NLO impact-factor for the forward η_c meson production

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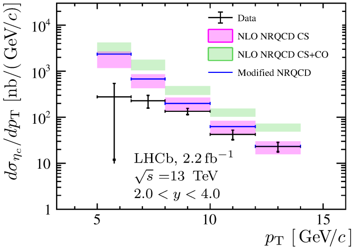
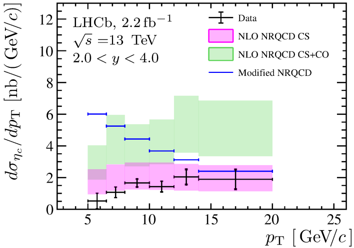
Motivation: inclusive heavy quarkonium production at moderate p_T

It is well known that quarkonium production p_T spectra at $p_T \gtrsim M$ are not described by collinear factorisation computations at NLO. Example: η_c production at LHCb [LHCb, '24] (left panel $d\sigma/dp_T$, right panel: ratio to J/ψ):



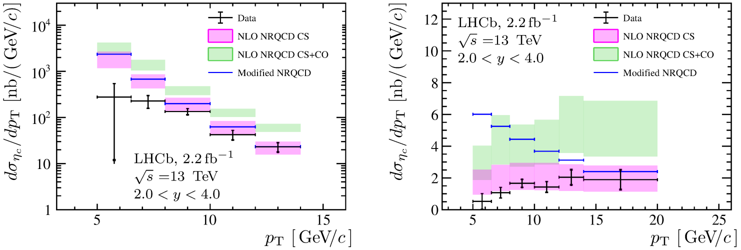


- NLO CF overshoots the data for $5 < p_T < 8$ GeV: nothing to do with TMD/Sudakov logs ($\ln p_T/M$) which contribute only at $p_T \ll M \simeq 3$ GeV
- Physical shape of the p_T-spectrum is reproduced in k_T-factorisation [Kniehl, Vasin, Saleev '06;...;MN,Saleev,Shipilova '12;...] and Saturation/CGC calculations [Kang, Ma, Venugopalan '13; ... Mantysaari *et al.* '24] **at LO in α_s**

The goal of the present work is to compute the NLO imact-factor for the forward production of η_c in the Colour-Singlet Model($c\bar{c}[^1S_0^{[1]}]$) in order to advance the High-Energy factorisation calculations of η_c p_T spectra beyond LLA.

Inclusive heavy quarkonium production at moderate p_T in CF

It is well known that quarkonium production p_T spectra at $p_T \gtrsim M$ are not described by collinear factorisation computations at NLO. Example: η_c production at LHCb [LHCb, '24] (left panel $d\sigma/dp_T$, right panel: ratio to J/ψ):



$p(P_1)+p(P_2) \to \eta_c(p)+X$, parton level e.g.: $g(x_1P_1)+g(x_2P_2) \to c\bar{c}[^1S_0^{[1]}]+X$,

with $S=(P_1+P_2)^2$ and $p^2=M^2$. Cross section in **collinear factorisaton**:

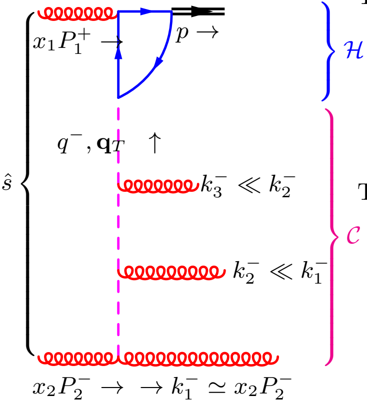
$$\frac{d\sigma}{d\mathbf{p}_T^2 dy} = \frac{M^2}{S}\int\limits_0^{\eta_{\max}} d\eta \int\limits_{z_{\min}}^{z_{\max}} dz\ f_i\left(\frac{M_T e^y}{\sqrt{S}z},\mu_F\right) f_j\left(\frac{M^2 z(1+\eta)}{M_T\sqrt{S}}e^{-y},\mu_F\right)\frac{d\hat{\sigma}_{ij}(\eta,z,\mathbf{p}_T^2)}{dz\,d\mathbf{p}_T^2},$$

where $\eta=\hat{s}/M^2-1$ with $\hat{s}=Sx_1x_2$, $z=\frac{p_+}{x_1P_1^+}$, $M_T=\sqrt{M^2+\mathbf{p}_T^2}$.

High-Energy Factorization, forward η_c hadroproduction

The **LLA** ($\sum\limits_n \alpha_s^n \ln^{n-1}$) formalism [Collins, Ellis, '91; Catani, Ciafaloni, Hautmann, '91,'94]

Physical picture in the **LLA**:



The LLA in $\ln(x_2 P_2^-/p^-)$:

$$\hat{\sigma}_{\text{HEF}}(p_-/(x_2 P_2^-), z) \propto \int\limits_0^\infty dq^- \int\limits_0^\infty d\mathbf{q}_{T1}^2 \mathcal{C}\left(\frac{q^-}{x_2 P_2^-}, \mathbf{q}_{T1}^2, \mu_F, \mu_R\right) \mathcal{H}(q^-, \mathbf{q}_{T1}^2, z),$$

The LLA in $\ln(\hat{s}/M^2)$:

$$\hat{\sigma}_{\text{HEF}}(\hat{s}/M^2, z) \propto \int\limits_0^\infty d\mathbf{q}_{T1}^2 \mathcal{C}\left(\frac{\hat{s}}{M^2}, \mathbf{q}_{T1}^2, \mu_F, \mu_R\right) \int\limits_0^\infty dq^- \mathcal{H}(q^-, \mathbf{q}_{T1}^2, z).$$

Two kinds of LLA are equivalent up to NLL terms because

$$\frac{\hat{s}}{M^2} = \frac{x_1 x_2 S}{M^2} = \frac{M^2 + \mathbf{p}_T^2}{M^2} \frac{x_2 P_2^-}{q^-} = \frac{M^2 + \mathbf{p}_T^2}{M^2} \frac{x_2 P_2^-}{p^-}.$$

The Leading Order

The LLA resummation formula for $\mathbf{p}_T^2$ and $z = p_-/q_-$-differential partonic cross section:

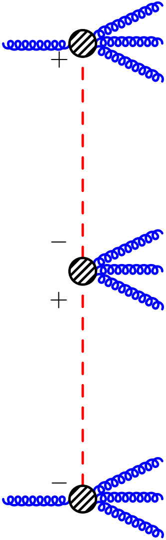
$$\frac{d\hat{\sigma}_{ig}^{(\mathrm{LLA})}}{dz d\mathbf{p}_T^2} = \frac{1}{2M^2} \int \frac{d^2\mathbf{q}_T}{\pi} \mathcal{C}_{ig}\left(\frac{\hat{s}}{M^2}, \mathbf{q}_T^2, \mu_F, \mu_R\right) \mathcal{H}_{gg}^{(\mathrm{LO})}(\mathbf{q}_T^2, z, \mathbf{p}_T^2),$$
$$= \frac{1}{2M^2} \mathcal{C}_{ig}\left(\frac{\hat{s}}{M^2}, \mathbf{p}_T^2, \mu_F, \mu_R\right) \mathcal{H}_{gg}^{(\mathrm{LO})}(\mathbf{p}_T^2) \delta(1-z),$$

with [Kniehl, Vasin, Saleev, '06]

$$\mathcal{H}_{gg}^{(\mathrm{LO})} = \frac{32\pi^3 \alpha_s^2(\mu_R) M^4}{N_c^2 (N_c^2 - 1)(M^2 + \mathbf{p}_T^2)^2} \frac{\left\langle \mathcal{O}\left[{}^1S_0^{[1]}\right]\right\rangle}{M^3} \delta(1-z) \delta(\mathbf{q}_T^2 - \mathbf{p}_T^2),$$

where $\left\langle \mathcal{O}\left[{}^1S_0^{[1]}\right]\right\rangle = 2N_c |R(0)|^2/(4\pi)$.

In this talk we will compute $\mathcal{H}_{gg}^{(\mathrm{NLO})}(\mathbf{q}_T^2, z, \mathbf{p}_T^2)$, which includes **virtual** and **real-emission** corrections.



- Reggeized gluon fields $R_\pm$ carry $(k_\pm, \mathbf{k}_T, k_\mp = 0)$: $\partial_\mp R_\pm = 0$.
- **Induced interactions** of particles and Reggeons [Lipatov '95, '97; Bondarenko, Zubkov '18]:

$$L = \frac{i}{g_s}\,\mathrm{tr}\left[R_+ \partial_\perp^2 \partial_- \left(W\left[A_-\right] - W^\dagger\left[A_-\right]\right) + (+ \leftrightarrow -)\right],$$

with $W_{x_\mp}[x_\pm, \mathbf{x}_T, A_\pm] = P\exp\left[\frac{-ig_s}{2}\int\limits_{-\infty}^{x_\mp} dx'_\mp A_\pm(x_\pm, x'_\mp, \mathbf{x}_T)\right] = \left(1 + ig_s\partial_\pm^{-1}A_\pm\right)^{-1}$.

- Expansion of the Wilson line generates **induced vertices**:

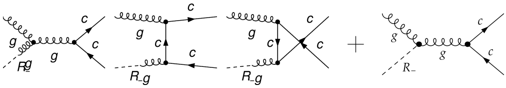
$$\mathrm{tr}\left[R_+\partial_\perp^2 A_- + (-ig_s)(\partial_\perp^2 R_+)(A_-\partial_-^{-1}A_-)\right.$$
$$\left. +(-ig_s)^2(\partial_\perp^2 R_+)(A_-\partial_-^{-1}A_-\partial_-^{-1}A_-) + O(g_s^3) + (+\leftrightarrow -)\right].$$

- The *Eikonal propagators* $\partial_\pm^{-1} \to -i/(k^\pm)$ lead to **rapidity divergences**, which are regularized by tilting the Wilson lines from the light-cone [Hentschinski, Sabio Vera, Chachamis *et. al.*, '12-'13; M.N. '19]:

$$n_\pm^\mu \to \tilde{n}_\pm^\mu = n_\pm^\mu + r n_\mp^\mu,\ r \ll 1:\ \tilde{k}^\pm = \tilde{n}^\pm k.$$

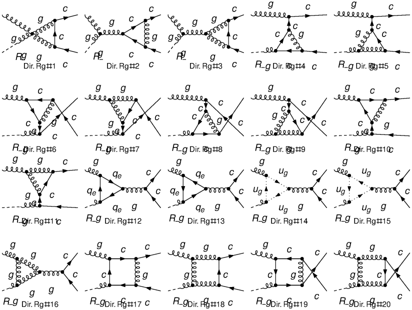
The terms for conversion of the result into any other regularisation scheme for RDs can be easily computed.

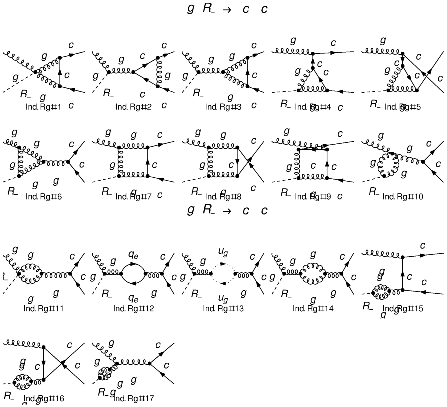
$Rg \to c\bar{c}\left[{}^1S_0^{[1]}\right]$ and $c\bar{c}\left[{}^3S_1^{[8]}\right]$ @ 1 loop

Interference with LO: 

Induced Rgg coupling diagrams:

Some Rg-coupling diagrams:





and so on...

- Diagrams had been generated using custom `FeynArts` model-file, projector on the $c\bar{c}\left[{}^1S_0^{[1]}\right]$-state is inserted
- heavy-quark momenta $= p_{\mathcal{Q}}/2 \Rightarrow$ need to resolve linear dependence of quadratic denominators in some diagrams before IBP
- IBP reduction to master integrals has been performed using `FIRE`
- Master integrals with linear and massless quadratic denominators are expanded in $r \ll 1$ using Mellin-Barnes representation. The differential equations technique is used when the integral depends on more than one scale of virtuality.
- In presence of the linear denominator the massive propagator can be converted to the massless one:

$$\frac{1}{((\tilde{n}_+ l) + k_+)(l^2 - m^2)} = \frac{1}{((\tilde{n}_+ l) + k_+)(l + \kappa\tilde{n}_+)^2} + \frac{2\kappa \cancel{\left[(\tilde{n}_+ l) + \frac{m^2 + \tilde{n}_+^2\kappa^2}{2\kappa}\right]}}{\cancel{((\tilde{n}_+ l) + k_+)}(l + \kappa\tilde{n}_+)^2(l^2 - m^2)}$$

$\Rightarrow$ all the masses can be moved to integrals with **only quadratic propagators**.

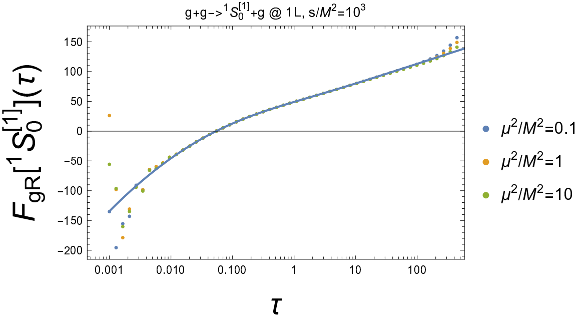
See [hep-ph/2408.06234] for details.

Result: $Rg \to c\bar{c}\left[{}^1S_0^{[1]}\right]$ @ 1 loop

Result [MN, '23, '24] for $2\Re\left[\frac{H_{\mathbf{1L\ x\ LO}}(\mathbf{q}_T)-\mathbf{(On\text{-}shell\ mass\ CT)}}{(\alpha_s/(2\pi))H_{\rm LO}(\mathbf{q}_T)}\right]$:

$$\left(\frac{\mu^2}{\mathbf{q}_T^2}\right)^{\epsilon}\left\{-\frac{N_c}{\epsilon^2}+\frac{1}{\epsilon}\left[N_c\left(\ln\frac{q_-^2}{\mathbf{q}_T^2 r}+\frac{25}{6}\right)-\frac{2n_F}{3}-\frac{3}{2N_c}\right]\right\}-\frac{10}{9}n_F+F_{{}^1S_0^{[1]}}(\mathbf{q}_T^2/M^2)$$

Cross-check against the Regge limit of one-loop amplitude ($\tau=\mathbf{q}_T^2/M^2$):



Points – the function $F_{{}^1S_0^{[1]}}(\tau)$ extracted form numerical results for interference between **exact** one-loop and tree-level QCD amplitudes of $g+g\to c\bar{c}[{}^1S_0^{[1]}]+g$ at $s=10^3M^2$. Solid line – analytic result from the EFT.

Real-emission correction

The real-emission contribution:

$$g(x_1 P_1) + R_-(q) \to c\bar{c}[{}^1S_0^{[1]}](p) + g(k),$$

to the coefficient function is given by:

$$\mathcal{H}_{gg}^{(\text{NLO, R})}(\mathbf{q}_T^2, z, \mathbf{p}_T^2) = \frac{\alpha_s(\mu_R)}{2\pi}\mathcal{H}_{gg}^{(\text{LO})}(\mathbf{p}_T^2) \int \frac{d\Omega_{2-2\epsilon}}{(2\pi)^{1-2\epsilon}} \frac{\tilde{H}_{Rg}(\mathbf{q}_T, \mathbf{p}_T, z)}{z(1-z)\mathbf{q}_T^2},$$

where the **function** $\tilde{H}_{Rg}(\mathbf{q}_T, \mathbf{p}_T, z)$ **is very complicated.** The following subtraction term ($O(\epsilon)$ terms not shown):

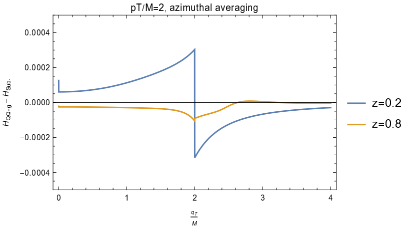
$$\boxed{\mathcal{J}_{Rj}^{(\text{sub.})} = \frac{2C_A}{\mathbf{k}_T^2}\left[\frac{1-z}{(1-z)^2 + r\frac{\mathbf{k}_T^2}{q_-^2}} + z(1-z) + 2\frac{\mathbf{k}_T^2\mathbf{p}_T^2 - (\mathbf{k}_T\mathbf{p}_T)^2}{z\mathbf{k}_T^2\mathbf{p}_T^2} - \frac{3\mathbf{k}_T^2\mathbf{p}_T^2 - 2(\mathbf{k}_T\mathbf{p}_T)^2}{\mathbf{k}_T^2\mathbf{p}_T^2}\right],}$$

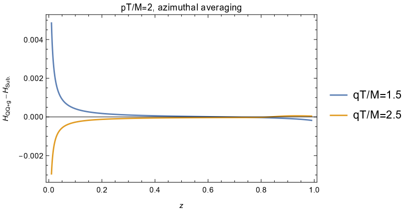
captures it's singular behaviour in:

- **Regge limit:** $z \to 1$, $\mathbf{k}_T = \mathbf{q}_T - \mathbf{p}_T$ – fixed,
- **Collinear limit:** $\mathbf{k}_T \to 0$, z-fixed
- **Soft limit:** $\mathbf{k}_T \to 0$, $z \to 1$

Real-emission correction, finite part

$$\mathcal{H}^{(\text{fin.})}_{gj}(\mathbf{q}_T^2, z, \mathbf{p}_T^2) = \frac{\alpha_s(\mu_R)}{2\pi}\mathcal{H}^{(\text{LO})}_{gg}(\mathbf{p}_T^2)\int\limits_0^{2\pi}\frac{d\phi}{2\pi}\left[\frac{\tilde{H}_{Rj}(\mathbf{q}_T, \mathbf{p}_T, z)}{z(1-z)\mathbf{q}_T^2} - \mathcal{J}^{(\text{sub.})}_{Rj}(\mathbf{q}_T, \mathbf{p}_T, z, r=0)\right]$$





This contribution is finite for $\mathbf{k}_T \to 0$ and $z \to 1$ and can be safely convoluted with the resummation factor or unintegrqted-PDF in $\mathbf{q}_T$ and gluon PDF in z.

Integrated subtraction term

$$\mathcal{H}_{gj}^{(\text{int. sub.})}(\mathbf{q}_T^2, z, \mathbf{p}_T^2) = \frac{\alpha_s(\mu_R)}{2\pi} \frac{\Omega_{2-2\epsilon}\mu^{2\epsilon}}{(2\pi)^{1-2\epsilon}} \int d^{2-2\epsilon}\mathbf{k}_T \mathcal{J}_{Rj}^{(\text{sub.})}(\mathbf{q}_T, \mathbf{q}_T - \mathbf{k}_T, z, r)$$
$$\times \mathcal{H}_{gg}^{(\text{LO})}(\mathbf{q}_T - \mathbf{k}_T, \mathbf{p}_T) = \mathcal{H}_{gj}^{(\text{int. sub. I})} + \mathcal{H}_{gj}^{(\text{int. sub. II})}$$

$$\mathcal{H}_{gj}^{(\text{int. sub. I})} = \frac{\alpha_s(\mu_R) C_A}{\pi} \int \frac{d^2\mathbf{k}_T}{\mathbf{k}_T^2} \left[\frac{1}{(1-z)_+} + (\ldots)\right.$$
$$\left. -\delta(1-z)\frac{1}{2}\ln\frac{r\mathbf{k}_T^2}{q_-^2}\right] \left[\delta^{(2)}(\mathbf{q}_T - \mathbf{k}_T - \mathbf{p}_T) - \frac{\mathbf{p}_T^2}{\mathbf{p}_T^2 + \mathbf{k}_T^2}\delta^{(2)}(\mathbf{q}_T - \mathbf{p}_T)\right] \mathcal{H}_{gg}^{(\text{LO})}(\mathbf{p}_T^2)$$

$$\mathcal{H}_{gj}^{(\text{int. sub. II})} = \frac{\alpha_s(\mu_R)}{2\pi} \mathcal{H}_{gg}^{(\text{LO})}(\mathbf{p}_T^2)\delta(\mathbf{p}_T^2 - \mathbf{q}_T^2)\frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}\left(\frac{\mu^2}{\mathbf{p}_T^2}\right)^\epsilon$$
$$\times \left\{-\frac{1}{\epsilon}P_{gg}(z) + \delta(1-z)\left[\frac{C_A}{\epsilon^2} + \frac{\beta_0}{2}\frac{1}{\epsilon} + \frac{C_A}{\epsilon}\ln\frac{r\mathbf{p}_T^2}{q_-^2} - \frac{\pi^2}{6}C_A\right] + O(\epsilon^2)\right\}.$$

Rapidity factorisation schemes

The $\ln r$-regularisation is equivalent to the cut in rapidity, **for HEF** we need to cut in "projectile light-cone component" k_-:

$$\mathcal{H}_{gj}^{(\text{HEF-sch.})}(\mathbf{q}_T^2, z, \mathbf{p}_T^2) = \frac{\bar{\alpha}_s}{2\pi}\mathcal{H}_{gg}^{(\text{LO})}(\mathbf{q}_T^2, \mathbf{p}_T^2, z)\left[-\frac{1}{\epsilon}\left(\frac{\beta_0}{2} - C_A\right) + \frac{4}{3}C_A - \frac{5}{6}\beta_0 - \frac{\pi^2}{3}C_A\right]$$
$$-\frac{\alpha_s C_A}{\pi}\mathcal{H}_{gg}^{(\text{LO})}(\mathbf{p}_T^2)\int\frac{d^2\mathbf{k}_T}{\mathbf{k}_T^2}\left[\delta^{(2)}(\mathbf{q}_T - \mathbf{k}_T - \mathbf{p}_T) - \frac{\mathbf{p}_T^2}{\mathbf{p}_T^2 + \mathbf{k}_T^2}\delta^{(2)}(\mathbf{q}_T - \mathbf{p}_T)\right]$$
$$\times\left(-\frac{1}{2}\ln r + \ln\frac{|\mathbf{k}_T|}{\Lambda_-}\right),$$

where $\Lambda_- \simeq q_- = (M^2 + \mathbf{p}_T^2)/(x_1 P_1^+)$. The **blue** terms come from R self-energy.

In BFKL we cut in $\ln(s_{\eta_c g}/s_0)$:

$$\mathcal{H}_{gj}^{(\text{BFKL-sch.})}(\mathbf{q}_T^2, z, \mathbf{p}_T^2) = \frac{\bar{\alpha}_s}{2\pi}\mathcal{H}_{gg}^{(\text{LO})}(\mathbf{q}_T^2, \mathbf{p}_T^2, z)\left[-\frac{1}{\epsilon}\left(\frac{\beta_0}{2} - C_A\right) + \frac{4}{3}C_A - \frac{5}{6}\beta_0\right]$$
$$-\frac{\alpha_s C_A}{\pi}\mathcal{H}_{gg}^{(\text{LO})}(\mathbf{p}_T^2)\int\frac{d^2\mathbf{k}_T}{\mathbf{k}_T^2}\left[\delta^{(2)}(\mathbf{q}_T - \mathbf{k}_T - \mathbf{p}_T) - \frac{\mathbf{p}_T^2}{\mathbf{p}_T^2 + \mathbf{k}_T^2}\delta^{(2)}(\mathbf{q}_T - \mathbf{p}_T)\right]$$
$$\times\left(-\frac{1}{2}\ln r + \ln\frac{x_1 P_1^+}{\sqrt{s_0}}\right).$$

Impact factor, HEF scheme

$$\mathcal{H}_{gg}^{(\text{NLO, analyt.})}(\mathbf{q}_T, z, \mathbf{p}_T) = \mathcal{H}_{gj}^{(\text{int. sub. I})} + \mathcal{H}_{gj}^{(\text{int. sub. II})} + \mathcal{H}_{gj}^{(\text{NLO, V})} + \mathcal{H}_{gj}^{(\text{HEF-sch.})}$$

$$= \frac{\alpha_s C_A}{\pi} \mathcal{H}_{gg}^{(\text{LO})}(\mathbf{p}_T^2) \int \frac{d^2\mathbf{k}_T}{\mathbf{k}_T^2} \left[\delta^{(2)}(\mathbf{q}_T - \mathbf{k}_T - \mathbf{p}_T) - \frac{\mathbf{p}_T^2}{\mathbf{p}_T^2 + \mathbf{k}_T^2} \delta^{(2)}(\mathbf{q}_T - \mathbf{p}_T)\right] \left[\frac{1}{(1-z)_+}\right.$$

$$\left. + z(1-z) + 2\frac{\mathbf{k}_T^2\mathbf{p}_T^2 - (\mathbf{k}_T\mathbf{p}_T)^2}{z\mathbf{k}_T^2\mathbf{p}_T^2} - \frac{3\mathbf{k}_T^2\mathbf{p}_T^2 - 2(\mathbf{k}_T\mathbf{p}_T)^2}{\mathbf{k}_T^2\mathbf{p}_T^2} + \delta(1-z)\ln\left(\frac{M^2 + \mathbf{p}_T^2}{\mathbf{k}_T^2}\right)\right]$$

$$+ \frac{\alpha_s C_A}{2\pi} \mathcal{H}_{gg}^{(\text{LO})}(\mathbf{p}_T^2)\delta(\mathbf{q}_T^2 - \mathbf{p}_T^2) \left\{ -\ln\frac{\mu_F^2}{\mathbf{p}_T^2} P_{gg}(z)\right.$$

$$\left. + \delta(1-z)\left[-\frac{\pi^2}{2}C_A + \frac{4}{3}C_A - \frac{5}{6}\beta_0 - 2C_F\left(2 + \frac{2}{3}\ln\frac{\mathbf{p}_T^2}{m_c^2}\right) + \beta_0 \ln\frac{\mu_R^2}{\mathbf{p}_T^2} + F_{{}^1S_0^{[1]}}(\mathbf{p}_T^2/M^2)\right]\right\}$$

This result should be added to the $\mathcal{H}_{gg}^{(\text{fin.})}$.

Impact factor, BFKL scheme

$$\mathcal{H}_{gg}^{(\text{NLO, analyt., BFKL})}(\mathbf{q}_T, z, \mathbf{p}_T) = \mathcal{H}_{gj}^{(\text{int. sub. I})} + \mathcal{H}_{gj}^{(\text{int. sub. II})} + \mathcal{H}_{gj}^{(\text{NLO, V})} + \mathcal{H}_{gj}^{(\text{BFKL-sch.})}$$
$$= \frac{\alpha_s C_A}{\pi}\mathcal{H}_{gg}^{(\text{LO})}(\mathbf{p}_T^2)\int \frac{d^2\mathbf{k}_T}{\mathbf{k}_T^2}\left[\delta^{(2)}(\mathbf{q}_T - \mathbf{k}_T - \mathbf{p}_T) - \frac{\mathbf{p}_T^2}{\mathbf{p}_T^2 + \mathbf{k}_T^2}\delta^{(2)}(\mathbf{q}_T - \mathbf{p}_T)\right]\left[\frac{1}{(1-z)_+}\right.$$
$$\left. + z(1-z) + 2\frac{\mathbf{k}_T^2\mathbf{p}_T^2 - (\mathbf{k}_T\mathbf{p}_T)^2}{z\mathbf{k}_T^2\mathbf{p}_T^2} - \frac{3\mathbf{k}_T^2\mathbf{p}_T^2 - 2(\mathbf{k}_T\mathbf{p}_T)^2}{\mathbf{k}_T^2\mathbf{p}_T^2} + \delta(1-z)\ln\left(\frac{\sqrt{s_0}}{|\mathbf{k}_T|}\right)\right]$$
$$+ \frac{\alpha_s C_A}{2\pi}\mathcal{H}_{gg}^{(\text{LO})}(\mathbf{p}_T^2)\delta(\mathbf{q}_T^2 - \mathbf{p}_T^2)\left\{-\ln\frac{\mu_F^2}{\mathbf{p}_T^2}P_{gg}(z)\right.$$
$$\left. + \delta(1-z)\left[-\frac{\pi^2}{6}C_A + \frac{4}{3}C_A - \frac{5}{6}\beta_0 - 2C_F\left(2 + \frac{2}{3}\ln\frac{\mathbf{p}_T^2}{m_c^2}\right) + \beta_0\ln\frac{\mu_R^2}{\mathbf{p}_T^2} + F_{{}^1S_0^{[1]}}(\mathbf{p}_T^2/M^2)\right]\right\}$$

This result should be added to **the same** $\mathcal{H}_{gg}^{(\text{fin.})}$.

Impact factor, $q + R$ channel

$$
\begin{aligned}
&\mathcal{H}_{qg}^{(\text{NLO, analyt.})}(\mathbf{q}_T, z, \mathbf{p}_T) \\
&= \frac{\alpha_s C_F}{2\pi} \mathcal{H}_{gg}^{(\text{LO})}(\mathbf{p}_T^2) \int \frac{d^2\mathbf{k}_T}{\mathbf{k}_T^2} \left[\delta^{(2)}(\mathbf{q}_T - \mathbf{k}_T - \mathbf{p}_T) - \frac{\mathbf{p}_T^2}{\mathbf{p}_T^2 + \mathbf{k}_T^2} \delta^{(2)}(\mathbf{q}_T - \mathbf{p}_T) \right] \\
&\times \left[\frac{(2-z)^2}{z} - \frac{4(1-z)}{z} \frac{(\mathbf{k}_T \mathbf{p}_T)^2}{\mathbf{k}_T^2 \mathbf{p}_T^2} \right] \\
&\qquad + \frac{\alpha_s}{2\pi} \mathcal{H}_{gg}^{(\text{LO})}(\mathbf{p}_T^2) \delta(\mathbf{q}_T^2 - \mathbf{p}_T^2) \left\{ - \ln \frac{\mu_F^2}{\mathbf{p}_T^2} P_{gq}(z) + C_F z \right\}.
\end{aligned}
$$

This result should be added to $\mathcal{H}_{qg}^{(\text{fin.})}$ computed using the subtraction term ($O(\epsilon)$ terms are not shown):

$$
\mathcal{J}_{qg} = \frac{C_F}{z\mathbf{k}_T^2} \left[(2-z)^2 - 4(1-z) \frac{(\mathbf{k}_T \mathbf{p}_T)^2}{\mathbf{k}_T^2 \mathbf{p}_T^2} \right],
$$

together with the complete NLO matrix element of the process $q + R \to q + g$: $\tilde{H}_{Rq}$.

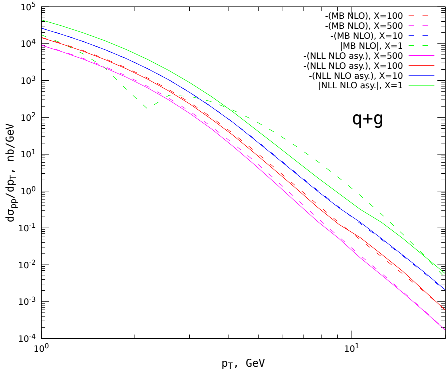
Numerical cross-check against NLO CF computation ($q+g$ channel)

Expansion of the NLL HEF result should reproduce the $\hat{s} \gg M^2$ asymptotics of the full NLO CF computation:

$$\left.\frac{d\hat{\sigma}_{qg}^{(\mathrm{LO+NLO})}}{dz\, d\mathbf{p}_T^2}\right|_{\hat{s}\gg M^2} = \frac{\hat{\alpha}_s}{\mathbf{q}_T^2} \widetilde{\otimes} \mathcal{H}_{qg}^{(\mathrm{NLO})}$$

$$= \hat{\alpha}_s \int \frac{d^2\mathbf{q}_T}{\pi \mathbf{q}_T^2} \left[\mathcal{H}_{qg}^{(\mathrm{NLO})}(\mathbf{q}_T, z, \mathbf{p}_T) - \mathcal{H}_{qg}^{(\mathrm{NLO})}(0, z, \mathbf{p}_T)\theta(\mu_F^2 - \mathbf{q}_T^2) \right].$$

Preliminary numerical comparison $\left(\mu_R = \mu_F = \sqrt{M^2 + p_T^2}\right)$:



Dashed lines – NLO CF results by *M. Butenschön*, solid lines – NLL HEF prediction.
The NLO CF computation is done with the cut on $\hat{s} > X\hat{s}_{\min}$ with $\hat{s}_{\min} = 2M_T[M_T + |\mathbf{p}_T|] - M^2$ – kilematical lower bound of $\hat{s}$ for given p_T.
Green – $X = 1$, blue – $X = 10$ and red – $X = 100$, magenta – $X = 500$.

Resummation function beyond DLA

The NLO result for the resummation function $\mathcal{C}_{gg}(x, \mathbf{q}_T, \mu_F, \mu_R)$ (1-loop virtual $g + R \to g$ + real $g + R \to g + g$ and $g + R \to q + \bar{q}$):

$$\mathcal{C}_{gg}^{(\mathrm{NLO})} = \frac{\hat{\alpha}_s^2}{\mathbf{q}_T^2} \left[\ln \frac{1}{x} \ln \frac{\mathbf{q}_T^2}{\mu_F^2} + \left(\frac{11}{12} - \frac{n_F}{6N_c} \right) \ln \frac{\mu_R^2}{\mu_F^2} + \left(\frac{n_F}{6N_c} - \frac{n_F}{6N_c^3} - \frac{11}{6} \right) \ln \frac{\mathbf{q}_T^2}{\mu_F^2} + R_{gg}^{(2)} \right],$$

where $R_{gg}^{(2)} = \frac{67}{36} - \frac{5n_F}{18N_c} - \frac{n_F}{12N_c^3}$, $\hat{\alpha}_s = \alpha_s(\mu_R) C_A / \pi$.

The **DLA** resums corrections $\sim \hat{\alpha}_s \left(\hat{\alpha}_s \ln \frac{1}{x} \ln \frac{\mathbf{q}_T^2}{\mu_F} \right)^n$. The **NDLA** has one $\ln 1/x$ or $\ln \mu_F^2$ less per power of α_s. The resummed expression in NDLA can be obtained from RG analysis (running of α_s+collinear factorisation/DGLAP, $n_F = 0$):

$$\mathcal{C}_{gg}^{(\mathrm{NDLA})} = \left(1 + 4C_A a_s R_{gg}^{(2)} \right) \left[1 + a_s \beta_0 \ln \frac{\mu_R^2}{\mu_F^2} \left(1 + \ln \frac{\mu_F^2}{\mathbf{q}_T^2} \frac{\partial}{\partial \ln \mu_F^2} \right) \right.$$
$$\left. + a_s \ln \frac{\mu_F^2}{\mathbf{q}_T^2} \left(\frac{22C_A}{3} + \frac{\beta_0}{2} \ln \frac{\mu_F^2}{\mathbf{q}_T^2} \frac{\partial}{\partial \ln \mu_F^2} \right) \right] \mathcal{C}_{gg}^{(\mathrm{DLA})}(x, \mathbf{q}_T^2, \mu_F, \mu_R),$$

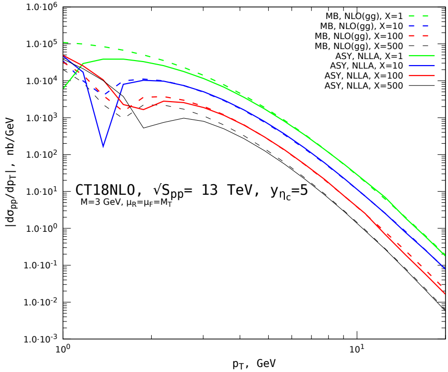
where $a_s = \alpha_s(\mu_R)/(4\pi)$.

Numerical cross-check against NLO CF computation ($g+g$ channel)

Expansion of the NLL HEF result should reproduce the $\hat{s} \gg M^2$ asymptotics of the full NLO CF computation:

$$\left.\frac{d\hat{\sigma}_{gg}^{(\mathrm{LO+NLO})}}{dz d\mathbf{p}_T^2}\right|_{\hat{s}\gg M^2} = \frac{\hat{\alpha}_s}{\mathbf{p}_T^2}\left[1 + \hat{\alpha}_s \ln\frac{\hat{s}}{M_T^2}\ln\frac{\mu_F^2}{\mathbf{p}_T^2} + (\mathrm{NDLA}\ \mathcal{C})\right]\mathcal{H}_{gg}^{(\mathrm{LO})} + \frac{\hat{\alpha}_s}{\mathbf{q}_T^2}\widetilde{\otimes}\mathcal{H}_{gg}^{(\mathrm{NLO})}.$$

Preliminary numerical comparison ($\mu_R = \mu_F = \sqrt{M^2 + p_T^2}$):



Dashed lines – NLO CF results by *M. Butenschön*, solid lines – NLL HEF prediction.

The NLO CF computation is done with the cut on $\hat{s} > X\hat{s}_{\min}$ with $\hat{s}_{\min} = 2M_T[M_T + |\mathbf{p}_T|] - M^2$ – kilematical lower bound of $\hat{s}$ for given p_T.

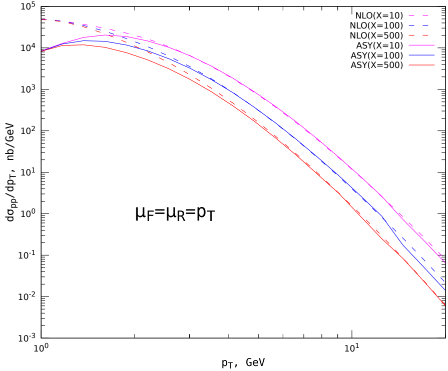
Green – $X=1$, blue – $X=10$ and red – $X=100$, black – $X=500$.

Numerical cross-check against NLO CF computation ($g+g$ channel)

Expansion of the NLL HEF result should reproduce the $\hat{s} \gg M^2$ asymptotics of the full NLO CF computation:

$$\left.\frac{d\hat{\sigma}_{gg}^{(\mathrm{LO+NLO})}}{dz d\mathbf{p}_T^2}\right|_{\hat{s}\gg M^2} = \frac{\hat{\alpha}_s}{\mathbf{p}_T^2}\left[1+\hat{\alpha}_s \ln\frac{\hat{s}}{M_T^2}\ln\frac{\mu_F^2}{\mathbf{p}_T^2}+(\mathrm{NDLA}\ \mathcal{C})\right]\mathcal{H}_{gg}^{(\mathrm{LO})}+\frac{\hat{\alpha}_s}{\mathbf{q}_T^2}\widetilde{\otimes}\mathcal{H}_{gg}^{(\mathrm{NLO})}.$$

Preliminary numerical comparison ($\mu_R=\mu_F=p_T$):



Dashed lines – NLO CF results by *M. Butenschön*, solid lines – NLL HEF prediction.

The NLO CF computation is done with the cut on $\hat{s} > X\hat{s}_{\min}$ with $\hat{s}_{\min} = 2M_T[M_T+|\mathbf{p}_T|]-M^2$ – kilematical lower bound of $\hat{s}$ for given p_T.

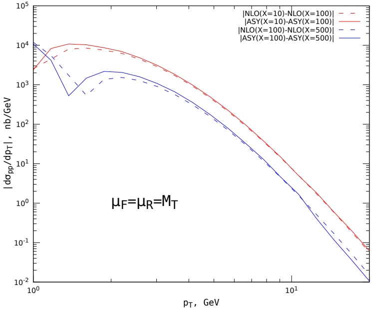
Magenta – $X=10$ and blue – $X=100$, red – $X=500$.

Numerical cross-check against NLO CF computation ($g + g$ channel)

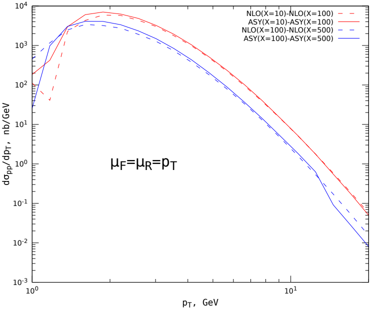
Constraining $\hat{s}$ from both sides:

$$X_1 < \frac{\hat{s}}{\hat{s}_{\min}} < X_2.$$

$\mu_R = \mu_F = \sqrt{M^2 + p_T^2}$



$\mu_R = \mu_F = p_T$



Red – $10 < \hat{s}/\hat{s}_{\min} < 100$, blue – $100 < \hat{s}/\hat{s}_{\min} < 500$. Solid lines – HEF prediction, dashed lines – NLO CF.

Conclusions and outlook

- The complete NLO HEF coefficient function (impact factor) for the $g + R \to c\bar{c}[^1S_0^{[1]}]$ process is computed, including one-loop and real-emission corrections in both $g + R$ and $q + R$ channels
- The computation for other NRQCD-factorisation intermediate states: $c\bar{c}[^1S_0^{[8]}, {}^3S_1^{[8]}, {}^3P_J^{[1,8]}]$ are in progress. The $c\bar{c}[^3S_1^{[1]}]$ is more challenging.
- The result in HEF scheme is useful for the resummation of $\ln \hat{s}/M^2$ corrections in CF coefficient function
- The result in BFKL scheme is useful for the study of double-η_c froduction at large rapidity separation
- The result in the "shockwave" scheme, corresponding to the cut in "projectile" light-cone component (k^+) is easy to obtain. However this is $1R$-exchange only.
- The same computaion technology can be applied to the central production vertices $RR \to c\bar{c}[n]$.

Thank you for your attention!

Perturbative instability of quarkonium total cross sections

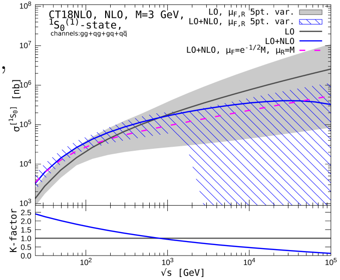
Inclusive η_c-hadroproduction (CSM)

[Mangano *et.al.*, '97, ..., Lansberg, Ozcelik, '20]

$$p+p \to c\bar{c}\left[{}^1S_0^{[1]}\right]+X,\ \text{LO: } g(p_1)+g(p_2) \to c\bar{c}\left[{}^1S_0^{[1]}\right],$$

$$\sigma(\sqrt{s_{pp}}) = f_i(x_1,\mu_F) \otimes f_j(x_2,\mu_F) \otimes \hat{\sigma}(z),$$

where $z = \frac{M^2}{\hat{s}}$ with $\hat{s} = (p_1+p_2)^2$.



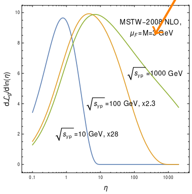
Inclusive J/ψ-photoproduction (CSM)

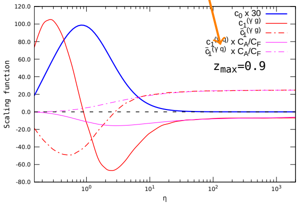
[Krämer, '96, ...,Colpani Serri *et.al*, '21]

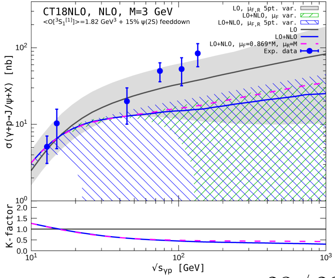
$$\gamma+p \to c\bar{c}\left[{}^3S_1^{[1]}\right]+X,\ \text{LO: } \gamma(q)+g(p_1) \to c\bar{c}\left[{}^3S_1^{[1]}\right]+g,$$

$$\sigma(\sqrt{s_{\gamma p}}) = f_i(x_1,\mu_F) \otimes \hat{\sigma}(\eta),$$

where $\eta = \frac{\hat{s}-M^2}{M^2}$ with $\hat{s} = (q+p_1)^2$, $z = \frac{pP}{qP}$.





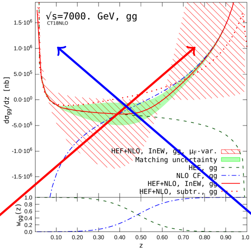


Matching with NLO

The HEF is valid in the **leading-power** in $M^2/\hat{s}$, so for $\hat{s} \sim M^2$ we match it with NLO CF by the *Inverse-Error Weighting Method* [Echevarria *et.al.*, 18'].

η_c-hadroproduction,

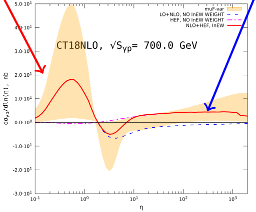
$z = M^2/\hat{s}$:

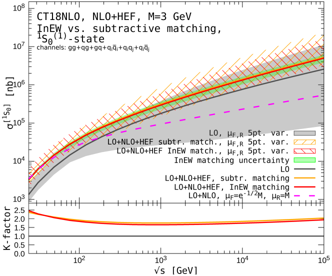


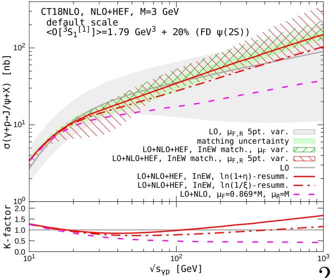
NLO **HEF**

J/ψ-photoproduction,

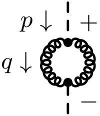
$\eta = (\hat{s} - M^2)/M^2$:







Rapidity divergences and regularization.

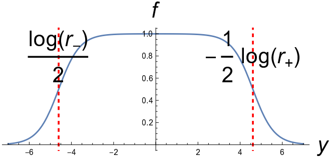


$$= g_s^2 C_A \delta_{ab} \int \frac{d^d q}{(2\pi)^D} \frac{\left(\mathbf{p}_T^2 (n_+ n_-)\right)^2}{q^2 (p-q)^2 q^+ q^-}, \quad \int \frac{dq^+ dq^- (\ldots)}{q^+ q^-} = \int\limits_{y_1}^{y_2} dy \int \frac{dq^2 (\ldots)}{q^2 + \mathbf{q}_T^2}$$

the regularization by explicit cutoff in rapidity was originally proposed [Lipatov, '95] ($q^{\pm} = \sqrt{q^2 + \mathbf{q}_T^2} e^{\pm y}$, $p^+ = p^- = 0$):

$$\delta_{ab} \mathbf{p}_T^2 \times \underbrace{C_A g_s^2 \int \frac{\mathbf{p}_T^2 d^{D-2} \mathbf{q}_T}{\mathbf{q}_T^2 (\mathbf{p}_T - \mathbf{q}_T)^2}}_{\omega^{(1)}(\mathbf{p}_T^2)} \times (y_2 - y_1) + \text{finite terms}$$

The square of regularized Lipatov vertex:



$$\Gamma_{+\mu-} \Gamma_{+\nu-} P^{\mu\nu} = \frac{16 \mathbf{q}_{T1}^2 \mathbf{q}_{T2}^2}{\mathbf{k}_T^2} f(y),$$

$$\longleftarrow f(y) = \frac{1}{(re^{-y} + e^{y})(re^{y} + e^{-y})},$$

$$\int\limits_{-\infty}^{+\infty} dy \ f(y) = -\log r + O(r)$$

The C_F coefficient

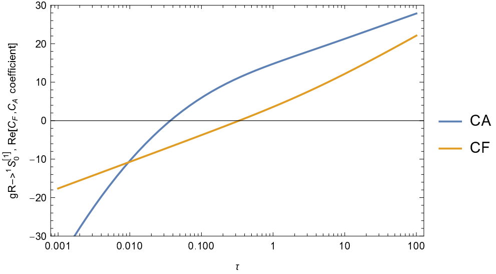
$$C_{gR}[{}^1S_0^{[8]}, C_F] = \frac{1}{6(\tau+1)^2}\Bigg\{-12\tau(\tau+1)\mathrm{Li}_2(-2\tau-1) + \frac{6L_2}{\tau}(-2L_1\tau + L_1 + 6\tau(\tau+1))$$

$$+\frac{1}{(2\tau+1)^2}\Bigg[(\tau+1)12\ln(2)(\tau+1)\left(6\tau^2+8\tau+3\right) - 8\tau^3\left(9\ln(\tau+1)+2\pi^2+15\right)$$

$$-4\tau^2\left(30\ln(\tau+1)+\pi^2+63\right) + 8\tau\left(-6\ln(\tau+1)+\pi^2-21\right)$$

$$+18(\tau+1)(2\tau+1)^2\ln(\tau) + 3\pi^2 - 36\Bigg]\Bigg\},$$

where $L_1 = L_1^{(+)} - L_1^{(-)} - L_2/2$ with $L_1^{(\pm)} = \sqrt{\tau(1+\tau)}\ln\left(\sqrt{1+\tau} \pm \sqrt{\tau}\right)$ and $L_2 = \sqrt{\tau(1+\tau)}\ln\left(1+2\tau+2\sqrt{\tau(1+\tau)}\right)$.

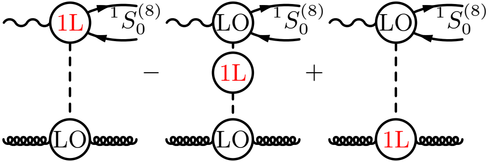


The C_A coefficient for $gR \to c\bar{c}\left[{}^1S_0^{[1]}\right]$

$$
\begin{aligned}
C_{gR}[{}^1S_0^{[1]}, C_A] = {} & \frac{2(\tau(\tau(\tau(7\tau+8)+2)-4)-1)}{(\tau-1)(\tau+1)^3}\mathrm{Li}_2(-\tau) \\
& -\frac{\tau(\tau(4\tau+5)+3)}{(\tau+1)^3}\mathrm{Li}_2(-2\tau-1)-\frac{L_2^2}{2\tau(\tau+1)^2} \\
& +\frac{1}{18(\tau-1)(\tau+1)^3}\Big\{-2\left(\tau^2-1\right)(18\ln(2)(\tau-1)\tau-67(\tau+2)\tau-67) \\
& +18[\ln(\tau)\left(-2\tau^4+\left(\tau\left(-\tau^3+\tau+3\right)+2\right)\tau\ln(\tau)+2\tau^2+\ln(\tau)\right) \\
& -(\tau-1)^2(\tau+1)^3\ln^2(\tau+1)+2(\tau-1)(\tau+1)^2\left(\tau+(\tau+1)^2\ln(\tau)\right)\ln(\tau+1)] \\
& +\pi^2(3\tau(\tau(\tau(15\tau+14)-3)-12)-6)\Big\},
\end{aligned}
$$

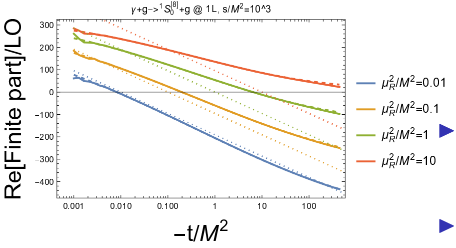
$R\gamma \to c\bar{c}\left[{}^1S_0^{[8]}\right]$ @ 1 loop, cross-check

In the combination of 1-loop results in the EFT:



the $\ln r$ cancels and it should reproduce the the Regge limit($s \gg -t$) of the *real part* of the $2 \to 2$ 1-loop QCD amplitude:

$$\gamma + g \to c\bar{c}\left[{}^1S_0^{(8)}\right] + g.$$



Solid lines – QCD, dashed lines – EFT, dotted lines – $-2C_A \ln(-t/\mu_R^2)\ln(s/M^2)$

- The $2 \to 2$ QCD 1-loop amplitude can be computed numerically using `FormCalc` (with some tricks, due to Coulomb divergence)
- The Regge limit of $1/\epsilon$ divergent part agrees with the EFT result
- For the finite part agreement within few % is reached, need to push to higher s

Cornell potential:

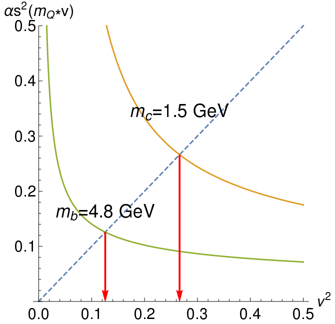
$$V(r) = -C_F \frac{\alpha_s(1/r)}{r} + \sigma r,$$

neglect linear part, because quarkonium is "small" (~ 0.3 fm) $\rightarrow$ Coulomb wavefunction (for effective mass $\frac{m_1 m_2}{m_1+m_2} = \frac{m_Q}{2}$):

$$R(r) = \frac{\sqrt{m_Q^3 \alpha_s^3 C_F^3}}{2} e^{-\frac{\alpha_s C_F}{2} m_Q r}$$

$$\langle v^2 \rangle = \frac{C_F^2 \alpha_s^2}{2}, \langle r \rangle = \frac{3}{2C_F} \frac{1}{m_Q v}$$

$$\Rightarrow \boxed{\alpha_s^2(m_Q v) \simeq v^2}$$



Non-relativistic QCD

The velocity-expansion for quarkonium eigenstate is carbon-copy of corresponding arguments from atomic physics (hierarchy of E-dipole/M-dipole with ΔS/M-dipole transitions):

$$\begin{aligned} |J/\psi\rangle &= O(1)\left|c\bar{c}\left[{}^3S_1^{(1)}\right]\right\rangle + O(v)\left|c\bar{c}\left[{}^3P_J^{(8)}\right]+g\right\rangle \\ &+ O(v^{3/2})\left|c\bar{c}\left[{}^1S_0^{(8)}\right]+g\right\rangle + O(v^2)\left|c\bar{c}\left[{}^3S_1^{(8)}\right]+gg\right\rangle + \ldots, \end{aligned}$$

for validity of this arguments, we should work in *non-relativistic EFT*, dynamics of which conserves number of heavy quarks. In such EFT, $Q\bar{Q}$-pair is produced in a point, by local operator:

$$\mathcal{A}_{\mathrm{NRQCD}} = \langle J/\psi + X|\, \chi^\dagger(0)\kappa_n\psi(0)\,|0\rangle\,,$$

Different operators "couple" to different Fock states:

$$\chi^\dagger(0)\psi(0) \leftrightarrow \left|c\bar{c}\left[{}^1S_0^{(1)}\right]\right\rangle,\ \chi^\dagger(0)\sigma_i\psi(0) \leftrightarrow \left|c\bar{c}\left[{}^3S_1^{(1)}\right]\right\rangle,$$

$$\chi^\dagger(0)\sigma_i T^a\psi(0) \leftrightarrow \left|c\bar{c}\left[{}^3S_1^{(8)}\right]\right\rangle,\ \chi^\dagger(0)D_i\psi(0) \leftrightarrow \left|c\bar{c}\left[{}^1P_1^{(8)}\right]\right\rangle, \ldots$$

squared NRQCD amplitude (=LDME):

$$\sum_X |\mathcal{A}|^2 = \langle 0|\, \underbrace{\psi^\dagger\kappa_n^\dagger\chi a^\dagger_{J/\psi}a_{J/\psi}\chi^\dagger\kappa_n\psi}_{\mathcal{O}_n^{J/\psi}}\,|0\rangle = \left\langle \mathcal{O}_n^{J/\psi}\right\rangle,$$

Effect of anomalous dimension beyond LO

Effect of taking **full LLA** for $\gamma_{gg}(N) = \frac{\hat{\alpha}_s}{N} + 2\zeta(3)\frac{\hat{\alpha}_s^4}{N^4} + 2\zeta(5)\frac{\hat{\alpha}_s^6}{N^6} + \dots$ together with NLO PDF.