q_{T} resummation on the Drell-Yan process and zero-bin subtraction beyond the leading power

Wan-Li Ju

INFN, Sezione di Milano

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Outline

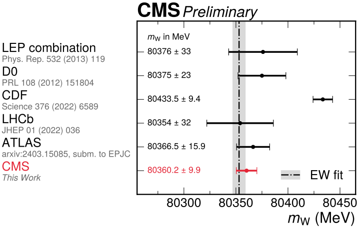
Challenges in precise measurement on the M_W and Γ_W at LHC

- PDF uncerntainties
- electroweak corrections
- modelling effects in the q_{T} distributions

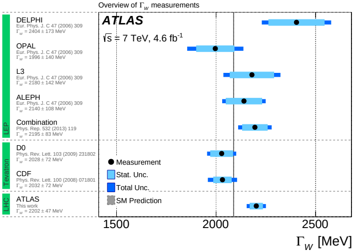
$$\mathcal{R}_{W/Z}(q_{\mathrm{T}}) \sim \left(\frac{\mathrm{d}\sigma_W}{\mathrm{d}q_{\mathrm{T}}}\right)\left(\frac{\mathrm{d}\sigma_Z}{\mathrm{d}q_{\mathrm{T}}}\right)^{-1}, \quad \mathcal{R}_{W/Z}(\Delta\phi) \sim \left(\frac{\mathrm{d}\sigma_W}{\mathrm{d}\Delta\phi}\right)\left(\frac{\mathrm{d}\sigma_Z}{\mathrm{d}\Delta\phi}\right)^{-1}$$

$\rightarrow$ QCD resummation

- … …



[CMS-PAS-SMP-23-002]



[2403.15085]

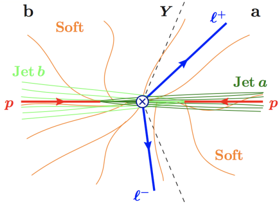
Resummation on the colorless particle hadroproduction

- Methodologies
  dQCD: [Catani:1988vd,Davies:1984hs,Collins&Soper&Sterman1984,Catani:2000vq,Bozzi:2005wk...]
  Momentum space: [Monni:2016ktx,Bizon:2017rah,Bizon:2019zgf,Bizon:2018foh,Ebert:2016gcn...]
  SCET: [Becher:2010tm,Chiu:2011qc,Chiu:2012ir,Li:2016axz...] $\to$ followed by this work
  ...
- Latest logarithmic accuracy at leading power:
  N3LL' [Re:2021con,Camarda:2021ict,Ju:2021lah...]
  N4LL [Camarda:2023dqn,Neumann:2022lft,Moos:2023yfa,Piloneta:2024aac...]
  ...
- Recent developments at next-to-leading power:
  Leptonic tensor: [Camarda:2021jsw,Ebert:2020dfc...]
  Hadronic tensor:
  [Ebert:2019zkb,Oleari:2020wvt,Cieri:2019tfv,Inglis-Whalen:2020rpi,Inglis-Whalen:2021bea,Ferrera:2023vsw...]

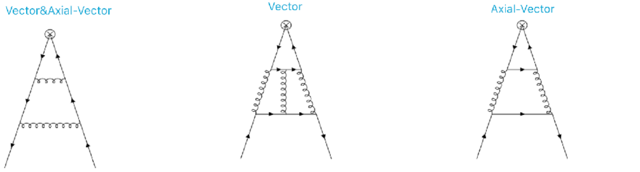
- Factorization of leading singular terms in the low q_{T} regime

$$\frac{\mathrm{d}^4\sigma}{\mathrm{d}^2\vec{q}_{\mathrm{T}}\mathrm{d}Y_{\ell\bar{\ell}}\mathrm{d}M^2_{\ell\bar{\ell}}\mathrm{d}\Omega} \sim \sum_{i,j} \mathcal{H}^V_{ij} \otimes \mathcal{B}^i_n \otimes \mathcal{B}^j_{\bar{n}} \otimes \mathcal{S}_{ij}$$

  - Soft function $\mathcal{S}_{ij}$ up to N^{3}LO [Zhu:2012ts,Li:2013mia,Angeles-Martinez:2018mqh,Catani:2014qha,Catani:2021cbl];
  - Beam functions $\mathcal{B}_{n(\bar{n})}$ up to N^{3}LO [Luo:2020epw,Luo:2019szz,Luo:2019bmw,Gutierrez-Reyes:2019rug,Catani:2022sgr];
  - Hard function $\mathcal{H}_{ij}$:
    $pp \to W \to \nu\bar{\ell}$: Non-singlet-diagrams
    $pp \to Z/\gamma^* \to \ell\bar{\ell}$:
    Singlet/non-singlet diagrams



[1004.2489]



(a) Non-singlet amplitude (b) Singlet amplitude induced by vector current (c) Singlet amplitude induced by axial-vector current

(1) Non-Singlet diagrams and Vector-current-induced singlet diagrams: [2202.04660,2105.11504...]

(2) Axial-vector-induced singlet diagrams: [Chetyrkin:1993ug,Chetyrkin:1993jm,Larin:1993tq,Chetyrkin:1993hk]

$$A_Z^\mu = g_A \left[\sum_{i=1,2,3} \Delta_i^{\text{ns}} + C_t \mathcal{O}_s \right] \text{, where } \mathcal{O}_s = \sum_{q_i=u,d,c,s,b} \bar{q}_i \gamma^\mu \gamma_5 q_i \,,$$

$$\Delta_1^{\text{ns}} = \bar{u}\gamma^\mu\gamma_5 u - \bar{d}\gamma^\mu\gamma_5 d \,, \qquad \Delta_2^{\text{ns}} = \bar{c}\gamma^\mu\gamma_5 c - \bar{s}\gamma^\mu\gamma_5 s \,, \qquad \Delta_3^{\text{ns}} = \frac{\mathcal{O}_s}{N_f} - \bar{b}\gamma^\mu\gamma_5 b \,.$$

$C_t(m_t, \mu)$: N^{4}LO[Chetyrkin:1993jm,Chetyrkin:1993ug,Rittinger:2012bha,2112.03795]
$\langle q_i \bar{q}_j | \mathcal{O}_s | 0 \rangle$ N^{3}LO[Bernreuther:2005rw,Gehrmann:2021ahy,Duhr:2021vwj]

Resummation and matching procedure

Resummation: solving (rapidity) renormalization group equation

$$\frac{d^4\sigma_{\text{res}}}{d^2\vec{q}_T dY_L dM_L^2 d\Omega_L} \sim \sum_{i,j=q,\bar{q},g} \int d^2\vec{b}_T\, e^{i\vec{b}_T\cdot\vec{q}_T}\, \mathcal{U}_V(\mu_h,\mu_{b_n},\mu_{b_{\bar{n}}},\mu_s)\,\mathcal{U}_R(\nu_{b_n},\nu_{b_{\bar{n}}},\nu_s) \times \widetilde{H}_{ij}^{V,\text{res}}\, \mathcal{B}_n^i(x_n,\vec{b}_T,\mu_{b_n},\nu_{b_n})\,\mathcal{B}_{\bar{n}}^j(x_{\bar{n}},\vec{b}_T,\mu_{b_{\bar{n}}},\nu_{b_{\bar{n}}}) \times \mathcal{S}_{ij}(\vec{b}_T,\mu_s,\nu_s),$$

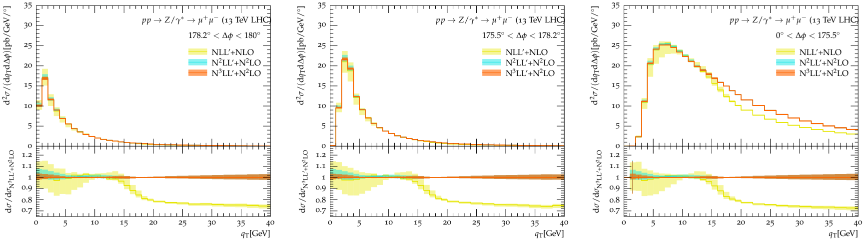
- $\mathcal{U}_V$ evolves the scales associated with virtuality-div renormalization
- $\mathcal{U}_R$ evolves the scales brought in by the rapidity-div renormalization

Matching onto the fixed-order results

$$\frac{\mathrm{d}\sigma_{\text{mat}}}{\mathrm{d}\Phi} = \left\{\frac{\mathrm{d}\sigma_{\text{res+lpc}}}{\mathrm{d}\Phi} - \frac{\mathrm{d}\sigma_{\text{exp+lpc}}}{\mathrm{d}\Phi}(\mu_R,\mu_F)\right\} f\left(\frac{q_{\mathrm{T}}}{q_{\mathrm{T}}^{\text{mat}}}\right) + \frac{\mathrm{d}\sigma_{\text{f.o.}}}{\mathrm{d}\Phi}(\mu_R,\mu_F)$$

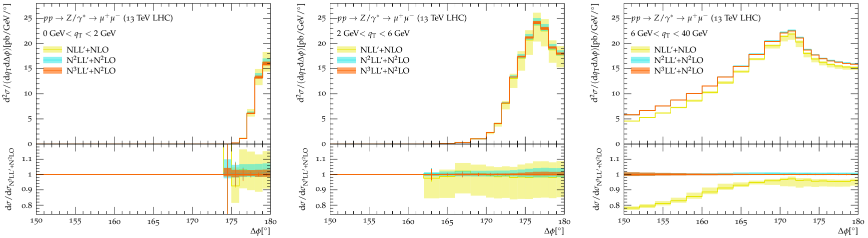
- Boosting the leptonic tensor to the physical position→leptonic power correction
- Transition function $f\left(q_{\mathrm{T}}/q_{\mathrm{T}}^{\text{mat}}\right)$ to switch of resummation in the tail domain

q_{T} distributions in slices of $\Delta\phi \in [178.2^\circ, 180^\circ]$, $\Delta\phi \in [175.5^\circ, 178.2^\circ]$, $\Delta\phi \in [0^\circ, 175.5^\circ]$.



- Scale Variations: $\mu_X \in [2\mu_X^{\mathrm{def}}, \mu_X^{\mathrm{def}}/2]$ and $\nu_X \in [2\nu_X^{\mathrm{def}}, \nu_X^{\mathrm{def}}/2]$
- Kinematic preference in the region where $q_{\mathrm{T}} \to 0$ and $\Delta\phi \to \pi$.

$\Delta\phi$ distributions in slices of $q_\mathrm{T} \in [0,2]$ GeV, $q_\mathrm{T} \in [2,6]$ GeV, $q_\mathrm{T} \in [6,40]$ GeV.



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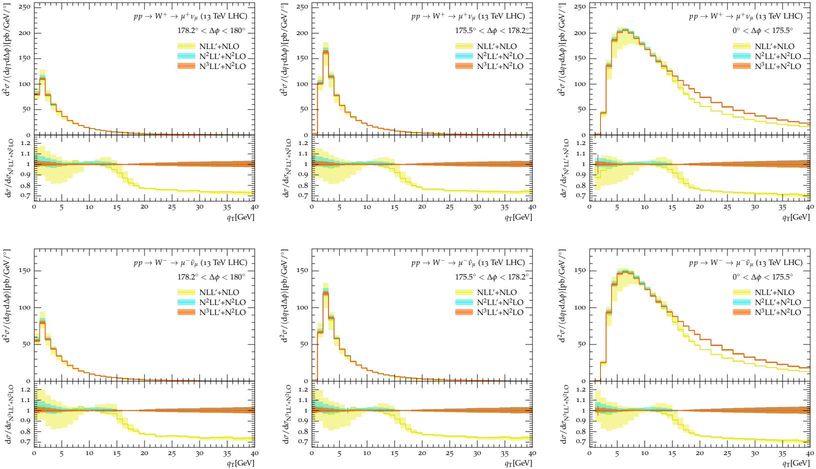
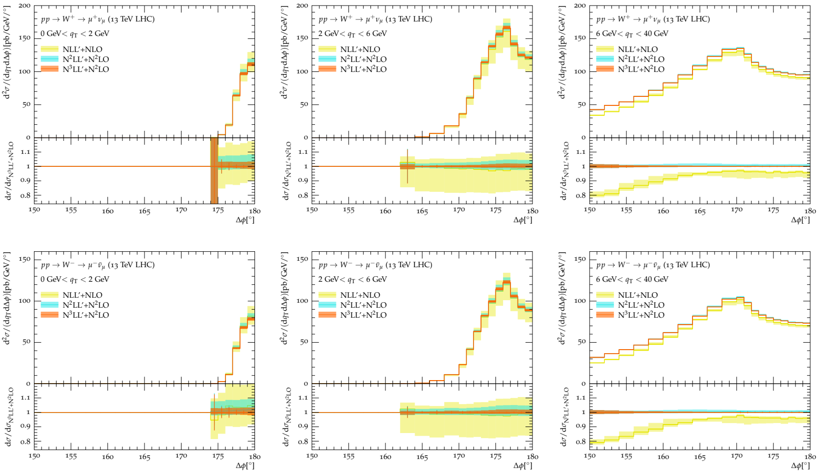
Numeric outputs

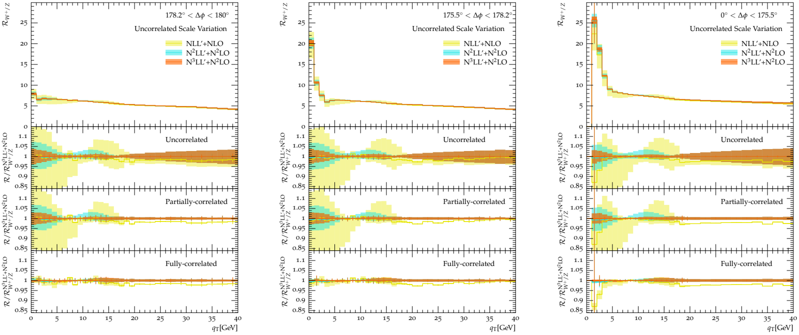
q_{T} distributions in slices of $\Delta\phi \in [178.2^\circ, 180^\circ]$, $\Delta\phi \in [175.5^\circ, 178.2^\circ]$, $\Delta\phi \in [0^\circ, 175.5^\circ]$.



$\Delta\phi$ distributions in slices of $q_{\mathrm{T}} \in [0, 2]$ GeV, $q_{\mathrm{T}} \in [2, 6]$ GeV, $q_{\mathrm{T}} \in [6, 40]$ GeV.



W/Z correlations



- The central values are close to each other.
- With the accuracy growing, the uncertainties decrease considerably.
- The error bands of higher accuracy are contained by those with lower accuracy.

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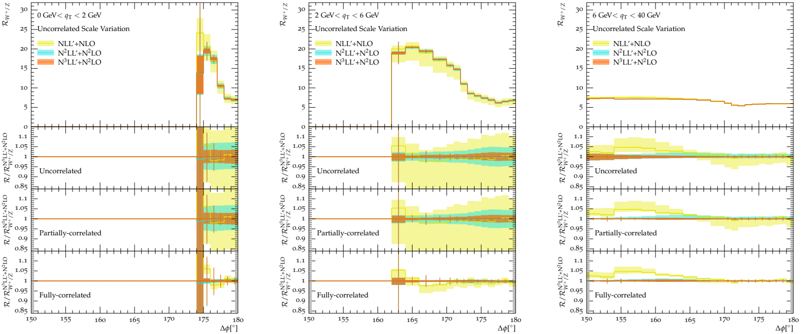

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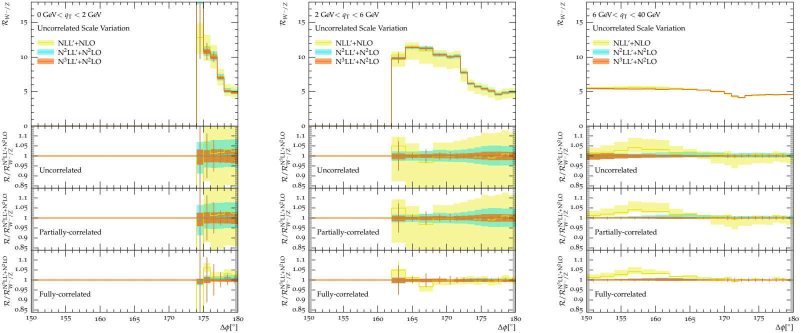

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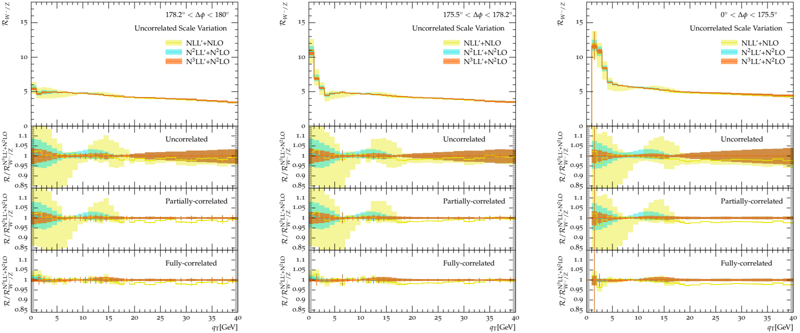

Outline

Expanding F.O. in the low $q_{\rm T}$ domain ($L_H \equiv \ln[\mu_h/Q]$):

$$\frac{d\sigma}{dY\,dq_{\rm T}^2} = \underbrace{\sum_m \frac{\Delta^{(m)}_{\rm LP}}{q_{\rm T}^2}\left(L_H\right)^m}_{\rm LP} + \overbrace{\sum_m \Delta^{(m)}_{\rm NLP}\left(L_H\right)^m}^{\rm NLP} + q_{\rm T}^2 \underbrace{\sum_m \Delta^{(m)}_{\rm N^2LP}\left(L_H\right)^m}_{\rm N^2LP} + \ldots .$$

N^{n}LP corrections will improve the coverage of the resummation range and also reduce the scale variation in matching procedures

$\Delta_{\rm N^kLP}$ can be derived by matching $\rm SCET_I$ onto $\rm SCET_{II}$ [BPS+M.Beneke, M.Neubert, 2000-now]:

$$\Delta_{\rm N^kLP} \sim \underbrace{\mathcal{O}_{n\bar n}}_{\mathcal{A}_{n(\bar n)},\xi_{n(\bar n)},A_{us},q_{us}} + \mathbf{T}\Big[\mathcal{O}'_{n\bar n}, \underbrace{\mathcal{O}_n^{\rm SCET_I}}_{\mathcal{A}_n,\xi_n,A_{us},q_{us}}\Big] + \mathcal{O}''_{n\bar n} + (\bar n \leftrightarrow n)$$

$$\to \mathbf{J}\left(\omega_n, \omega_{\bar n}\right) \underbrace{\mathcal{O}_{n\bar n}^{\rm SCET_{II}}\left(\omega_n, \omega_{\bar n}\right)}_{\mathcal{A}_n,\xi_n,\mathcal{A}_{\bar n},\xi_{\bar n},A_s,q_s} + \ldots \text{ Zero Bin Subtraction??}$$

Alternative strategies:

[Ian Balitsky: Balitsky:2017gis,Balitsky:2020jzt,Balitsky:2021fer,Balitsky:2017flc...]

[Alexey Vladimirov: 2211.04494,2211.13209,2306.09495,2307.13054,2109.09771,2306.15052...]

[Michael Luke:Inglis-Whalen:2021bea,Inglis-Whalen:2022vyn].......

[Iain W. Stewart: Ebert:2020dfc,2112.07680.........

Zero-bin Subtraction at Leading power

Leading power beam function with exponential regulator [Li:2016axz,Luo:2019hmp]

$$\mathcal{B}_+ \sim \underbrace{\mathcal{Z}_{\mathcal{B}_+}^{-1}}_{\text{renormalization}} \quad \overbrace{\mathcal{Z}_0^{-1}}^{\text{zero bin subtrahend}} \quad \sum_j \mathcal{I}_{ij}^{\text{BARE}} \otimes f_{j/N_\pm}^{\text{BARE}} \,,$$

- Zero bin subtrahend $\mathcal{Z}_0^{-1}$ at LP is equal to soft function
- $\mathcal{I}_{ij}$ accounts for collinear contributions with the virtuality $\sim \mathcal{O}(q_\mathrm{T}^2) \gg \Lambda_H^2$.

Proposal for NLP zero bin subtrahend [Michael Luke:Inglis-Whalen:2021bea,Inglis-Whalen:2022vyn].......

$$\hat{\mathrm{T}}_n + \hat{\mathrm{T}}_{\bar{n}} - \frac{1}{2}\left(\hat{\mathrm{T}}_n \otimes \hat{\mathrm{T}}_{\bar{n}} + \hat{\mathrm{T}}_{\bar{n}} \otimes \hat{\mathrm{T}}_n\right)$$

How to construct zero bin subtraction beyond NLP??

NLO q_{T} distributions for the process $pp \to B + X$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Y_B \mathrm{d}q_{\mathrm{T}}^2 \mathrm{d}M_B} \propto \sum_{i,j} \int_{k_+^{\min}}^{k_+^{\max}} \frac{\mathrm{d}k_+}{k_+} \frac{f_{i/N}(\xi_n)}{\xi_n} \frac{f_{j/\bar{N}}(\xi_{\bar{n}})}{\xi_{\bar{n}}} \overline{\sum_{\mathrm{col,pol}}} \left|\mathcal{M}(i+j \to B+k)\right|^2$$

- k^μ denotes the momentum of the emitted parton with $k_+ = k\cdot n$ and $k_- = k\cdot\bar{n}$
- Boundaries are subject to the colliding energy $\sqrt{s}$:

$$k_+^{\max} = \sqrt{s} - m_{\mathrm{T}} e^{-Y_H}\,, \quad k_+^{\min} = q_{\mathrm{T}}^2 / \left(\sqrt{s} - m_{\mathrm{T}} e^{+Y_H}\right)$$

- Momentum fractions:

$$\xi_n = (k_+ + m_{\mathrm{T}} e^{-Y_H})/\sqrt{s}\,, \qquad \xi_{\bar{n}} = (k_- + m_{\mathrm{T}} e^{+Y_H})/\sqrt{s}$$

Asymptotic expansion in the small parameter $\lambda \equiv q_{\mathrm{T}}/M_B$

A variety scalings are encompassed:

$$k_+ \to k_+^{\max} \Rightarrow n\text{-collinear mode}: \; k^\mu \equiv (k_+, k_-, k_\perp) \sim m_H\left(1, \lambda^2, \lambda\right)$$
$$k_+ \to k_+^{\min} \Rightarrow \bar{n}\text{-collinear mode}: \; k^\mu \equiv (k_+, k_-, k_\perp) \sim m_H\left(\lambda^2, 1, \lambda\right)$$
$$k_+ \to q_{\mathrm{T}} \Rightarrow \text{soft mode}: \; k^\mu \equiv (k_+, k_-, k_\perp) \sim m_H\,(\lambda, \lambda, \lambda)$$

Zero-bin subtraction beyond LP

Asymptotic expansion in the small parameter $\lambda \equiv q_{\mathrm{T}}/M_B$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Y_B \mathrm{d}q_{\mathrm{T}}^2 \mathrm{d}M_B} \propto \sum_{i,j} \int_{k_+^{\min}}^{k_+^{\max}} \frac{\mathrm{d}k_+}{k_+} \frac{f_{i/N}(\xi_n)}{\xi_n} \frac{f_{j/\bar{N}}(\xi_{\bar{n}})}{\xi_{\bar{n}}} \overline{\sum_{\mathrm{col,pol}}} \left|\mathcal{M}(i+j \to B+k)\right|^2$$

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$$k_+ \to q_{\mathrm{T}} \Rightarrow \text{soft mode}: \; k^\mu \equiv (k_+, k_-, k_\perp) \sim m_H \left(\lambda, \lambda, \lambda\right)$$

Auxiliary cutoffs in the integration range:

$$m_B e^{-Y_B} \gtrsim \nu_n \sim \mathcal{O}(m_B) \gg q_{\mathrm{T}}\,, \quad m_B e^{Y_B} \gtrsim \nu_{\bar{n}} \sim \mathcal{O}(m_B) \gg q_{\mathrm{T}}$$

Categorization of the phase space integral:

$$\int_{\nu_n}^{k_+^{\max}} \mathrm{d}k_+ \Rightarrow n\text{-col}\,, \quad \int_{k_+^{\min}}^{\frac{q_{\mathrm{T}}^2}{\nu_{\bar{n}}}} \mathrm{d}k_+ \Rightarrow \bar{n}\text{-col}\,, \quad \int_{\frac{q_{\mathrm{T}}^2}{\nu_{\bar{n}}}}^{\nu_n} \mathrm{d}k_+ \Rightarrow \text{soft} + \text{col}\,.$$

Asymptotic expansion in the small parameter $\lambda \equiv q_{\mathrm{T}}/M_B$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Y_B \mathrm{d}q_{\mathrm{T}}^2 \mathrm{d}M_B} \propto \sum_{i,j} \int_{k_+^{\mathrm{min}}}^{k_+^{\mathrm{max}}} \frac{\mathrm{d}k_+}{k_+} \frac{f_{i/N}(\xi_n)}{\xi_n} \frac{f_{j/\bar{N}}(\xi_{\bar{n}})}{\xi_{\bar{n}}} \overline{\sum_{\mathrm{col,pol}}} \left|\mathcal{M}(i+j \to B+k)\right|^2$$

Extrapolation to eliminate the auxiliary cutoffs:

$$\int_{\nu_n}^{k_+^{\mathrm{max}}} \mathrm{d}k_+ \Rightarrow \overbrace{\lim_{\tau\to 0} \int_0^{k_+^{\mathrm{max}}} \mathrm{d}k_+ \underbrace{\mathcal{R}(k_+, k_-, \tau)}_{\text{rapidity}-\text{div}-\text{regulator}}}^{n-\text{Beam Function}} - \lim_{\tau\to 0} \int_0^{\nu_n} \mathrm{d}k_+ \mathcal{R}(k_+, k_-, \tau)$$

$$\int_{k_+^{\mathrm{min}}}^{\frac{q_{\mathrm{T}}^2}{\nu_{\bar{n}}}} \mathrm{d}k_+ \Rightarrow \underbrace{\lim_{\tau\to 0} \int_{k_+^{\mathrm{min}}}^{+\infty} \mathrm{d}k_+ \mathcal{R}(k_+, k_-.\tau)}_{\bar{n}-\text{Beam Function}} - \lim_{\tau\to 0} \int_{\frac{q_{\mathrm{T}}^2}{\nu_{\bar{n}}}}^{+\infty} \mathrm{d}k_+ \mathcal{R}(k_+, k_-, \tau)$$

$$\lim_{\tau\to 0} \int_{\frac{q_{\mathrm{T}}^2}{\nu_{\bar{n}}}}^{\nu_n} \mathrm{d}k_+ \mathcal{R}(k_+, k_-, \tau)$$

Zero-bin-subtrahend

Zero-bin subtraction beyond LP

Asymptotic expansion in the small parameter $\lambda \equiv q_{\rm T}/M_B$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Y_B \mathrm{d}q_{\rm T}^2 \mathrm{d}M_B} \sim \sum_{\omega}^{\infty} (q_{\rm T}^2)^{\omega} \left\{ \left[\widetilde{\mathcal{G}}_c^{(\omega)} - \widetilde{\mathcal{G}}_{c0,(\omega)}^{\langle \mathrm{NS} \rangle} \right] + \left[\widetilde{\mathcal{G}}_{\bar{c}}^{(\omega)} - \widetilde{\mathcal{G}}_{\bar{c}0,(\omega)}^{\langle \mathrm{NS} \rangle} \right] \right\}$$

where

eg., $\widetilde{\mathcal{I}}^{\rho,\sigma}_{[\kappa],\{\alpha,\beta\}} \sim \frac{(k_-)^{\rho}\,(k_+)^{\sigma}\; f_{i/N} f_{j/\bar{N}}}{(k_+ + m_{\rm T} e^{-Y_H})^{\alpha} (k_- + m_{\rm T} e^{Y_H})^{\beta}}$

$$(q_{\rm T}^2)^{\omega}\; \widetilde{\mathcal{G}}_c^{(\omega)} \sim \lim_{\tau\to 0} \int_0^{\tilde{k}_+^{\max}} \frac{\mathrm{d}k_+}{k_+} \mathcal{R}(k_-, k_+, \tau)\, \widehat{\mathbf{T}}_c^{(\omega)}\; I^{[\kappa]}_{\{\rho,\sigma\}}\,,$$

$$(q_{\rm T}^2)^{\omega}\; \widetilde{\mathcal{G}}_{c0}^{(\omega)} \sim \lim_{\tau\to 0} \int_0^{\infty} \frac{\mathrm{d}k_+}{k_+} \mathcal{R}(k_-, k_+, \tau) \sum_{\overline{\omega}=(\rho+\sigma)}^{2\omega} \left(1 - \frac{\delta^{2\omega}_{\overline{\omega}}}{2}\right) \widehat{\mathbf{T}}_s^{(\overline{\omega})} \widehat{\mathbf{T}}_c^{(\omega)} I^{[\kappa]}_{\{\rho,\sigma\}}\,,$$

- At NLP dual scalings agree with [Inglis-Whalen:2020rpi,Inglis-Whalen:2021bea]
- With analytic rap regulator, dual scalings are reduced to [Jantzen:2011nz]

Zero-bin subtraction beyond LP

Difference between the soft and dual scalings.

(a) Subtrahends in the soft scaling

(b) Subtrahends in the dual scaling

1. supposing $|\mathcal{M}|^2 \propto \frac{1}{(k_- k_+)(k_+ + m_\mathrm{T} e^{-Y_H})(k_- + m_\mathrm{T} e^{Y_H})}$
2. zero-bin subtrahend:

$$\widetilde{\mathcal{G}}_{c0}^{(\omega)} \sim \lim_{\tau \to 0} \int_0^\infty \frac{\mathrm{d}k_+}{k_+} \mathcal{R}(k_-, k_+, \tau) \frac{(k_-)^\lambda (k_+)^\eta}{(m_\mathrm{T} e^{-Y_H})^{\alpha_1} (m_\mathrm{T} e^{+Y_H})^{\beta_2}} f_{i/N}^{(\alpha_2)} f_{j/\bar{N}}^{(\beta_2)}$$

3. As examples, the red stars highlight the subleading (NLP) power corrections, the blue triangles represent the sub-subleading (N^{2}LP) ones, and the green dots indicate the sub-sub-subleading (N^{3}LP) ones.

Application at NLO up to NNLP: exponential rapidity regulator

Power expansion with exponential rapidity regulator $\mathcal{R} = \exp(-\tau b_0 k_0)$ here $b_0 = 2e^{-\gamma_{\mathrm{E}}}$ [Li:2016axz,Li:2016ctv] for the process $pp \to H + X$

$$\frac{\mathrm{d}\sigma_H}{\mathrm{d}Y_H \mathrm{d}q_{\mathrm{T}}^2} = \left.\frac{\mathrm{d}\sigma_H}{\mathrm{d}Y_H \mathrm{d}q_{\mathrm{T}}^2}\right|_{b.c.} + \left.\frac{\mathrm{d}\sigma_H^{\langle\exp\rangle}}{\mathrm{d}Y_H \mathrm{d}q_{\mathrm{T}}^2}\right|_{c} - \left.\frac{\mathrm{d}\sigma_H^{\langle\exp\rangle}}{\mathrm{d}Y_H \mathrm{d}q_{\mathrm{T}}^2}\right|_{c0} + (c \leftrightarrow \bar{c})$$

where

$$\begin{aligned}\left.\frac{\mathrm{d}\sigma_H^{\langle\exp\rangle}}{\mathrm{d}Y_H \mathrm{d}q_{\mathrm{T}}^2}\right|_{c} \equiv \frac{\alpha_s^3 C_t^2}{192\pi^2 s v^2} & \sum_{i,j=\{g,q,\bar{q}\}} \sum_{\omega=-1}^{\infty} \left(\frac{q_{\mathrm{T}}^2}{m_H^2}\right)^{\omega} \int_{x_n}^{1} \mathrm{d}z_n \left[\mathbf{F}_{i/N}\left(\frac{x_n}{z_n}\right)\right]^{\mathbf{T}} \\ & \cdot \left\{\mathbf{R}_{sc}^{(\omega),ij}(z_n) + \mathbf{P}_{sc}^{(\omega),ij}\left[\frac{1}{1-z_n}\right]_+ + \left(\mathbf{T}_c^{(\omega),ij} + \mathbf{D}_{sc}^{(\omega),ij}\right)\delta(1-z_n)\right. \\ & \left. + \mathbf{B}_{sc}^{(\omega),ij}(z_n)\,\delta(x_n - z_n)\right\} \cdot \mathbf{F}_{j/\bar{N}}(x_{\bar{n}})\end{aligned}$$

$$\left.\frac{\mathrm{d}\sigma_H^{\langle\exp\rangle}}{\mathrm{d}Y_H \mathrm{d}q_{\mathrm{T}}^2}\right|_{c0} \equiv \frac{\alpha_s^3 C_t^2}{192\pi^2 s v^2} \sum_{i,j=\{g,q,\bar{q}\}} \sum_{\omega=-1}^{\infty} \left(\frac{q_{\mathrm{T}}^2}{m_H^2}\right)^{\omega} \left[\mathbf{F}_{i/N}(x_n)\right]^{\mathbf{T}} \cdot \mathbf{T}_c^{(\omega),ij} \cdot \mathbf{F}_{j/\bar{N}}(x_{\bar{n}})$$

Power expansion with exponential rapidity regulator $\mathcal{R} = \exp(-\tau b_0 k_0)$ here $b_0 = 2e^{-\gamma_{\mathrm{E}}}$ [Li:2016axz,Li:2016ctv] for the process $pp \to H + X$

$$\frac{\mathrm{d}\sigma_H}{\mathrm{d}Y_H \mathrm{d}q_{\mathrm{T}}^2} = \frac{\mathrm{d}\sigma_H}{\mathrm{d}Y_H \mathrm{d}q_{\mathrm{T}}^2}\bigg|_{b.c.} + \frac{\mathrm{d}\sigma_H^{\langle\exp\rangle}}{\mathrm{d}Y_H \mathrm{d}q_{\mathrm{T}}^2}\bigg|_{c} - \frac{\mathrm{d}\sigma_H^{\langle\exp\rangle}}{\mathrm{d}Y_H \mathrm{d}q_{\mathrm{T}}^2}\bigg|_{c0} + (c \leftrightarrow \bar{c})$$

where

$$\left[\mathbf{F}_{i/n}(\xi_n)\right]^{\mathbf{T}} \equiv \left[f_{i/n}(\xi_n)\,, \xi_n\, f'_{i/n}(\xi_n)\,, \frac{(\xi_n)^2}{2!}\, f''_{i/n}(\xi_n)\,, \ldots\,, \frac{(\xi_n)^k}{k!}\, f^{(k)}_{i/n}(\xi_n)\,, \ldots\right]$$

$$\mathbf{F}_{j/\bar{n}}(\xi_{\bar{n}}) \equiv \left[f_{j/\bar{n}}(\xi_{\bar{n}})\,, \xi_{\bar{n}}\, f'_{j/\bar{n}}(\xi_{\bar{n}})\,, \frac{(\xi_{\bar{n}})^2}{2!}\, f''_{j/\bar{n}}(\xi_{\bar{n}})\,, \ldots\, \frac{(\xi_{\bar{n}})^k}{k!}\, f^{(k)}_{j/\bar{n}}(\xi_{\bar{n}})\,, \ldots\right]^{\mathbf{T}}$$

with

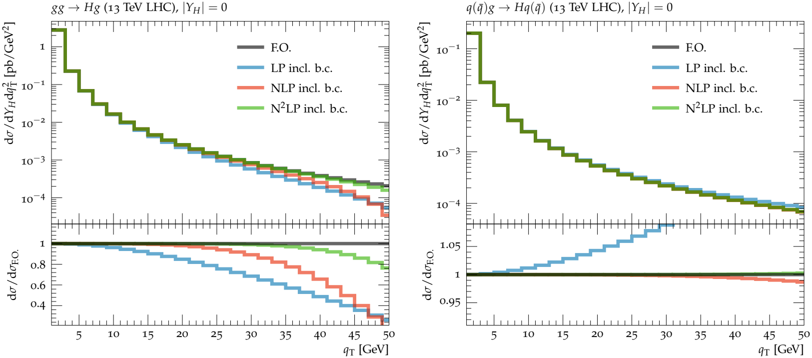
$$\mathbf{T}_c^{(1),gg} = -L_\tau\, \mathbf{P}_c^{(1),gg} + \begin{bmatrix} \frac{3r_n^2}{q_{\mathrm{T}}^2\tilde{\tau}^2} - 3r_n^2 - \frac{7r_n}{2q_{\mathrm{T}}\tilde{\tau}} + \frac{5r_n}{2q_{\mathrm{T}}\tilde{\tau}} & -\frac{r_n^2}{q_{\mathrm{T}}^2\tilde{\tau}^2} + r_n^2 + \frac{7r_n}{2q_{\mathrm{T}}\tilde{\tau}} - \frac{r_n}{2q_{\mathrm{T}}\tilde{\tau}} & \frac{r_n^2}{q_{\mathrm{T}}^2\tilde{\tau}^2} - r_n^2 \\ & & \frac{r_n}{q_{\mathrm{T}}\tilde{\tau}} \end{bmatrix}$$

....

Same as those derived via pure-rapidity regulator/momentum-cutoffs.

Definition of the power series in the low q_T domain

$$\frac{\mathrm{d}\sigma_H}{\mathrm{d}Y_H\,\mathrm{d}q_\mathrm{T}^2} = \underbrace{\overbrace{\underbrace{\sum_m \frac{\Delta_\mathrm{LP}^{(m)}}{q_\mathrm{T}^2}\,(L_H)^m}_{\mathrm{LP}} + \sum_m \Delta_\mathrm{NLP}^{(m)}\,(L_H)^m}^{\mathrm{NLP}} + \sum_m q_\mathrm{T}^2\,\Delta_{\mathrm{N^2LP}}^{(m)}\,(L_H)^m}_{\mathrm{N^2LP}} + \ldots .$$

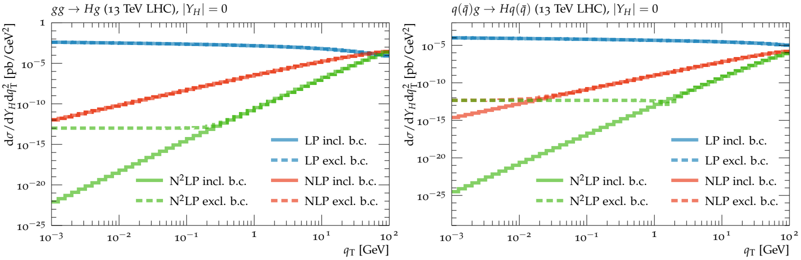


Difference between F.O. and Apps

$$\left.\frac{d\sigma_H}{dY_H\,dq_T^2}\right|_{\text{F.O.}} - \left.\frac{d\sigma_H}{dY_H\,dq_T^2}\right|_{\text{LP}} \sim \sum_m \Delta^{(m)}_{\text{NLP}}\,(L_H)^m$$

$$\left.\frac{d\sigma_H}{dY_H\,dq_T^2}\right|_{\text{F.O.}} - \left.\frac{d\sigma_H}{dY_H\,dq_T^2}\right|_{\text{NLP}} \sim \sum_m q_T^2\,\Delta^{(m)}_{\text{N}^2\text{LP}}\,(L_H)^m$$

$$\left.\frac{d\sigma_H}{dY_H\,dq_T^2}\right|_{\text{F.O.}} - \left.\frac{d\sigma_H}{dY_H\,dq_T^2}\right|_{\text{N}^2\text{LP}} \sim \sum_m q_T^4\,\Delta^{(m)}_{\text{N}^3\text{LP}}\,(L_H)^m$$

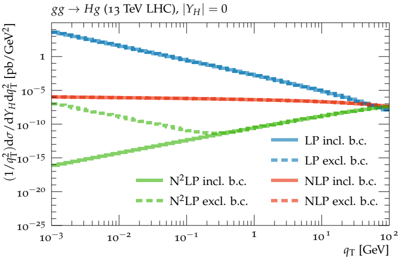


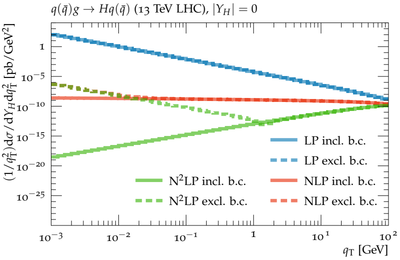
Difference between F.O. and App.

$$\left.\frac{d\sigma_H}{q_T^2 dY_H dq_T^2}\right|_{\text{F.O.}} - \left.\frac{d\sigma_H}{q_T^2 dY_H dq_T^2}\right|_{\text{LP}} \sim \sum_m \frac{1}{q_T^2} \Delta^{(m)}_{\text{NLP}} (L_H)^m$$

$$\left.\frac{d\sigma_H}{q_T^2 dY_H dq_T^2}\right|_{\text{F.O.}} - \left.\frac{d\sigma_H}{q_T^2 dY_H dq_T^2}\right|_{\text{NLP}} \sim \sum_m \Delta^{(m)}_{\text{N}^2\text{LP}} (L_H)^m$$

$$\left.\frac{d\sigma_H}{q_T^2 dY_H dq_T^2}\right|_{\text{F.O.}} - \left.\frac{d\sigma_H}{q_T^2 dY_H dq_T^2}\right|_{\text{N}^2\text{LP}} \sim \sum_m q_T^2 \Delta^{(m)}_{\text{N}^3\text{LP}} (L_H)^m$$





Outline

Upshot

- Resummation-improved q_{T} and $\Delta\phi$ spectra are delivered on the Drell-Yan process up to N^{3}LL'+N^{2}LO with the inclusion of the complete non-singlet and singlet contributions in the hard sector. Theoretical uncertainties are reduced to percent level in the asymptotic regime of $\mathrm{d}\sigma/\mathrm{d}\mathcal{Q}(\mathcal{Q} = q_{\mathrm{T}}/\Delta\phi)$ and also the W/Z correlation $\mathcal{R}_{W/Z}(\mathcal{Q})$.
- Preparation is made for NLP resummation. An algorithm to organize the zero-bin subtrahend is derived via a mathematically well defined approach. We demonstrate that at NLO this method is applicable onto the power expansion at an arbitrary power accuracy with a generic choice of rapidity regularisation scheme. We look forward to generalizing this method to NNLO and beyond.

Thanks for your attention!