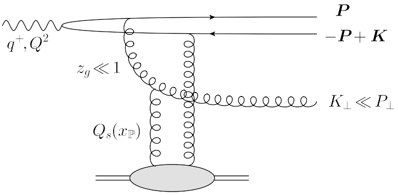
TMD factorisation for diffractive jets in photon-nucleus interactions at small x

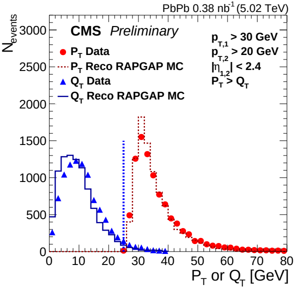
Edmond Iancu

IPhT, Université Paris-Saclay

with A.H. Mueller, S. Hauksson, D.N. Triantafyllopoulos, S.-Y. Wei

2112.06353 (PRL) & 2207.06268 & 2304.12401 & 2402.14748





Outline

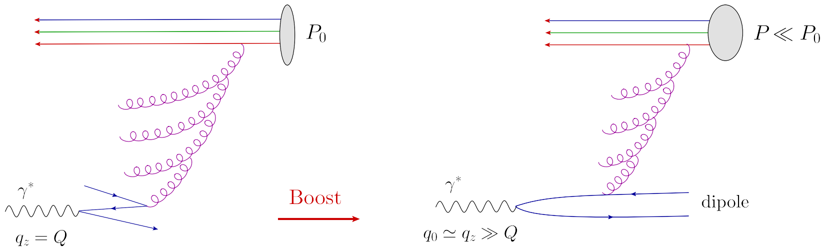
- Diffraction in γA interactions: DIS or AA UPCs
- Controlled by strong scattering $\Longrightarrow$ sensitive to unitarity corrections
  - multiple scattering, gluon saturation
- Large $A \gg 1$ and/or small $x_{\mathbb{P}} \ll 1$: high gluon density, smallish $\alpha_s(Q_s)$

$$Q_s^2(A, x_{\mathbb{P}}) \sim A^{1/3} \left(\frac{1}{x_{\mathbb{P}}}\right)^{\lambda} \qquad \text{with} \qquad \lambda \simeq 0.25$$

- CGC effective theory: pQCD + all order resummations of high-density effects
- For large virtuality and/or transverse momenta $Q^2, P_\perp^2 \gg Q_s^2(A, x_{\mathbb{P}})$
  - collinear (TMD) factorisation emerges from CGC calculations
- Explicit expressions for the quark and gluon diffractive TMDs/PDFs
- Transparent physical picture for the Pomeron

Colour dipole picture *(talks by Jamal J.-M., Paul C., Michael F.)*

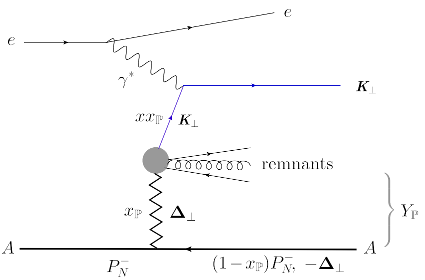
- Factorisation scheme for DIS at small-x: saturation, higher order corrections
- Start in the Breit frame: the target $(p,\, A)$ is a left mover: $P_N^- \gg M_N$
- Large boost in the positive z direction: ultrarelativistic photon $q^+ \gg Q$

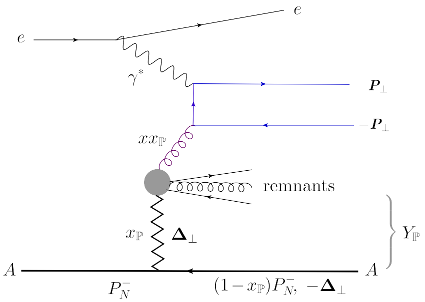


- The $q\bar{q}$ pair can now be seen as a part of the photon wavefunction
- γ^* fluctuates into a $q\bar{q}$ color dipole, which then scatters off the target

Collinear factorisation for diffractive jets

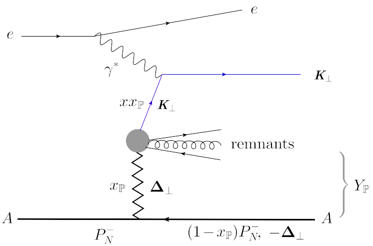
- Elastic scattering ⇒ "Pomeron" exchange ⇒ rapidity gap: $Y_{\mathbb{P}} = \ln \frac{1}{x_{\mathbb{P}}}$
  - $x_{\mathbb{P}} \ll 1$: longitudinal momentum fraction taken by the Pomeron
- Diffractive SIDIS: γ^* absorbed by a quark constituent of the Pomeron
- Diffractive Dijets: photon-gluon fusion with a gluon from the Pomeron
  - $x \leq 1$: splitting fraction w.r.t. the Pomeron

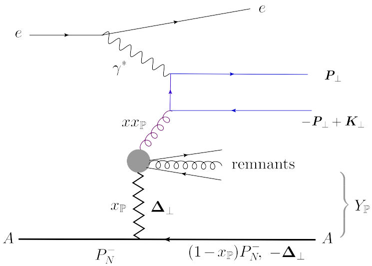




Collinear factorisation for diffractive jets

- Diffractive PDFs: $xq_{\mathbb{P}}(x, x_{\mathbb{P}}, Q^2)$, $xG_{\mathbb{P}}(x, x_{\mathbb{P}}, P_\perp^2)$
- Non-perturbative quantities, but DGLAP evolution with the hardest scale

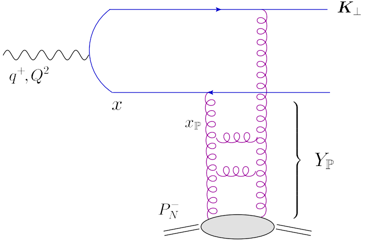


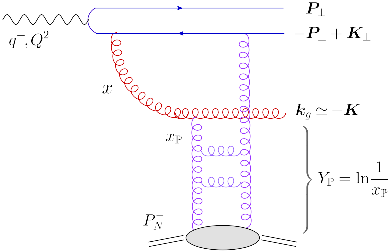


- What is the nature/structure of the Pomeron ?
- Can one predict the dependencies upon $x_{\mathbb{P}}$, or x ?
- Or the $K_\perp$-distribution of the measured jet/hadron in diff-SIDIS ?
- Or the momentum imbalance $K_\perp$ between the two jets in diff-Dijets ?

Colour dipole picture

- The would-be Pomeron constituents now belong to the photon wavefunction
- The Pomeron: Elastic scattering in the t-channel
  - 2 (or more) gluon exchanges in a colour-singlet state

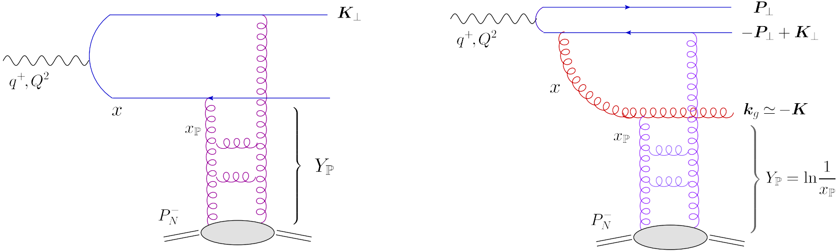




- Large rapidity phase-space $Y_{\mathbb{P}} = \ln \frac{1}{x_{\mathbb{P}}}$ for the high energy evolution
  - JIMWLK evolution of the scattering amplitude: $T(K_\perp, Y_{\mathbb{P}})$
  - saturation dual to strong scattering: $T \sim 1$ when $K_\perp \sim Q_s(A, Y_{\mathbb{P}})$

Colour dipole picture

- The would-be Pomeron constituents now belong to the photon wavefunction
- The Pomeron: Elastic scattering in the t-channel
  - 2 (or more) gluon exchanges in a colour-singlet state

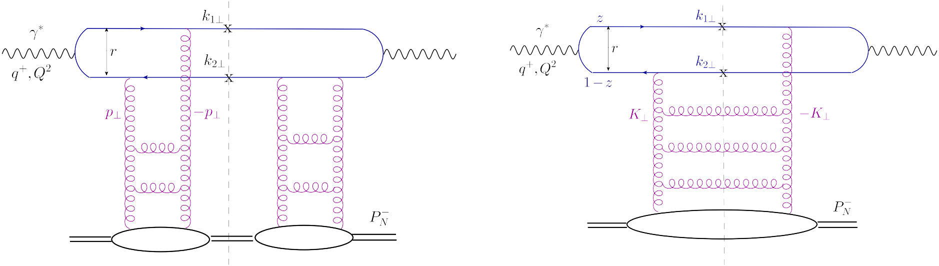


- Emergent TMD factorisation when $Q^2 \gg Q_s^2$ (SIDIS) & $P_\perp^2 \gg Q_s^2$ (dijets)
  - hard factors $\times$ quark or gluon diffractive TMDs
- Operator definitions clarified in *Hatta, Xiao, Yuan, 2205.08060*

Diffraction is strong scattering

- Why should saturation be important when $Q^2,\ P_\perp^2 \gg Q_s^2$?
- Elastic scattering is either strong, or strongly suppressed !

$$\sigma_{el} \propto |T|^2 \longleftrightarrow \sigma_{tot} \propto 2\mathrm{Im}\,T$$

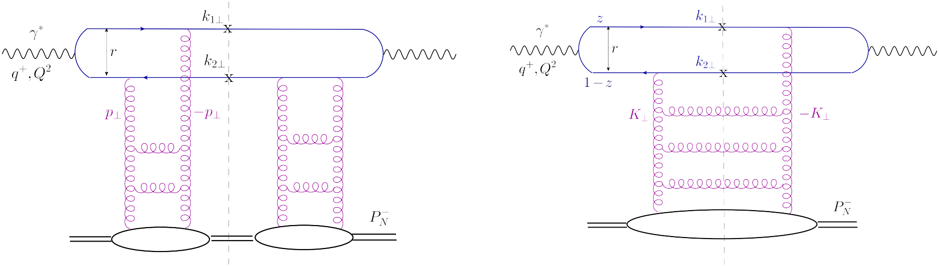


$$T_{q\bar{q}}(r,x) \simeq \begin{cases} r^2 Q_s^2(A,x), & \text{for } rQ_s \ll 1 \ \text{(color transparency)} \\ 1, & \text{for } rQ_s \gtrsim 1 \ \text{(black disk/saturation)} \end{cases}$$

Diffraction is strong scattering

- Why should saturation be important when $Q^2,\ P_\perp^2 \gg Q_s^2$?
- Elastic scattering is either strong, or strongly suppressed !

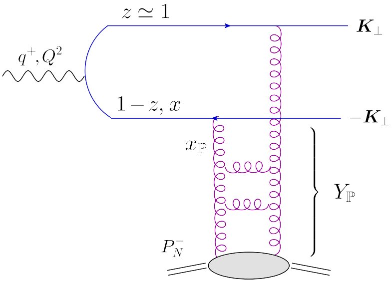
$$\sigma_{el} \propto |T|^2 \longleftrightarrow \sigma_{tot} \propto 2\mathrm{Im}\,T$$



- F_{2D} controlled by large dipoles, $r \sim 1/Q_s$, even when $Q^2 \gg Q_s^2$
  - "aligned jet": $r^2 z(1-z)Q^2 \sim 1 \Rightarrow$ large r when $z(1-z) \ll 1$
- The produced fermions have semi-hard momenta: $k_\perp \sim Q_s$

TMD factorisation for diff-SIDIS

- Leading order in α_s & leading twist in $K_\perp^2/Q^2 \ll 1$



$$\frac{\mathrm{d}\sigma^{\gamma_T^* A \to qAX}}{\mathrm{d}\ln(1/x)\,\mathrm{d}^2\boldsymbol{K}} = \frac{8\pi^2\alpha_{em}e_f^2}{Q^2}\,\mathcal{Q}_{\mathbb{P}}(x, x_{\mathbb{P}}, K_\perp^2)$$

$$x = \frac{Q^2}{Q^2 + M_{q\bar{q}}^2} = \frac{x_{\mathrm{Bj}}}{x_{\mathbb{P}}} \equiv \beta$$

- $\mathcal{Q}_{\mathbb{P}}(x, x_{\mathbb{P}}, K_\perp^2)$: quark diffractive TMD

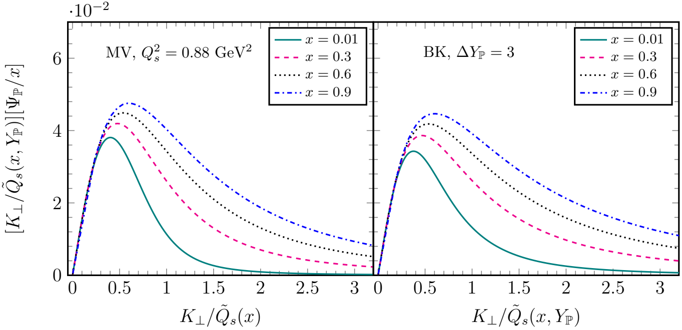
- Unintegrated quark distribution of the Pomeron:

$$\mathcal{Q}_{\mathbb{P}}(x, x_{\mathbb{P}}, K_\perp^2) \simeq \frac{x}{2\pi}\begin{cases} 1 & \text{for} \quad K_\perp \ll \tilde{Q}_s(x, Y_{\mathbb{P}}) \\ \dfrac{\tilde{Q}_s^4(x, Y_{\mathbb{P}})}{K_\perp^4} & \text{for} \quad K_\perp \gg \tilde{Q}_s(x, Y_{\mathbb{P}})\,. \end{cases}$$

- Effective saturation momentum: $\tilde{Q}_s^2(x, Y_{\mathbb{P}}) \equiv (1-x)Q_s^2(Y_{\mathbb{P}})$

Numerical results *(2402.14748)*

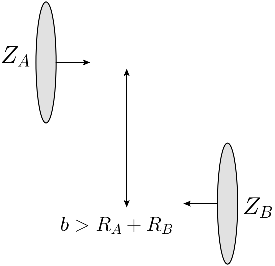
- The bulk of the distribution lies at saturation: $K_\perp \lesssim \tilde{Q}_s(x, Y_{\mathbb{P}})$
- Left: McLerran-Venugopalan model. Right: BK evolution of T_{gg}
- $\mathcal{Q}_{\mathbb{P}}(x, x_{\mathbb{P}}, K_\perp^2)$ multiplied by $K_\perp$: Pronounced maximum at $K_\perp \simeq \tilde{Q}_s$

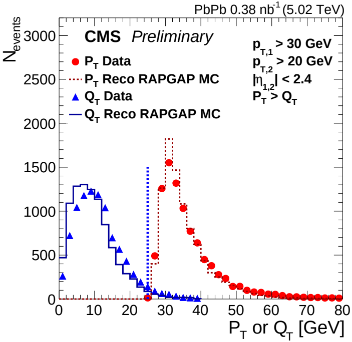


- BK evolution: increasing $Q_s(Y_{\mathbb{P}})$, approximate geometric scaling
  - a function of the ratio $K_\perp/\tilde{Q}_s(x, Y_{\mathbb{P}})$ alone

Diffractive jets in UPCs at the LHC

- A jet with $K_\perp \sim 1 \div 2$ GeV: interesting for the EIC ... but not for the LHC
- Pb+Pb UPCs: *ATLAS-CONF-2022-021* and *CMS arXiv:2205.00045*

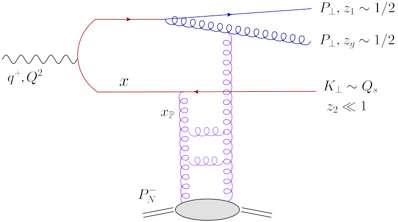


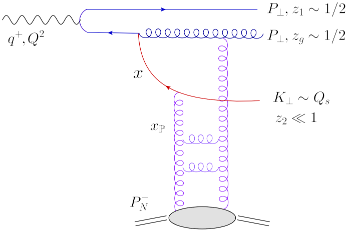


- Diffractive di-jets with large $P_\perp \geq 30$ GeV and imbalance $K_\perp \sim 10$ GeV
- How to produce a hard jets ($P_\perp \gg Q_s(A, x_{\mathbb{P}})$) via elastic scattering ?

TMD factorisation for 2+1 jets

- "Hard jets": cross-section with leading-twist power-tail at large $P_\perp \gg Q_s$
  - $1/P_\perp^2$ for single jets, $1/P_\perp^4$ for dijets ...
- Naturally produced via a hard splitting: e.g. $q \to q + g$

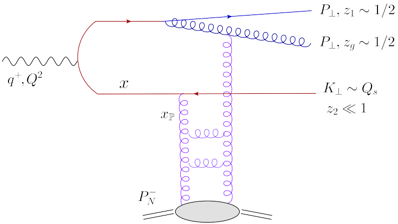


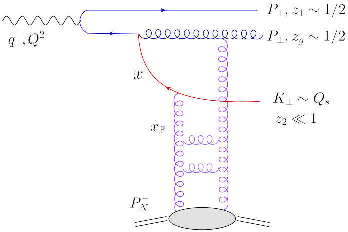


- A system of 3 jets ($q\bar{q}g$) which must suffer strong elastic scattering
  - hard quark-gluon pair in the final state: $P_\perp \gg Q_s$
  - semi-hard antiquark $K_\perp \sim Q_s$ to ensure strong scattering
- TMD factorisation: antiquark "transferred" to the target *(2402.14748)*

TMD factorisation for 2+1 jets

- "Hard jets": cross-section with leading-twist power-tail at large $P_\perp \gg Q_s$
  - $1/P_\perp^2$ for single jets, $1/P_\perp^4$ for dijets ...
- Naturally produced via a hard splitting: e.g. $q \to q + g$



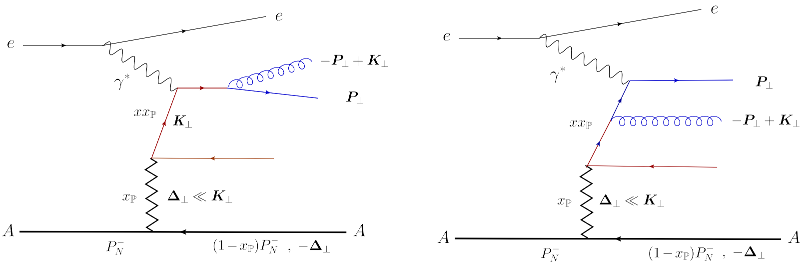


$$\frac{d\sigma^{\gamma_T^* A \to qg(\bar{q})A}}{dz_1 dz_g d^2\boldsymbol{P} d^2\boldsymbol{K}\, d\ln(1/x)} = H_T(z_1, z_g, P_\perp^2, Q^2)\, \mathcal{Q}_{\mathbb{P}}(x, x_{\mathbb{P}}, K_\perp^2)$$

- Hard factor H_T decaying like $1/P_\perp^4$
- Same quark diff-TMD as in SIDIS: controls the dijet imbalance: $K_\perp \sim Q_s$

Target picture for diffractive dijets

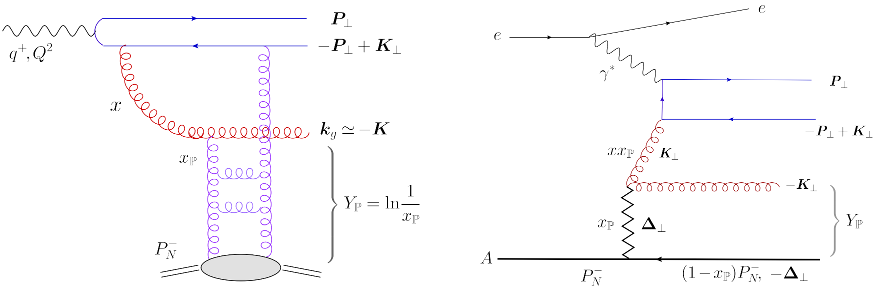
- The corresponding processes in collinear factorisation
  - dijet imbalance $K_\perp$ = intrinsic $K_\perp$ of a quark from the Pomeron
  - however, TMD factorisation was not addressed in that context



- We have computed these processes in the colour dipole picture
  - "top-down" proof for TMD factorisation
  - explicit expression for the quark diffractive TMD
  - universality of the quark diff-TMD (SIDIS & dijets)

Diffractive $q\bar{q}$ jets *(2112.06353 & 2207.06268)*

- A hard $q\bar{q}$ pair $P_\perp \gg Q_s$ plus a semi-hard gluon: $K_\perp \sim Q_s \ll P_\perp$
  - the exclusive production of hard $q\bar{q}$ jets is higher twist
  - effective gluon-gluon dipole with transverse size $R \sim 1/Q_s$



- TMD factorisation involving the gluon diff-TMD $\mathcal{G}_{\mathbb{P}}(x, x_{\mathbb{P}}, K_\perp)$
  - the unintegrated gluon distribution of the Pomeron

The gluon diffractive TMD

- Same behaviour in $K_\perp$ as the quark diff-TMD, but different behaviour in x:

$$\mathcal{G}_{\mathbb{P}}(x, x_{\mathbb{P}}, K_\perp^2) \simeq (1-x) \begin{cases} 1, & K_\perp \lesssim \tilde{Q}_s(x) \\ \dfrac{\tilde{Q}_s^4(x)}{K_\perp^4}, & K_\perp \gg \tilde{Q}_s(x) \end{cases}$$

- Effective saturation momentum: $\tilde{Q}_s^2(x, Y_{\mathbb{P}}) = (1-x) Q_s^2(Y_{\mathbb{P}})$
- Diffractive PDFs: integrate over $K_\perp$ up to the resolution scale $Q^2 \gg Q_s^2$
  - rapidly converging and effectively cut off at $K_\perp \sim \tilde{Q}_s(x)$

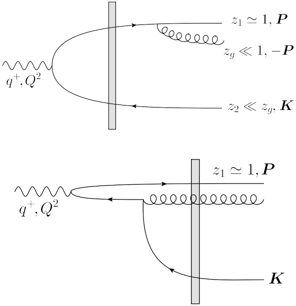
$$xG_{\mathbb{P}}(x, x_{\mathbb{P}}, Q^2) \equiv \int^{Q^2} \mathrm{d}K_\perp^2 \, \mathcal{G}_{\mathbb{P}}(x, x_{\mathbb{P}}, K_\perp^2) \, \propto \, (1-x)^2 \, Q_s^2(Y_{\mathbb{P}})$$

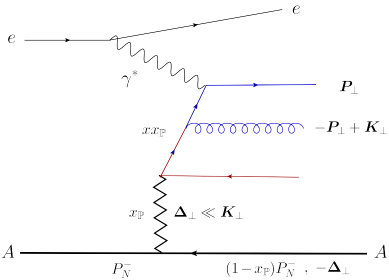
$$xq_{\mathbb{P}}(x, x_{\mathbb{P}}, Q^2) \, \propto \, x(1-x) \, Q_s^2(Y_{\mathbb{P}})$$

- Initial conditions for DGLAP evolution with Q^2: controlled by saturation

Diffractive SIDIS at large $P_\perp$

- Start with $2+1$ jets and measure only the hard quark $(P_\perp \gg Q_s)$
  - integrate out $\bar{q}$, g: NLO corrections to SIDIS

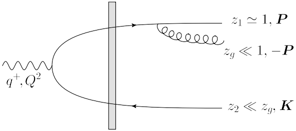


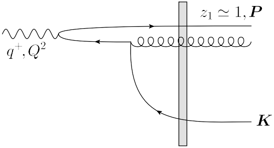


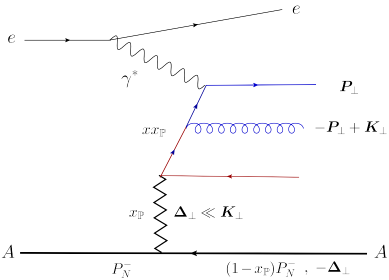
$$\Delta Q_{\mathbb{P}}(x, x_{\mathbb{P}}, P_\perp) = \frac{\alpha_s C_F}{2\pi^2} \frac{1}{P_\perp^2} \int_x^{\xi_M} \mathrm{d}\xi \, \frac{1+\xi^2}{1-\xi} \, \frac{x}{\xi} q_{\mathbb{P}}^{(0)} \left(\frac{x}{\xi}, x_{\mathbb{P}}, P_\perp^2 \right)$$

- TMD factorisation preserved with new contributions to the quark diff-TMD
- $\mathcal{O}(\alpha_s)$ but leading-twist: a dominant contribution at large $P_\perp$

- Start with $2+1$ jets and measure only the hard quark $(P_\perp \gg Q_s)$
  - integrate out $\bar{q}$, g: NLO corrections to SIDIS





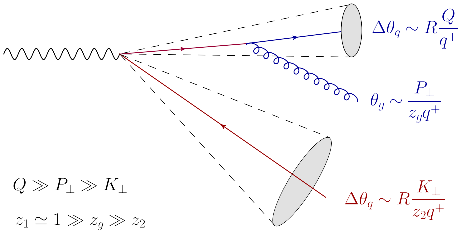


$$\Delta Q_{\mathbb{P}}(x, x_{\mathbb{P}}, P_\perp) = \frac{\alpha_s C_F}{2\pi^2}\frac{1}{P_\perp^2}\int_x^{\xi_M} \mathrm{d}\xi\, \frac{1+\xi^2}{1-\xi}\, \frac{x}{\xi} q_{\mathbb{P}}^{(0)}\left(\frac{x}{\xi}, x_{\mathbb{P}}, P_\perp^2\right)$$

- Integrate over $P_\perp \le Q$: one step in the DGLAP evolution of diff-PDF
- Log dependence upon rapidity cutoff ξ_M: CSS evolution for diff-TMD

CSS evolution for diffractive TMDs

- The upper limit ξ_M depends upon the jet definition *(cf. talk by Paul Caucal)*
- The gluon must be emitted outside the measured quark jet : $\theta_g > \theta_q$



- The quark has virtuality $Q^2 \gg P_\perp^2$ $\Rightarrow$ larger angle than naively expected

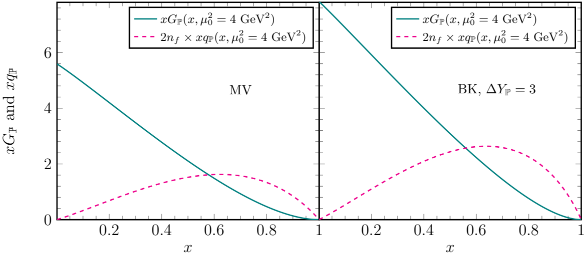
$$\theta_g \sim \frac{P_\perp}{z_g q^+} > \theta_q \sim \frac{Q}{q^+} \quad \Longrightarrow \quad z_g < \frac{P_\perp}{Q} \quad \Longrightarrow \quad 1 - \xi_M \simeq \frac{P_\perp}{Q}$$

- "Diagonal" version of the CSS equation: the same as for inclusive TMDs

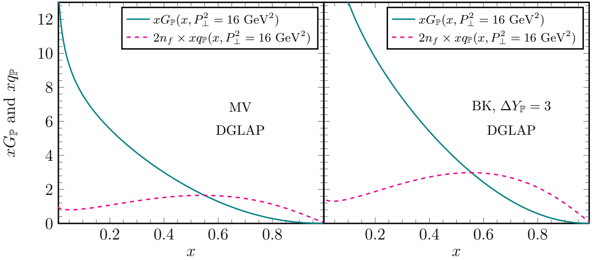
Caucal, E.I., 2406.04238; Caucal, E.I., Mueller, Yuan, 2408.03129

Diffractive PDFs from the CGC

- Initial conditions for DGLAP (MV, or MV+BK): gluon & quark



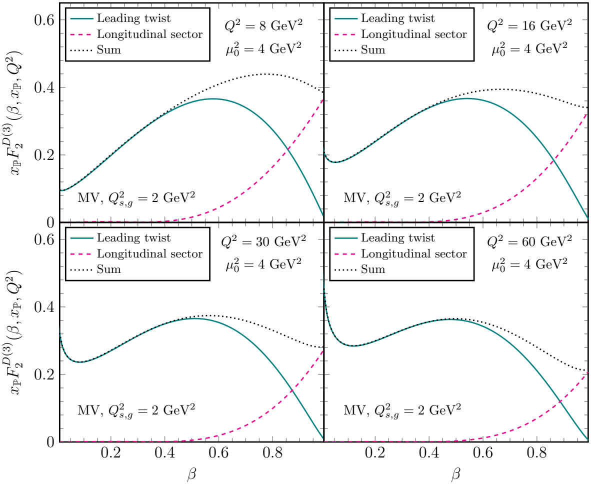
- DGLAP evolution from $\mu_0^2 = 4$ GeV^2 up to $P_\perp^2 = 16$ GeV^2



Diffractive structure function: nuclear target

$$F_2^{D(3)}(\beta, x_{\mathbb{P}}, Q^2) = 2\sum_f e_f^2\, x q_{\mathbb{P}}(x, x_{\mathbb{P}}, Q^2)\big|_{x=\beta} + F_L^{D(3)}$$

- Factorisation scale $\mu_0^2 \sim 2Q_s^2 = 4$ GeV^2 (turn on DGLAP)



Conclusions

- Diffraction in γA (EIC, UPC): the best laboratory to study gluon saturation
- For small $x_{\mathbb{P}} \lesssim 10^{-2}$ and/or large $A \sim 200$: CGC effective theory
- Emergence of TMD/collinear factorisation when $Q^2,\, P_\perp^2 \gg Q_s^2(A, x_{\mathbb{P}})$
- Top-down approach:
  - start with the CGC result for the cross-section (LO, NLO...)
  - demonstrate TMD factorisation at leading twist
- Explicit expressions for the diffractive TMDs, controlled by saturation
- "Diagonal" version of the CSS evolution
  - UV and rapidity renormalisation scales are identified with each other and with the largest transverse/virtuality scale in the problem
- Phenomenology (EIC, UPCs): finally, the smoking gun for gluon saturation ?

Back-up: CSS evolution in diagonal form

- The gluon TMD $\mathcal{F}_g(x, K_\perp^2, P_\perp^2)$ as an example: inclusive or diffractive
- Initial condition at $P_\perp^2 = K_\perp^2$ from the CGC: $\mathcal{F}_g^{(0)}(x, K_\perp^2)$
  - including BK/JIMWLK evolution down to x (or $x_{\mathbb{P}}$)
- CSS evolution from $K_\perp^2$ up to $P_\perp^2$:

$$\frac{\partial \mathcal{F}_g(x, K_\perp^2, P_\perp^2)}{\partial \ln P_\perp^2} = \frac{\alpha_s N_c}{2\pi}\left\{\frac{1}{K_\perp^2}\int_{\Lambda^2}^{K_\perp^2} \mathrm{d}\ell_\perp^2\, \mathcal{F}_g(x, \ell_\perp^2, P_\perp^2) - \int_{K_\perp^2}^{P_\perp^2} \frac{\mathrm{d}\ell_\perp^2}{\ell_\perp^2} \mathcal{F}_g(x, K_\perp^2, P_\perp^2)\right\}$$

$$+\beta_0 \frac{\alpha_s N_c}{\pi} \mathcal{F}_g(x, K_\perp^2, P_\perp^2)$$

- Diagonal version $\mu_F^2 = \zeta = P_\perp^2$ of the two RG+CS equations *(Collins-11)*:
- The "natural" relation between TMD and PDF is consistent with DGLAP

$$xG(x, P_\perp^2) = \pi \int_{\Lambda^2}^{P_\perp^2} \mathrm{d}K_\perp^2\, \mathcal{F}_g(x, K_\perp^2, P_\perp^2)$$

(Ebert, Stewart et al, 2201.07237; del Rio, Prokudin et al, 2402.01836)

Back-up: CSS equation in coordinate space

- The CSS equation is usually written/solved in transverse coordinate space

$$\tilde{\mathcal{F}}_g(x, b_\perp^2, Q^2) \equiv \int \frac{\mathrm{d}^2\boldsymbol{K}}{(2\pi)^2}\, \mathrm{e}^{-i\boldsymbol{K}\cdot\boldsymbol{b}}\, \mathcal{F}_g(x, K_\perp^2, Q^2)$$

- After the Fourier transform, the real piece "disappears" ...
  - it provides the lower limit $1/b_\perp^2$ on the transverse integration
  - coarse-graining of the $K_\perp$–distribution

$$\frac{\partial \tilde{\mathcal{F}}_g(x, b_\perp^2, Q^2)}{\partial \ln Q^2} = \frac{N_c}{\pi}\left\{-\frac{1}{2}\int_{1/b_\perp^2}^{Q^2} \frac{\mathrm{d}\ell_\perp^2}{\ell_\perp^2}\, \alpha_s(\ell_\perp^2) + \beta_0 \alpha_s(Q^2)\right\} \tilde{\mathcal{F}}_g$$

- Local in both Q^2 and $b_\perp^2$ $\Longrightarrow$ trivial to solve

$$\tilde{\mathcal{F}}_g(x, b_\perp^2, Q^2) = \tilde{\mathcal{F}}_0(x, b_\perp^2)\left\{-\frac{N_c}{\pi}\int_{1/b_\perp^2}^{Q^2} \frac{\mathrm{d}\ell_\perp^2}{\ell_\perp^2}\, \alpha_s(\ell_\perp^2)\left[\frac{1}{2}\ln\frac{Q^2}{\ell_\perp^2} - \beta_0\right]\right\}$$

- ... but the Fourier transform back to momentum space can be tricky !