Quarkonium-pair production at NLL in TMD factorisation at the LHC

Resummation, Evolution, Factorization 2024, Saclay (France)

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TMDs → 3D structure of the nucleon

Correlations between k_T and the polarisation of the nucleon/parton

2 components
- collinear (x)
- transversal $(\vec{k}_{\perp})$ → generate q_T (final-state)

Quark TMDs extracted from data

↪ SIDIS, DY processes

↪ Precision era!

Gluon TMDs → lack of data

↪ Extremely poorly known!

↪ How to measure them?

Inclusive **quarkonium** production

Prog.Part.Nucl.Phys. 122 (2022) 103906

Nucleon \ Quark	U	L	T	
U	f_1		$h_1^{\perp}$	
L		g_{1L}	$h_{1L}^{\perp}$	
T	$f_{1T}^{\perp}$	g_{1T}	h_1	$h_{1T}^{\perp}$

Nucleon \ Gluon	U	C	L
U	f_1		$h_1^{\perp}$
L		g_{1L}	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	h_1, $h_{1T}^{\perp}$

A. Bacchetta et al. (JHEP 08 (2008) 023)

Quarkonium production mechanisms

Experimental point of view:

- quarkonium production observed in different experiments
- J/ψ (and Υ): easy to produce and detect
  $\hookrightarrow$ plenty of experimental data

Theoretical point of view:

- Not clear how to treat quarkonium production
- 3 common models $\rightarrow$ Colour Singlet Model (CSM)
  $\rightarrow$ Colour Octet Mechanism (COM)
  $\rightarrow$ Colour Evaporation Model (CEM)
- ✗ single J/ψ: no consensus ...
- ✓ double-J/ψ: **consensus!** (Including $\Upsilon + \Upsilon$)

J.-P. Lansberg, Phys.Rept. 889 (2020) 1-106

CSM: description of **double-**J/ψ production

Study of gluon TMDs → TMD factorisation ($q_T \ll Q$)

q_T: transverse momentum of the double-J/ψ; Q: scale of the process

P.J. Mulders, J. Rodrigues, PRD 63 094021

General factorised cross section

↪ partonic scattering amplitude $\hat{\mathcal{M}}$ (*perturbative*)

↪ k_T-dependent correlators Φ (*non-perturbative*)

$$d\sigma = \int dx_1 dx_2 d^2\vec{k}_{T1} d^2\vec{k}_{T2} \delta^{(2)}(\vec{k}_{T1} + \vec{k}_{T2} - \vec{q}_T)$$

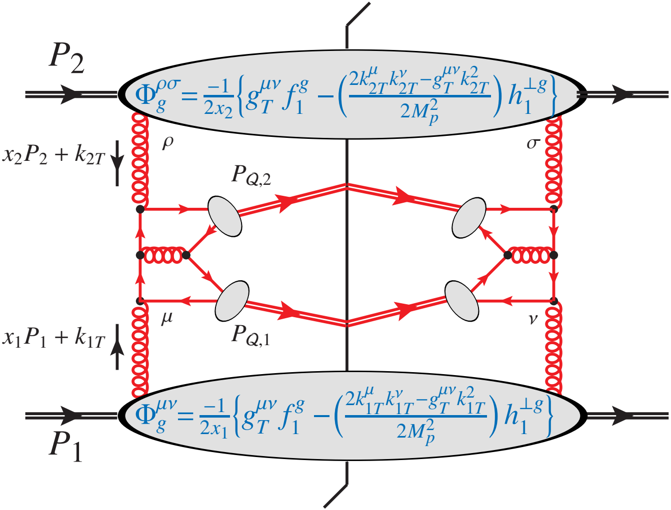
$$\times \Phi_g^{\mu\nu}(x_1, \vec{k}_{T1}) \Phi_g^{\rho\sigma}(x_2, \vec{k}_{T2}) \left[\hat{\mathcal{M}}_{\mu\rho} \hat{\mathcal{M}}^*_{\nu\sigma} \right]\Big|_{\substack{k_1 = x_1 P_1 \\ k_2 = x_2 P_2}} + \mathcal{O}\left(\frac{q_T^2}{Q^2}\right)$$

where:

$$\Phi_g^{\mu\nu}(x, \vec{k}_T) = \frac{1}{2x}\left[-g_T^{\mu\nu} f_1^g(x, \vec{k}_T^2) + \left(\frac{k_T^\mu k_T^\nu}{M_p^2} - g_T^{\mu\nu} \frac{\vec{k}_T^2}{2M_p^2} \right) h_1^{\perp g}(x, \vec{k}_T^2) \right]$$

LO Feynman diagram for $p(P_1) + p(P_2) \to \mathcal{Q}(P_{Q,1}) + \mathcal{Q}(P_{Q,1}) + X$

F. Scarpa, D. Boer, M.G. Echevarria, J.-P. Lansberg, M. Schlegel, EPJC (2020) 80:87



Hadronic cross section

The general formula for the cross section of gluon fusion is:

$$
\begin{aligned}
d\sigma^{gg} \propto\ & F_1 \times \mathcal{C}[f_1^g f_1^g] \\
& + F_2 \times \mathcal{C}[w_2 h_1^{\perp g} h_1^{\perp g}] \\
& + (F_3 \times \mathcal{C}[w_3 f_1^g h_1^{\perp g}] + F_3' \times \mathcal{C}[w_3' h_1^{\perp g} f_1^g]) \cos(2\Phi_{CS}) \\
& + (F_4 \times \mathcal{C}[w_4 h_1^{\perp g} h_1^{\perp g}]) \cos(4\Phi_{CS})
\end{aligned}
$$

- first two members: azimuthally independent
- third member: $\cos(2\Phi_{CS})$-modulation
- fourth member: $\cos(4\Phi_{CS})$-modulation

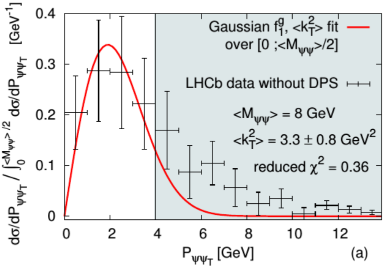
$\mathcal{C}[f_1^g f_1^g]$ drives the q_T dependence

J.-P. Lansberg, C. Pisano, F. Scarpa, M. Schlegel, PLB 784(2018) 217

First "fit" of f_1^g

JPL, C. Pisano, F. Scarpa, M. Schlegel, PLB 784(2018)217

- f_1^g modelled as a Gaussian in $\vec{k}_T$: $f_1^g(x, \vec{k}_T^2) = \frac{g(x)}{\pi\langle k_T^2\rangle} \exp\left(\frac{-\vec{k}_T^2}{\langle k_T^2\rangle}\right)$
  where $g(x)$ is the usual collinear PDF
- First experimental determination [with a pure colorless final state] of $\langle k_T^2\rangle$
  by fitting $\mathcal{C}[f_1^g f_1^g]$ over the normalised LHCb $d\sigma/dP_{\psi\psi_T}$ spectrum at 13 TeV
  from which we have subtracted the DPS yield determined by LHCb



Slide courtesy of Jean-Philippe Lansberg

- Integration over $\phi \Rightarrow \cos(n\phi)$-terms cancel out
- $F_2 \ll F_1 \Rightarrow$ only $\mathcal{C}[f_1^g f_1^g]$ contributes to the cross-section
- No evolution so far: $\langle k_T^2\rangle \sim 3$ GeV^2 accounts both for non-perturbative and perturbative broadenings at a scale close to $M_{\psi\psi} \sim 8$ GeV
- Disentangling such (non-)perturbative effects requires data at different scales

- Implementation in impact parameter space (b_T)

  J. Collins, Camb.Monogr.Part.Phys.Nucl.Phys.Cosmol. 32 (2011) 1-624

- S_{NP} typically chosen to be a Gaussian: EPJC 80:87

$$S_{NP}(b_T;Q) = A \ln \frac{Q}{Q_{NP}} b_T^2 \quad \text{with} \quad Q_{NP} = 1\,\text{GeV}$$

- We obtain the following expression for the convolutions:

$$\mathcal{C}[w\, f\, g](x_1, x_2, \vec{q}_T; Q) = \int_0^\infty \frac{db_T}{2\pi}\, b_T^n\, J_m(b_T\, q_T)\, e^{-S_A(b_T^*;Q^2,Q)}\, e^{-S_{NP}(b_T;Q)}$$
$$\times\ \hat{f}(x_1, \mathrm{b}_T^*; \mu_b^2, \mu_b)\, \hat{g}(x_2, \mathrm{b}_T^*; \mu_b^2, \mu_b)$$

where $\hat{f}$ and $\hat{g}$ are substituted by:

$$\hat{f}_1^g(x, \mathrm{b}_T^2) \equiv \int d^2\mathrm{q}_T\ e^{i\mathrm{b}_T\cdot\mathrm{q}_T}\ f_1^g(x, \mathrm{q}_T^2)$$

$$\hat{h}_1^{\perp g}(x, \mathrm{b}_T^2) \equiv \int d^2\mathrm{q}_T\ \frac{(\mathrm{b}_T\cdot\mathrm{q}_T)^2 - \frac{1}{2}\mathrm{b}_T^2\mathrm{q}_T^2}{\mathrm{b}_T^2 M_h^2}\ e^{i\mathrm{b}_T\cdot\mathrm{q}_T}\ h_1^{\perp g}(x, \mathrm{q}_T^2)$$

Switching on TMD evolution: 2020 results

F. Scarpa, D. Boer, M.G. Echevarria, J.-P. Lansberg, M. Schlegel, EPJC (2020) 80:87

First implementation of TMD evolution: evolution effects are measurable!

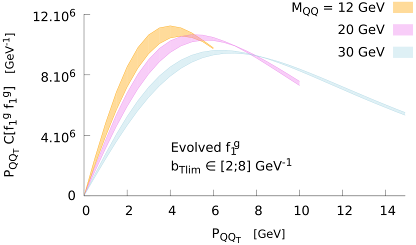
However:

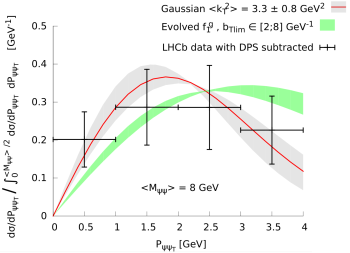
- No x-dependence
- No rapidity cuts

To be implemented!
Updates: x-dependence, experimental cuts and PDF uncertainty (MSTW2008LO)

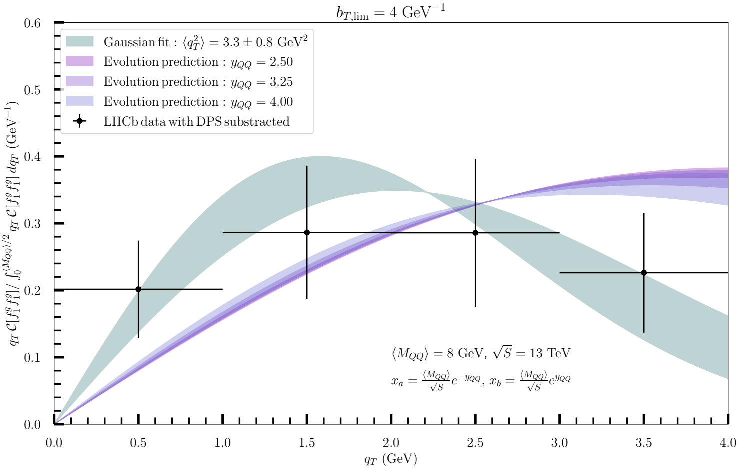
D. Boer, J. Bor, ACS, J.P. Lansberg,

in preparation



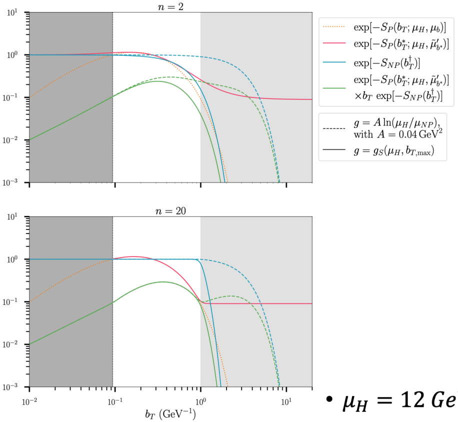


Theoretical predictions: first update



S_A and S_{NP} continuity

Slide courtesy of Jelle Bor



- We want to trust perturbative physics when we can, to study S_{NP}

$$b_T^* = \frac{b_T}{\left(1 + (b_T/b_{T,\max})^n\right)^{1/n}}, \quad b'_T = \left(b_T^n + b_{T,\min}^n\right)^{1/n}$$

- Remove the 'kink' at the same order n

$$S_{NP}(b_T) = g\, b_T^{\dagger 2} \quad b_T^{\dagger 2} = \left(b_T^n + b_{T,\max}^n\right)^{2/n} - b_{T,\max}^2$$

$$g_S(\mu_H, b_{T,\max}) = \frac{\left|\frac{\partial}{\partial b_T} S_A(b_T; \mu_H, \mu_b)\big|_{b_T = b_{T,\max}}\right|}{2^{2/n}\, b_{T,\max}}$$

- Nonperturbative physics is dependent on perturbative physics!

- $\mu_H = 12\ GeV$

$$S_{NP}(b_T; x, \mu_H, b_{T,\max}) = (g_S + g_{f_A}(x_a, b_{T,\max}) + g_{f_B}(x_a, b_{T,\max}))\, b_T^{\dagger 2}$$

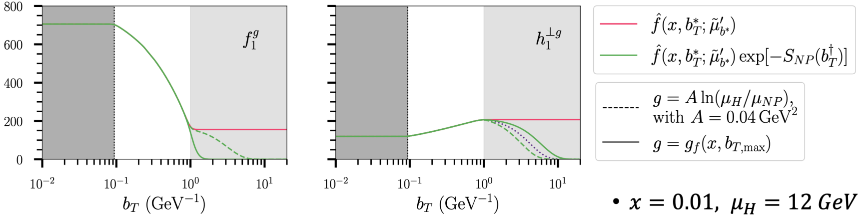
Similarly for the perturbative tails!

Slide courtesy of Jelle Bor

- Extra factor g_f needed to remove other 'kinks'
- Exception of absolute value when $g_f < \Lambda^2_{QCD}$:
  $\Longrightarrow g_f = \Lambda^2_{QCD}$

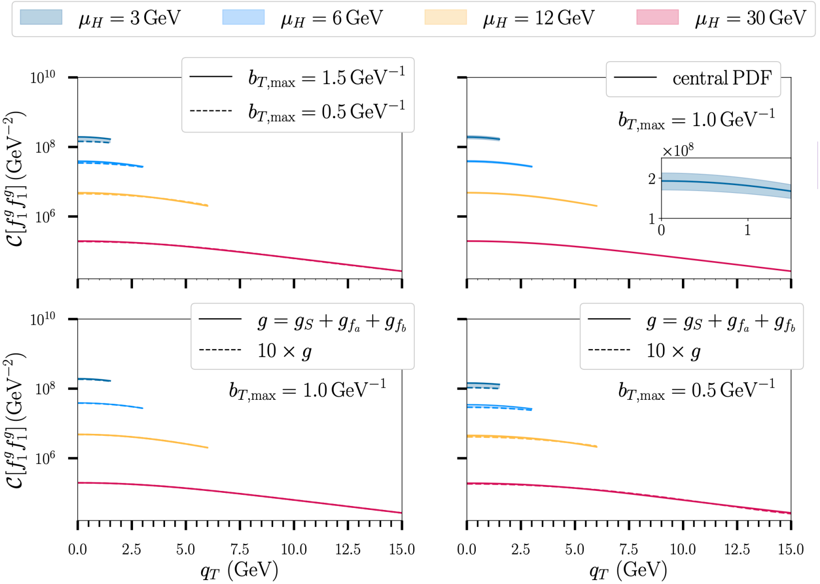
$$g_f(x, b_{T,\max}) = \frac{\left| \frac{\partial}{\partial b_T} \ln f(x, b_T; \mu_b) \big|_{b_T = b_{T,\max}} \right|}{2^{2/n}\, b_{T,\max}}$$

- MHST20lo_as130



- $x = 0.01,\ \mu_H = 12\ GeV$

$C[f_1^g f_1^g]$ results with this approach: no more uncertainties?



Perturbative scale uncertainty: C_1, C_2, C_3

- $\mu \to C\mu$ with $C = [1/2 : 2]$
  - $C_1 \times \mu_b = b_0/b_T$ and $C_2 \times \mu_H = M_{QQ} = Q$
  - $C_3 \times \mu_b$ in the perturbative tails

Melis et al. 2015

$$S_A(b_T^*; C_2 Q, C_1 \mu'_{b*}) = \frac{1}{2}\frac{C_A}{\pi}\int_{C_1^2 \mu'^2_{b*}}^{C_2^2 Q^2} \frac{d\mu'^2}{\mu'^2}\Bigg(\alpha_s(\mu') + \frac{\alpha_s(\mu')^2}{4\pi}$$

$$\times \left[\left(\frac{67}{9} - \frac{\pi^2}{3}\right) N_c - \frac{20}{9} T_R n_f + 2\beta_0 \ln C_1\right] \ln \frac{C_2^2 Q^2}{\mu'^2}$$

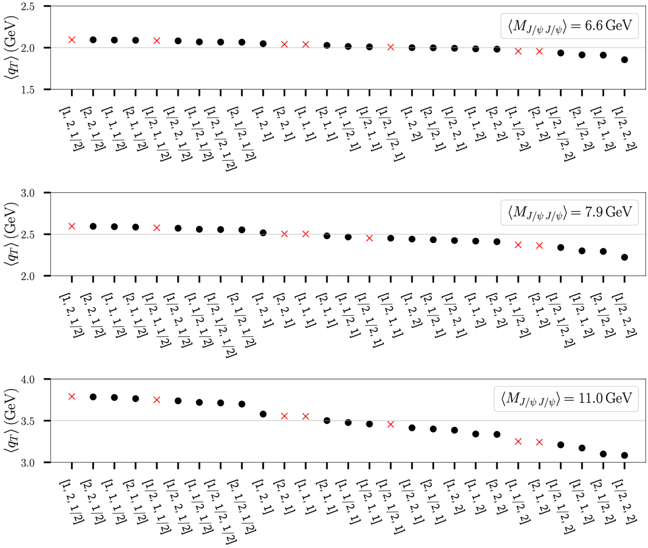
$$- \alpha_s(\mu') \left[\frac{\beta_0}{6} + 2 \ln \frac{C_2}{C_1}\right]\Bigg)$$

$$C_3 = \frac{C_1}{C_2}$$ Collins et al.1984

$$\hat{f}_1^g(x, b_T^*; \mu'_{b*}) = f_1^g(x; C_3 \mu'_{b*}) + \mathcal{O}(\alpha_s) + \mathcal{O}(b_T \Lambda_{\rm QCD})$$

Scale variation: C_1, C_2, C_3

Collins: $C_3 = C_1/C_2$

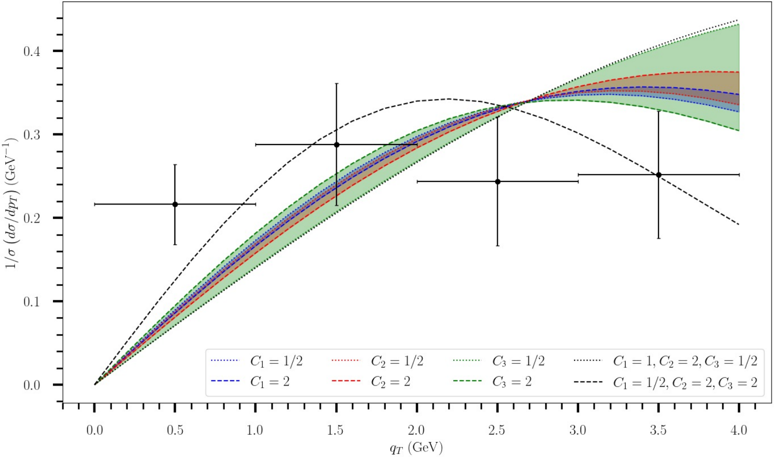


Updated results

$M_{QQ} = 7.9\text{GeV}$

Latest LHCb results

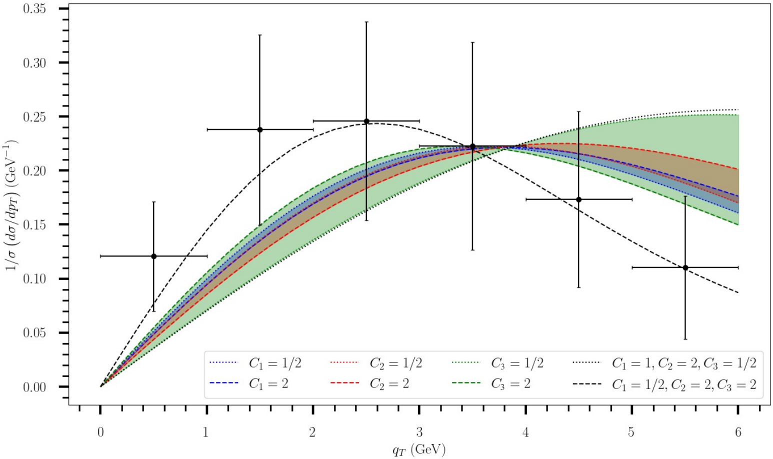
JHEP 03 (2024) 088



Latest LHCb results

JHEP 03 (2024) 088

$M_{QQ} = 11.0\text{GeV}$

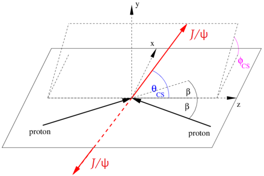


Computation of azimuthal modulations (average)

The corresponding expressions for $\cos(2\Phi_{CS})$ and $\cos(4\Phi_{CS})$:

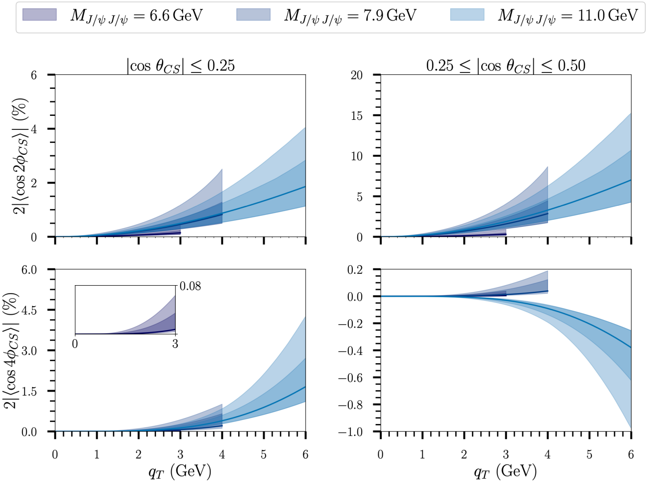
$$\langle\cos(2\phi_{CS})\rangle = \frac{1}{2}\frac{F_3\mathcal{C}[w_3 f_1^g h_1^{\perp g}] + F_3'\mathcal{C}[w_3' h_1^{\perp g} f_1^g]}{F_1\mathcal{C}[f_1^g f_1^g] + F_2\mathcal{C}[w_2 h_1^{\perp g} h_1^{\perp g}]}$$

$$\langle\cos(4\phi_{CS})\rangle = \frac{1}{2}\frac{F_4\mathcal{C}[w_4 h_1^{\perp g} h_1^{\perp g}]}{F_1\mathcal{C}[f_1^g f_1^g] + F_2\mathcal{C}[w_2 h_1^{\perp g} h_1^{\perp g}]}$$



- The hard-scattering coefficients (F_1, F_2, F_3, F_3', F_4) give the explicit dependence on $M_{\psi\psi}$ and θ_{CS}
- Modulations due to $h_1^{\perp g}$
- Set scale $Q^2 = M_{\psi\psi}^2$
- TMD evolution applied within the convolutions

Azimuthal modulations: updated results



LHCb (2024): $\langle \cos 2\phi_{CS} \rangle = -0.029 \pm 0.050(\text{stat.}) \pm 0.009(\text{syst.})$

$\langle \cos 4\phi_{CS} \rangle = -0.087 \pm 0.052(\text{stat.}) \pm 0.013(\text{syst.})$

Summary and outlook

- Quarkonium production is a great tool for many purposes
  $\hookrightarrow$ exploration of nucleon structure through gluon TMDs
- Double J/ψ production gives the possibility to investigate gluon TMD induced effects (Υ updates soons)
- Work in progress: extension of S_{NP} parametrisation

  D. Boer, J. Bor, ACS, J.P. Lansberg, in preparation
- New LHCb results comparisons! Scale uncertainty study. Improved predictions $\rightarrow$ good agreement with data!
- **FUTURE** Studies can be made considering polarised protons (FT LHC) $\rightarrow$ access to more gluon TMDs
- Di-J/ψ production: most promising $\rightarrow$ gluon Sivers function

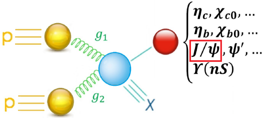
  C. Hadjidakis et al. Phys.Rept. 911 (2021) 1

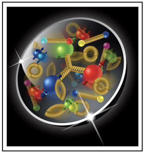
Backup slides

Inclusive production of J/ψ pairs in pp collisions (gluon fusion)

$\Downarrow$

Azimuthal modulations of the cross section for inclusive production of quarkonium pairs in hadronic collisions





$\hookrightarrow$ **understanding the internal structure of nucleons**

$\hookrightarrow$ gluon dynamics poorly known

Results $\longrightarrow$ future measurements at LHCb

$\hookrightarrow$ Transverse Momentum Dependent PDFs (TMDs)

Gluon TMDs

2 independent collinear partonic distributions:

- $f_1^g(x)$ *unpolarised*
- $g_1^g(x)$ *circular*

Unpolarised protons → 2 TMDs:

- f_1^g: unpolarised gluon TMD
- $h_1^{\perp g}$: linearly polarised gluon TMD

		Gluon		
		U	**C**	**L**
Nucleon	**U**	f_1		$h_1^{\perp}$
	L		g_{1L}	$h_{1L}^{\perp}$
	T	$f_{1T}^{\perp}$	g_{1T}	$h_1, h_{1T}^{\perp}$

Why di-J/ψ production?

- **Single J/ψ production**: a lot of data at low p_T ✓
  ↪ but gluon in the final state → presence of soft gluons (non-perturbative) between Initial State Interactions (ISIs) and Final State Interactions (FSIs) can be problematic
  ↪ **no** TMD factorisation ✗
- **Single η_c production**: no gluon in the final state ✓
  ↪ but no data at low p_T ✗
- **Double J/ψ production**:
  ▹ data at low $p_T^{\psi\psi}$ ✓
  ▹ no gluon in the final state ✓
  ↪ gluon fusion: ISI can be encapsulated in the TMDs
  ↪ consider CSM: no FSIs

→ **Safe to assume TMD factorisation**

F. Scarpa *et al*, *Eur. Phys. J.C.* 80 no.2, (2020) 87

Gluon TMDs and correlators

TMD correlator parametrisation for an unpolarised proton

- ▷ unpolarised: $f_1^g \longrightarrow$
- ▷ linearly polarised: $h_1^{\perp g} \longrightarrow$

		Gluon U	Gluon C	Gluon L
Nucleon	U	f_1		$h_1^\perp$
Nucleon	L		g_{1L}	$h_{1L}^\perp$
Nucleon	T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

$$\Phi_g^{\mu\nu}(x, \vec{k}_T) = -\frac{1}{2x}\Big[g_T^{\mu\nu} f_1^g(x, \vec{k}_T^2) - \left(\frac{k_T^\mu k_T^\nu}{M_H^2} + g_T^{\mu\nu} \frac{\vec{k}_T^2}{2M_H^2} \right) h_1^{\perp g}(x, \vec{k}_T^2) \Big]$$

$\hookrightarrow$ Second term goes to 0 if $k_T = 0$

P.J. Mulders and J. Rodrigues (Phys.Rev.D 63 (2001) 094021)

- Implementing evolution is more easily done in impact parameter space (b_T), where convolutions become simple products:

$$d\sigma_{UU}^{gg} \propto \int d^2\mathrm{b}_T\, e^{-i\mathrm{b}_T\cdot\mathrm{q}_T}\, \hat{W}(\mathrm{b}_T, Q) + \mathcal{O}(\mathrm{q}_T^2/Q^2)$$

$$\hat{W}(\mathrm{b}_T, Q) = \hat{f}(x_1, \mathrm{b}_T; \zeta_f, \mu)\, \hat{g}(x_2, \mathrm{b}_T; \zeta_g, \mu)\, \mathcal{H}(Q; \mu)\,.$$

- The convolutions are rewritten by Fourier transforming:

$$\mathcal{C}[w\, f\, g](x_1, x_2, \vec{q}_T) = \int d^2\vec{k}_{T1} \int d^2\vec{k}_{T2}\, \delta^{(2)}(\vec{k}_{T1} + \vec{k}_{T2} - \vec{q}_T)$$
$$\times\, w_{n,m}(\vec{k}_{T1}, \vec{k}_{T2})\, f(x_1, \vec{k}_{T1})\, g(x_2, \vec{k}_{T2})$$
$$\Rightarrow \int_0^\infty \frac{db_T}{2\pi}\, b_T^n\, J_m(b_T\, q_T)\, \hat{f}(x_1, \mathrm{b}_T)\, \hat{g}(x_2, \mathrm{b}_T)$$

Introduction Evolution (2)

$$\mathcal{C}[w\,f\,g](x_1, x_2, \vec{q}_T; Q) = \int_0^\infty \frac{db_T}{2\pi}\, b_T^n\, J_m(b_T\, q_T)$$
$$\times\; e^{-S_A(b_T;Q^2,Q)}\, \hat{f}(x_1, \mathrm{b}_T; \mu_b^2, \mu_b)\, \hat{g}(x_2, \mathrm{b}_T; \mu_b^2, \mu_b)$$

- S_A contains $\ln Qb_T$
- Expressions (based on pQCD) are valid when:
  $b_0/Q \leq b_T \leq b_{T,\max}$
- At lower limit $\mu_b = \; b_0/b_T$ becomes larger than Q, i.e. evolution should stop ($S_A = 0$)
- At upper limit perturbation theory starts to fail, which is not exactly known. Common to take $b_{T,\max} = 0.5\,\mathrm{GeV}^{-1}$ or $b_{T,\max} = 1.5\,\mathrm{GeV}^{-1}$.
- This effectively boils down to a different resummation:
  $\mu_b(b_T)/Q \to \mu_b(b_T^*)/Q$

- We need to add a component that takes over as $b_T > b_{T,\text{max}}$:

$$\hat{W}(b_T, Q) \equiv \hat{W}(b_T^*, Q) e^{-S_{NP}(b_T, Q)}$$

- There are different parameterizations for S_{NP} in the literature, but typically it is chosen to be a Gaussian:

$$S_{NP}(b_T; Q) = A \ln \frac{Q}{Q_{NP}} b_T^2 \quad \text{with} \quad Q_{NP} = 1\,\text{GeV}$$

- We obtain the following expression for the convolutions:

$$\mathcal{C}[w\, f\, g](x_1, x_2, \vec{q}_T; Q) = \int_0^\infty \frac{db_T}{2\pi}\, b_T^n\, J_m(b_T\, q_T)\, e^{-S_A(b_T^*; Q^2, Q)}\, e^{-S_{NP}(b_T; Q)} \times\ \hat{f}(x_1, \mathrm{b}_T^*; \mu_b^2, \mu_b)\, \hat{g}(x_2, \mathrm{b}_T^*; \mu_b^2, \mu_b)$$

- The solution of the evolution equations results in:

$$\hat{f}_1^g(x_1, \mathrm{b}_T; \zeta, \mu) = e^{-\frac{1}{2}S_A(b_T;\zeta,\mu)}\hat{f}_1^g(x_1, \mathrm{b}_T; \mu_b^2, \mu_b)$$
$$\hat{h}_1^{\perp g}(x_1, \mathrm{b}_T; \zeta, \mu) = e^{-\frac{1}{2}S_A(b_T;\zeta,\mu)}\hat{h}_1^{\perp g}(x_1, \mathrm{b}_T; \mu_b^2, \mu_b)$$

- $\mu \sim Q$ avoids large logarithms in $\mathcal{H}$
- TMDs should be evaluated at their natural scale: $\sqrt{\zeta_0} \sim \mu_0 \ll \sqrt{\zeta} \sim \mu$
- $\Rightarrow$ take $\sqrt{\zeta_0} \sim \mu_0 \sim \mu_b \equiv b_0/b_T$ (with $b_0 = 2e^{-\gamma_E}$), in order to minimize both logarithms of μb_T and ζb_T^2 in S_A, and then evolved up to $\sqrt{\zeta} \sim \mu \sim Q$

Perturbative tails

- The large transverse momentum perturbative tail of the TMDs can be written as:

$$\hat{f}_1^g(x, b_T; \mu_b^2, \mu_b) = f_{g/P}(x; \mu_b) + \mathcal{O}(\alpha_s) + \mathcal{O}(b_T \Lambda_{\text{QCD}})$$

$$\hat{h}_1^{\perp g}(x, b_T; \mu_b^2, \mu_b) = -\frac{\alpha_s(\mu_b)}{\pi} \int_x^1 \frac{dx'}{x'} \left(\frac{x'}{x} - 1 \right) \left\{ C_A \, f_{g/P}(x'; \mu_b) + C_F \sum_{i=q,\bar{q}} f_{i/P}(x'; \mu_b) \right\} + \mathcal{O}(\alpha_s^2) + \mathcal{O}(b_T \Lambda_{\text{QCD}})$$

P. Sun et al. (Phys.Rev.D 84 (2011) 094005)

b_T-Domains

- To ensure $b_0/Q \leq b_T$ we take:

$$b_c(b_T) = \sqrt{b_T^2 + \left(\frac{b_0}{Q}\right)^2}$$

- For $b_T \leq b_{T,\text{max}}$:

$$b_T^*(b_c(b_T)) = \frac{b_c(b_T)}{\sqrt{1 + \left(\frac{b_c(b_T)}{b_{T,\text{max}}}\right)^2}}$$

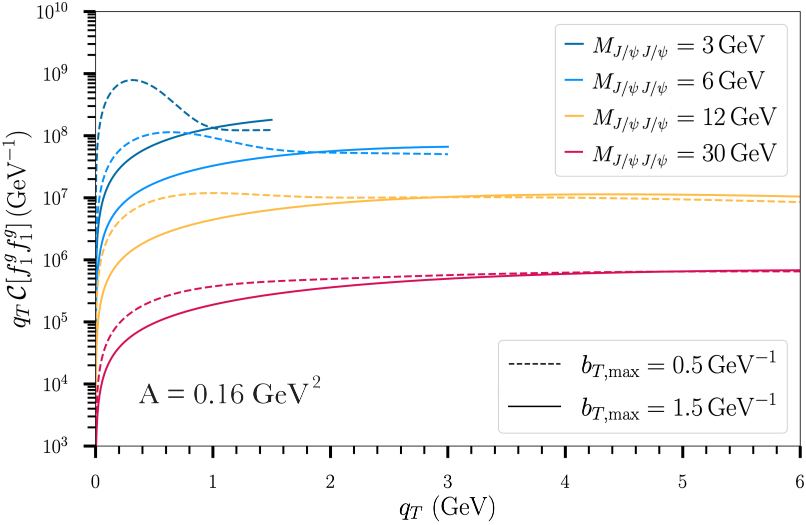
J. Collins et al. (Phys.Rev.D 94 (2016) 3, 034014)

$$S_{NP}(b_T; Q) = A \ln \frac{Q}{Q_{NP}} b_T^2 \quad \text{with} \quad Q_{NP} = 1\,\text{GeV}$$

$b_{T,\text{lim}}$ (GeV^{-1})	r (fm $\sim 1/(0.2$ GeV$)$)	A (GeV^2)
2	0.2	0.64
4	0.4	0.16
8	0.8	0.04

Table 1: Values of the parameter A for $b_{T,\text{lim}}$ and r determined at $Q = 12$ GeV. A is defined at which $\exp(-S_{NP})$ becomes negligible ($\sim 10^{-3}$). To estimate the uncertainty associated with the S_{NP} we vary $b_{T,\text{lim}}$ spanning roughly from $b_{T,\text{max}} = 1.5\ \text{GeV}^{-1}$ to the charge radius of the proton. r is the range over which the interactions occur from the centre of the proton.

Limitations of Gaussian ansatz for S_{NP}



Convolutions (as simple products)

Explicit formulae of the convolutions used:

$$\mathcal{C}[f_1^g f_1^g] = \int_0^\infty \frac{db_T}{2\pi}\, b_T\, J_0(b_T q_T)\, \hat{f}_1^g(x_a, \mathrm{b}_T^2)\, \hat{f}_1^g(x_b, \mathrm{b}_T^2)\,,$$

$$\mathcal{C}[w_2 h_1^{\perp g} h_1^{\perp g}] = \int_0^\infty \frac{db_T}{2\pi}\, b_T\, J_0(b_T q_T)\, \hat{h}_1^{\perp g}(x_a, \mathrm{b}_T^2)\, \hat{h}_1^{\perp g}(x_b, \mathrm{b}_T^2)\,,$$

$$\mathcal{C}[w_3 f_1^g h_1^{\perp g}] = \int_0^\infty \frac{db_T}{2\pi}\, b_T\, J_2(b_T q_T)\, \hat{f}_1^g(x_a, \mathrm{b}_T^2)\, \hat{h}_1^{\perp g}(x_b, \mathrm{b}_T^2)\,,$$

$$\mathcal{C}[w_3' h_1^{\perp g} f_1^g] = \int_0^\infty \frac{db_T}{2\pi}\, b_T\, J_2(b_T q_T)\, \hat{h}_1^{\perp g}(x_a, \mathrm{b}_T^2)\, \hat{f}_1^g(x_b, \mathrm{b}_T^2)\,,$$

$$\mathcal{C}[w_4 h_1^{\perp g}\; h_1^{\perp g}] = \int_0^\infty \frac{db_T}{2\pi}\, b_T\, J_4(b_T q_T)\, \hat{h}_1^{\perp g}(x_a, \mathrm{b}_T^2)\, \hat{h}_1^{\perp g}(x_b, \mathrm{b}_T^2)$$

$$F_1 = \frac{\mathcal{N}}{\mathcal{D} M_\Psi^2} \sum_{n=0}^{6} f_{1,n} (\cos\theta_{CS})^{2n} \qquad F_2 = \frac{2^4 3 M_\Psi^2 \mathcal{N}}{\mathcal{D} M_{\Psi\Psi}^4} \sum_{n=0}^{4} f_{2,n} (\cos\theta_{CS})^{2n}$$

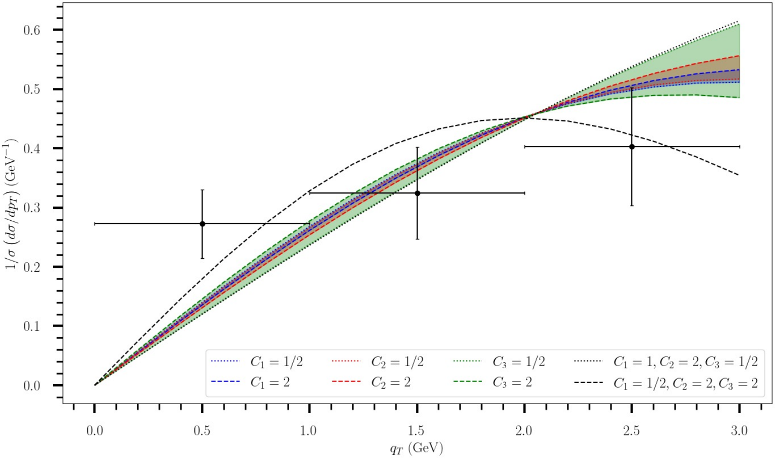
$$F_3' = F_3 = \frac{-2^3(1-\alpha^2)\mathcal{N}}{\mathcal{D} M_{\Psi\Psi}^2} \sum_{n=0}^{5} f_{3,n} (\cos\theta_{CS})^{2n}$$

$$F_4 = \frac{(1-\alpha^2)^2 \mathcal{N}}{\mathcal{D} M_{\Psi\Psi}^2} \sum_{n=0}^{6} f_{4,n} (\cos\theta_{CS})^{2n} \tag{1}$$

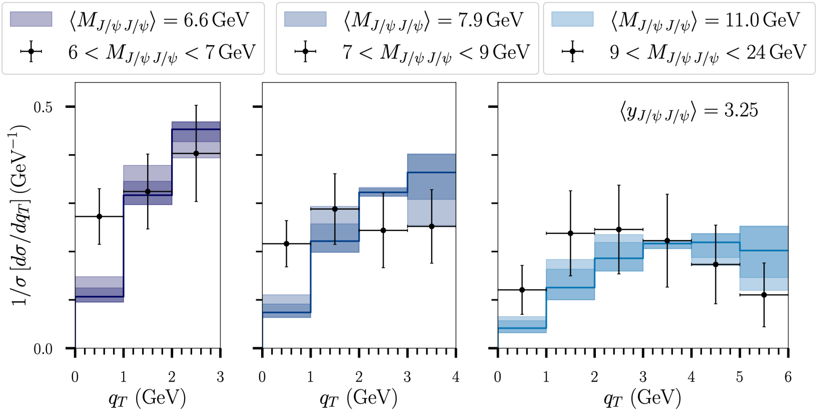
with: $\alpha = \frac{2M_\Psi}{M_{\Psi\Psi}}$, $\mathcal{N} = 2^{11} 3^{-4} (N_c^2 - 1)^{-2} \pi^2 \alpha_s^4 |R_\Psi(0)|^4$, $\mathcal{D} = M_{\Psi\Psi}^4 (1 - (1-\alpha^2)\cos\theta_{CS}^2)^4$ and $R_\Psi(0)$ is the J/ψ radial wave function at the origin and $N_c = 3$.

Updated results: $M_{QQ} = 6.6\text{GeV}$

Latest LHCb results (2024)

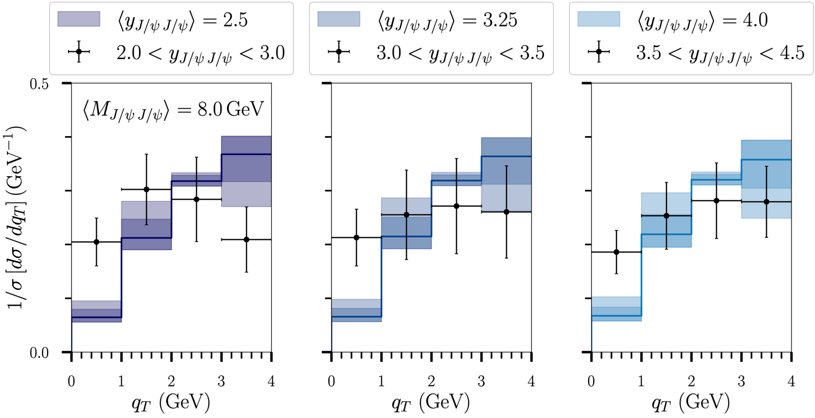


Switching on TMD evolution: updated results for J/ψ



Latest LHCb results (2024)

Switching on TMD evolution: updated results for J/ψ



Latest LHCb results (2024)