Breakdown of collinear factorisation at the leading twist in exclusive $\pi^0\gamma$ photoproduction: Collinear-to-soft Glauber exchanges

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Based on 2311.09146 and 2409.16067 with J. Schönleber, L. Szymanowski and S. Wallon

Original motivation: Extraction of chiral-odd GPDs at *leading* twist.

- $\gamma N \to \rho^0_T \pi^+ N'$:
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- $\gamma N \to \gamma M N'$:
  - $M = \rho^0$: R. Boussarie, B. Pire, L. Szymanowski, S. Wallon: [1609.03830]
  - $M = \pi^\pm$: G. Duplančić, K. Passek-Kumerički, B. Pire, L. Szymanowski, S. Wallon: [1809.08104]
  - $M = \pi^\pm$, $\rho^{0,\pm}$, wider kinematical coverage, various observables:
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Richer kinematics of 3-body final state processes allows the sensitivity of GPDs wrt x to be probed (beyond moment-type dependence, e.g. in DVCS)
J. Qiu, Z. Yu: [2305.15397]

$$\gamma(q) + N(p_1) \to \gamma(k) + M(p_M) + N'(p_2)$$

$$\mathcal{A} = \int_{-1}^{1} dx \int_{0}^{1} dz \; T(x,\xi,z) \; H(x,\xi,t) \; \Phi_M(z)$$

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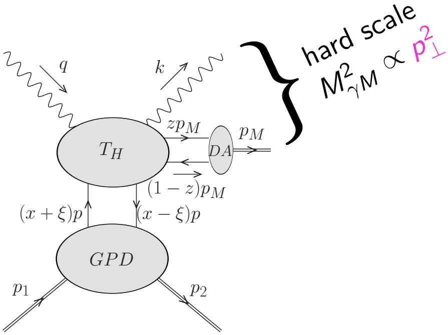
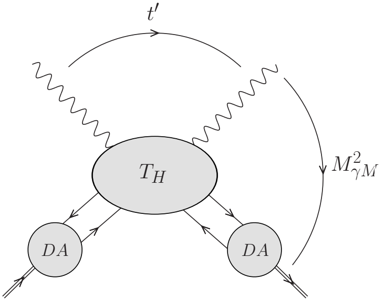
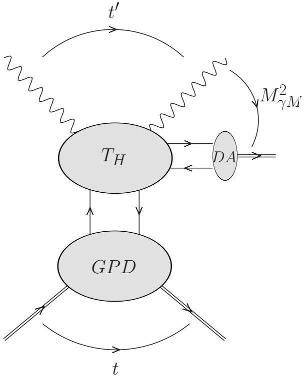
- *Fully differential* cross-section differential covering $S_{\gamma N}$ from $\sim 4\,\text{GeV}^2$ to $20000\,\text{GeV}^2$.
- Good statistics at various experiments, particularly at *JLab*.
- Polarisation asymmetries also sizeable.
- *Small* ξ limit of quark GPDs can be studied at collider experiments.





large angle factorisation
à la Brodsky Lepage



We thus argue *collinear factorisation* of the amplitude at large $M^2_{\gamma M}$, t', u', and small t.

$$t = (p_2 - p_1)^2\,, \qquad u' = (p_M - q)^2\,,$$
$$t' = (k - q)^2\,, \qquad S_{\gamma N} = (q + p_1)^2\,.$$

- Recently, factorisation has been proved for the process $\pi N \to \gamma\gamma N'$ by J. Qiu, Z. Yu [2205.07846].
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Issues with exclusive $\pi^0\gamma$ photoproduction...

- Because of the quantum numbers of π^0 ($J^{PC} = 0^{-+}$), the exclusive photoproduction of $\pi^0\gamma$ is also sensitive to *gluon GPD contributions*.

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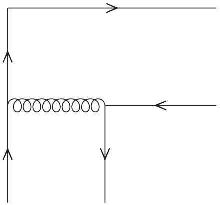
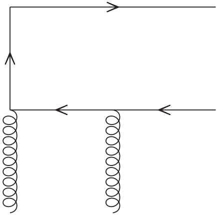
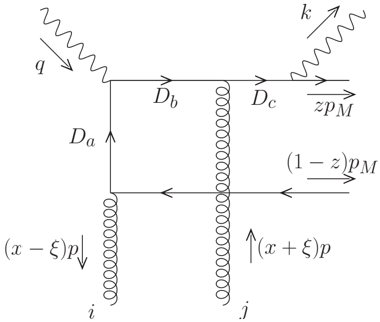
- Diagrams amount to connecting photons to the following two topologies.





Result assuming collinear factorisation

Specific diagram



$$CF \sim \frac{\mathrm{Tr}\left[\not{p}_M \gamma^5 \not{\epsilon}_k \left(\not{k} + z\not{p}_M\right) \gamma^j \left(\not{q} - (x-\xi)\not{p} - \bar{z}\not{p}_M\right) \not{\epsilon}_q \left(-(x-\xi)\not{p} - \bar{z}\not{p}_M\right) \gamma^i\right]}{\left[2z\, kp_M\right]\left[-2\left(x-\xi\right) qp - 2\bar{z}\, qp_M + 2\bar{z}\left(x-\xi\right) pp_M + i\epsilon\right]\left[2\bar{z}\left(x-\xi\right) pp_M + i\epsilon\right]}$$

$$\xrightarrow{x\to\xi,\bar{z}\to 0} \propto \frac{x-\xi}{\left[(x-\xi) + A\bar{z} - i\epsilon\right]\left[\bar{z}\left(x-\xi\right) + i\epsilon\right]}, \qquad A \equiv \frac{qp_M}{qp} > 0.$$

(Assuming p_M is along minus direction)

Result assuming collinear factorisation
Specific diagram

Need to dress coefficient function CF with gluon GPD $\left(\frac{H_g(x)}{(x-\xi+i\epsilon)(x+\xi-i\epsilon)}\right)$, and DA $(z\bar{z})$. This gives

$$\mathcal{A} \sim \frac{\bar{z}\,(x-\xi)\,H_g(x)}{(x-\xi+i\epsilon)\,[(x-\xi)+A\bar{z}-i\epsilon]\,[\bar{z}\,(x-\xi)+i\epsilon]}$$

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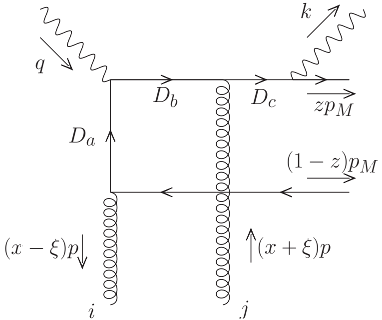
The integral over z and x diverges if the GPD $H_g(x)$ is non-vanishing at $x=\xi$:

$$\int_{-1}^{1} dx \int_{0}^{1} dz \frac{1}{[(x-\xi)+A\bar{z}-i\epsilon]\,[x-\xi+i\epsilon]}$$

$$\supset \int_{-1}^{1} dx \frac{\ln(x-\xi-i\epsilon)}{[x-\xi+i\epsilon]} \quad \Longrightarrow \text{ divergent imaginary part!}$$



$$\int_{-1}^{1} dx \int_{0}^{1} dz \frac{1}{\left[(x-\xi)+A\bar{z}-i\epsilon\right]\left[x-\xi+i\epsilon\right]}$$

$\implies$ The "*pinching*" is caused by propagators D_a and D_b.

What about the sum of diagrams?

$$\sum \mathcal{A} \sim \frac{z\bar{z}\left(x^2-\xi^2\right)\left[-\alpha\left[\left(x^2-\xi^2\right)^2(1-2z\bar{z})+8x^2\xi^2 z\bar{z}\right]-\left(1+\alpha^2\right)z\bar{z}\left(x^4-\xi^4\right)\right]H_g(x)}{z\bar{z}\left[x-\xi+i\epsilon\right]^2\left[\bar{z}\left(x+\xi\right)-\alpha z\left(x-\xi\right)-i\epsilon\right]\left[z\left(x-\xi\right)+\alpha\bar{z}\left(x+\xi\right)-i\epsilon\right]}$$

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$$\xrightarrow{x\to\xi,\bar{z}\to 0}\propto \frac{\left[-\alpha\left[\left(x^2-\xi^2\right)^2(1-2z\bar{z})+\ 8x^2\xi^2 z\bar{z}\ \right]-\left(1+\alpha^2\right)z\bar{z}\left(x^4-\xi^4\right)\right]H_g(x)}{\left[x-\xi+i\epsilon\right]\left[2\xi\bar{z}-\alpha\left(x-\xi\right)-i\epsilon\right]\left[\left(x-\xi\right)+2\xi\alpha\bar{z}-i\epsilon\right]}$$

Result assuming collinear factorisation
Full Amplitude

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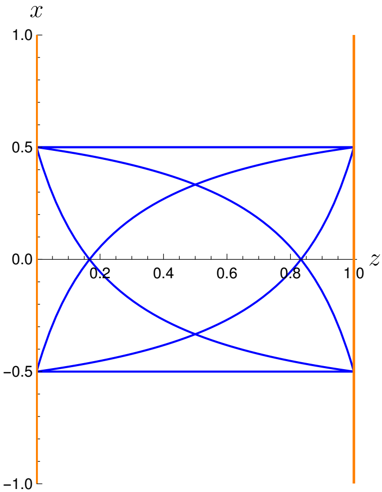
Full amplitude (anti)-symmetric in $x \to -x$ and $z \to \bar{z}$ for (anti)-symmetric GPD. (only symmetric result shown above)

$\Longrightarrow$ *divergence survives*, and actually adds up.

'Phase Space' for amplitude



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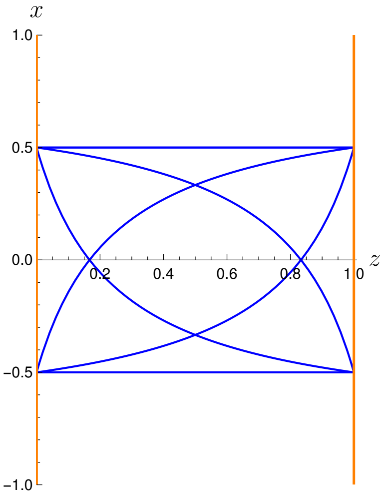

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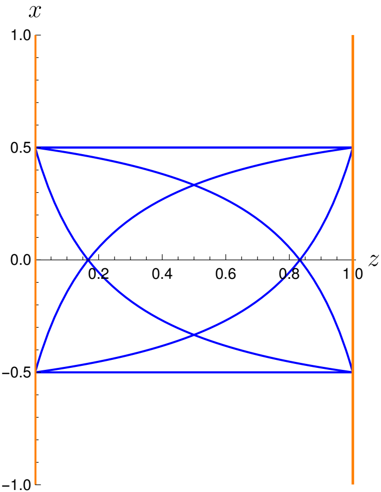

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Result assuming collinear factorisation

Singularity structure of the full amplitude

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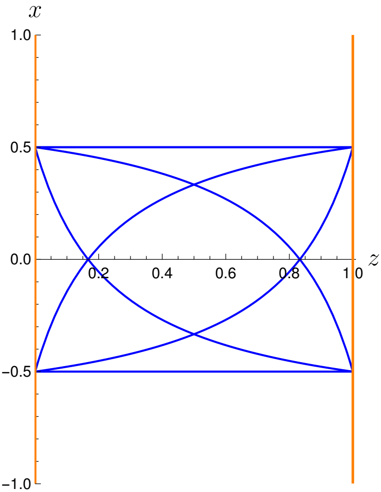

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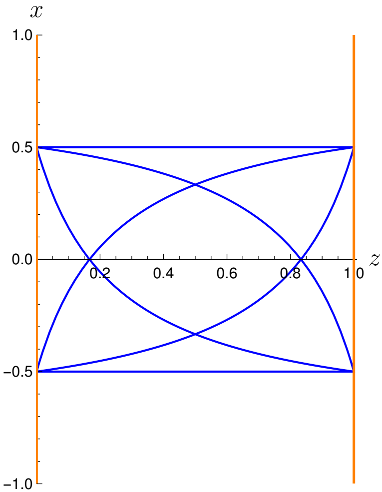

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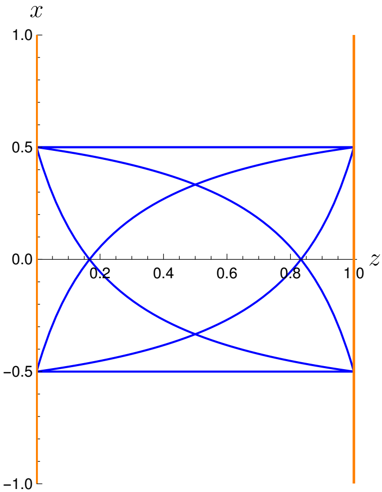

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- *Can this divergence be understood from a theoretical point of view?*
  YES! $\Longrightarrow$ [**S. N.**, J. Schönleber, L. Szymanowski, S. Wallon: 2311.09146, 2409.16067]

- How to obtain the dominant contribution of an amplitude (in QCD) given external specific kinematics (e. g. collinear)?
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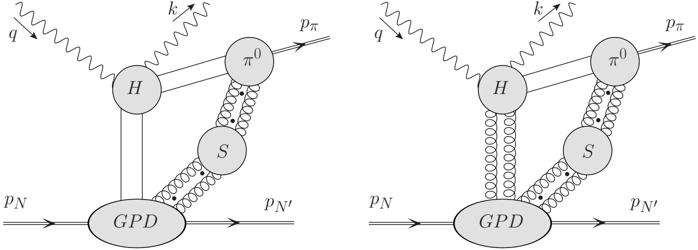
- Basic idea is to identify regions of loop momenta of partons (also number of partons), which gives the dominant contribution to the full amplitude.
- Collect all contributions to the *smallest* α:

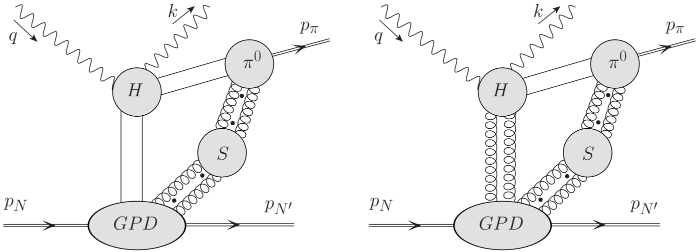
$$\mathcal{A} = Q^{\beta} \sum_{\alpha} f_{\alpha} \lambda^{\alpha} \,, \qquad \lambda = \frac{\Lambda_{\mathrm{QCD}},\ m_{\pi},\ m_{N}}{Q} \ll 1$$



In both of the above cases, the power counting is [**S. N.**, J. Schönleber, L. Szymanowski, S. Wallon: 2311.09146]:

$$\mathcal{A} \sim Q^{-1}\lambda^{\alpha}\,, \qquad \lambda = \frac{\Lambda_{\rm QCD},\, m_\pi,\, m_N}{Q} \ll 1\,, \qquad \alpha = 1$$

Reduced diagram analysis
Classic Collinear pinch



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Collinear factorisation at *all orders* and *leading power* provided:

- the above collinear pinch diagrams (standard) are the *only ones contributing to the leading power of $\alpha = 1$*
- the the soft factor S factorises into *process-independent Wilson lines*

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Given z, $\omega_S \in \mathbb{R}^{dL}$ such that the set

$$\mathcal{D} = \{ j \in \{1, ..., n\} \mid D_j(\omega_S, z) = 0 \}$$

is non-empty, a necessary condition for a pinch at ω_S is that for $j \in \mathcal{D}$, there exist real and non-negative numbers α_j such that

- $\forall i \in \{1, ..., dL\} \; : \; \sum_{j \in \mathcal{D}} \alpha_j \frac{\partial D_j}{\partial \omega_i}(\omega_S; z) = 0.$
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Note: Existence of pinch does *not* imply existence of a singularity: Need to also perform *power counting*.

Pinches

Soft pinch always present

Consider the bubble integral, with massless internal lines:

$$I_1(p^2) = \lim_{\epsilon \to 0^+} \int d^4k \, \frac{1}{(k^2 + i\epsilon)((p-k)^2 + i\epsilon)}.$$

Pinches

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According to the Landau conditions, there is *always* a pinch related to soft momentum k, independent of p.

This is because when $k = 0$, both the propagator $k^2 + i\epsilon$ and its first derivative are zero.

$\Longrightarrow$ *Landau conditions for a pinch at $k = 0$ are satisfied.*

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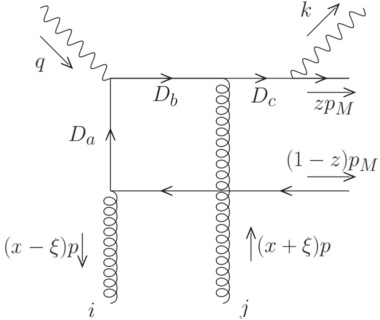
However, note that the power counting does not give an IR divergence for $p^2 \neq 0$. Take $k^\mu \sim \lambda$ (i.e. all components scale as λ):

$$\Longrightarrow \frac{[\lambda^4]}{[\lambda^2][1]} \sim \lambda^2$$

Reduced diagram analysis

Other leading pinch surfaces?



Divergence obtained when $(x-\xi)\,p$ and $(1-z)\,p_M$ lines become soft:

$\implies$ D_a becomes soft and D_b becomes collinear with respect to q.

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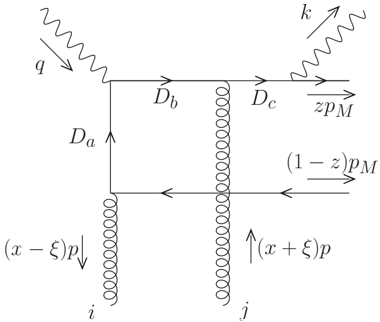

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Is there a *leading* pinch diagram that corresponds to this region?
Yes!

Reduced diagram analysis
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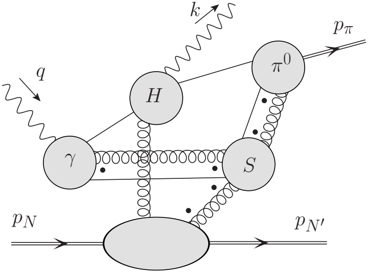

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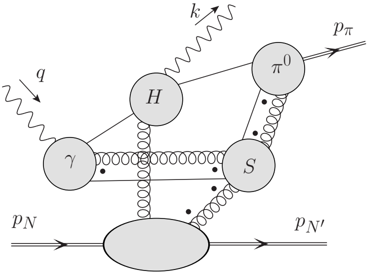

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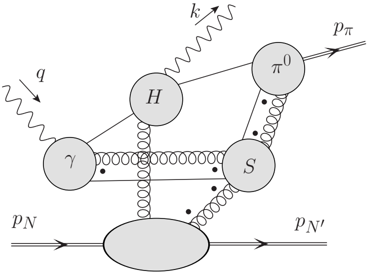
$\Longrightarrow$ power counting is the same as the collinear region!

Reduced diagram analysis
Other leading pinch surfaces?



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Note: Corresponding reduced diagram for quark GPD case is power suppressed.

What exactly does the pinch surface correspond to?

- Use Sudakov basis $(+, -, \perp)$ (set $Q = 1$ for simplicity):

$$\bar{n}\text{-coll.}: \quad k \sim \left(1, \lambda^2, \lambda\right) \qquad n\text{-coll.}: k \sim \left(\lambda^2, 1, \lambda\right)$$

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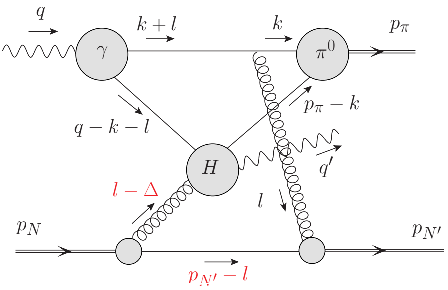
- Key Question: Is there a Glauber pinch that contributes at leading power?

Glauber pinch



(Notation: $(+,-,\perp)$)

$p_N,\, p_{N'},\, \Delta \sim \left(1, \lambda^2, \lambda\right), \quad \Delta^+ < 0.$

$p_\pi \sim \left(\lambda^2, 1, \lambda\right)$

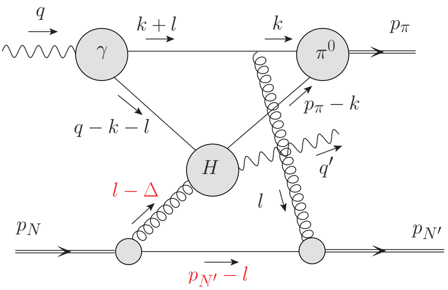
$q,\, q' \sim (1,1,1), \quad q^2,\, q'^2 \sim \lambda^2$

[Loop] $l \sim (\lambda, \lambda, \lambda)$

[Loop] $k \sim (\lambda, \lambda, \lambda)$

Recall: Soft loop momenta r and k *always* need to be considered.

Glauber pinch



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Recall: Soft loop momenta r and k *always* need to be considered.

- *l^- pinch*:

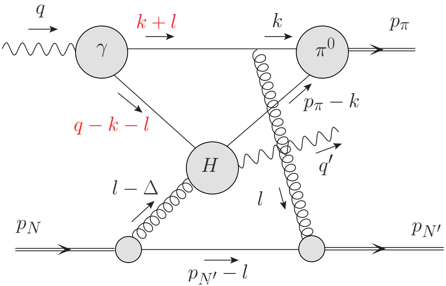
$$(l-\Delta)^2 + i0 = -2\Delta^+ l^- + \mathcal{O}(\lambda^2) + i0$$

$$\implies l^- = \mathcal{O}(\lambda^2) - i0\,.$$

$$(p_{N'} - l)^2 + i0 = -2p_{N'}^+ l^- + \mathcal{O}(\lambda^2) + i0$$

$$\implies l^- = \mathcal{O}(\lambda^2) + i0\,.$$

Glauber pinch



l^+ pinch:

$$(q-k-l)^2 + i0 = -2q^+k^- - 2q^-l^+ + \mathcal{O}(\lambda) + i0$$
$$\implies l^+ = \mathcal{O}(\lambda) + i0\,.$$

$$(k+l)^2 + i0 = 2l^+k^- + \mathcal{O}(\lambda^2) + i0$$
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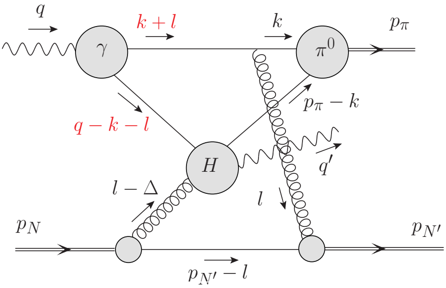
Glauber pinch



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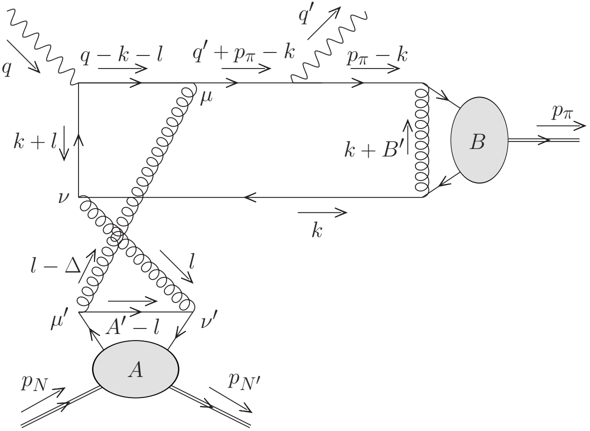
Conclusion: l^+ is pinched to be $\mathcal{O}(\lambda)$, and l^- is pinched to be $\mathcal{O}(\lambda^2)$.
$\implies$ Glauber pinch, since $l^+l^- \ll |l_\perp|^2$.

Glauber pinch is leading



Explicit 2-loop analysis shows that the Glauber pinch demonstrated previously is leading, i.e. it scales as λ^α, with $\alpha = 1$.

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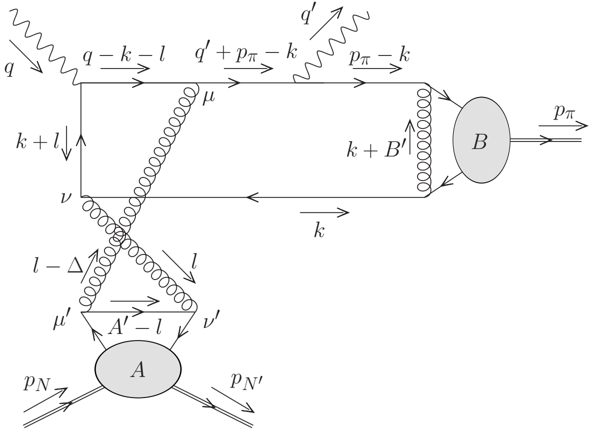

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Glauber pinch persists for any routing of the loop momentum l

Very similar to the exclusive double diffractive process, where the Glauber gluon is pinched between the two pairs of incoming and outgoing collinear hadrons.

$$p(p_{N_1}) + p(p_{N_2}) \longrightarrow p(p_{N_1'}) + p(p_{N_2'}) + \gamma(k_1) + \gamma(k_2)$$

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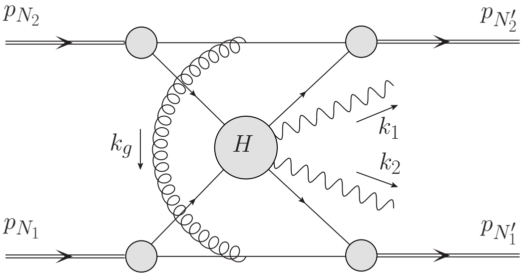

Here, the Glauber pinch corresponds to $k_g \sim (\lambda^2, \lambda^2, \lambda)$

Glauber pinch

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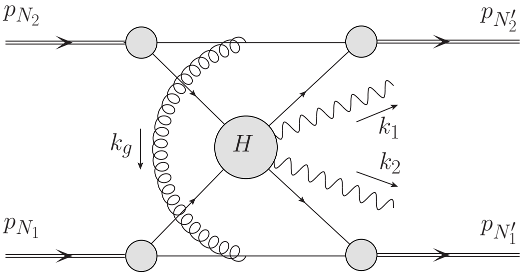

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Instead, in our case, the Glauber gluon (which corresponds to one of the active partons) is pinched between *a pair of collinear hadrons*, and *a soft line joining the outgoing pion and the incoming photon*.

Conclusions

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- Compute $\gamma N \to \gamma\pi^0 N$ in high-energy (k_T) factorisation. [ongoing]

BACKUP SLIDES

More about pinches
Soft pinch always present

Consider the *triangle* integral, with *massless* internal lines:

$$I_2 = \lim_{\epsilon \to 0^+} \int d^4k \, \frac{1}{(k^2 + i\epsilon)((k - p_1)^2 + i\epsilon)((k + p_2)^2 + i\epsilon)}.$$

Again, Landau conditions predict the existence of a pinch at $k = 0$.

If $p_1^2 = m_1^2$ and $p_2^2 = m_2^2$, then the power counting predicts a *logarithmic divergence*:

$$\implies \frac{[\lambda^4]}{[\lambda^2][\lambda][\lambda]} \sim \lambda^0$$

This is of course the well-known soft singularity of triangle integrals, where the massless particle connects to two on-shell legs.

More about pinches
Collinear pinch

Consider the bubble integral, with *massless* internal lines:

$$I_1(p^2) = \lim_{\epsilon \to 0^+} \int d^4k \, \frac{1}{(k^2 + i\epsilon)((p-k)^2 + i\epsilon)}.$$

We apply the Landau conditions:

$$k^2 = 0, \qquad p^2 - 2p \cdot k = 0, \qquad \alpha_1 k + \alpha_2 (k - p) = 0$$
$$\alpha_1, \alpha_2 \geq 0, \quad \alpha_1 + \alpha_2 > 0$$

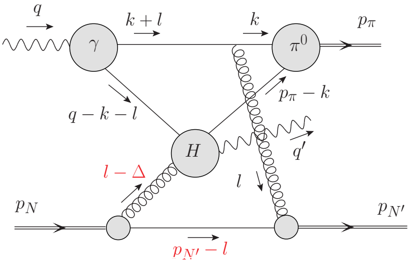
This implies

$$k^2 = 0, \qquad p^2 - 2p \cdot k = 0, \qquad k = \alpha p,$$

where $1 \geq \alpha \geq 0$. This only has a solution if $p^2 = 0$. This is of course nothing but the well-known collinear singularity.

The power counting indicates a *logarithmic divergence*:

$\implies \dfrac{[\lambda^4]}{[\lambda^2][\lambda^2]} \sim \lambda^0$, as expected

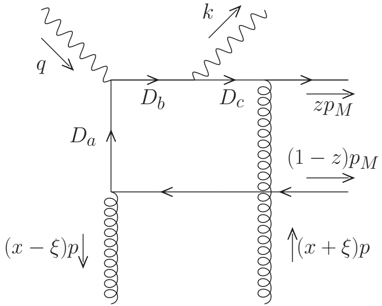


Start with $k \sim (\lambda_s, \lambda_s, \lambda_s)$, where $\lambda_s \ll 1$, but completely general wrt λ. *Study pole in k^+*:

$$k^2 + i0 = 2k^+k^- - |k_\perp|^2 + i0\,,$$
$$\implies k^+ = \mathcal{O}(\lambda_s) - \mathrm{sgn}(k^-)\, i0\,.$$

$$(p_\pi - k)^2 + i0 = -2p_\pi^- k^+ + \mathcal{O}(\max(\lambda^2, \lambda_s^2)) + i0\,,$$
$$\implies k^+ = \mathcal{O}(\max(\lambda^2, \lambda_s^2)) + i0\,.$$

Non-analyticity at $k^- = 0$, and k^+ pinched to be $\mathcal{O}(\lambda_s)$ for $\lambda_s \geq \lambda^2$, or k^+ pinched to be $\mathcal{O}(\lambda^2)$ for $\lambda_s \leq \lambda^2$



$$D_a = ((x-\xi)p + \bar{z}p_M)^2 + i\epsilon$$
$$= s\bar{\alpha}\bar{z}\left[x-\xi+i\epsilon\right] \ ,$$

$$D_b = (k + zp_M - (x+\xi)p)^2 + i\epsilon$$
$$= -s\left[z\left(x-\xi-i\epsilon\right) + \alpha\bar{z}\left(x+\xi-i\epsilon\right)\right] \ ,$$

$$D_c = (zp_M - (x+\xi)p)^2 + i\epsilon$$
$$= -s\bar{\alpha}z\left[x+\xi-i\epsilon\right]$$

$\Longrightarrow$ *pinching of poles in the propagators (D_a and D_b) in the limit of $z \to 1$*