Incoherent Diffractive production of jets in electron-nucleus DIS at high energies

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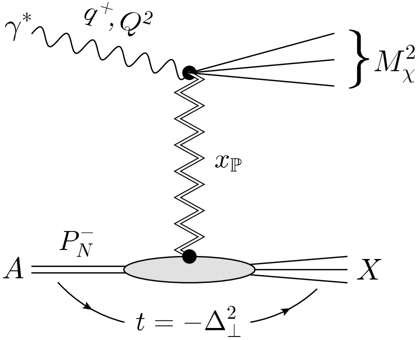
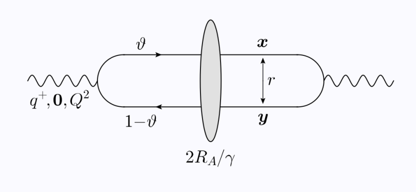
Incoherent DIS reaction



Generic incoherent reaction $\gamma^* A \to \chi X$

Time scales in DIS in the dipole picture



γ^*

- Right mover off shell $q^\mu = (q^+, \mathbf{0}, -Q^2/2q^+)$
- Projectile lifetime $\tau \sim 2q^+/Q^2$

A

- Left mover $p^\mu = (M_N^2/2P_N^-, \mathbf{0}, P_N^-)$ per nucleon
- Contracted length $L \sim 2R_A M_N/P_N^- \sim A^{1/3}/P_N^-$

$$\tau \gg L \iff xA^{1/3} \ll 1$$

Coherent vs. incoherent diffraction

Diffraction: rapidity gap

Target average to be taken with CGC wave-function

Total diffraction: $\langle T(\boldsymbol{x},\boldsymbol{y})T(\bar{\boldsymbol{x}},\bar{\boldsymbol{y}})\rangle$

Coherent diffraction:

$\langle T(\boldsymbol{x},\boldsymbol{y})\rangle\langle T(\bar{\boldsymbol{x}},\bar{\boldsymbol{y}})\rangle$

Incoherent diffraction:

$\langle T(\boldsymbol{x},\boldsymbol{y})T(\bar{\boldsymbol{x}},\bar{\boldsymbol{y}})\rangle - \langle T(\boldsymbol{x},\boldsymbol{y})\rangle\langle T(\bar{\boldsymbol{x}},\bar{\boldsymbol{y}})\rangle$

Homogeneous target:

$\sim \delta^2(\boldsymbol{\Delta})$ (smeared to $1/R_A$)
Momentum transfer from the target to projectile is negligible

Non-zero momentum transfer
$\boldsymbol{\Delta}$ conjugates to difference of impact parameters $\boldsymbol{B}$

Incoherent Diffraction

Variance of scattering amplitude determined by target fluctuations

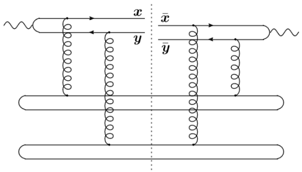
- Particle number fluctuations (Pomeron loops)
- Shape fluctuations (Hot spots)
- Color fluctuations (MV model, JIMWLK)
  - Q_s sets the scale for color fluctuations
  - Power-law tail for $\Delta_\perp > Q_s$
- ...

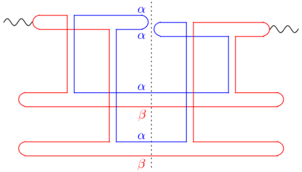
Incoherent correlator at 4-gluon exchange

Assume Gaussian CGC WF

Only pieces connecting DA with CCA survive

Illustration for 4-gluon exchange:





Projectile:

- Either quark or gluon dipole
- Elastic

Nuclear target:

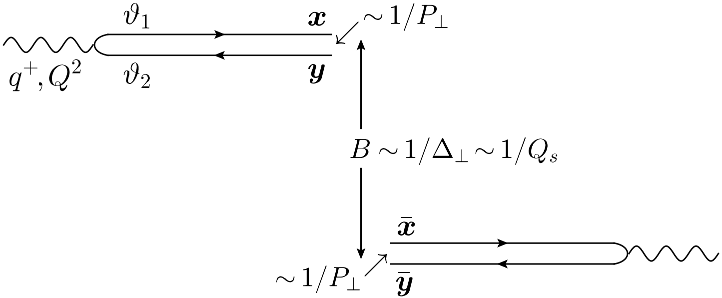
- Colorless substructures scatter inelastically
- Exchange of color among them

Incoherent scattering is $1/N_c^2$ suppressed

$q\bar{q}$ contribution: 2 hard jets

Only 2 hard jets: momentum transfer is equal to pair imbalance $\boldsymbol{\Delta} = \boldsymbol{K}$

$$P_\perp \gg \Delta_\perp, Q_s \longleftrightarrow r, \bar{r} \ll B, 1/Q_s$$



Expand the QCD correlator for $r, \bar{r} \ll B, 1/Q_s$, other than that B is arbitrary

2 hard jets averaged cross sections

There is a $\boldsymbol{P} \cdot \boldsymbol{\Delta}$ angular dependence.
We focus on averaged (over angle) cross section

$$\frac{\mathrm{d}\sigma^{\gamma_\lambda^* A \to q\bar{q}X}}{\mathrm{d}\vartheta_1 \mathrm{d}\vartheta_2 \mathrm{d}^2\boldsymbol{P} \mathrm{d}^2\boldsymbol{\Delta}} \simeq \frac{S_\perp \alpha_{\mathrm{em}} N_c}{4\pi^3} \sum e_f^2 \, \delta_\vartheta \frac{2C_F}{N_c^3} H_\lambda(\vartheta_1, \vartheta_2, P_\perp, \bar{Q}) \, \mathcal{G}^{(+)}(\Delta_\perp),$$

Hard factors H_λ are the same as in the coherent case. $\bar{Q}^2 \equiv \vartheta_1 \vartheta_2 Q^2$

$$\mathcal{G}^{(+)}(\Delta_\perp) = \int_0^\infty \mathrm{d}B B J_0(\Delta_\perp B) \Phi(B) [F_+(B)]^2,$$

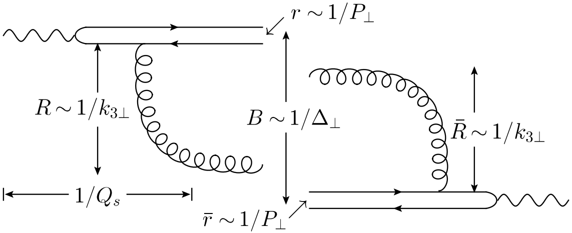
where $\Phi(B) = \frac{\mathcal{S}_g(B) - 1 - \ln \mathcal{S}_g(B)}{\ln^2 \mathcal{S}_g(B)}$ and $F_+(B) = \nabla^2 \ln \mathcal{S}_g(B)$,

$$H_\lambda \sim \frac{1}{P_\perp^6} \quad \text{for} \quad P_\perp \sim Q$$

$$\mathcal{G}^{(+)}(\Delta_\perp) \simeq \begin{cases} Q_s^2 & \text{for} \quad \Delta_\perp \ll Q_s, \\ \dfrac{Q_s^4}{\Delta_\perp^2} & \text{for} \quad \Delta_\perp \gg Q_s, \end{cases}$$

$q\bar{q}g$ contribution: 2 hard jets + 1 semi-hard jet

Diagrammatic soft gluon:



- Momentum transfer determined by pair imbalance and semi-hard jet momentum $\boldsymbol{\Delta} = \boldsymbol{K} + \boldsymbol{k}$

Again $P_\perp \gg \Delta_\perp, Q_s$

- Size of scattering dipole is large $R \sim 1/k_\perp \sim 1/Q_s$
- No expansion in the dipole-dipole QCD correlator

Tensor structure

Cross section tensor structure: $H^{ij}_{(g)}(\boldsymbol{P})G^{ij}(\Delta) + H^{ij}_{(q)}(\boldsymbol{P})Q^{ij}(\Delta)$

- Soft gluon

$$H^{ij}_{(g)}(\boldsymbol{P}) = H_{(g)}(P_\perp)\delta^{ij} + H_{1(g)}(P_\perp)\hat{P}^{ij}$$
$$G^{ij}(\boldsymbol{\Delta}) = G(\Delta_\perp)\delta^{ij} + 0 \times \hat{\Delta}^{ij}$$

- Soft quark

$$H^{ij}_{(q)}(\boldsymbol{P}) = H_{(q)}(P_\perp)\delta^{ij} + 0 \times \hat{P}^{ij}$$
$$Q^{ij}(\boldsymbol{\Delta}) = Q(\Delta_\perp)\delta^{ij} + Q_1(\Delta_\perp)\hat{\Delta}^{ij}$$

No $\boldsymbol{P} \cdot \boldsymbol{\Delta}$ angular dependence in total incoherent diffractive 2+1 jets cross section

[NB: $\hat{V}^{ij} = V^i V^j / V_\perp^2 - \delta^{ij}/2$]

2+1 jets cross section: soft gluon

$$\frac{\mathrm{d}\sigma^{\gamma_\lambda^* A\to q\bar{q}(g)X}}{\mathrm{d}\vartheta_1\,\mathrm{d}\vartheta_2\,\mathrm{d}^2\boldsymbol{P}\,\mathrm{d}^2\boldsymbol{\Delta}\,\mathrm{d}^2\boldsymbol{k}_3\,\mathrm{d}\ln 1/x} = \alpha_{\mathrm{em}}\alpha_s \sum e_f^2\,\delta_\vartheta\, H_\lambda^{(g)}(\vartheta_1,\vartheta_2,P_\perp,\bar{Q})\,\frac{S_\perp(N_c^2-1)}{4\pi^3}\,\frac{\mathcal{G}_{\mathrm{inc}}(x,x_{\mathbb{P}},\boldsymbol{k}_3,\boldsymbol{\Delta})}{2\pi(1-x)}$$

The hard factors are the same as in the coherent diffraction and the inclusive dijet production.

$$\mathcal{G}_{\mathrm{inc}}(x,x_{\mathbb{P}},\boldsymbol{k}_3,\boldsymbol{\Delta}) = \frac{1}{\pi}\int \frac{\mathrm{d}^2\boldsymbol{B}}{2\pi}\,\frac{\mathrm{d}^2\boldsymbol{R}}{2\pi}\,\frac{\mathrm{d}^2\bar{\boldsymbol{R}}}{2\pi}\,e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{B}-i\boldsymbol{k}_3\cdot(\boldsymbol{R}-\bar{\boldsymbol{R}})}$$
$$\times\,\frac{\cos 2\phi_{R\bar{R}}}{2}\mathcal{M}^2K_2(\mathcal{M}R)\mathcal{M}^2K_2(\mathcal{M}\bar{R})\mathcal{W}_g(\boldsymbol{R},\bar{\boldsymbol{R}},\boldsymbol{B})$$

$$\mathcal{M}^2 \equiv \frac{x}{1-x}\,k_{3\perp}^2.$$

$\mathcal{W}_g(\boldsymbol{R},\bar{\boldsymbol{R}},\boldsymbol{B})$ is the connected piece of the total diffractive CGC correlator in the adjoint representation.

2+1 jets cross section: soft quark

$$\frac{\mathrm{d}\sigma^{\gamma_\lambda^* A\to (q)\bar{q}gX}}{\mathrm{d}\vartheta_2\,\mathrm{d}\vartheta_3\,\mathrm{d}^2\boldsymbol{P}\,\mathrm{d}^2\boldsymbol{\Delta}\,\mathrm{d}^2\boldsymbol{k}_1\,\mathrm{d}\ln 1/x} = 2\alpha_{\mathrm{em}}\alpha_s C_F \sum e_f^2\, \delta_\vartheta H_\lambda^{(q)}(\vartheta_2,\vartheta_3,P_\perp,\tilde{Q})\, \frac{S_\perp N_c}{4\pi^3}\, \frac{\mathcal{Q}_{\mathrm{inc}}(x,x_{\mathbb{P}},\boldsymbol{k}_1,\boldsymbol{\Delta})}{2\pi(1-x)}$$

The hard factors are the same as in the coherent diffraction.

$$\mathcal{Q}_{\mathrm{inc}}(x,x_{\mathbb{P}},\boldsymbol{k}_1,\boldsymbol{\Delta}) = \frac{1}{2\pi}\int \frac{\mathrm{d}^2\boldsymbol{B}}{2\pi}\,\frac{\mathrm{d}^2\boldsymbol{R}}{2\pi}\,\frac{\mathrm{d}^2\bar{\boldsymbol{R}}}{2\pi}\, e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{B}-i\boldsymbol{k}_1\cdot(\boldsymbol{R}-\bar{\boldsymbol{R}})} \times \frac{\boldsymbol{R}\cdot\bar{\boldsymbol{R}}}{R\bar{R}}\, \mathcal{M}^2 K_1(\mathcal{M}R)\mathcal{M}^2 K_1(\mathcal{M}\bar{R})\mathcal{W}_q(\boldsymbol{R},\bar{\boldsymbol{R}},\boldsymbol{B})$$

$$\mathcal{M}^2 \equiv \frac{x}{1-x}\,k_{1\perp}^2.$$

$\mathcal{W}_q(\boldsymbol{R},\bar{\boldsymbol{R}},\boldsymbol{B})$ is the connected piece of the total diffractive CGC correlator in the fundamental representation.

Incoherent diffractive TMDs

Gluon sector:

$$\frac{\mathrm{d}xG_{\mathbb{P}}^{\mathrm{inc}}(x,x_{\mathbb{P}},\boldsymbol{k}_3,\boldsymbol{\Delta})}{\mathrm{d}^2\boldsymbol{k}_3\mathrm{d}^2\boldsymbol{\Delta}} \equiv \frac{S_\perp(N_c^2-1)}{4\pi^3}\frac{\mathcal{G}_{\mathrm{inc}}(x,x_{\mathbb{P}},\boldsymbol{k}_3,\boldsymbol{\Delta})}{2\pi(1-x)}$$

Quark sector:

$$\frac{\mathrm{d}xq_{\mathbb{P}}^{\mathrm{inc}}(x,x_{\mathbb{P}},\boldsymbol{k}_1,\boldsymbol{\Delta})}{\mathrm{d}^2\boldsymbol{k}_1\mathrm{d}^2\boldsymbol{\Delta}} \equiv \frac{S_\perp N_c}{4\pi^3}\frac{\mathcal{Q}_{\mathrm{inc}}(x,x_{\mathbb{P}},\boldsymbol{k}_1,\boldsymbol{\Delta})}{2\pi(1-x)}$$

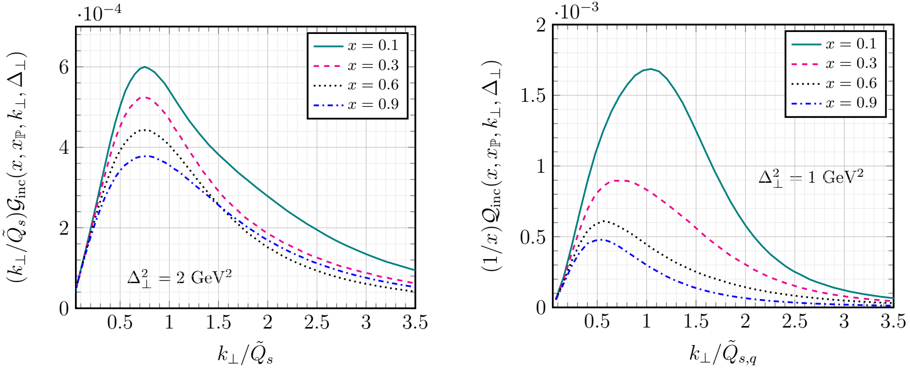
Assuming fixed $\Delta_\perp$ (and neglecting logarithmic dependences)

$$\mathcal{G}_{\mathrm{inc}} \sim \frac{1}{\pi N_c^2 Q_s^2}\times\begin{cases}1 & \text{for} \quad k_\perp \ll \tilde{Q}_s(x,x_{\mathbb{P}}),\\ \dfrac{\tilde{Q}_s^4(x,x_{\mathbb{P}})}{k_\perp^4} & \text{for} \quad k_\perp \gg \tilde{Q}_s(x,x_{\mathbb{P}}),\end{cases}$$

- Similar limits for $(1/x)\mathcal{Q}_{\mathrm{inc}}$ but it vanishes at small $k_\perp$
- Effective saturation momentum $\tilde{Q}_s^2 \equiv (1-x)Q_s^2$

$k_\perp\mathcal{G}_{\mathrm{inc}}$ and $k_\perp\mathcal{Q}_{\mathrm{inc}}$ have a maximum at $k_\perp \sim \tilde{Q}_s$

Gluon and Quark distributions

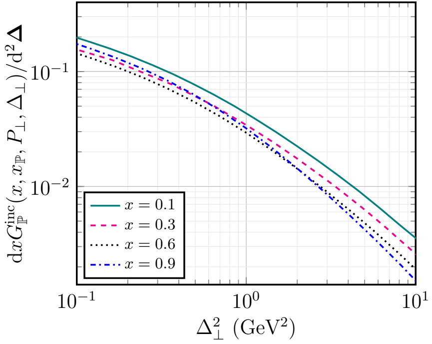


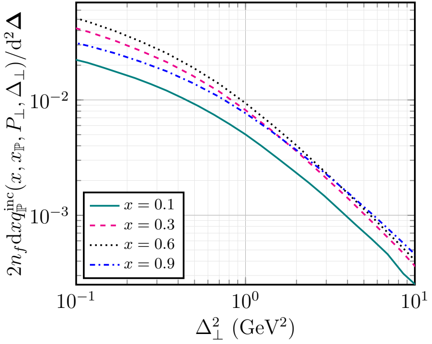
Fixed $\Delta_\perp$ and Q_s. Scattering amplitude is given by the MV model

- Pronunced maximun at $k_\perp \sim \tilde{Q}_s$

Incoherent DPDFs

$$\frac{\mathrm{d}xG_{\mathbb{P}}^{\mathrm{inc}}(x,x_{\mathbb{P}},P_\perp,\Delta_\perp)}{\mathrm{d}^2\boldsymbol{\Delta}} \equiv \int \mathrm{d}^2\boldsymbol{k}\, \frac{\mathrm{d}xG_{\mathbb{P}}^{\mathrm{inc}}(x,x_{\mathbb{P}},\boldsymbol{k},\boldsymbol{\Delta})}{\mathrm{d}^2\boldsymbol{k}\,\mathrm{d}^2\boldsymbol{\Delta}}$$ (and similar for quark DPDF)





$$\frac{\mathrm{d}xG_{\mathbb{P}}^{\mathrm{inc}}(x,x_{\mathbb{P}},P_\perp,\Delta_\perp)}{\mathrm{d}^2\boldsymbol{\Delta}}, \frac{N_c}{x}\frac{\mathrm{d}xq_{\mathbb{P}}^{\mathrm{inc}}(x,x_{\mathbb{P}},P_\perp,\Delta_\perp)}{\mathrm{d}^2\boldsymbol{\Delta}} \sim \frac{S_\perp}{8\pi^4} \times \begin{cases} \ln\dfrac{Q_s^2(x_{\mathbb{P}})}{\Lambda^2} & \text{for} \quad \Delta_\perp \ll Q_s(x_{\mathbb{P}}), \\ \dfrac{Q_s^4(x_{\mathbb{P}})}{\Delta_\perp^4} & \text{for} \quad \Delta_\perp \gg Q_s(x_{\mathbb{P}}). \end{cases}$$

2+1 jets cross sections

- Parametrically larger than 2 jets: $\alpha_s(P_\perp)/P_\perp^4$ vs. $1/P_\perp^6$
  Total 2+1 incoherent diffractive cross section:

$$\frac{\mathrm{d}\sigma^{\gamma_\lambda^* A\to q\bar{q}gX}}{\mathrm{d}\Pi} = \frac{\mathrm{d}\sigma^{\gamma_\lambda^* A\to q\bar{q}(g)X}}{\mathrm{d}\Pi} + 2\,\frac{\mathrm{d}\sigma^{\gamma_\lambda^* A\to (q)\bar{q}gX}}{\mathrm{d}\Pi}$$

  Well behaved:

$$\frac{\mathrm{d}\sigma^{\gamma_\lambda^* A\to q\bar{q}gX}}{\mathrm{d}\Pi} \sim \frac{S_\perp \alpha_{\mathrm{em}}}{4\pi^3 N_c} \sum e_f^2\, \delta_\vartheta\, \frac{\alpha_s N_c}{\pi} \frac{1}{P_\perp^4} \times \begin{cases} \ln \dfrac{Q_s^2}{\Lambda^2} & \text{for} \quad \Delta_\perp \ll Q_s, \\ \dfrac{Q_s^4}{\Delta_\perp^4} & \text{for} \quad \Delta_\perp \gg Q_s. \end{cases}$$

$\left[\mathrm{d}\Pi = (2\pi)^2 P_\perp \mathrm{d}P_\perp \Delta_\perp \mathrm{d}\Delta_\perp \mathrm{d}\vartheta_1 \mathrm{d}\vartheta_2\right]$

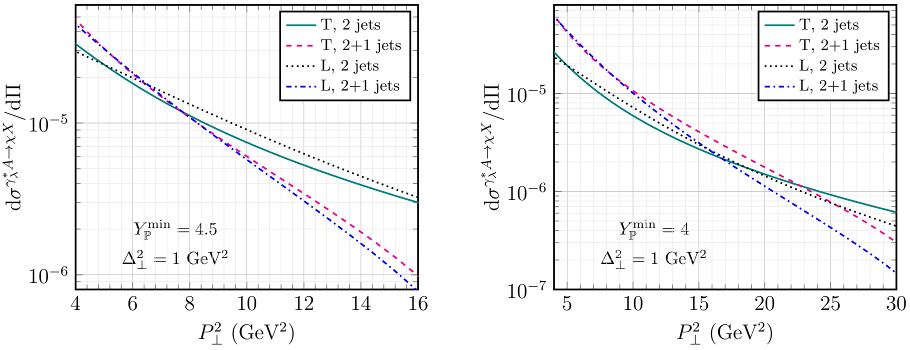
Difractive dijet production with a minimum rapidity gap

- Fix Q^2 and s, so that $Y_{\rm Bj} = \ln 1/x_{\rm Bj} = 6.1$ is fixed (EIC in DIS?)
- Require a minimum rapidity gap $Y_{\mathbb{P}}^{\rm min}$
  We consider two cases:

$$Y_{\mathbb{P}}^{\rm min} \simeq 4.5 \Longleftrightarrow x_{\mathbb{P}}^{\rm max} \simeq 1.1 \times 10^{-2}$$
$$Y_{\mathbb{P}}^{\rm min} \simeq 4 \Longleftrightarrow x_{\mathbb{P}}^{\rm max} \simeq 1.8 \times 10^{-2}$$

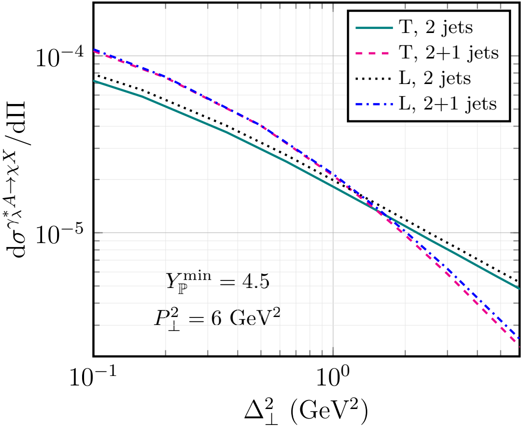
- This implies a minimum value of the splitting fraction $x_{\rm min}(P_\perp)$
- Fix $\vartheta_1 = \vartheta_2 = 1/2$
- Then $P_\perp^2$ cannot exceed a max value

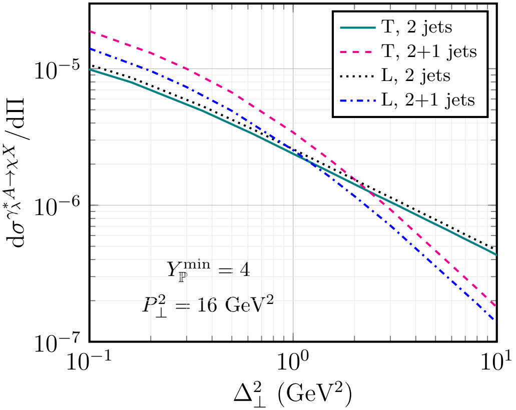
Incoherent diffractive dijet cross sections



Fixed momentum transfer $\Delta_\perp^2$. A factor $\alpha_{em}(S_\perp/4\pi^3)\delta_\vartheta$ has been neglected
Values for transversely and longitudinally polarized virtual photons.

Incoherent diffractive dijet cross sections



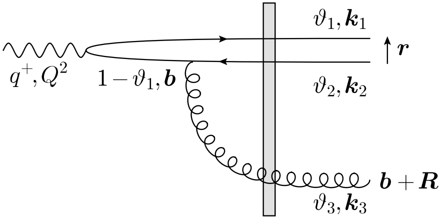


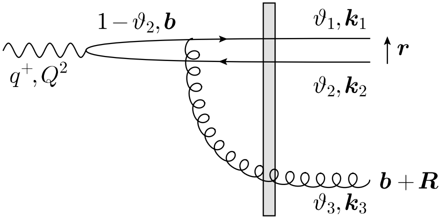
Fixed hard jet momentum $P_\perp^2$.

Conclusions

- Incoherent diffraction at hard momenta in γA collisions in CGC.
- Analytical description of the cross sections for producing hard dijets.
- Incoherent DTMDs and DPDFs from "first principles", for sufficiently large rapidity gaps and/or large nuclei.
- In incoherent diffraction both the 2 jets and 2+1 jets contributions are relevant for the tipical kinematics of the EIC.

Soft gluon diagrams and hard factors

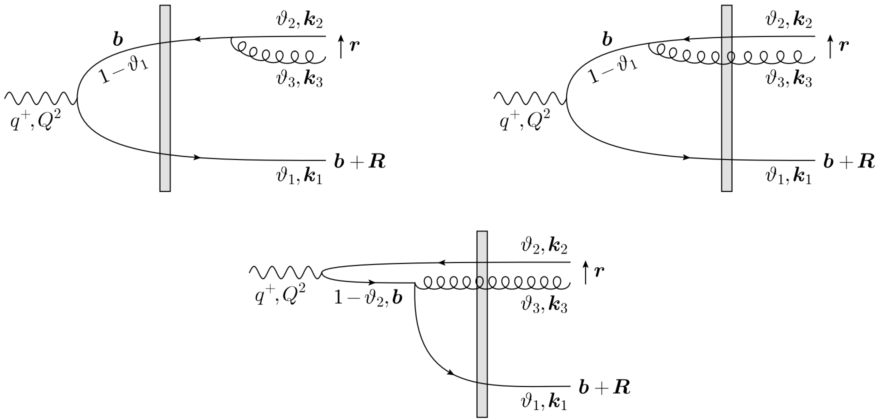




$$H_T^{ij(g)}(\vartheta_1, \vartheta_2, \boldsymbol{P}, \bar{Q}) = \left(\vartheta_1^2 + \vartheta_2^2\right)\left[\frac{(P_\perp^4 + \bar{Q}^4)}{(P_\perp^2 + \bar{Q}^2)^4}\,\delta^{ij} - \frac{4\bar{Q}^2 P_\perp^2}{(P_\perp^2 + \bar{Q}^2)^4}\,\hat{P}^{ij}\right]$$

$$H_L^{(g)}(\vartheta_1, \vartheta_2, P_\perp, \bar{Q}) = 8\vartheta_1\vartheta_2\,\frac{\bar{Q}^2 P_\perp^2}{(P_\perp^2 + \bar{Q}^2)^4}.$$

Soft quark diagrams and hard factors



$$H_T^{(q)}(\vartheta_2, \vartheta_3, P_\perp, \tilde{Q}) = \vartheta_2 \frac{(P_\perp^2 + \tilde{Q}^2)^2 + \vartheta_2^2 \tilde{Q}^4 + \vartheta_3^2 P_\perp^4}{P_\perp^2 (P_\perp^2 + \tilde{Q}^2)^3} \qquad H_L^{(q)}(\vartheta_2, \vartheta_3, P_\perp, \tilde{Q}) = 4\vartheta_2^2 \vartheta_3 \frac{\tilde{Q}^2}{(P_\perp^2 + \tilde{Q}^2)^3}$$

QCD correlators in the Gaussian approximation

Adjoint representation

$$\mathcal{W}_g(\boldsymbol{R}, \bar{\boldsymbol{R}}, \boldsymbol{B}) = \frac{2}{N_c^2} f^2 \mathcal{S}_g(R)\, \mathcal{S}_g(\bar{R}) \left[\frac{e^{-F} - 1 + F}{F^2} + \frac{e^{-(F-f)} - 1 + (F - f)}{(F-f)^2} \right]$$

Fundamental representation

$$\mathcal{W}_q(\boldsymbol{R}, \bar{\boldsymbol{R}}, \boldsymbol{B}) = \frac{1}{N_c^2} f^2 \mathcal{S}(R) \mathcal{S}(\bar{R}) \frac{e^{-F} - 1 + F}{F^2}$$

$$F = \frac{1}{2} \ln \frac{\mathcal{S}_g(R)\mathcal{S}_g(\bar{R})}{\mathcal{S}_g(|\boldsymbol{B} + \boldsymbol{R} - \bar{\boldsymbol{R}}|)\mathcal{S}_g(B)} \qquad \text{and} \qquad f = \frac{1}{2} \ln \frac{\mathcal{S}_g(|\boldsymbol{B} + \boldsymbol{R}|)\mathcal{S}_g(|\boldsymbol{B} - \bar{\boldsymbol{R}}|)}{\mathcal{S}_g(|\boldsymbol{B} + \boldsymbol{R} - \bar{\boldsymbol{R}}|)\mathcal{S}_g(B)}$$

$\mathcal{S}(R) = [\mathcal{S}_g(R)]^{C_F/N_c} \simeq \sqrt{\mathcal{S}_g(R)}$ (in the large N_c limit)

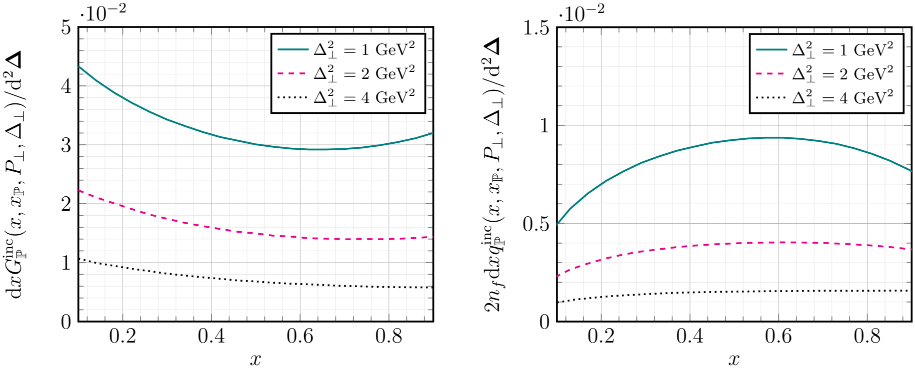
2 jets cross section in the aligned jets configuration

- Change of variable from $\vartheta_1 \to \beta \equiv \frac{\bar{Q}^2}{\bar{Q}^2+P_\perp^2}$
- if (and only if) aligned jet configuration ($\vartheta \ll 1$):

$$\frac{\mathrm{d}\sigma^{\gamma_T^* A\to q\bar{q}X}}{\mathrm{d}^2\boldsymbol{P}\mathrm{d}^2\boldsymbol{\Delta}\,\mathrm{d}\ln 1/\beta} = \frac{4\pi^2\alpha_{\mathrm{em}}}{Q^2}\sum e_f^2\, 2\,\frac{\mathrm{d}xq_{\mathbb{P}}^{\mathrm{inc}}(x, x_{\mathbb{P}}, \boldsymbol{P}, \boldsymbol{\Delta})}{\mathrm{d}^2\boldsymbol{P}\mathrm{d}^2\boldsymbol{\Delta}}\Bigg|_{x=\beta}$$

TMD factorization require the outgoing state to be in the aligned jets configuration

DPDFs as a function of x



DPDFs at fixed momentum transfer $\Delta_\perp^2$. Quark DPDFs multiplied by $2n_f$, where $n_f = 3$ is the number of flavors.

The hard scale $P_\perp^2$ has been taken equal to infinity.

MV model with a gluon saturation scale $Q_s^2 = 2\,\mathrm{GeV}^2$