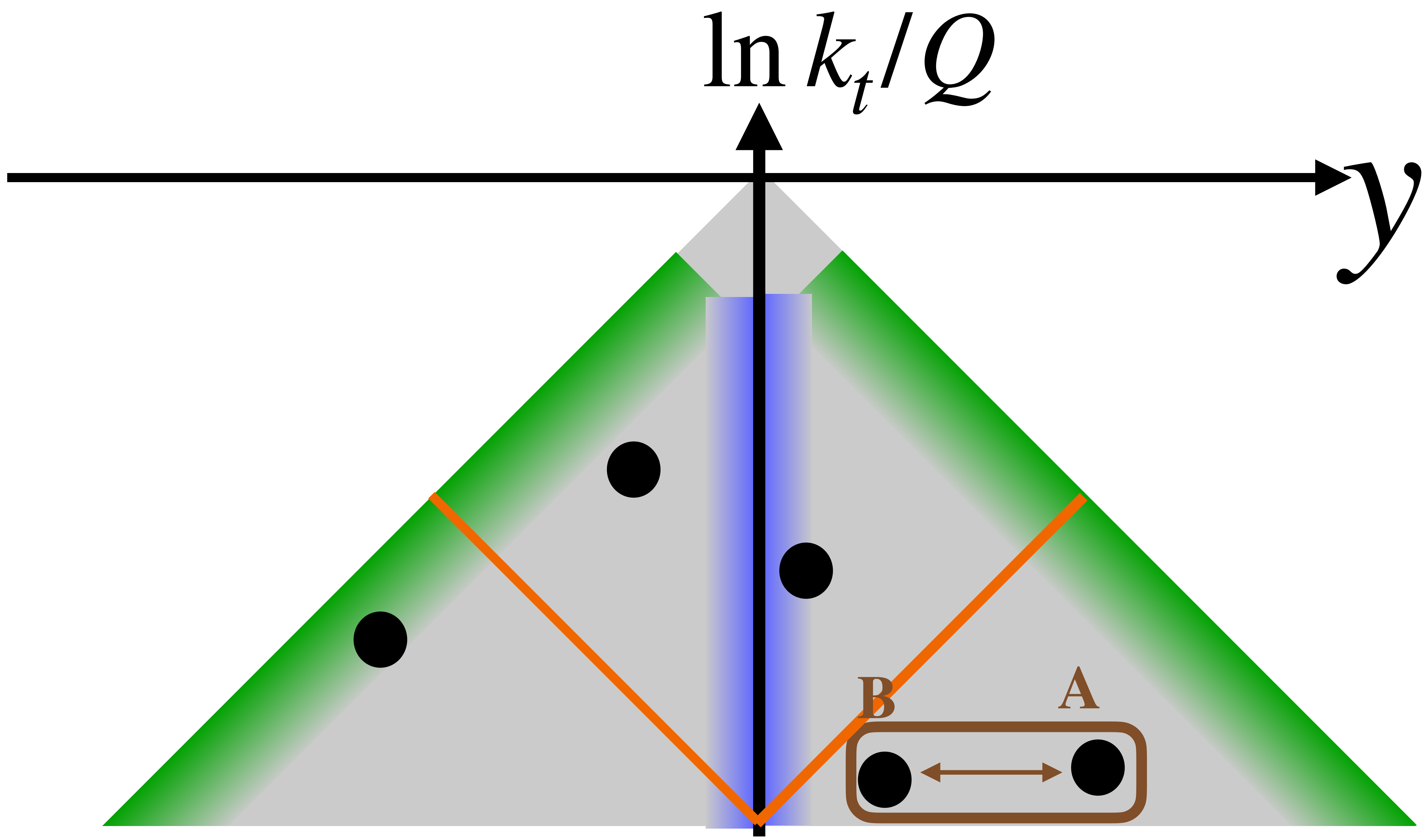
Constraints **kinematic mapping $\Phi_n \rightarrow \Phi_{n+1}$** and **ordering variable**: **emissions well separated in rapidity are independent from each other, even if they have similar transverse momentum**

How to build a **NLL** parton shower?

- Standard showers implement **local transverse momentum k_t conservation** and **transverse momentum ordering**: emission **A** will change substantially after emission **B**!

Global k_t conservation
PanScales
FHP 2003.06400 ,
Alaric 2208.06057,
Apollo 2403.19452

Ordering variable to enforce some angular ordering
Deductor 2011.04777,
PanScales

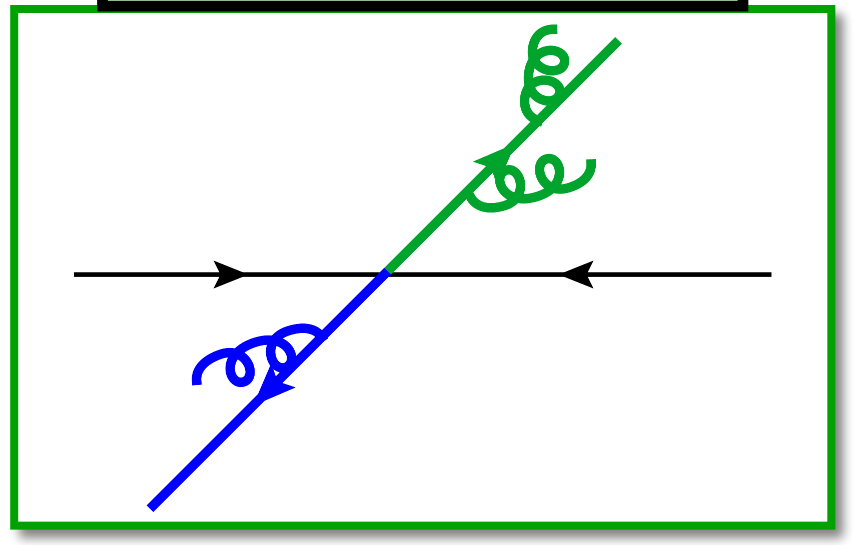


Constraints **kinematic mapping $\Phi_n \to \Phi_{n+1}$** and **ordering variable**: **emissions well separated in rapidity are independent from each other, even if they have similar transverse momentum**

Status of **NLL PanScales** showers

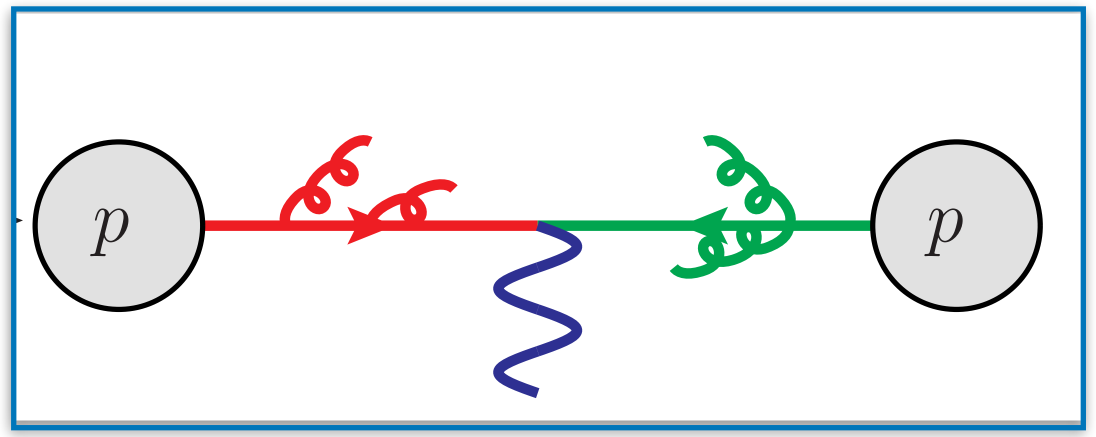
- This enabled the **PanScales** to devise the **first** showers with **general** NLL accuracy for

$e^+e^- \to j_1 j_2$



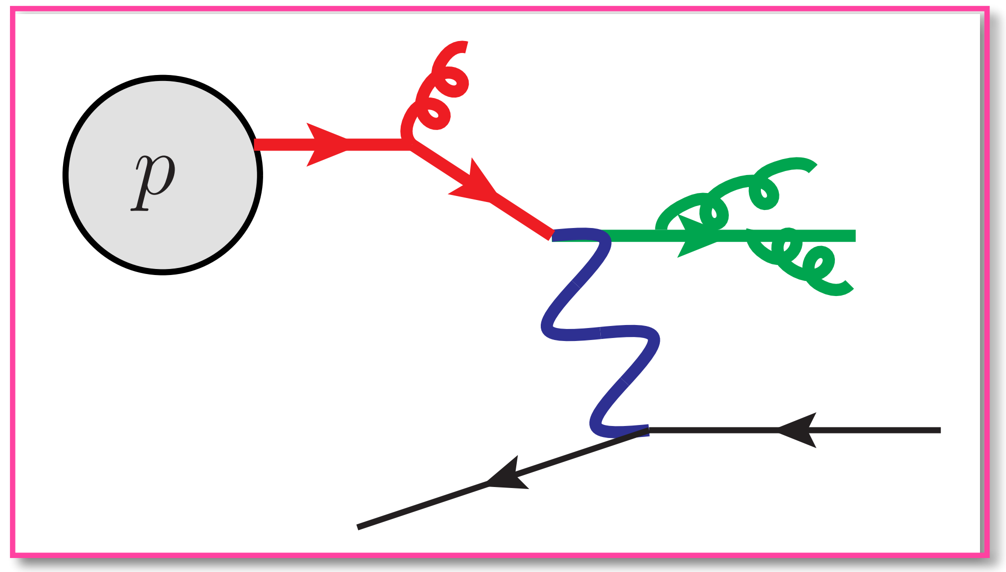
Dasgupta, Dreyer, Hamilton, Monni, Salam, Soyez, 2002.11114

pp → colour singlet



van Beekveld, **SFR**, Soto-Ontoso, Salam, Soyez, Verheyen, 2205.02237, + Hamilton 2207.09467

DIS & VBF



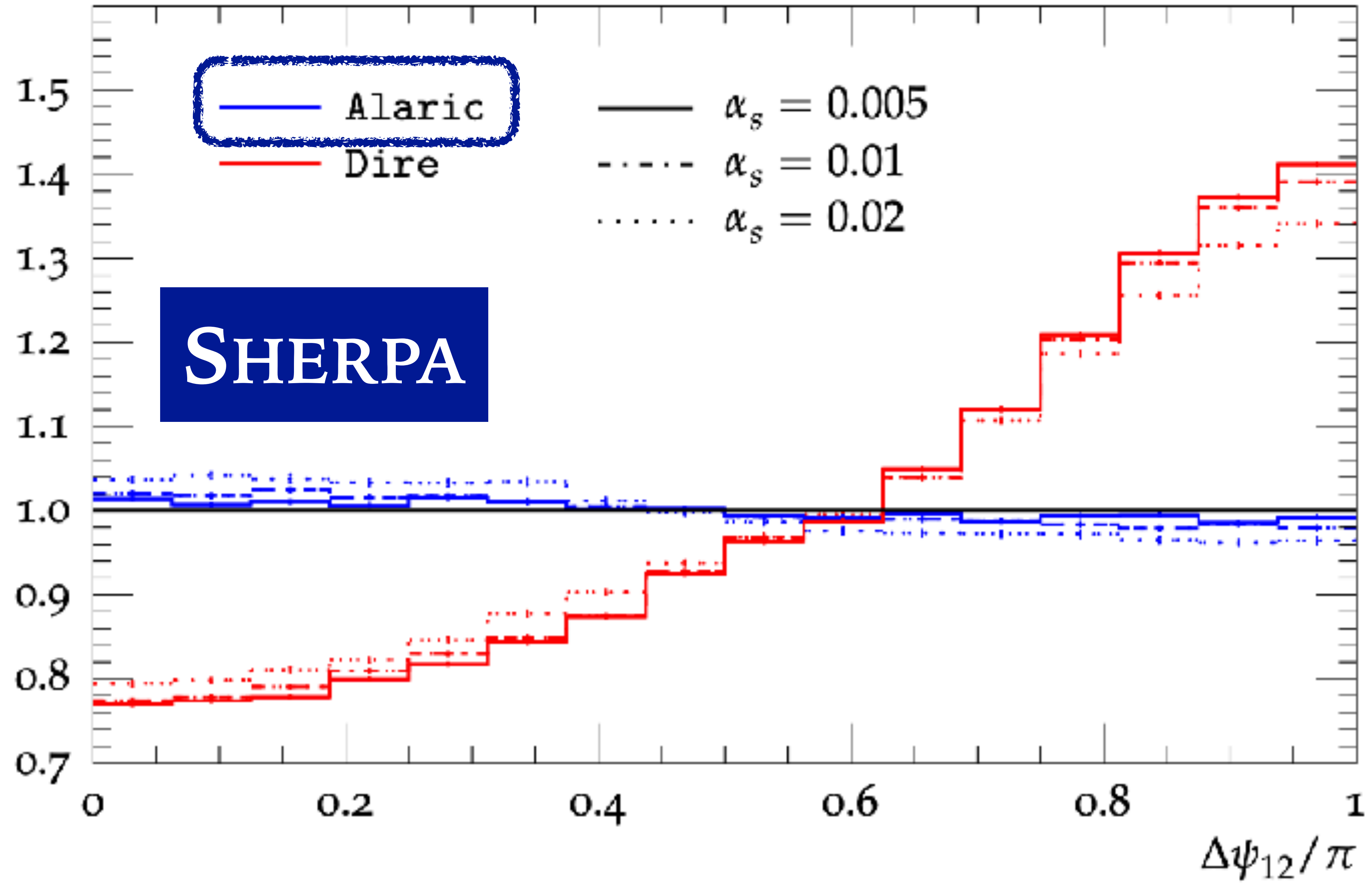
van Beekveld, **SFR**, 2305.08645

…with **subleading colour** (2011.10054) and **spin correlations** (2103.16526, 2111.01161)

What can be available in Shower Monte Carlo generators?

- Showers routinely used to interpret LHC (and LEP) data are **not NLL**!
- **Many groups** are independently formulating new showers with **NLL accuracy for e^+e^-**

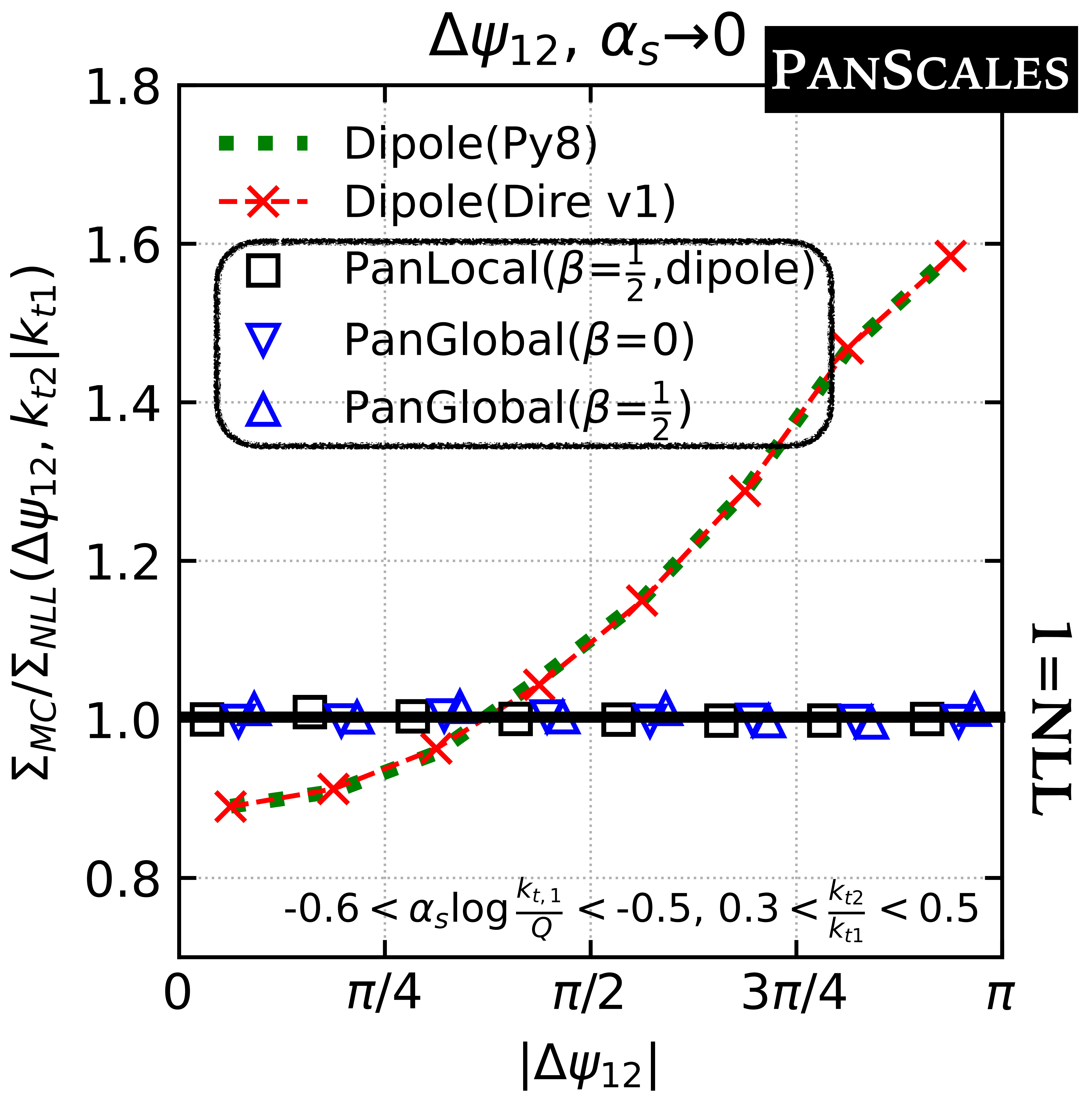
Herren et al. 2208.06057



DEDUCTOR
Nagy&Soper, 2011.04777

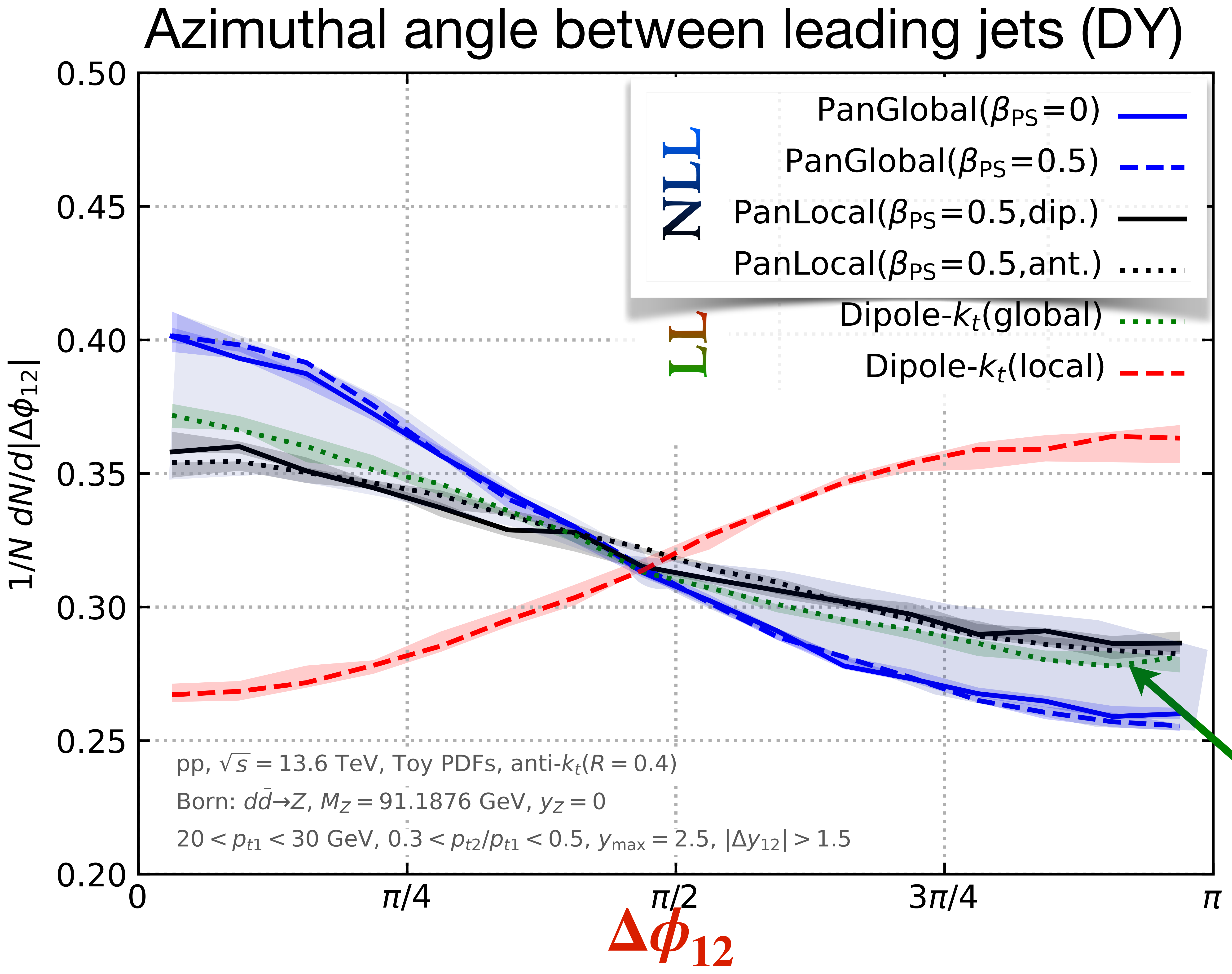
CVOLVER
Forshaw et. al, 2003.06400

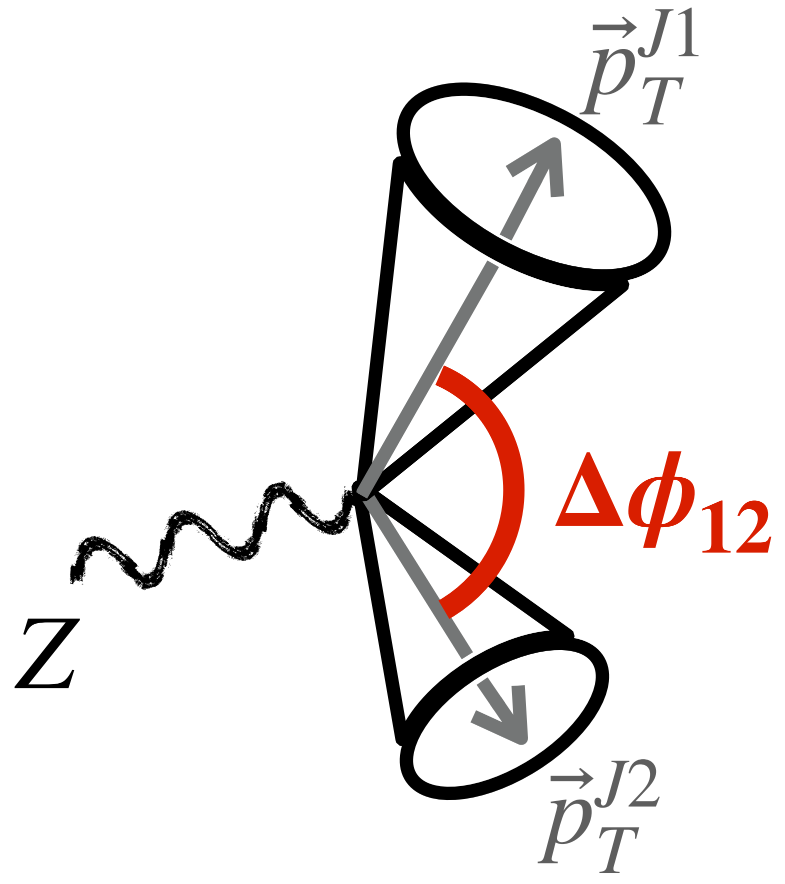
Dasgupta et al. 2002.11114



Exploratory phenomenology for Drell-Yan at the LHC

$m_{\ell\ell} = 91.2\,\mathrm{GeV}$



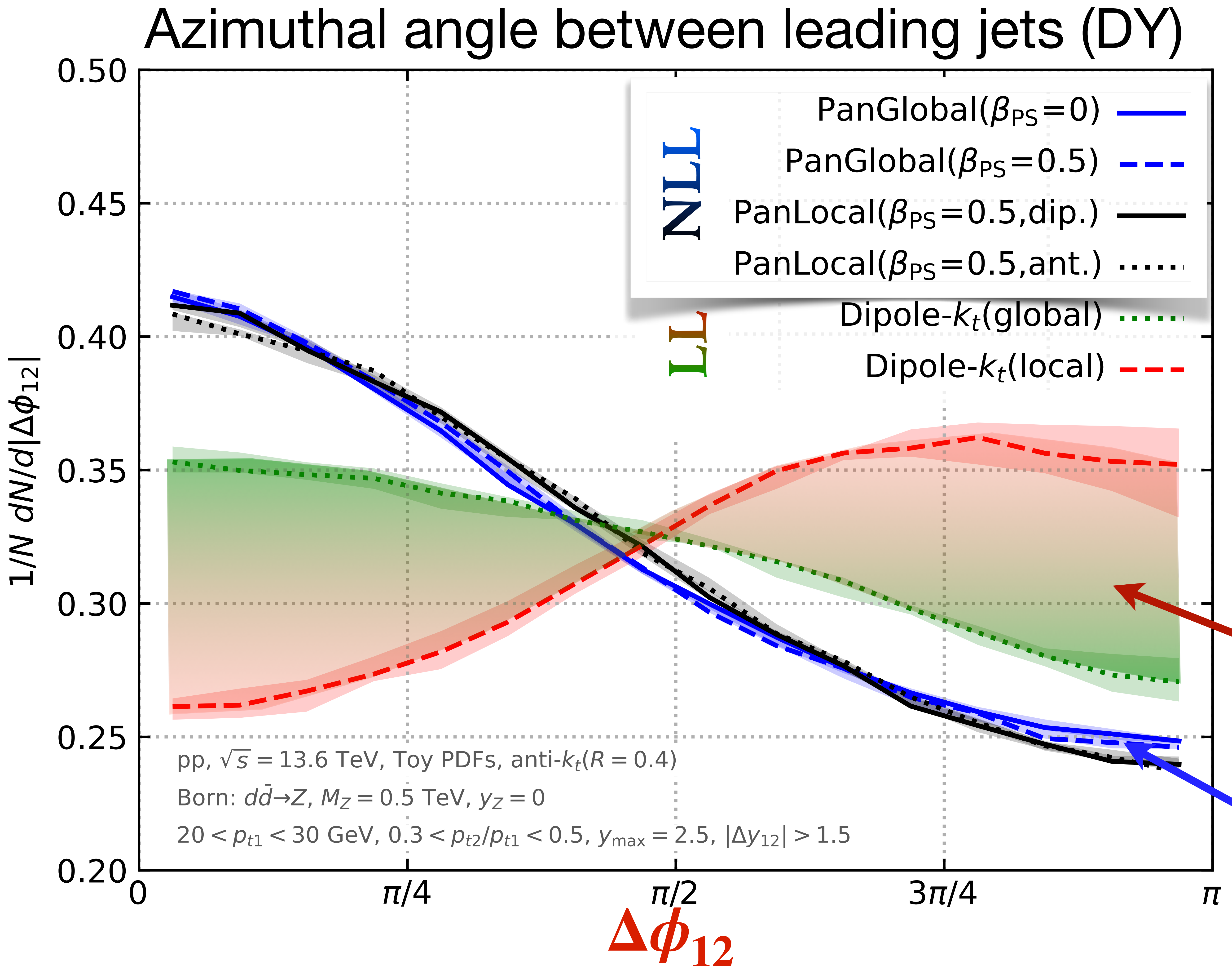


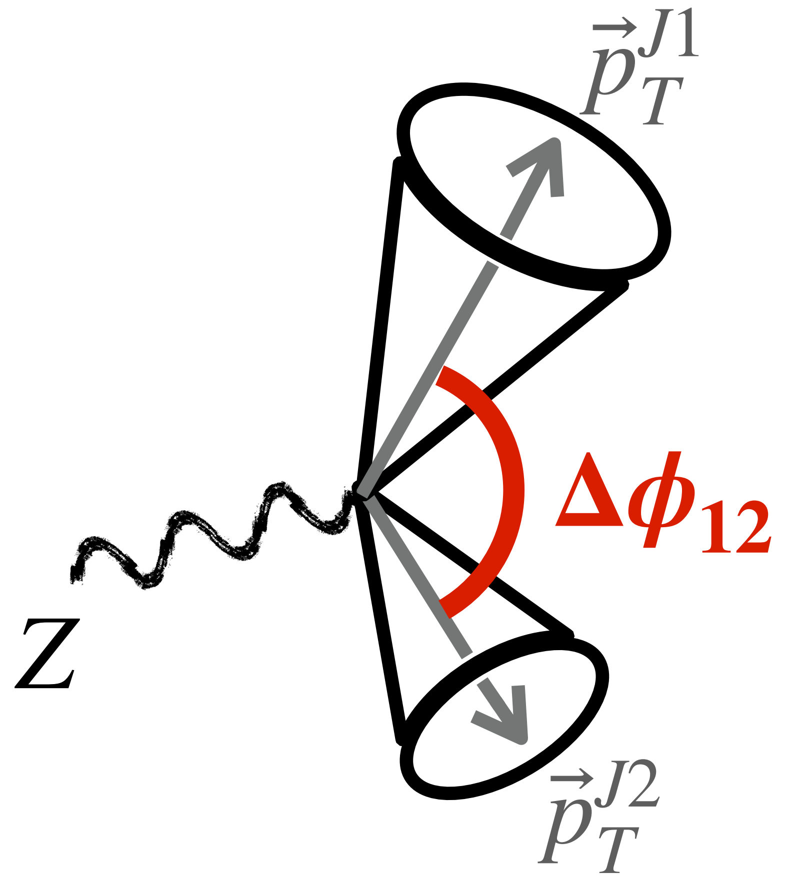
PanScales for $pp \to$ colour singlet: 2207.09467, van Beekveld, **SFR**, Hamilton, Salam Soto Ontoso, Soyez, Verheyen:

This LL shower lives within the span of the NLL showers

Exploratory phenomenology for **high-mass** Drell-Yan at the LHC

$m_{\ell\ell} = 500\,\mathrm{GeV}$





PanScales for $pp \to$ colour singlet: 2207.09467, van Beekveld, **SFR**, Hamilton, Salam Soto Ontoso, Soyez, Verheyen:

NLL/LL discrepancies at larger scales

LL showers

NLL showers