Angular distributions of Drell-Yan leptons in the TMD factorization approach

Based on **S.Piloñeta** and **A.Vladimirov** **arXiv 2407.06277** (July, 2024)

Resummation, Evolution and Factorization 2024

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Introduction

- The TMD factorization theorem is a powerful tool
  - Includes the parton's transverse momentum
    - Transverse momentum dependent distributions (TMDs)
  - Deeper knowledge about inelastic processes ⟶ unveil hadron 3D structure!
  - Its current form has limitations
    - Some interesting observables are power-suppressed
      - An update is needed to study them

TMD-with-KPCs factorization theorem

- Incorporates Kinematic Power Corrections (KPCs)

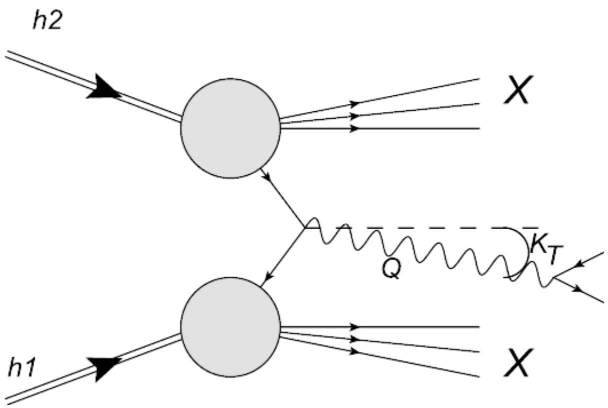
We expect that they are the largest in the TMD factorization regime $q_T \ll Q$

Unpolarized Drell-Yan as a starting point

Kinematics and Collins-Soper frame

- **Feasibility** of using the **TMD-with-KPCs factorization** ⟶ **Drell-Yan reaction**

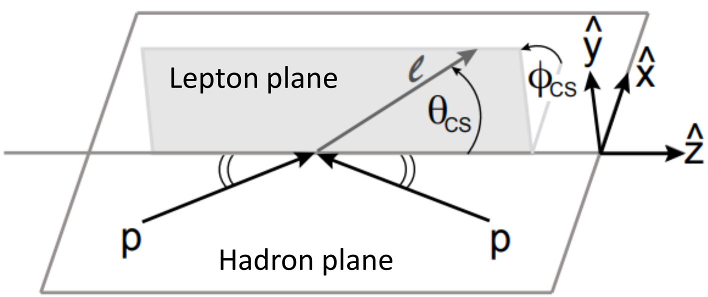
$$h_1(p_1) + h_2(p_2) \;\rightarrow\; \gamma^*/Z(q) + X \;\rightarrow\; \ell^-(l) + \ell^+(l') + X$$



Kinematics

- q^μ ⟶ gauge boson momentum
  - ⟶ $q^2 = Q^2$ ⟶ large scale
- $p_1^\mu = \bar{n}^\mu p_1^+,\; p_2^\mu = n^\mu p_2^-$
- $p_1^2 = p_2^2 = 0$

- **Starting point** for **testing** the **TMD-with-KPCs** factorization theorem
  - **Angular distributions** of the **lepton pair**
    - **Collins-Soper (CS) frame**
      - ⟶ Dilepton rest frame
      - ⟶ Angles θ and ϕ



Unpolarized Drell-Yan as a starting point

Cross-section decomposition and angular distributions

- **Angular decomposition** of the **cross section**

$$\frac{d\sigma}{dp_T dy dQ d\Omega} = \frac{3}{16\pi} \frac{d\sigma^{U+L}}{dp_T dy dQ} \Big[(1+\cos^2\theta) + \frac{1-3\cos^2\theta}{2} \boxed{A_0} + \sin 2\theta \cos\phi \boxed{A_1} + \frac{\sin^2\theta\cos 2\phi}{2} \boxed{A_2} + \sin\theta\cos\phi \boxed{A_3} + \cos\theta \boxed{A_4} + \sin^2\theta \sin 2\phi \boxed{A_5} + \sin 2\theta \sin\phi \boxed{A_6} + \sin\theta\sin\phi \boxed{A_7} \Big]$$

$\frac{d\sigma^{U+L}}{dp_T dy dQ}$ → **Unpolarized angle-integrated** cross-section

$\boxed{A_{0,\ldots,7}}$ → **Angular distributions**

- Only **four** at **LP** $\{ A_U,\ A_2,\ A_4,\ A_5 \}$
- The **rest** are **power-suppressed**

TMD factorization with **resummed KPCs**...

Good approximation?

The hierarchy of the TMD factorization theorem

- **General structure** of the **TMD factorization theorem**

$$
\begin{aligned}
W^{\mu\nu} = \frac{1}{N_c}\int \frac{d^2b}{(2\pi)^2}\, e^{-i(bq_T)} \Big\{ \\
& \boxed{\Phi_2 \times \Phi_2 \quad \text{LP}} \\
&+ \frac{1}{Q}\boxed{\Big(D\ \ \Phi_2\times\Phi_2 + \quad \Phi_2\times\Phi_3\Big)\ \text{NLP}} \\
&+ \frac{1}{Q^2}\boxed{\Big(D^2\,\Phi_2\times\Phi_2 + D\ \ \Phi_2\times\Phi_3 + \quad \Phi_3\times\Phi_3 + \quad \Phi_2\times\Phi_4 + \frac{\Phi_2\times\Phi_2}{b^2}\Big)\ \text{NNLP}} \\
&+ \frac{1}{Q^3}\Big(D^3\,\Phi_2\times\Phi_2 + D^2\,\Phi_2\times\Phi_3 + D\ \ \Phi_3\times\Phi_3 + D\ \ \Phi_2+\Phi_4 + D\frac{\Phi_2\times\Phi_2}{b^2} + \ldots\Big) \\
&+ \frac{1}{Q^4}\Big(D^4\,\Phi_2\times\Phi_2 + \ldots\Big) \\
&+ \ \ldots \Big\}^{\mu\nu}
\end{aligned}
$$

Notation

Φ_n → TMD of twist-n

$\times$ → Integral convolution

$D \sim \dfrac{\partial}{\partial b^\mu}$ → transverse derivative

The hierarchy of the TMD factorization theorem

- **General structure** of the **TMD factorization theorem**

$$W^{\mu\nu} = \frac{1}{N_c}\int \frac{d^2b}{(2\pi)^2}\, e^{-i(bq_T)}\Big\{$$

$$\Phi_2 \times \Phi_2$$

$$+\frac{1}{Q}\Big(D\ \ \Phi_2\times\Phi_2 + \Phi_2\times\Phi_3\Big)$$

$$+\frac{1}{Q^2}\Big(D^2\,\Phi_2\times\Phi_2 + D\ \ \Phi_2\times\Phi_3 + \Phi_3\times\Phi_3 + \Phi_2\times\Phi_4 + \frac{\Phi_2\times\Phi_2}{b^2}\Big)$$

$$+\frac{1}{Q^3}\Big(D^3\,\Phi_2\times\Phi_2 + D^2\,\Phi_2\times\Phi_3 + D\ \ \Phi_3\times\Phi_3 + D\ \ \Phi_2+\Phi_4 + D\frac{\Phi_2\times\Phi_2}{b^2} + \ldots\Big)$$

$$+\frac{1}{Q^4}\Big(D^4\,\Phi_2\times\Phi_2 + \ldots\Big)$$

$$+\ \ldots\ \Big\}^{\mu\nu}$$

Notation

- Φ_n → TMD of twist-n
- $\times$ → Integral convolution
- $D \sim \frac{\partial}{\partial b^\mu}$ → transverse derivative

Each **column**
- → Series of KPCs to the first term
- → Unique non-perturbative content
- → Independent contributions

The hierarchy of the TMD factorization theorem

- **General structure** of the **TMD factorization theorem**

$$
\begin{aligned}
W^{\mu\nu} = \frac{1}{N_c}\int \frac{d^2b}{(2\pi)^2}\, e^{-i(bq_T)} \Bigg\{ & \Phi_2 \times \Phi_2 \\
& + \frac{1}{Q}\Big(D\, \Phi_2 \times \Phi_2 + \Phi_2 \times \Phi_3 \Big) \\
& + \frac{1}{Q^2}\Big(D^2\, \Phi_2 \times \Phi_2 + D\, \Phi_2 \times \Phi_3 + \Phi_3 \times \Phi_3 + \Phi_2 \times \Phi_4 + \frac{\Phi_2 \times \Phi_2}{b^2} \Big) \\
& + \frac{1}{Q^3}\Big(D^3\, \Phi_2 \times \Phi_2 + D^2\, \Phi_2 \times \Phi_3 + D\, \Phi_3 \times \Phi_3 + D\, \Phi_2 + \Phi_4 + D\frac{\Phi_2 \times \Phi_2}{b^2} + \ldots \Big) \\
& + \frac{1}{Q^4}\Big(D^4\, \Phi_2 \times \Phi_2 + \ldots \Big) \\
& + \ldots \Bigg\}^{\mu\nu}
\end{aligned}
$$

$\boxed{k_T/Q}$ $\boxed{k^2_T/Q^2}$ $\boxed{k^3_T/Q^3}$ $\boxed{k^4_T/Q^4}$

Notation

- Φ_n → TMD of twist-n
- $\times$ → Integral convolution
- $D \sim \frac{\partial}{\partial b^\mu}$ → transverse derivative

KPCs following the **LP term**

- Same TMDs and perturbative part as LP
- All power-suppressed terms containing twist-2 TMDs
- Restoration of gauge and frame invariance

Hadron tensor in TMD-with-KPCs factorization theorem

- Theory of **KPCs** for **TMD factorization** developed in [AV, 2307.13054v2]
  - ➢ **KPCs** series to the **LP term** derived and summed at all powers
  - ➢ **Drell-Yan unpolarized cross-section** computed in this framework

- Starting point ⟶ **EW hadron tensor**
  ⟶ **Unpolarized** $f_{1;f/p}$ and **Boer-Mulders** $h^{\perp}_{1;f/p}$ distributions

$$\boxed{q_\mu W^{\mu\nu} = 0}$$

$$
\begin{aligned}
W^{\mu\nu}_{GG'} = {} & \frac{p_1^+ p_2^-}{N_c} C_0\left(\frac{Q^2}{\mu^2}\right) \int d^4k_1 d^4k_2 \delta(k_1^2)\delta(k_2^2)\delta^{(4)}(k_1+k_2-q) \\
& \int d\xi_1 d\xi_2 \delta(k_1^+ - \xi_1 p_1^+)\delta(k_2^- - \xi_2 p_2^-) \sum_q \Big[\\
& \mathrm{Tr}\left(\gamma^\mu_G \not{k}_2 \gamma^\nu_{G'} \not{k}_1\right) \boxed{f_{1,q/h_1} f_{1,\bar{q}/h_1}} + \mathrm{Tr}\left(\gamma^\mu_G \not{k}_1 \gamma^\nu_{G'} \not{k}_2\right) \boxed{f_{1,\bar{q}/h_1} f_{1,q/h_1}} - \frac{\epsilon_T^{\mu_1\alpha} k_1^\alpha k_1^{\nu_1} \epsilon_T^{\mu_2\beta} k_2^\beta k_2^{\nu_2}}{M^2} \Big(\\
& \mathrm{Tr}\left(\gamma^\mu_G \sigma^{\mu_2\nu_2} \gamma^\nu_{G'} \sigma^{\mu_1\nu_1}\right) \boxed{h^\perp_{1,q/h_1} h^\perp_{1,\bar{q}/h_1}} + \mathrm{Tr}\left(\gamma^\mu_G \sigma^{\mu_1\nu_1} \gamma^\nu_{G'} \sigma^{\mu_2\nu_2}\right) \boxed{h^\perp_{1,\bar{q}/h_1} h^\perp_{1,q/h_1}} \Big) \Big]
\end{aligned}
$$

Hadron tensor in TMD-with-KPCs factorization theorem

- Theory of KPCs for TMD factorization developed in [AV, 2307.13054v2]
  - KPCs series to the LP term derived and summed at all powers
  - Drell-Yan unpolarized cross-section computed in this framework

- Starting point $\Rightarrow$ EW hadron tensor
  - $\Rightarrow$ Unpolarized $f_{1;f/p}$ and Boer-Mulders $h^{\perp}_{1;f/p}$ distributions
  - $\Rightarrow$ Same coefficient function as LP

$$W^{\mu\nu}_{GG'} = \frac{p_1^+ p_2^-}{N_c} \boxed{C_0\left(\frac{Q^2}{\mu^2}\right)} \int d^4k_1 d^4k_2 \delta(k_1^2)\delta(k_2^2)\delta^{(4)}(k_1+k_2-q)$$

$$\int d\xi_1 d\xi_2 \delta(k_1^+ - \xi_1 p_1^+)\delta(k_2^- - \xi_2 p_2^-) \sum_q \Big[$$

$$\mathrm{Tr}\left(\gamma^\mu_G \not{k}_2 \gamma^\nu_{G'} \not{k}_1\right) \boxed{f_{1,q/h_1} f_{1,\bar{q}/h_1}} + \mathrm{Tr}\left(\gamma^\mu_G \not{k}_1 \gamma^\nu_{G'} \not{k}_2\right) \boxed{f_{1,\bar{q}/h_1} f_{1,q/h_1}} - \frac{\epsilon_T^{\mu_1\alpha} k_1^\alpha k_1^{\nu_1} \epsilon_T^{\mu_2\beta} k_2^\beta k_2^{\nu_2}}{M^2} \Big($$

$$\mathrm{Tr}\left(\gamma^\mu_G \sigma^{\mu_2\nu_2} \gamma^\nu_{G'} \sigma^{\mu_1\nu_1}\right) \boxed{h^\perp_{1,q/h_1} h^\perp_{1,\bar{q}/h_1}} + \mathrm{Tr}\left(\gamma^\mu_G \sigma^{\mu_1\nu_1} \gamma^\nu_{G'} \sigma^{\mu_2\nu_2}\right) \boxed{h^\perp_{1,\bar{q}/h_1} h^\perp_{1,q/h_1}} \Big) \Big]$$

$$\boxed{q_\mu W^{\mu\nu} = 0}$$

Hadron tensor in TMD-with-KPCs factorization theorem

- Theory of KPCs for TMD factorization developed in [AV, 2307.13054v2]
  - KPCs series to the LP term derived and summed at all powers
  - Drell-Yan unpolarized cross-section computed in this framework

- Starting point → EW hadron tensor
  - → Unpolarized $f_{1;f/p}$ and Boer-Mulders $h^{\perp}_{1;f/p}$ distributions
  - → Same coefficient function as LP
  - → Main difference with LP → Convolution integral

$$W^{\mu\nu}_{GG'} = \frac{p_1^+ p_2^-}{N_c} C_0\left(\frac{Q^2}{\mu^2}\right) \int d^4k_1 d^4k_2 \delta(k_1^2)\delta(k_2^2)\delta^{(4)}(k_1+k_2-q) \qquad q_\mu W^{\mu\nu} = 0$$

$$\int d\xi_1 d\xi_2 \delta(k_1^+ - \xi_1 p_1^+)\delta(k_2^- - \xi_2 p_2^-) \sum_q \Big[$$

$$\mathrm{Tr}\left(\gamma^\mu_G \not{k}_2 \gamma^\nu_{G'} \not{k}_1\right) f_{1,q/h_1} f_{1,\bar{q}/h_1} + \mathrm{Tr}\left(\gamma^\mu_G \not{k}_1 \gamma^\nu_{G'} \not{k}_2\right) f_{1,\bar{q}/h_1} f_{1,q/h_1} - \frac{\epsilon_T^{\mu_1\alpha} k_1^\alpha k_1^{\nu_1} \epsilon_T^{\mu_2\beta} k_2^\beta k_2^{\nu_2}}{M^2} \Big($$

$$\mathrm{Tr}\left(\gamma^\mu_G \sigma^{\mu_2\nu_2} \gamma^\nu_{G'} \sigma^{\mu_1\nu_1}\right) h^{\perp}_{1,q/h_1} h^{\perp}_{1,\bar{q}/h_1} + \mathrm{Tr}\left(\gamma^\mu_G \sigma^{\mu_1\nu_1} \gamma^\nu_{G'} \sigma^{\mu_2\nu_2}\right) h^{\perp}_{1,\bar{q}/h_1} h^{\perp}_{1,q/h_1} \Big)\Big]$$

Angular decomposition of the lepton tensor

- Information about

  ↳ Detected lepton pair + Measurement

  ↓

  Six-dimensional phase-space ➡ $dPS = d^4q\, d\Omega$ → Lepton-pair **angular part**

  Boson momentum $q = l + l'$

- Express lepton tensor via CS-frame angles θ and ϕ

  ↳ Leptons momenta decomposition [Arnold, Metz, Schlegel, 0809.2262v2]

  $$l^\mu = \frac{q^\mu + \Delta_l^\mu}{2} \qquad l'^\mu = \frac{q^\mu - \Delta_l^\mu}{2}$$

  Parametrization

  $$\Delta_l^\mu = \bar{n}^\mu q^+ \frac{Q}{\tau}\left(\cos\theta - \frac{Q}{|\boldsymbol{q}_T|}\sin\theta\cos\phi\right) + n^\mu q^- \frac{Q}{\tau}\left(-\cos\theta - \frac{Q}{|\boldsymbol{q}_T|}\sin\theta\cos\phi\right)$$
  $$+ q^\mu \frac{\tau}{|\boldsymbol{q}_T|}\sin\theta\cos\phi - \tilde{q}^\mu \frac{Q}{|\boldsymbol{q}_T|}\sin\theta\sin\phi$$

Angular decomposition of the lepton tensor

- Information about
  - Detected lepton pair + Measurement
    - Six-dimensional phase-space $dPS = d^4q \, d\Omega$
      - $d\Omega$: Lepton-pair angular part
      - d^4q: Boson momentum $q = l + l'$

- Express lepton tensor via CS-frame angles θ and ϕ
  - Convenient decomposition for the lepton tensor

$$L^{\mu\nu}_{GG'} = (-Q^2)\left(\underbrace{(v^2+a^2)_l}_{\text{Combination of EW coupling constants}} \sum_{n=U,0,1,2,5,6} \underbrace{S_n(\theta,\phi)}_{\text{Set of independent angular polynomials}}\mathfrak{L}^{\mu\nu}_n + (av)_l \sum_{n=3,4,7} S_n(\theta,\phi)\underbrace{\mathfrak{L}^{\mu\nu}_n}_{\text{Set of independent tensors}}\right)$$

Angular structure functions

- Differential cross-section computation using the previous hadron and lepton tensors

  The result is conveniently expressed as

$$\frac{d\sigma}{d^4q\,d\Omega} = \frac{3}{16\pi} \sum_{n=U,0,\ldots,7} S_n(\theta,\phi)\Sigma_n,$$

Structure functions $\rightarrow$ $A_n = \frac{\Sigma_n}{\Sigma_U}$ $\rightarrow$ $\Sigma_U = \frac{d\sigma}{d^4q}$ Normalization

- We have explicitly obtained the theoretical expressions for the various Σ_n

  We can codify them in a general way as

$\Sigma_n \sim$ Constants $\times$ Gauge boson propagator $\times$ Some combination of vector (v) and axial (a) coupling constants (1) $\times$ Convolution integral involving f_1 or $h_1^\perp$ distributions (2)

1 $\sim (v^2+a^2) \quad (v^2-a^2) \quad (av)$ $\rightarrow$ Some due to the Z-boson

2 Two possibilities

- $\mathcal{C}[A, f_1 f_2]$ $\rightarrow$ Symmetric flavor $f_{q_1} f_{\bar{q}_2} + f_{\bar{q}_1} f_{q_2}$
- $\mathcal{C}[A, \{f_1 f_2\}_A]$ $\rightarrow$ Anti-symmetric flavor $f_{q_1} f_{\bar{q}_2} - f_{\bar{q}_1} f_{q_2}$

Perturbative and non-perturbative set-up

- The KPCs have been implemented in ***artemide***
- New global fit is required

Technical difficulties and $\uparrow\uparrow$ computation time → Complex task

- First exploratory study

We use the input of ART23

→ Perturbative elements (N^4LL) and evolution

→ Unpolarized TMD distribution f_1

ATLAS data for A_2 angular distribution → Boer-Mulders distribution $h_1^\perp$ extraction

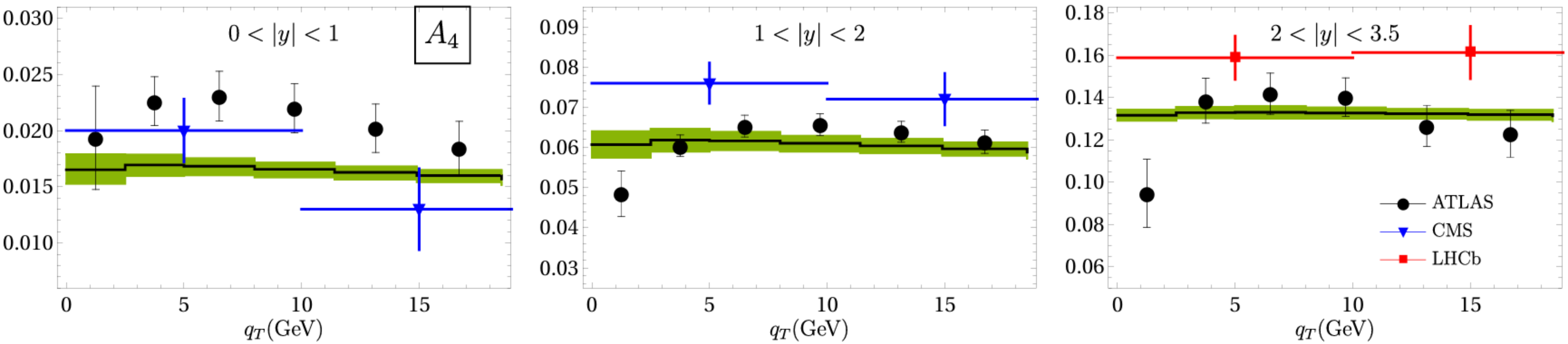
More details later!

We expect to describe the data sufficiently well without significant modifications of the models for TMD distributions

Angular distribution A_4

- Leading power
- Proportional to the difference between quark and anti-quark distributions
- Inclusion in standard f_1 extractions

➢ Comparison as a function of q_T with **ATLAS**, CMS and **LHCb** measurements

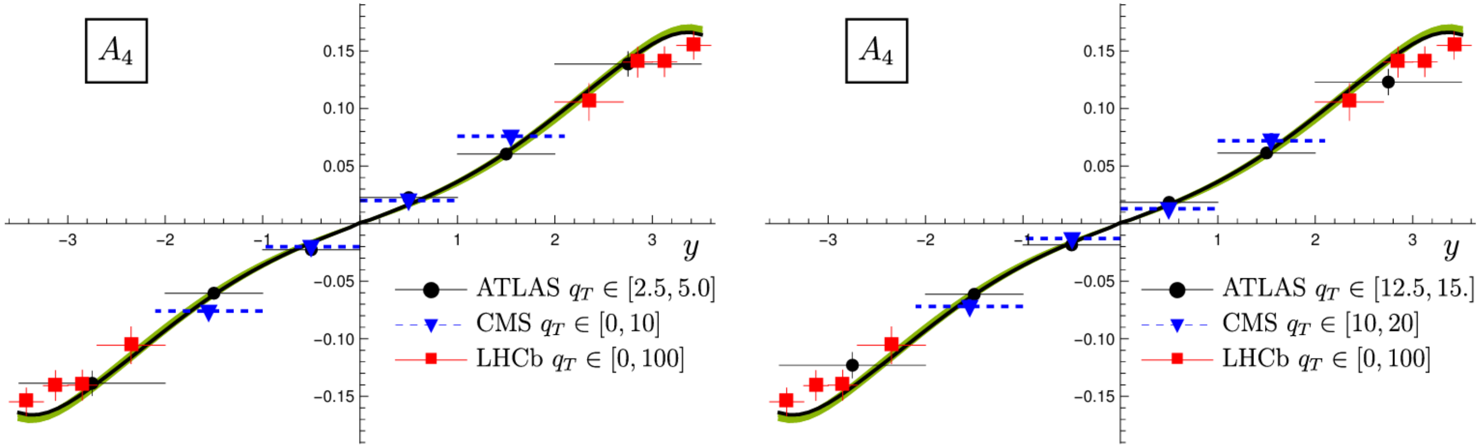


- Theory prediction agrees very well with the measurements

Angular distribution A_4

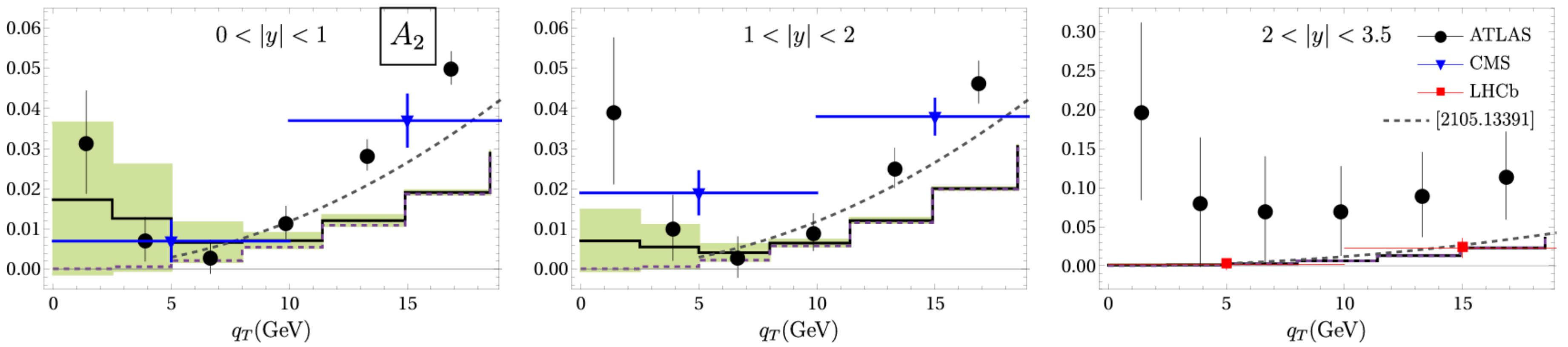
- **Leading power**
- Proportional to the **difference between quark and anti-quark** distributions
- **Inclusion** in standard f_1 **extractions**

➢ Comparison as a **function** of **y** with **ATLAS**, **CMS** and **LHCb** measurements



- The **agreement** is **even more transparent**

Angular distribution A_2



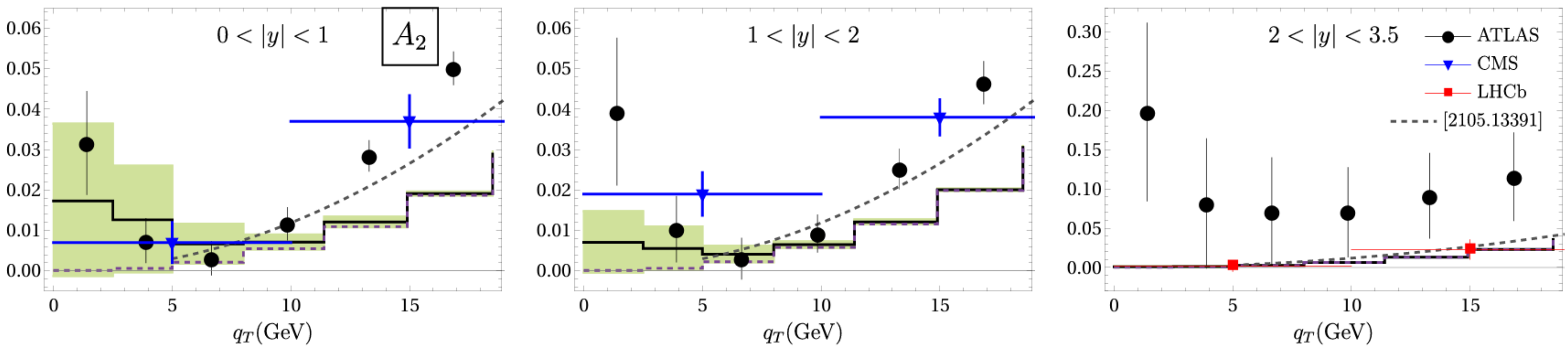
- Contains both **Boer-Mulders** and **unpolarized** distributions

$$A_2 \sim \frac{\boldsymbol{\Delta}^2 - 2(\boldsymbol{t}\boldsymbol{\Delta})^2}{2M^2}\left(\boxed{h_1^\perp h_1^\perp} - \frac{2M^2}{Q^2}\boxed{f_1 f_1}\right)$$

Dominant at $q_T \to 0$ Dominant at larger q_T

- **Additional contributions** from q_T^2/Q^2 corrections at large q_T
  - We can quantify them using ATLAS measurement
    - 2% - 4% for Σ_U at $q_T \sim 20\,\mathrm{GeV}$

- We **extract** the **Boer-Mulders** function from the data for the A_2 **angular coefficient** measured at **ATLAS**



- We **model** it with the following form → $\tilde{h}_1^\perp(x,b) = \dfrac{2^{\alpha+1}}{\Gamma(\alpha+1)} \dfrac{N}{\cosh(\lambda b)}\, x \ln^\alpha(1/x)$

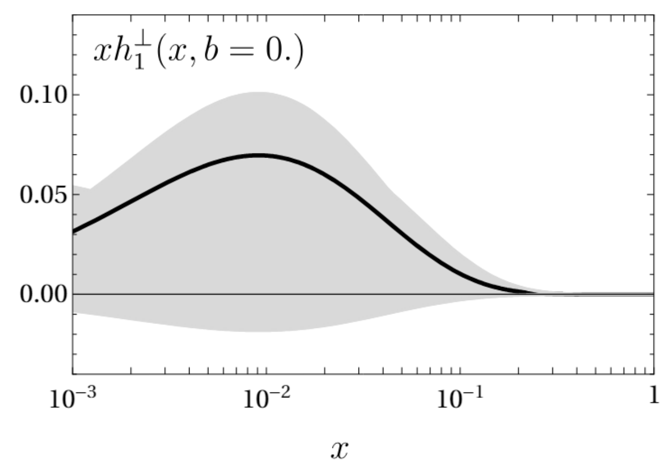
Fix the parameters → **Data points** with $q_T < 10\,\text{GeV}$ → $N_{pt} = 12$

Insufficient → Fix $\lambda = 0.2\text{GeV}$

Fit

$$\chi^2/N_{pt} = 1.16$$

$N = -0.27^{+0.34}_{-0.12}$ and $\alpha = 9.4^{+5.4}_{-0.9}$



Lam-Tung relation

$$\boxed{\Sigma_{\mathrm{LT}} = \Sigma_2 - \Sigma_0}$$

- If we use the TMD factorization theorem the relation does not hold at LP
- TMD-with-KPCs factorization theorem expression

$$\Sigma_{\mathrm{LT}} = \frac{4\pi\alpha_{\mathrm{em}}^2}{3N_c s} \sum_{q,G,G'} Q^4 \Delta_G^* \Delta_{G'} \Big\{ z_{+\ell}^{GG'} z_{+q}^{GG'} \mathcal{C}[2\frac{((\boldsymbol{t}\boldsymbol{k}_1) - (\boldsymbol{t}\boldsymbol{k}_2))^2 - (\boldsymbol{k}_1 - \boldsymbol{k}_2)^2}{Q^2}, f_1 f_1]$$
$$+ z_{+\ell}^{GG'} r_{+q}^{GG'} \mathcal{C}[\frac{\boldsymbol{k}_1^2 + \boldsymbol{k}_2^2 - ((\boldsymbol{t}\boldsymbol{k}_1) - (\boldsymbol{t}\boldsymbol{k}_2))^2}{M^2} + \frac{\boldsymbol{k}_1^2 + \boldsymbol{k}_2^2}{M^2} \frac{((\boldsymbol{t}\boldsymbol{k}_1) - (\boldsymbol{t}\boldsymbol{k}_2))^2 - (\boldsymbol{k}_1 - \boldsymbol{k}_2)^2}{Q^2}, h_1^\perp h_1^\perp] \Big\}$$

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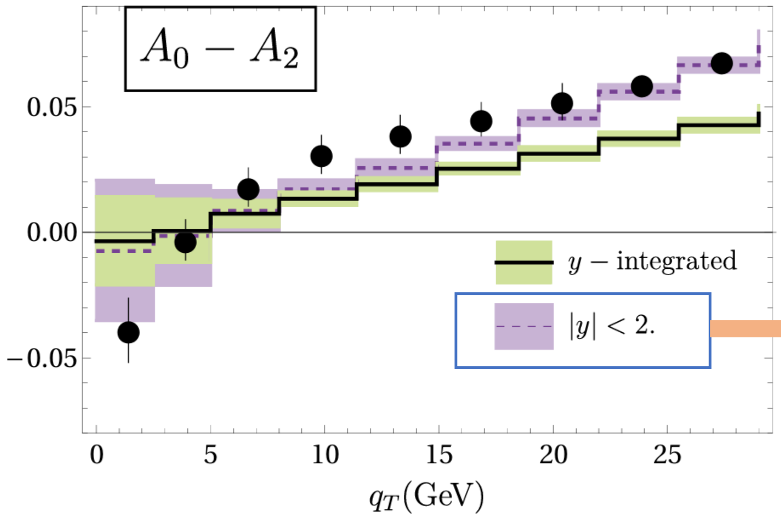
$$+ z_{+\ell}^{GG'} r_{+q}^{GG'} \mathcal{C}[\frac{\boldsymbol{k}_1^2 + \boldsymbol{k}_2^2 - ((\boldsymbol{t}\boldsymbol{k}_1) - (\boldsymbol{t}\boldsymbol{k}_2))^2}{M^2} + \frac{\boldsymbol{k}_1^2 + \boldsymbol{k}_2^2}{M^2} \frac{((\boldsymbol{t}\boldsymbol{k}_1) - (\boldsymbol{t}\boldsymbol{k}_2))^2 - (\boldsymbol{k}_1 - \boldsymbol{k}_2)^2}{Q^2}, h_1^\perp h_1^\perp] \Big\}$$



Prediction < ATLAS measurement → large x ✗

Theory drops to 0 at large y

Integrating to restrict the y-region

Good agreement

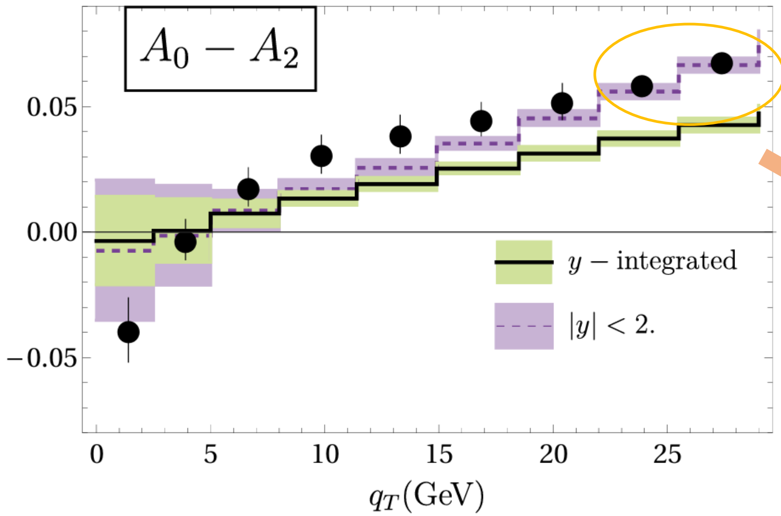
Lam-Tung relation

$$\boxed{\Sigma_{\rm LT} = \Sigma_2 - \Sigma_0}$$

- If we use the TMD factorization theorem the relation does not hold at LP
- TMD-with-KPCs factorization theorem expression

$$\Sigma_{\rm LT} = \frac{4\pi\alpha_{\rm em}^2}{3N_c s}\sum_{q,G,G'} Q^4 \Delta_G^* \Delta_{G'}\Big\{ z_{+\ell}^{GG'} z_{+q}^{GG'}\, \mathcal{C}[2\frac{((\boldsymbol{t}\boldsymbol{k}_1)-(\boldsymbol{t}\boldsymbol{k}_2))^2-(\boldsymbol{k}_1-\boldsymbol{k}_2)^2}{Q^2}, f_1 f_1]$$

$$+ z_{+\ell}^{GG'} r_{+q}^{GG'}\, \mathcal{C}[\frac{\boldsymbol{k}_1^2+\boldsymbol{k}_2^2-((\boldsymbol{t}\boldsymbol{k}_1)-(\boldsymbol{t}\boldsymbol{k}_2))^2}{M^2} + \frac{\boldsymbol{k}_1^2+\boldsymbol{k}_2^2}{M^2}\frac{((\boldsymbol{t}\boldsymbol{k}_1)-(\boldsymbol{t}\boldsymbol{k}_2))^2-(\boldsymbol{k}_1-\boldsymbol{k}_2)^2}{Q^2}, h_1^\perp h_1^\perp]\Big\}$$



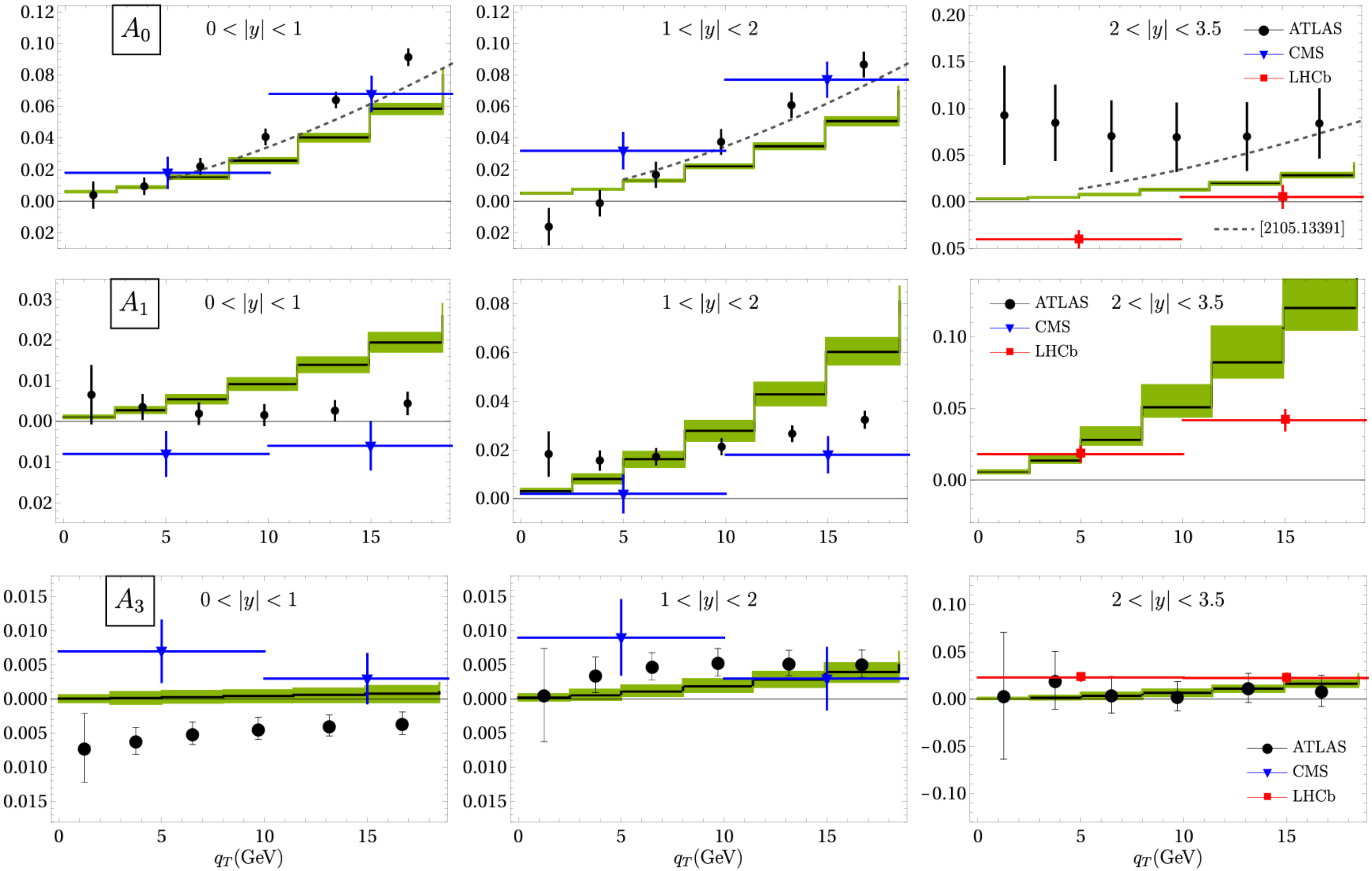
- Y-term suppressed in Lam-Tung relation

  Smaller q_T^2/Q^2 corrections

Good agreement in contrast to A_0 and A_2 independently

Description only posible due to KPCs inclusion

Angular distribution $A_{0,1,3}$



Conclusions

- First practical application of the TMD-with-KPCs factorization theorem
  - The angular distributions of the Drell-Yan leptons can be satisfactorily described
- The Boer-Mulders has been extracted using the A_2 data from ATLAS
  - The peculiar shape of A_2 at low-q_T is an evidence of the Boer-Mulders distribution
- We have tested the method proposed in [AV, 2307.13054v2]
  - Good description of the Lam-Tung relation

But... this is not all! Next step: SIDIS

- The same thing can be done with SIDIS
- Cross-section decomposition over structure functions

$$\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} \sim \Big\{ \boxed{F_{UU,T}} + \varepsilon \boxed{F_{UU,L}} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h}$$

$$+\,\varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h} + \ldots$$

We have already computed a theoretical expression in the TMD-with-KPCs factorization theorem framework

Currently working in the plots ⟶ We expect the effect of KPCs to be greater

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Thank you for your attention!