# *Gluon tomography through diffractive processes in a saturation framework*

Michael Fucilla

Université Paris-Saclay, CNRS/IN2P3, IJCLab

in collaboration with

R. Boussarie, A. V. Grabovsky, E. Li, L. Szymanowski and S. Wallon

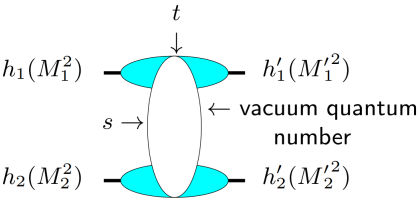
based on

**[Boussarie, Grabovsky, Szymanowski, Wallon. JHEP 11 (2016) 149]**
**[M. F., Grabovsky, Li, Szymanowski, Wallon. JHEP 03 (2023) 159]**
**[M. F., Grabovsky, Li, Szymanowski, Wallon. JHEP 02 (2024) 165]**

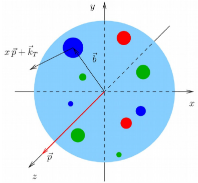
REF2024, IPhT Saclay, 14-18 October 2024

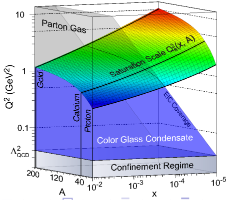
This project is supported by the European Union's Horizon 2020 research and innovation programme under Grant agreement no. 824093

- One of the important longstanding theoretical questions raised by QCD is its behaviour in the perturbative **Regge limit** $s \gg -t$




- Diffractive processes: **rapidity gap** between two clusters in the detector
- **Hard Pomeron** exchange at the **amplitudes** level
- Diffractive processes are strongly sensitive to the gluon **saturation dynamics** and are excellent to explore **hadron tomography**



# The dipole gluon Wigner function

- **Diffractive processes** $\to$ golden channels to probe the GTMD gluon distribution $xW\left(x, \vec{q}_\perp, \vec{\Delta}_\perp\right)$
- Dipole gluon Wigner function

$$xW\left(x, \vec{q}_\perp, \vec{b}_\perp\right) = \frac{2}{P^+(2\pi)^3} \int dz^+ d^2\vec{z}_\perp \int \frac{d^2\vec{\Delta}_\perp}{(2\pi)^2} e^{i\vec{q}_\perp \cdot \vec{z}_\perp - ixP^- z^+}$$
$$\times \left\langle P + \frac{\vec{\Delta}_\perp}{2} \middle| \mathrm{Tr}\left[U_+ F_a^{+i}\left(\vec{b}_\perp + \frac{z}{2}\right) U_- F_a^{+i}\left(\vec{b}_\perp - \frac{z}{2}\right)\right] \middle| P - \frac{\vec{\Delta}_\perp}{2} \right\rangle$$

- Gluon **GTMD distribution** at small-$x$

**[Dominguez, Marquet, Xiao Yuan (2011)]**
**[Hatta, Xiao, Yuan (2016)]**

$$xW\left(x, \vec{q}_\perp, \vec{\Delta}_\perp\right) \approx \frac{2N_c}{\alpha_s}\left(q_\perp^2 - \frac{\Delta_\perp^2}{4}\right) S_Y\left(\vec{q}_\perp, \vec{\Delta}_\perp\right)$$

- Fourier transform of the dipole $S$-matrix

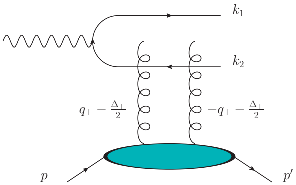
$$S_Y\left(\vec{q}_\perp, \vec{\Delta}_\perp\right) = \int \frac{d^2\vec{r}_\perp d^2\vec{b}_\perp}{(2\pi)^4} e^{i\vec{\Delta}_\perp \cdot \vec{b}_\perp + i\vec{q}_\perp \cdot \vec{r}_\perp} \left\langle \frac{1}{N_c} \mathrm{Tr}\, V\left(\vec{b}_\perp + \frac{\vec{r}_\perp}{2}\right) V^\dagger\left(\vec{b}_\perp - \frac{\vec{r}_\perp}{2}\right) \right\rangle_Y$$

$Y = \ln(1/x)$

# *Diffractive dijet production*

- **Diffractive dijet electroproduction** can be sensitive to both $q_\perp$ and $\Delta_\perp$ dependence of $W\left(x, \vec{q}_\perp, \vec{\Delta}_\perp\right)$

  **[Hatta, Xiao and Yuan (2016)]**




- Possibility of investigating angular correlations between impact parameter and dipole size predicted by small-$x$ evolutions
- Enhanced sensitivity to **gluonic saturation** from contributions beyond the leading order (e.g. two hard quark anti-quark jets accompanied by a softer gluon jet, in the *correlation limit*)

  **[Iancu, Mueller, Triantafyllopoulos (2022)]**
- Even better access to the Wigner distribution is possible via **diffractive dijet photoproduction** in ultraperipheral pA collision

  **[Hagiwara, Hatta, Pasechnik, Tasevsky, Teryaev (2017)]**

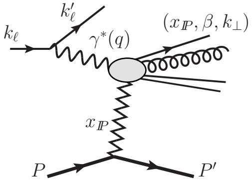
# *Semi-inclusive diffractive DIS (SIDDIS)*

- At small-$x$, the quark and gluon *TMD distribution functions* are directly related to the *color dipole* S-matrix in the CGC formalism

[**Paul's talk**] and [**Jamal's talk**]

- A similarly connection between the **diffractive parton distribution functions (DPDFs)** and the color dipole exists
- One of the best processes to investigate this connection is the **semi-inclusive diffractive DIS (SIDDIS)**

**[Hatta, Xiao and Yuan (2022)]**




- pQCD motivated initial inputs for collinear QCD evolution of DPDFs
- Study the matching between small-$x$ and moderate-$x$ regime

- **Goal**: extend all these studies at the full NLO level

# *Diffractive dijet/hadron(s) production at the NLO*

- **Diffractive dijet/hadron(s) production at NLO**

$$\gamma^{(*)}(p_\gamma) + P(p_0) \to V_L(p_V) + P(p_0') \qquad V = (\rho, \phi, \omega)$$

**[Boussarie, Grabovsky, Ivanov, Szymanowski, Wallon (2017)]**

**[Ivanov, Kotsky, Papa (2004)] [Mäntysaari, Pentalla (2022)]**

$$\gamma^{(*)}(p_\gamma) + P(p_0) \to V_T(p_V) + P(p_0') \qquad V = (\rho, \phi, \omega) \qquad (LO)$$

**[Boussarie, M.F., Szymanowski, Wallon (2024)]**

**[Samuel's talk]**

$$\gamma^{(*)}(p_\gamma) + P(p_0) \to j_1(p_{h1}) + j_2(p_{h2}) + P(p_0')$$

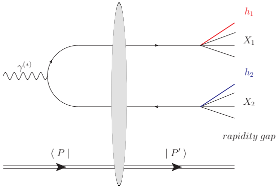
**[Boussarie, Grabovsky, Szymanowski, Wallon (2016)]**

$$\gamma^{(*)}(p_\gamma) + P(p_0) \to h_1(p_{h1}) + h_2(p_{h2}) + X + P(p_0') \qquad (X = X_1 + X_2)$$

**[M. F., Grabovsky, Li, Szymanowski, Wallon (2023)]**

$$\gamma^{(*)}(p_\gamma) + P(p_0) \to h_1(p_{h1}) + X + P(p_0')$$

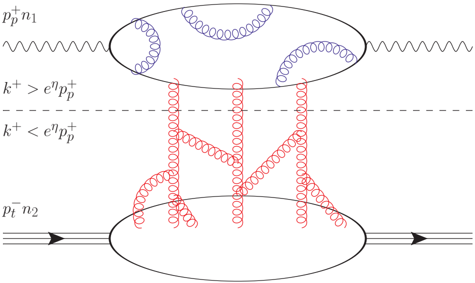
**[M. F., Grabovsky, Li, Szymanowski, Wallon (2024)]**




- General kinematics $(t, Q^2)$ and photon polarization
- Rapidity gap between $(h_1 h_2 X)$ and $P'$
- $\vec{p}_{h_1}^{\,2}, \vec{p}_{h_2}^{\,2} \gg \Lambda_{\mathrm{QCD}}^2$

# *Shockwave approach*

- High-energy approximation $s = (p_p + p_t)^2 \gg \{Q^2\}$




$$p_p = p_p^+ n_1 - \frac{Q^2}{2p_p^+} n_2$$

$$p_t = \frac{m_t^2}{2p_t^-} n_1 + p_t^- n_2$$

$$p_p^+ \sim p_t^- \sim \sqrt{\frac{s}{2}}$$

$$n_1^2 = n_2^2 = 0 \quad n_1 \cdot n_2 = 1$$

- Separation of the gluonic field into "fast" (quantum) part and "slow" (classical) part through a rapidity parameter $\eta < 0$

**[McLerran, Venugopalan (1994)] [I. Balitsky (1996-2001)]**

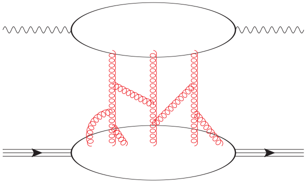
$$\mathcal{A}^\mu(k^+, k^-, \vec{k}) = A^\mu(k^+ > e^\eta p_p^+, k^-, \vec{k}) + b^\mu(k^+ < e^\eta p_p^+, k^-, \vec{k})$$

$e^\eta \ll 1$

# *Shockwave approach*

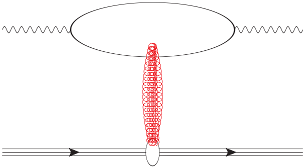
- Large longitudinal boost: $\Lambda = \sqrt{\frac{1+\beta}{1-\beta}} \sim \frac{\sqrt{s}}{m_t}$

$$\begin{cases} b^+(x^+, x^-, \vec{x}) & = \Lambda^{-1} b_0^+(\Lambda x^+, \Lambda^{-1} x^-, \vec{x}) \\ b^-(x^+, x^-, \vec{x}) & = \Lambda b_0^-(\Lambda x^+, \Lambda^{-1} x^-, \vec{x}) \\ b^i(x^+, x^-, \vec{x}) & = b_0^i(\Lambda x^+, \Lambda^{-1} x^-, \vec{x}) \end{cases}$$



$b_0^\mu(x)$

$\xrightarrow{\text{boost}}$



$b^\mu(x^+, x^-, \vec{x}) = \delta(x^+)\mathbf{B}(\vec{x}) n_2^\mu + \mathcal{O}\left(\Lambda^{-1}\right)$

***Shockwave*** approximation

- Light-cone gauge $A \cdot n_2 = 0$

$A \cdot b = 0 \implies$ *Simple effective Lagrangian*

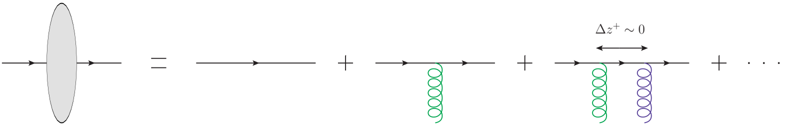
# *Shockwave approach*

- Interactions with the simple shockwave field

  *i.* Independence from $x^-$ $\implies$ conservation of $p^+$ (**eikonal** approx.)

  *ii.* $\delta(x^+)$ $\implies$ interactions at a **single transverse coordinate**.

- Quark line through the shockwave

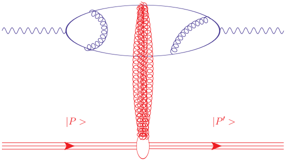



$$V_{\vec{z}_i} = 1 + ig \int_{-\infty}^{+\infty} dz_i^+ b_\eta^- \left(z_i^+, \vec{z}_i\right) + (ig)^2 \int_{-\infty}^{+\infty} dz_i^+ dz_j^+ b_\eta^- \left(z_i^+, \vec{z}_i\right) b_\eta^- \left(z_j^+, \vec{z}_i\right) \theta\left(z_{ij}^+\right) + \cdots$$

- Multiple interactions with the target $\rightarrow$ ***path-ordered Wilson lines***

$$V_{\vec{z}}^{\eta} = \mathcal{P} \exp\left[ig \int_{-\infty}^{+\infty} dz_i^+ b_\eta^- \left(z_i^+, \vec{z}\right)\right]$$

- Factorization in the Shockwave approximation

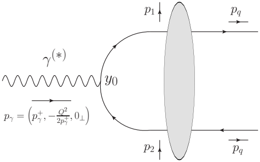



$$\mathcal{M}^{\eta} = N_c \int d^d \boldsymbol{z}_1 d^d \boldsymbol{z}_2 \, \Phi^{\eta}(\boldsymbol{z}_1, \boldsymbol{z}_2) \langle P' \left| \mathcal{U}^{\eta}_{12}(\boldsymbol{z}_1, \boldsymbol{z}_2) \right| P \rangle$$

- **Dipole operator** $\mathcal{U}^{\eta}_{ij} = 1 - \frac{1}{N_c} \operatorname{Tr}\left(V^{\eta}_{\vec{z}_i} V^{\eta\dagger}_{\vec{z}_j}\right)$
- Evolution equations
  - **Balitsky-JIMWLK** evolution equations
    **[Balitsky (1995)]**
    **[Jalilian-Marian, Iancu, McLerran, Weigert, Kovner, Leonidov]**
  - Large $N_c$ → **Balitky-Kovchegov (BK)** non-linear equation
    **[Balitsky (1995)] [Kovchegov (1999)]**
  - Evolution at the NLO
    **[Balitsky, Chirilli (2007)] [Kovner, Lublinsky, Mulian (2013)]**

# *Open diffractive $q\bar{q}$ production*

- Sudakov decomposition for the momenta: $p_i^\mu = x_i p_\gamma^+ n_1^\mu + \frac{\vec{p}_i^{\,2}}{2x_i p_\gamma^+} n_2^\mu + p_{i\perp}^\mu$




- LO $S$-matrix element

$$S_0 = \int d^D y_0 \; [\bar{u}(p_q, y_0)]^{nk} \left(-iee_q\right) \gamma^\alpha \theta(p_\gamma^+) \frac{\varepsilon_\alpha}{\sqrt{2p_\gamma^+}} e^{-ip\cdot y_0} \; [v(p_{\bar{q}}, y_0)]^{kl} \; \frac{\delta_{ln}}{\sqrt{N_c}}$$

$$\left[\bar{u}_\alpha^{nk}\left(p_q, z_0\right)\right]_{z_0^+<0} = \frac{(-i)^{d/2}}{2(2\pi)^{d/2}} \left(\frac{p_q^+}{-z_0^+}\right)^{d/2} \theta\left(p_q^+\right) \theta\left(-z_0^+\right)$$

$$\times \int d^d z_1 V_{\boldsymbol{z}_1}^{nk} \frac{\bar{u}(p_q)}{\sqrt{2p_q^+}} \gamma^+ \frac{-z_0^+ \gamma^- + \hat{z}_{10\perp}}{-z_0^+} \; \exp\left\{ip_q^+\left(z_0^- - \frac{\vec{z}_{10}^{\,2}}{2z_0^+} + i0\right) - i\vec{p}_q\cdot\vec{z}_{10}\right\}$$

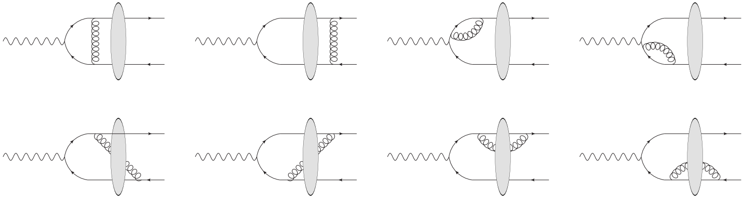
- LO scattering amplitude

$$\mathcal{M}_0^\alpha \propto N_c \delta(p_q^+ + p_{\bar{q}}^+ - p_\gamma^+) \int d^d\vec{p}_1 d^d\vec{p}_2 \; \delta(\vec{p}_{q1} + \vec{p}_{\bar{q}2}) \, \Phi_0^\alpha(\vec{p}_1, \vec{p}_2) \widetilde{\mathcal{U}}_{12}$$

# One loop $\gamma^{(*)} \to q\bar{q}$ impact factor

- One loop diagrams for the open $q$-$\bar{q}$ pair production

**[Boussarie, Grabovsky, Szymanowski, Wallon (2016)]**



- Color factor and Wilson line operators of top diagrams: $C_F N_c\, \mathcal{U}_{12}$
- Color factor and Wilson line operators of bottom diagrams:

$$-\frac{N_c^2}{2}(\mathcal{U}_{13} + \mathcal{U}_{32} - \mathcal{U}_{12} - \mathcal{U}_{13}\mathcal{U}_{32}) - C_F N_c\, \mathcal{U}_{12}$$

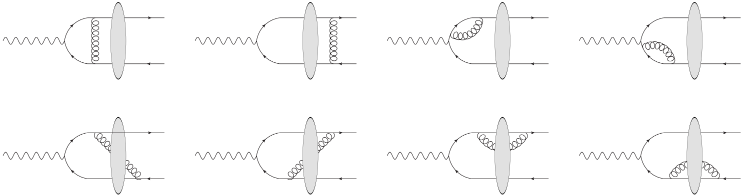
- Virtual reduced matrix element

$$T_1^{\alpha} = -\alpha_s \frac{N_c \Gamma(1-\epsilon)}{(4\pi)^{1+\epsilon}} \int d^d\vec{p}_1 d^d\vec{p}_2 \left\{ \delta\left(\vec{p}_{q1} + \vec{p}_{\bar{q}2}\right) \left(\frac{N_c^2 - 1}{N_c}\right) \tilde{\mathcal{U}}_{12}\left(\vec{p}_1, \vec{p}_2\right) \Phi_{V1}^{\alpha}\left(\vec{p}_1, \vec{p}_2\right) \right.$$

$$\left. + N_c \int \frac{d^d\vec{p}_3}{(2\pi)^d} \delta\left(\vec{p}_{q1} + \vec{p}_{\bar{q}2} - \vec{p}_3\right) \left[\tilde{\mathcal{U}}_{13} + \tilde{\mathcal{U}}_{32} - \tilde{\mathcal{U}}_{12} - \widetilde{\mathcal{U}_{13}\mathcal{U}_{32}}\right] \left(\vec{p}_1, \vec{p}_2, \vec{p}_3\right) \Phi_{V2}^{\alpha}\left(\vec{p}_1, \vec{p}_2, \vec{p}_3\right) \right\}$$

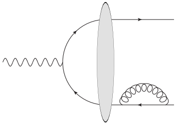
# *One loop $\gamma^{(*)} \to q\bar{q}$ impact factor: UV divergences*

- One loop diagrams for the open $q$-$\bar{q}$ pair production



- **UV-divergences** manifest themselves when $\vec{p}_g^{\,2} \to \infty$
- Renormalization in non-covariant gauges is typically challenging but all running coupling effects are encoded in the dynamics of the target
- Just dressing of the external quark lines

$$\Phi_{\text{dress}} \propto \left( \frac{1}{2\epsilon_{IR}} - \frac{1}{2\epsilon_{UV}} \right)$$

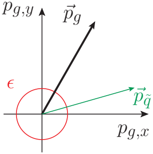


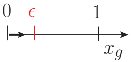
- $\epsilon_{IR} = \epsilon_{UV}$ mixes **UV** and **IR** divergences

# One loop $\gamma^{(*)} \to q\bar{q}$ impact factor: Rapidity divergences

***Rapidity divergences*** $(x_g \to 0)$

$$p_g^\mu = x_g p_\gamma^+ n_1^\mu + \frac{\vec{p}_g^{\,2}}{2 x_g p_\gamma^+} n_2^\mu + p_{g\perp}^\mu$$

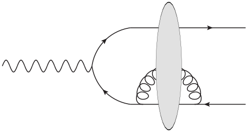



*i.* $x_g \to 0$

*ii.* $\vec{p}_g$ generic

- Coming from $\Phi_{V_2}$ (double dipole part of the virtual contribution)



- Regularized by **longitudinal cut-off**: $|p_g^+| = |x_g| p_\gamma^+ > \alpha p_\gamma^+ \implies \ln \alpha$ terms

- **Dim-reg** for transverse momentum integration $(d = D - 2 = 2 + 2\epsilon)$

- Artificial separation of projectile and target through rapidity regions generates the $x_g \to 0$ divergences

- B-JIMWLK equation for the dipole operator in momentum space

$$\frac{\partial \tilde{\mathcal{U}}^{\alpha}_{12}}{\partial \ln \alpha} = 2\alpha_s N_c \mu^{2-d} \int \frac{d^d\vec{k}_1 d^d\vec{k}_2 d^d\vec{k}_3}{(2\pi)^{2d}} \delta\left(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 - \vec{p}_1 - \vec{p}_2\right) \left(\widetilde{\mathcal{U}^{\alpha}_{13}\mathcal{U}^{\alpha}_{32}} + \tilde{\mathcal{U}}^{\alpha}_{13} + \tilde{\mathcal{U}}^{\alpha}_{32} - \tilde{\mathcal{U}}^{\alpha}_{12}\right)$$

$$\times \left[ 2\frac{\left(\vec{k}_1 - \vec{p}_1\right)\cdot\left(\vec{k}_2 - \vec{p}_2\right)}{\left(\vec{k}_1 - \vec{p}_1\right)^2 \left(\vec{k}_2 - \vec{p}_2\right)^2} + \frac{\pi^{\frac{d}{2}}\Gamma\left(1-\frac{d}{2}\right)\Gamma^2\left(\frac{d}{2}\right)}{\Gamma(d-1)} \left( \frac{\delta\left(\vec{k}_2 - \vec{p}_2\right)}{\left[\left(\vec{k}_1 - \vec{p}_1\right)^2\right]^{1-\frac{d}{2}}} + \frac{\delta\left(\vec{k}_1 - \vec{p}_1\right)}{\left[\left(\vec{k}_2 - \vec{p}_2\right)^2\right]^{1-\frac{d}{2}}} \right) \right]$$

- $\eta$ rapidity divide, which separates the upper and the lower impact factors

$$\Phi_0 \tilde{\mathcal{U}}^{\alpha}_{12} \to \Phi_0 \tilde{\mathcal{U}}^{\eta}_{12} + 2\ln\left(\frac{e^{\eta}}{\alpha}\right) \mathcal{K}_{BK} \Phi_0 \tilde{\mathcal{W}}_{123}$$

- Provides a counterterm to the $\ln\alpha$ divergence in the virtual double dipole
- The new double dipole $\times$ dipole part of the virtual impact factor

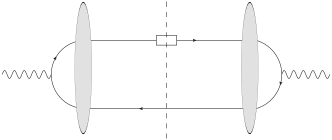
$$\tilde{\Phi}_{V_2} = \Phi_0 \tilde{\mathcal{U}}^{\alpha}_{12} + \Phi_{V2} \tilde{\mathcal{W}}^{\alpha}_{123}$$

is finite and indepedent of $\alpha$

# *SIDDIS: LO cross-section*

- Sudakov decomposition for the momenta: $p_i^\mu = x_i p_\gamma^+ n_1^\mu + \frac{\vec{p}^{\,2}}{2x_i p_\gamma^+} n_2^\mu + p_\perp^\mu$

$$\begin{array}{ccccc} & + & - & \perp & \\ p_\gamma = & \Bigg( p_\gamma^+ \; , & -\dfrac{Q^2}{2p_\gamma^+} \; , & \vec{0} & \Bigg) \end{array}$$



- Collinearity $(p_q^+, \vec{p}_q) = (x_q/x_h)(p_h^+, \vec{p}_h)$

$$\frac{d\sigma_{0JI}^{q\to h}}{dx_h d^d p_{h\perp}} = \frac{2\alpha_{\rm em} Q^2}{(2\pi)^{4d} N_c \, x_h^d} \sum_q Q_q^2 \int_{x_h}^1 dx_q \; x_q^{1+d} (1-x_q)^2 D_q^h \left(\frac{x_h}{x_q}\right) f_{JI}$$

$$f_{JI} = \int d^d p_{\bar{q}\perp} d^d p_{2\perp} d^d p_{2'\perp} \frac{\mathbf{F}\left(\frac{x_q}{2x_h} p_{h\perp} + \frac{1}{2} p_{\bar{q}\perp} - p_{2\perp}\right)}{\vec{p}_{\bar{q}2}^{\,2} + x_q(1-x_q)Q^2} \frac{\mathbf{F}^*\left(\frac{x_q}{2x_h} p_{h\perp} + \frac{1}{2} p_{\bar{q}\perp} - p_{2'\perp}\right)}{\vec{p}_{\bar{q}2'}^{\,2} + x_q(1-x_q)Q^2} \delta_{JI}$$

$J, I \to$ photon polarization of the amplitudes

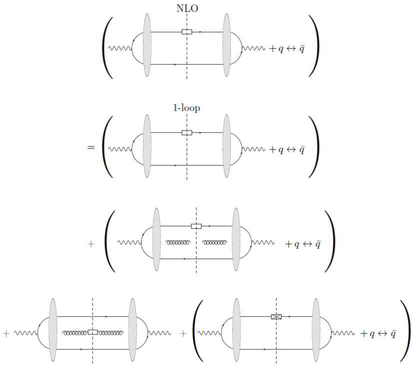
$$\delta_{LL} = 1, \qquad \delta_{TL} = \frac{(1-2x_q)}{2x_q(1-x_q)Q} \left(\vec{p}_{\bar{q}2'} \cdot \vec{\varepsilon}_T\right)^*$$

$$\delta_{TT} = \left[(1-2x_q)^2 g_\perp^{ri} g_\perp^{lk} - g_\perp^{rk} g_\perp^{li} + g_\perp^{rl} g_\perp^{ik}\right] \frac{\varepsilon_{Ti} \; p_{\bar{q}2\perp r} \left(\varepsilon_{Tk} \; p_{\bar{q}2'\perp l}\right)^*}{4x_q^2(1-x_q)^2 Q^2}$$

# *SIDDIS: NLO cross-section in a nutshell*

- Different *fragmentation mechanisms*

   *i.* Quark fragmentation
   *ii.* Anti-quark fragmentation
   *iii.* Gluon fragmentation



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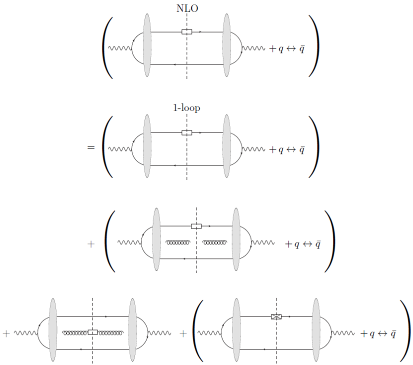
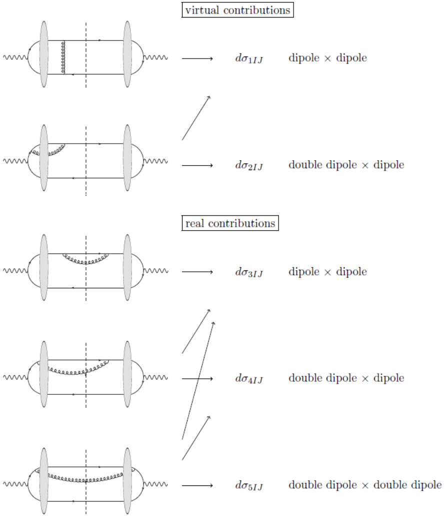
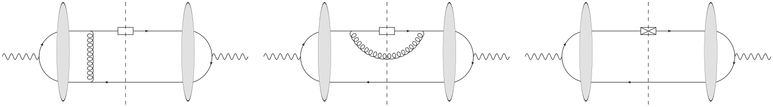



- Operator structure classification

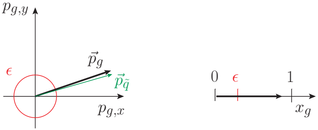


- Divergent contributions



$d\sigma_{1IJ}$ $d\sigma_{3IJ}$ $d\sigma_{\mathrm{counter}}$

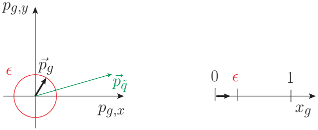
- Collinear divergence




*i.* $\vec{p}_g \to \vec{p}_{\tilde{q}} = \frac{x_g}{x_q}\vec{p}_q$

*ii.* $x_g$ generic

- Soft divergence

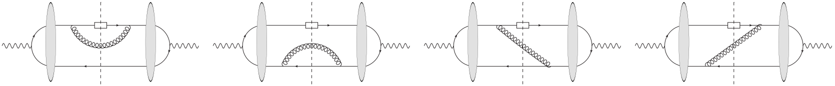



*i.* $\vec{p}_g \equiv x_g \vec{u}$

*ii.* $x_g \to 0$ and $\vec{u}$ generic

- Soft and collinear divergence ($x_g \to 0$ and $\vec{u} \to \frac{\vec{p}_q}{x_q}$)

- Divergences in the quark fragmentation channel



(1): soft + collinear ($qg$) (2): soft + collinear ($\bar{q}g$) (3): soft (4): soft

- In the **soft limit** the four diagrams combines leaving the LO cross-section times a soft current factor

$$\frac{d\sigma_{JI}^{q\to h}}{dx_h\, d^d p_{h\perp}}\Bigg|_{\text{Soft}} = \frac{8\alpha_{\text{em}} Q^2}{(2\pi)^{4d} N_c x_h^d} \sum_q Q_q^2 \int_{x_h}^1 dx'_q (x'_q)^{1+d} (1-x'_q)^2 D_q^h\left(\frac{x_h}{x'_q}, \mu_F\right) \int d^d p_{2\perp} \int d^d p_{2'\perp}$$

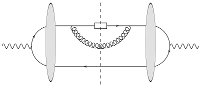
$$\times \frac{\alpha_s C_F}{\mu^{2\epsilon}} \int d^d p_{\bar{q}\perp} \frac{\mathbf{F}\left(\frac{x'_q}{2x_h} p_{h\perp} + \frac{1}{2} p_{\bar{q}\perp} - p_{2\perp}\right)}{\vec{p}_{\bar{q}2}^{\,2} + x'_q(1-x'_q)Q^2} \frac{\mathbf{F}^*\left(\frac{x'_q}{2x_h} p_{h\perp} + \frac{1}{2} p_{\bar{q}\perp} - p_{2'\perp}\right)}{\vec{p}_{\bar{q}2'}^{\,2} + x'_q(1-x'_q)Q^2} \delta_{JI}$$

$$\times \int_\alpha^{1-x'_q} \frac{dx_g}{x_g^{1-2\epsilon}} \int \frac{d^d p_{g\perp}}{(2\pi)^d} \frac{\left(\frac{\vec{p}_q}{x'_q} - \frac{\vec{p}_{\bar{q}}}{1-x'_q}\right)^2}{\left(\vec{p}_g - \frac{\vec{p}_q}{x'_q}\right)^2 \left(\vec{p}_g - \frac{\vec{p}_{\bar{q}}}{1-x'_q}\right)^2}$$

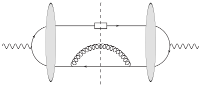
- The soft (and soft + collinear) singularities cancel when this contribution is combined with the virtual corrections

# *SIDDIS: collinear singularities*

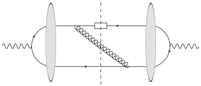
- Divergences in the quark fragmentation channel



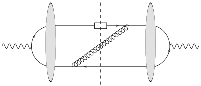
(1): soft + collinear ($qg$)



(2): soft + collinear ($\bar{q}g$)



(3): soft

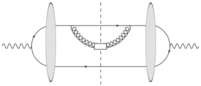


(4): soft

- Soft subtraction

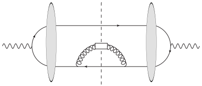
$$d\sigma_{1,V} + d\sigma_{3,\mathrm{soft}} + (d\sigma_3^{(3)} - d\sigma_{3,\mathrm{soft}}^{(3)}) + ((d\sigma_3^{(4)} - d\sigma_{3,\mathrm{soft}}^{(4)}))$$
$$+ \underbrace{(d\sigma_3^{(1)} - d\sigma_{3,\mathrm{soft}}^{(1)})}_{d\sigma_{3,\mathrm{collinear}}^{q(1)}} + \underbrace{(d\sigma_3^{(2)} - d\sigma_{3,\mathrm{soft}}^{(2)})}_{d\sigma_{3,\mathrm{collinear}}^{q(2)}} + d\sigma_{\mathrm{counter}}$$

- Divergences: $g$-fragmentation



(5): collinear $\longrightarrow d\sigma_{3,\mathrm{collinear}}^{g(5)}$

- Divergences: $g$-fragmentation



(6): collinear $\longrightarrow d\sigma_{3,\mathrm{collinear}}^{g(6)}$

# *SIDDIS: NLO renormalized fragmentation functions*

- NLO renormalized quark FFs

$$D_q^{h_1}\left(\frac{x_{h_1}}{x_q}\right) = D_q^{h_1}\left(\frac{x_{h_1}}{x_q}, \mu_F\right) - \frac{\alpha_s}{2\pi}\left(\frac{1}{\hat{\epsilon}} + \ln\frac{\mu_F^2}{\mu^2}\right)\left[\left[P_{qq}\otimes D_q^{h_1}\right]\left(\frac{x_{h_1}}{x_q}, \mu_F\right) + \left[P_{gq}\otimes D_g^{h_1}\right]\left(\frac{x_{h_1}}{x_q}, \mu_F\right)\right]$$

$$\downarrow$$

- Divergent contributions from the NLO fragmentation functions

$$\left.\frac{d\sigma_{JI}^{q\to h}}{dx_h d^d p_{h\perp}}\right|_{\mathrm{ct}} = -\frac{2\alpha_{\mathrm{em}}Q^2}{(2\pi)^{4d}N_c\, x_h^d}\sum_q Q_q^2\int_{x_h}^1 dx_q\; x_q^{1+d}(1-x_q)^2 f_{JI}\frac{\alpha_s}{2\pi}\left(\frac{1}{\hat{\epsilon}} + \ln\left(\frac{\mu_F^2}{\mu^2}\right)\right)$$
$$\int_{\frac{x_h}{x_q}}^1 \frac{d\beta}{\beta}\left[D_q^h\left(\frac{x_h}{\beta x_q}, \mu_F\right)P_{qq}(\beta) + D_g^h\left(\frac{x_h}{\beta x_q}, \mu_F\right)P_{gq}(\beta)\right]$$

- Finite part of the cross sections

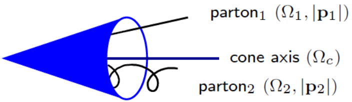
$$d\sigma^h = \left(\sum_{\{q\}} D_q^h\otimes d\hat{\sigma}_{q(\bar{q}),\mathrm{fin}}^{1-\mathrm{loop}} + \sum_{\{q\}} D_q^h\otimes d\hat{\sigma}_{q(\bar{q}g),\mathrm{fin}}^{\mathrm{Born}} + (q\leftrightarrow\bar{q})\right) + D_g^h\otimes d\hat{\sigma}_{g(q\bar{q}),\mathrm{fin}}^{\mathrm{Born}}$$

- In a similar fashion the cross sections for the diffractive di-hadron photo- or electroproduction are obtained

**[M. F., Grabovsky, Li, Szymanowski, Wallon (2023)]**

# *Diffractive dijet*

- LO diffractive dijet cross section **[Hatta, Xiao, Yuan (2016)]**
- NLO cross section using the **cone jet algorithm**
  **[Boussarie, Grabovsky, Szymanowski, Wallon (2016)]**
- $|\vec{p}_i|$ = *transverse energy deposit* in the calorimeter cell $i$ of parameter $(y_i, \phi_i)$ in $y$-$\phi$ plane
- Define the transverse energy of the jet: $p_J = |\vec{p}_1| + |\vec{p}_2|$




$$\Omega_c \begin{cases} y_J = \dfrac{|\mathbf{p}_1|\, y_1 + |\mathbf{p}_2|\, y_2}{p_J} \\[2ex] \phi_J = \dfrac{|\mathbf{p}_1|\, \phi_1 + |\mathbf{p}_2|\, \phi_2}{p_J} \end{cases}$$

- $\Omega_i = (y_i, \phi_i)$ in the $y$-$\phi$ plane

$$\text{If distances } |\Omega_i - \Omega_c|^2 \equiv (y_i - y_J)^2 + (\phi_i - \phi_J)^2 < R^2 \qquad (i = 1, 2)$$
$$\Longrightarrow \text{ partons 1 and 2 are in the same cone } \Omega_c$$

- Applying this (in the small $R^2$ limit) cancel soft and collinear divergences
- Same observables using the exclusive $k_t$ jet algorithm
  **[Boussarie, Grabovsky, Szymanowski, Wallon (2019)]**

# *Summary and prospects*

- Fully general NLO results at small-$x$ for
  - **Diffractive dijet production**
    **[Boussarie, Grabovsky, Szymanowski, Wallon (2016)]**
  - **Dihadron production in SIDDIS**
    **[M. F., Grabovsky, Li, Szymanowski, Wallon (2023)]**
  - **Semi-inclusive diffractive DIS (SIDDIS)**
    **[M. F., Grabovsky, Li, Szymanowski, Wallon (2024)]**
- Exploiting particular kinematic configurations
  - **Correlation limit** of dijet or dihadron production in SIDDIS at NLO
  - SIDDIS in the **TMD limit** ($Q^2 \gg \vec{p}_h^{\,2}$) at NLO level

  **[Boussarie, M.F., Szymanowski, Wallon, Yuan. (ongoing work)]**
- This has several advantages:
  - Exploring the connection with diffractive distributions and GTMD
  - Interplay between small-$x$ and Sudakov effects
  - More manageable analytical expression of small-$x$ cross sections suitable for the numerical implementation

Thanks for your attention

# Backup

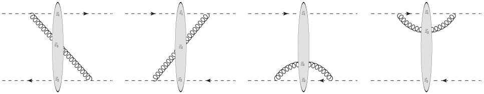
- ***Balitsky-JIMWLK evolution equations*** for the dipole
  [Balitsky — Jalilian-Marian, Iancu, McLerran, Weigert, Kovner, Leonidov]

$$\frac{\partial \mathcal{U}_{12}^{\eta}}{\partial \eta} = \frac{\alpha_s N_c}{2\pi^2} \int d^2\vec{z}_3 \left( \frac{\vec{z}_{12}^{\,2}}{\vec{z}_{23}^{\,2}\,\vec{z}_{31}^{\,2}} \right) \Big[ \underbrace{\mathcal{U}_{13}^{\eta} + \mathcal{U}_{32}^{\eta} - \mathcal{U}_{12}^{\eta}}_{\text{BFKL}} - \mathcal{U}_{13}^{\eta}\mathcal{U}_{32}^{\eta} \Big]$$

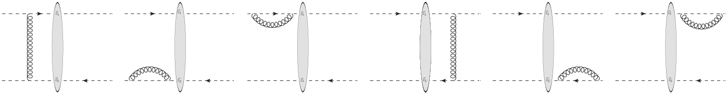
$$\frac{\partial \mathcal{U}_{13}^{\eta}\mathcal{U}_{32}^{\eta}}{\partial \eta} = \cdots \qquad\qquad \longleftarrow \quad \textit{Balitsky hierarchy}$$

$$\vdots$$

- Double dipole contribution and Dipole contribution

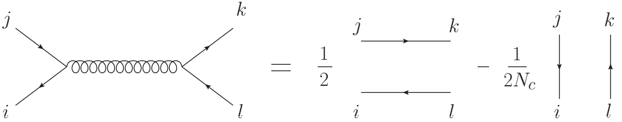


- Dipole contribution



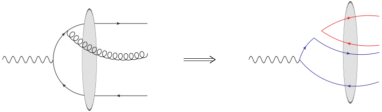
# *Balitsky-Kovchegov evolution equation*

- Large-$N_c$ limit **[G. 't Hooft (1974)]**




$$\boxed{t^a_{ij} t^a_{kl} = \frac{1}{2}\left(\delta_{il}\delta_{jk} - \frac{1}{N_c}\delta_{ij}\delta_{kl}\right)}$$

- Double dipole → Dipole × dipole



$$\langle \mathcal{U}^\eta_{13}\mathcal{U}^\eta_{32}\rangle \to \langle \mathcal{U}^\eta_{13}\rangle \, \langle \mathcal{U}^\eta_{32}\rangle$$

- Hierarchy of equations broken → closed non-linear **BK-equation**

**[I. I. Balitsky (1995)] [Y. V. Kovchegov (1999)]**

$$\frac{\partial \langle \mathcal{U}^\eta_{12}\rangle}{\partial \eta} = \frac{\alpha_s N_c}{2\pi^2} \int d^2\vec{z}_3 \left(\frac{\vec{z}^{\,2}_{12}}{\vec{z}^{\,2}_{23}\vec{z}^{\,2}_{31}}\right) \left[\langle \mathcal{U}^\eta_{13}\rangle + \langle \mathcal{U}^\eta_{32}\rangle - \langle \mathcal{U}^\eta_{12}\rangle - \langle \mathcal{U}^\eta_{13}\rangle\langle \mathcal{U}^\eta_{32}\rangle\right]$$

with $\langle \mathcal{U}^\eta_{12}\rangle \equiv \langle P'|\mathcal{U}^\eta_{12}|P\rangle$

- LO diffractive dijet cross section (transverse photon initiated)

**[Boussarie, Grabovsky, Szymanowski, Wallon (2016)]**
**[Hatta, Xiao and Yuan (2016)]**

$$\frac{d\sigma^{\gamma P}}{dy_1 dy_2 d^2P_\perp d^2\Delta_\perp} = \sum_f \alpha_{em} e_f^2 \delta(x_\gamma - 1) 2z(1-z)(z^2 + (1-z)^2) N_c \int d^2k_{1g\perp} d^2k_{2g\perp}$$

$$\times S_Y\left(\vec{k}_{1g\perp}, \vec{\Delta}_\perp\right) S_Y^*\left(\vec{k}_{2g\perp}, \vec{\Delta}_\perp\right) \left[\frac{(\vec{P}_\perp - \vec{k}_{1g\perp})}{(\vec{P}_\perp - \vec{k}_{1g\perp})^2 + \epsilon_f^2} \cdot \frac{(\vec{P}_\perp - \vec{k}_{2g\perp})}{(\vec{P}_\perp - \vec{k}_{2g\perp})^2 + \epsilon_f^2}\right]$$

$$\vec{P}_\perp = \frac{\vec{k}_{2\perp} - \vec{k}_{1\perp}}{2} \qquad \vec{\Delta}_\perp = -\vec{k}_{1\perp} - \vec{k}_{2\perp} \qquad \epsilon = z(1-z)Q^2 + m_f^2$$

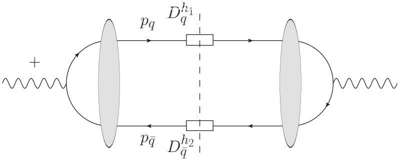
$z \rightarrow$ longitudinal fraction of the photon momenta carried by the quark jet

$y_1, y_2 \rightarrow$ rapidities of the jets, $z = \frac{|\vec{P}_\perp|}{s} e^{y_1}$ and $\bar{z} = 1 - z = \frac{|\vec{P}_\perp|}{s} e^{y_2}$

# *Dihadron in SIDDIS: LO cross-section*

- Sudakov decomposition for the momenta: $p_i^\mu = x_i p_\gamma^+ n_1^\mu + \frac{\vec{p}^{\,2}}{2 x_i p_\gamma^+} n_2^\mu + p_\perp^\mu$

$$\begin{array}{cccc} & + & - & \perp \\ p_\gamma = & \Big( p_\gamma^+ \;, & -\dfrac{Q^2}{2p_\gamma^+} \;, & \vec{0} \Big) \end{array}$$




- Collinearity $(p_q^+, \vec{p}_q) = (x_q/x_{h_1})(p_{h_1}^+, \vec{p}_{h_1})$ and $(p_{\bar{q}}^+, \vec{p}_{\bar{q}}) = (x_{\bar{q}}/x_{h_2})(p_{h_2}^+, \vec{p}_{h_2})$

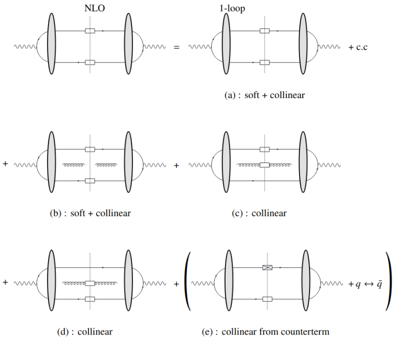
$$\frac{d\sigma_{0JI}^{h_1 h_2}}{dx_{h_1} dx_{h_2} d^d\vec{p}_{h_1} d^d\vec{p}_{h2}} = \sum_q \int_{x_{h_1}}^1 \frac{dx_q}{x_q} \int_{x_{h_2}}^1 \frac{dx_{\bar{q}}}{x_{\bar{q}}} \left(\frac{x_q}{x_{h1}}\right)^d \left(\frac{x_{\bar{q}}}{x_{h2}}\right)^d$$
$$D_q^{h_1}\left(\frac{x_{h_1}}{x_q}\right) D_{\bar{q}}^{h_2}\left(\frac{x_{h_2}}{x_{\bar{q}}}\right) \frac{d\hat{\sigma}_{JI}}{dx_q dx_{\bar{q}} d^d\vec{p}_q d^d\vec{p}_{\bar{q}}} + (h_1 \leftrightarrow h_2)$$

$J, I \rightarrow$ photon polarization for respectively the complex conjugated amplitude and the amplitude.

# Dihadron in SIDDIS: NLO cross-section in a nutshell

- Different *fragmentation mechanisms*

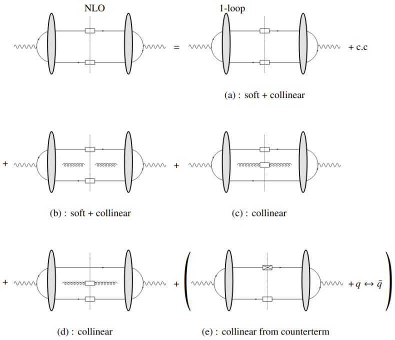
  i. Quark fragmentation
  ii. Anti-quark fragmentation
  iii. Gluon fragmentation



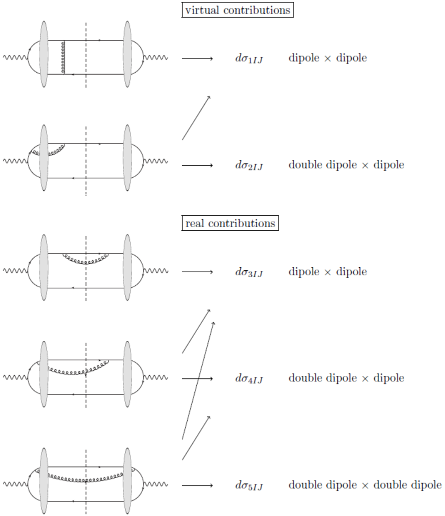
# *Dihadron in SIDDIS: NLO cross-section in a nutshell*

- Different *fragmentation mechanisms*

  *i.* Quark fragmentation
  *ii.* Anti-quark fragmentation
  *iii.* Gluon fragmentation

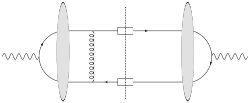



- Operator structure classification

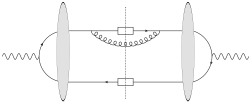


# Dihadron in SIDDIS: IR singularities

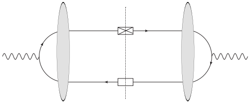
- Divergent contributions



$d\sigma_{1IJ}$

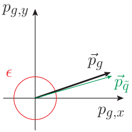


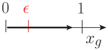
$d\sigma_{3IJ}$



$d\sigma_{\text{counter}}$

- Collinear divergence

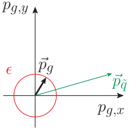


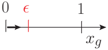




*i.* $\vec{p}_g \to \vec{p}_{\tilde{q}} = \frac{x_g}{x_q}\vec{p}_q$

*ii.* $x_g$ generic

- Soft divergence



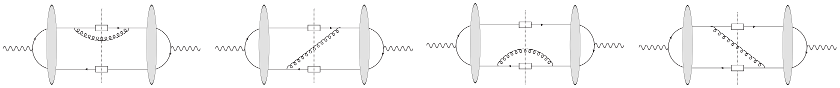

*i.* $\vec{p}_g \equiv x_g \vec{u}$

*ii.* $x_g \to 0$ and $\vec{u}$ generic

- Soft and collinear divergence ($x_g \to 0$ and $\vec{u} \to \frac{\vec{p}_q}{x_q}$)

# *Dihadron in SIDDIS: IR singularities*

- Divergences: $q\bar{q}$-fragmentation

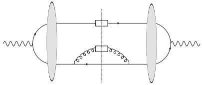


(1): soft + collinear ($qg$) (2): soft (3): soft + collinear ($\bar{q}g$) (4): soft

- Treatment of divergences in a nutshell

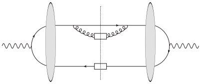
$$d\sigma_1 + d\sigma_{3,\mathrm{soft}} + \underbrace{(d\sigma_3^{(1)} - d\sigma_{3,\mathrm{soft}}^{(1)})}_{d\sigma_{3,\mathrm{collinear}}^{(1)}} + (d\sigma_3^{(2)} - d\sigma_{3,\mathrm{soft}}^{(2)}) + \underbrace{(d\sigma_3^{(3)} - d\sigma_{3,\mathrm{soft}}^{(3)})}_{d\sigma_{3,\mathrm{collinear}}^{(3)}} + ((d\sigma_3^{(4)} - d\sigma_{3,\mathrm{soft}}^{(4)})) + d\sigma_{\mathrm{counter}}$$

- Divergences: $qg$-fragmentation



(5): collinear $\longrightarrow d\sigma_{3,\mathrm{collinear}}^{\bar{q}g(5)}$

- Divergences: $\bar{q}g$-fragmentation



(6): collinear $\longrightarrow d\sigma_{3,\mathrm{collinear}}^{\bar{q}g(6)}$

# Dihadron: NLO renormalized fragmentation functions

- Renormalized quark FFs (similar for the anti-quark)

$$\tilde{D}_q^{h_1}\left(\frac{x_{h_1}}{x_q}\right) = D_q^{h_1}\left(\frac{x_{h_1}}{x_q}, \mu_F\right) - \frac{\alpha_s}{2\pi}\left(\frac{1}{\hat{\epsilon}} + \ln\frac{\mu_F^2}{\mu^2}\right)\left[[P_{qq}\otimes D_q^{h_1}]\left(\frac{x_{h_1}}{x_q}, \mu_F\right) + [P_{gq}\otimes D_g^{h_1}]\left(\frac{x_{h_1}}{x_q}, \mu_F\right)\right]$$

$$\downarrow$$

$$d\sigma_{LL}^{h_1 h_2}\Big|_{\text{ct}} = \frac{4\alpha_{\text{em}}Q^2}{(2\pi)^{4(d-1)}N_c}\sum_q Q_q^2 \int_{x_{h_1}}^1 dx_q \int_{x_{h_2}}^1 dx_{\bar{q}} x_q x_{\bar{q}} \left(\frac{x_q}{x_{h_1}}\right)^d \left(\frac{x_{\bar{q}}}{x_{h_2}}\right)^d \delta(1 - x_q - x_{\bar{q}})$$

$$\times \mathcal{F}_{LL}\left(-\frac{\alpha_s}{2\pi}\right)\left(\frac{1}{\hat{\epsilon}} + \ln\frac{\mu_F^2}{\mu^2}\right)\Bigg\{\underbrace{[P_{qq}\otimes D_q^{h_1}]\left(\frac{x_{h_1}}{x_q}, \mu_F\right)}_{(1)} D_{\bar{q}}^{h_2}\left(\frac{x_{h_2}}{x_{\bar{q}}}, \mu_F\right)$$

$$\underbrace{+[P_{gq}\otimes D_g^{h_1}]\left(\frac{x_{h_1}}{x_q}, \mu_F\right)}_{(6)} D_{\bar{q}}^{h_2}\left(\frac{x_{h_2}}{x_{\bar{q}}}, \mu_F\right) + \left[\left(q, x_q, x_{h_1}\right) \leftrightarrow \left(\bar{q}, x_{\bar{q}}, x_{h_2}\right)\right]\Bigg\} + (h_1 \leftrightarrow h_2)$$

- Finite part of the cross sections

$$d\sigma_{h_1,h_2} = \sum_{(a,b)} D_a^{h_1}\otimes D_b^{h_2}\otimes d\hat{\sigma}_{ab} \qquad (a,b) = \{(q,\bar{q}),(q,g),(g,\bar{q})\}$$

- Extension to the **semi-inclusive diffractive DIS (SIDDIS)** at the NLO
  **[M.F., Grabovsky, Li, Szymanowski, Wallon (2024)]**