Studying nucleon structures via Target fragmentation: Fracture function and Nucleon energy correlator

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REF 2024
Paris, France

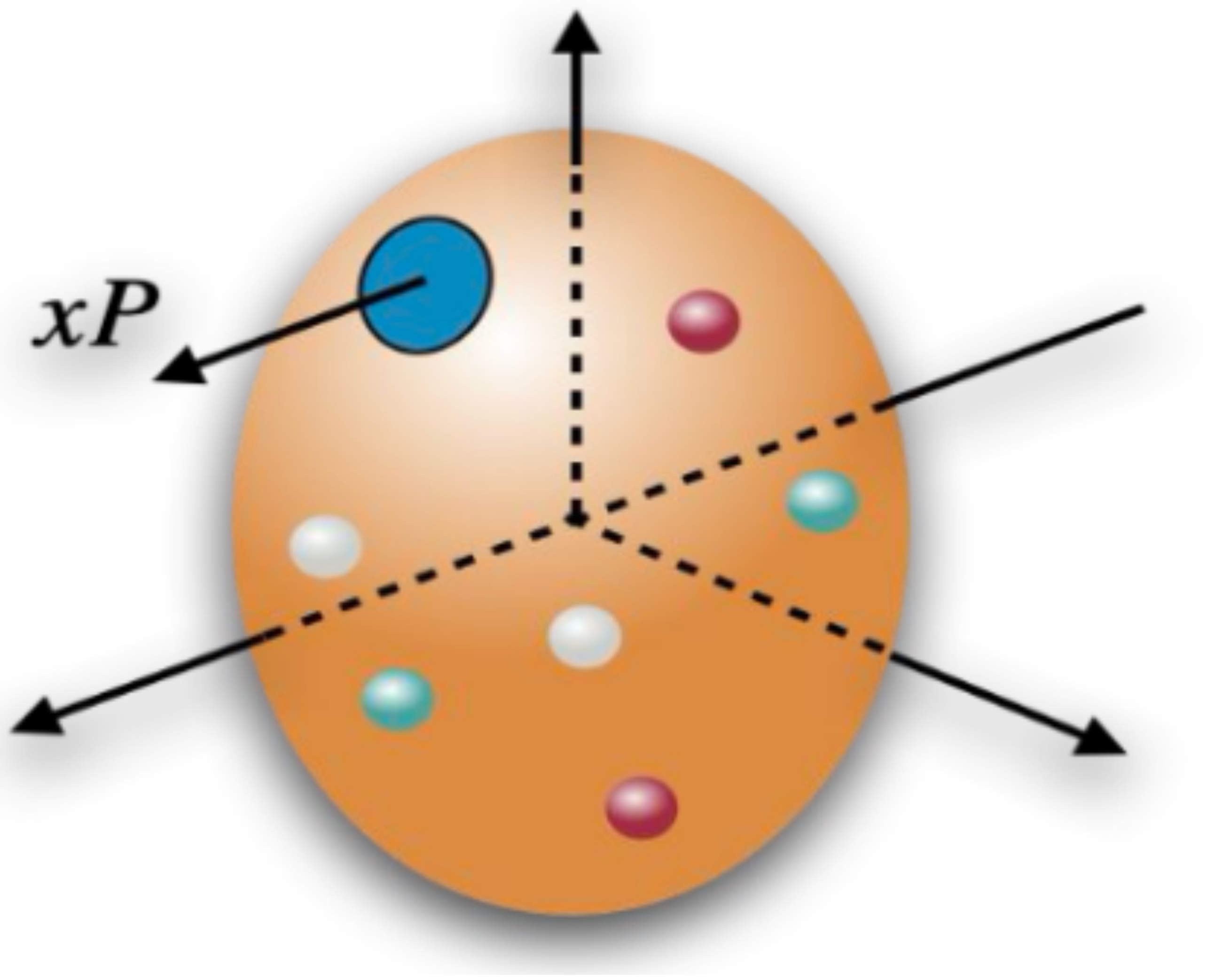
Oct 17th, 2024

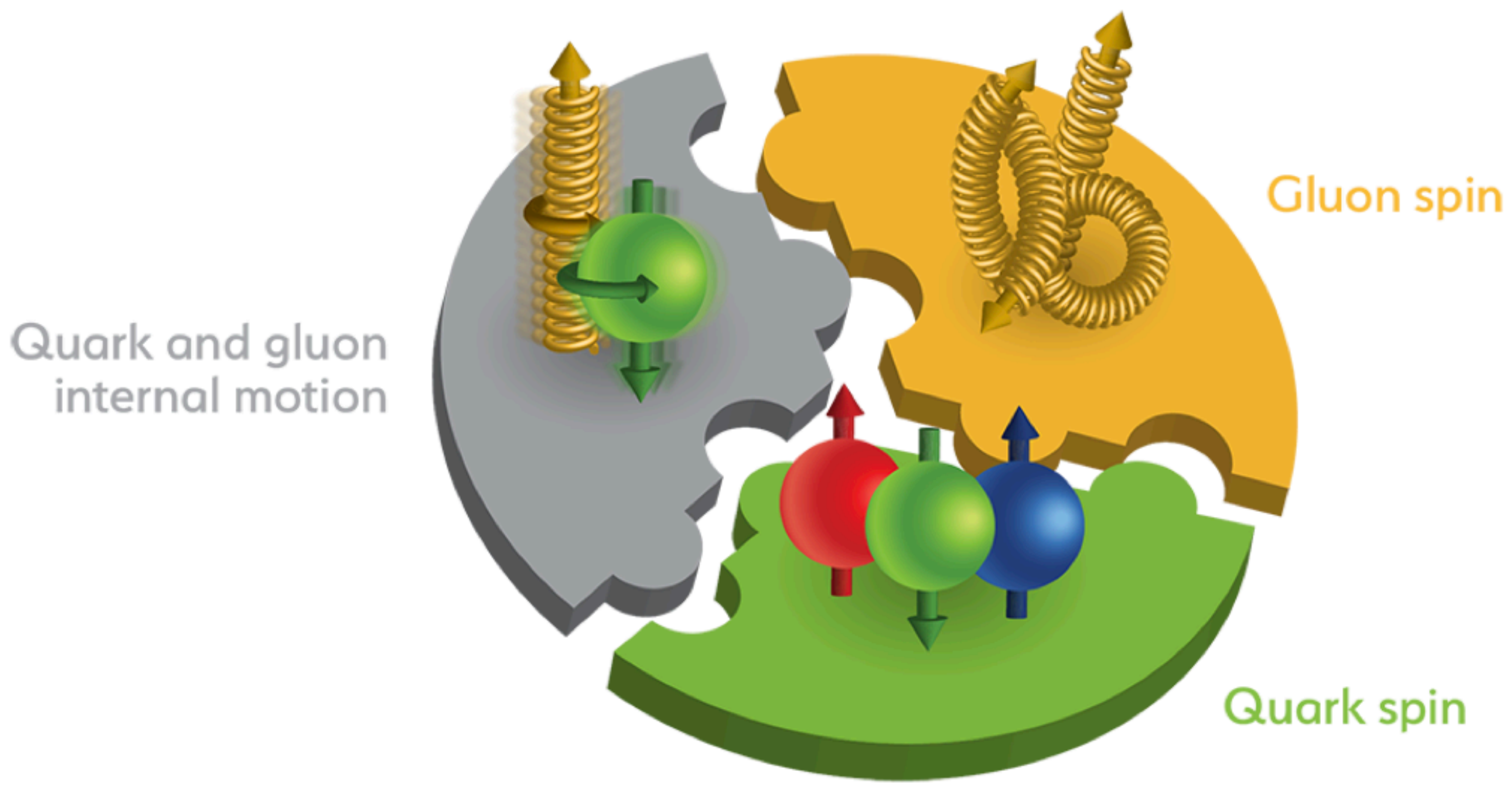
[1] K.B Chen, J.P Ma, X.B.Tong:

JHEP 08 (2024) 227, 2406.08559
JHEP 05 (2024) 298, 2402.15112
PRD 108 (2023) 9, 9, 2308.11251

[2] H. Mantysaari, Y. Tawabutr, X.Tong, work in preparation

Nucleon tomography



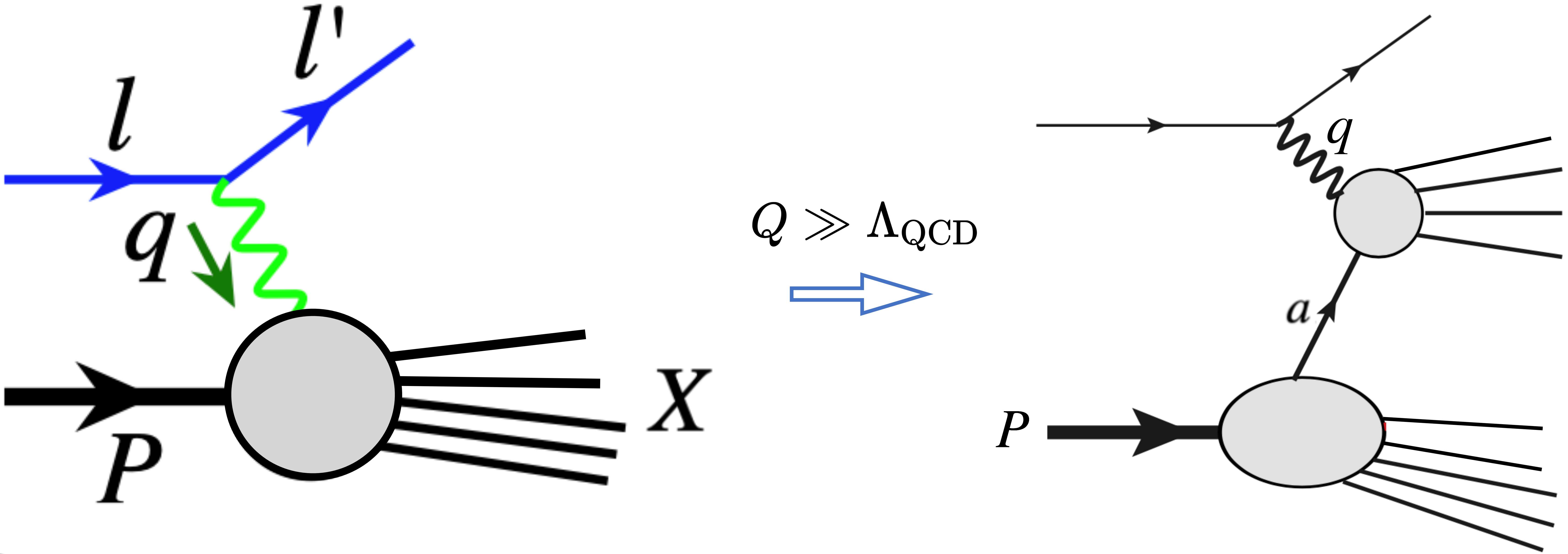


Questions:

- How is the momentum/spin of a nucleon distributed among quarks and gluons?
- Are there correlations between their momentum and spin orientations?

⇨ One of the main goals in HERA, JLab, Compass, EIC, EicC…

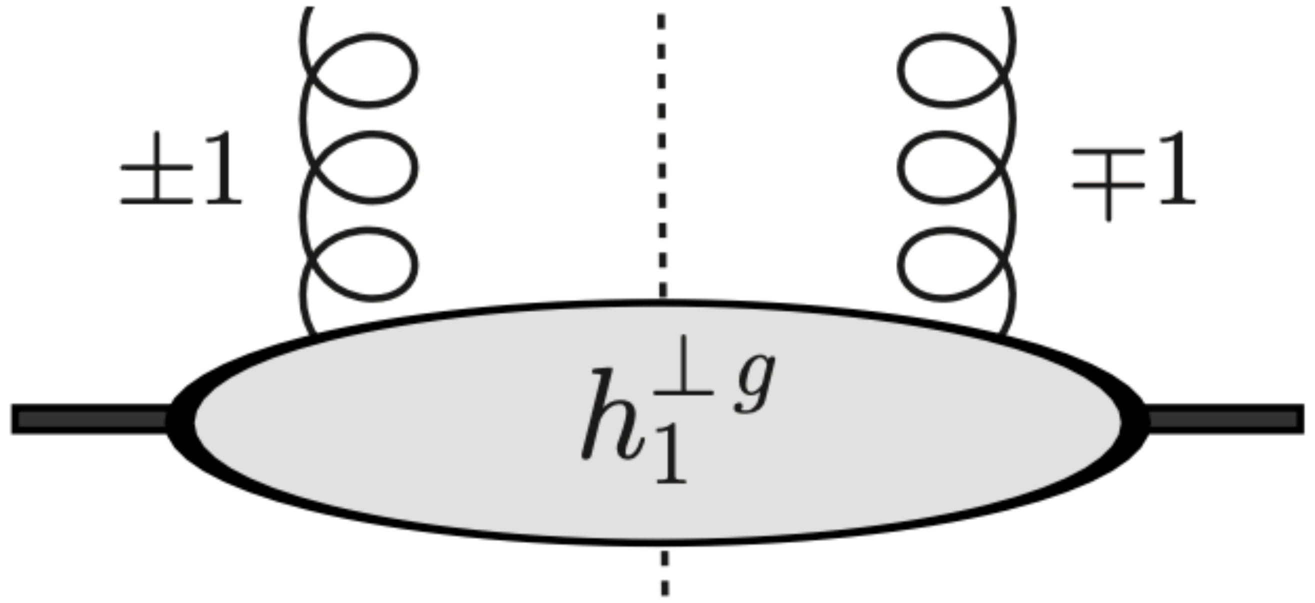
Inclusive Deep-Inelastic Scattering



- Inclusive DIS:
  - Dominated by the hard scattering on a collinear parton
    - Soft gluons cancel in the inclusive sum
  - Simple collinear factorization structure : $\sigma \propto H(Q) \otimes f_{a/P}(x, \mu^2)$
  - However, too inclusive, lose information:
    - Fragmentation
    - Transverse-momentum dependence

Vanishing correlations in inclusive DIS

- Linearly polarized gluons



- Require a transverse reference direction

e.g. the initial parton k_T [Mulders, Rodrigues, 2001]

$$\langle P|F^{+\mu}F^{+\nu}|P\rangle \propto g_\perp^{\mu\nu} f_1^g - \underbrace{\frac{1}{M^2}\left(k_\perp^\mu k_\perp^\nu + g_\perp^{\mu\nu}\frac{\boldsymbol{k}_\perp^2}{2}\right)h_1^{\perp g}}_{}$$

Cos 2ϕ correlation

Vanish after the k_T-integration

No associated collinear PDF

- Single transverse spin asymmetry

$$A_N \propto d\sigma(\vec{S}_\perp) - d\sigma(-\vec{S}_\perp)$$

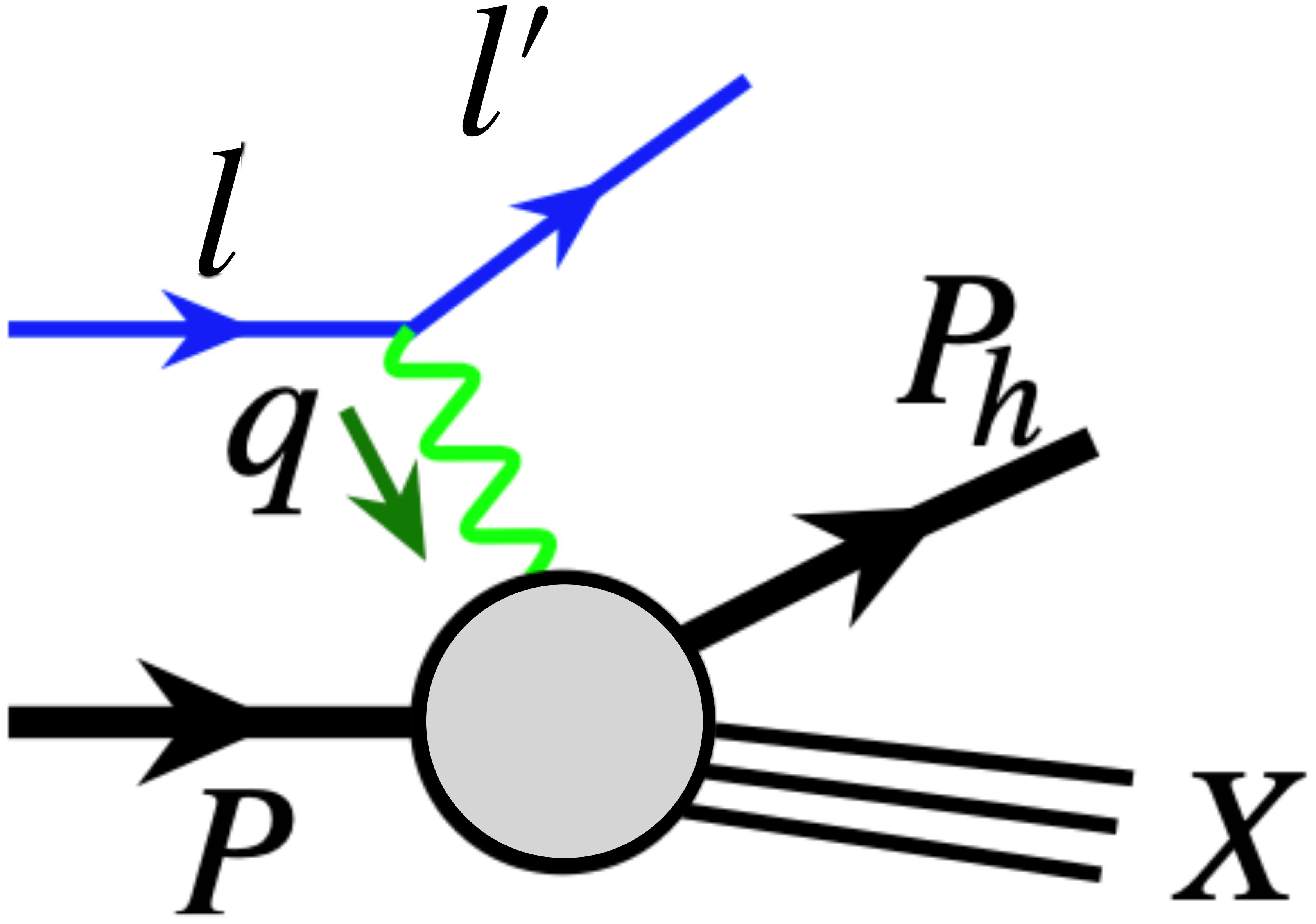
- T-odd effects
- Require final/initial-state interactions

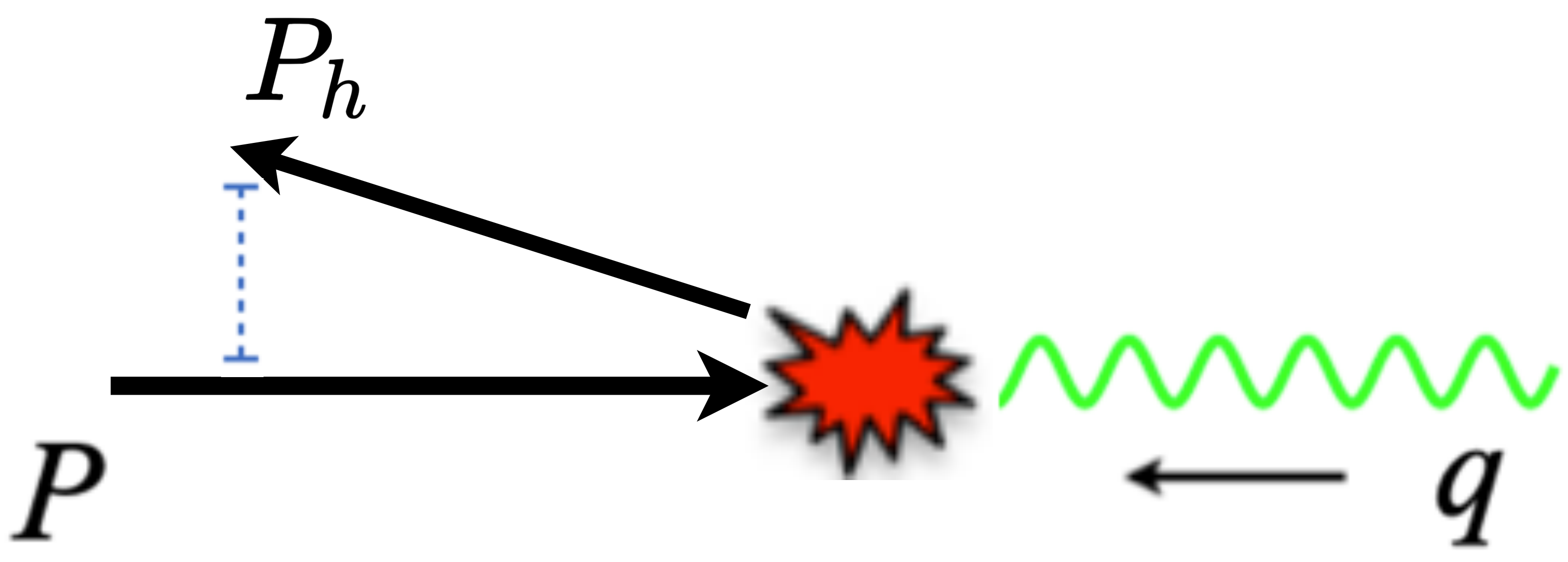
Otherwise, prohibited by time reversal invariance

- Sivers TMDs: [Sivers, 1992]

$$\langle P|\bar{\psi}\gamma^+\psi|P\rangle \propto \underbrace{\frac{k_\perp \times S_T}{M}}_{k_T\text{-odd}} f_{1T}^\perp(x, k_\perp^2)$$

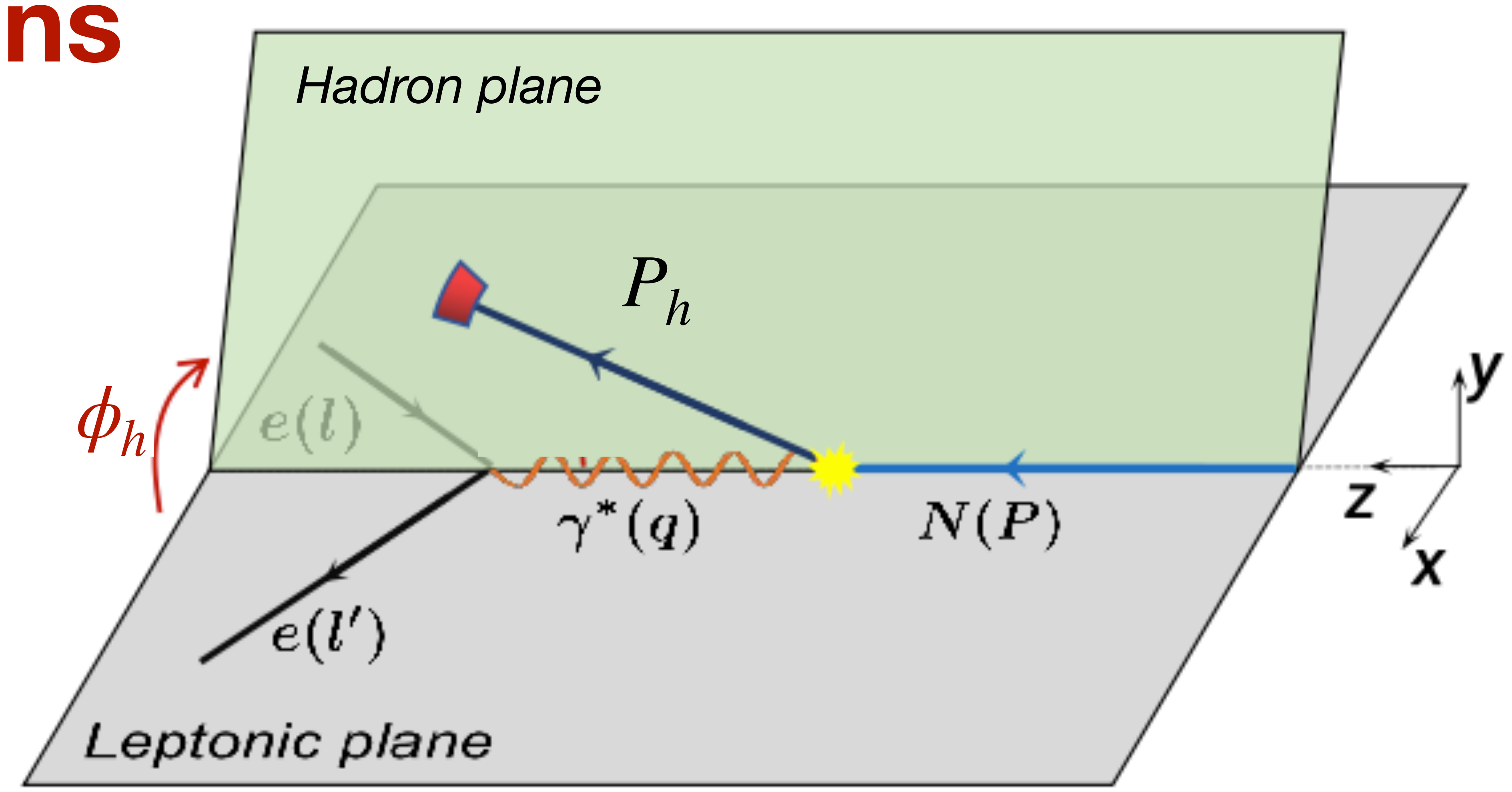
Semi-inclusive Deep-Inelastic Scattering





- SIDIS: a final-state hadron (P_h) is detected
  - Fragmentation, final-state interactions
  - A tunable transverse momentum, $\vec{P}_{h\perp}$
    - The intrinsic parton k_T
    - Azimuthal correlations with the parton/nulceon polarization
- 18 structure functions:

SIDIS structure functions



- SIDIS differential cross section   Bacchetta et al JHEP 02 (2007) 093.

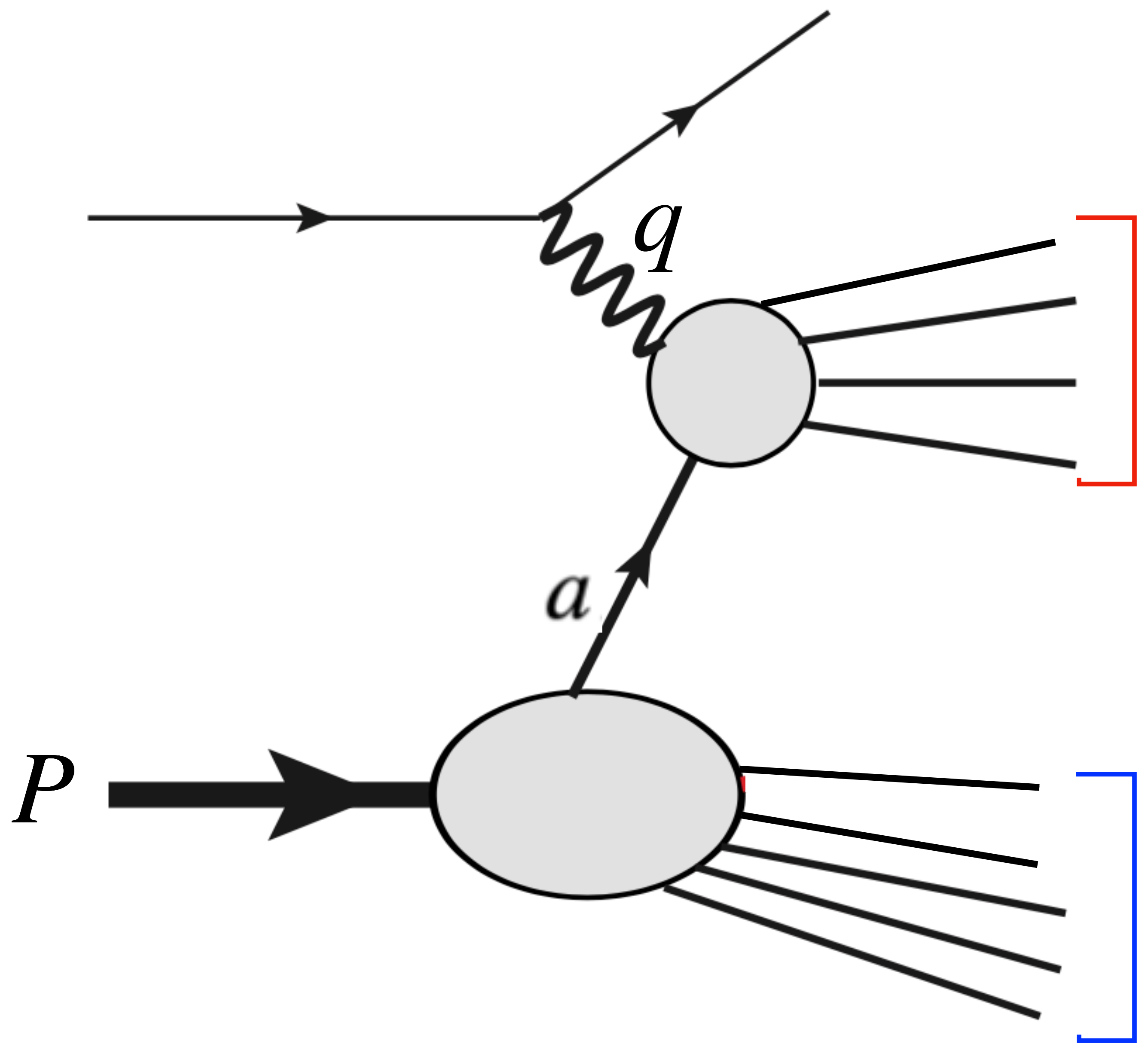
- 18 structure functions:

$$
\begin{aligned}
&\frac{d\sigma}{dx_B\,dy\,dz\,dP_{hT}^2\,d\phi_h\,d\phi_S} \\
&= \frac{\alpha^2}{x_B y Q^2}\frac{y^2}{2(1-\epsilon)}\left(1+\frac{\gamma^2}{2x_B}\right) \\
&\times\Big\{F_{UU,T}+\epsilon F_{UU,L}+\sqrt{2\epsilon(1+\epsilon)}F_{UU}^{\cos\phi_h}\cos\phi_h+\boxed{\epsilon F_{UU}^{\cos 2\phi_h}}\cos 2\phi_h+\lambda_e\sqrt{2\epsilon(1-\epsilon)}F_{LU}^{\sin\phi_h}\sin\phi_h \\
&+S_L\left[\sqrt{2\epsilon(1+\epsilon)}F_{UL}^{\sin\phi_h}\sin\phi_h+\epsilon F_{UL}^{\sin 2\phi_h}\sin 2\phi_h\right]+\lambda_e S_L\left[\sqrt{1-\epsilon^2}F_{LL}+\sqrt{2\epsilon(1-\epsilon)}F_{LL}^{\cos\phi_h}\cos\phi_h\right] \\
&+S_T\Big[\Big(\underline{F_{UT,T}^{\sin(\phi_h-\phi_S)}}+\epsilon F_{UT,L}^{\sin(\phi_h-\phi_S)}\Big)\sin(\phi_h-\phi_S)+\epsilon F_{UT}^{\sin(\phi_h+\phi_S)}\sin(\phi_h+\phi_S) \\
&+\epsilon F_{UT}^{\sin(3\phi_h-\phi_S)}\sin(3\phi_h-\phi_S)\quad+\sqrt{2\epsilon(1+\epsilon)}F_{UT}^{\sin\phi_S}\sin\phi_S+\sqrt{2\epsilon(1+\epsilon)}F_{UT}^{\sin(2\phi_h-\phi_S)}\sin(2\phi_h-\phi_S)\Big] \\
&+\lambda_e S_T\Big[\sqrt{1-\epsilon^2}F_{LT}^{\cos(\phi_h-\phi_S)}\cos(\phi_h-\phi_S) \\
&+\sqrt{2\epsilon(1-\epsilon)}F_{LT}^{\cos\phi_S}\cos\phi_S+\sqrt{2\epsilon(1-\epsilon)}F_{LT}^{\cos(2\phi_h-\phi_S)}\cos(2\phi_h-\phi_S)\Big]\Big\}
\end{aligned}
$$

Boer-Mulder partons like linearly polarized gluons

Single transverse spin, from e.g. Sivers effects,

Two regions in SIDIS

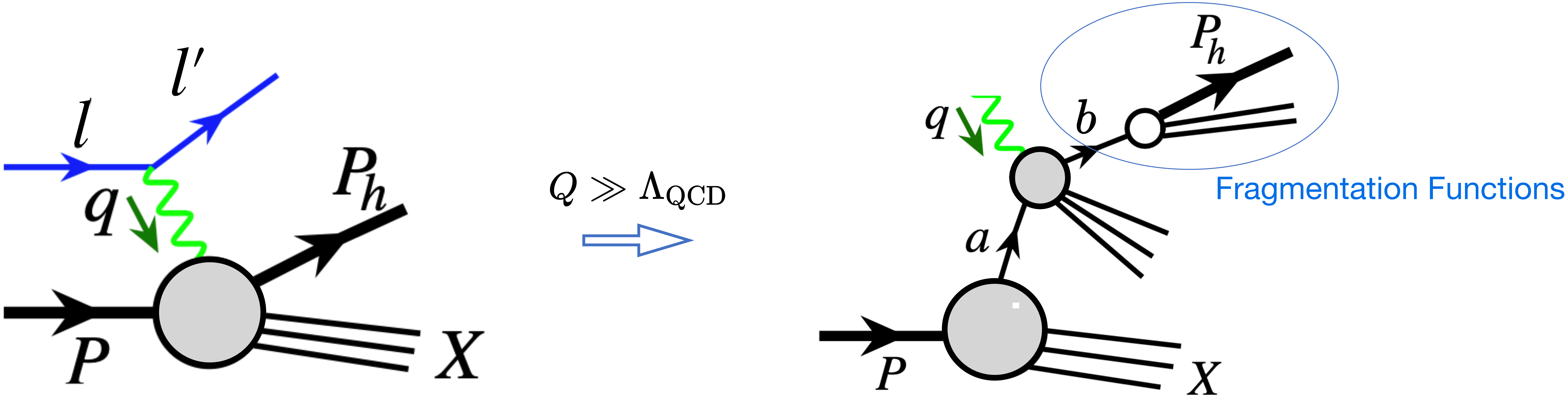


Current fragmentation

Target fragmentation

➡ Different interpretations for an azimuthal correlation

Current Fragmentation Region (CFR)



✦ Collinear factorization: $P_{h_\perp} \gg \Lambda_{\rm QCD}$

$$\sigma \propto H(Q, P_{h\perp}) \otimes f_{a/P}(x, \mu^2) \otimes D_{h/b}(z, \mu^2)$$

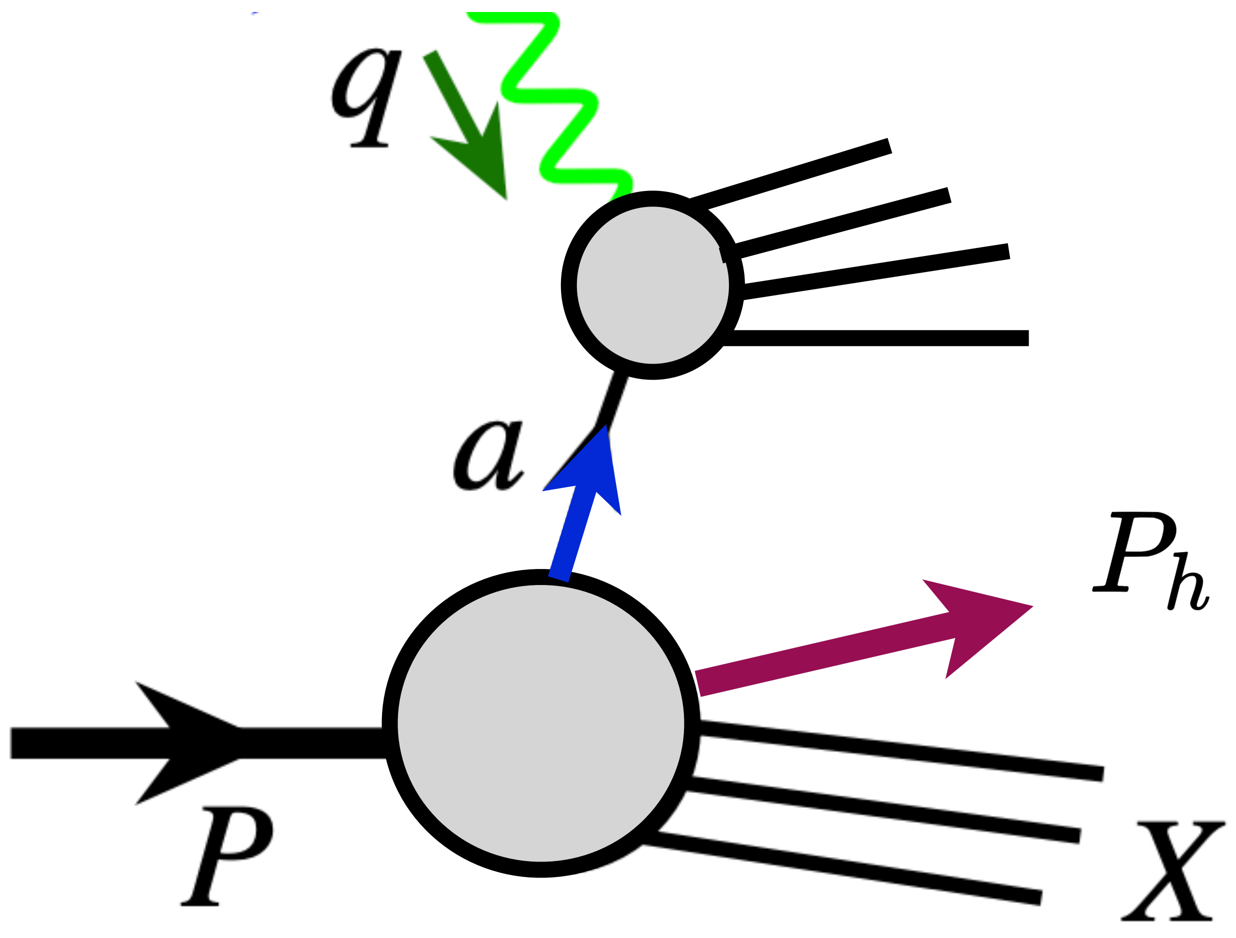
✦ TMD factorization: $P_{h\perp} \ll Q$

$$\sigma \propto H(Q) \otimes f_{a/P}(x, k_\perp, \mu^2) \otimes D_{h/b}(z, p_\perp, \mu^2)$$

- Because of k_T, there are more TMDs than collinear PDFs
  - Accommodate Sivers-effects, Boer-Mulder partons,, etc.
- However, soft-gluon radiations play an important role
  - Sudakov effects
  - May generate asymmetries contaminate the interpretations

See e.g.. Hatta-Xiao-Yuan-Zhou, PRD 104, 054037 (2021).

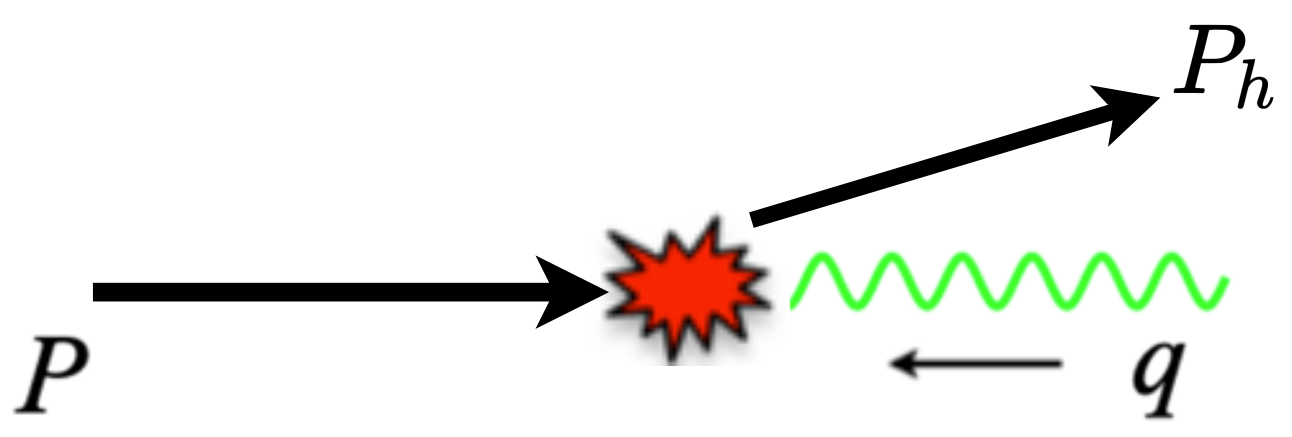
Target Fragmentation Region (TFR)



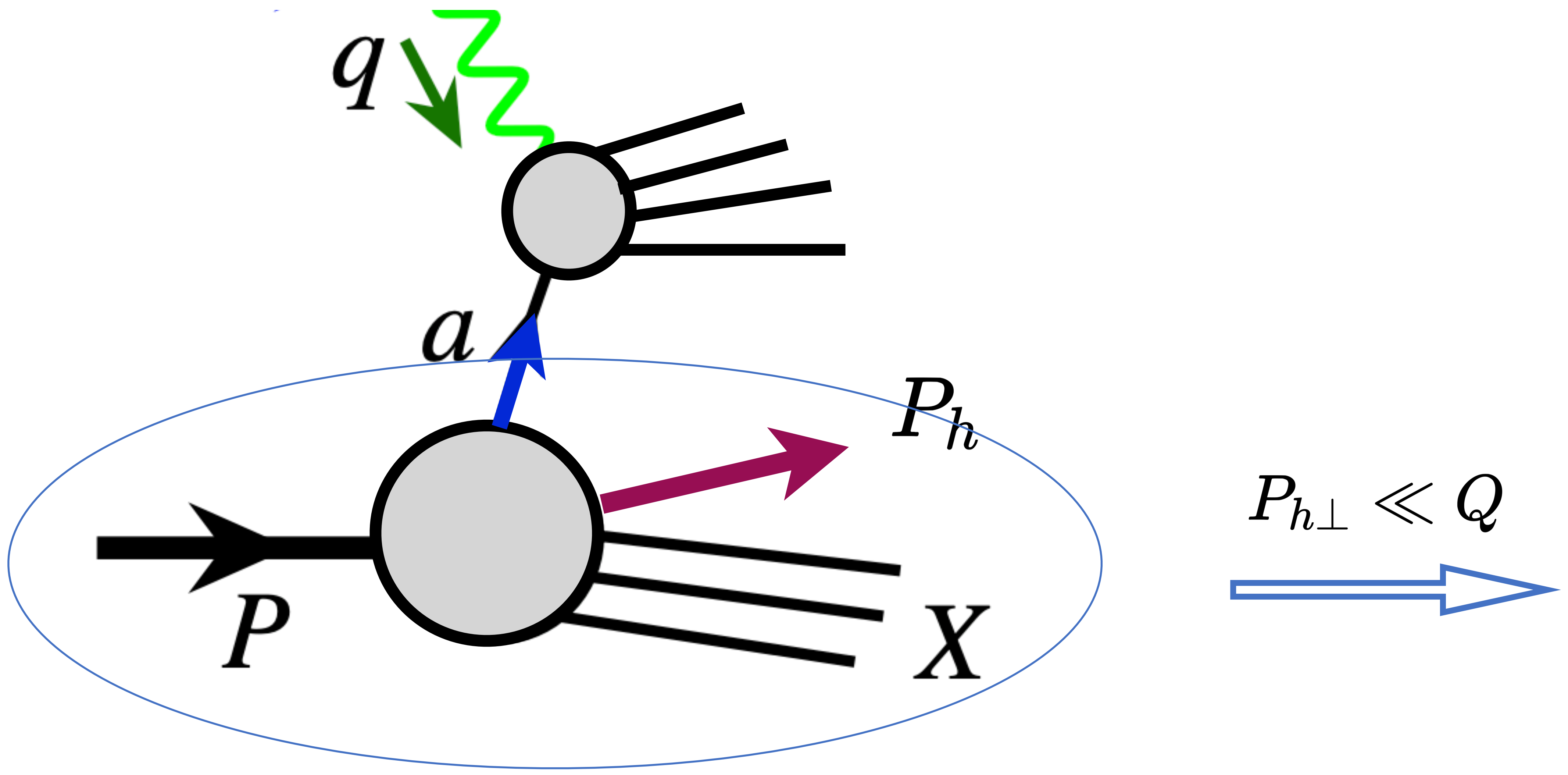
- Is there a probe:
  - ☑ Free of soft-gluon contributions, like inclusive DIS
  - ☑ Accommodate various correlation effects like TMDs

Target Fragmentation Region (TFR)

- Measured in the forward region of the incoming target



✦ Fragmented from **the target remnants**, after a parton was struck out



Strong correlations between

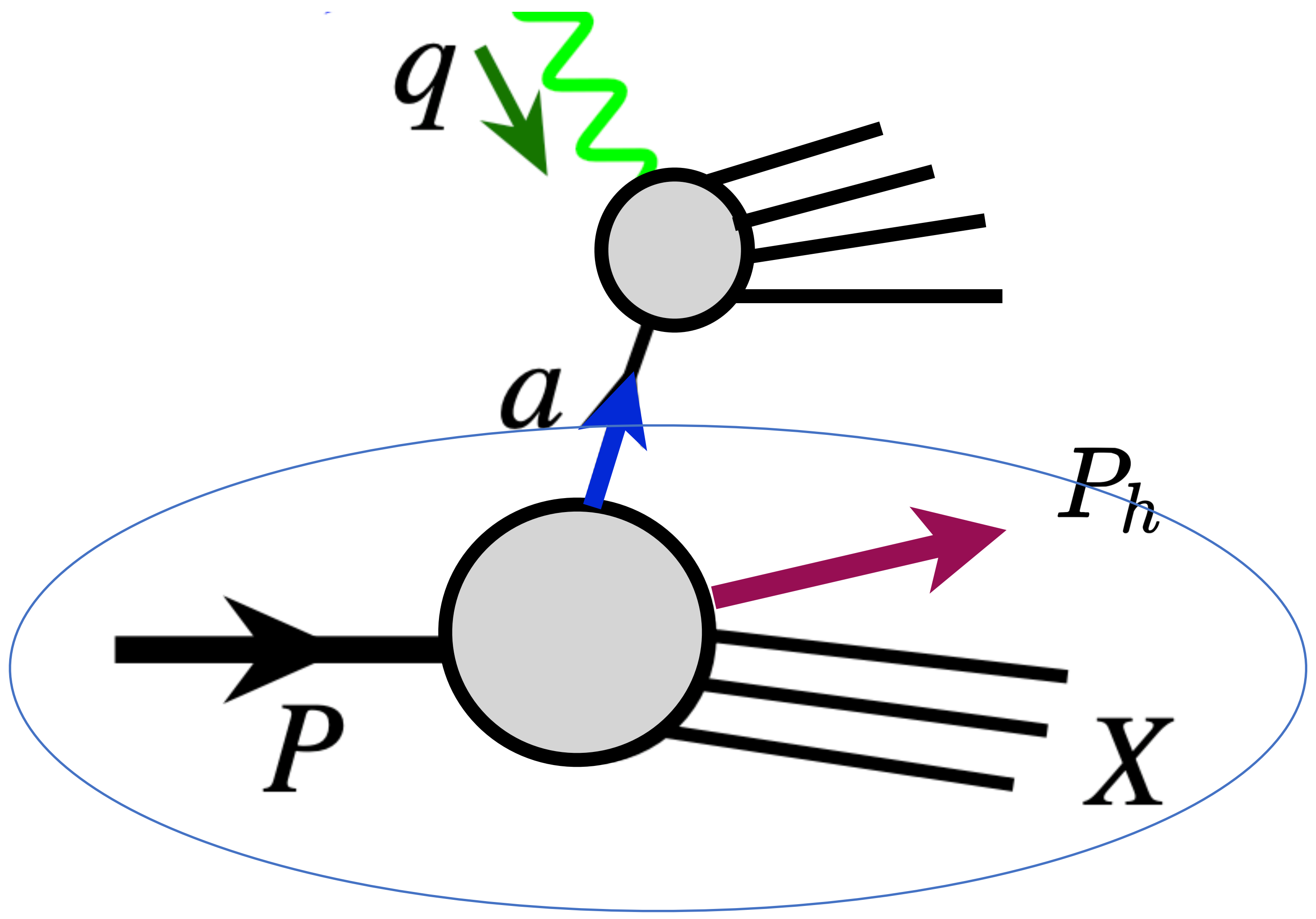
the initial patrons and the final hadron

Fracture functions

Operator definition for collinear quark:

$$\mathcal{F}_{ij}(x, \xi_h, P_{h\perp}) \qquad \xi_h = \frac{P_h^+}{P^+}$$

$$= \frac{1}{2\xi_h(2\pi)^3} \int \frac{d\lambda}{2\pi} e^{-ixP^+\lambda} \sum_X \langle PS|\bar{\psi}_j(\lambda n)\mathcal{L}_n^\dagger(\lambda n)|XP_h\rangle\langle P_h X|\mathcal{L}_n(0)\psi_i(0)|PS\rangle$$



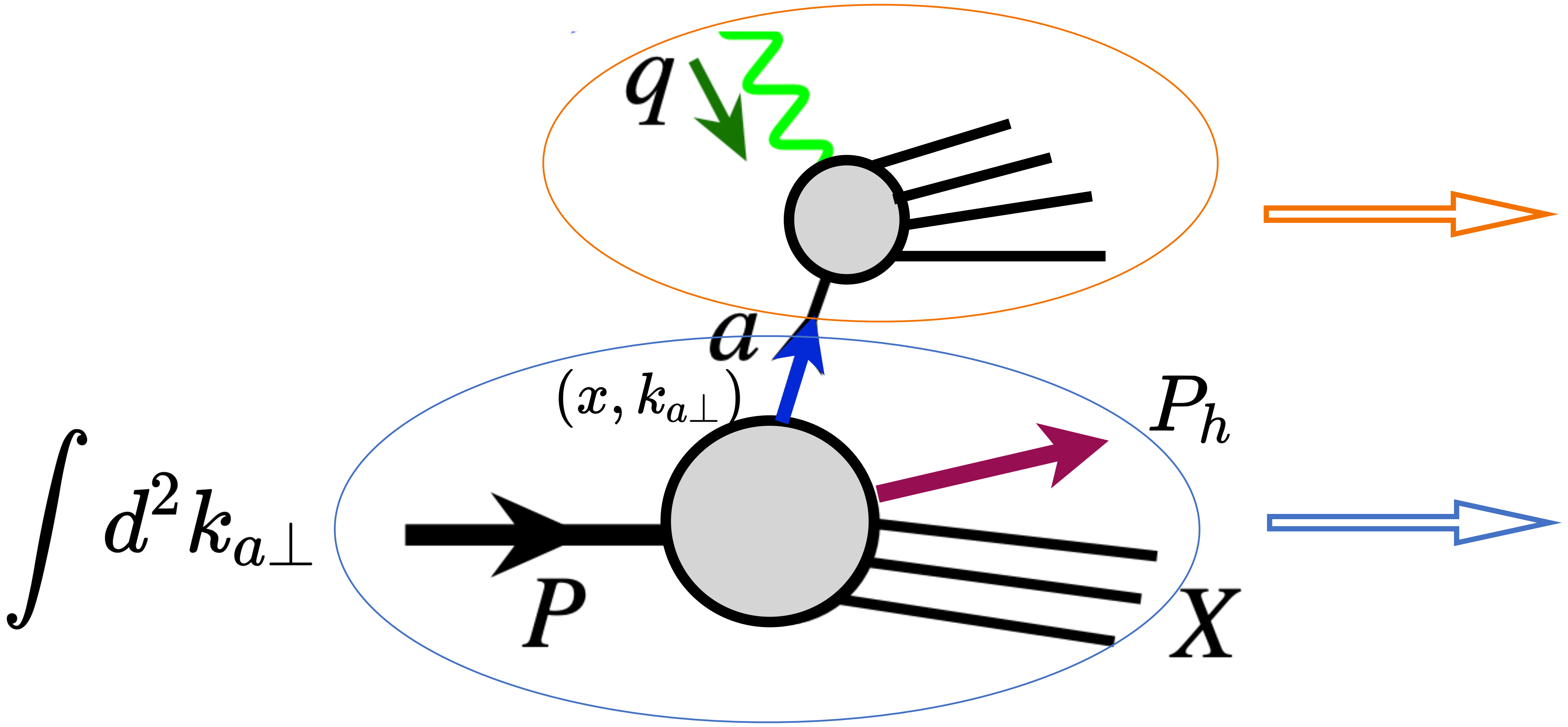
Fracture **functions**

Trentadue-Veneziano
PLB 323 (1994) 201

- Describe the partonic ***structure*** of the target once it ***fragments*** into a given hadron h.
- Conditional probability; A combination between PDF and FFs
- h=P, also called diffractive PDFs

[Berera-Soper *PRD 53 (1996) 6162*]

SIDIS Factorization in the TFR



Same as inclusive DIS ($k_{a\perp} \ll Q$, thus neglected)

$k_{a\perp}$-integrated fracture functions

- **Collinear factorization:** $\sigma \propto \underline{H(Q)} \otimes \mathcal{F}_a(x, \xi_h, P_{h\perp}, \underline{\mu})$ *Collins PRD 57 (1998) 3051*

Soft gluons cancel

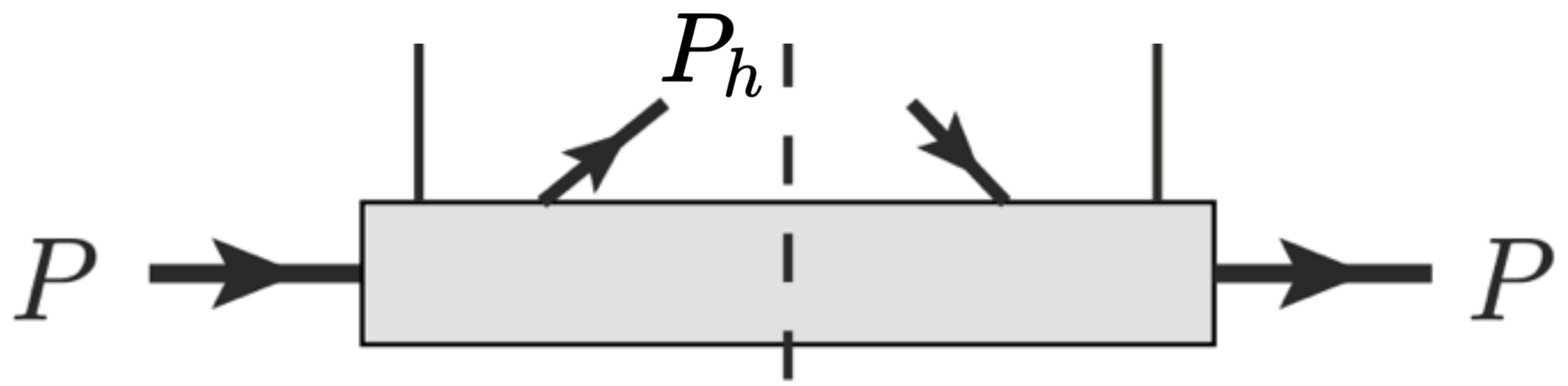
DGLAP evolutions

✦ Collinear fracture functions: probe the nucleon structure by the correlations between the initial state and final state

➡ **Azimuthal correlations** between $P_{h\perp}$ and **the polarizations of partons/nucleon**

Quark contributions

Anselmino-Barone-Kotzinian, PLB 699 (2011) 108



Quark polarization

Nucleon polarization

	U	L	T
U	u_1		t_1^h
L		l_{1L}	t_{1L}^h
T	u_{1T}^h	l_{1T}^h	t_1, t_{1T}^h

Unpolarized

Quark helicity

Sivers-like

$P_{h\perp} \times S_T$

T-odd

Worm-gear-like

$P_{h\perp} \cdot S_T$

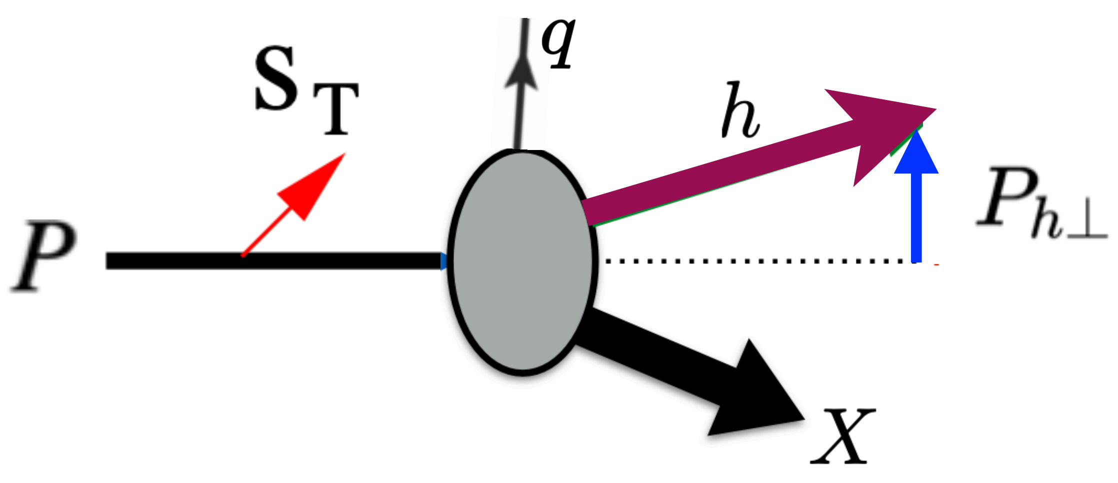
T-even

$$\mathcal{F}_{ij} \propto \sum_X \langle PS|\bar{\psi}_j(\lambda n)\mathcal{L}_n^\dagger(\lambda n)|XP_h\rangle\langle P_h X|\mathcal{L}_n(0)\psi_i(0)|PS\rangle$$

- The decomposition is analogous to TMDs, e.g.,

$$\mathcal{F}^{[\gamma^+]} = u_1 + \frac{P_{h\perp} \times S_T}{M} u_{1T}^h$$
$$\mathcal{F}^{[\gamma^+\gamma_5]} = S_L l_{1L} + \frac{P_{h\perp} \cdot S_T}{M} l_{1T}^h$$

- Eight quark fracture functions at twist 2



➡ **Azimuthal correlations** between $P_{h\perp}$ and **the nucleon transverse spin**

Quark contributions

Anselmino-Barone-Kotzinian, PLB 699 (2011) 108

Quark polarization

Nucleon polarization	**U**	**L**	**T**
U	u_1		$\underline{t_1^h}$
L		l_{1L}	t_{1L}^h
T	$\underline{u_{1T}^h}$	$\underline{l_{1T}^h}$	t_1, t_{1T}^h

Unpolarized

Boer-Mulder-like

Quark helicity

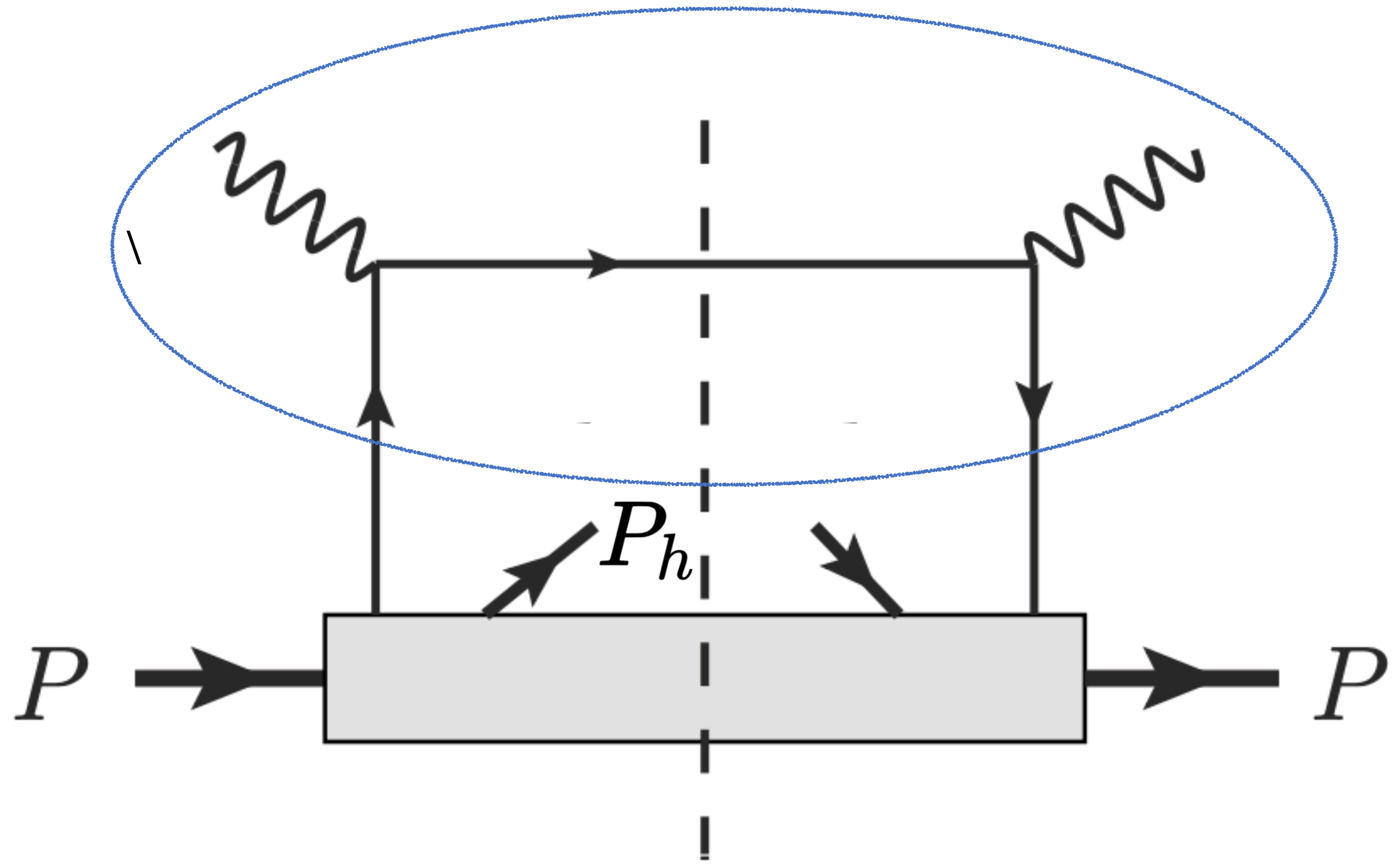
Sivers-like

Worm-gear-like

Chiral odd,

Do not contribute!

- Quark scattering is chiral even



- Four structure functions

$$F_{UU,T} = x_B u_1, \quad F_{UT,T}^{\sin(\phi_h-\phi_S)} = \frac{|\vec{P}_{h\perp}|}{M} x_B u_{1T}^h,$$

$$F_{LL} = x_B l_{1L}, \quad F_{LT}^{\cos(\phi_h-\phi_S)} = \frac{|\vec{P}_{h\perp}|}{M} x_B l_{1T}^h.$$

Gluonic contributions

Chen-Ma-Tong, JHEP 05 (2024) 298

$$\mathcal{M}^{\alpha\beta} \propto \sum_X \langle PS|(G^{+\alpha}(\lambda n)\mathcal{L}_n^\dagger(\lambda n))^a\,|XP_h\rangle\,\langle P_h X|\big(\mathcal{L}_n(0)G^{+\beta}(0)\big)^a\,|PS\rangle$$

Gluon polarization

Nucleon polarization

	U	**Circularly polarized**	**Linearly polarized**
U	u_{1g} Unpolarized		t_{1g}^h
L		l_{1gL} Gluon helicity	t_{1gL}^h
T	u_{1gT}^h Sivers	l_{1gT}^h Worm-gear	t_{1gT}, t_{1gT}^h

yield non-zero contributions

Do not mix with quark!

- Eight gluonic fracture functions at twist 2

- $P_{h\perp}$ define the transverse reference direction

$$\mathcal{M}_U^{\alpha\beta} = -\frac{1}{2}g_\perp^{\alpha\beta}u_{1g} + \frac{1}{2M^2}\underline{\left(P_{h\perp}^\alpha P_{h\perp}^\beta + \frac{1}{2}g_\perp^{\alpha\beta}P_{h\perp}^2\right)}t_{1g}^h$$

Similar tensor structures to the TMD

$$\mathcal{M}_L^{\alpha\beta} = S_L\left(i\frac{\epsilon_\perp^{\alpha\beta}}{2}l_{1gL} + \frac{\tilde{P}_{h\perp}^{\{\alpha}P_{h\perp}^{\beta\}}}{4M^2}\underline{t_{1gL}^h}\right)$$

$$\mathcal{M}_T^{\alpha\beta} = \frac{g_\perp^{\alpha\beta}}{2}\frac{P_{h\perp}\cdot\tilde{S}_\perp}{M}u_{1gT}^h + \frac{P_{h\perp}\cdot S_\perp}{M}i\frac{\epsilon_\perp^{\alpha\beta}}{2}l_{1gT}^h - \frac{P_{h\perp}\cdot S_\perp}{M}\frac{\tilde{P}_{h\perp}^{\{\alpha}P_{h\perp}^{\beta\}}}{4M^2}\underline{t_{1gT}^h} + \frac{\tilde{P}_{h\perp}^{\{\alpha}S_\perp^{\beta\}} + \tilde{S}_\perp^{\{\alpha}P_{h\perp}^{\beta\}}}{8M}\underline{t_{1gT}}$$

➡ **Azimuthal correlations** between $P_{h\perp}$ and **the gluon spin**

Gluonic contributions

Chen-Ma-Tong, *JHEP* 05 (2024) 298

$$\mathcal{M}^{\alpha\beta} \propto \sum_X \langle PS| (G^{+\alpha}(\lambda n)\mathcal{L}_n^{\dagger}(\lambda n))^a \,|X P_h\rangle\, \langle P_h X| \big(\mathcal{L}_n(0) G^{+\beta}(0)\big)^a \,|PS\rangle$$

Gluon polarization

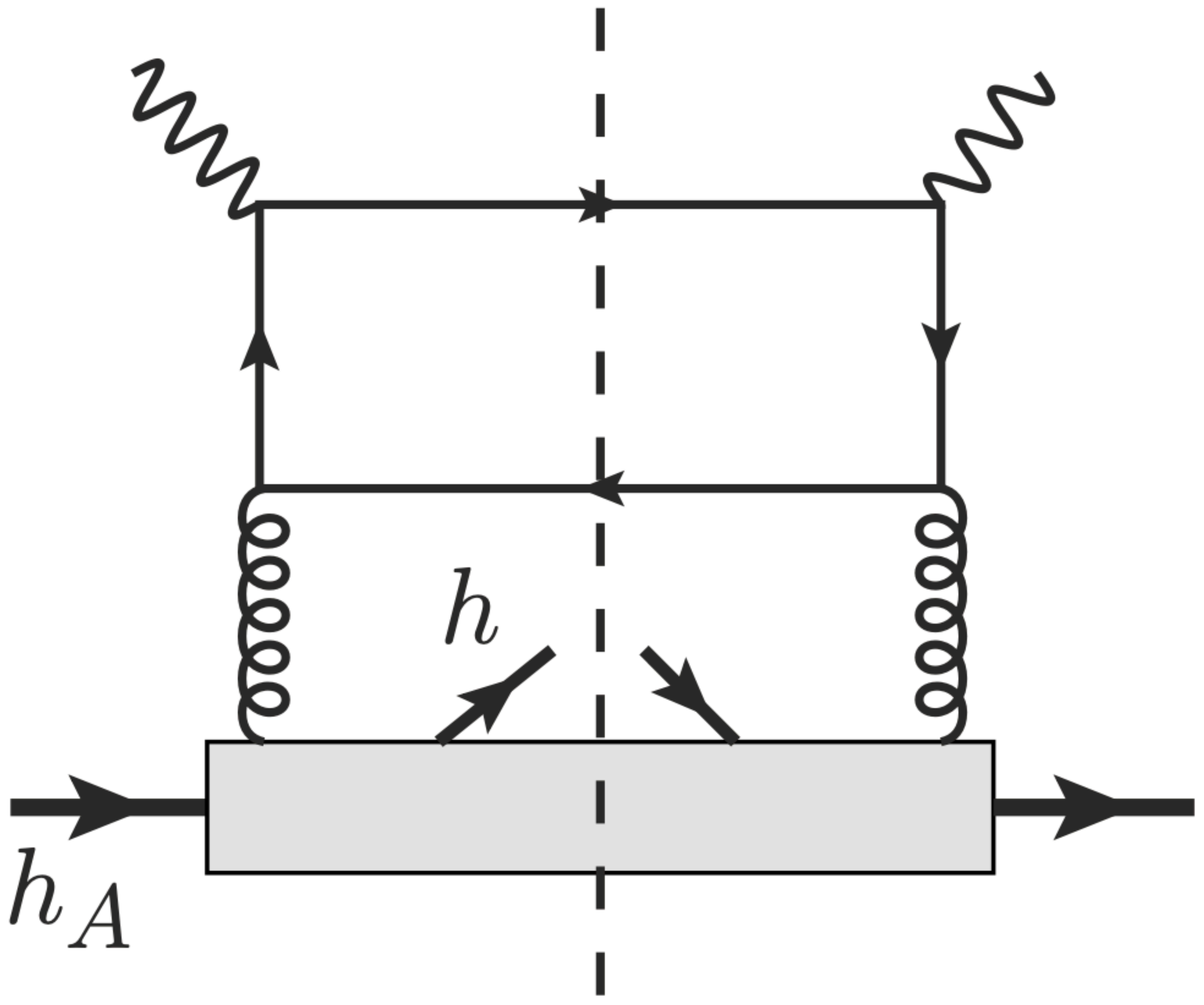
Nucleon polarization

	U	**Circularly polarized**	**Linearly polarized**
U	u_{1g} Unpolarized		t_{1g}^h
L		l_{1gL} Gluon helicity	t_{1gL}^h
T	u_{1gT}^h Sivers	l_{1gT}^h Worm-gear	t_{1gT}, t_{1gT}^h

yield non-zero contributions

Do not mix with quark!

- Start from one loop



Novel contributions from linearly polarized gluons

- Four azimuthal correlations uniquely generated by gluons:

$$F_{UU}^{\cos 2\phi_h} = -\frac{\alpha_s T_F}{2\pi} x_B \sum_{q,\bar{q}} e_q^2 \int_{x_B}^{1-\xi_h} \frac{dz}{z}\left(\frac{x_B}{z}\right)^2 \frac{P_{h\perp}^2}{2M^2} t_{1g}^h\left(z, \xi_h, P_{h\perp}\right),$$

T-even, linear polarized gluons in an unpolarized target

$$F_{UL}^{\sin 2\phi_h} = \frac{\alpha_s T_F}{2\pi} x_B \sum_{q,\bar{q}} e_q^2 \int_{x_B}^{1-\xi_h} \frac{dz}{z}\left(\frac{x_B}{z}\right)^2 \frac{P_{h\perp}^2}{2M^2} \underline{t_{1gL}^h}\left(z, \xi_h, P_{h\perp}\right),$$

$$F_{UT}^{\sin(3\phi_h-\phi_S)} = \frac{\alpha_s T_F}{2\pi} x_B \sum_{q,\bar{q}} e_q^2 \int_{x_B}^{1-\xi_h} \frac{dz}{z}\left(\frac{x_B}{z}\right)^2 \frac{P_{h\perp}^3}{4M^3} \underline{t_{1gT}^h}\left(z, \xi_h, P_{h\perp}\right),$$

T-odd, single spin asymmetry

$$F_{UT}^{\sin(\phi_h+\phi_S)} = \frac{\alpha_s T_F}{2\pi} x_B \sum_{q,\bar{q}} e_q^2 \int_{x_B}^{1-\xi_h} \frac{dz}{z}\left(\frac{x_B}{z}\right)^2 \frac{P_{h\perp}}{2M}\left[\underline{t_{1gT}}\left(z, \xi_h, P_{h\perp}\right) + \frac{P_{h\perp}^2}{2M^2} t_{1gT}^h\left(z, \xi_h, P_{h\perp}\right)\right]$$

- Free of quark contributions to all loops

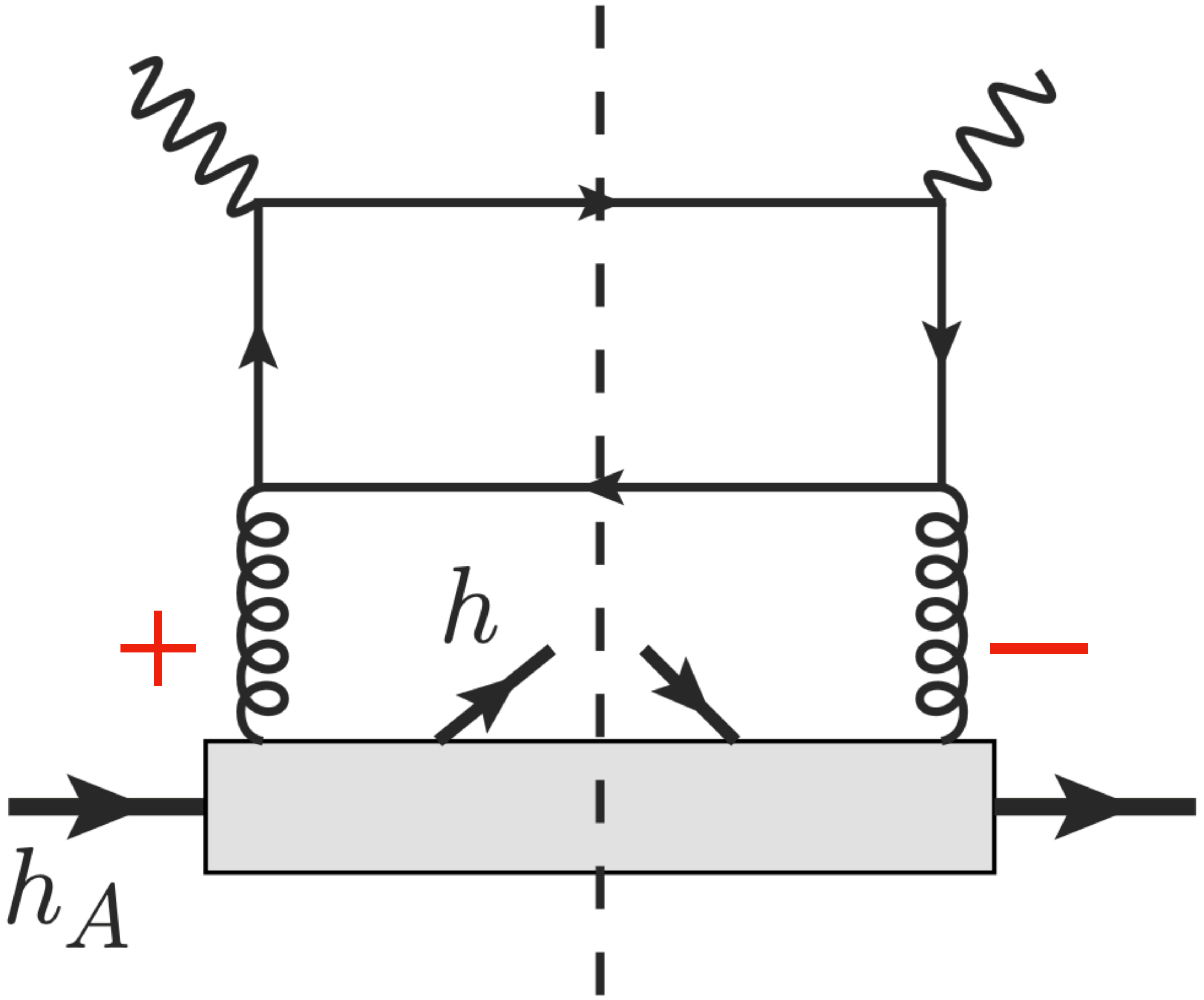
➡ Provide unique probes into gluon dynamics

Gluon polarization

Nucleon polarization

	U	L	T
U	u_{1g}		t_{1g}^h
L		l_{1gL}	t_{1gL}^h
T	u_{1gT}^h	l_{1gT}^h	t_{1gT}, t_{1gT}^h

Hadrons in TFR: a polarizing filter for gluons



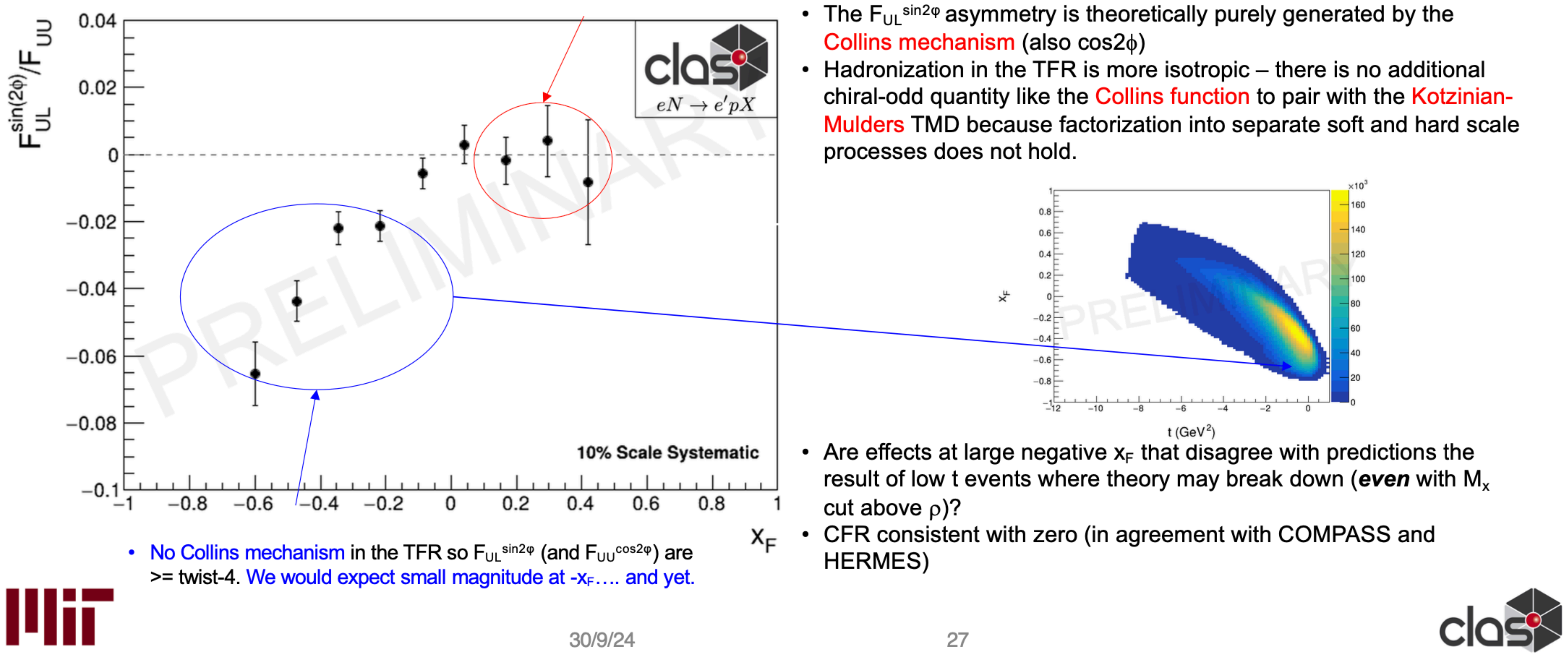
- The TFR hadron dose not directly participate in the hard scattering
- However, it can serves as a polarizing filter to select the initial gluons

See also the applications in explaining the near-side ridge in pp collisions in, Guo-Liu-Yuan, *2408.14693*

A signal of the Gluonic mechanism

- A significant $F_{UL}^{\sin(2\phi_h)}/F_{UU}$ asymmetry is observed at CLAS12

  ➡ see a slide form Timothy B. Hayward :



- No Collins mechanism in the TFR so $F_{UL}^{\sin 2\varphi}$ (and $F_{UU}^{\cos 2\varphi}$) are >= twist-4. We would expect small magnitude at $-x_F$…. and yet.

- The $F_{UL}^{\sin 2\varphi}$ asymmetry is theoretically purely generated by the Collins mechanism (also cos2ϕ)
- Hadronization in the TFR is more isotropic – there is no additional chiral-odd quantity like the Collins function to pair with the Kotzinian-Mulders TMD because factorization into separate soft and hard scale processes does not hold.

- Are effects at large negative x_F that disagree with predictions the result of low t events where theory may break down (***even*** with M_x cut above ρ)?
- CFR consistent with zero (in agreement with COMPASS and HERMES)

30/9/24 27

Nucleon energy correlator

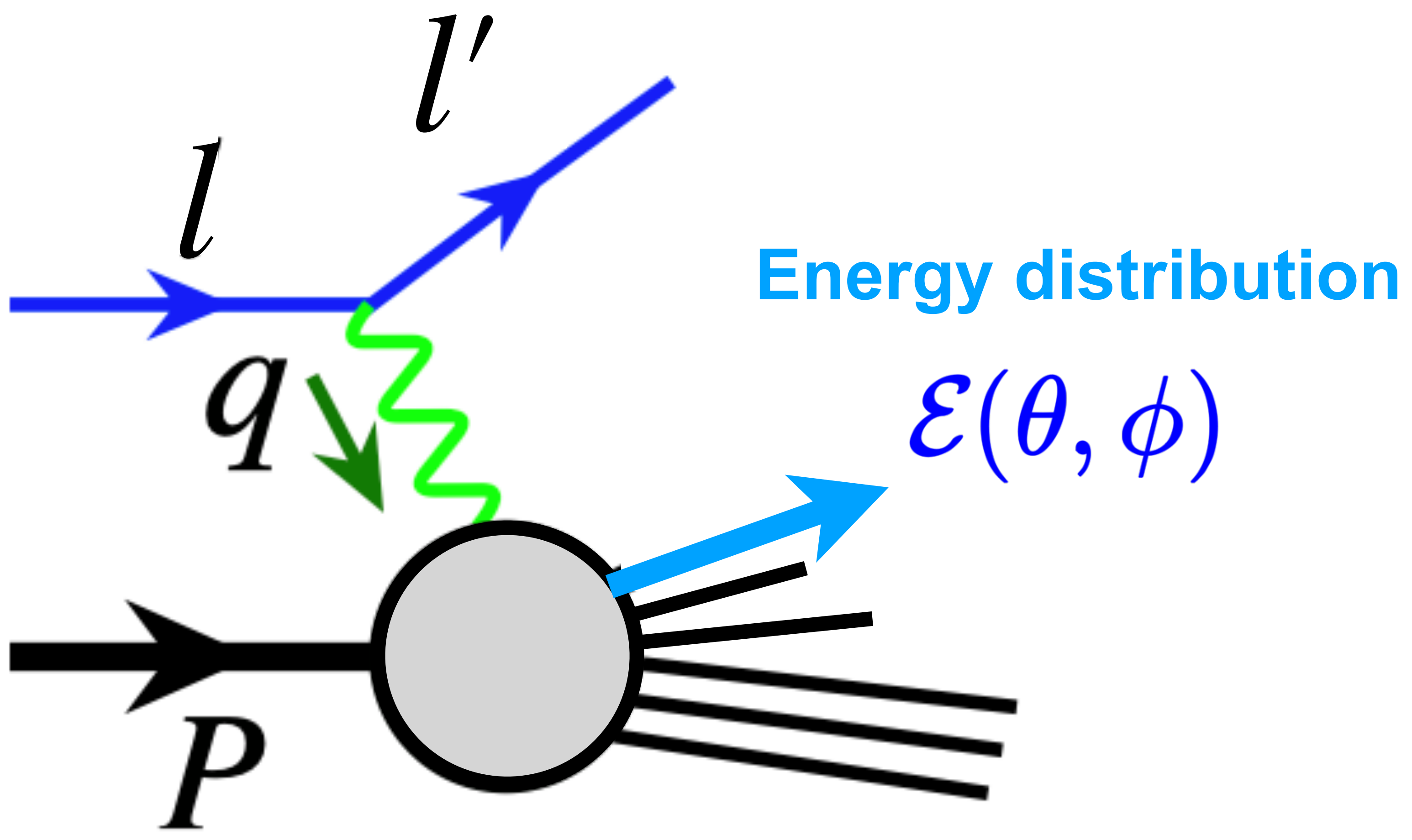
[Liu-Zhu PRL 130, 2023]

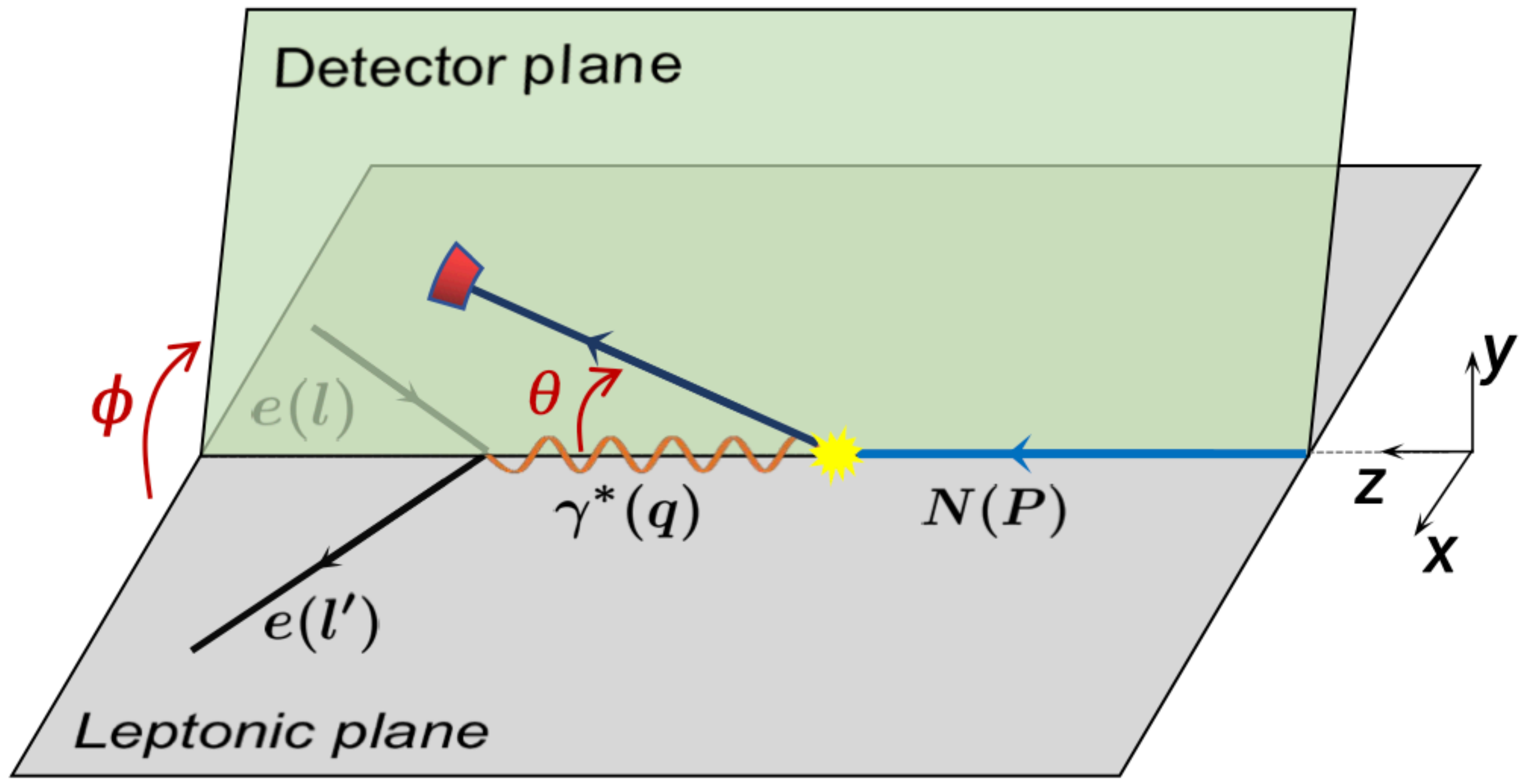
A function to describe the **DIS energy pattern** in the **target fragmentation region**

- Describe the same parton physics as fracture functions?
- But was studied completely independent of fracture functions

➡ What is the connection?

Energy pattern in the DIS





$$\Sigma(\theta,\phi)_{\mathrm{DIS}} = \sum_{\underline{h}} \int d\sigma^{e+N\to e+h+X} \frac{E_h}{\underline{E_N}} \delta\left(\theta^2 - \theta_h^2\right)\delta(\phi - \phi_h)$$

Reduce collinear singularities

Reduce soft singularities

Meng-Olness-Soper NPB 371 (1992) 79
Li-Makris-Vitev PRD 103 (2021) 094005
Kang-Lee-Shao-Fan JHEP 03 (2024) 153
Liu-Zhu，PRL130 (2023) 091901
Chen-Ma-Tong，JHEP 08 (2024) 227
…..

- An extension of the energy pattern in e^+e^- annihilation:

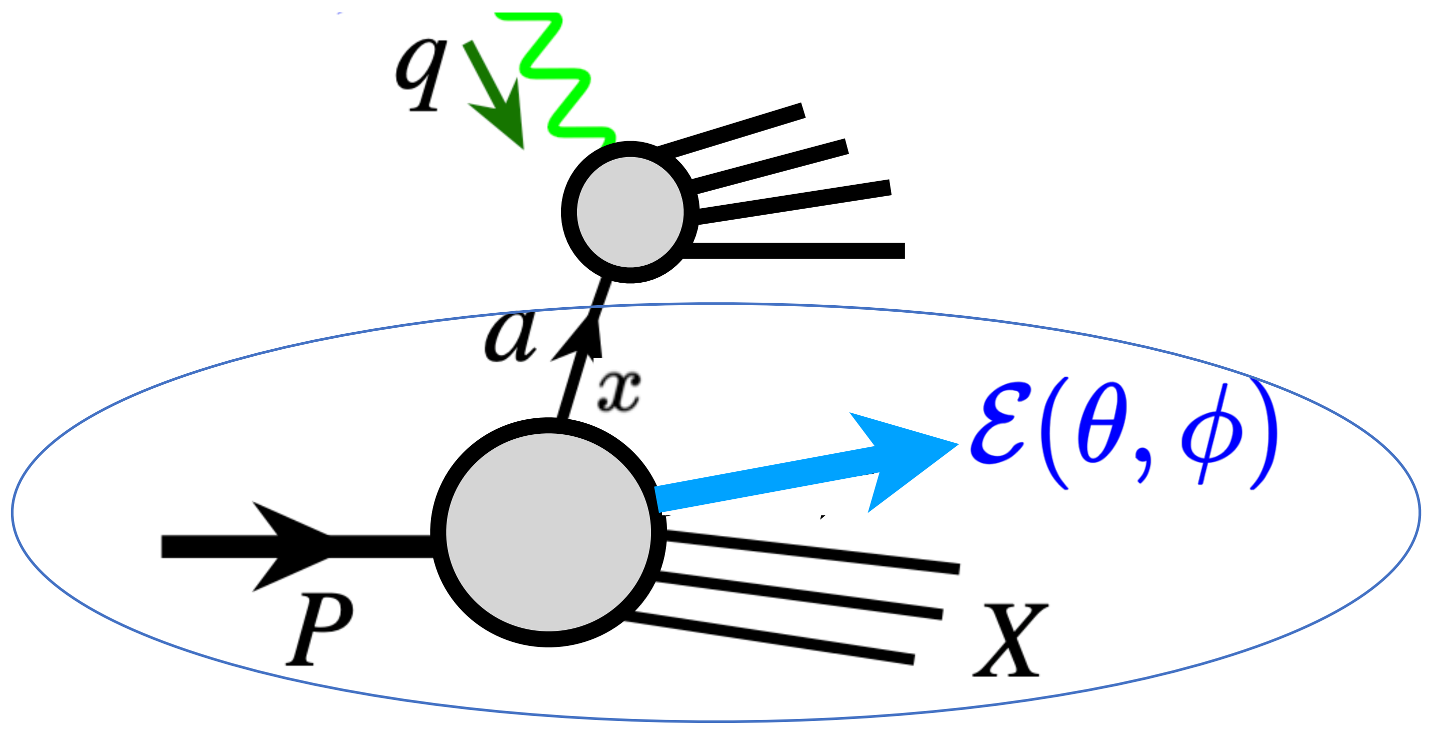
  Basham-Brown-Ellis-Love, PRD 17 (1978) 2298.

  - Cleaner probe ; Minimize the non-perturbative effects in the final state
  - Does not require event-by-event analysis, unlike jet algorithms
- 18 structure functions, one-to-one correspond to SIDIS

The connection

Nucleon energy correlator (NEC)

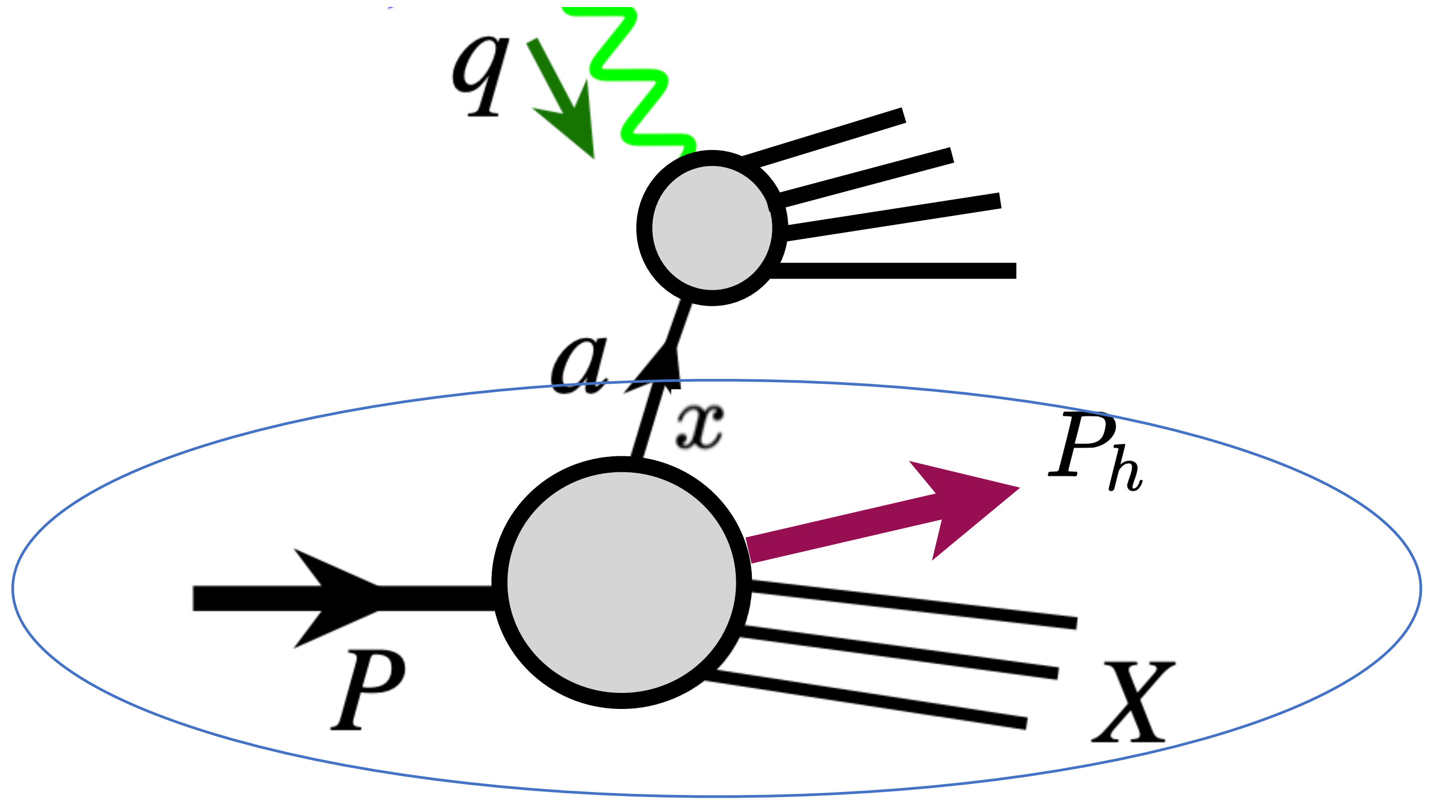
[Liu-Zhu PRL 130, 2023]



➡ The probability of finding a parton while observing an energy flow from target remnants

Fracture functions

[Trentadue-Veneziano *PLB 323 (1994) 201]*



➡ The probability of finding a parton while observing a specific hadron h from target remnants

★ An energy sum rule: [Chen-Ma-Tong, *JHEP* 08 (2024) 227]

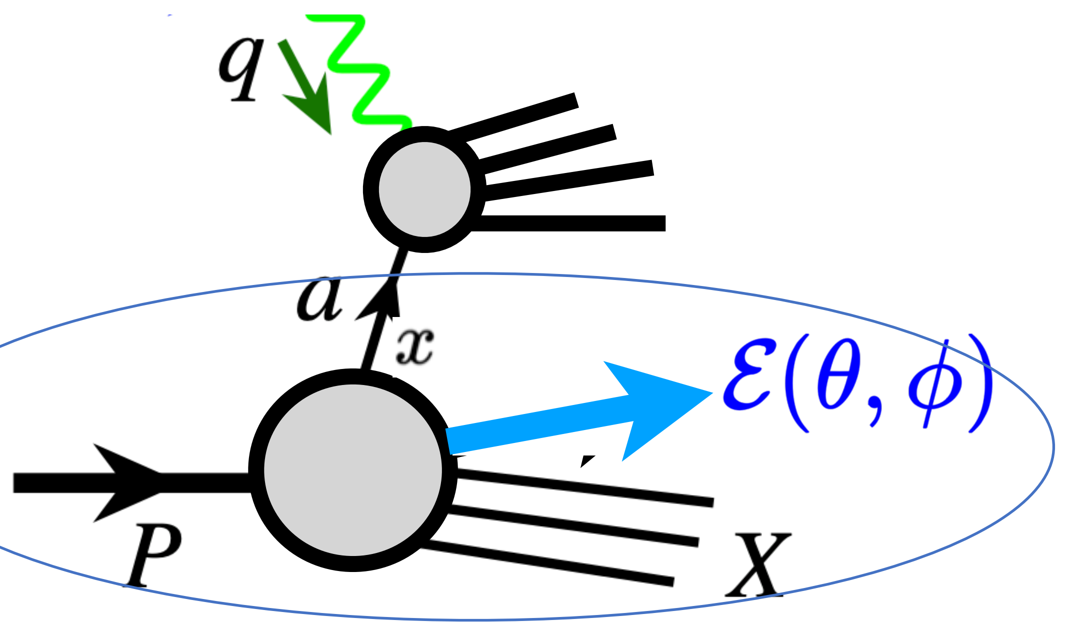
$$\mathcal{M}_{\rm NEC}(x,\theta,\phi) = \sum_h \int_0^{1-x} \xi_h d\xi_h \int d^2\boldsymbol{P}_{h\perp}\delta\left(\theta^2-\theta_h^2\right)\delta(\phi-\phi_h)\mathcal{M}_{\rm FrF}(x,\xi_h,\boldsymbol{P}_{h_\perp})$$

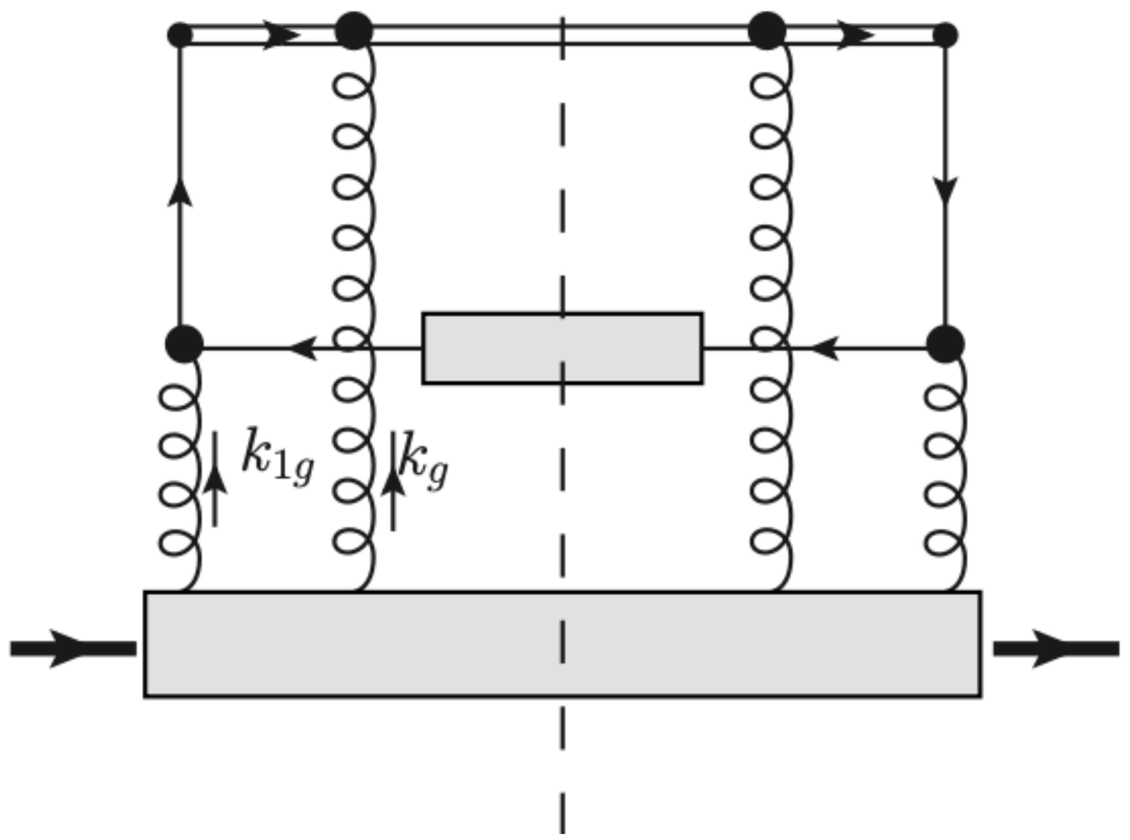
- One-to-one correspondence; Same evolutions
- Fracture functions are the parent functions of nucleon energy correlator
- Applications
  - ☑ Derive the behaviors of nucleon energy correlators
  - ☑ Derive the factorization formula of the energy pattern from SIDIS

Nucleon energy corrolators for Odderons

—the energy pattern from a subset of hadrons

Mantysaari-Tawabutr-Tong: in preparation





✦ Single transverse spin asymmetry: $\Sigma_{UT}^{\sin(\phi-\phi_S)} = \sum_{a=q,\bar{q}} e_a^2 x_B f_T^{t,a}(x_B,\theta)$

quark Sivers NEC at small x:

$$f_T^{t,q}(x,\theta) = \frac{N_c}{\theta^2(2\pi)^4}\int_0^{1-x}\frac{d\xi}{\xi}(\boldsymbol{k}_\perp^2)^2\int d^2\boldsymbol{k}_{g\perp}\left[\frac{\boldsymbol{k}_{g\perp}+\boldsymbol{k}_\perp}{\epsilon_f^2+(\boldsymbol{k}_{g\perp}+\boldsymbol{k}_\perp)^2}-\frac{\boldsymbol{k}_\perp}{\epsilon_f^2+\boldsymbol{k}_\perp^2}\right]^2\frac{\boldsymbol{k}_\perp\cdot\boldsymbol{k}_{g\perp}}{|\boldsymbol{k}_\perp|}\underline{O_{1T,x_g}^{\perp}(\boldsymbol{k}_{g\perp}^2)}$$

$\epsilon_f^2 \equiv (x/\xi)\boldsymbol{k}_\perp^2$

Spin-dependent odderon

[J. Zhou 2014]

However, this inclusive measurement is not feasible, due to C-odd nature of the odderon:

$$\sum_{a=q,\bar{q}} e_a^2 f_T^{t,a}(x,\theta) = 0$$

See the pomeron case in

Liu-Pan-Yuan -Zhu PRL. 130 (2023) 181901

➡ Measure the energy flow from a subset of charged hadrons, like $\mathbb{S} = \{\pi^+\},\{\pi^-\}$

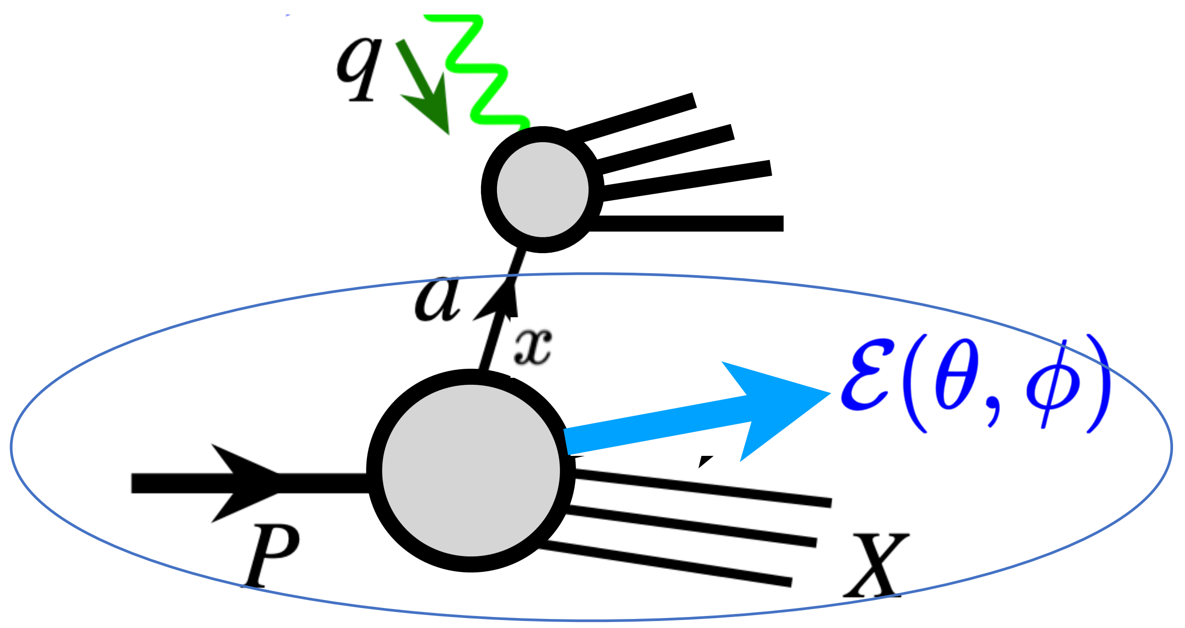
$$f_{T,\mathbb{S}}^{t,q}(x,\theta) = \Big(\sum_{h\in\mathbb{S}}\int_0^1 z d_{h/\bar{q}}(z)dz\Big) f_T^{t,q}(x,\theta)$$

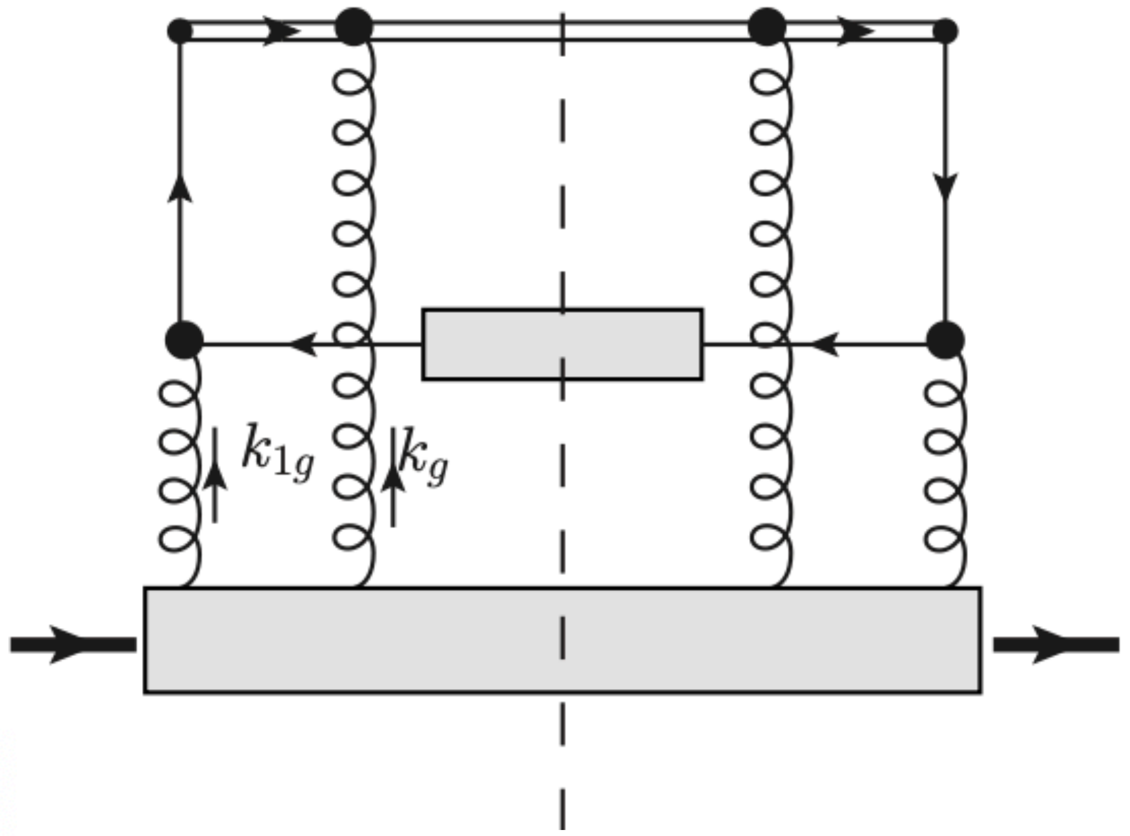
⇨ Derived from the quark Sivers fracture function using the sum rule

Nucleon energy corrolators for Odderons

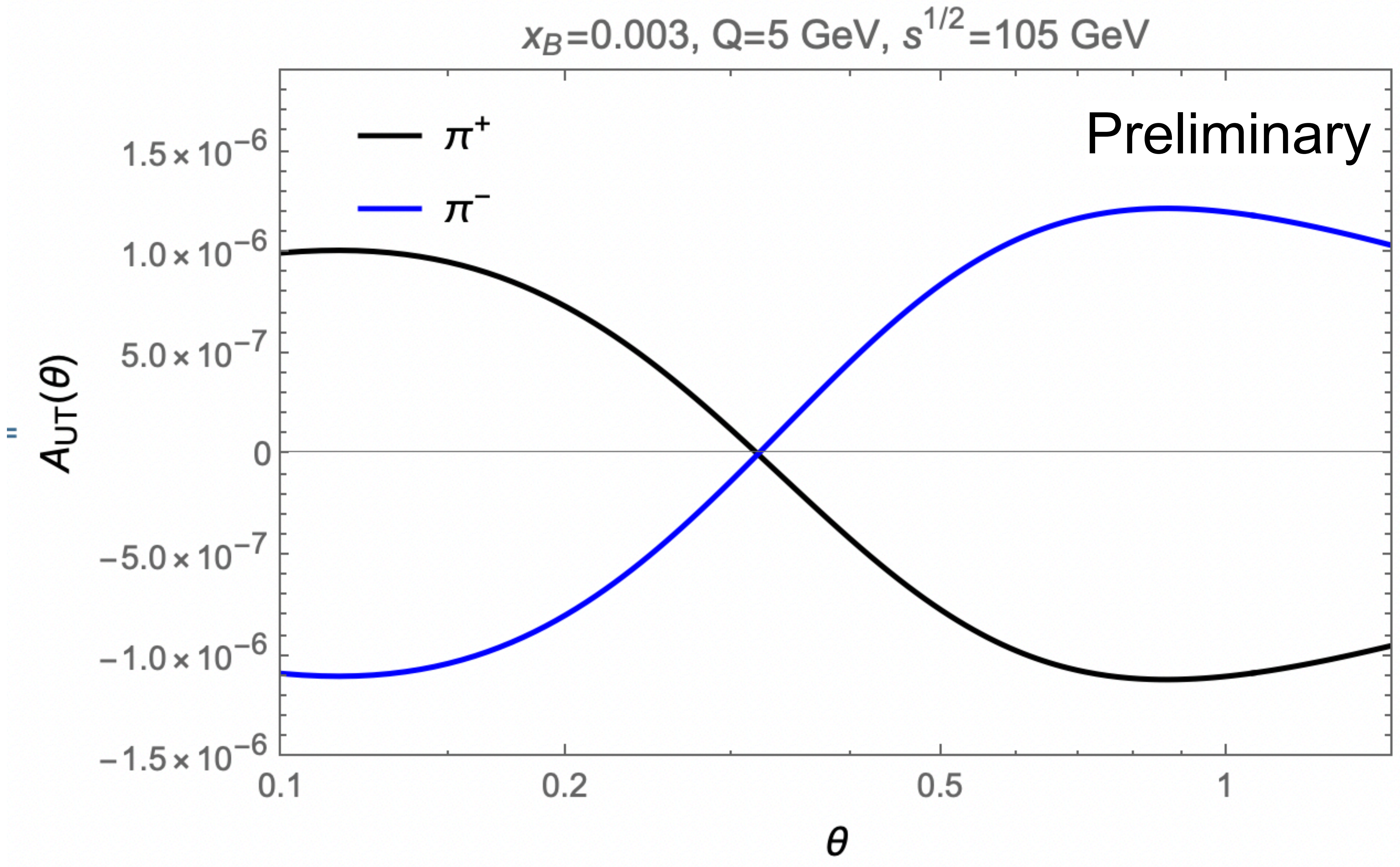
—the energy pattern from a subset of hadrons

Mantysaari-Tawabutr-Tong: in preparation





- ✦ Single transverse spin asymmetry: $\Sigma_{UT}^{\sin(\phi-\phi_S)} = \sum_{a=q,\bar{q}} e_a^2 x_B f_T^{t,a}(x_B,\theta)$



➡ The sign changes from π^+ to π^-

Summary

- Fracture function—SIDIS in the TFR
  - Probe the nucleon structure through the correlation between the initial state and the final state, complementary to TMDs
  - Sensitive to the linearly polarized gluons
  - Hadrons in the TFR: a polarizing filter for gluons
- Nucleon energy correlator—the DIS energy pattern in the TFR
  - An energy sum rule:
    - Fracture functions are the parent functions of the NEC
  - The application for a subset of hadrons:
    - Spin-dependent odderon& Quark Sivers-NEC :

Backup:

Nucleon energy correlator

Liu-Zhu PRL 130, 2023

$$\mathcal{M}^{q}_{ij,\mathrm{EEC}}(x,\theta,\phi) = \int \frac{d\eta^-}{2\pi} e^{-ixP^+\eta^-} \langle PS|\bar{\psi}_j(\eta^-)\mathcal{L}^{\dagger}_{n}(\eta^-)\mathcal{E}(\theta,\phi)\mathcal{L}_n(0)\psi_i(0)|PS\rangle$$

$$\mathcal{E}(\theta,\phi)|X\rangle = \sum_{a\in X} \delta(\theta^2-\theta_a^2)\delta(\phi-\phi_a)\frac{E_a}{E_N}|X\rangle$$

$$\mathcal{E}(\theta,\phi) = \sum_h \int \frac{d^3\boldsymbol{P}_h}{2E_h(2\pi)^3}\frac{E_h}{E_N}\delta(\theta^2-\theta_h^2)\delta(\phi-\phi_h)a_h^{\dagger}a_h$$

Fracture functions

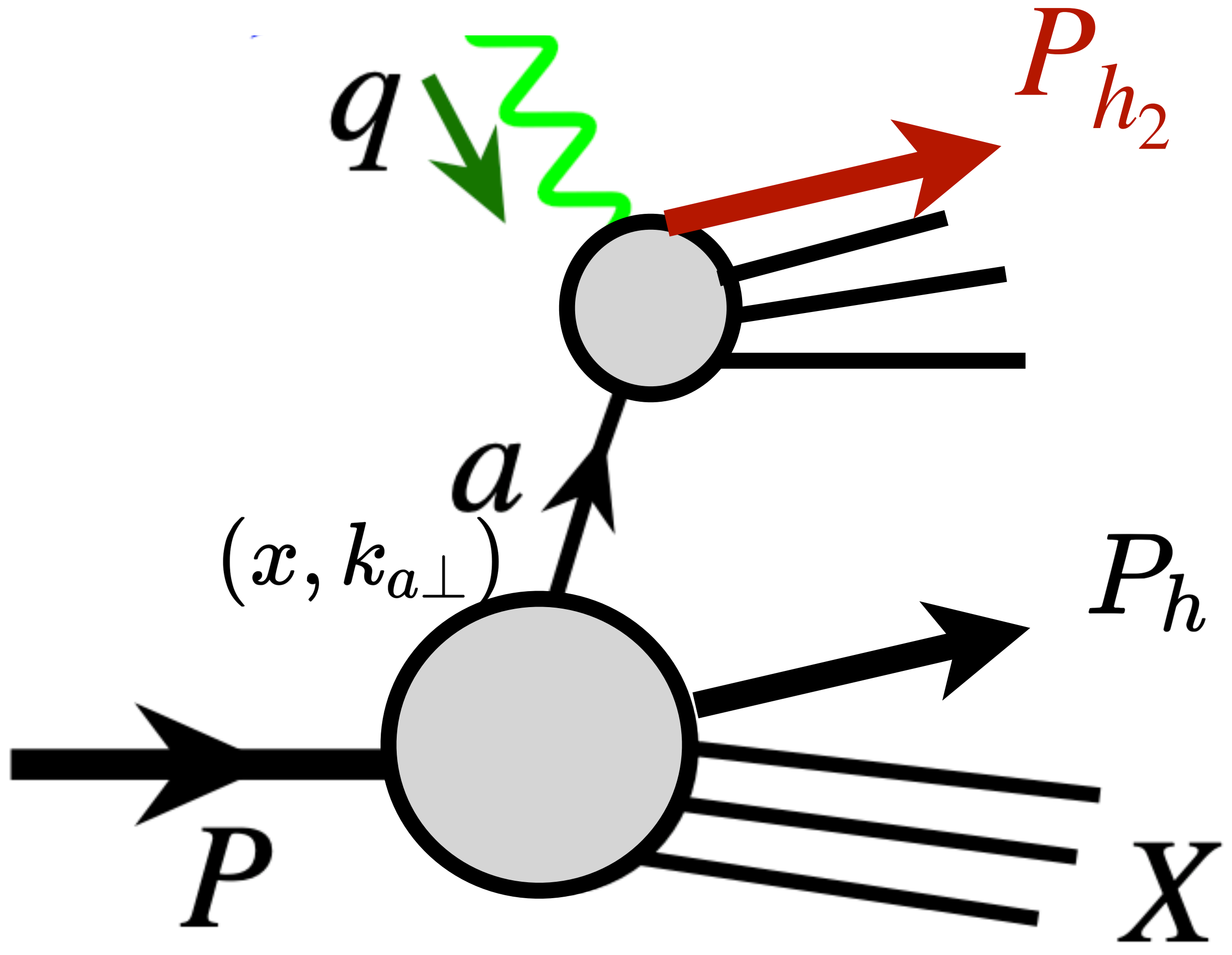
$$\mathcal{M}_{\mathrm{FrF},ij}(x,\xi_h,\vec{P}_{h\perp}) = \frac{1}{2\xi_h(2\pi)^3}\int\frac{d\lambda}{2\pi}e^{-ixP^+\lambda}\sum_X\langle PS|\bar{\psi}_j(\lambda n)\mathcal{L}^{\dagger}_n(\lambda n)|XP_h\rangle\langle P_hX|\mathcal{L}_n(0)\psi_i(0)|PS\rangle$$

$$\sum_X\int\frac{d^3\boldsymbol{P}_X}{2E_X(2\pi)^3}|P_hX\rangle\langle XP_h| = a_h^{\dagger}a_h$$

Backup:

TMD fracture functions & Dihadron production

$$\int d^2 k_{a\perp}$$



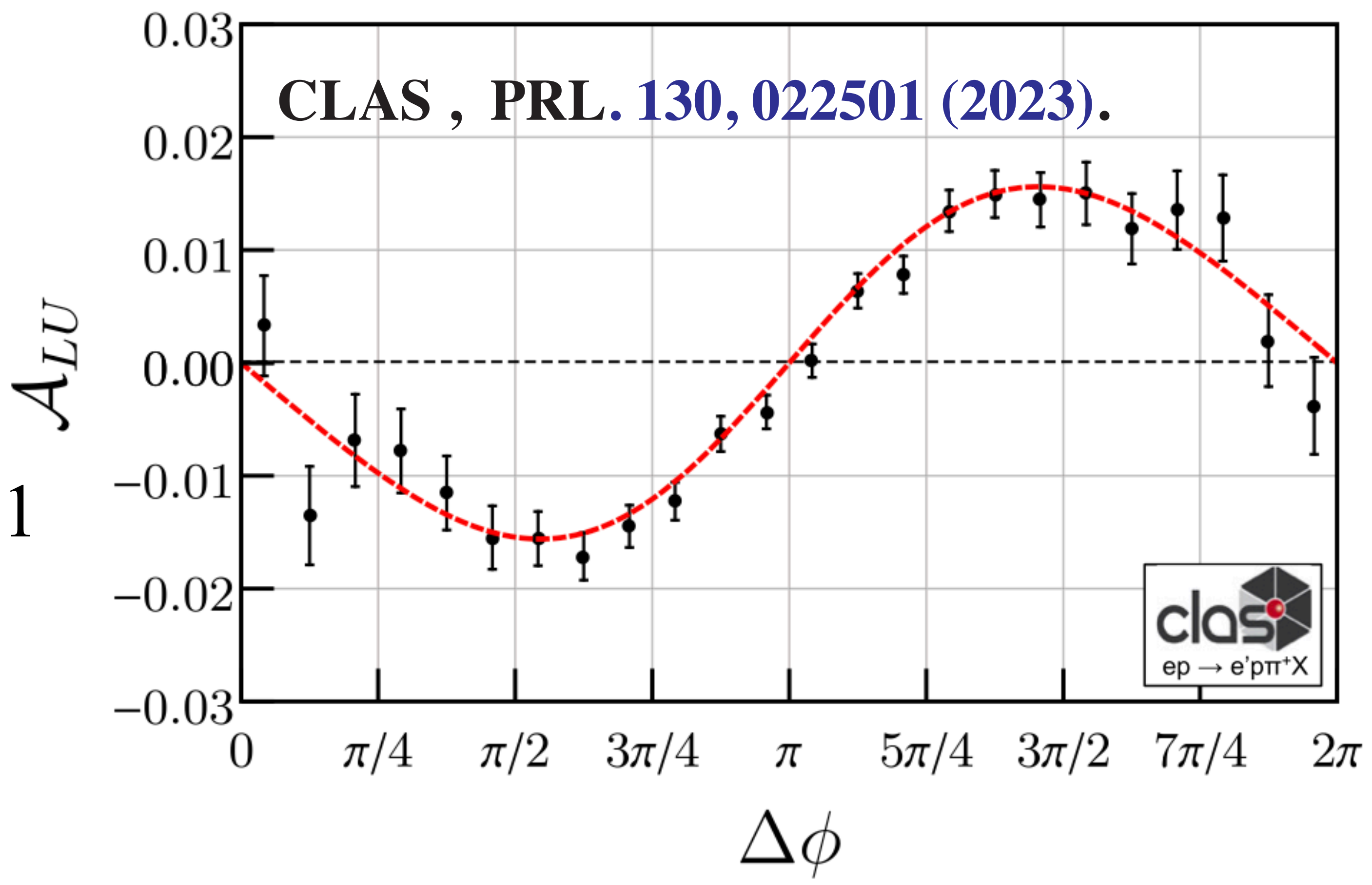
The hadron $P_{h_2\perp}$ in the CFR can resolve the initial $k_{a\perp}$

$k_{a\perp}$ un-integrated →TMD *fracture* functions

★**TMD factorization:** $\sigma \propto H(Q) \otimes \mathcal{F}_a(x, k_\perp, \xi, P_{h\perp}) \otimes D(z, p_\perp)$

- Spin-dependence: Anselmino-Barone-Kotzinian, PLB 699 (2011) 108
  PLB 706, 46 (2011);
  PLB 713, 317 (2012).
- TMD Evolutions: Chen-Ma-Tong, JHEP 10 (2019) *285*
- Diffraction and small-x: Iancu-Mueller-Triantafyllopoulos, PRL. 128 (2022) 202001
  Hatta-Xiao-Yuan PRD 106 (2022) 094015
  Hatta-Yuan PLB 854 (2024) 138738

•Beam-spin asymmetry



-An explanation: *Guo-Yuan* 2312.01008

Backup:

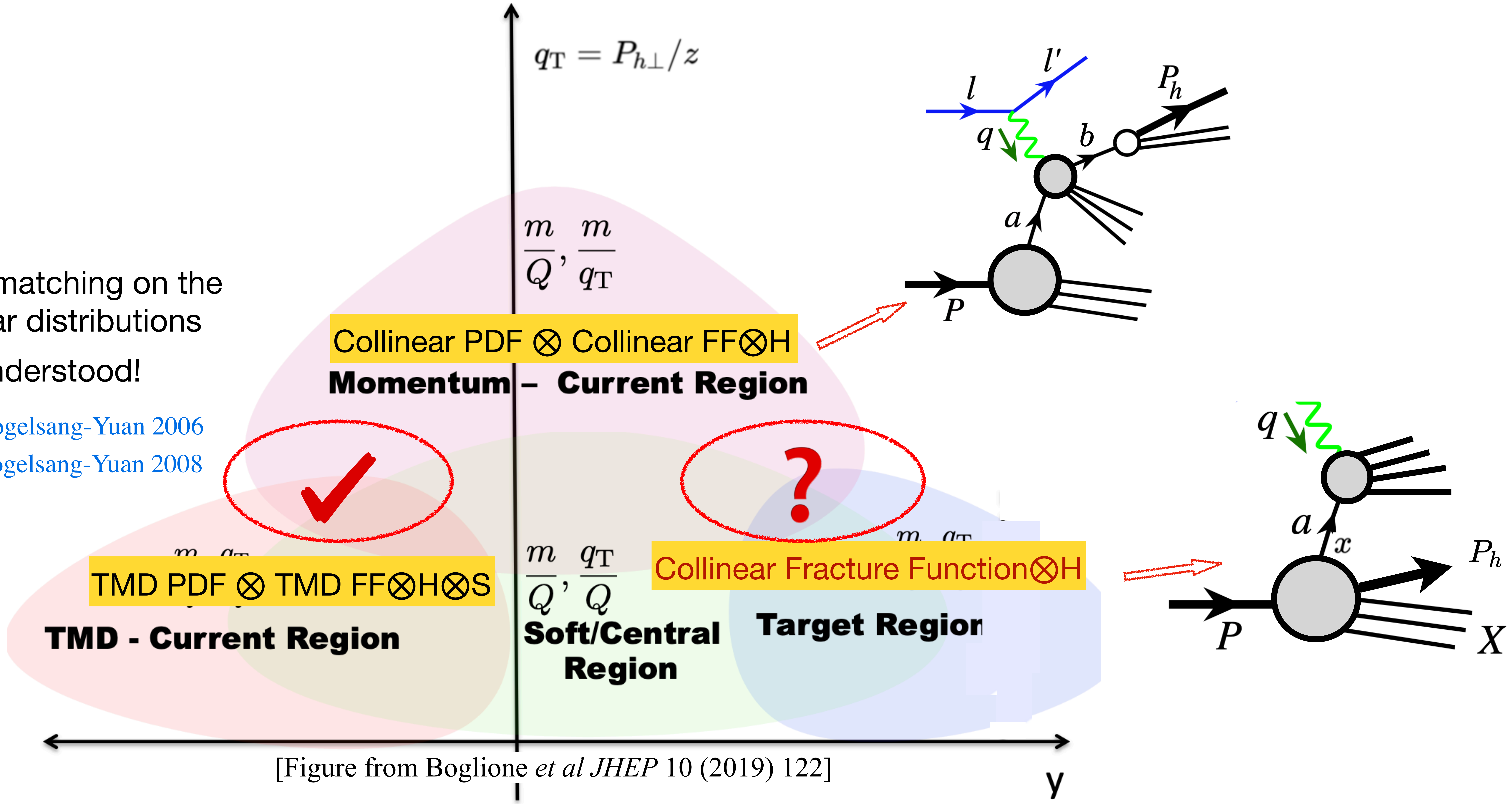
Matching of Fracture Functions at large $P_{h\perp}$

TMDs matching on the collinear distributions

Well-understood!

Ji-Qiu-Vogelsang-Yuan 2006

Koike-Vogelsang-Yuan 2008



$q_T = P_{h\perp}/z$

$\frac{m}{Q}, \frac{m}{q_T}$

Collinear PDF ⊗ Collinear FF⊗H

Momentum – Current Region

TMD PDF ⊗ TMD FF⊗H⊗S

TMD - Current Region

$\frac{m}{Q}, \frac{q_T}{Q}$

Soft/Central Region

Collinear Fracture Function⊗H

Target Region

y

[Figure from Boglione *et al JHEP* 10 (2019) 122]

In the intermediate $P_{h\perp}$ region, how are the two approaches should be consistent

Chen-Ma-Tong JHEP 11 (2021) 038

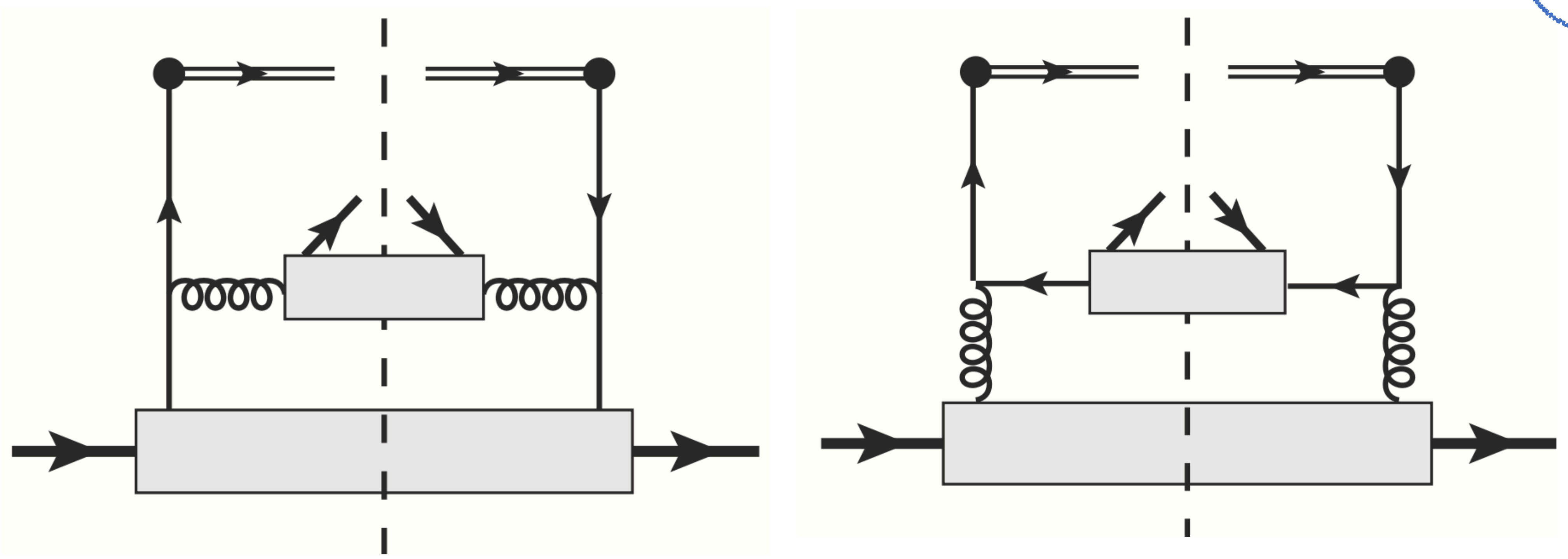
Backup:

Quark fracture functions at large $P_{h\perp}$

Chen-Ma-Tong JHEP 11 (2021) 038

- Unpolarized/helicity quark:

$$u_1, l_{1L} \propto \frac{1}{P_{h\perp}^2} \otimes \text{twist-2 FFs} \otimes \text{twist-2 PDFs}$$

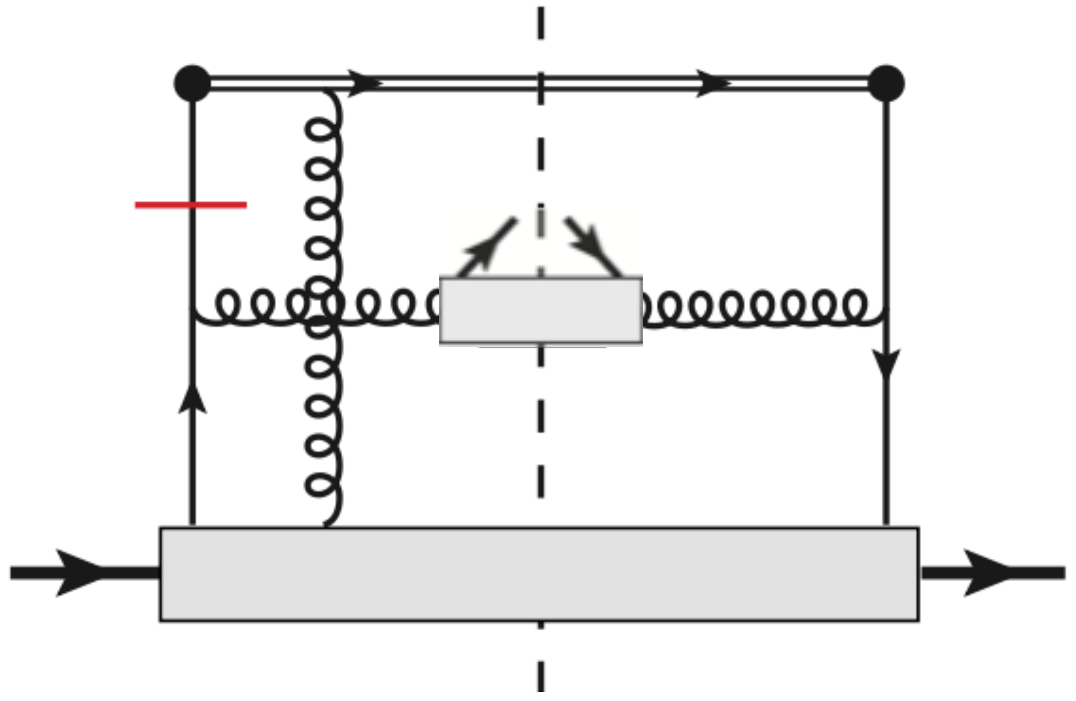


$$u_1^q\left(x, \xi_h, \boldsymbol{P}_{h\perp}^2\right) = \int_{\frac{\xi_h}{1-x}}^1 \frac{dz}{z^2} \int_x^1 dy \delta(x + \xi_h/z - y) \frac{\alpha_s z^2}{2\pi^2 \xi_h \boldsymbol{P}_{h\perp}^2} \times \left[C_F \frac{x^2+y^2}{y^2} d_{h/g}(z) q(y) + T_F\left(1-\frac{x}{y}\right)\left[\frac{x^2}{y^2} + \left(1-\frac{x}{y}\right)^2\right] d_{h/\bar{q}}(z) g(y)\right]$$

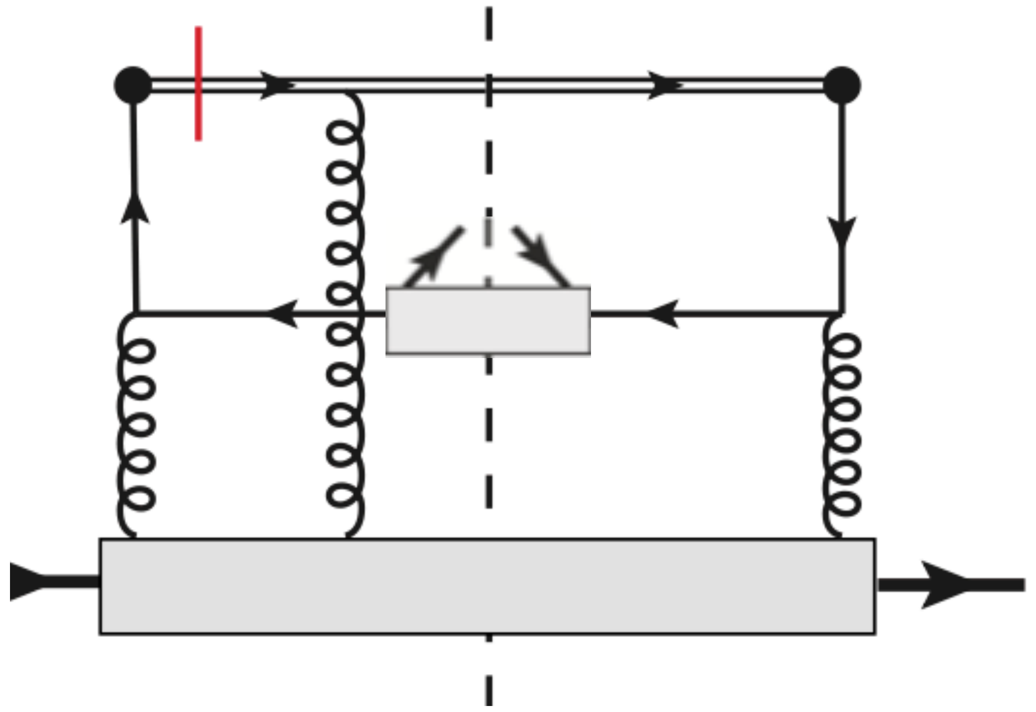
- Sivers & Worm-gear quark:

$$u_{1T}, l_{1T} \propto \frac{1}{P_{h\perp}^4} \otimes \text{twist-2 FFs} \otimes \text{twist-3 multi-parton distributions}$$

Follows the Efremov-Teryaev-Qiu-Sterman mechanism



Quark-gluon-quark



Three-gluon

Sivers-type: T-odd, single spin asymmetry

Need a non-trivial phase to be generated in the perturbative region

Worm-gear-type: T-even, double spin asymmetry