HELICITY TMD AT NLO

ALESSIA BONGALLINO

Universidad del País Vasco

Resummation, Evolution, Factorization 2024

Based on

- arXiv:2409.18078v1 [hep-ph] A. Bacchetta, **A.B.**, M. Cerutti, M. Radici, L. Rossi
- Work in progress, **A.B.**, M. G. Echevarria, G. Schnell

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RECENT RESULTS ON HELICITY TMD

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PARTON DISTRIBUTION FUNCTION $g_1(x, k_\perp)$

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FRAGMENTATION FUNCTION $G_1(z, k_\perp)$

Work in progress,
A.B., M. G. Echevarria, G. Schnell

HELICITY TMD g_1

HELICITY PDF g_1

Collinear component $g_1(x)$ is well known

$$g_1^q(x) = q^+ - q^-$$

- How the polarization of the proton reflects on its internal structure?

J. J. Ethier and E. R. Nocera, Ann. Rev. Nucl. Part. Sci. 70, 43 (2020)

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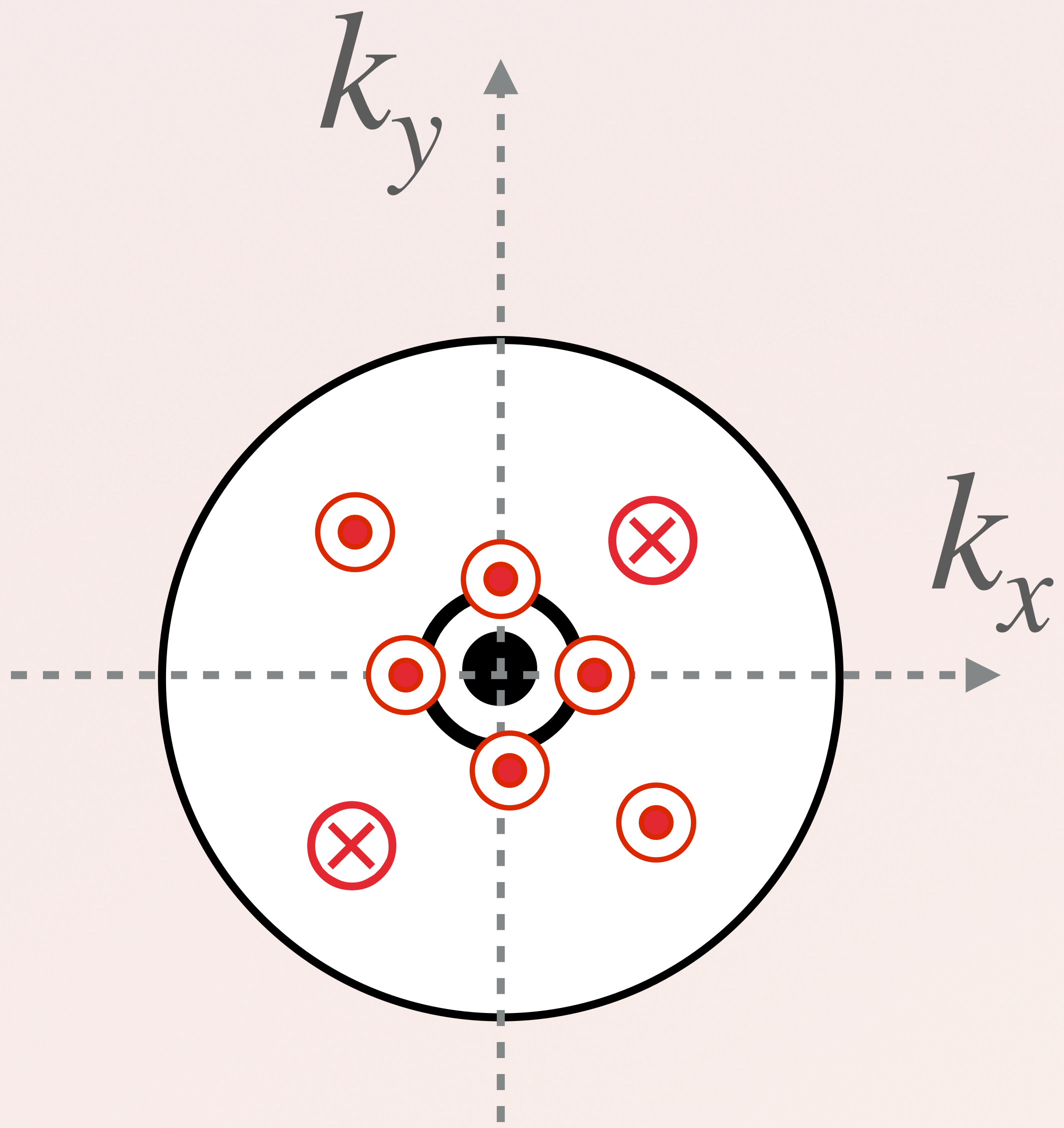

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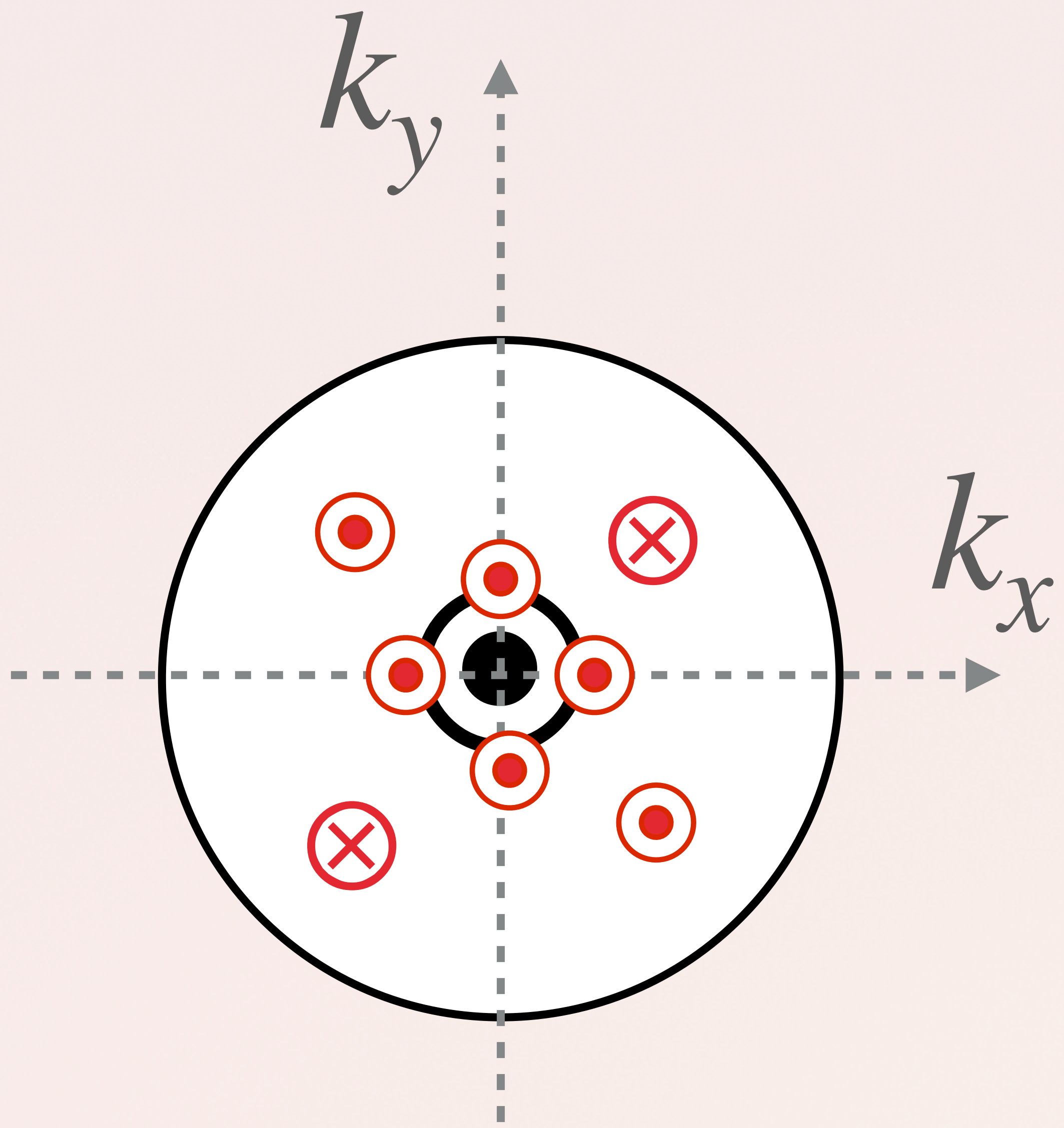

- How the polarization of the proton reflects on its internal structure in 3 dimensions?
- How the polarization of the quark **distorts** their transverse momentum?
- Do quarks with spin parallel to the proton's spin have smaller or larger transverse momentum?

HELICITY EXTRACTION: PROCESS AND OBSERVABLE

Analysis of longitudinally polarized process

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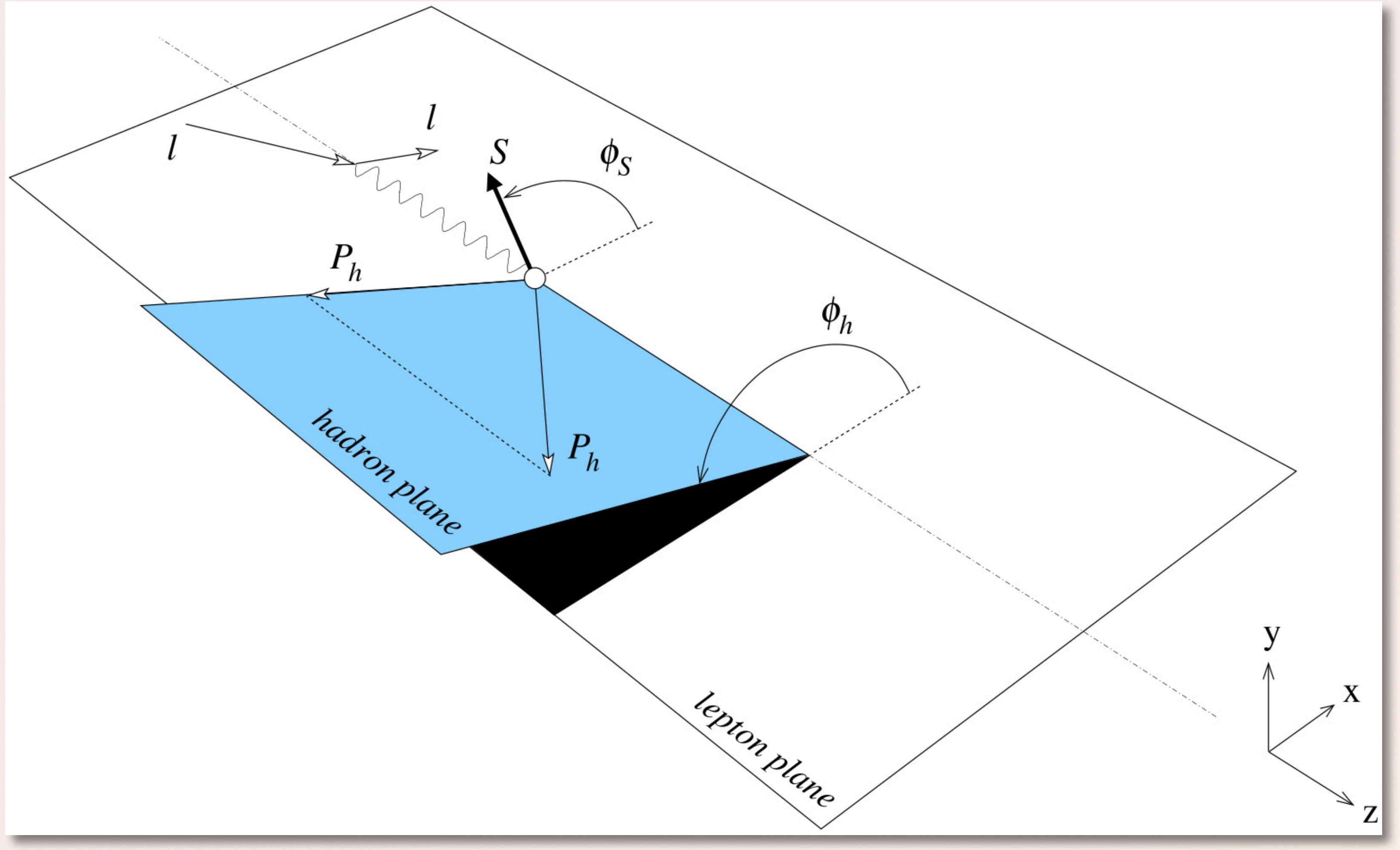
SIDIS

$$\ell^{\rightleftarrows}(l) + N^{\leftrightarrows}(P) \rightarrow \ell(l') + h(P_h) + X$$



A. Bacchetta et al., Phys.Rev.D 70 (2004), 117504

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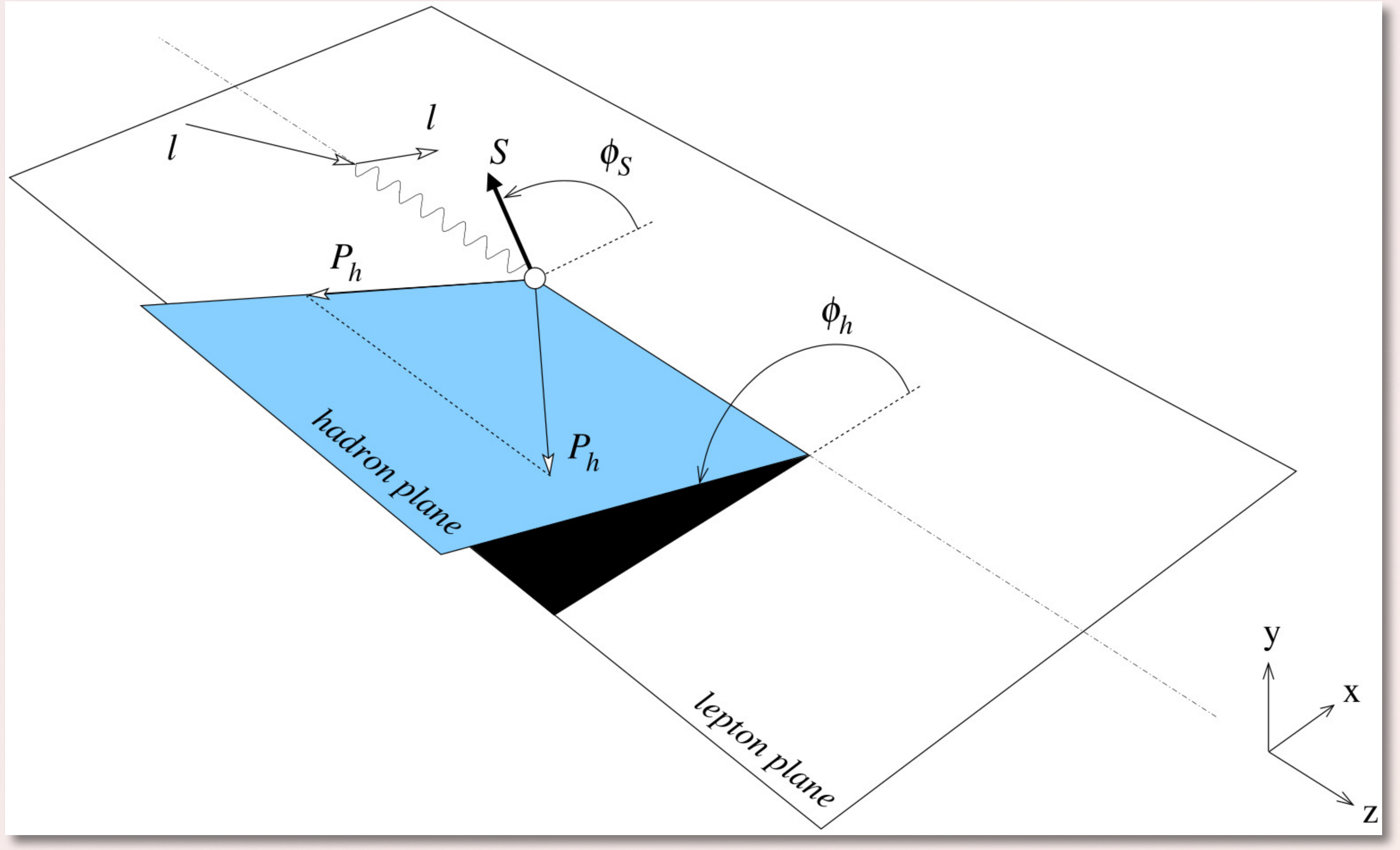

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DOUBLE SPIN ASYMMETRY

$$A_1 = \frac{d\sigma^{\rightarrow\leftarrow} - d\sigma^{\rightarrow\rightarrow} + d\sigma^{\leftarrow\rightarrow} - d\sigma^{\leftarrow\leftarrow}}{d\sigma^{\rightarrow\leftarrow} + d\sigma^{\rightarrow\rightarrow} + d\sigma^{\leftarrow\rightarrow} + d\sigma^{\leftarrow\leftarrow}}$$

M. Diehl and S. Sapeta, Eur. Phys. J. C 41, 515 (2005)

INTERPRETATION IN TERMS OF TMDS

$$A_1(x, z, Q, |\boldsymbol{P}_{hT}|) = \frac{\sum_{a=q,\bar{q}} e_a^2 \int_0^{+\infty} d|\boldsymbol{b}_T|^2 J_0\left(\frac{|\boldsymbol{b}_T||\boldsymbol{P}_{hT}|}{z}\right) \hat{g}_1^a(x, |\boldsymbol{b}_T|^2, Q)\, \hat{D}_1^{a\to h}(z, |\boldsymbol{b}_T|^2, Q)}{\sum_{a=q,\bar{q}} e_a^2 \int_0^{+\infty} d|\boldsymbol{b}_T|^2 J_0\left(\frac{|\boldsymbol{b}_T||\boldsymbol{P}_{hT}|}{z}\right) \hat{f}_1^a(x, |\boldsymbol{b}_T|^2, Q)\, \hat{D}_1^{a\to h}(z, |\boldsymbol{b}_T|^2, Q)}$$

Invariant mass Q of exchanged γ^* is the hard scale of the process

Power corrections of the type $\boldsymbol{P}_{hT}^2/Q^2$, $\boldsymbol{P}_{hT}^2/z^2Q^2$, M_h^2/Q^2 are neglected

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J. C. Collins, D. E. Soper, and G. F. Sterman, Nucl. Phys. B 250, 199 (1985)

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The evolution of the TMDs follows the CSS approach **consistently**:

$$\hat{f}_1(x, |\boldsymbol{b}_T|^2, Q) = \left[C^f \otimes f_1\right](x, b_\star(|\boldsymbol{b}_T|^2))\, f_{NP}(x, |\boldsymbol{b}_T|^2, Q_0)\, e^{S(\mu_{b_\star}^2, Q^2)}\, e^{g_K(\boldsymbol{b}_T)\ln(Q^2/Q_0^2)}$$

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MAPTMD22

A. Bacchetta et al., JHEP 10 (2022), 127

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MAPTMD22

A. Bacchetta et al., JHEP 10 (2022), 127

Known only up to NLO

D. Gutiérrez-Reyes et al., Phys. Lett. B 769, 84 (2017)

COLLINEAR SETS

Choice of the collinear sets:

$$\hat{f}_1(x, |\boldsymbol{b}_T|^2, Q) = \left[C^f \otimes f_1\right](x, b_\star(|\boldsymbol{b}_T|^2))\, f_{NP}(x, |\boldsymbol{b}_T|^2, Q_0)\, e^{S(\mu_{b_\star}^2, Q^2)}\, e^{g_K(\boldsymbol{b}_T)\ln(Q^2/Q_0^2)}$$

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Same datasets as MAPTMD22 for unpolarized collinear functions

✦ $f_1(x) \rightarrow$ MMHT2014 set L.A. Harland-Lang et al., Eur.Phys.J.C 75

✦ $D_1(z) \rightarrow$ DSS14, D. de Florian, et al., *Phys. Rev. D* 91 (2015) 014035

DSS17 sets D. de Florian, et al., Phys. Rev. D 95 (2017) 094019

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$g_1(x) \rightarrow$ NNPDFpol1.1 set

E. R. Nocera et al. (NNPDF), Nucl. Phys. B 887, 276 (2014)

CHOICE OF PARAMETERIZATION

Parameterization of the nonperturbative part:

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MAPTMD22

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$$f_{NP}^{MAP22}(x, k_\perp^2, Q_0) = \frac{\exp\left(-\frac{k_\perp^2}{g_{1A}(x)}\right) + k_\perp^2 \lambda^2 \exp\left(-\frac{k_\perp^2}{g_{1B}(x)}\right) + \lambda_2^2 \exp\left(-\frac{k_\perp^2}{g_{1C}(x)}\right)}{\pi\left(g_{1A}(x) + \lambda^2 g_{1B}(x)^2 + \lambda_2^2\, g_{1C}(x)\right)}$$

$$g_{\{1A,1B,1C\}}(x) = N_{\{1,2,3\}} \frac{(1-x)^{\alpha^2_{\{1,2,3\}}}\, x^{\sigma_{\{1,2,3\}}}}{(1-\hat{x})^{\alpha^2_{\{1,2,3\}}}\, \hat{x}^{\sigma_{\{1,2,3\}}}}$$

CHOICE OF PARAMETERIZATION

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$$k_{norm}(x) \to \int d^2\boldsymbol{k}_\perp\, g_{NP} = 1$$

$\omega_1(x) \to$ crucial to satisfy $|g_1| \leq f_1$

CHOICE OF PARAMETERIZATION

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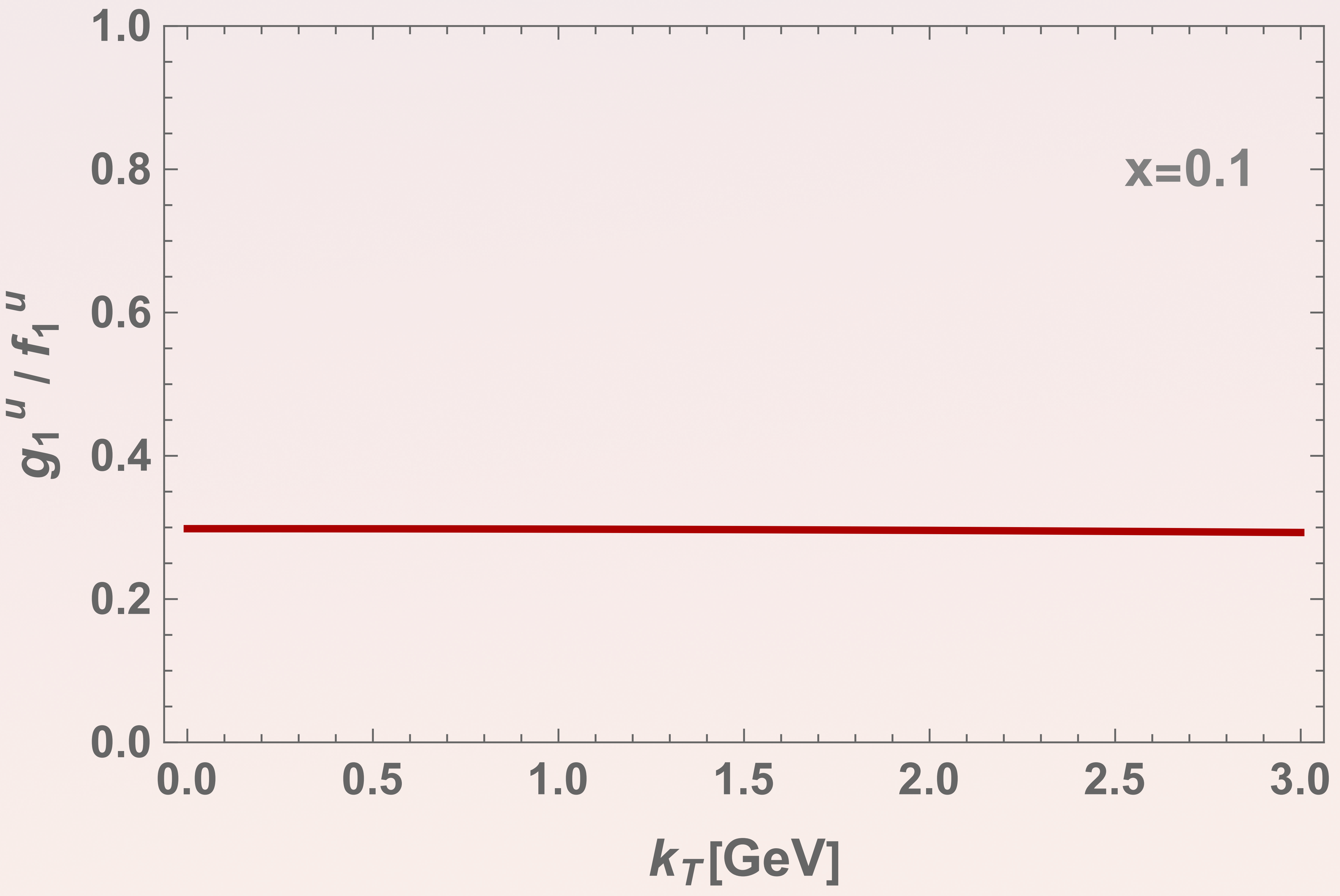
$$\omega_1(x) \to \infty \implies g_1 \simeq f_1$$



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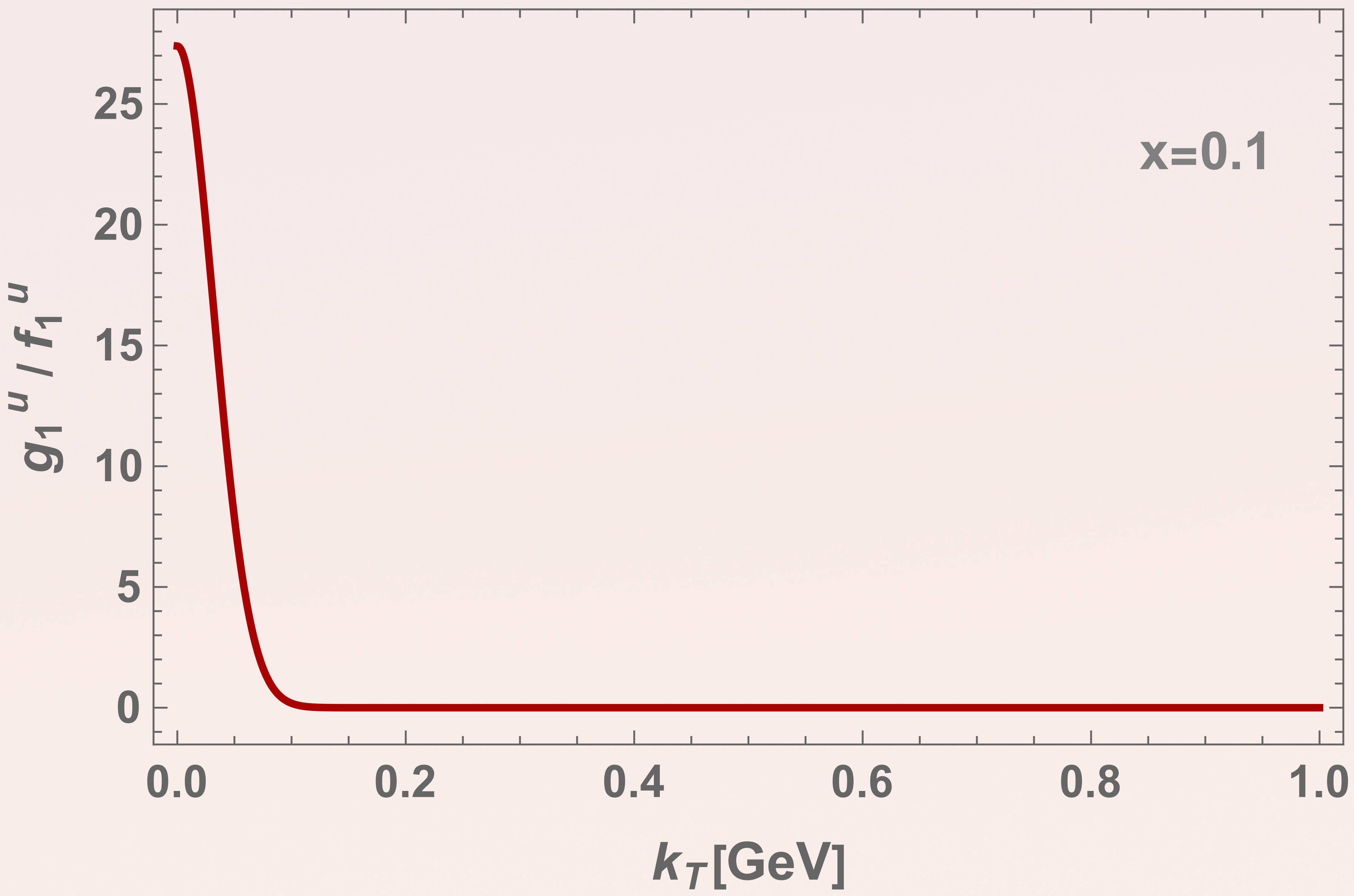

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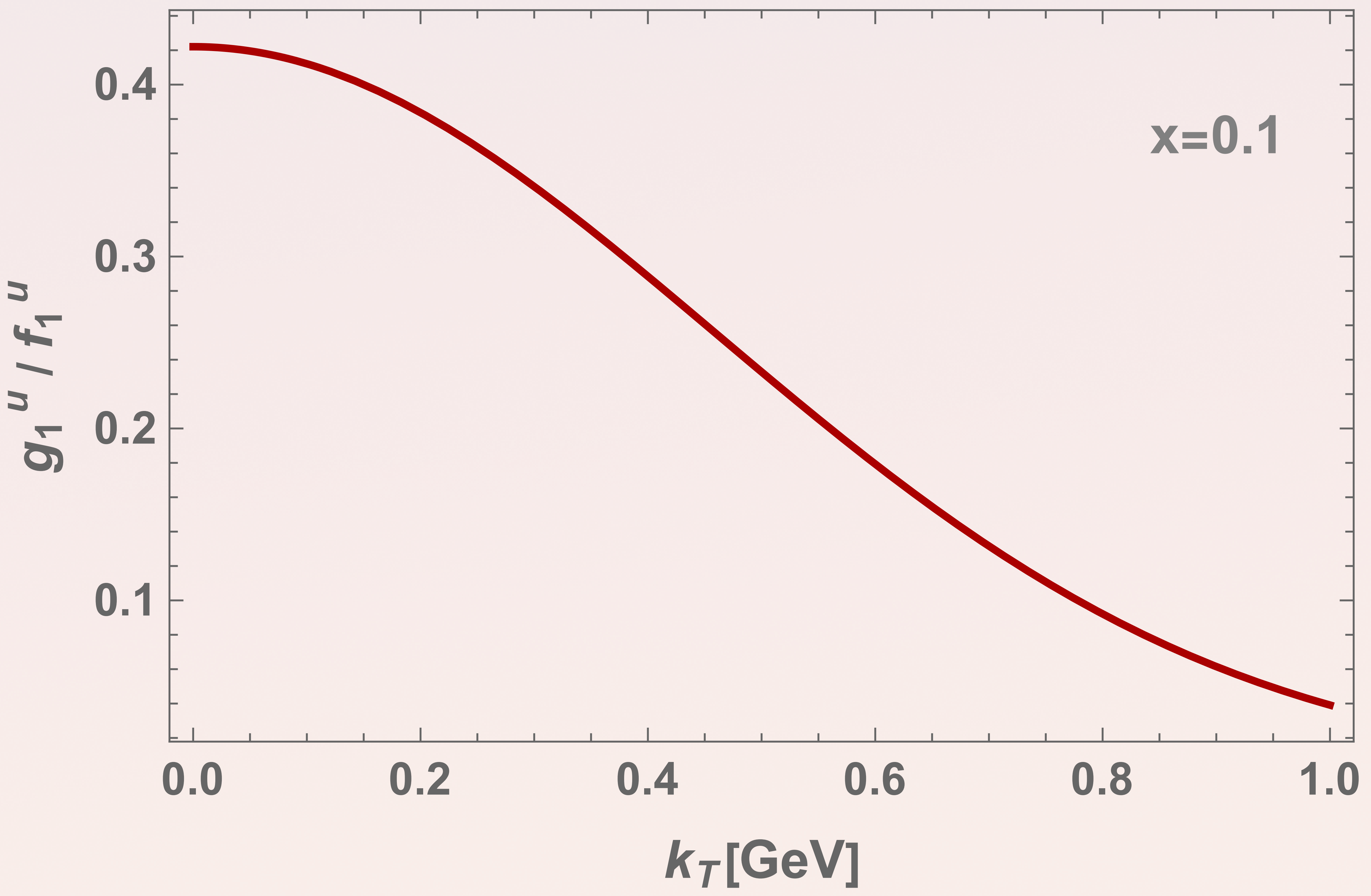
Ratio taken from our fit



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- $N_{1g}, \alpha_{1g}, \sigma_{1g}$ are the **free parameters** of the fit, $\hat{x} = 0.1$.

RESULTS

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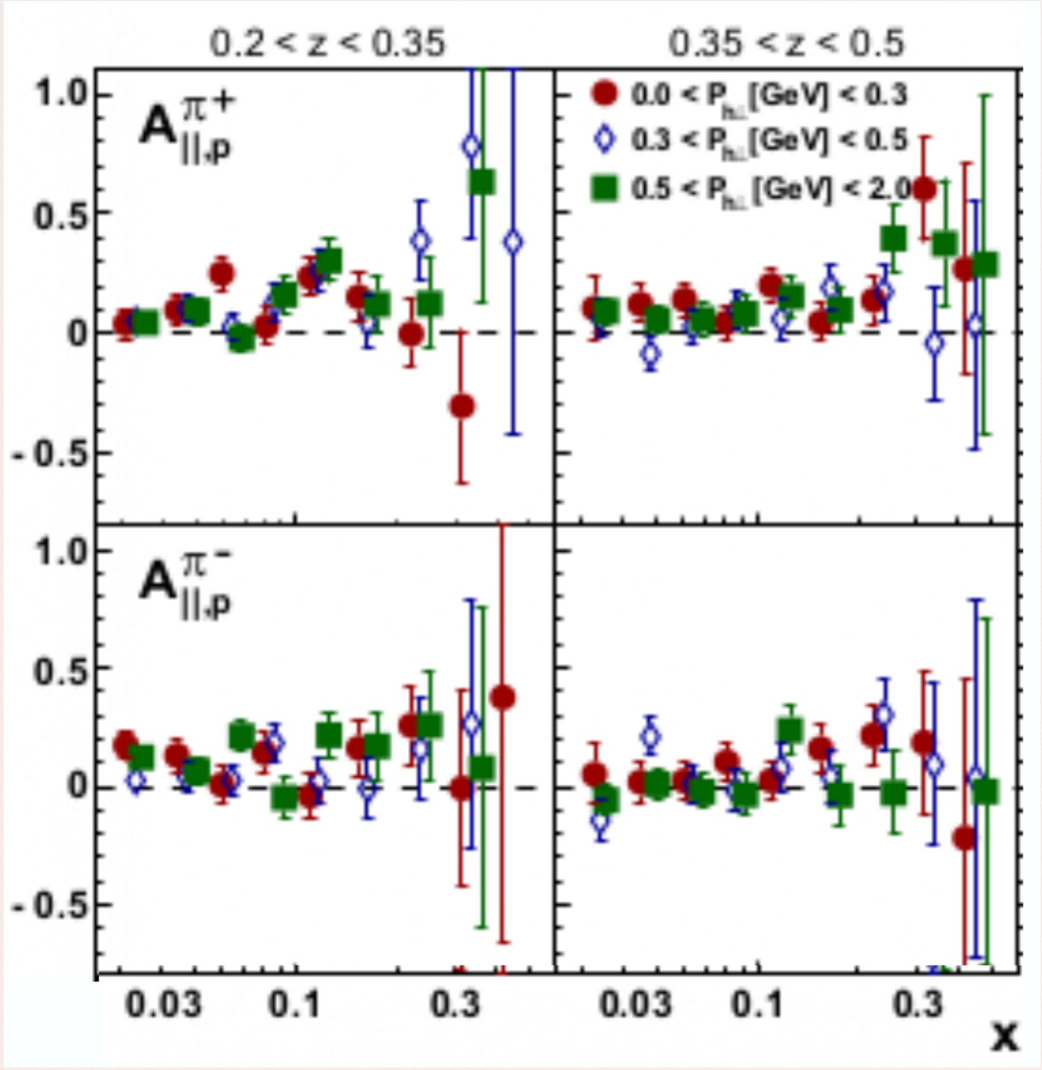

A. Airapetian et al. (HERMES), Phys. Rev. D 99, 112001 (2019)

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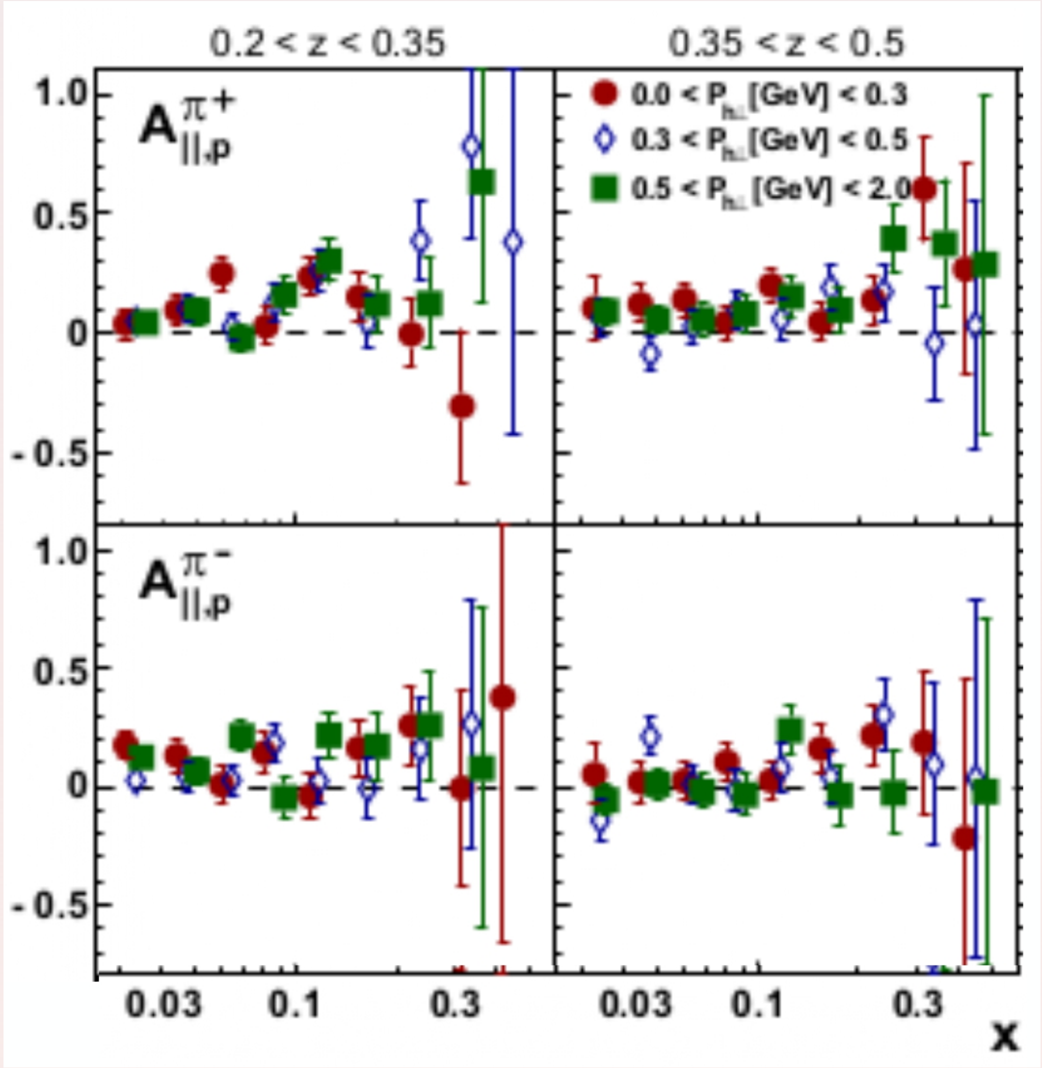

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NOT INCLUDED

➤ COMPASS Collaboration deuteron target data

➤ CLAS6 Collaboration data

FITTING FRAMEWORK

https://github.com/MapCollaboration/NangaParbat

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Experiment	N_{dat}	$\chi^2_{\text{NLL}}/N_{\text{dat}}$	$\chi^2_{\text{NNLL}}/N_{\text{dat}}$
HERMES $(d \to \pi^+)$	47	1.34	1.30
HERMES $(d \to \pi^-)$	47	1.10	1.08
HERMES $(d \to K^+)$	46	1.26	1.25
HERMES $(d \to K^-)$	45	0.93	0.89
HERMES $(p \to \pi^+)$	53	1.17	1.21
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Total	291	1.11	1.09

Parameters	N_{1g}	α_{1g}	σ_{1g}
NLL	0.70 ± 0.54	27.81 ± 27.70	0.42 ± 0.86
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- 291 fitted data points
- Perturbative order: **NLO**
- Perturbative accuracy: **NLL** & **N2LL**
- 3 fitted parameters
- Error analysis with bootstrap method

Parameters	N_{1g}	α_{1g}	σ_{1g}
NLL	0.70 ± 0.54	27.81 ± 27.70	0.42 ± 0.86
NNLL	0.87 ± 0.72	6.73 ± 6.58	3.04 ± 3.09

REPLICA METHOD

$$f_1(x) \rightarrow \text{MMHT2014 set}, \; D_1(z) \rightarrow \text{DSS14, DSS17 sets}$$
$$g_1(x) \rightarrow \; \textbf{NNPDFpol1.1:} \; \text{100 MC members}$$

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$f_1(x) \to$ MMHT2014 set, $D_1(z) \to$ DSS14, DSS17 sets

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$\downarrow$

100 replicas of A_1 data points to be fitted

i-th replica of $g_1(x)$ and the extracted g_1 TMD
associated with the **same replica** of unpolarized TMDs

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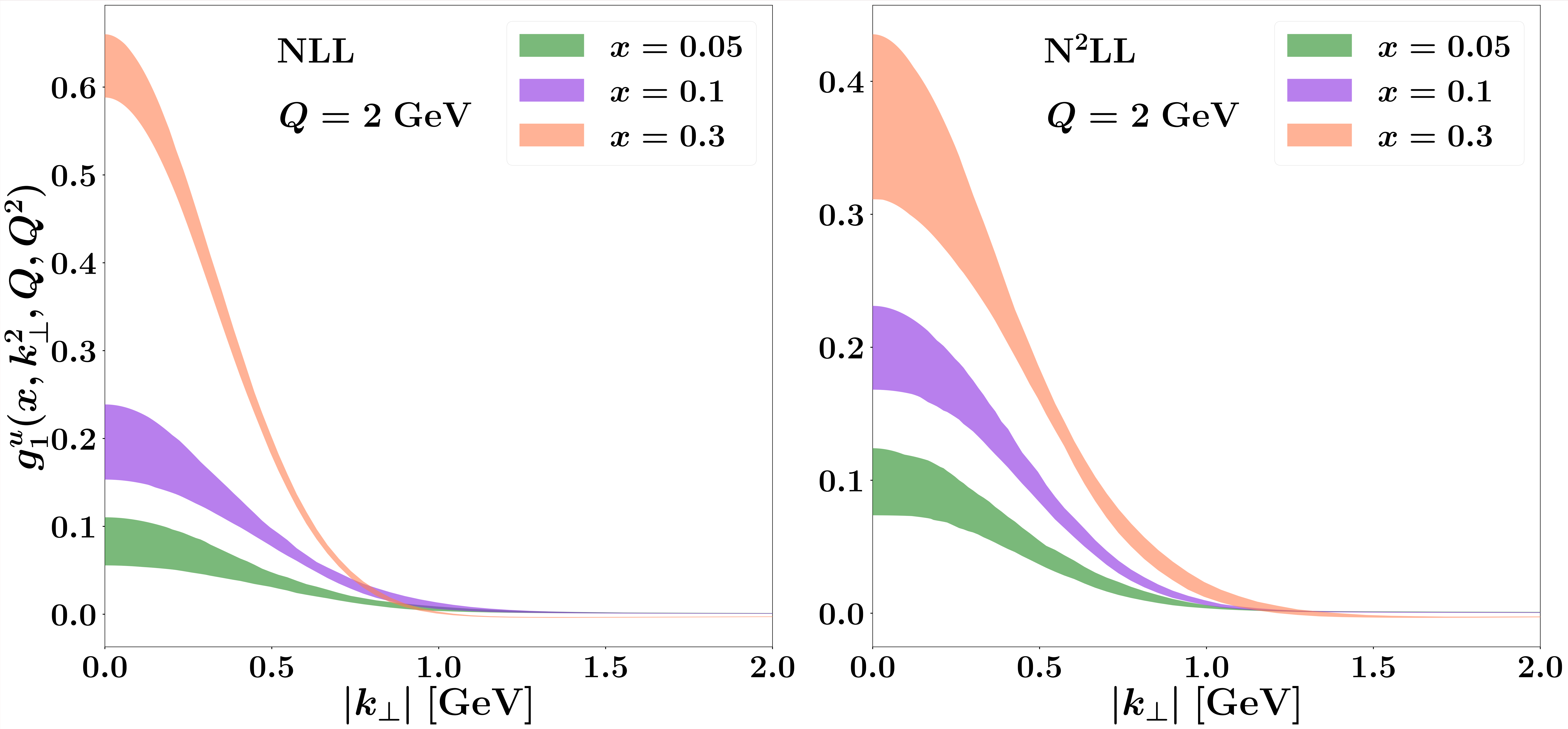
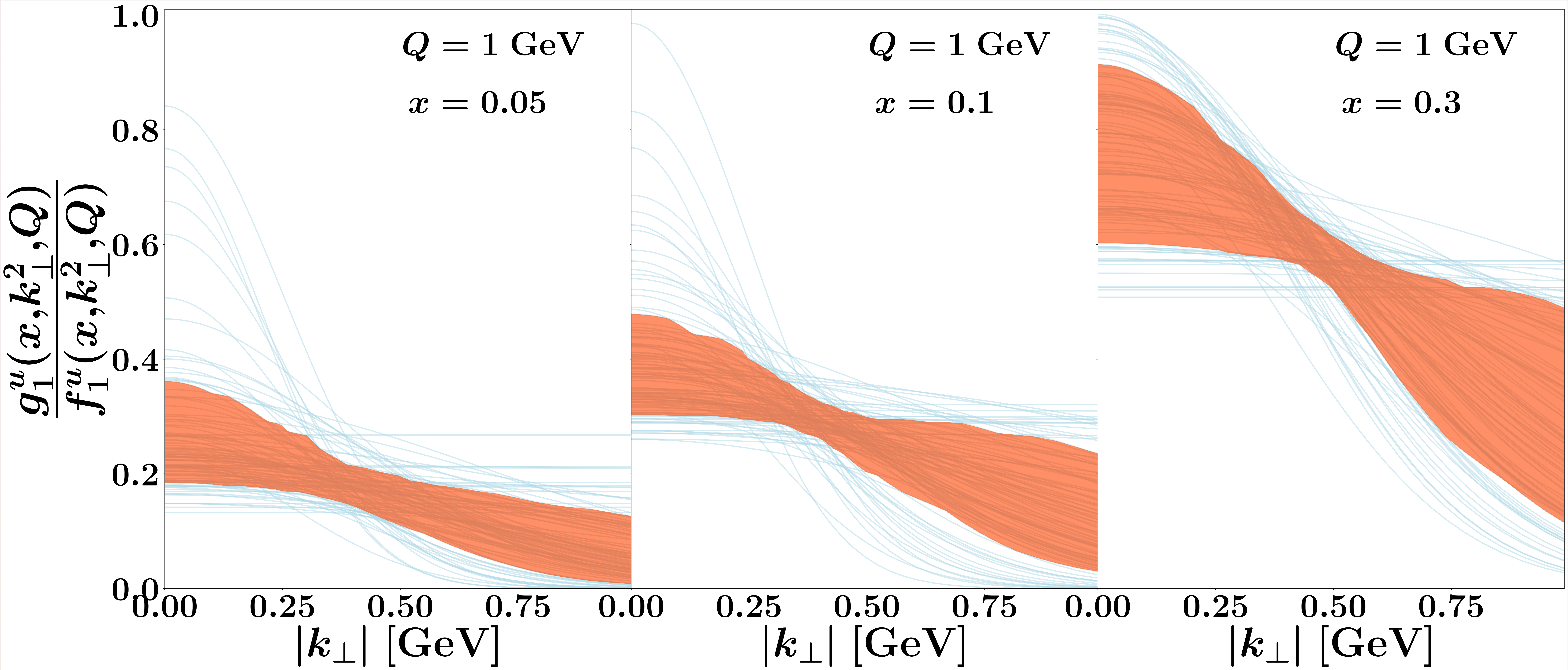
$\downarrow$

Uncertainty of extracted **collinear** PDF propagated onto **TMD**'s uncertainty

EXTRACTION OF THE HELICITY TMD



RATIO PLOT AT NNLL



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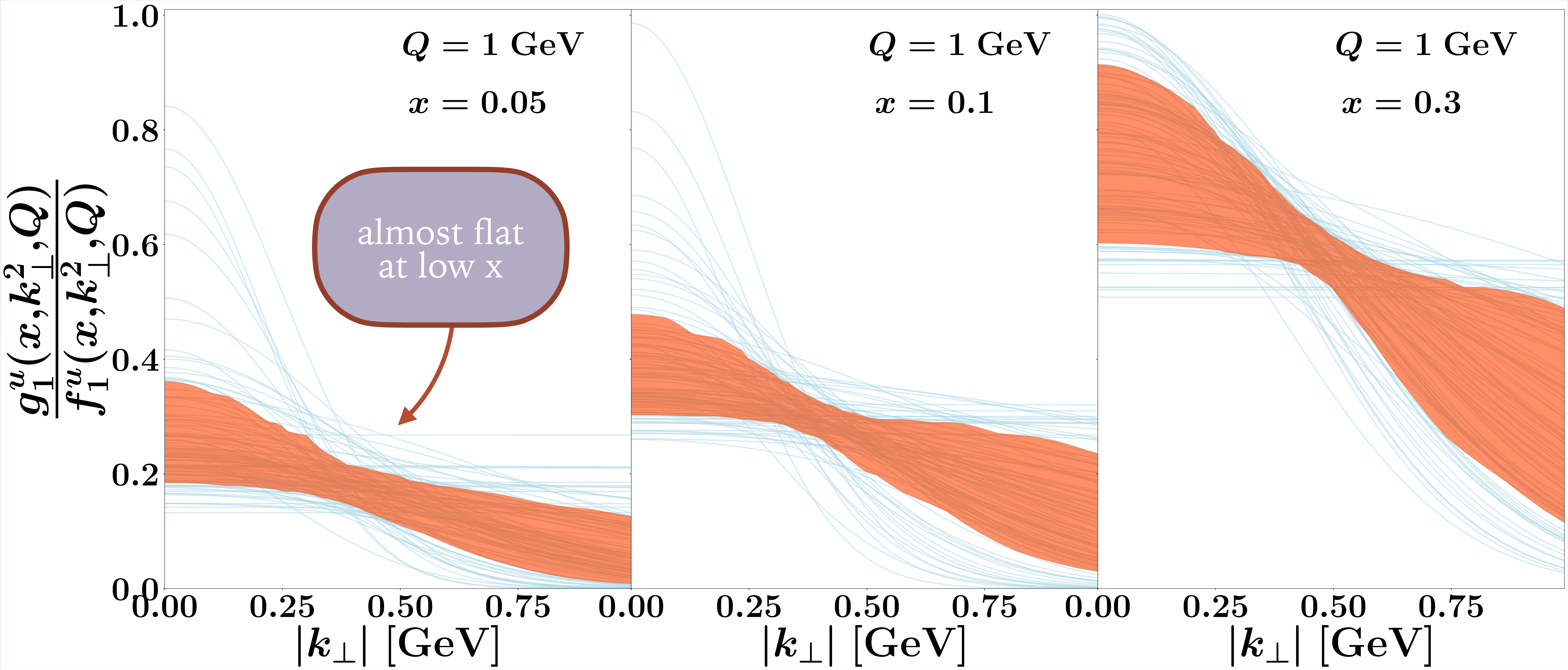
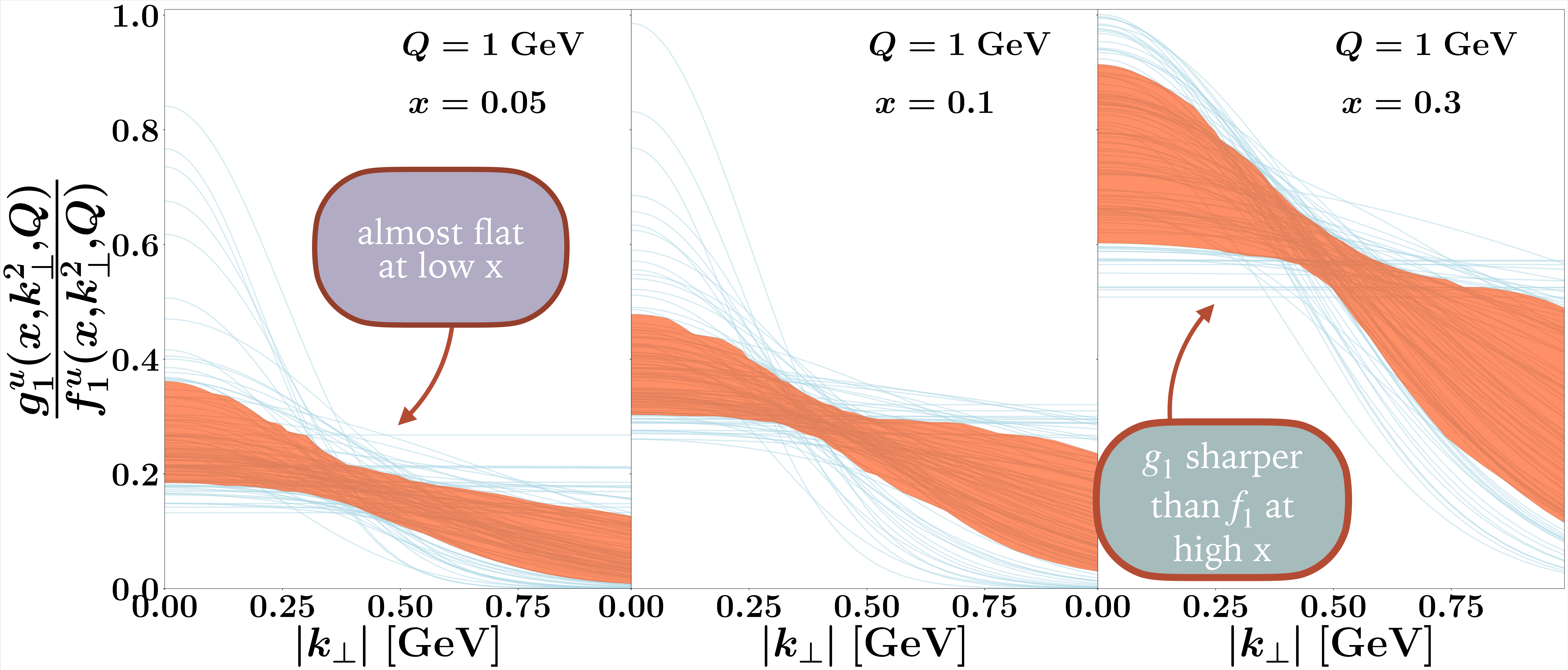
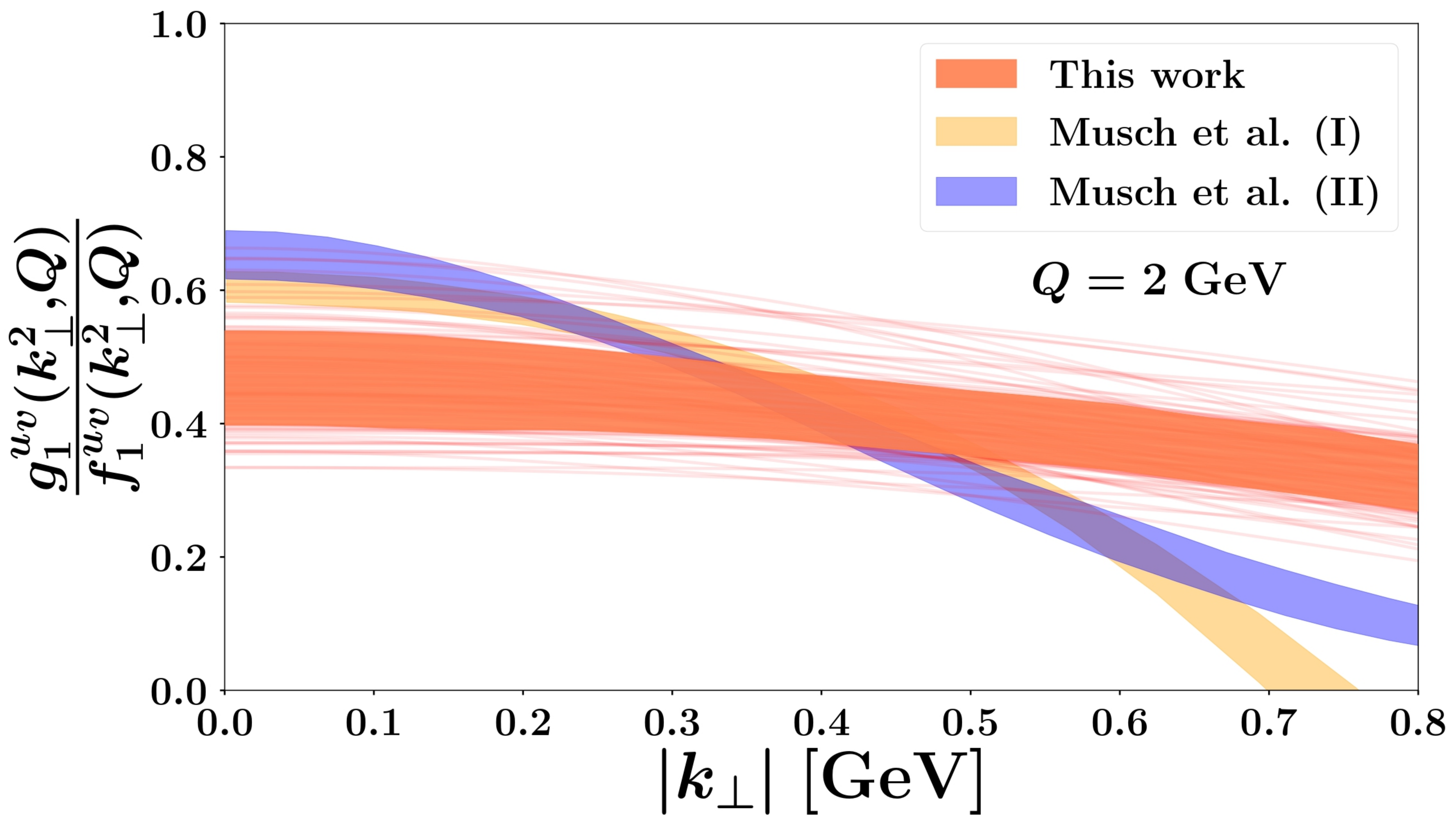


RATIO PLOT AT NNLL



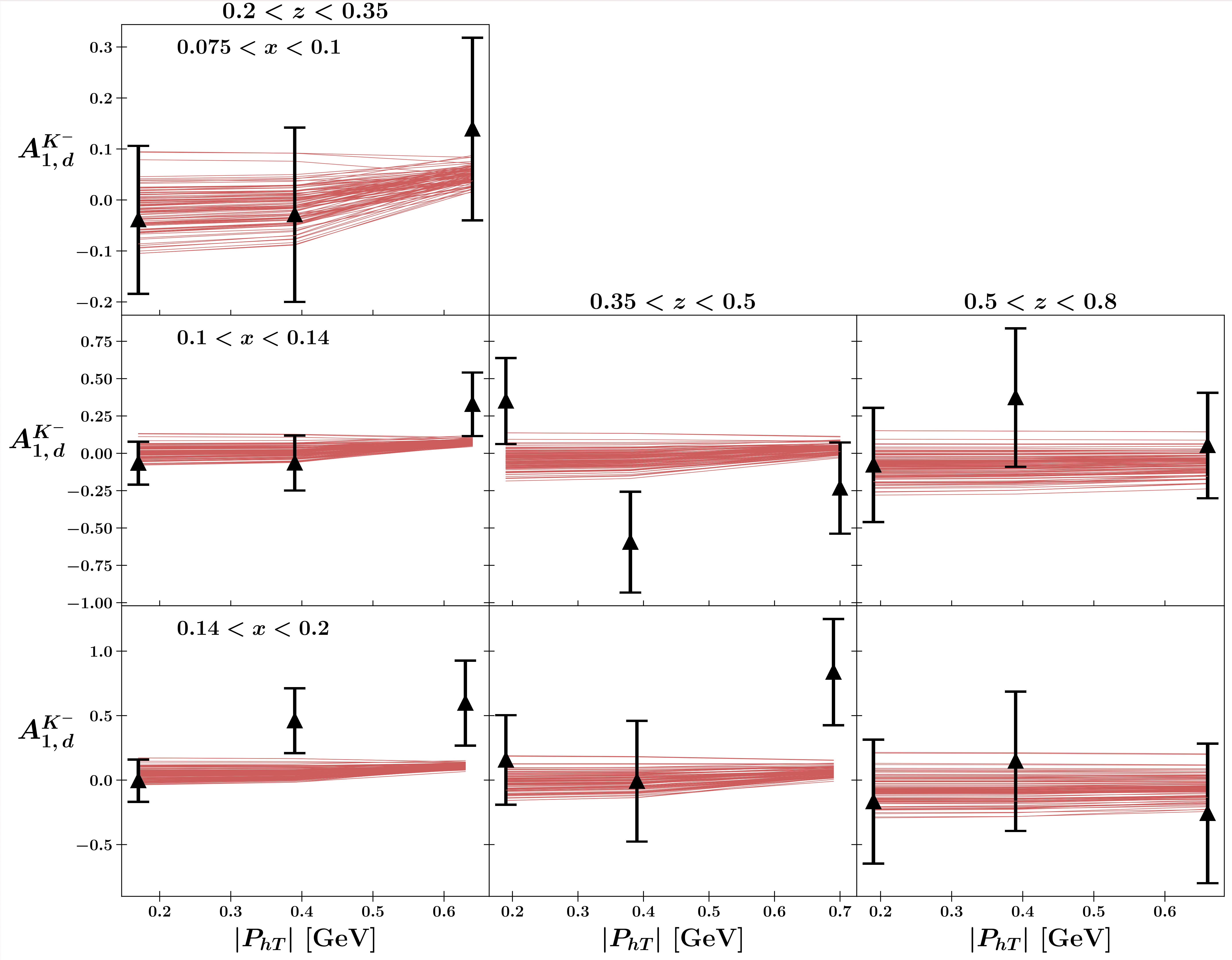
COMPARISON WITH LATTICE



B. U. Musch et al., Phys. Rev. D 83, 094507 (2011)

- g_1/f_1 TMD ratio for u_v integrated over x , at NNLL
- Yellow and blue bands correspond to two lattice predictions
- Milder slope but fair agreement

DATA/THEORY COMPARISON



Experiment	N_{dat}	$\chi^2_{\text{NLL}}/N_{\text{dat}}$	$\chi^2_{\text{NNLL}}/N_{\text{dat}}$
HERMES $(d \to \pi^+)$	47	1.34	1.30
HERMES $(d \to \pi^-)$	47	1.10	1.08
HERMES $(d \to K^+)$	46	1.26	1.25
HERMES $(d \to K^-)$	45	0.93	0.89
HERMES $(p \to \pi^+)$	53	1.17	1.21
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Total	291	1.11	1.09

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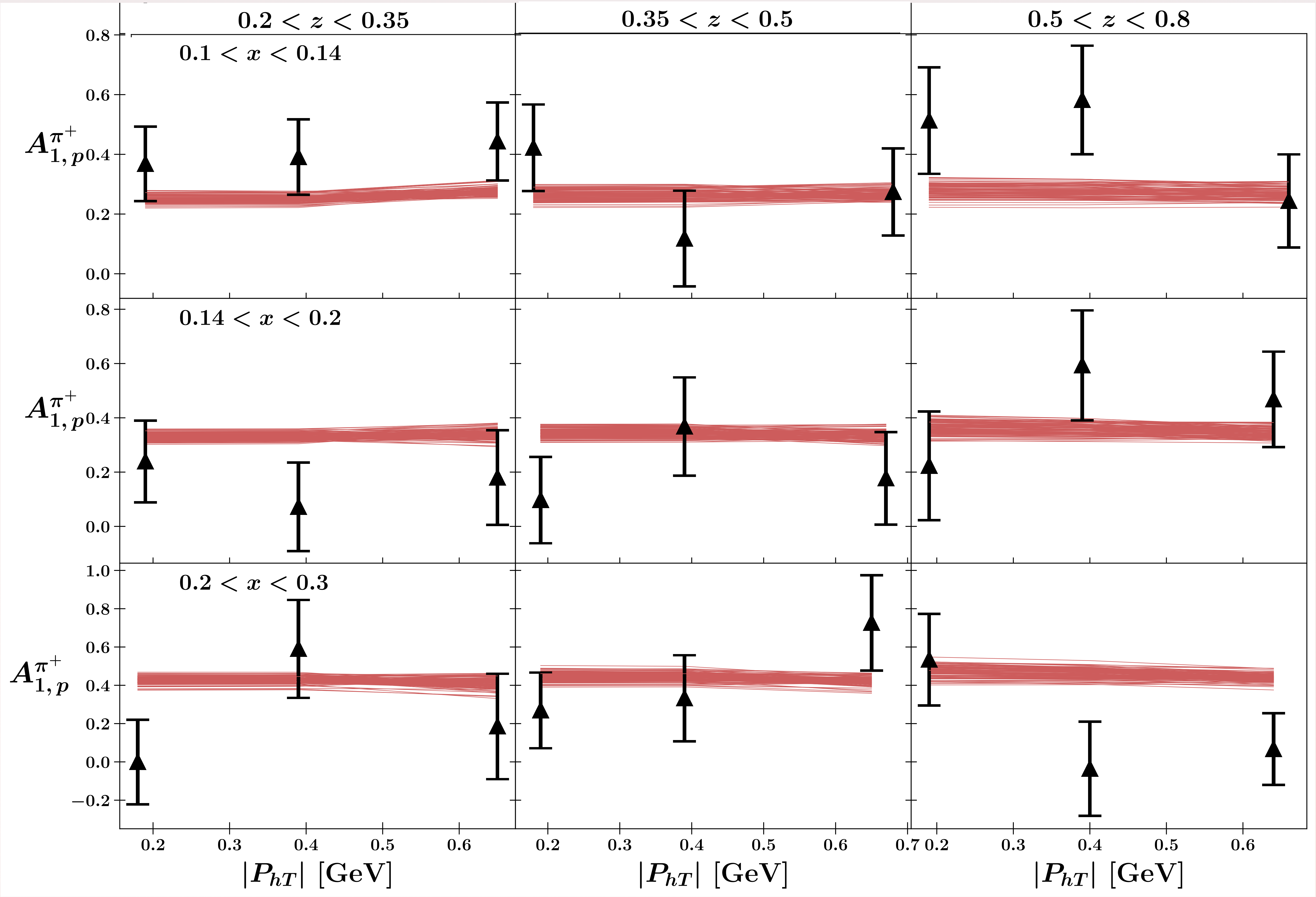

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Total	291	1.11	1.09

Largest χ^2 are obtained for π^+channels (observed also in MAPTMD22 extraction)

due to smaller exp. uncertainties

HELICITY TMD G_1

MATCHING COEFFICIENTS COMPUTATION

MATCHING COEFFICIENTS COMPUTATION

M. G. Echevarria et al., JHEP 09 (2016), 004

At small-b_T :

$$G_{1,\,f\to N}(z,\mathbf{b}_T;\mu,\zeta) = \sum_{f'} \mathscr{C}_{f\to f'}(z,\mathbf{b}_T;\mu,\zeta,\mu_b) \otimes \frac{G_{1,\,f'\to N}(z,\mu_b)}{z^{2-2\varepsilon}} + \mathcal{O}(\mu b_T)$$

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Explicitating the relation of the TMD with the unsubtracted*:

$$\mathscr{C}^{[1]}_{f\to f'}(z,\mathbf{b}_T;\mu,\zeta) = G^{un\,[1]}_{1,f\to f'}(z,\mathbf{b}_T;\mu,\zeta) - \frac{S^{[1]}G^{un\,[0]}_{1,f\to f'}}{2} + (Z^{[1]}_{ren.} - Z^{[1]}_{wfr})G^{un\,[0]}_{1,f\to f'} - \frac{G^{[1]}_{1,f\to f'}(z,\mu,\zeta)}{z^{2-2\varepsilon}}$$

*only for $g\to g, q\to q$

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M. G. Echevarria et al., JHEP 09 (2016), 004

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Fixes rapidity divergence $\ln\frac{\delta^+}{p^+}$

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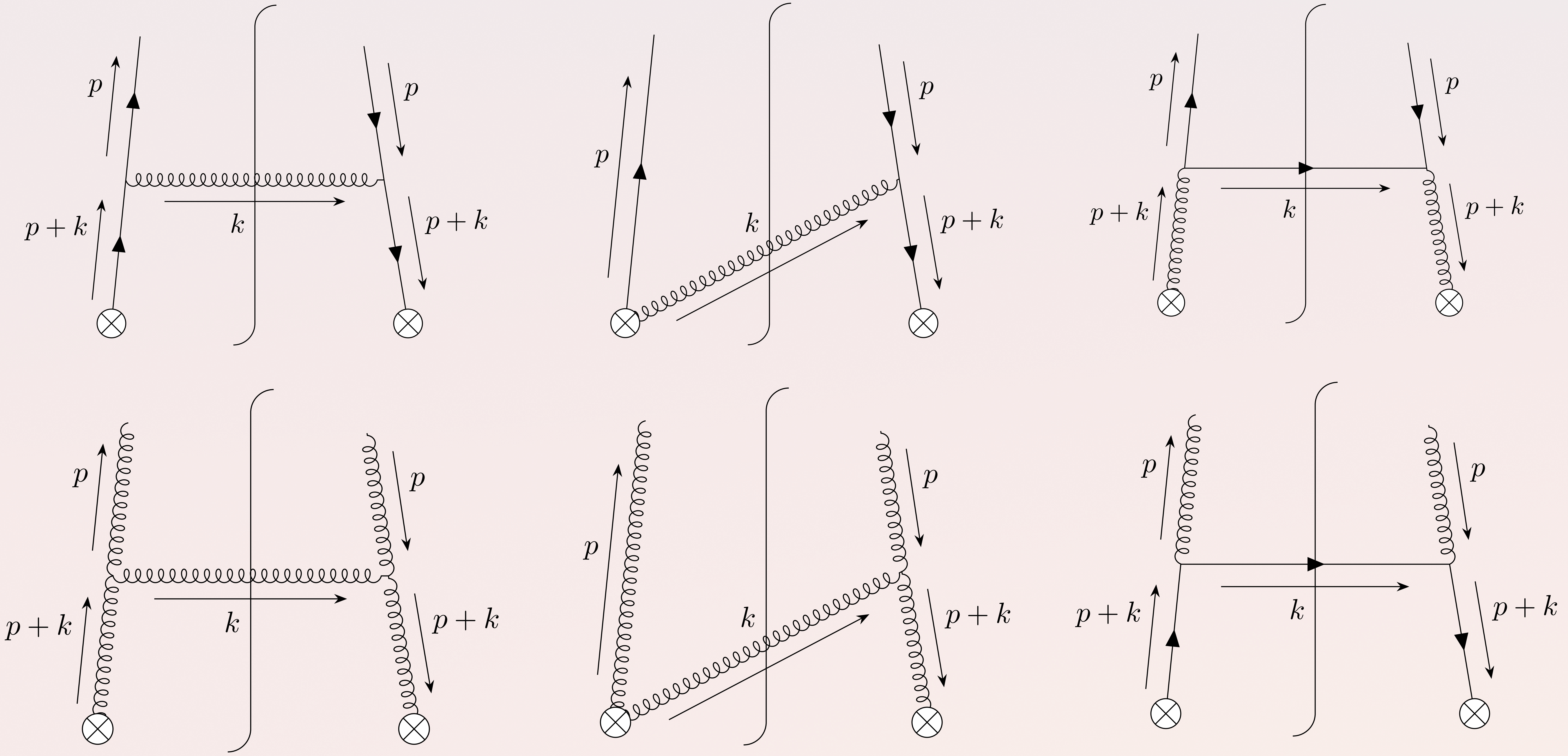
Fixes rapidity divergence $\ln\frac{\delta^+}{p^+}$

Fixes UV div.

Fixes IR div.

*only for $g\to g, q\to q$

HELICITY TMD FF AT NLO



Lorentz structures

$$\Gamma = \gamma^+\gamma_5$$
$$\Gamma^{\mu\nu} = i\epsilon_T^{\mu\nu} \equiv i\epsilon^{+-\mu\nu}$$

Scheme choice

$$(\gamma^+\gamma_5)_{Larin^+} = \frac{i\epsilon_T^{\alpha\beta}}{2!}\gamma^+\gamma_\alpha\gamma_\beta$$

D. Gutiérrez-Reyes, et al., Phys. Lett. B 769, 84 (2017)

RESULTS FOR MATCHING COEFFICIENTS AT NLO

$$\mathscr{C}^{[1]}_{q\to q}(z,\mathbf{b}_T;\mu,\zeta) = \frac{C_F\alpha_s}{4\pi}\frac{1}{z^2}\left[-2L_T\left(\frac{2z}{(1-z)_+}+(1-z)\right)+2(1-z)+\delta(1-z)\left(-L_T^2+2L_Tl_\zeta-\zeta_2\right)\right]$$

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$$\mathscr{C}^{[1]}_{g\to q}(z,\mathbf{b}_T;\mu,\zeta) = \frac{T_r\alpha_s}{4\pi}\frac{1}{z^2}\left(-2L_T(2z-1)+4(1-z)\right)$$

$$\mathscr{C}^{[1]}_{g\to g}(z,\mathbf{b}_T;\mu,\zeta) = \frac{C_A\alpha_s}{4\pi}\frac{1}{z^2}\left[-2L_T\left(\frac{1+z}{(1-z)_+}+(3-4z)\right)-8(1-z)+\delta(1-z)\left(-L_T^2+2L_Tl_\zeta-\zeta_2\right)\right]$$

RESULTS FOR MATCHING COEFFICIENTS AT NLO

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via **Gribov-Lipatov** relation

$$\Delta_{f\to f'}(z,\delta) = \frac{-1}{z}N_{ff'}\,\Phi_{f\leftarrow f'}(z^{-1},\delta)$$

M. G. Echevarria et al., JHEP 09 (2016), 004

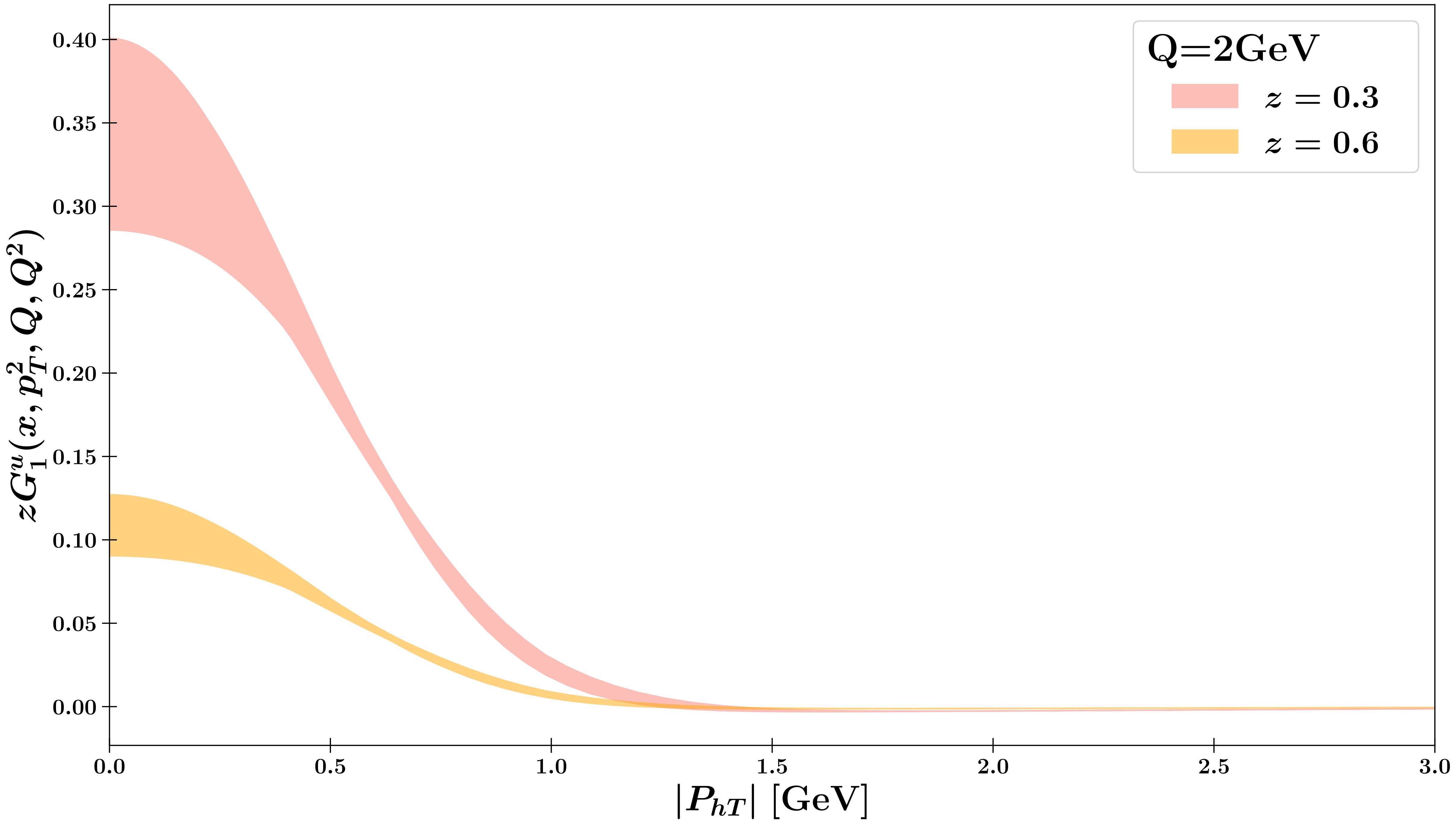
$$N_{qq} = N_{gg} = 1$$

$$N_{qg} = -(1-\epsilon)\frac{C_F}{T_r}$$

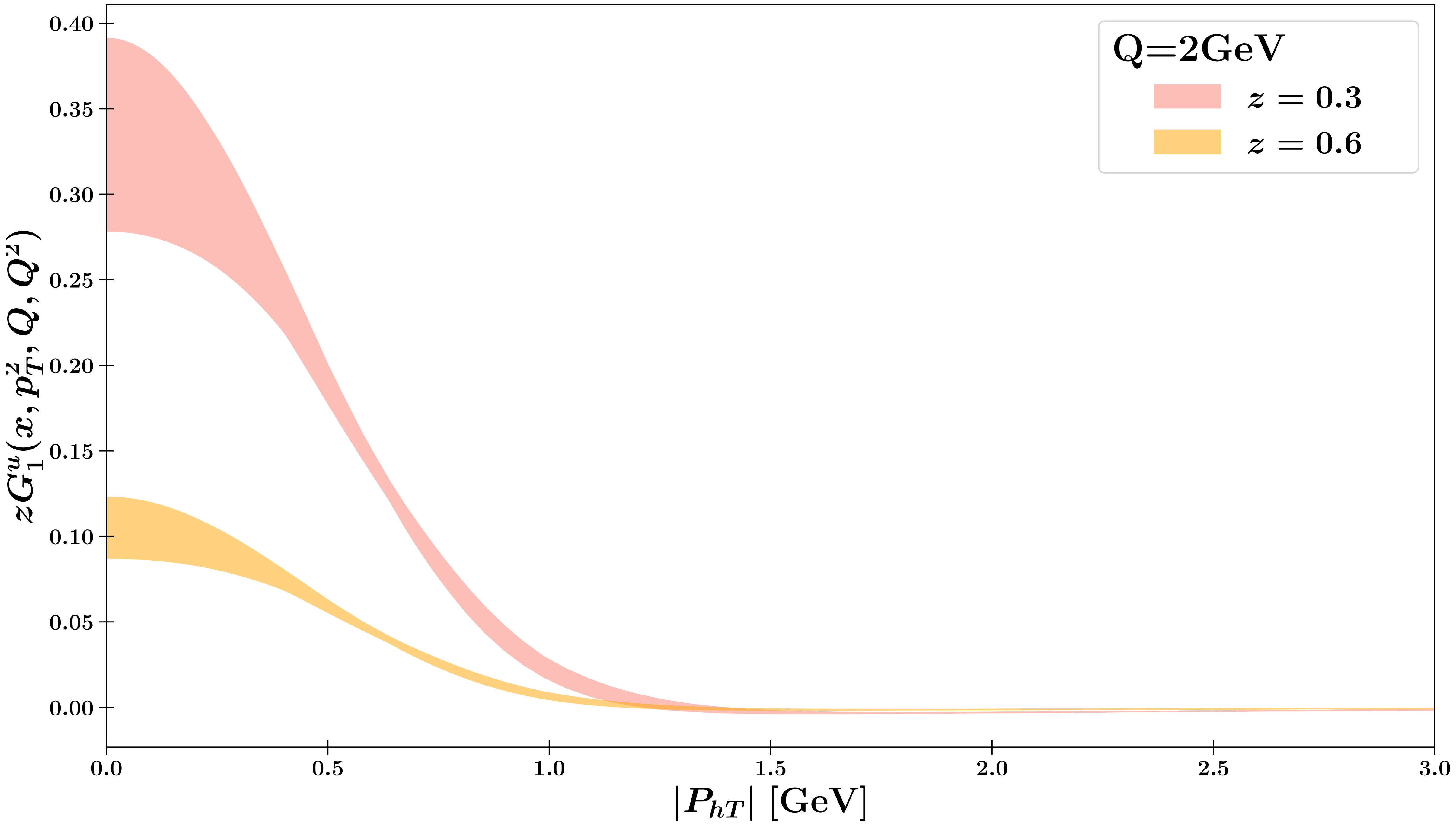
$$N_{gq} = \frac{-1}{1-\epsilon}\frac{T_r}{C_F}$$

EFFECT OF THE MATCHING COEFFICIENTS

NLL



NNLL



CONCLUSIONS

CONCLUSIONS AND FUTURE IMPROVEMENTS

HELICITY TMD PDF

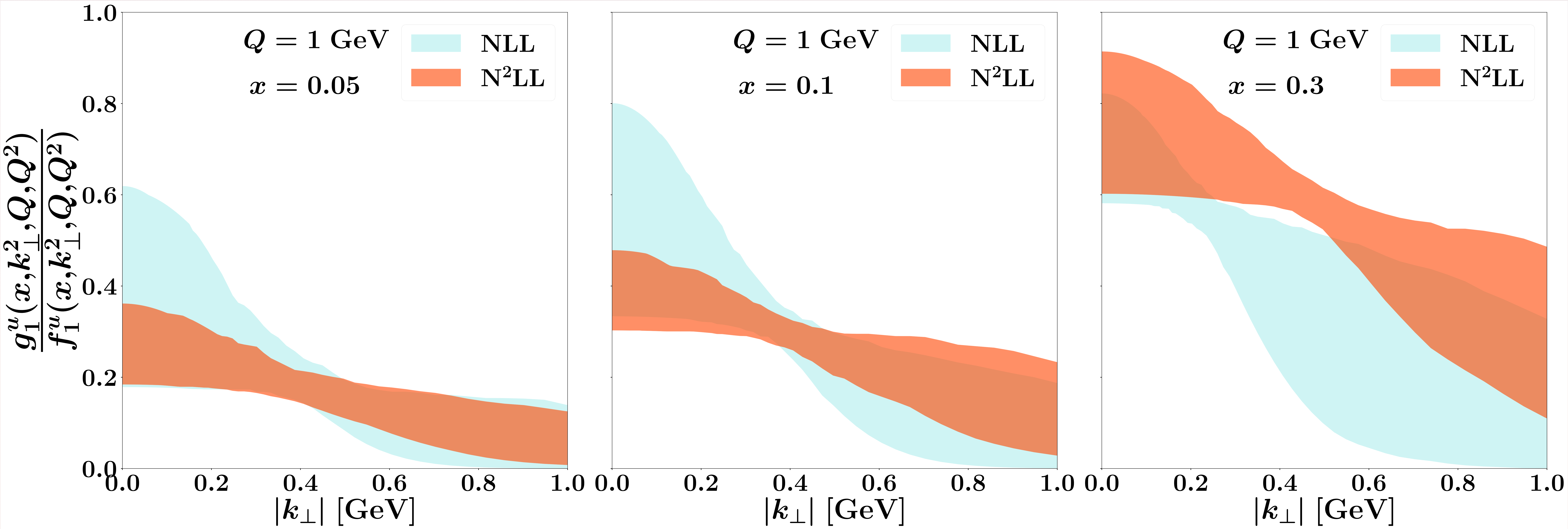
- First extraction of the helicity TMD PDF for quarks at NLO with numerator-denominator compatibility, reaching NNLL perturbative accuracy
- Positivity constraint fulfilled by construction
- Helicity TMD shows a x-dependence and different behaviour from the unpolarized at large x
- Free parameters poorly constrained for limited size of the experimental dataset: new data will refine the model

HELICITY TMD FF

- First computation of the matching coefficients of the helicity TMD FF at NLO
- Phenomenological analysis in the TMD context can be done with data from Λ-production experiments (CC NOMAD, SIDIS COMPASS, HERMES, LEP)

BACKUP SLIDES

RATIO PLOT NLL VS NNLL



NP PART OF THE UNPOLARIZED TMD PDF f_1

$$f_{NP}^{MAP22}(x, k_\perp^2, Q_0) = \frac{\exp\left(-\frac{k_\perp^2}{g_{1A}(x)}\right) + k_\perp^2 \lambda^2 \exp\left(-\frac{k_\perp^2}{g_{1B}(x)}\right) + \lambda_2^2 \exp\left(-\frac{k_\perp^2}{g_{1C}(x)}\right)}{\pi \left(g_{1A}(x) + \lambda^2 g_{1B}(x)^2 + \lambda_2^2\, g_{1C}(x)\right)}$$

The x-dependent gaussian widths are

$$g_{\{1A,1B,1C\}}(x) = N_{\{1,2,3\}} \frac{(1-x)^{\alpha^2_{\{1,2,3\}}}\, x^{\sigma_{\{1,2,3\}}}}{(1-\hat{x})^{\alpha^2_{\{1,2,3\}}}\, \hat{x}^{\sigma_{\{1,2,3\}}}}$$

Parameter	Average over replicas
g_2 [GeV]	0.248 ± 0.008
N_1 [GeV^2]	0.316 ± 0.025
α_1	1.29 ± 0.19
σ_1	0.68 ± 0.13
λ [GeV^{-1}]	1.82 ± 0.29
N_3 [GeV^2]	0.0055 ± 0.0006
β_1	10.23 ± 0.29
δ_1	0.0094 ± 0.0012
γ_1	1.406 ± 0.084
λ_F [GeV^{-2}]	0.078 ± 0.011
N_{3B} [GeV^2]	0.2167 ± 0.0055
N_{1B} [GeV^2]	0.134 ± 0.017
N_{1C} [GeV^2]	0.0130 ± 0.0069
λ_2 [GeV^{-1}]	0.0215 ± 0.0058
α_2	4.27 ± 0.31
α_3	4.27 ± 0.13
σ_2	0.455 ± 0.050
σ_3	12.71 ± 0.21
β_2	4.17 ± 0.13
δ_2	0.167 ± 0.006
γ_2	0.0007 ± 0.0110

EXPRESSION OF THE $k_{norm}(x)$ FACTOR

$$k_{norm}(x) = w_1(x)\frac{\dfrac{g_{1A}(x)}{g_{1A}(x)+w_1(x)} + \lambda^2\dfrac{g_{1B}^2(x)w_1(x)}{(g_{1B}(x)+w_1(x))^2} + \lambda_2^2\dfrac{g_{1C}(x)}{g_{1C}(x)+w_1(x)}}{g_{1A}(x)+\lambda^2 g_{1B}^2(x)+\lambda_2^2 g_{1C}(x)}$$

$$Z_q^{[1]}(\mu,\zeta) = \frac{\alpha_s C_F}{2\pi}\left(-\frac{1}{\varepsilon_{UV}^2} - \frac{1}{\varepsilon_{UV}}\left(2 + \ln\frac{\mu^2}{\zeta}\right)\right)$$

$$Z_g^{[1]}(\mu,\zeta) = \frac{\alpha_s C_A}{2\pi}\left(-\frac{1}{\varepsilon_{UV}^2} - \frac{1}{\varepsilon_{UV}}\left(1 + \ln\frac{\mu^2}{\zeta}\right)\right)$$

WFR

$$Z_2 = \frac{\alpha_s C_F}{4\pi}\left(-\frac{1}{\varepsilon_{UV}} + \frac{1}{\varepsilon_{IR}}\right)$$

$$Z_3 = \frac{\alpha_s}{4\pi}\left(\frac{1}{\varepsilon_{UV}} - \frac{1}{\varepsilon_{IR}}\right)\left(\frac{5}{3}C_A - \frac{4}{3}T_r n_f\right) \equiv \frac{\alpha_s}{4\pi}\left(\frac{1}{\varepsilon_{UV}} - \frac{1}{\varepsilon_{IR}}\right)2\left(\frac{\beta_0}{2} - C_A\right)$$

UV RENORMALIZATION CONSTANTS

$$S^{[1]} = \frac{\alpha_s C_F}{2\pi}\left[-\frac{2}{\varepsilon_{UV}^2} + \frac{2}{\varepsilon_{UV}}\ln\frac{\delta^+\delta^-}{\mu^2} + L_T^2 + 2L_T\ln\frac{\delta^+\delta^-}{\mu^2} + \frac{\pi^2}{6}\right]$$

$$\ln(\delta^+\delta^-/\mu^2) \to \ln\left(\frac{\delta^+}{p^+}\right)^2 \ln\left(\frac{\zeta}{\mu^2}\right)$$

SOFT FUNCTION

δ-REGULARIZATION

$$W_n(y) = P \exp\left[ig \int_{-\infty}^{0} d\lambda\, \bar{n} \cdot A(y + \lambda \bar{n}) e^{-\delta^+ \lambda}\right] = P \int \frac{d^n k}{(2\pi)^n} e^{-iky} \frac{-g\, \bar{n}^\mu t^a}{k^+ - i\delta^+} \tilde{A}^a_\mu(k)$$

$$B^\mu_{n\perp} = \frac{1}{g}\left[W_n^\dagger(y) i D^\mu_{n\perp} W_n(y)\right] \qquad B^{(0)\mu\nu}_{n\perp}(k) = g^{\mu\nu}_\perp - \frac{k^\mu_\perp \bar{n}^\nu}{k^+ - i\delta}$$

QUARK AND GLUON FUNCTIONS

$$G_{q\to N}(z, \mathbf{b_T}) = \frac{1}{4zN_c} \sum_X \frac{1}{2} \int \frac{d\xi^-}{2\pi} e^{-ip\cdot\xi/2z} < 0 | T \left[W_n^{T\dagger} q_j\right]_a (\xi) | X, N > \gamma^+_{ij}\gamma_5 < X, N | \bar{T} \left[\bar{q}_i W_n^T\right]_a 0\ | 0 >$$

$$G_{g\to N}(z, \mathbf{b}_T) = \frac{-p^+ z^{-2}}{(d-2)(d-3)(N_c^2 - 1)} i\epsilon^\perp_{\mu\nu} \sum_X \frac{1}{2} \int \frac{d\xi^-}{2\pi} e^{-ip\cdot\xi/2z} < 0 | T \left[B^\mu_{n\perp}\right] (\xi) | X, N > < X, N | \bar{T} \left[B^\nu_{n\perp}\right] (0) | 0 >$$