university of groningen

faculty of science and engineering

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Hadronic Wilson loop correlators

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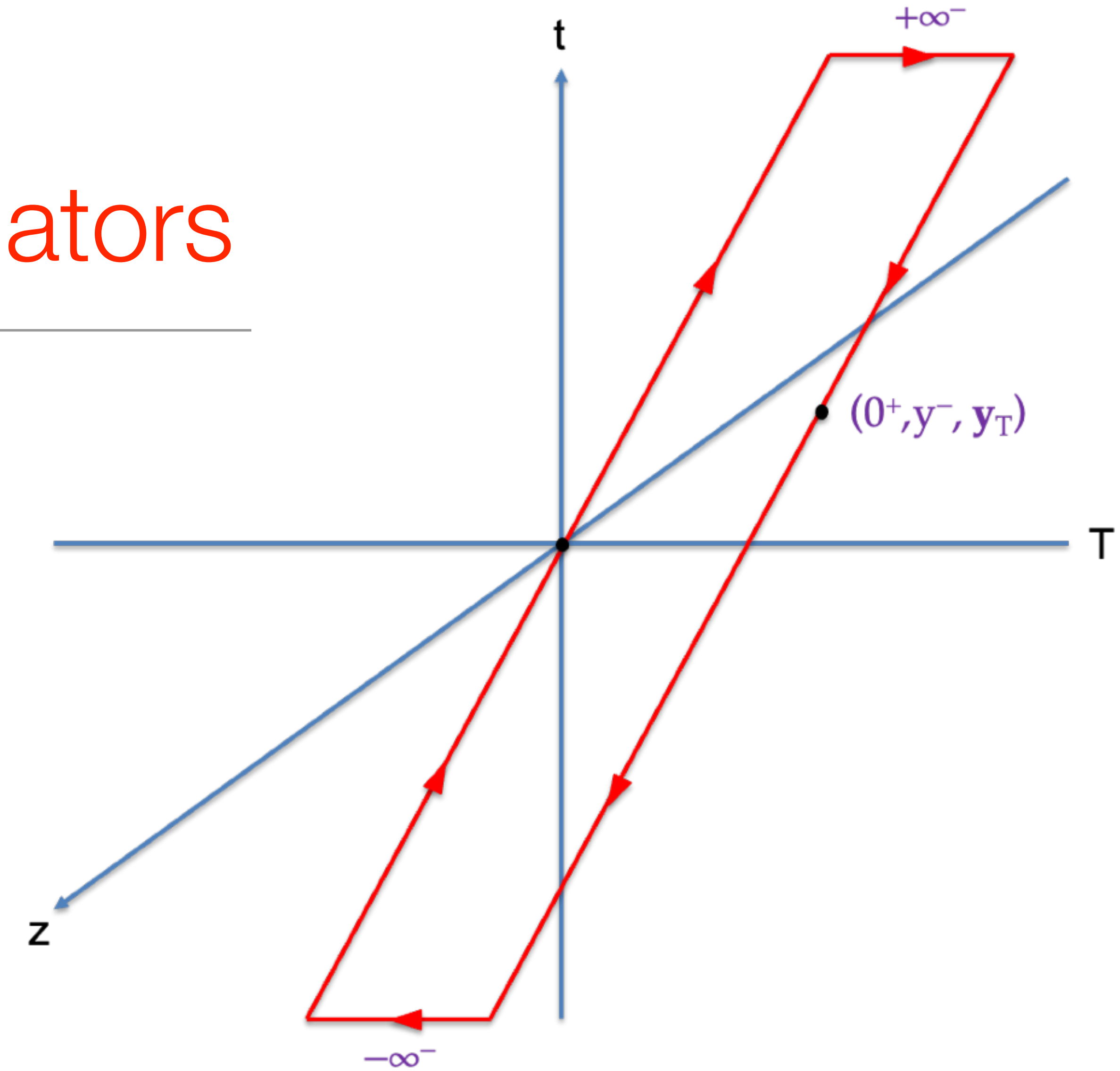


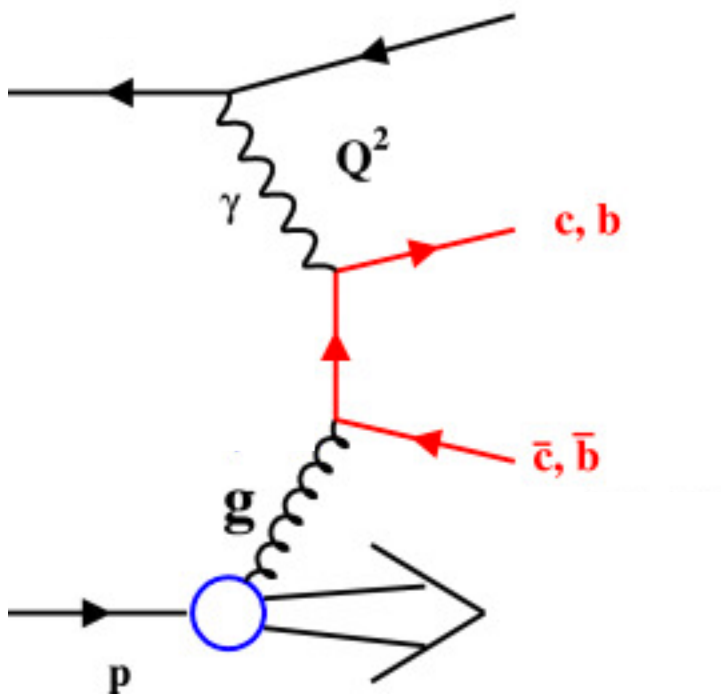
REF2024, Institut de Physique Théorique, Saclay, October 15, 2024

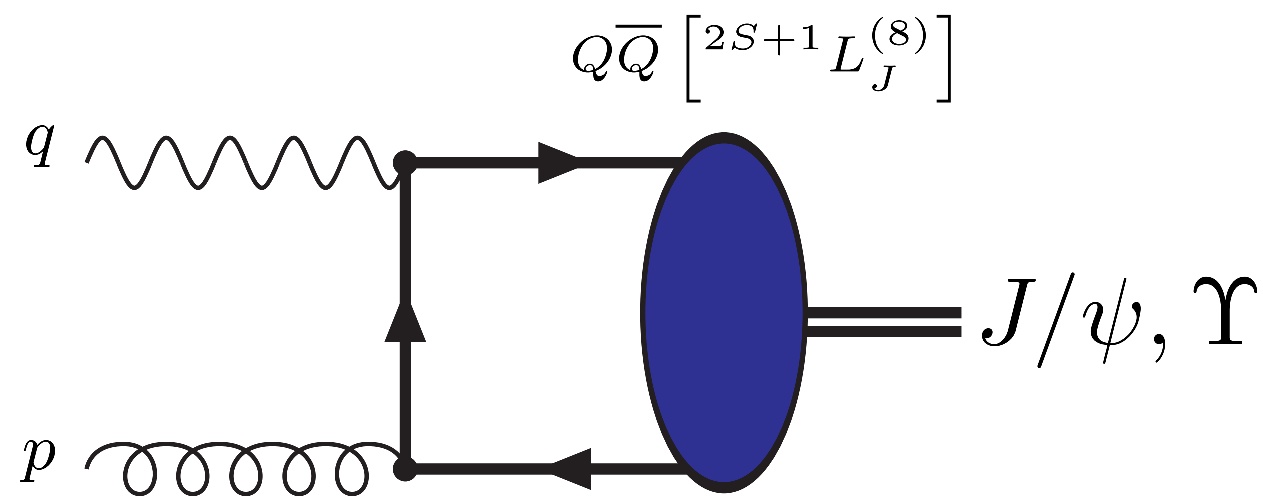
Overview

- Gluon TMDs: gauge links & small-x limit
- Wilson loop TMDs: evolution & phenomenology
- Wilson loop GTMDs: phenomenology
- Odderons

Gluon TMDs

Probing gluon TMDs



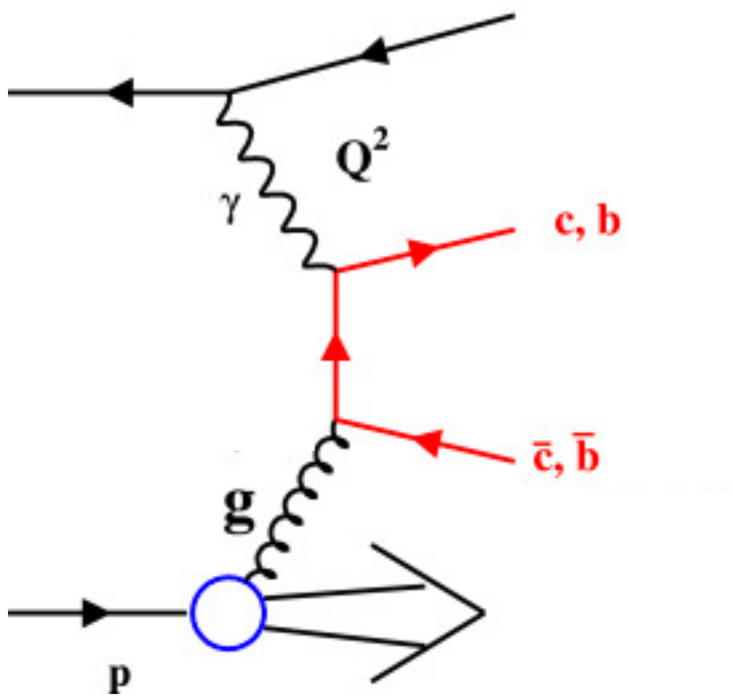


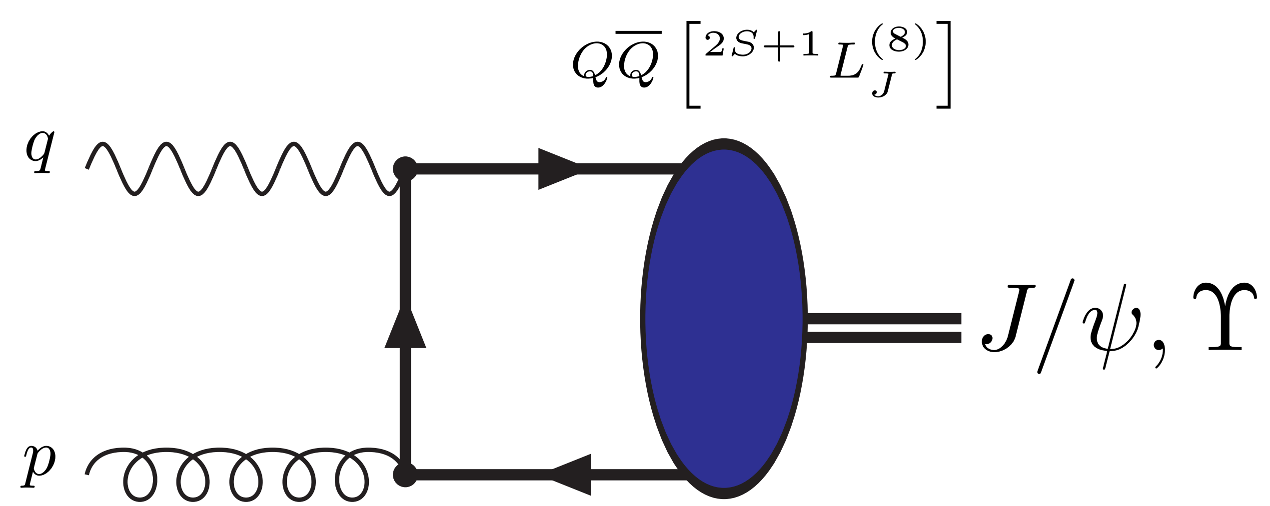
$ep \to e' Q\bar{Q} X$

$e\, p \to e'\, \mathcal{Q}\, X$

Open heavy quark pair production and quarkonium production are arguably the simplest processes that are sensitive to the transverse momentum of gluons

Probing gluon TMDs





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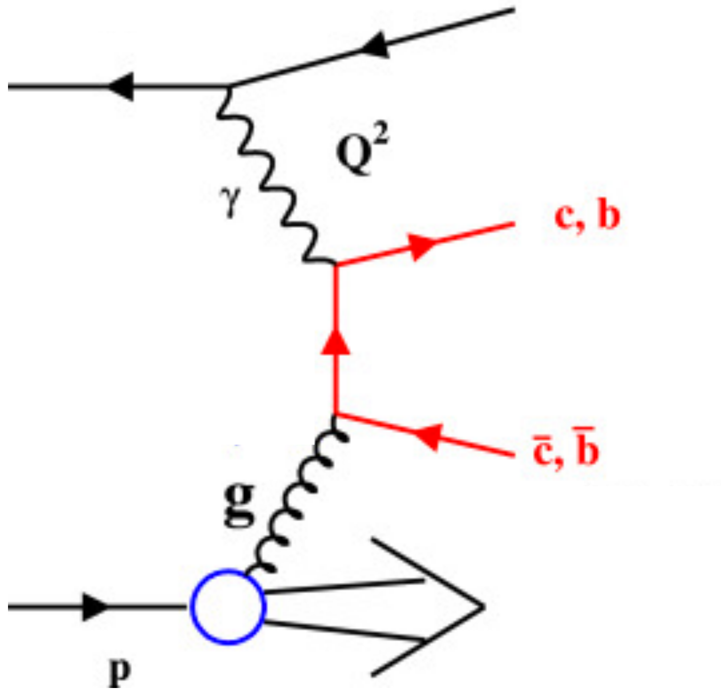

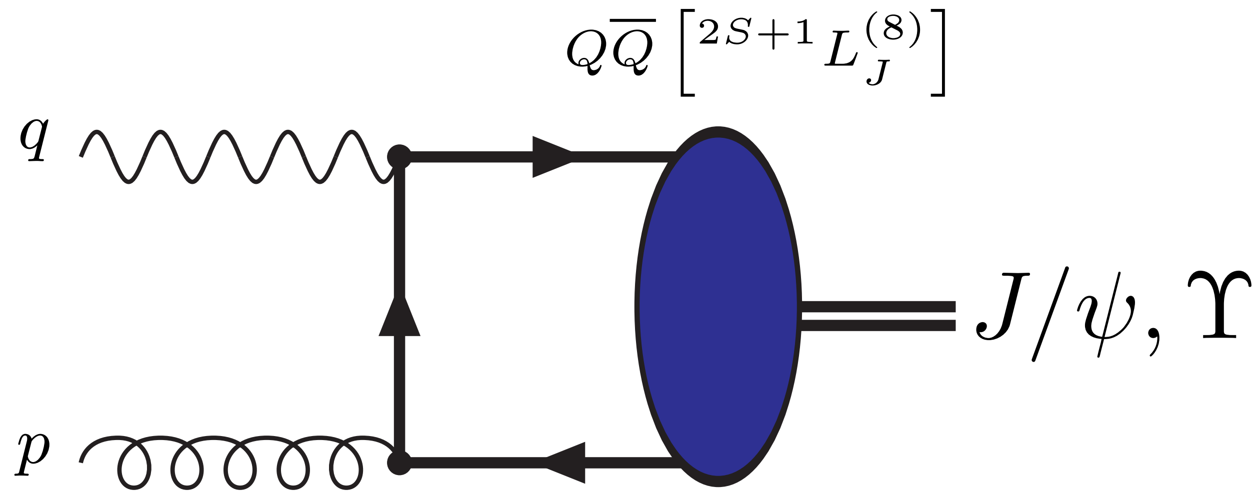

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Nuclei can also help boost the gluon density, but cannot be polarized in a collider

Gluons TMDs

Gluon TMD correlator: $\Gamma_g^{\mu\nu}(x, p_T) \propto \langle P|F^{+\nu}(0)\,\mathcal{U}\,F^{+\mu}(\xi^-, \xi_T)\,\mathcal{U}'|P\rangle$

transverse momentum dependent (TMD)

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For unpolarized protons at leading twist (LT):

$$\Gamma_U^{\mu\nu}(x, \boldsymbol{p}_T) = \frac{x}{2}\left\{-g_T^{\mu\nu}\, f_1^g(x, \boldsymbol{p}_T^2) + \left(\frac{p_T^\mu p_T^\nu}{M_p^2} + g_T^{\mu\nu}\frac{\boldsymbol{p}_T^2}{2M_p^2}\right) h_1^{\perp\, g}(x, \boldsymbol{p}_T^2)\right\}$$

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unpolarized gluon TMD

linearly polarized gluon TMD

Gluons inside *unpolarized* protons can be polarized!

Mulders, Rodrigues '01

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unpolarized gluon TMD

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Gluons inside *unpolarized* protons can be polarized!

For transversely polarized protons:

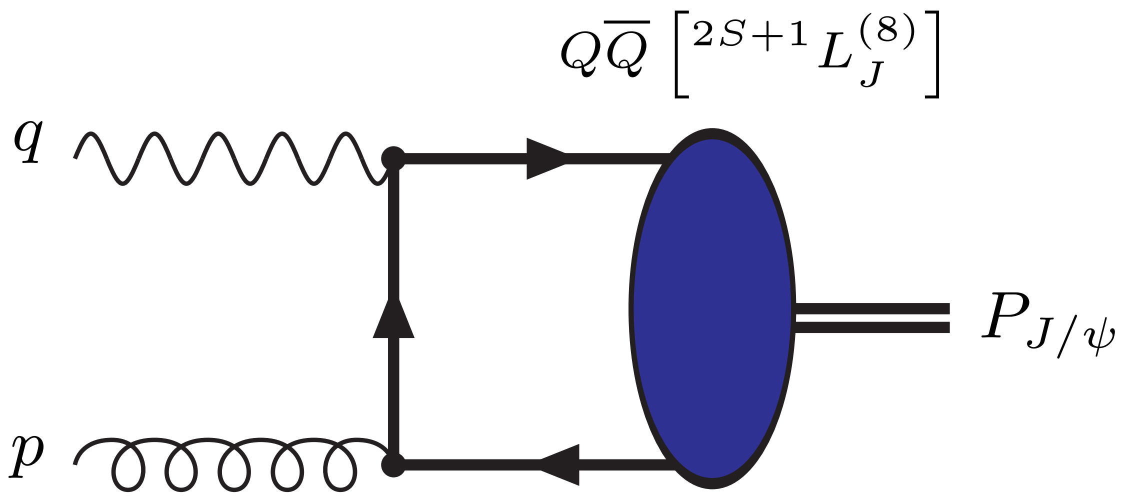
gluon Sivers TMD

$$\Gamma_T^{\mu\nu}(x, \boldsymbol{p}_T) = \frac{x}{2}\left\{ g_T^{\mu\nu}\frac{\epsilon_T^{\rho\sigma} p_{T\rho} S_{T\sigma}}{M_p} f_{1T}^{\perp\, g}(x, \boldsymbol{p}_T^2) + \ldots \right\}$$

Quarkonium production

$e\,p \to e'\,\mathcal{Q}\,X$ with $\mathcal{Q}$ either a J/ψ or a Υ meson

Mukherjee, Rajesh, 2017; Sun, Zhang, 2017; Bacchetta, DB, Pisano, Taels, 2018; Kishore, Mukherjee, 2018; Kishore, Mukherjee, Siddiqah, 2021; …



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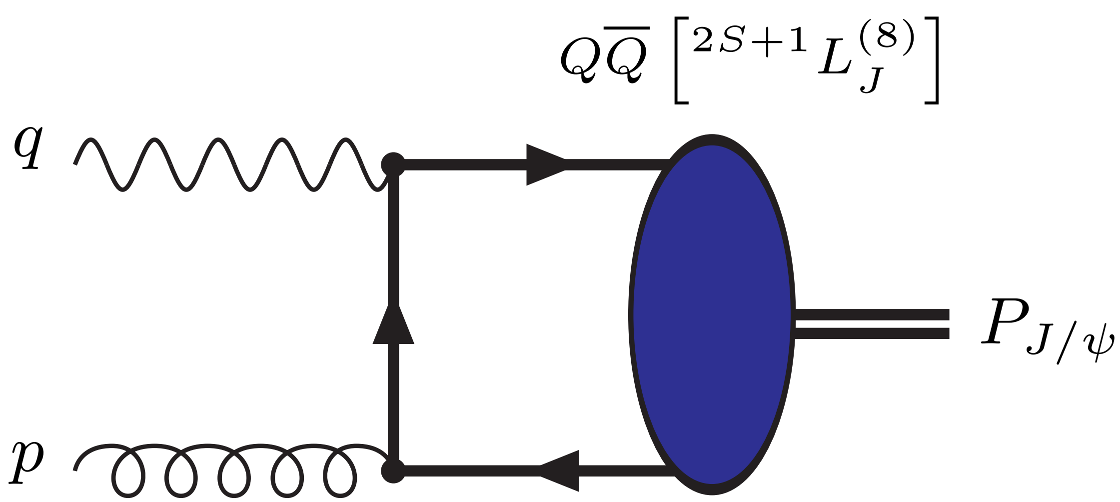

A cos($2\phi_T$) asymmetry probes $h_1^{\perp\,g}$

$$\langle\cos 2\phi_T\rangle = \frac{(1-y)\,\mathcal{B}_T^{\gamma^* g\to\mathcal{Q}}}{[1+(1-y)^2]\mathcal{A}_{U+L}^{\gamma^* g\to\mathcal{Q}} - y^2\mathcal{A}_L^{\gamma^* g\to\mathcal{Q}}} \times \frac{\boldsymbol{q}_T^2}{2M_p^2}\,\frac{h_1^{\perp\,g}(x,\boldsymbol{q}_T^2)}{f_1^g(x,\boldsymbol{q}_T^2)}.$$

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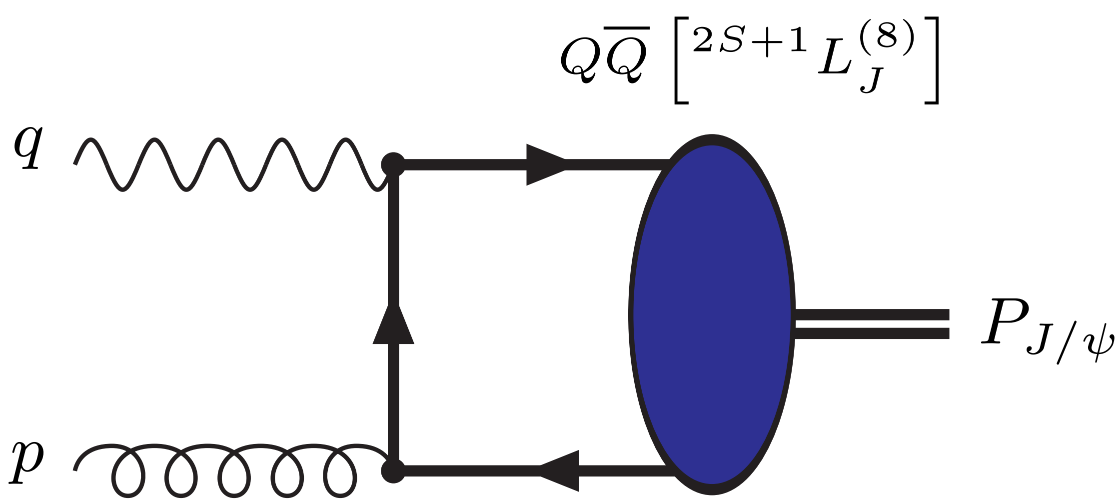

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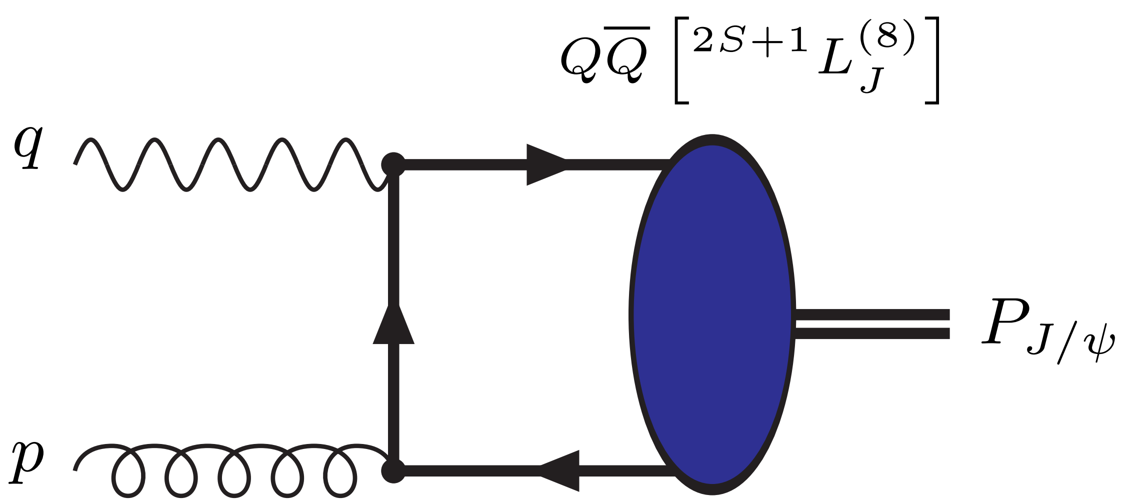
In LO NRQCD the prefactor of the asymmetry depends on two quite uncertain Color Octet (CO) Long Distance Matrix Elements (LDMEs)

One can cancel out the CO LDMEs by considering ratios with spin asymmetries

Quarkonium production in ep

$e\,p^{\uparrow} \to e'\,\mathcal{Q}\,X$ with $\mathcal{Q}$ either a J/ψ or a Υ meson

Godbole, Misra, Mukherjee, Rawoot, 2012/3; Godbole, Kaushik, Misra, Rawoot, 2015; Mukherjee, Rajesh, 2017; Rajesh, Kishore, Mukherjee, 2018; …



Using LO NRQCD the Sivers asymmetry is:

$$A^{\sin(\phi_S-\phi_T)} = \frac{|\boldsymbol{q}_T|}{M_p}\,\frac{f_{1T}^{\perp\,g}(x,\boldsymbol{q}_T^2)}{f_1^g(x,\boldsymbol{q}_T^2)}$$

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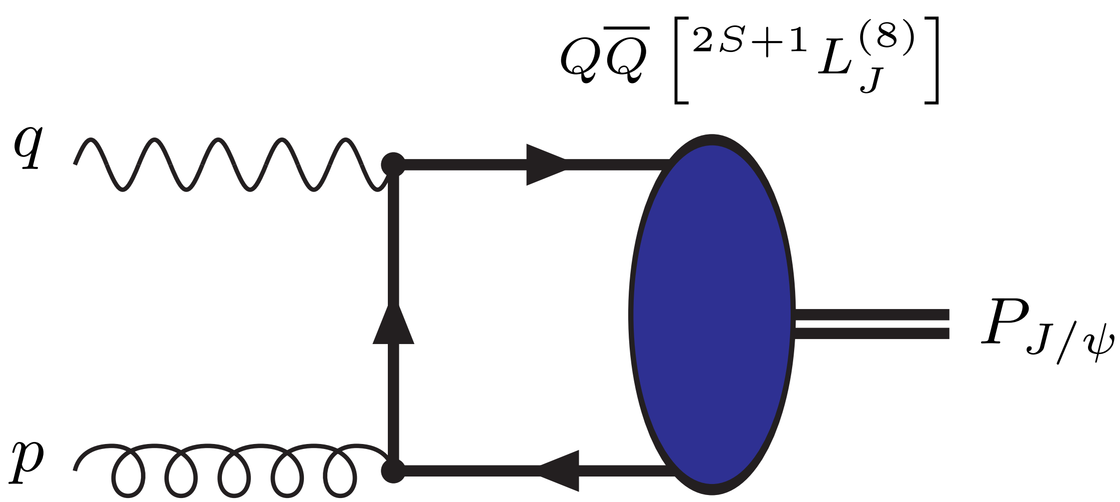

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CO NRQCD LDMEs cancel out in ratios of asymmetries at LO

Bacchetta, DB, Pisano, Taels, 2018

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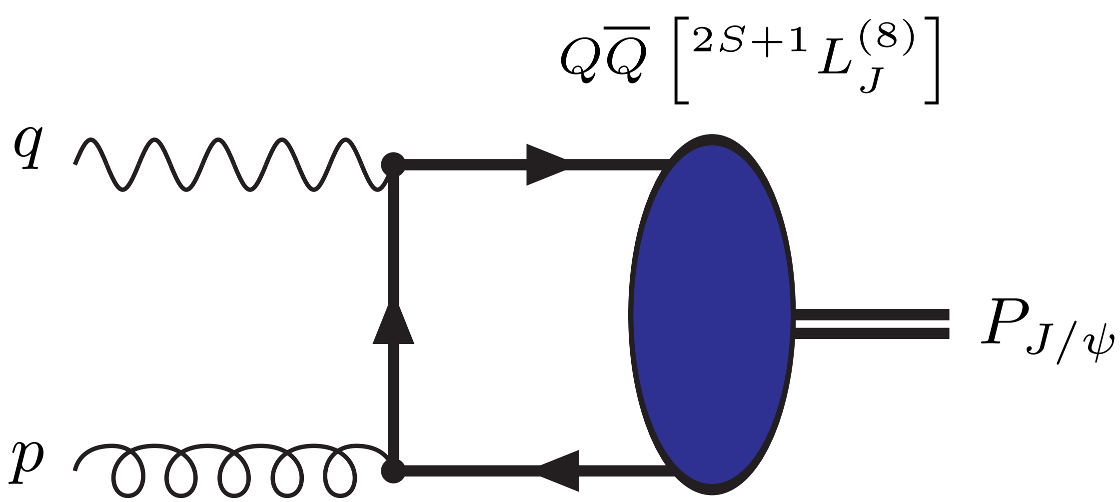

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Higher order corrections and shape functions will complicate this simple picture of course

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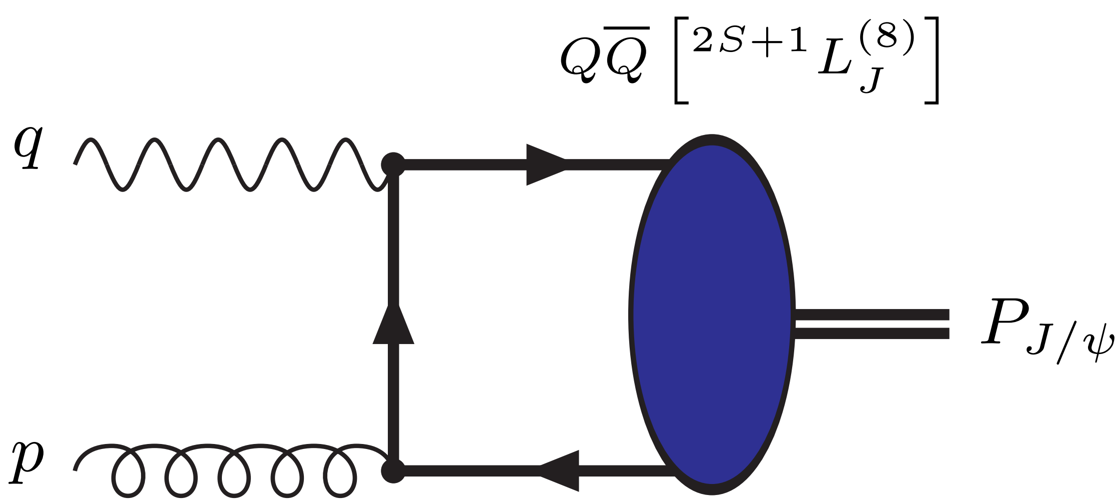

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Bacchetta, DB, Pisano, Taels, 2018

Higher order corrections and shape functions will complicate this simple picture of course

In addition, TMDs are process dependent: which functions are probed here?

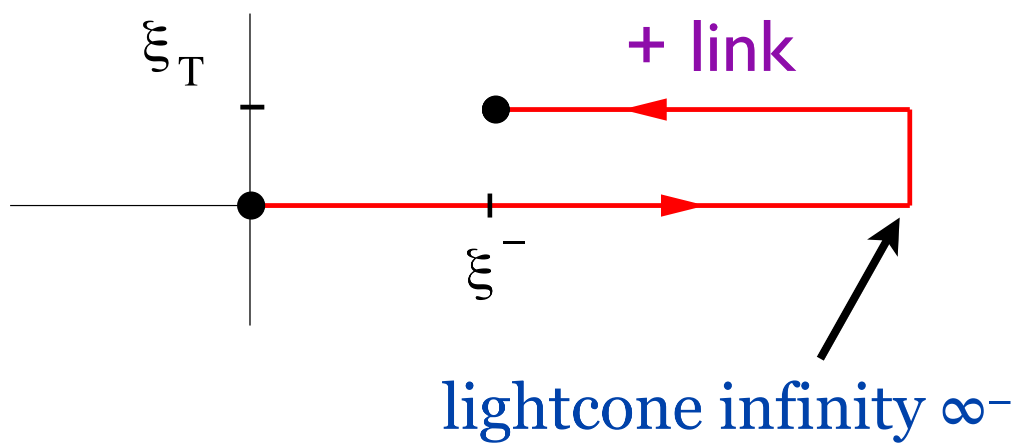
Process dependence of gluon TMDs

Operator structure of gluon TMDs

Gluon TMD correlators depend on two gauge links:

$$\Gamma_g^{\mu\nu[\mathcal{U},\mathcal{U}']}(x,k_T) \equiv \mathrm{F.T.}\langle P|\mathrm{Tr}_c\left[F^{+\nu}(0)\,\mathcal{U}_{[0,\xi]}\,F^{+\mu}(\xi)\,\mathcal{U}'_{[\xi,0]}\right]|P\rangle$$

$$\mathcal{U}_{\mathcal{C}}[0,\xi] = \mathcal{P}\exp\left(-ig\int_{\mathcal{C}[0,\xi]} ds_\mu\, A^\mu(s)\right)$$



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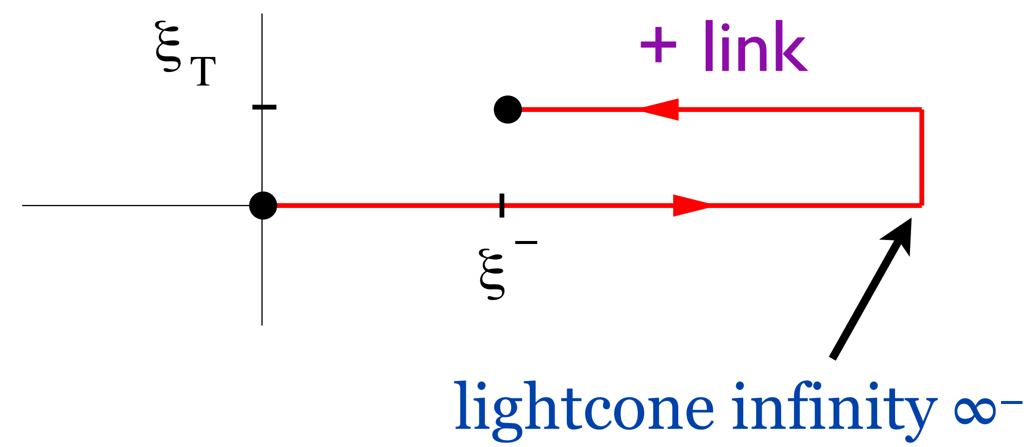
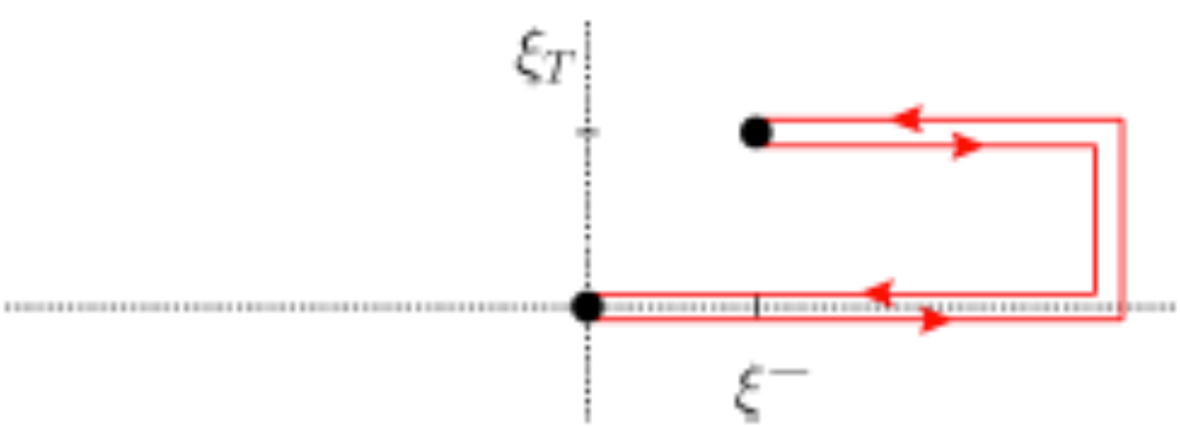
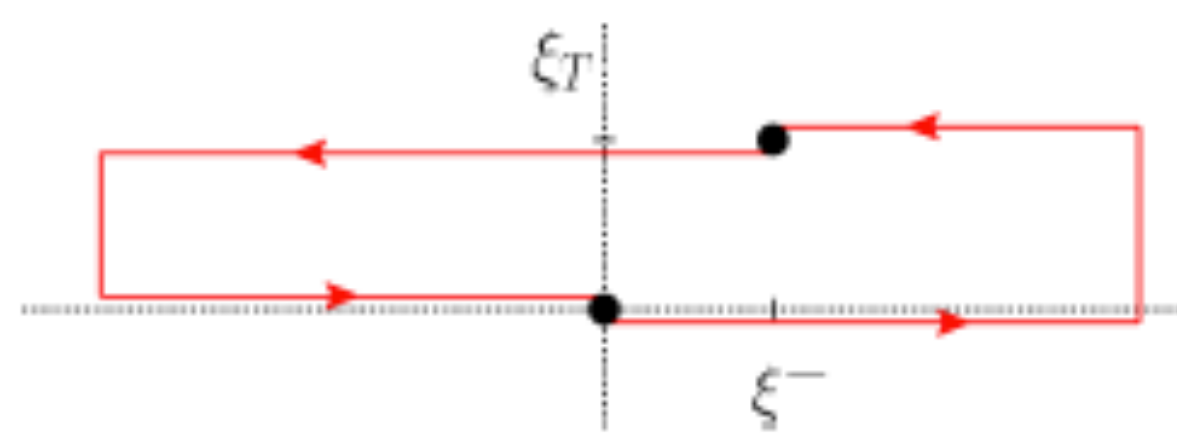


For most gluon TMDs there are only 2 link combinations of interest: [+,+] & [+,−]





[−,−] & [−,+] are related to them by parity and time reversal

WW vs DP

For unpolarized gluons there are two gluon TMDs of relevance

$$xG^{(1)}(x,k_\perp) = 2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \langle P|\mathrm{Tr}\left[F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[+]\dagger}F^{+i}(0)\mathcal{U}^{[+]}\right]|P\rangle \qquad [+,+]$$

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At small x the two correspond to the Weizsäcker-Williams (WW) and dipole (DP) distributions, which are generally different in magnitude and width:

$$xG^{(1)}(x,k_\perp) = -\frac{2}{\alpha_S}\int \frac{d^2v}{(2\pi)^2}\frac{d^2v'}{(2\pi)^2}\, e^{-ik_\perp\cdot(v-v')}\left\langle \mathrm{Tr}\left[\partial_i U(v)\right]U^\dagger(v')\left[\partial_i U(v')\right]U^\dagger(v)\right\rangle_{x_g} \qquad \mathrm{WW}$$

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Dominguez, Marquet, Xiao, Yuan, 2011

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Dominguez, Marquet, Xiao, Yuan, 2011

Explains Kharzeev, Kovchegov & Tuchin's "tale of two gluon distributions" (2003)

WW vs DP

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Dominguez, Marquet, Xiao, Yuan, 2011

Explains Kharzeev, Kovchegov & Tuchin's "tale of two gluon distributions" (2003)

This is for the isotropic ($g_T^{\mu\nu}$) term but holds also for the anisotropic term

Wilson loop correlator

The *leading twist* [+,−] correlator becomes a Wilson loop correlator at small x:

$$\Gamma^{[+,-]\,ij}(x,\boldsymbol{k}_T) \;\xrightarrow{x\to 0}\; \frac{k_T^i k_T^j}{2\pi L}\,\Gamma_0^{[\Box]}(\boldsymbol{k}_T)$$

a single Wilson loop matrix element

DB, Cotogno, van Daal, Mulders, Signori & Ya-Jin Zhou, 2016

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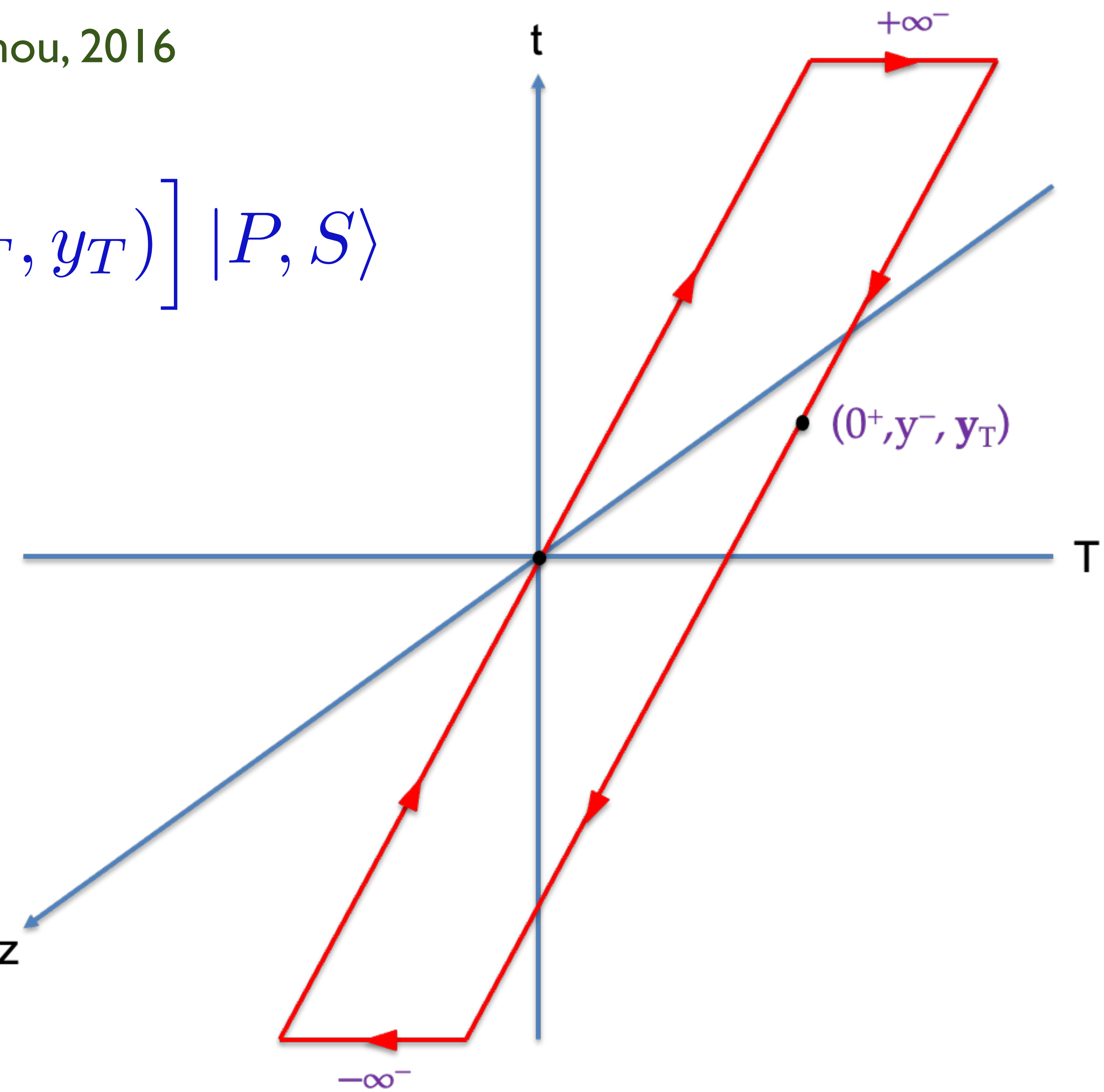
$$U^{[\square]} = U^{[+]}_{[0,y]}U^{[-]}_{[y,0]}$$



Measures flux through the loop

Large k_T corresponds to narrow loop

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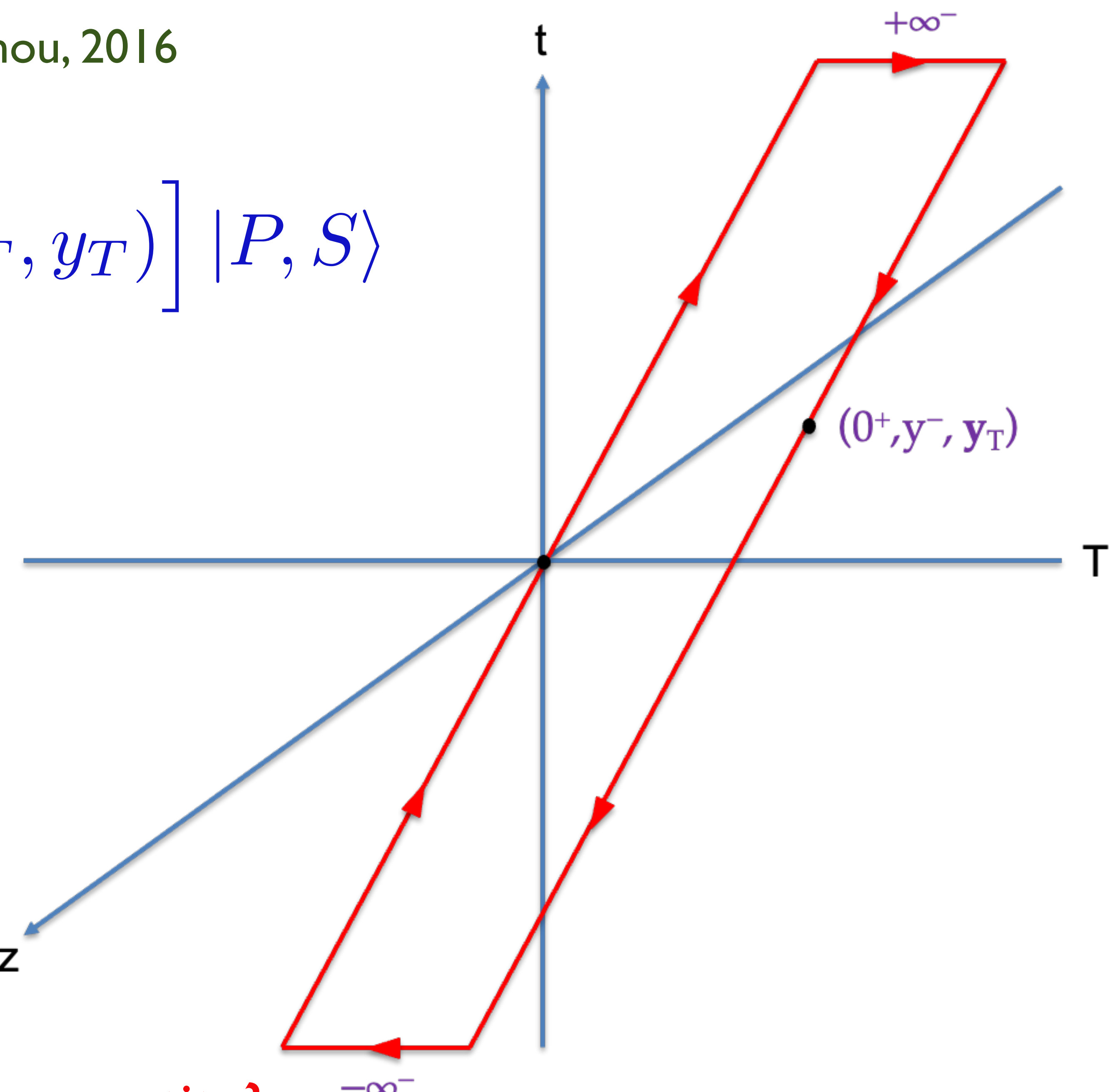
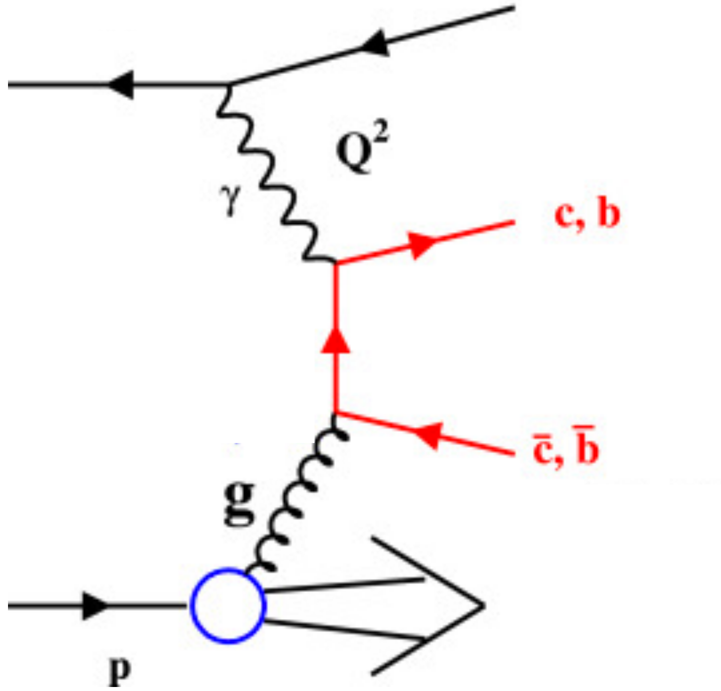


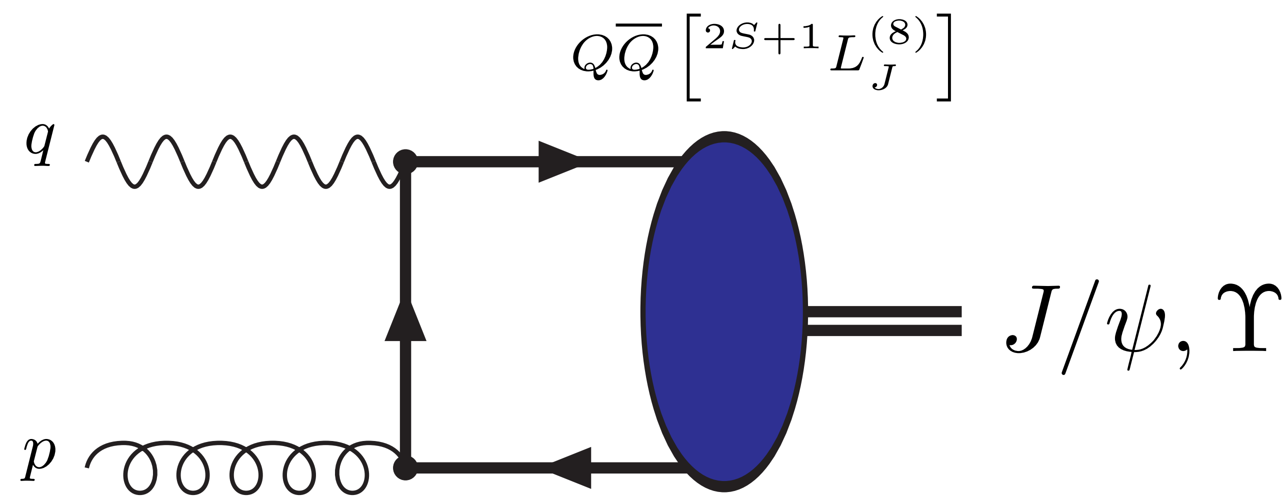
Measures flux through the loop

Large k_T corresponds to narrow loop

What are the processes that probe this quantity?

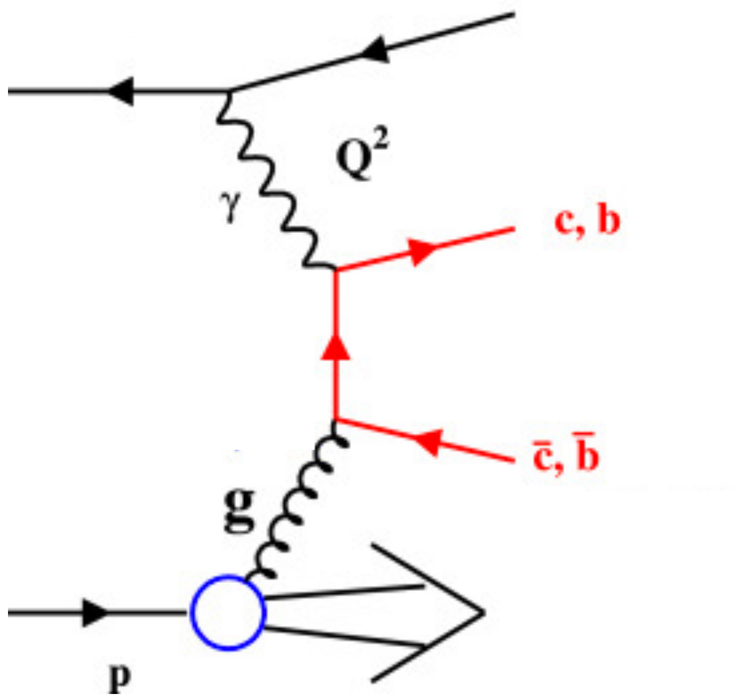
Probing gluon TMDs using heavy quarks

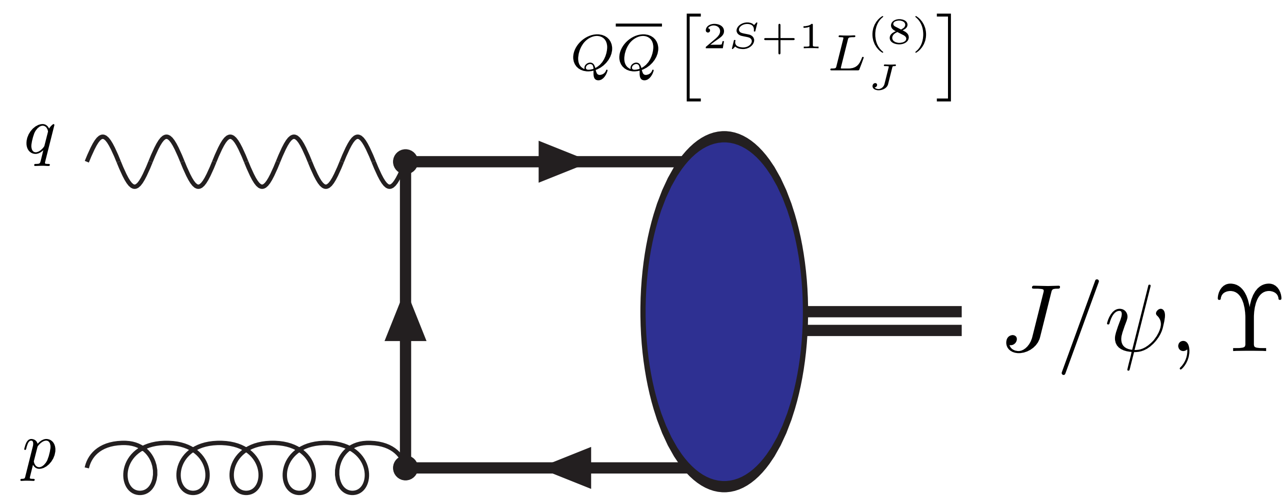




Open heavy quark pair production and single quarkonium production: $[+, +]$

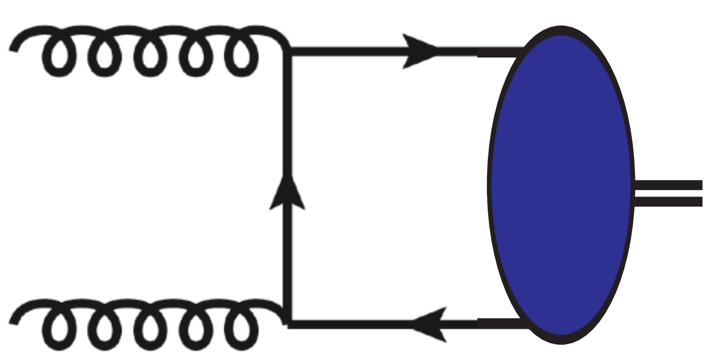
Probing gluon TMDs using heavy quarks





Open heavy quark pair production and single quarkonium production: $[+,+]$

In pp collisions one probes the [−,−] correlator through gluon-gluon fusion

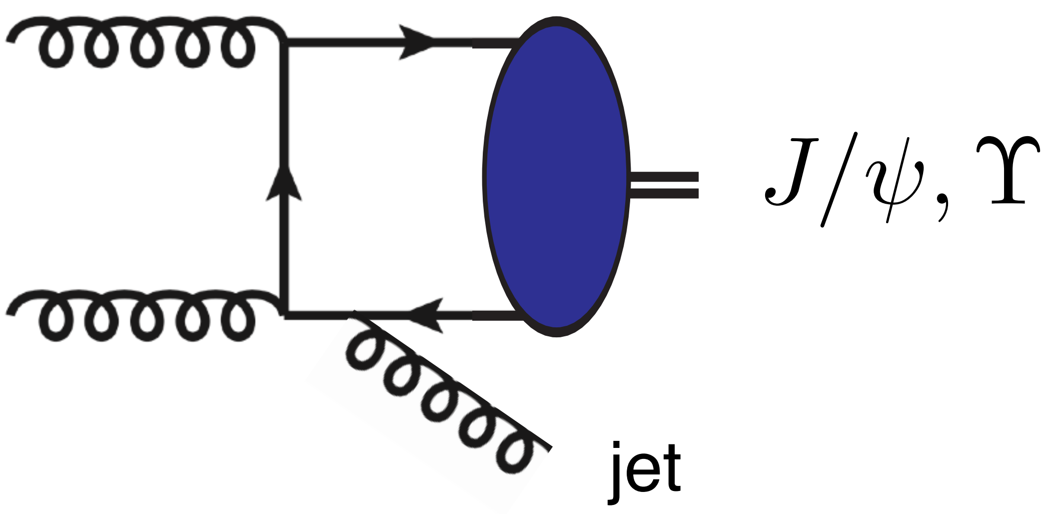


$\eta_c, \eta_b, \chi_{c0}, \chi_{b0}$ $\quad [-,-]$

$J/\psi, \Upsilon$

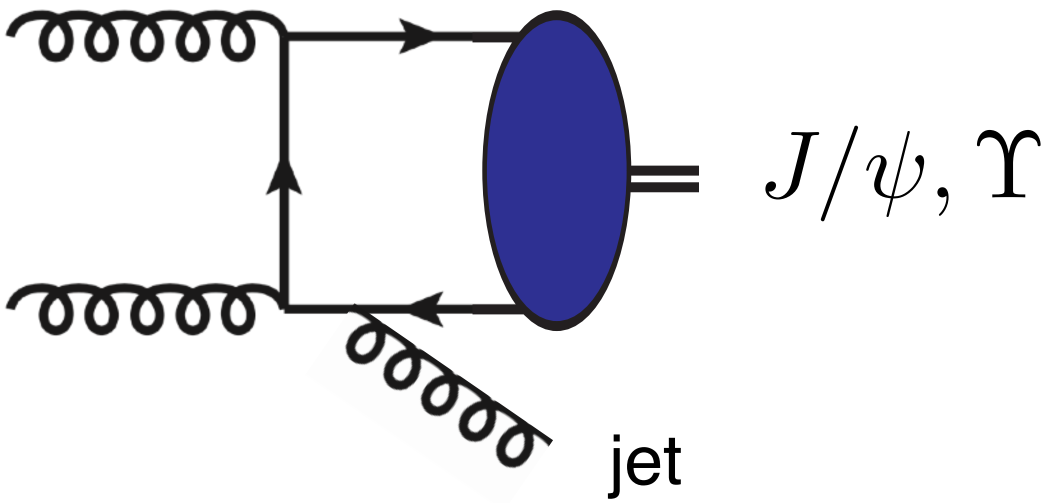
CS not allowed by Landau−Yang theorem
CO complicated link structure & possibly factorization breaking

Probing gluon TMDs using heavy quarks



CS allowed, nevertheless complicated link structure & possibly factorization breaking

Probing gluon TMDs using heavy quarks



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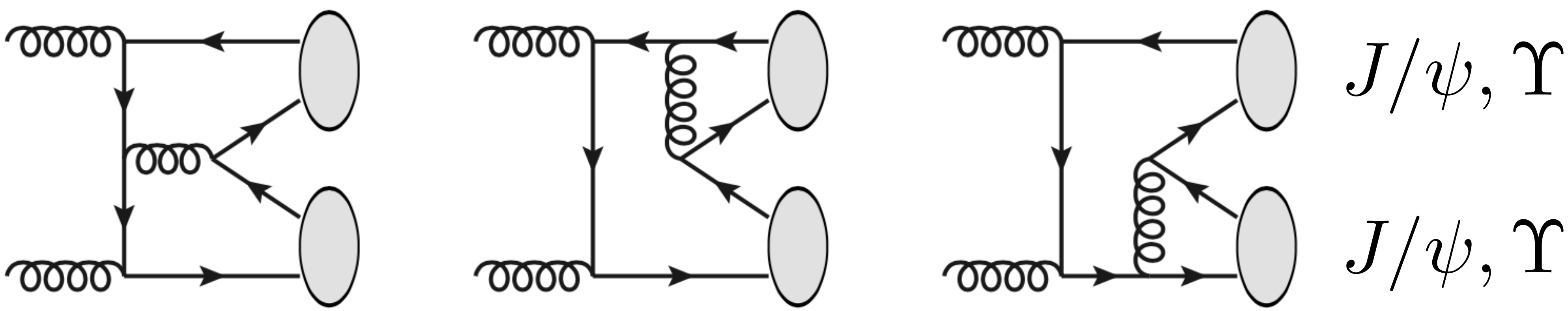
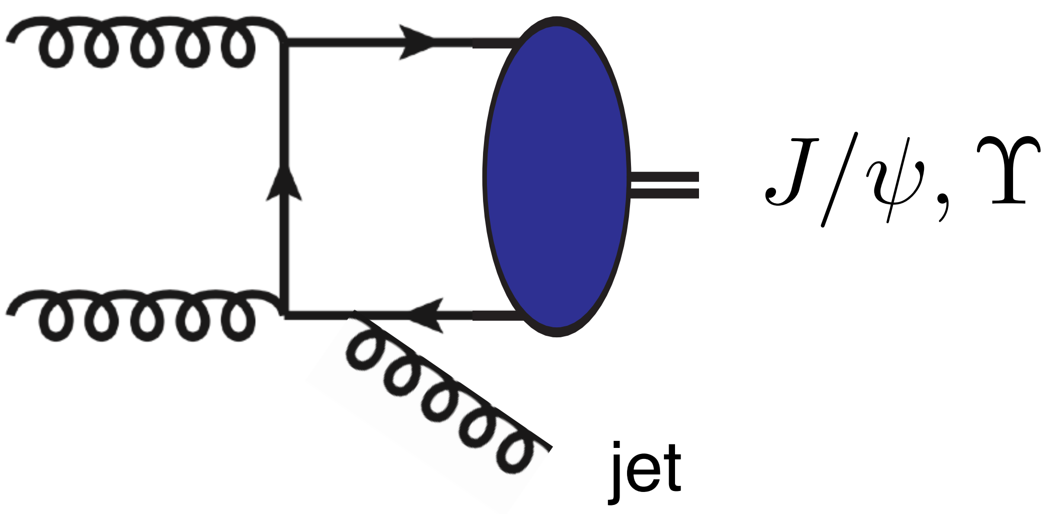


CS–CS » CO–CO

$[-,-]$

[Scarpa et al., 2020]

Probing gluon TMDs using heavy quarks



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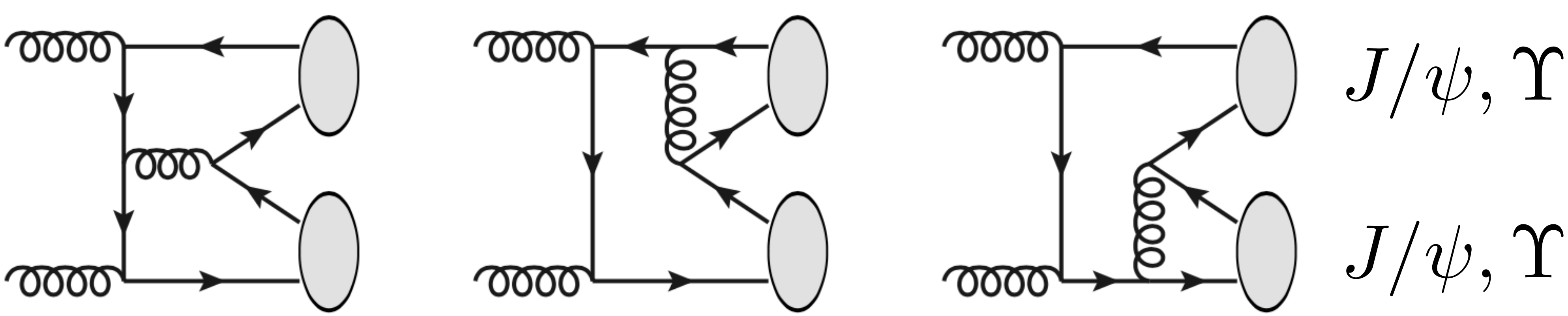

CS–CS » CO–CO

$[-,-]$

[Scarpa et al., 2020]

$pp \to Q\bar{Q}\, X$

Never CS

TMD factorization is a concern here

[Rogers, Mulders, 2010; Catani, Grazzini, Torre, 2015]

Processes that probe gluon TMDs

$f_1^{g\,[+,+]}$	$pp \to \gamma\, J/\psi\, X$
	$pp \to \gamma\, \Upsilon\, X$
$f_1^{g\,[+,-]}$	$pp \to \gamma\, \mathrm{jet}\, X$
$h_1^{\perp\, g\,[+,+]}$	$e\,p \to e'\, Q\, \overline{Q}\, X$
	$e\,p \to e'\, \mathrm{jet}\, \mathrm{jet}\, X$
	$pp \to \eta_{c,b}\, X$
	$pp \to H\, X$
$h_1^{\perp\, g\,[+,-]}$	$pp \to \gamma^*\, \mathrm{jet}\, X$
$f_{1T}^{\perp\, g\,[+,+]}$	$e\,p^\uparrow \to e'\, Q\, \overline{Q}\, X$
	$e\,p^\uparrow \to e'\, \mathrm{jet}\, \mathrm{jet}\, X$
$f_{1T}^{\perp\, g\,[-,-]}$	$p^\uparrow p \to \gamma\, \gamma\, X$
$f_{1T}^{\perp\, g\,[+,-]}$	$p^\uparrow A \to \gamma^{(*)}\, \mathrm{jet}\, X$
	$p^\uparrow\, A \to h\, X\ \ (x_F < 0)$

process dependence of the TMDs $\Gamma^{[U,U']}$:
[+,+] = ± [−,−]
[+,−] = ± [−,+]
with + if T-even
and − if T-odd

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with + if T-even

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Small x

-

Unpolarized case

Wilson loop correlator

The *leading twist* [+,−] correlator becomes a Wilson loop correlator in the small-x limit:

$$\Gamma^{[+,-]\,ij}(x,\boldsymbol{k}_T) \xrightarrow{x\to 0} \frac{k_T^i k_T^j}{2\pi L}\Gamma_0^{[\Box]}(\boldsymbol{k}_T)$$

a single Wilson loop matrix element

DB, Cotogno, van Daal, Mulders, Signori & Ya-Jin Zhou, 2016

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As a consequence, the DP $h_1^{\perp g}$ becomes maximal when x → 0

$$\Gamma_U^{ij}(x,\boldsymbol{k}_T) = \frac{x}{2}\left[-g_T^{ij}\,f_1(x,\boldsymbol{k}_T^2) + \frac{k_T^{ij}}{M^2}\,h_1^\perp(x,\boldsymbol{k}_T^2)\right] \xrightarrow{x\to 0} \frac{k_T^i k_T^j}{2M^2}\,e(\boldsymbol{k}_T^2)$$

$$\lim_{x\to 0} x f_1(x,\boldsymbol{k}_T^2) = \frac{\boldsymbol{k}_T^2}{2M^2}\lim_{x\to 0} x h_1^\perp(x,\boldsymbol{k}_T^2) = \frac{\boldsymbol{k}_T^2}{2M^2}\,e(\boldsymbol{k}_T^2)$$

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DB, Cotogno, van Daal, Mulders, Signori & Ya-Jin Zhou, 2016

In line with MV model calculation:

$$x h_{1,DP}^{\perp g}(x,k_\perp) = 2x f_{1,DP}^{g}(x,k_\perp)$$

Metz, Zhou, 2011

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CGC gluons are maximally linear polarized (amount probed depends on process)

Linear gluon polarization at small x

The DP $h_1^{\perp g}$ becomes maximal when $x \to 0$, but is this stable under evolution?

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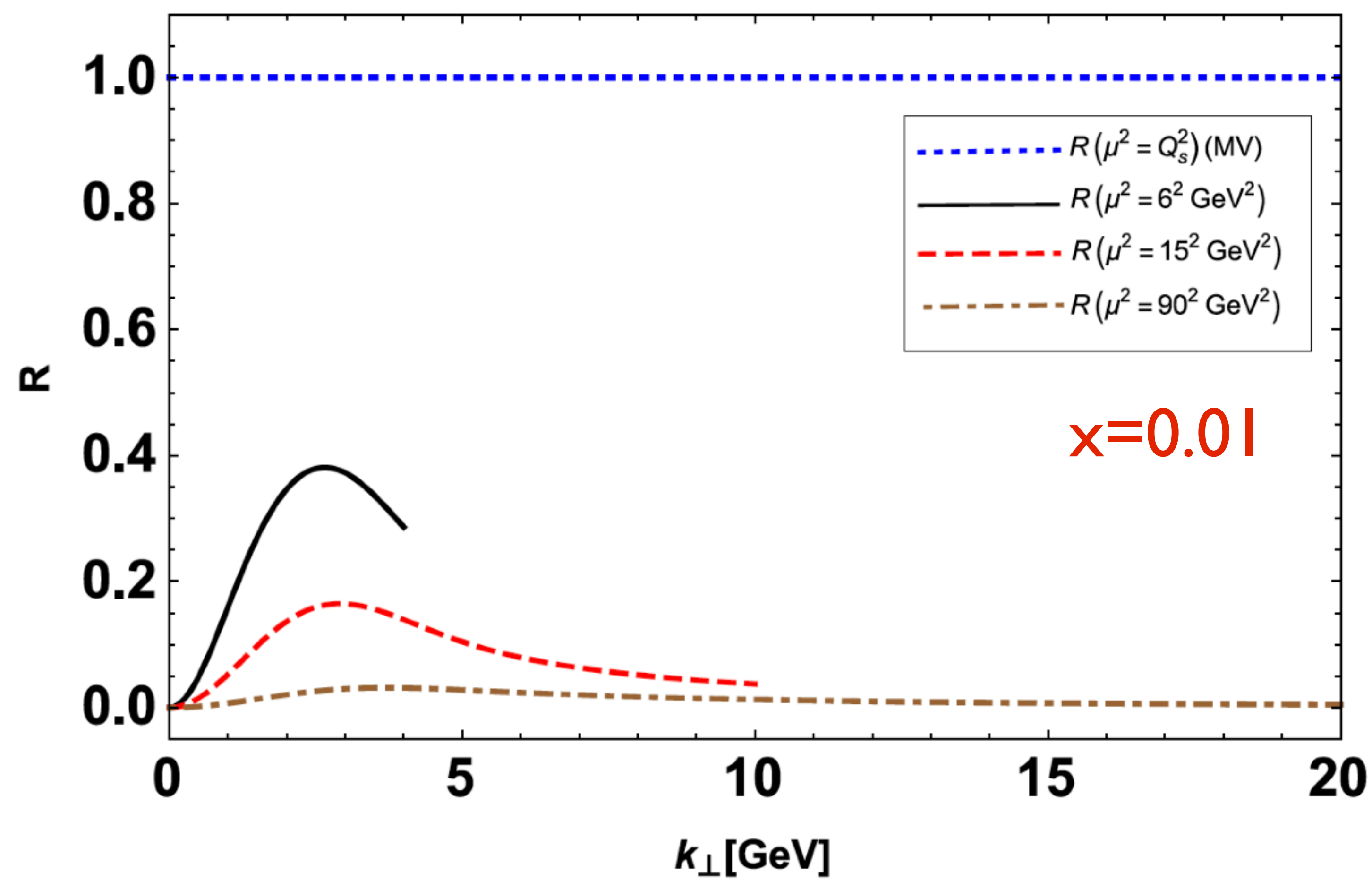
Under TMD (scale) evolution however there is Sudakov suppression:

$$R = h_1^{\perp g} / f_1^g$$

MV model

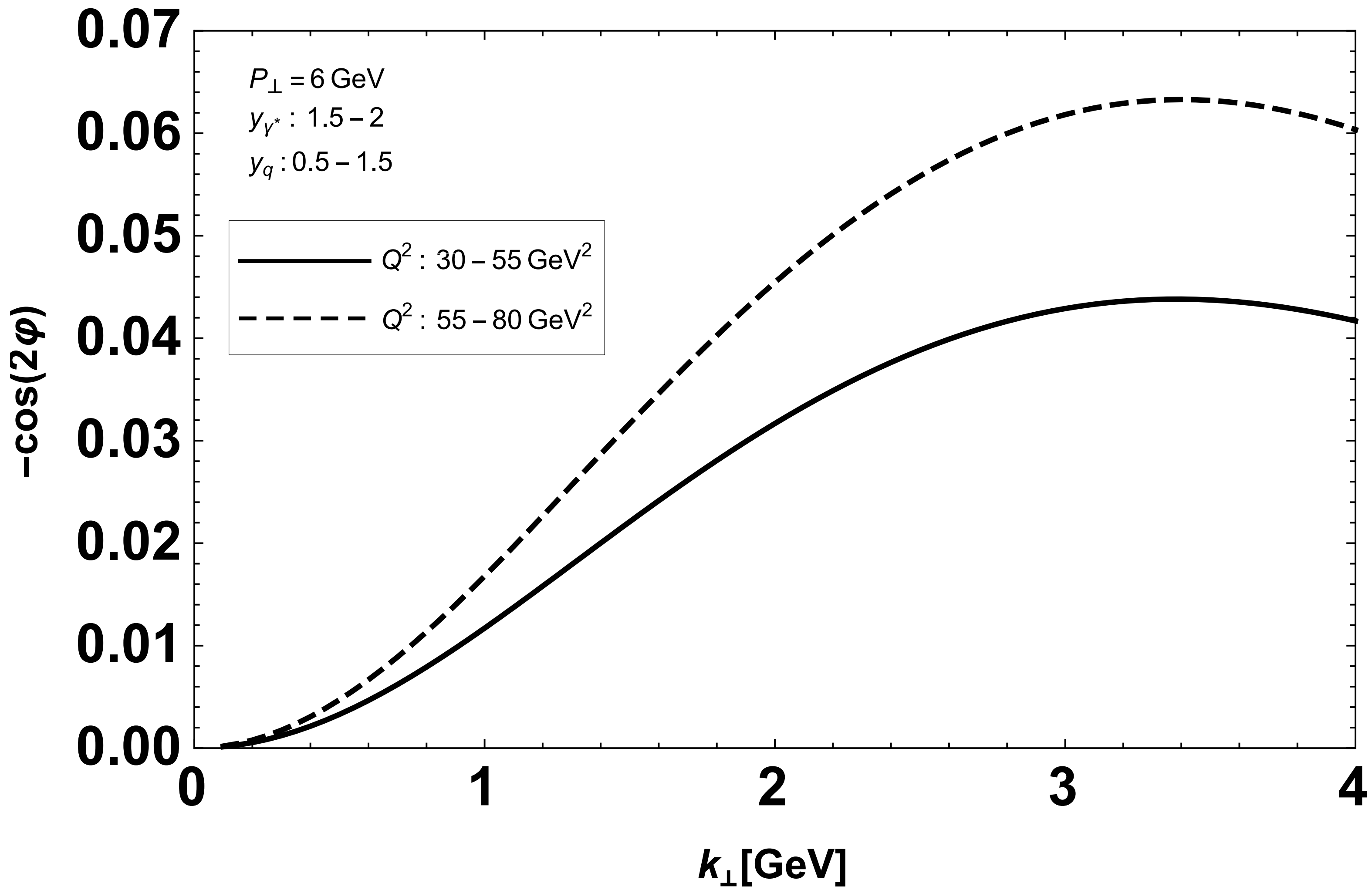


Still it may be accessible

DB, Mulders, Jian Zhou & Ya-Jin Zhou, 2017

Sudakov suppression of linear gluon polarization

$pA \to \gamma^*$ *jet X* offers a good opportunity to study the DP linear gluon polarization



Despite the DP linear gluon polarization becoming maximal at small x, there is amplitude and Sudakov suppression of the cos(2ϕ) asymmetry in $pA \to \gamma^*$ *jet X*:

~5% asymmetry at RHIC (p-Au)

DB, Mulders, Jian Zhou & Ya-Jin Zhou, 2017

Small x
-
Polarized case

Dipole gluon Sivers effect

The d-type gluon Sivers function $f_{1T}^{\perp\, g\,[+,-]}$ at small x is part of:

$$\Gamma_{(d)}^{(T-\mathrm{odd})} \equiv \left(\Gamma^{[+,-]} - \Gamma^{[-,+]}\right) \propto \mathrm{F.T.}\, \langle P, S_T | \mathrm{Tr} \left[U^{[\Box]}(0_T, y_T) - U^{[\Box]\dagger}(0_T, y_T) \right] | P, S_T \rangle$$

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This corresponds with the *spin-dependent odderon* J. Zhou, 2013

$$\Gamma^{\mu\nu}_{\text{T-odd}}(x, k_T; S_T) = \frac{k_T^\mu k_T^\nu N_c}{2\pi^2 \alpha_s x} \frac{\epsilon_T^{\alpha\beta} S_{T\alpha} k_{T\beta}}{M} O_{1T}^{\perp}(x, k_T^2)$$

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Similar relations hold for spin-1 hadrons as well

DB, Cotogno, van Daal, Mulders, Signori & Ya-Jin Zhou, 2016

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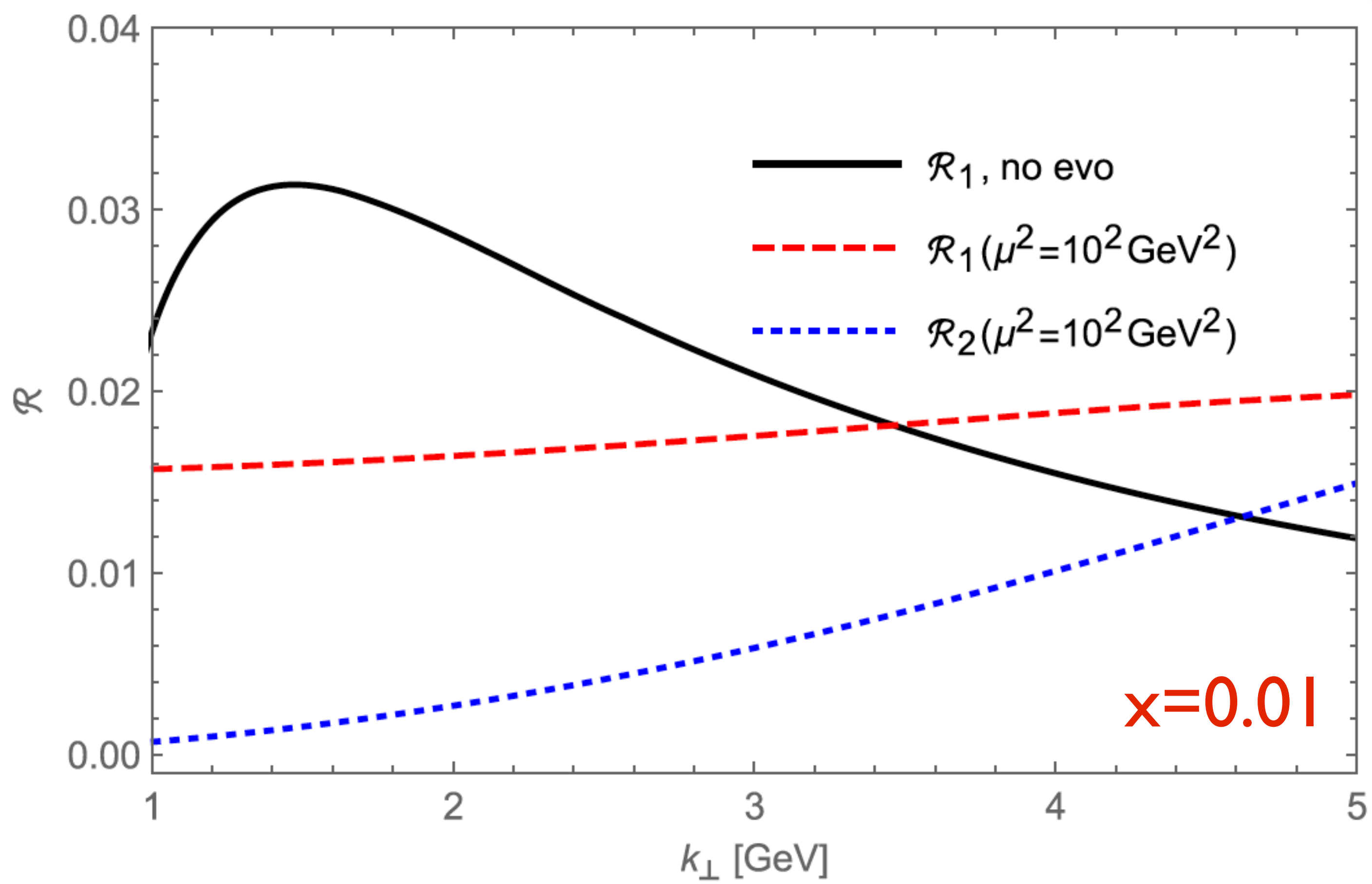
Presumably preserved under x evolution, but ratio to f_1 rapidly drops

Kovchegov, Szymanowski, Wallon, 2004; Hatta, Iancu, Itakura, McLerran, 2005; …

T-odd gluon TMDs at small x - scale evolution



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DB, Hagiwara, Jian Zhou & Ya-Jin Zhou, 2022

Scale evolution does not preserve the small-x equality of T-odd dipole gluon TMDs

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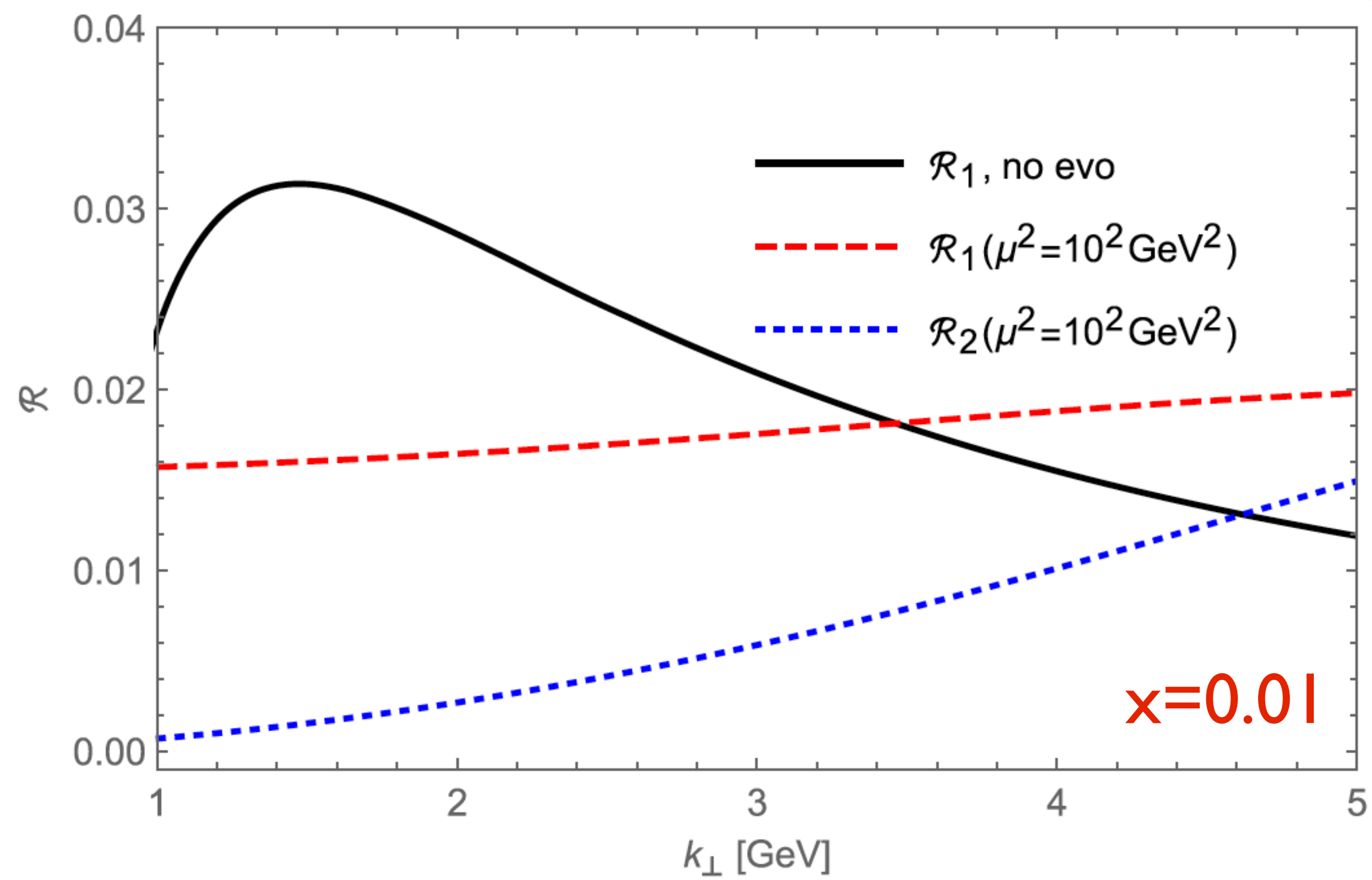

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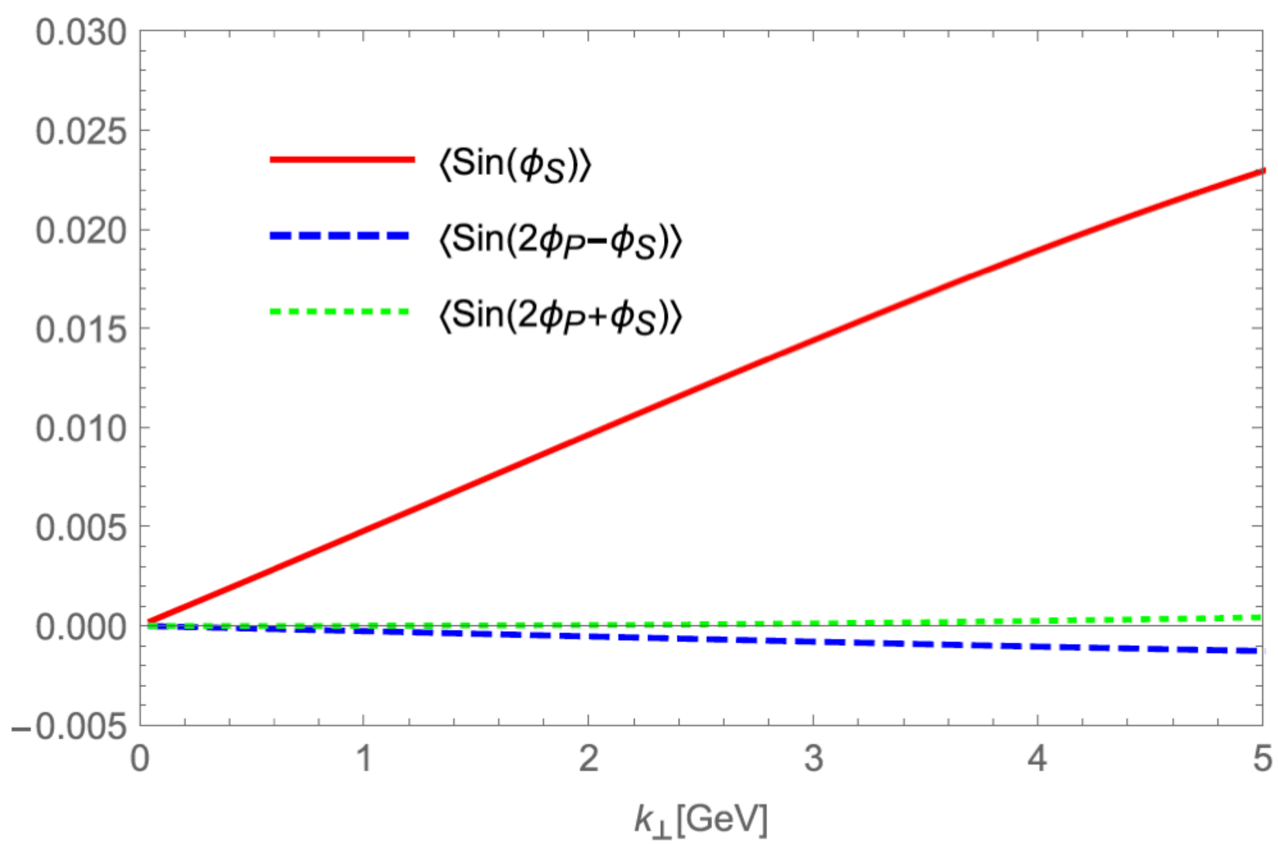
Scale evolution does not preserve the small-x equality of T-odd dipole gluon TMDs

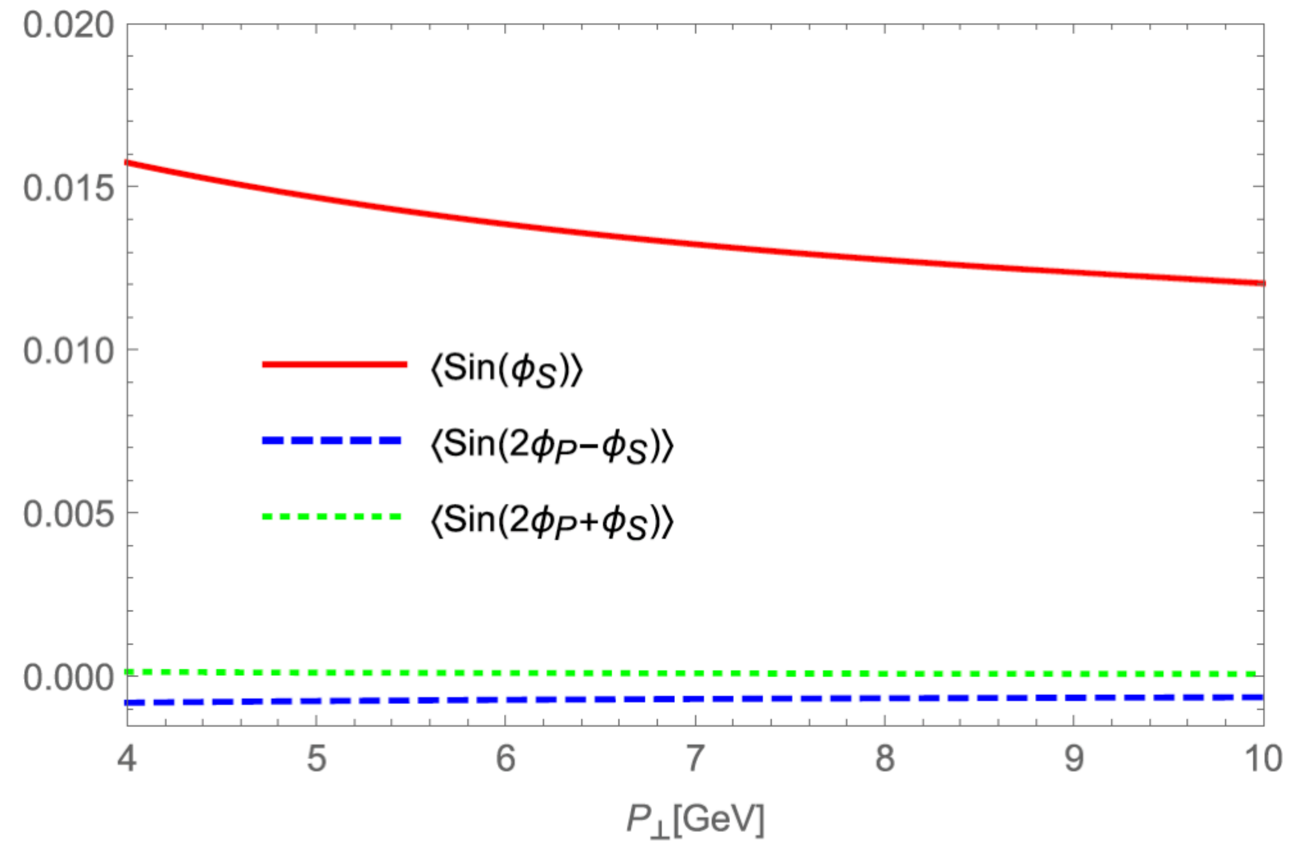
Not the MV model since applied to polarized protons at not too small x
Rather a diquark model that is used as a source for the gluon distributions

Szymanowski and J. Zhou, 2016

T-odd DP gluon asymmetries at small x

$$\frac{d\sigma^{p^\uparrow p\to\gamma^* qX}}{dP.S} = \sum_q x_q f_1^q(x_q)\Big\{H_{UU}\Big[xf_1^g(x,\boldsymbol{k}_\perp^2) + \sin(\phi_S)\frac{|\boldsymbol{k}_\perp|}{M}xf_{1T}^{\perp g}(x,\boldsymbol{k}_\perp^2)\Big] + H_{UT}\cos(2\phi_P)\frac{|\boldsymbol{k}_\perp|^2}{2M^2}xh_1^{\perp g}(x,\boldsymbol{k}_\perp^2)$$
$$+\frac{1}{2}H_{UT}\sin(2\phi_P-\phi_S)\frac{|\boldsymbol{k}_\perp|}{M}xh_1^g(x,\boldsymbol{k}_\perp^2) + \frac{1}{2}H_{UT}\sin(2\phi_P+\phi_S)\frac{|\boldsymbol{k}_\perp|}{M}\frac{|\boldsymbol{k}_\perp|^2}{2M^2}xh_{1T}^{\perp g}(x,\boldsymbol{k}_\perp^2)\Big\},$$





Gluon Sivers asymmetry largest, but small (% level) in this model

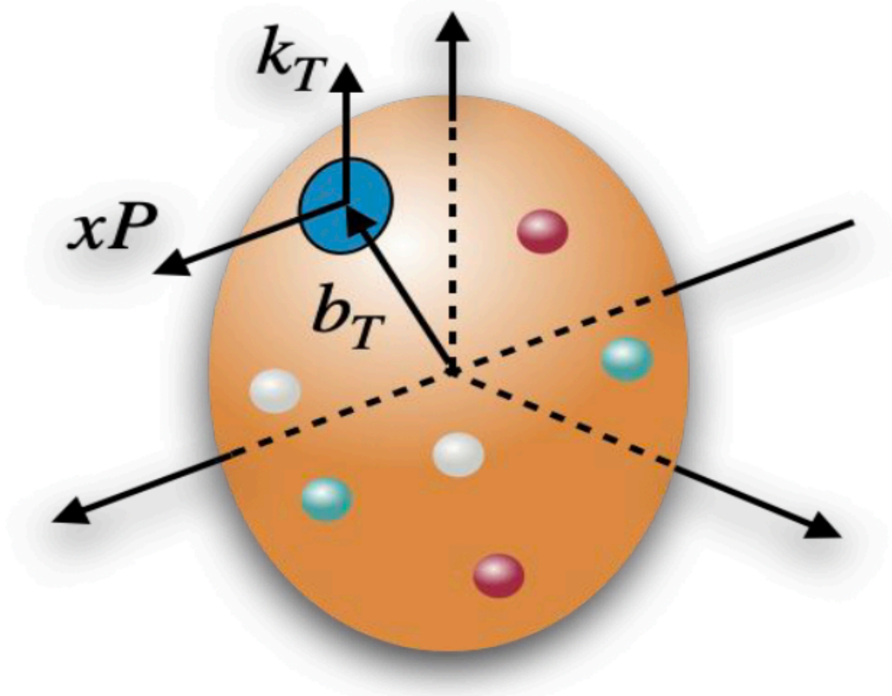
DB, Hagiwara, Jian Zhou & Ya-Jin Zhou, 2022

Gluon GTMDs

3D momentum and spatial distributions

TMDs - 3D momentum structure (x & k_T)
GPDs - 3D spatial structure (ξ & t or z & b_T)

GTMDs - combined 5D (or 6D) structure



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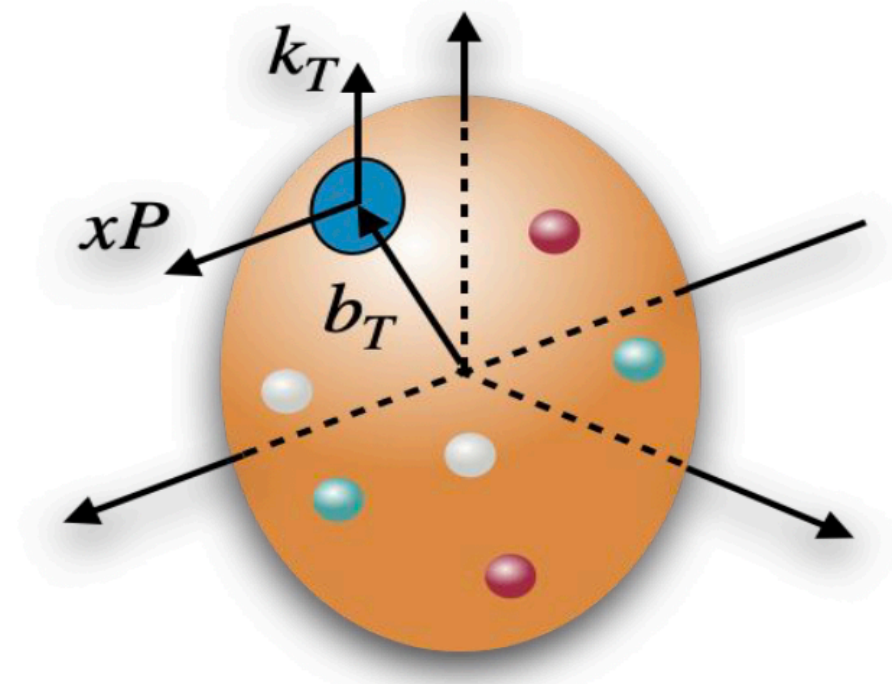
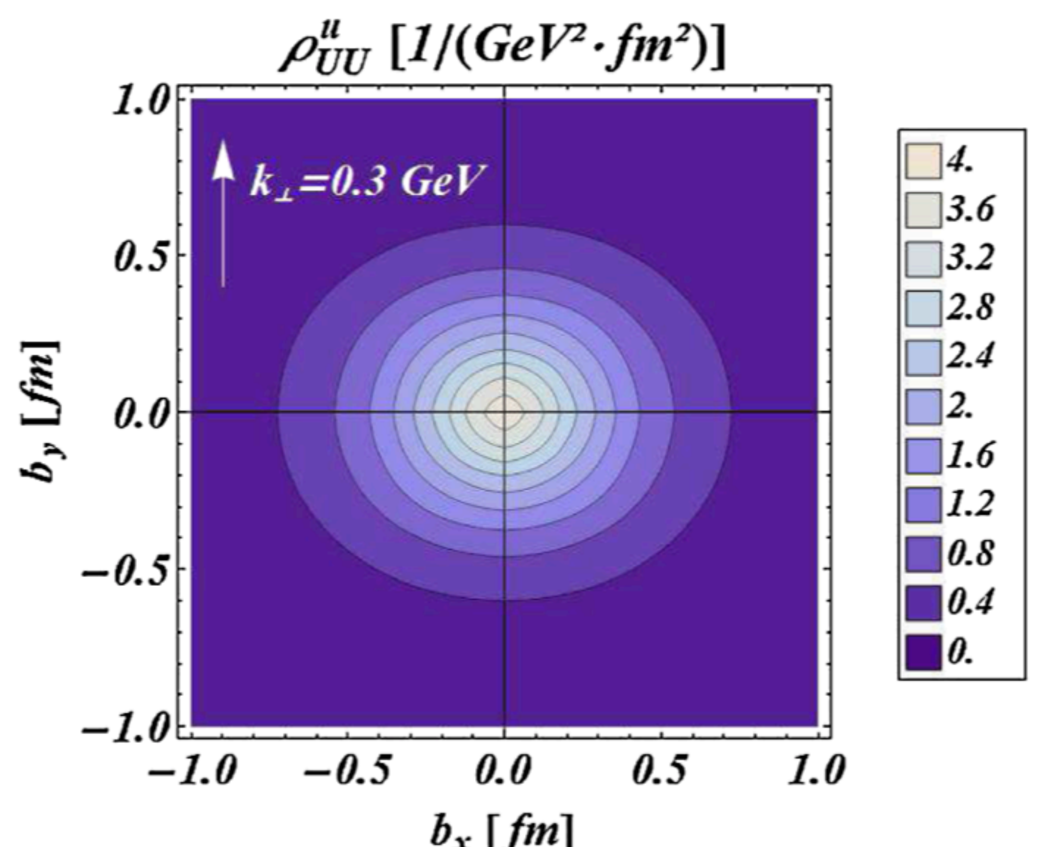


TMDs - 3D momentum structure (x & k_T)
GPDs - 3D spatial structure (ξ & t or z & b_T)

GTMDs - combined 5D (or 6D) structure

Teaches us about orbital angular momentum

Lorcé, Pasquini, 2011; Hatta, 2011; …



GTMDs - 5D parton distributions

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GTMDs can be seen as:

- off-forward TMDs
- transverse momentum dependent GPDs
- Fourier transforms of Wigner distributions

$$G(x, \boldsymbol{k}_T, \boldsymbol{\Delta}_T) \stackrel{FT}{\longleftrightarrow} W(x, \boldsymbol{k}_T, \boldsymbol{b}_T)$$

Meißner, Metz, Schlegel, 2009 Ji, 2003; Belitsky, Ji & Yuan, 2004

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GTMDs combine all properties of TMDs and GPDs, such as the gauge link and process dependence & translation non-invariance

Gluon GTMDs for unpolarized protons

For unpolarized protons there are 2 (real valued) gluon TMDs:

$$\Gamma_U^{ij}(x, \boldsymbol{k}_T) \;=\; \frac{x}{2}\left[-\,g_T^{ij}\,f_1(x, \boldsymbol{k}_T^2) + \frac{k_T^{ij}}{M^2}\,h_1^{\perp}(x, \boldsymbol{k}_T^2)\right]$$

$$a_T^{ij} \equiv a_T^i a_T^j - \frac{1}{2}a_T^2 \delta_T^{ij}$$

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For GTMDs one has one more vector so more anisotropic terms can arise

For unpolarized protons there are 4 (complex valued) gluon GTMDs

$$G^{[U,U']\,ij}(x,\boldsymbol{k}_T,\boldsymbol{\Delta}_T) = x\left(\delta_T^{ij}\,\mathcal{F}_1+\frac{k_T^{ij}}{M^2}\,\mathcal{F}_2+\frac{\Delta_T^{ij}}{M^2}\,\mathcal{F}_3+\frac{k_T^{[i}\Delta_T^{j]}}{M^2}\,\mathcal{F}_4\right)$$

DB, van Daal, Mulders, Petreska, 2018

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Also for GTMDs gauge links *[U,U']* matter → WW and DP versions at small x

Dipole gluon GTMD

In the x→0 the dipole gluon GTMD becomes a correlator of a single Wilson loop:

$$G^{[+,-]\,ij}(\boldsymbol{k},\boldsymbol{\Delta}) = \frac{2N_c}{\alpha_s}\left[\frac{1}{2}\left(\boldsymbol{k}^2 - \frac{\boldsymbol{\Delta}^2}{4}\right)\delta_T^{ij} + k_T^{ij} - \frac{\Delta_T^{ij}}{4} - \frac{k_T^{[i}\Delta_T^{j]}}{2}\right] G^{[\square]}(\boldsymbol{k},\boldsymbol{\Delta})$$

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All gluon polarization states (linear & circular) become related:

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Real part of $G^{[\square]}(k,\Delta)$ only depends on k^2, Δ^2 and $(k\cdot\Delta)^2$

It means that also the δ_T^{ij} term can be anisotropic now

Elliptic Wigner distributions

$$xW(x,\boldsymbol{b},\boldsymbol{k}) = x\mathcal{W}_0(x,\boldsymbol{b}^2,\boldsymbol{k}^2) + 2\cos(\phi_b-\phi_k)\,x\mathcal{W}_1(x,\boldsymbol{b}^2,\boldsymbol{k}^2)$$
$$+\,2\cos 2(\phi_b-\phi_k)\,x\mathcal{W}_2(x,\boldsymbol{b}^2,\boldsymbol{k}^2) + \,\ldots$$

The cos $2(\phi_b-\phi_k)$ part is called the elliptic Wigner distribution

Hatta, Xiao, Yuan, 2016; J. Zhou, 2016; Mäntysaari, Mueller, Schenke, 2019; Salazar, Schenke, 2019

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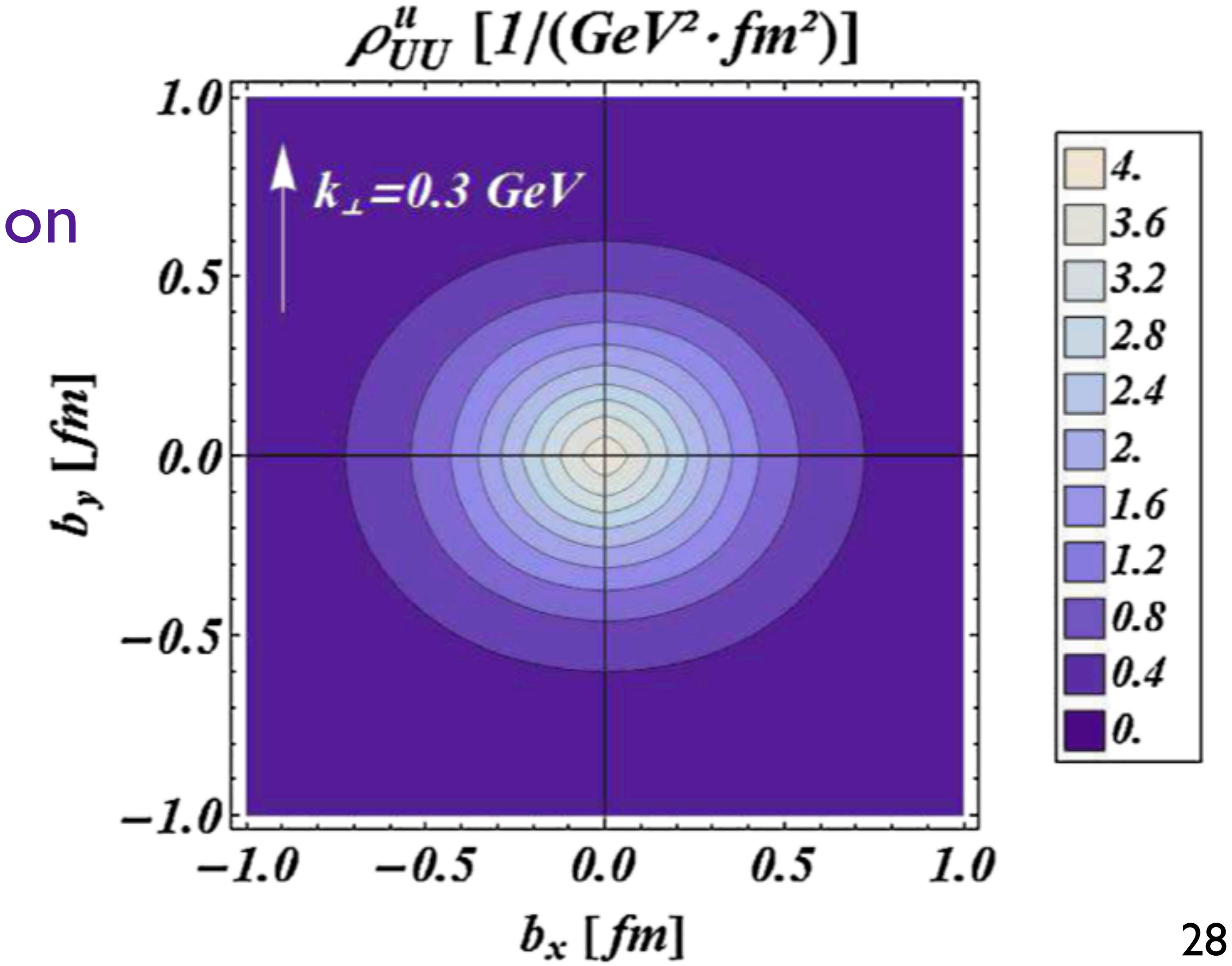
A nonzero elliptic quark Wigner distribution in the lightcone constituent quark model:

Lorcé, Pasquini, 2011



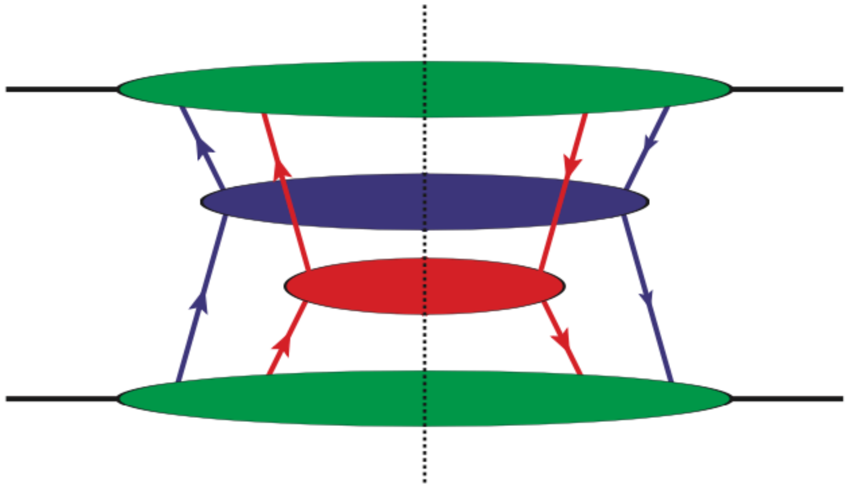
Due to quark orbital angular momentum

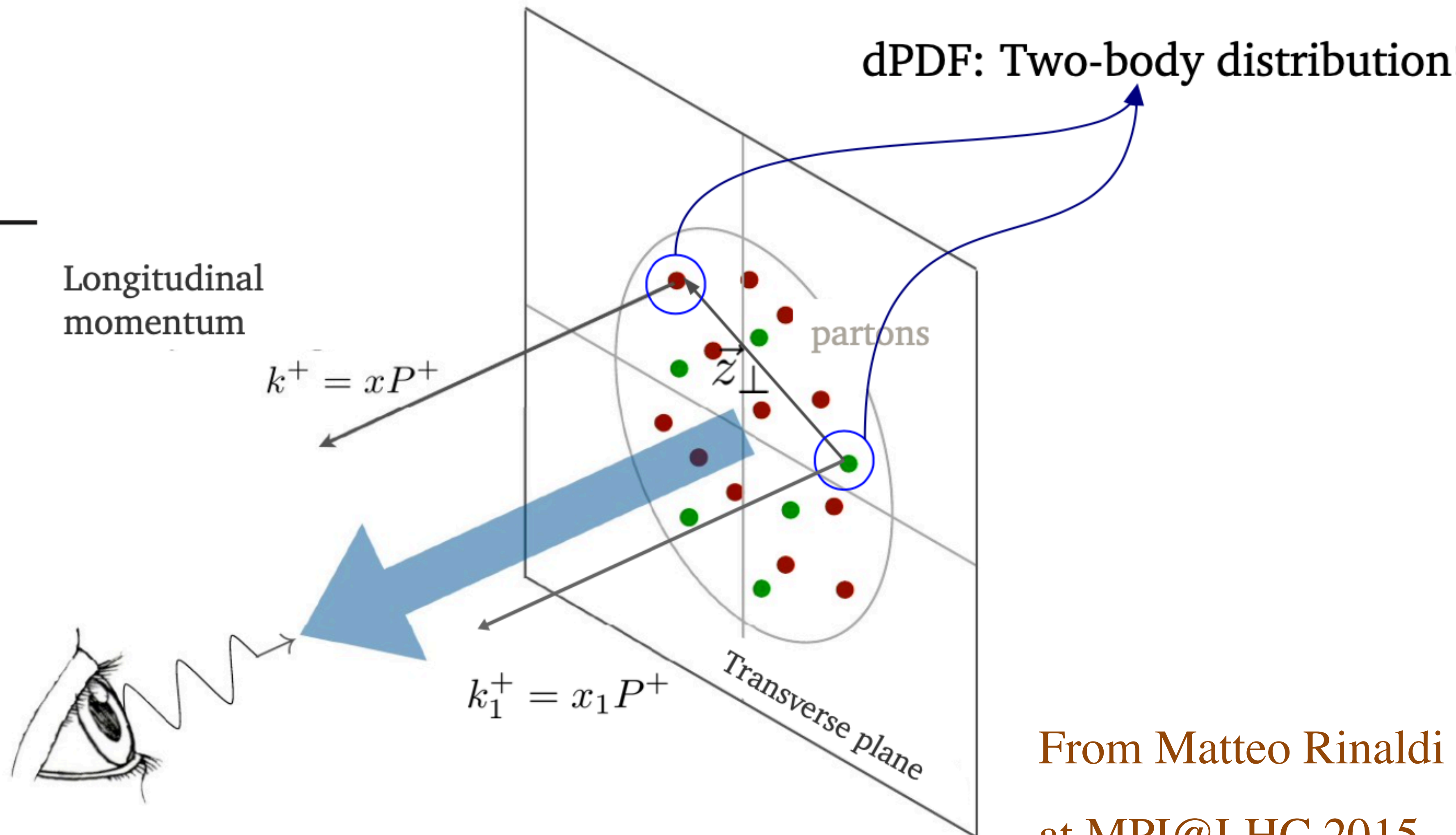
Lorcé, Pasquini, 2011; Hatta, 2011

Accessing gluon GTMDs

Double Parton Scattering

DPS: 2 hard scatterings off partons in the same hadron simultaneously





From Matteo Rinaldi at MPI@LHC 2015

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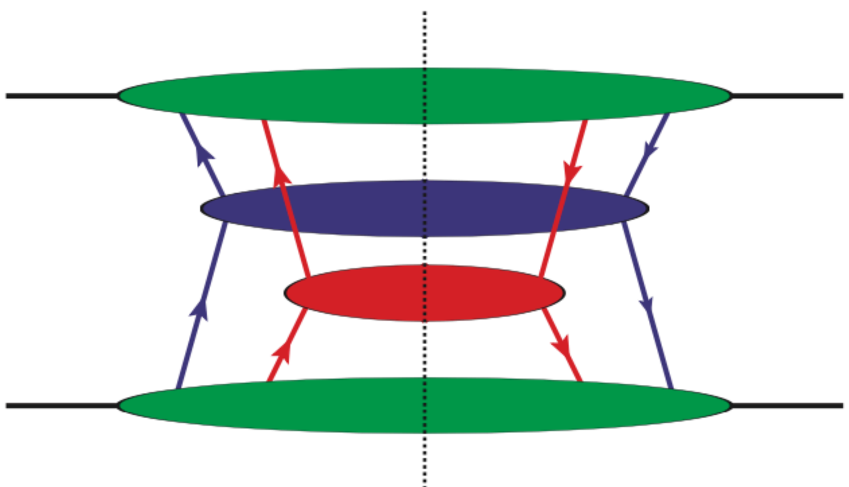
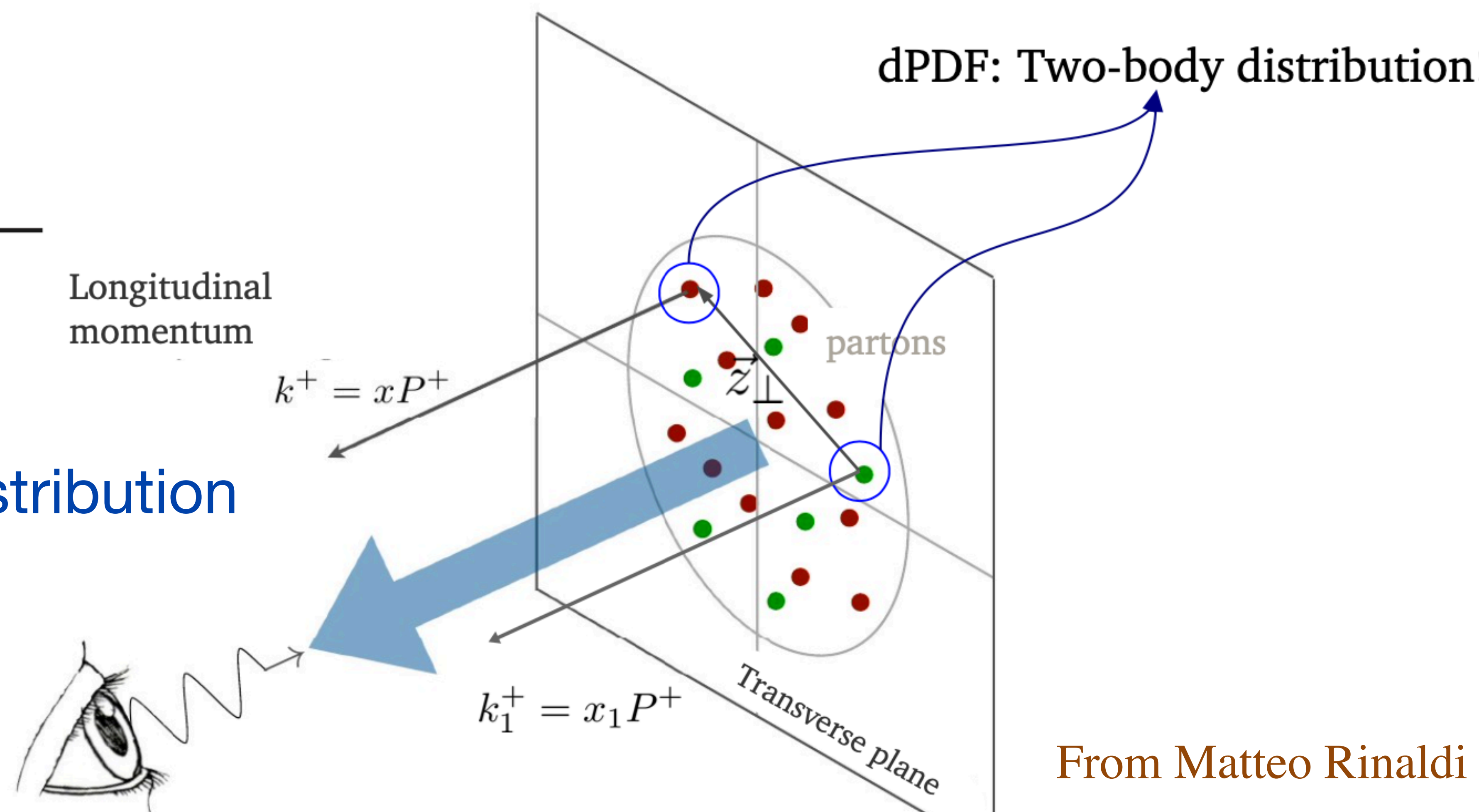


DPD = Double Parton Distribution

$$F(x, x'; z_\perp)$$



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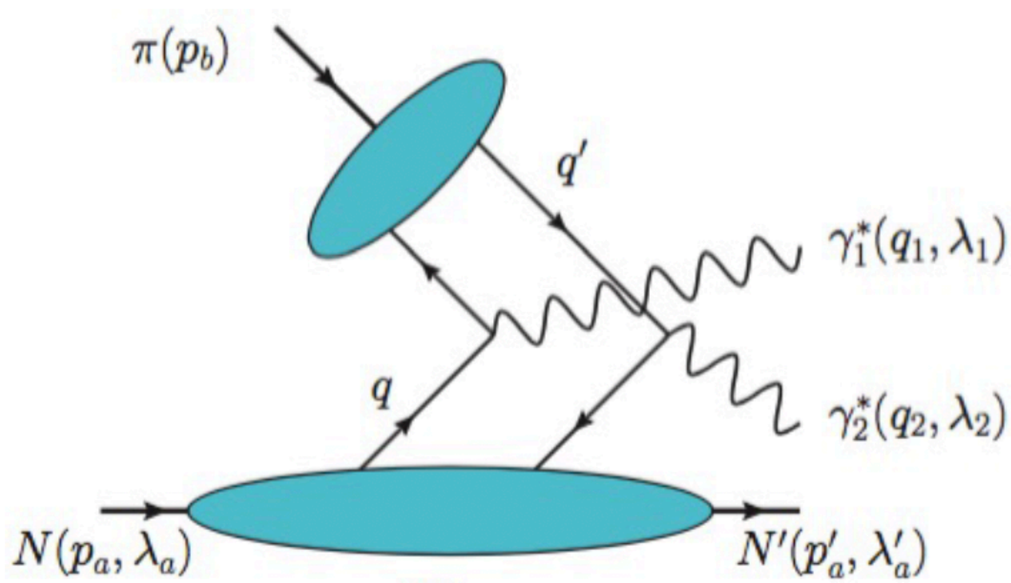
DPDs capture the spin, color & flavor correlations between partons

GTMDs from exclusive DPS

If the hadron stays intact, then there is a connection to GTMDs:



DPD → GTMD^2

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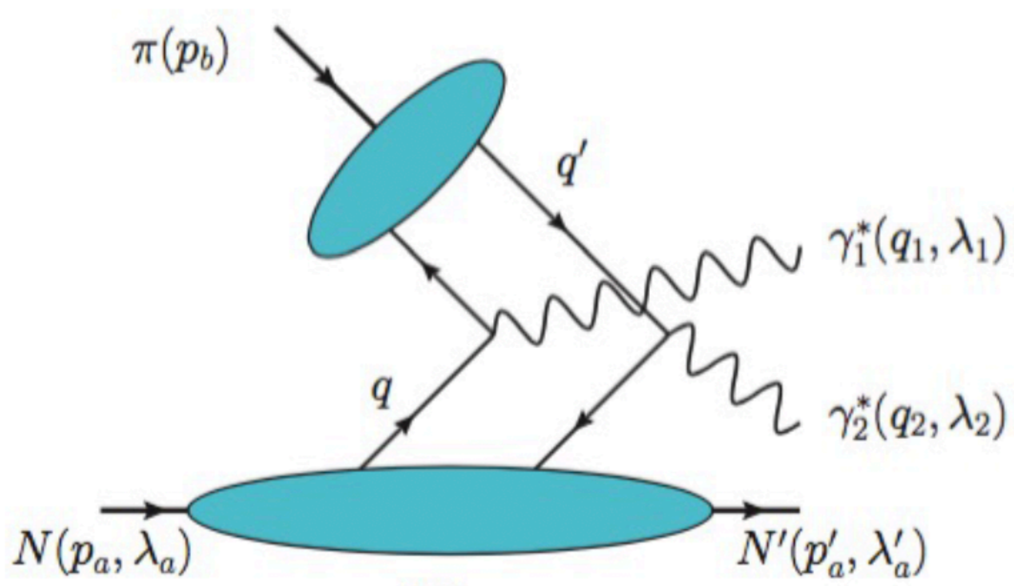

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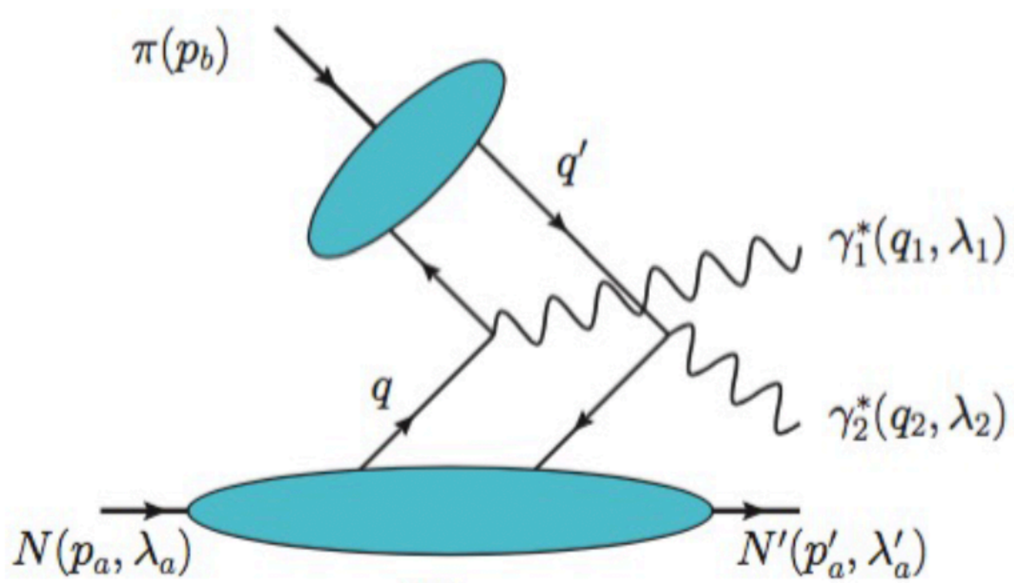

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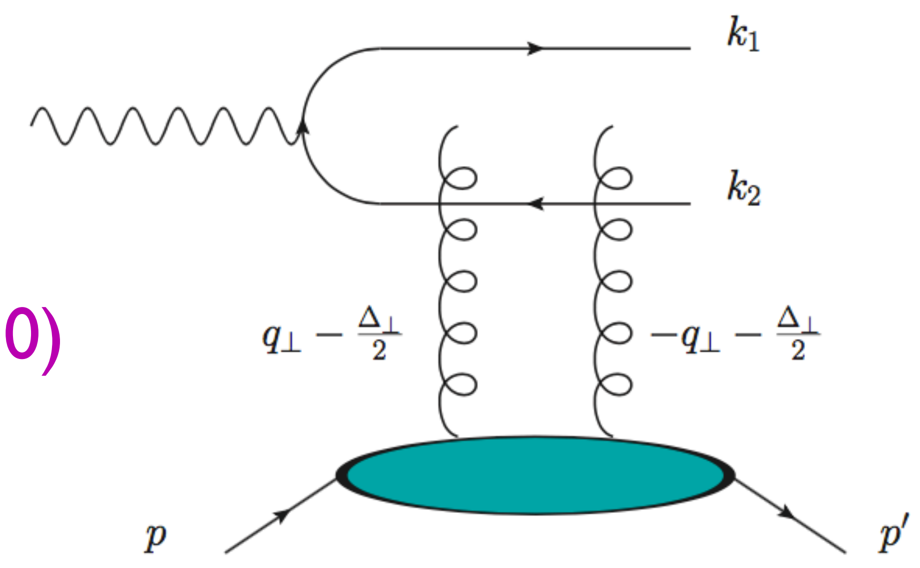
Also exclusive coherent diffractive processes have been suggested, which involve 2 DP gluon GTMDs, rather than 4 WW ones

Diffractive dijet production

Probe gluon GTMDs via hard diffractive dijet production in eA ($\Delta_\perp \neq 0, \xi = 0$)

Altinoluk, Armesto, Beuf, Rezaeian, 2016; Hatta, Xiao, Yuan, 2016



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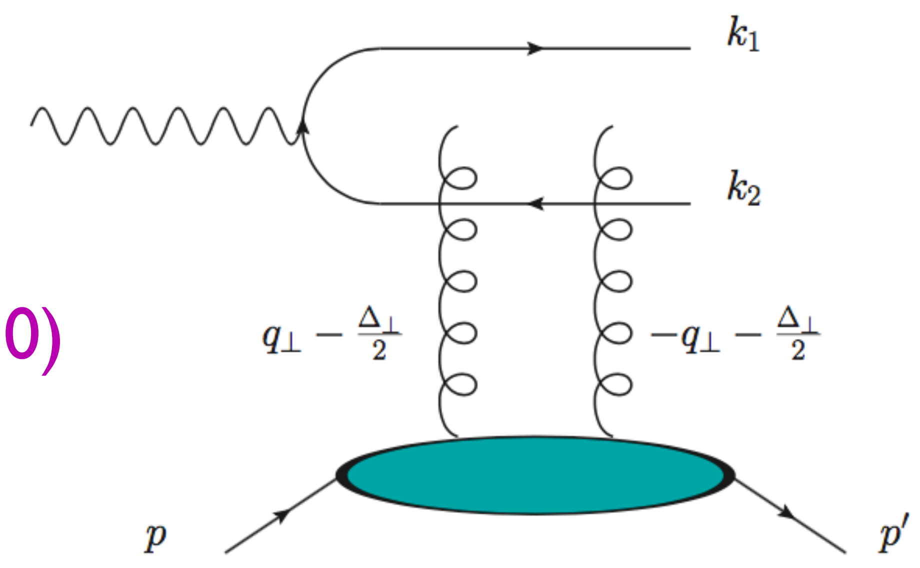

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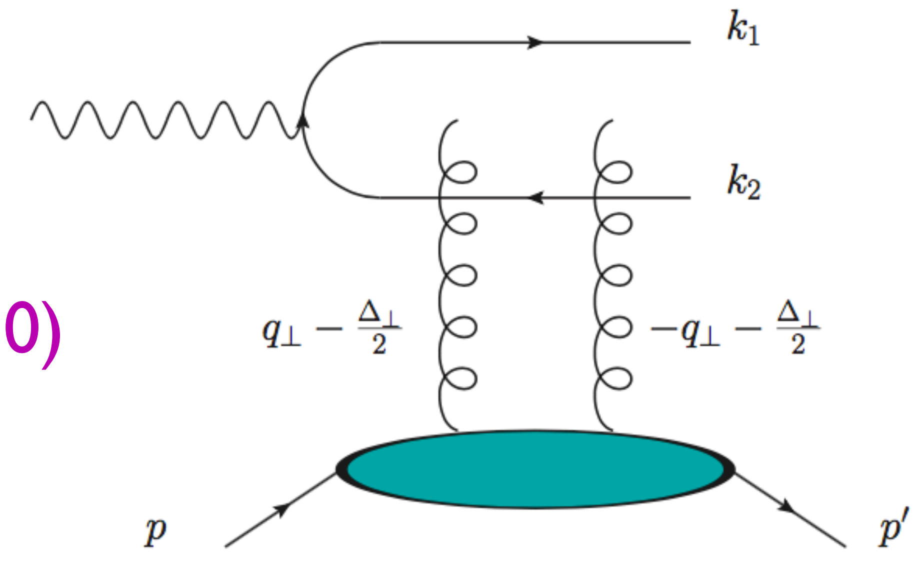

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$$\frac{d\sigma}{dy_1 dy_2 d^2\boldsymbol{k}_{1\perp} d^2\boldsymbol{k}_{2\perp}} \propto \int d^2\boldsymbol{q}_\perp d^2\boldsymbol{q}'_\perp \mathcal{F}^{[\Box]}(\boldsymbol{q}_\perp, \boldsymbol{\Delta}_\perp)\mathcal{F}^{[\Box]}(\boldsymbol{q}'_\perp, \boldsymbol{\Delta}_\perp)\mathcal{A}(\boldsymbol{K}_\perp, \boldsymbol{q}_\perp, \boldsymbol{q}'_\perp, \epsilon_f^2)$$

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$$\epsilon_f^2 = z(1-z)Q^2$$

$$G^{[\Box]} \to \mathcal{F}^{[\Box]} \qquad S^{[\Box]}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp) \to 1 - S^{[\Box]}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp)$$

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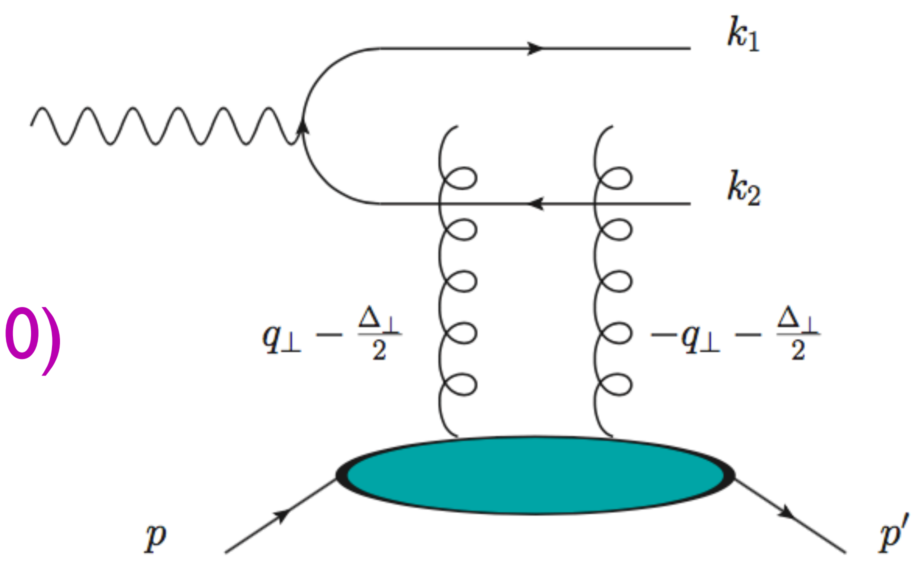

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$$\frac{d\sigma_T^{\gamma^* p \to jjp}}{dK_\perp d\Delta_\perp^2} = \frac{(2\pi)^4 \alpha_{em}}{16 N_c} \sum_f e_f^2 \int dz \left[z^2 + (1-z)^2\right] \frac{\mathcal{A}_\mathrm{T}^2(K_\perp, \Delta_\perp, z, Q, y)}{K_\perp}$$

$$\mathcal{A}_\mathrm{T}(K_\perp, \Delta_\perp, z, Q, y) = \int \frac{d^2 q_\perp}{(2\pi)^3} \left[\frac{K_\perp \cdot (K_\perp - q_\perp)}{z(1-z)Q^2 + (K_\perp - q_\perp)^2}\right] \mathcal{F}_0^{[\Box]}(x, q_\perp, \Delta_\perp)\Big|_{x = s/(yQ^2)}$$

By varying Q^2 and $K_\perp$ one probes differently weighted integrals over the GTMD

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Idem for the longitudinal photon polarization (which requires $Q^2 \neq 0$):

$$\frac{d\sigma_L^{\gamma^* p\to jjp}}{dK_\perp d\Delta_\perp^2} = \frac{(2\pi)^4 \alpha_{em}}{4 N_c} \sum_f e_f^2 \int dz\, z^2 (1-z)^2 \frac{\mathcal{A}_\mathrm{L}^2(K_\perp, \Delta_\perp, z, Q, y)}{K_\perp}$$

$$\mathcal{A}_\mathrm{L}(K_\perp, \Delta_\perp, z_i, Q) = \int \frac{d^2 q_\perp}{(2\pi)^3} \left[\frac{Q K_\perp}{z(1-z)Q^2 + (K_\perp - q_\perp)^2}\right] \mathcal{F}_0^{[\Box]}(x, q_\perp, \Delta_\perp)\Big|_{x=s/(yQ^2)}$$

DB, Setyadi, 2021

Diffractive J/ψ production

Again the transverse momentum dependence of the GTMD is probed indirectly

$$\mathcal{A}_{T,L} = \frac{\pi i}{2N_c} \int_0^1 dz \int d^2 r_\perp \left(\Psi_V^* \Psi_\gamma\right)_{T,L}(r_\perp, z) \int d^2 q_\perp J_0\left(|q_\perp + \delta_\perp| r_\perp\right) \mathcal{F}_0^{[\Box]}(x, q_\perp, \Delta_\perp)$$

$$\delta_\perp = (\frac{1}{2} - z)\Delta_\perp$$

Kowalski, Teaney, 2003; Kowalski, Motyka, Watt, 2006; many others

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$$J_0\left(|q_\perp + \delta_\perp|r_\perp\right) \approx 1 - \frac{(q_\perp + \delta_\perp)^2 r_\perp^2}{4}$$

$$\mathcal{A}_{T,L} \approx \frac{\pi i}{8N_c}\int_0^1 dz \int d^2r_\perp r_\perp^2 \left(\Psi_V^*\Psi_\gamma\right)_{T,L}(r_\perp, z)\int d^2q_\perp q_\perp^2 \mathcal{F}_0^{[\Box]}(x, q_\perp, \Delta_\perp)$$

$$= \frac{\pi^3 i\alpha_s}{N_c}\int_0^1 dz \int d^2r_\perp r_\perp^2 \left(\Psi_V^*\Psi_\gamma\right)_{T,L}(r_\perp, z)\, xH_g(x, \Delta_\perp)$$

This yields an expression in terms of a GPD (requires regularization)

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$$\mathcal{A}_{T,L} \approx \frac{\pi i}{8N_c}\int_0^1 dz \int d^2 r_\perp r_\perp^2 \left(\Psi_V^* \Psi_\gamma\right)_{T,L}(r_\perp, z) \int d^2 q_\perp q_\perp^2 \mathcal{F}_0^{[\Box]}(x, q_\perp, \Delta_\perp)$$

$$= \frac{\pi^3 i \alpha_s}{N_c}\int_0^1 dz \int d^2 r_\perp r_\perp^2 \left(\Psi_V^* \Psi_\gamma\right)_{T,L}(r_\perp, z)\, x H_g(x, \Delta_\perp)$$

This yields an expression in terms of a GPD (requires regularization)

Diffractive J/ψ production

Again the transverse momentum dependence of the GTMD is probed indirectly

$$\mathcal{A}_{T,L} = \frac{\pi i}{2N_c} \int_0^1 dz \int d^2 r_\perp \left(\Psi_V^* \Psi_\gamma\right)_{T,L}(r_\perp, z) \int d^2 q_\perp J_0\left(|q_\perp + \delta_\perp| r_\perp\right) \mathcal{F}_0^{[\square]}(x, q_\perp, \Delta_\perp)$$

$$\delta_\perp = (\frac{1}{2} - z)\Delta_\perp$$

Kowalski, Teaney, 2003; Kowalski, Motyka, Watt, 2006; many others

By varying *t* one probes (slightly) differently weighted integrals
Using different quarkonia also changes the weights ($r_\perp \sim 1/M_V$)

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See also Bertone, 2022

This yields an expression in terms of a GPD (requires regularization)

Phenomenology

MV-like model

We consider the MV-like model for the isotropic part:

$$\mathcal{F}^{[\Box]}(\boldsymbol{k}_\perp, \boldsymbol{\Delta}_\perp) = 4N_c \int \frac{d^2\boldsymbol{r}_\perp d^2\boldsymbol{b}_\perp}{(2\pi)^2} \, e^{-i\boldsymbol{k}_\perp\cdot\boldsymbol{r}_\perp} e^{i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp} \, e^{-\epsilon_r r_\perp^2} \, \left[1 - \exp\left(-\frac{1}{4} r_\perp^2 \chi Q_s^2(b_\perp) \ln\left[\frac{1}{r_\perp^2\Lambda^2} + e\right]\right)\right]$$

Similar to Hagiwara, Hatta, Pasechnik, Tasevsky & Teryaev, 2017; Salazar, Schenke, 2019

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χ sets the normalization of Q_s and is x dependent (of GBW form)

$$\chi(x) = \bar{\chi}\left(\frac{x_0}{x}\right)^\lambda \qquad x_0 = 3\times 10^{-4} \quad \lambda = 0.29$$

Q_s is proportional to the proton (Gaussian) or nuclear (Woods-Saxon) profile

For details see DB, Setyadi, 2023

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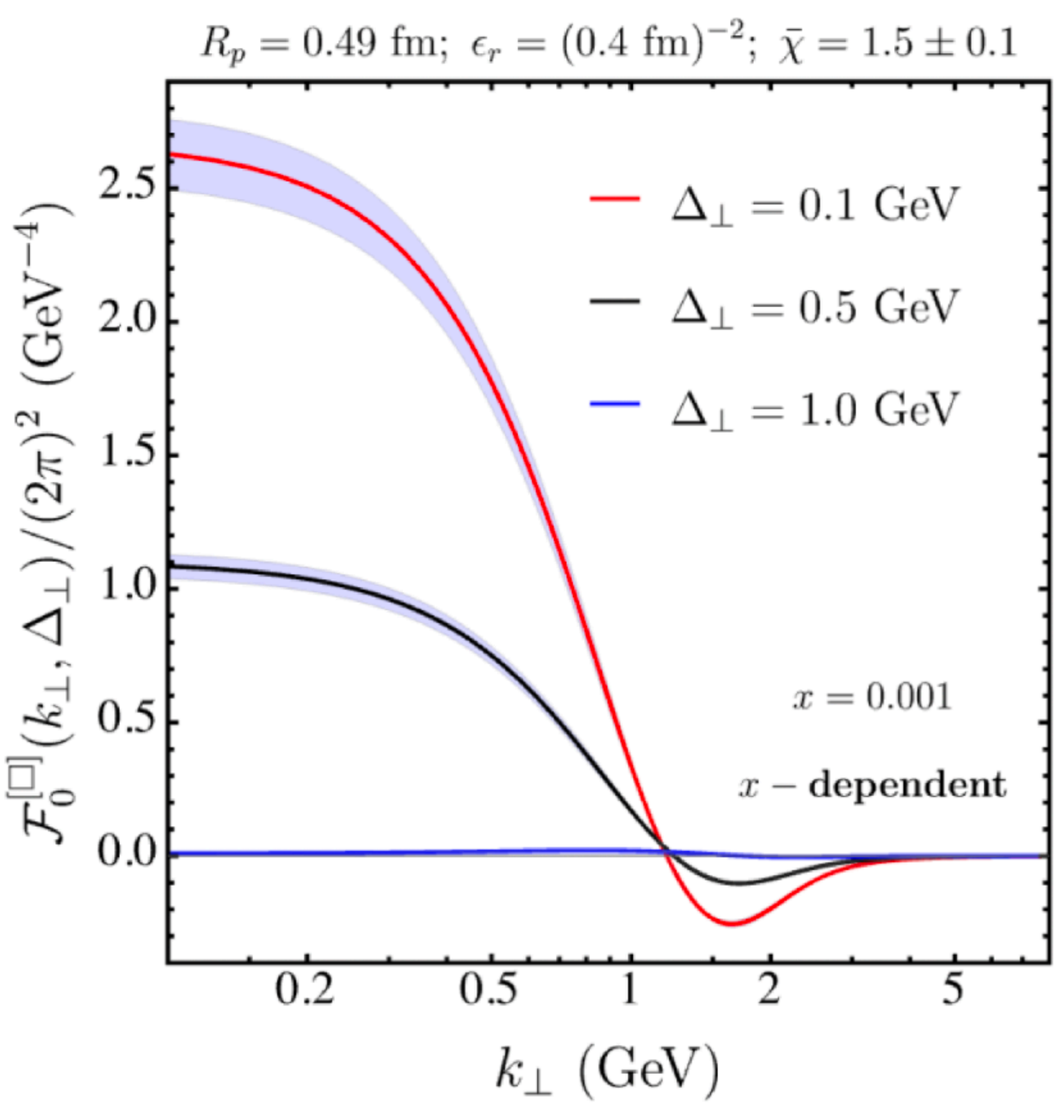
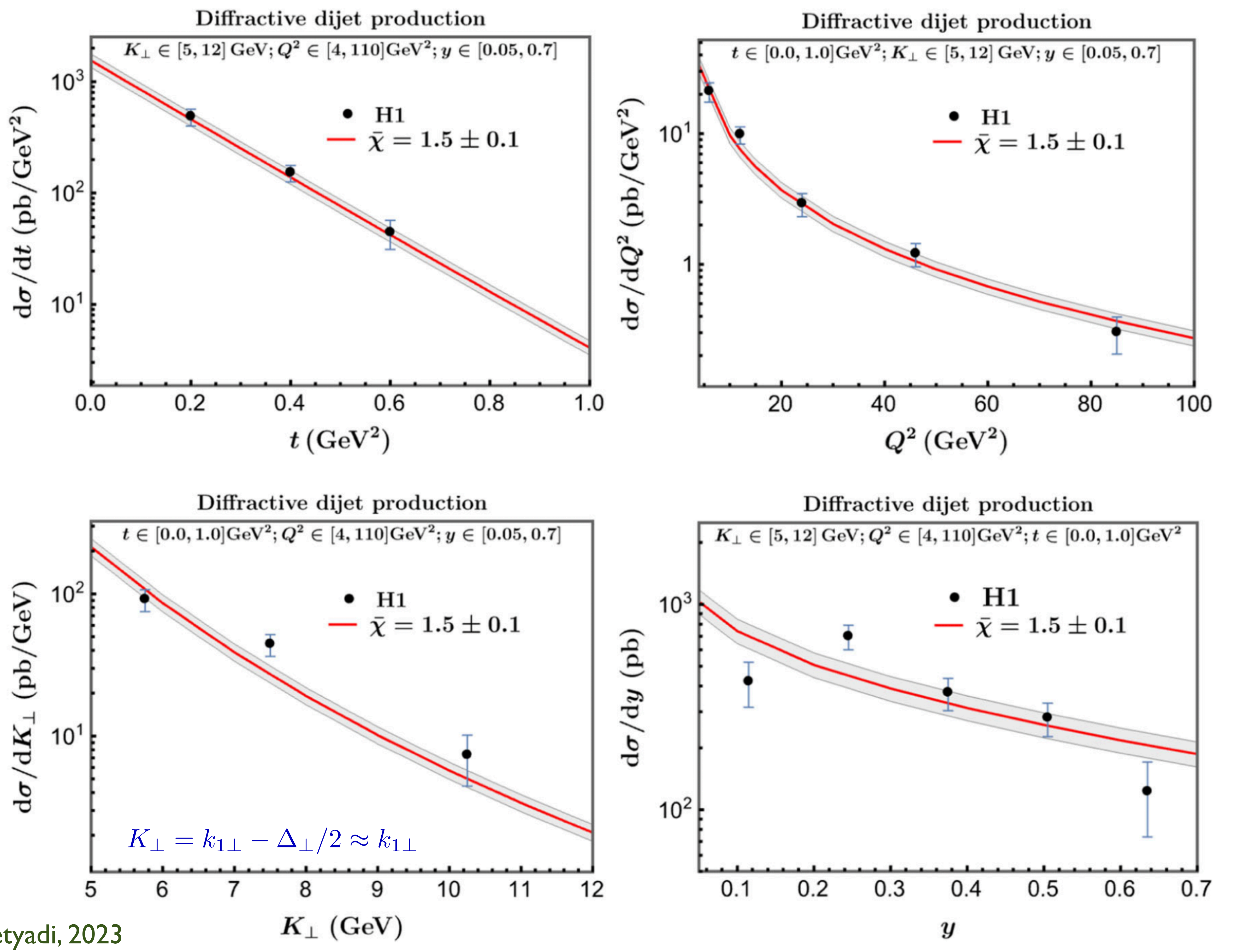
For details see DB, Setyadi, 2023

Dominant contribution from:

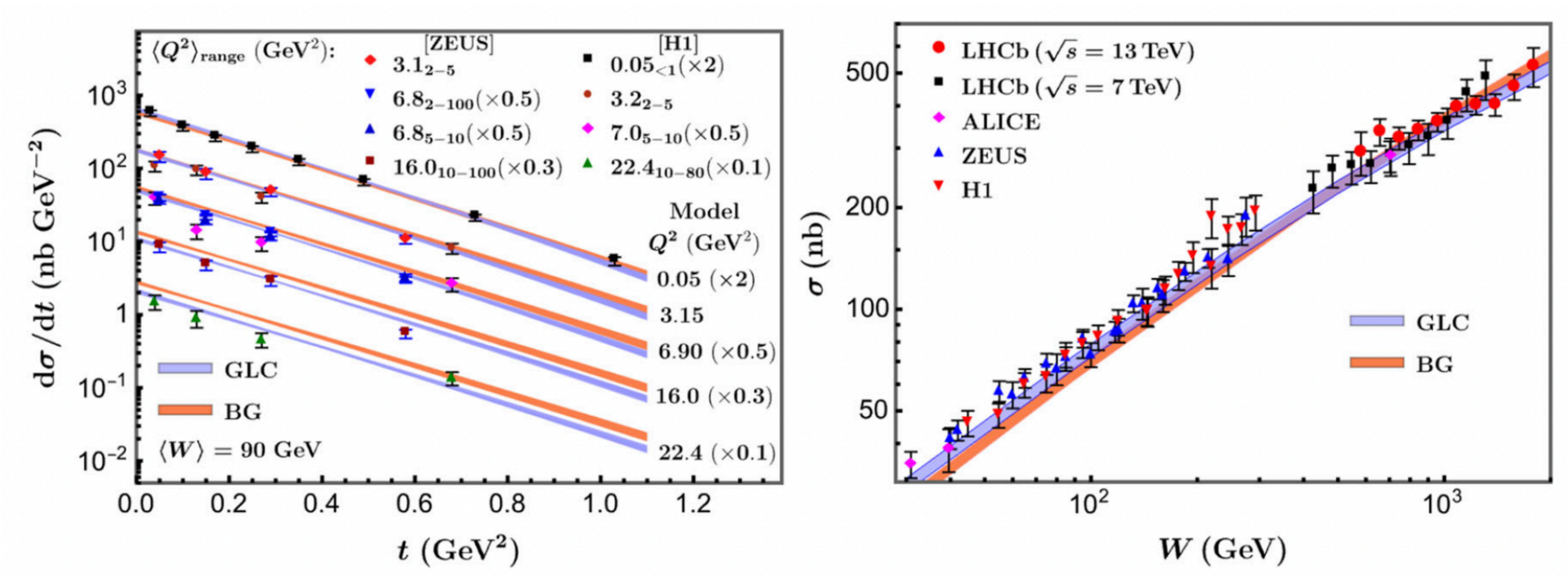
$$\Delta_\perp \ll K_\perp \text{ or } M_V$$



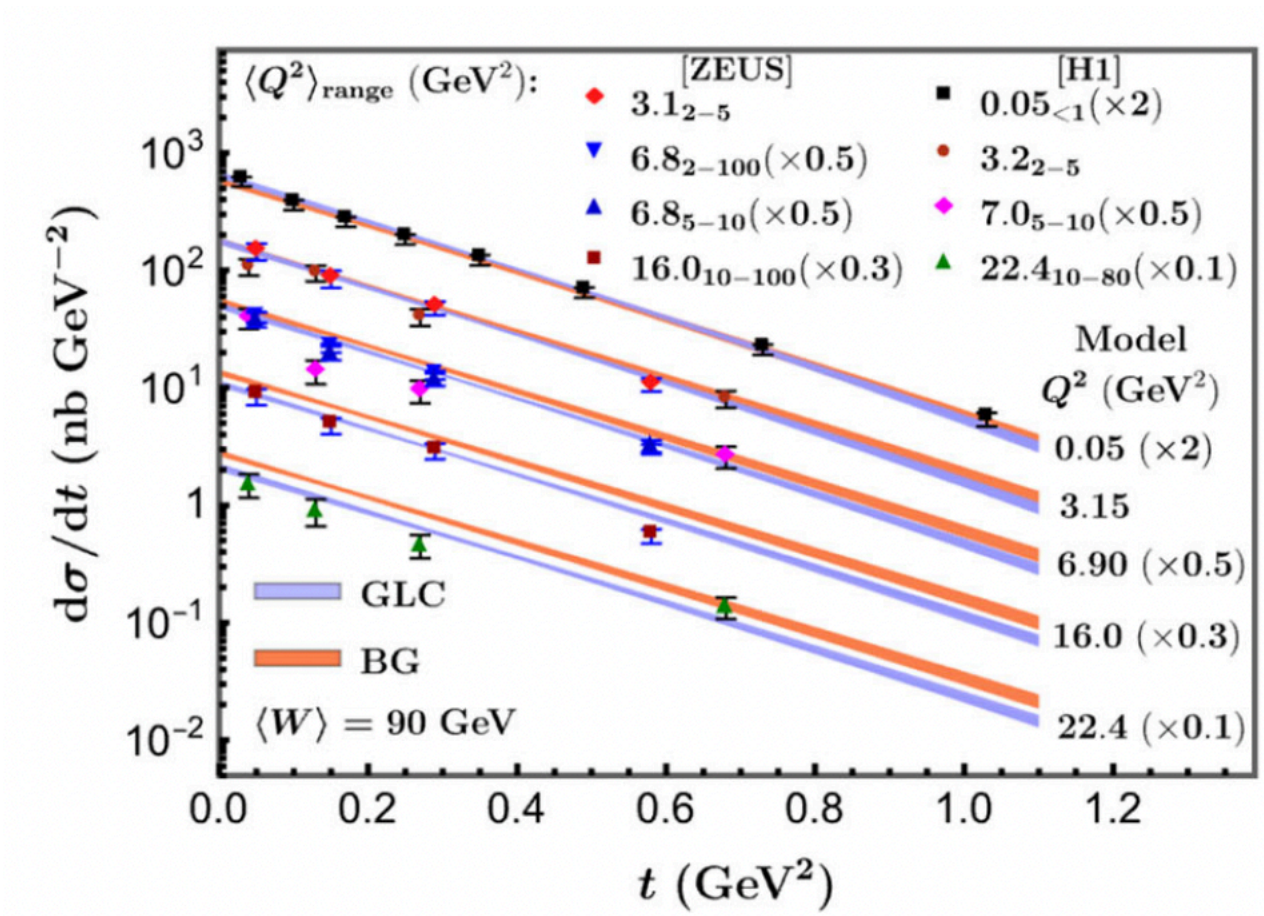
Best fit of H1 dijet data with Rp = 0.49 fm, λ = 0.29, and $\epsilon_r = (0.4\ \mathrm{fm})^{-2}$

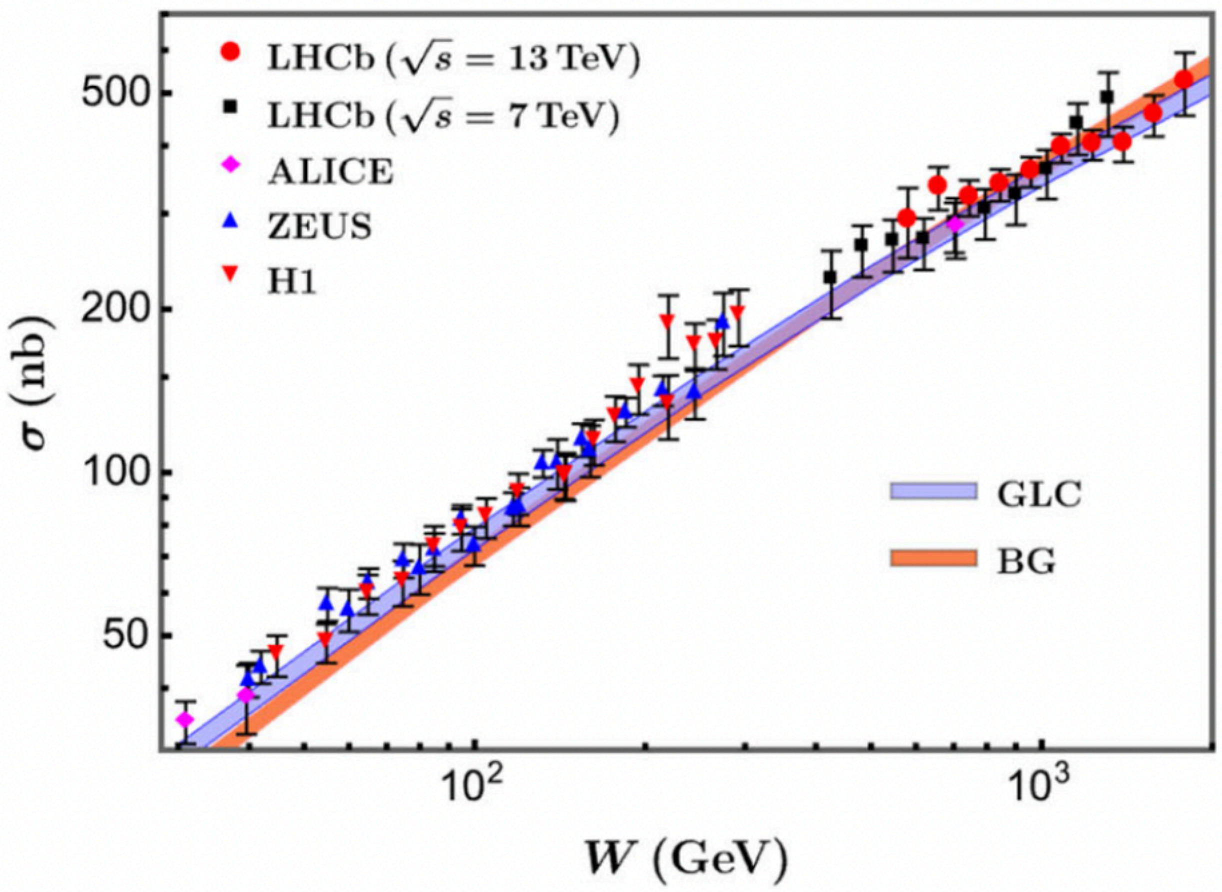


DB, Setyadi, 2023



Diffractive J/ψ production data of HERA (H1 & ZEUS) prefer smaller Rp (0.40-0.41 fm), smaller λ (0.22), and is more sensitive to ϵ_r





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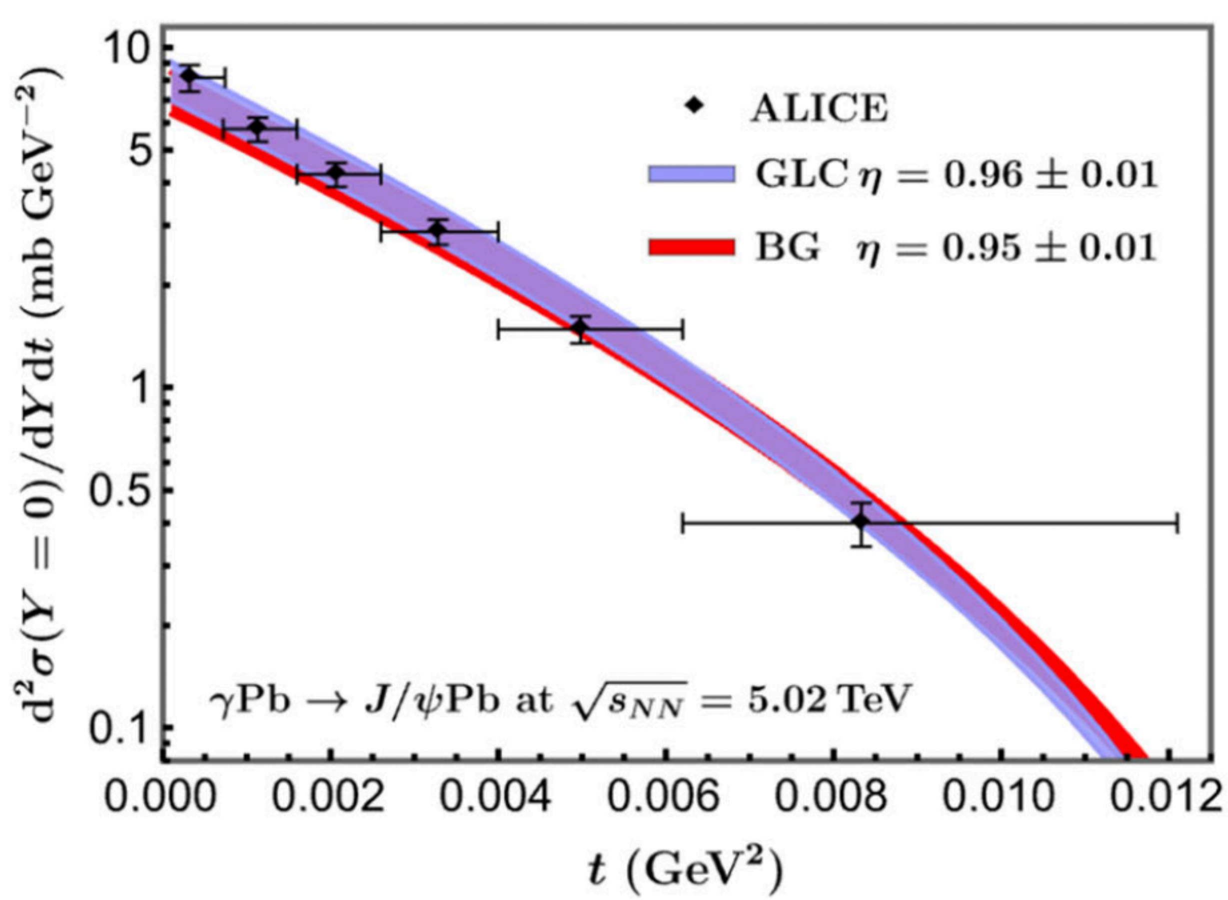
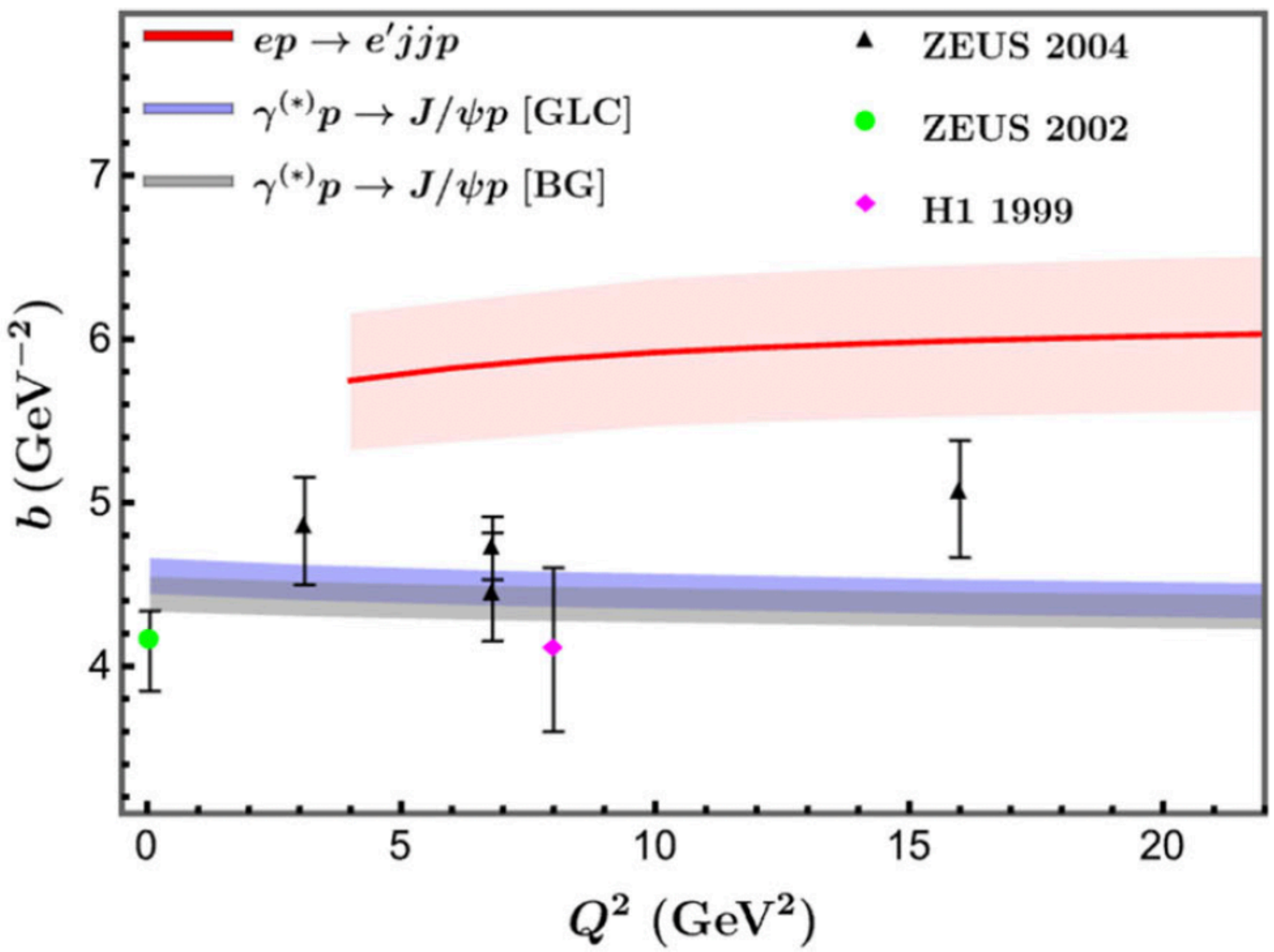


Description of ALICE UPC data qualitatively fine with an A dependence somewhat smaller than $A^{1/3}$, but this is dependent on the profile functions

Tension between diffractive dijet and J/ψ production

There is tension between the dijet and J/ψ data regarding the steepness of the t-slope (dictated by R_p)



DB, Setyadi, 2023

UPC data from RHIC and LHC and especially EIC data can shed further light on these issues, in order to check whether a common GTMD description is possible

Odderons

Odderon GTMDs

$S^{[\Box]}$ can also have an imaginary part:

$$S^{[\Box]}(\boldsymbol{x},\boldsymbol{y}) = \mathcal{P}(\boldsymbol{x},\boldsymbol{y}) + i\mathcal{O}(\boldsymbol{x},\boldsymbol{y})$$

$$\mathcal{P}(\boldsymbol{x},\boldsymbol{y}) \equiv \frac{1}{2N_c}\mathrm{Tr}\left(U^{[\Box]} + U^{[\Box]\dagger}\right) \qquad \mathcal{O}(\boldsymbol{x},\boldsymbol{y}) \equiv \frac{1}{2iN_c}\mathrm{Tr}\left(U^{[\Box]} - U^{[\Box]\dagger}\right)$$

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This "odderon" operator is C-odd and T-odd

$$\begin{aligned} G_{(d)}^{(\text{T-odd})\,ij}(\boldsymbol{k},\boldsymbol{\Delta}) &\equiv \frac{1}{2}\left(G^{[+,-]\,ij}(\boldsymbol{k},\boldsymbol{\Delta}) - G^{[-,+]\,ij}(\boldsymbol{k},\boldsymbol{\Delta})\right) \\ &= \frac{N_c}{\alpha_s}\left[\frac{1}{2}\left(\boldsymbol{k}^2 - \frac{\boldsymbol{\Delta}^2}{4}\right)\delta_T^{ij} + k_T^{ij} - \frac{\Delta_T^{ij}}{4} - \frac{k_T^{[i}\Delta_T^{j]}}{2}\right] \\ &\quad \times \left(G^{[\Box]}(\boldsymbol{k},\boldsymbol{\Delta}) - G^{[\Box^\dagger]}(\boldsymbol{k},\boldsymbol{\Delta})\right) \end{aligned}$$

$$G^{[\Box]}(\boldsymbol{k},\boldsymbol{\Delta}) - G^{[\Box^\dagger]}(\boldsymbol{k},\boldsymbol{\Delta}) \propto \int \frac{d^2\boldsymbol{x}\, d^2\boldsymbol{y}}{(2\pi)^4}\, e^{-i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{y})+i\boldsymbol{\Delta}\cdot\frac{\boldsymbol{x}+\boldsymbol{y}}{2}}\, \langle\mathcal{O}(\boldsymbol{x},\boldsymbol{y})\rangle$$

Odderon GTMDs

Hermiticity and PT constraints imply:

$$G^{[\Box]*}(\boldsymbol{k},\boldsymbol{\Delta}) = G^{[\Box]}(\boldsymbol{k},-\boldsymbol{\Delta}) \qquad G^{[\Box]*}(\boldsymbol{k},\boldsymbol{\Delta}) = G^{[\Box^\dagger]}(-\boldsymbol{k},-\boldsymbol{\Delta})$$

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Odderon (for $\xi = 0$) involves only odd harmonics $\cos[(2n+1)(\phi_k - \phi_\Delta)]$

$$\begin{aligned} xW(x,\boldsymbol{b},\boldsymbol{k}) = {} & x\mathcal{W}_0(x,\boldsymbol{b}^2,\boldsymbol{k}^2) + 2\cos(\phi_b-\phi_k)\, x\mathcal{W}_1(x,\boldsymbol{b}^2,\boldsymbol{k}^2) \\ & + 2\cos 2(\phi_b-\phi_k)\, x\mathcal{W}_2(x,\boldsymbol{b}^2,\boldsymbol{k}^2) + \ldots \end{aligned}$$

$\mathcal{W}_1$ leads to odd harmonics in dihadron production through double parton scattering in pA collisions (not in exclusive DPS in this case, but using large N_c)

DB, van Daal, Mulders, Petreska, 2018

Lappi, Schenke, Schlichting, Venugopalan, 2016

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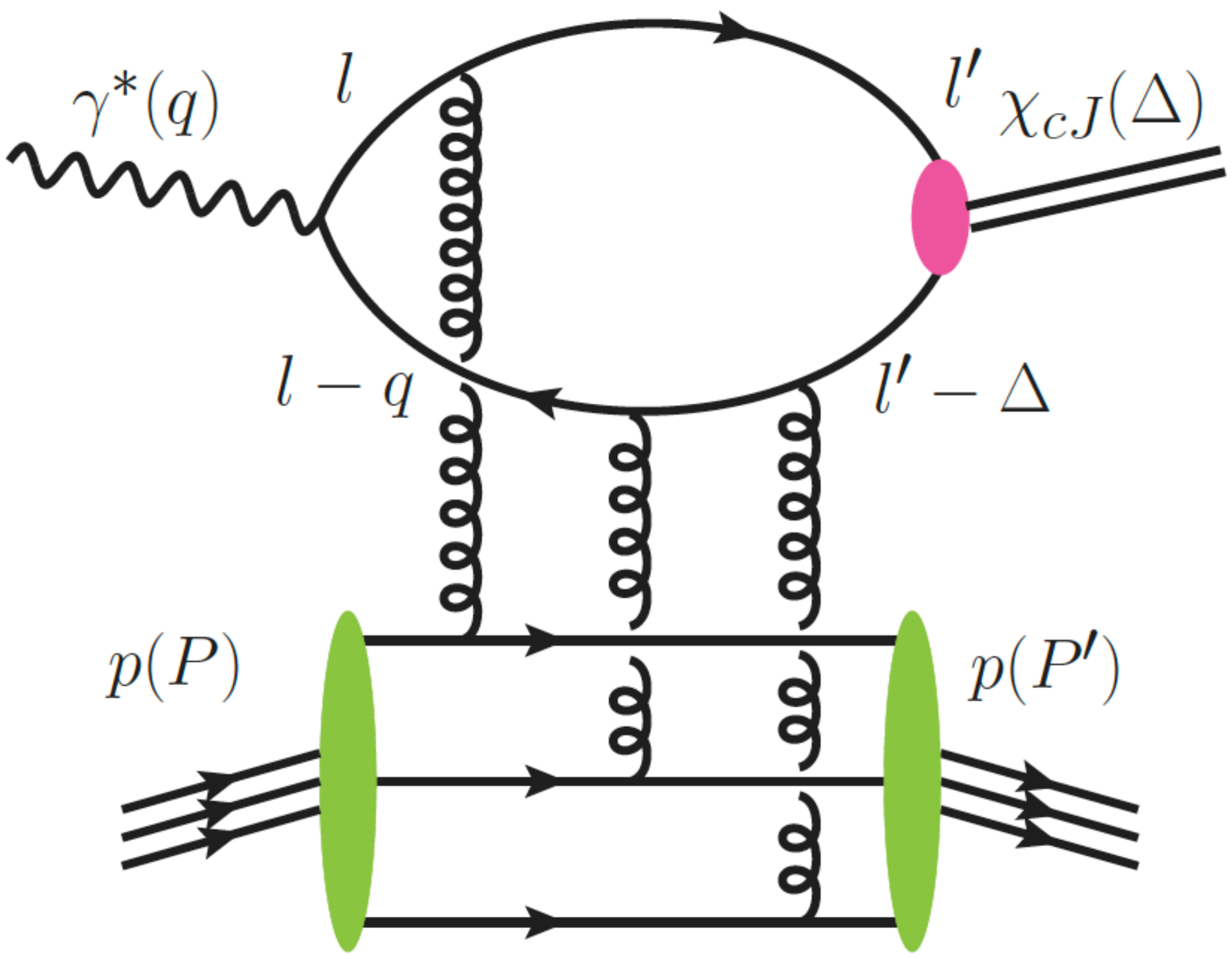
For $\xi \neq 0$ odd powers of $k \cdot \Delta$ can appear in the real parts as well

Exclusive χ_c production at EIC

This process also probes the odderon:



C-even final state requires
C-odd t-channel exchange

Constructive interference
with photon t-channel exchange
for |t| ~ 1 GeV²

Benić, Dumitru, Kaushik, Motyka, Stebel, 2024

Spin dependent odderon

$$G^{[\Box]}(\boldsymbol{k}, \boldsymbol{\Delta}) - G^{[\Box^\dagger]}(\boldsymbol{k}, \boldsymbol{\Delta}) \text{ only depends on odd powers of } \boldsymbol{k} \cdot \boldsymbol{\Delta}$$

Therefore, no odderon in the forward limit for unpolarized protons (at LT)

For polarized protons the forward limit does not need to vanish however

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The d-type gluon Sivers function $f_{1T}^{\perp\, g\, [+,-]}$ at small x is part of:

$$\left(\Gamma^{[+,-]} - \Gamma^{[-,+]}\right) \propto \text{F.T.} \langle P, S_T | \text{Tr}\left[U^{[\Box]}(0_T, y_T) - U^{[\Box]\dagger}(0_T, y_T)\right] |P, S_T\rangle$$

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It can be probed for instance in $p^\uparrow p \to h^\pm X$ at $x_F<0$

$p^{\uparrow}p \to h^{\pm} X$ at $x_F < 0$

It is the only relevant contribution to A_N in backward ($x_F < 0$) charged hadron production in $p^{\uparrow}p$ or $p^{\uparrow}A$ (in contrast to the many contributions at $x_F > 0$)

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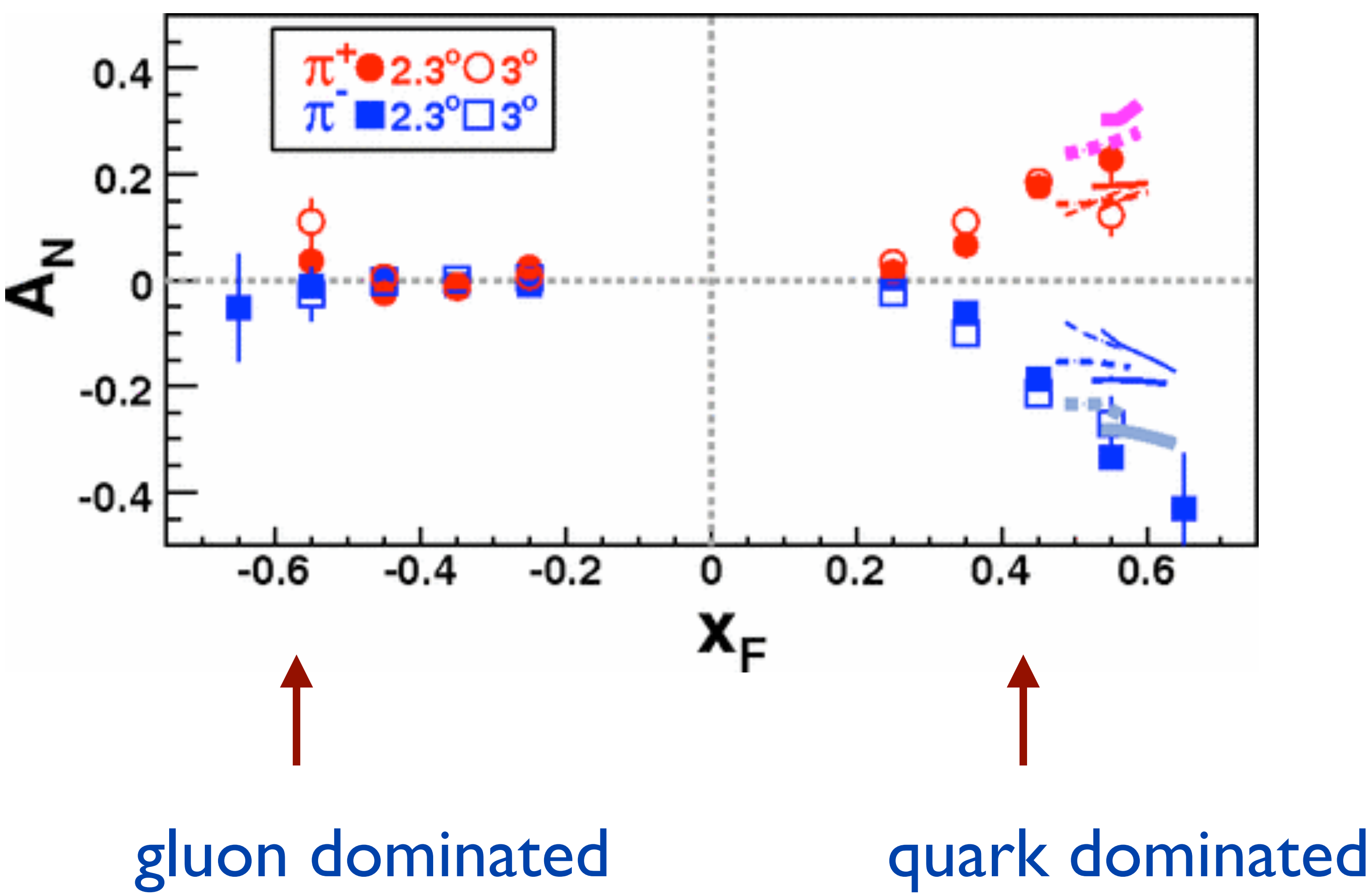
Backward charged hadron production at RHIC

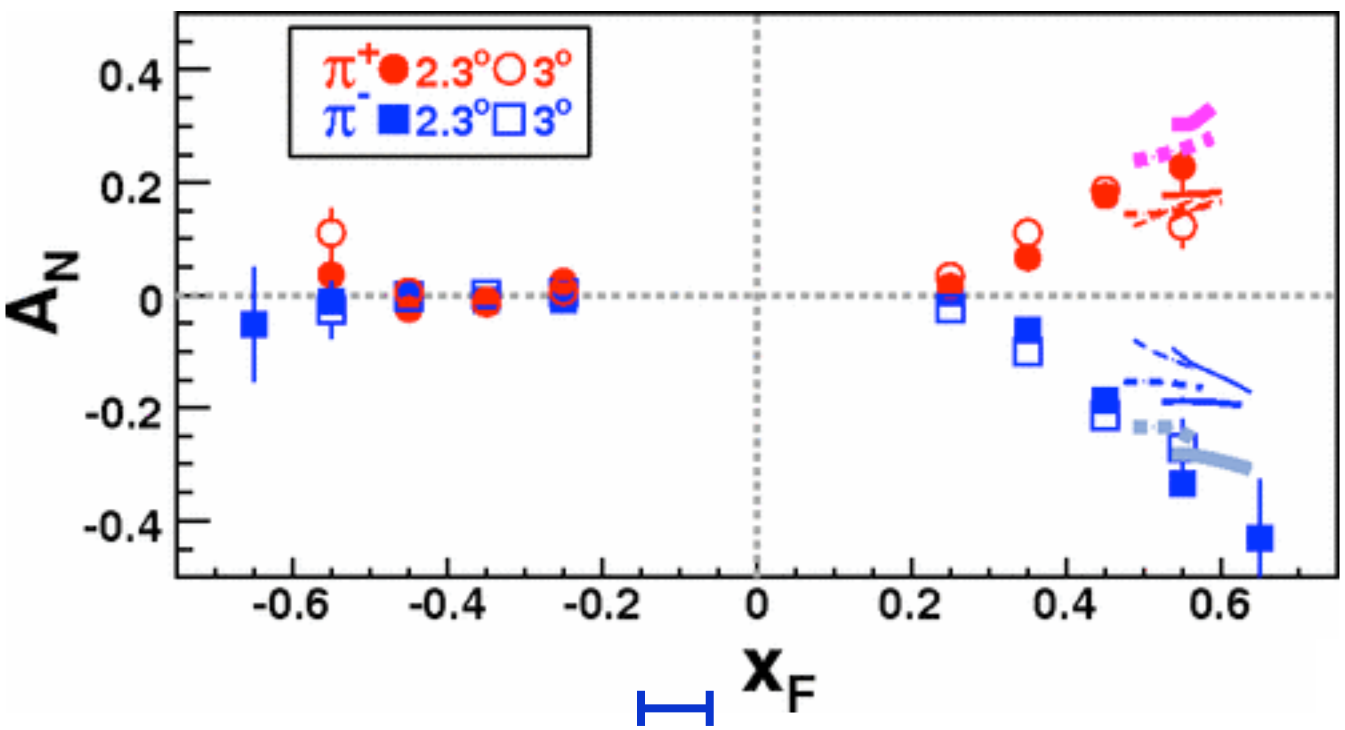


BRAHMS, 2008 $\sqrt{s}$ = 62.4 GeV
low p_T, up to roughly 1.2 GeV
where gg channel dominates

$$x_F = \frac{2p_z}{\sqrt{s}}$$

The asymmetry in the gluon dominated region is smaller and needs more precision

$p^{\uparrow}p \rightarrow h^{\pm} X$ at $x_F < 0$



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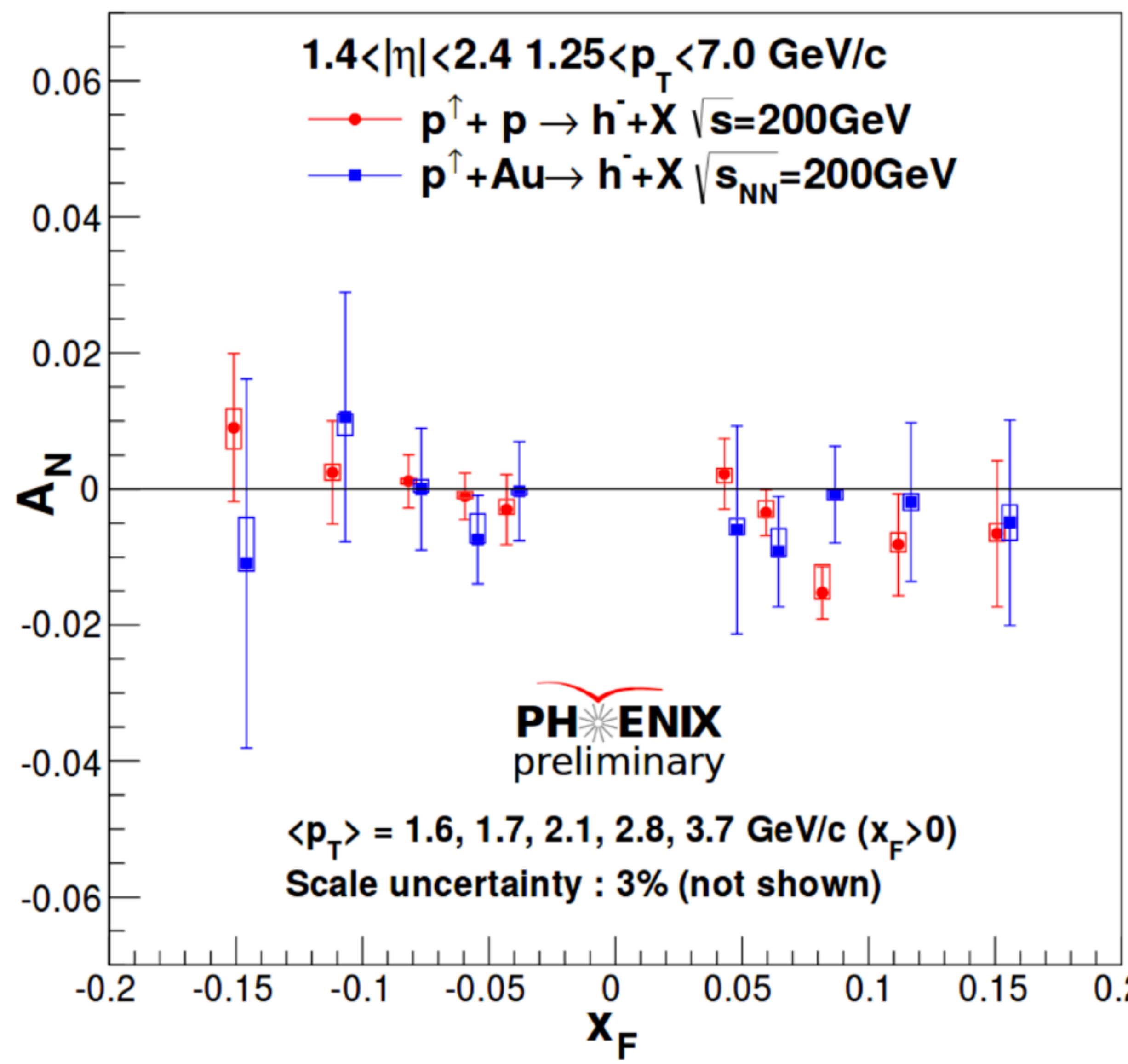
PHENIX, 2017
$\sqrt{s}$ = 200 GeV
p_T between 1.25 and 7 GeV



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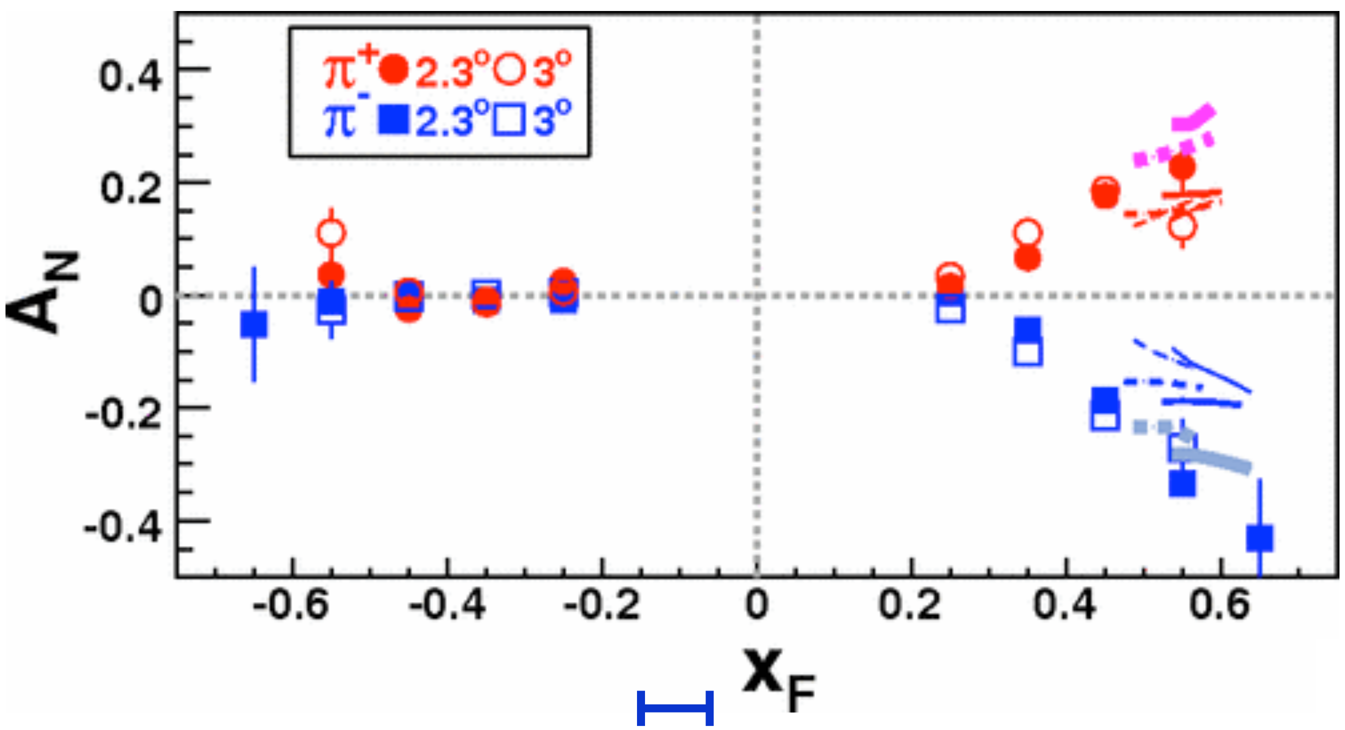

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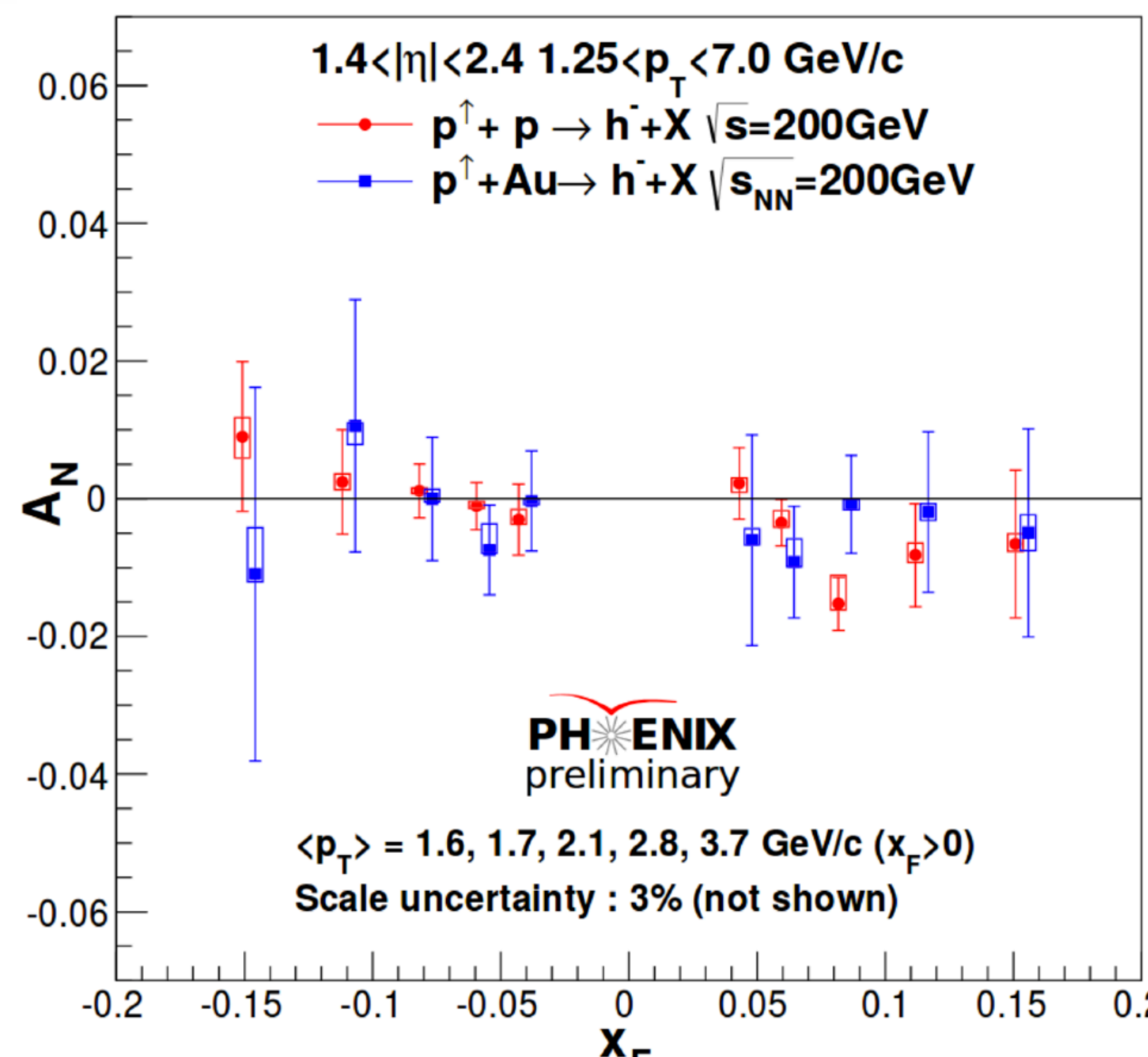

More data at larger negative x_F needed

Conclusions

Conclusions

- TMDs and GTMDs are process dependent, with WW and DP versions at small x
- The DP gluon (G)TMDs become a Wilson loop correlator in the small-x limit, leading to maximally polarized states, which are preserved under x evolution (at least for linear gluon polarization), but not scale evolution (Sudakov suppression)
- CGC gluons are maximally polarized, but the amount of polarization observed depends on the process and kinematics
- $p\,A \to \gamma^*\,jet\,X$ offers a good opportunity to study the DP gluon TMD and exclusive coherent diffractive dijet & J/ψ production in ep the DP gluon GTMD
- The imaginary parts of the DP GTMDs are odderon quantities, which lead at small x to odd harmonics in forward dihadron production in pA through DPS
- There is no odderon in the forward limit for unpolarized protons (at leading twist), but for polarized protons there is: can be probed in $p^{\uparrow}p \to h^{\pm}\,X$ at $x_F<0$

Back-up slides

Parallels between quarks and gluons

$$\Phi_U(x,\boldsymbol{k}) = \frac{1}{2}\left[\not{\bar{n}}\, f_1(x,\boldsymbol{k}^2) + \frac{\sigma_{\mu\nu}k_T^\mu \bar{n}^\nu}{M}\, h_1^\perp(x,\boldsymbol{k}^2)\right],$$

$$\Phi_L(x,\boldsymbol{k}) = \frac{1}{2}\left[\gamma^5\not{\bar{n}}\, S_L\, g_1(x,\boldsymbol{k}^2) + \frac{i\sigma_{\mu\nu}\gamma^5\bar{n}^\mu k_T^\nu S_L}{M}\, h_{1L}^\perp(x,\boldsymbol{k}^2)\right],$$

$$\Phi_T(x,\boldsymbol{k}) = \frac{1}{2}\left[\frac{\not{\bar{n}}\,\epsilon_T^{S_T k_T}}{M}\, f_{1T}^\perp(x,\boldsymbol{k}^2) + \frac{\gamma^5\not{\bar{n}}\,\boldsymbol{k}\cdot\boldsymbol{S}_T}{M}\, g_{1T}(x,\boldsymbol{k}^2)\right.$$
$$\left. + i\sigma_{\mu\nu}\gamma^5\bar{n}^\mu S_T^\nu\, h_1(x,\boldsymbol{k}^2) - \frac{i\sigma_{\mu\nu}\gamma^5\bar{n}^\mu k_T^{\nu\rho} S_{T\rho}}{M^2}\, h_{1T}^\perp(x,\boldsymbol{k}^2)\right]$$

$$\Gamma_U^{ij}(x,\boldsymbol{k}) = x\left[\delta_T^{ij}\, f_1(x,\boldsymbol{k}^2) + \frac{k_T^{ij}}{M^2}\, h_1^\perp(x,\boldsymbol{k}^2)\right],$$

$$\Gamma_L^{ij}(x,\boldsymbol{k}) = x\left[i\epsilon_T^{ij} S_L\, g_1(x,\boldsymbol{k}^2) + \frac{\epsilon_T^{\{i}{}_\alpha k_T^{j\}\alpha} S_L}{2M^2}\, h_{1L}^\perp(x,\boldsymbol{k}^2)\right],$$

$$\Gamma_T^{ij}(x,\boldsymbol{k}) = x\left[\frac{\delta_T^{ij}\,\epsilon_T^{S_T k_T}}{M}\, f_{1T}^\perp(x,\boldsymbol{k}^2) + \frac{i\epsilon_T^{ij}\,\boldsymbol{k}\cdot\boldsymbol{S}_T}{M}\, g_{1T}(x,\boldsymbol{k}^2)\right.$$
$$\left. - \frac{\epsilon_T^{k_T\{i} S_T^{j\}} + \epsilon_T^{S_T\{i} k_T^{j\}}}{4M}\, h_1(x,\boldsymbol{k}^2) - \frac{\epsilon_T^{\{i}{}_\alpha k_T^{j\}\alpha S_T}}{2M^3}\, h_{1T}^\perp(x,\boldsymbol{k}^2)\right]$$

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$$\Phi_T(x,\boldsymbol{k}) = \frac{1}{2}\left[\frac{\not{n}\, \epsilon_T^{S_T k_T}}{M}\, f_{1T}^\perp(x,\boldsymbol{k}^2) + \frac{\gamma^5 \not{n}\, \boldsymbol{k}\cdot\boldsymbol{S}_T}{M}\, g_{1T}(x,\boldsymbol{k}^2) \right.$$
$$\left. + i\sigma_{\mu\nu}\gamma^5 \bar{n}^\mu S_T^\nu\, h_1(x,\boldsymbol{k}^2) - \frac{i\sigma_{\mu\nu}\gamma^5 \bar{n}^\mu k_T^{\nu\rho} S_{T\rho}}{M^2}\, h_{1T}^\perp(x,\boldsymbol{k}^2)\right]$$

For quarks the BM & Sivers TMDs are T-odd and the h-type functions are chiral-odd

$$\Gamma_U^{ij}(x,\boldsymbol{k}) = x\left[\delta_T^{ij}\, f_1(x,\boldsymbol{k}^2) + \frac{k_T^{ij}}{M^2}\, h_1^\perp(x,\boldsymbol{k}^2)\right],$$

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$$\Gamma_T^{ij}(x,\boldsymbol{k}) = x\left[\frac{\delta_T^{ij}\, \epsilon_T^{S_T k_T}}{M}\, f_{1T}^\perp(x,\boldsymbol{k}^2) + \frac{i\epsilon_T^{ij}\, \boldsymbol{k}\cdot\boldsymbol{S}_T}{M}\, g_{1T}(x,\boldsymbol{k}^2) \right.$$
$$\left. - \frac{\epsilon_T^{k_T\{i} S_T^{j\}} + \epsilon_T^{S_T\{i} k_T^{j\}}}{4M}\, h_1(x,\boldsymbol{k}^2) - \frac{\epsilon_T^{\{i}{}_\alpha k_T^{j\}\alpha S_T}}{2M^3}\, h_{1T}^\perp(x,\boldsymbol{k}^2)\right]$$

Parallels between quarks and gluons

$$\Phi_U(x,\boldsymbol{k}) = \frac{1}{2}\left[\not{n}\, f_1(x,\boldsymbol{k}^2) + \frac{\sigma_{\mu\nu}k_T^\mu \bar{n}^\nu}{M}\, h_1^\perp(x,\boldsymbol{k}^2)\right],$$

$$\Phi_L(x,\boldsymbol{k}) = \frac{1}{2}\left[\gamma^5\not{n}\, S_L\, g_1(x,\boldsymbol{k}^2) + \frac{i\sigma_{\mu\nu}\gamma^5 \bar{n}^\mu k_T^\nu S_L}{M}\, h_{1L}^\perp(x,\boldsymbol{k}^2)\right],$$

$$\Phi_T(x,\boldsymbol{k}) = \frac{1}{2}\left[\frac{\not{n}\,\epsilon_T^{S_T k_T}}{M}\, f_{1T}^\perp(x,\boldsymbol{k}^2) + \frac{\gamma^5\not{n}\,\boldsymbol{k}\cdot\boldsymbol{S}_T}{M}\, g_{1T}(x,\boldsymbol{k}^2) + i\sigma_{\mu\nu}\gamma^5\bar{n}^\mu S_T^\nu\, h_1(x,\boldsymbol{k}^2) - \frac{i\sigma_{\mu\nu}\gamma^5\bar{n}^\mu k_T^{\nu\rho} S_{T\rho}}{M^2}\, h_{1T}^\perp(x,\boldsymbol{k}^2)\right]$$

For quarks the BM & Sivers TMDs are T-odd and the h-type functions are chiral-odd

$$\Gamma_U^{ij}(x,\boldsymbol{k}) = x\left[\delta_T^{ij}\, f_1(x,\boldsymbol{k}^2) + \frac{k_T^{ij}}{M^2}\, h_1^\perp(x,\boldsymbol{k}^2)\right],$$

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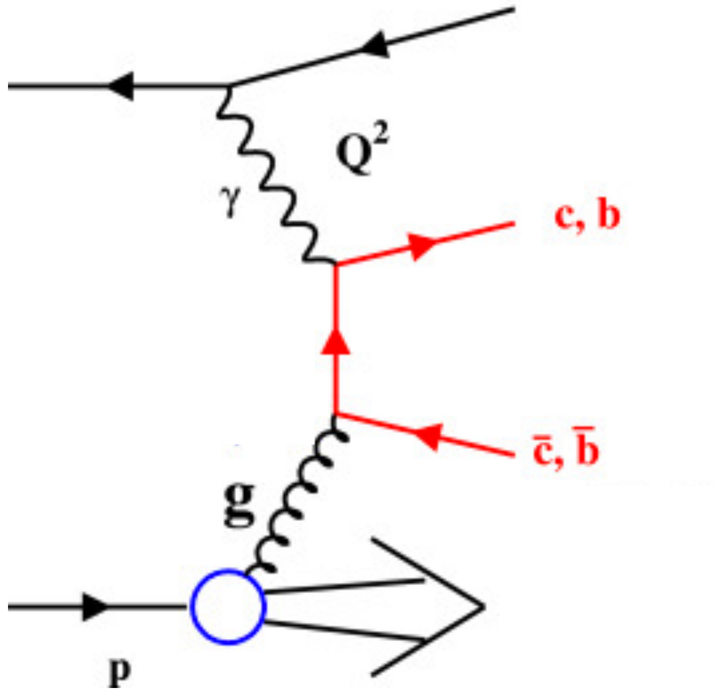
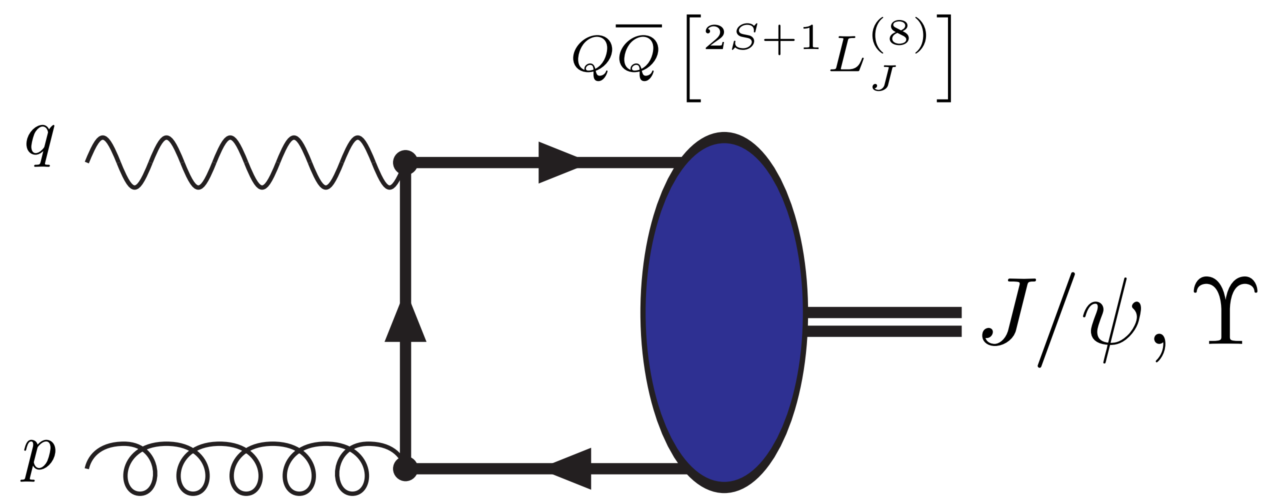
For gluons $h_1^\perp$ is T-even and h_1 is k_T-odd, T-odd and unrelated to transversity

Probing gluon TMDs using heavy quarks in ep

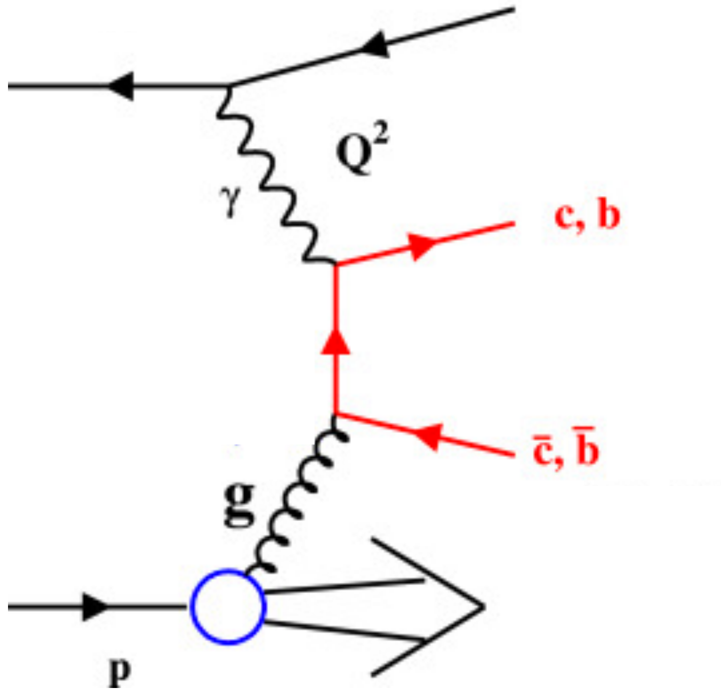


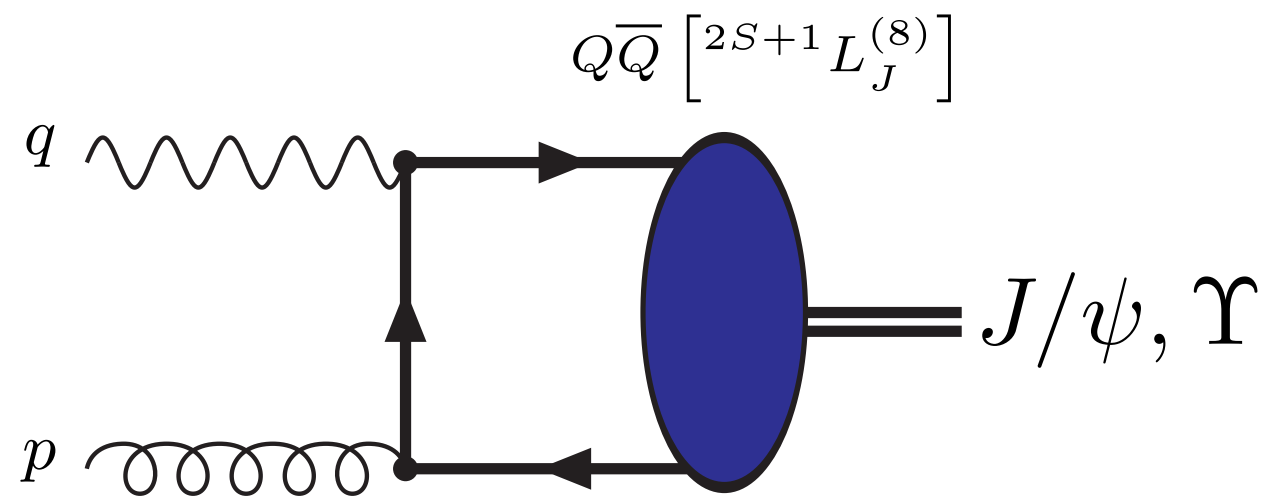


$$ep \to e' Q\bar{Q} X \qquad\qquad e\,p \to e'\,\mathcal{Q}\,X$$

Open heavy quark pair production and single quarkonium production: $[+,+]$

Probing gluon TMDs using heavy quarks in ep





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TMD factorization of the heavy quark pair or dijet production will involve a new 6 Wilson line soft factor complicating the description

Zhu, Sun, Yuan, 2013; del Castillo, Echevarria, Makris, Scimemi, 2020

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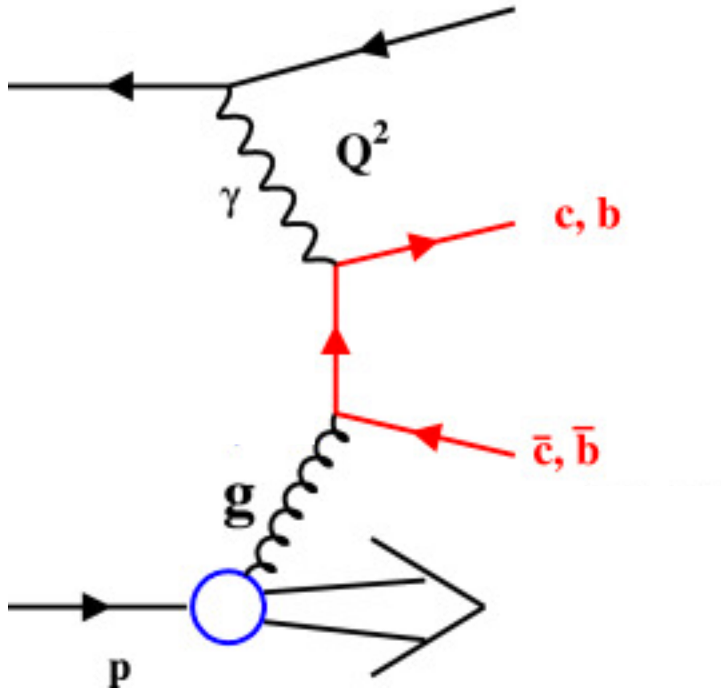

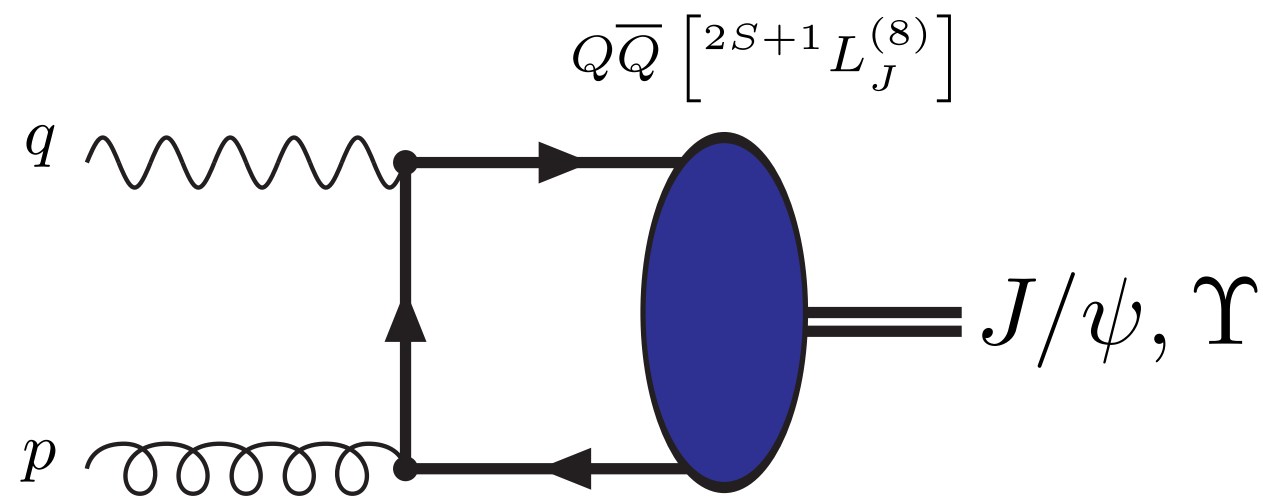

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TMD factorization of quarkonium production will involve new shape functions

Echevarria, 2019; Fleming, Makris & Mehen, 2019; Boer, D'Alesio, Murgia, Pisano, Taels, 2020; Boer, Bor, Maxia, Pisano, Yuan 2023

Open heavy quark electro-production

For the WW gluon distributions the equality does not hold, except at large $k_\perp$

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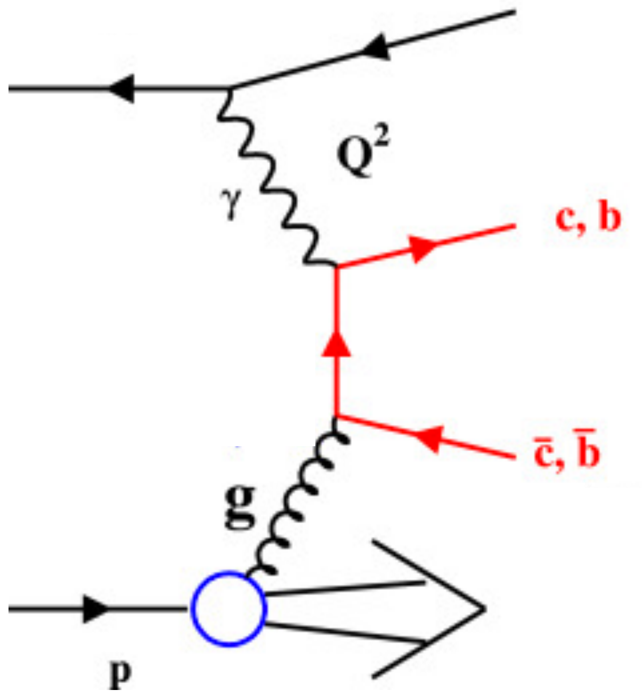
Unpolarized open heavy quark production at EIC probes the WW $h_1^{\perp\, g}(x, p_T^2)$



The heavy quarks will not be exactly back-to-back in the transverse plane:

$$K_\perp = (K_{Q\perp} - K_{\bar{Q}\perp})/2$$

$$q_T = K_{Q\perp} + K_{\bar{Q}\perp}$$

$$|q_T| \ll |K_\perp|$$

ϕ_T, $\phi_\perp$ are the angles of q_T, $K_\perp$

WW linear gluon polarization shows up as a $\cos 2\phi_T$ or $\cos 2(\phi_T - \phi_\perp)$ distribution

Boer, Brodsky, Mulders & Pisano, 2010

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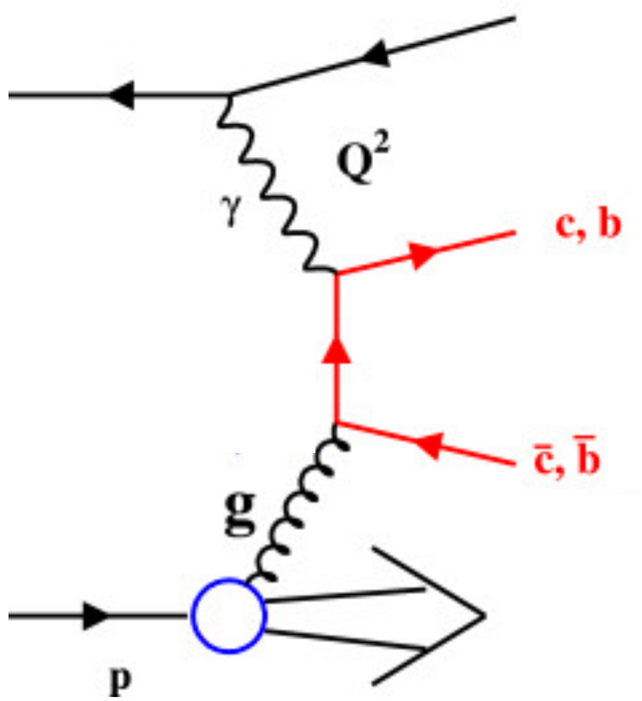

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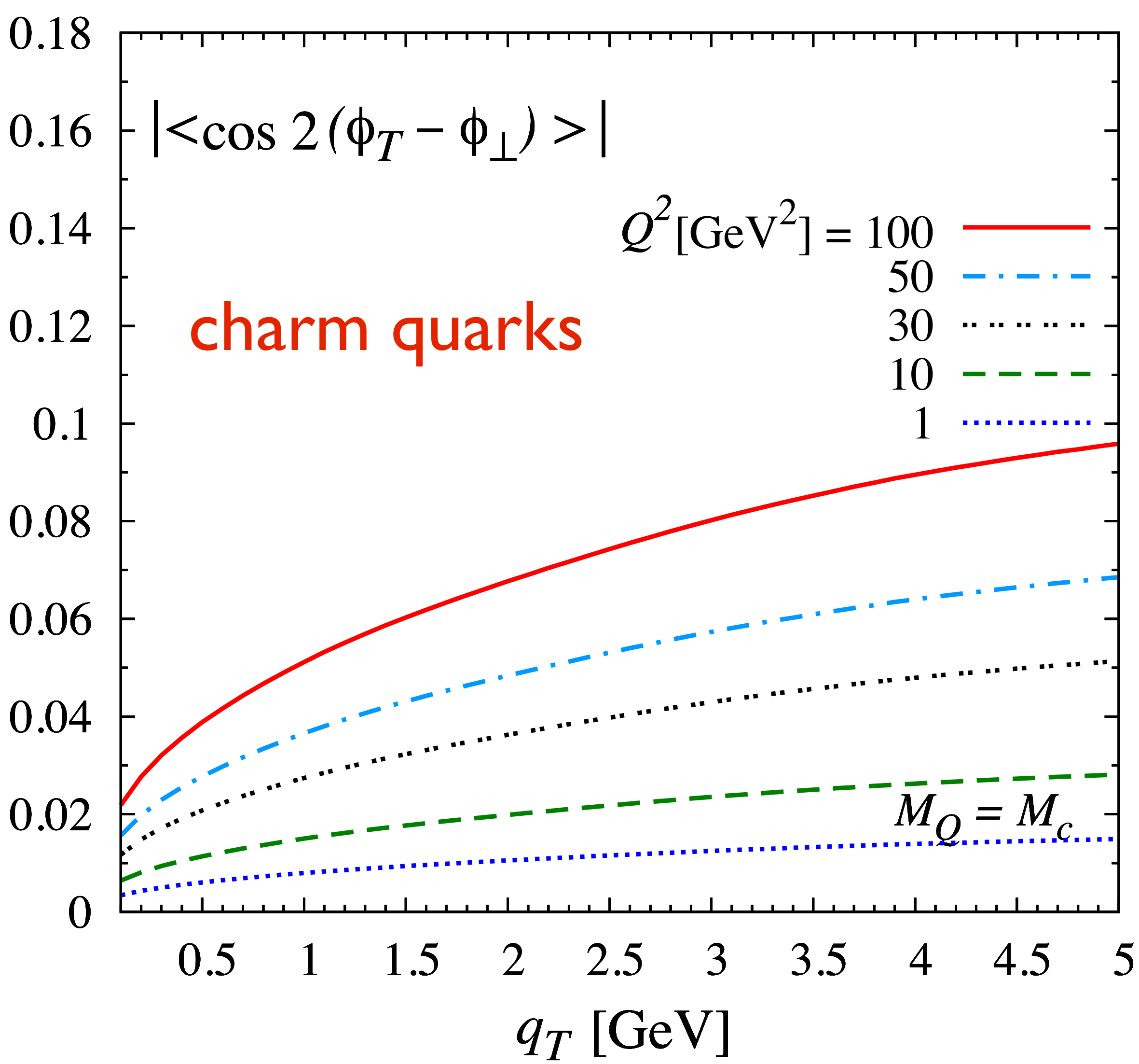
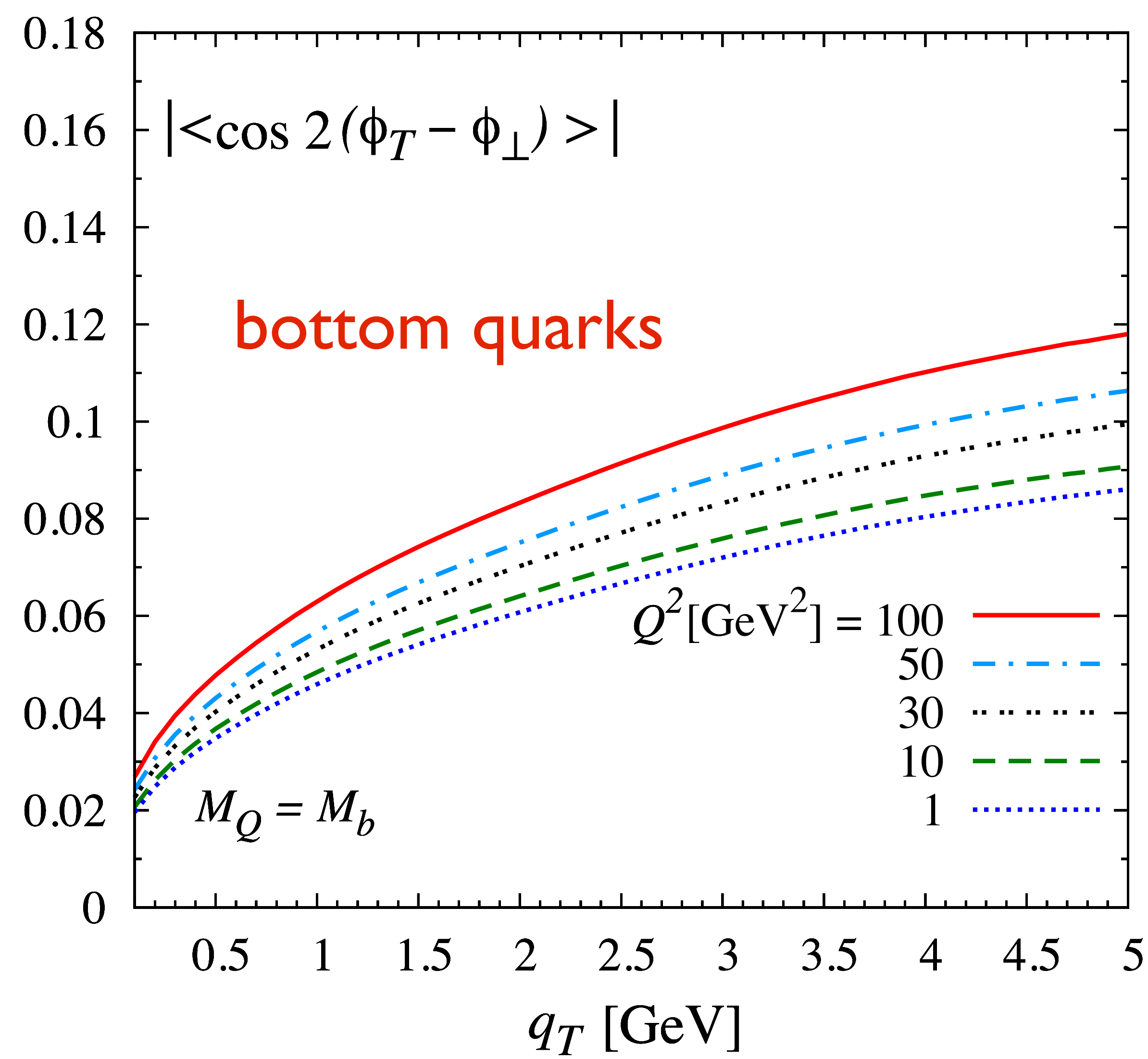
$h_1^{\perp g}$ (WW) is also accessible in inclusive dijet production at EIC

Metz, Zhou 2011; Pisano, Boer, Brodsky, Buffing, Mulders, 2013; Dumitru, Lappi, Skokov, 2015; …

Asymmetries in heavy quark pair production





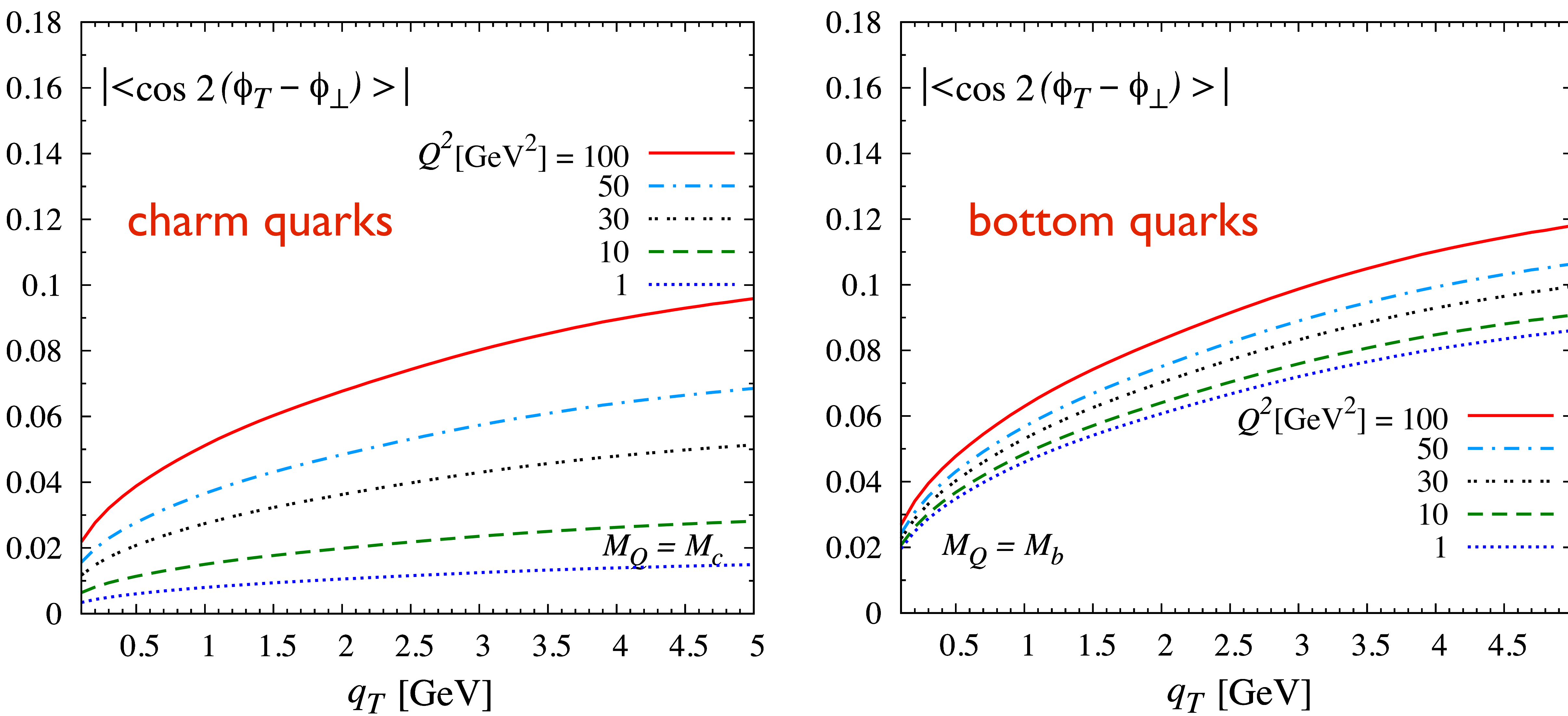
small x
MV model

$$|\boldsymbol{K}_\perp| = 10\,\mathrm{GeV}$$
$$z = 0.5$$
$$y = 0.3$$

Up to 10% asymmetries at EIC

Boer, Pisano, Mulders, Zhou, 2016

Asymmetries in heavy quark pair production



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Up to 10% asymmetries at EIC

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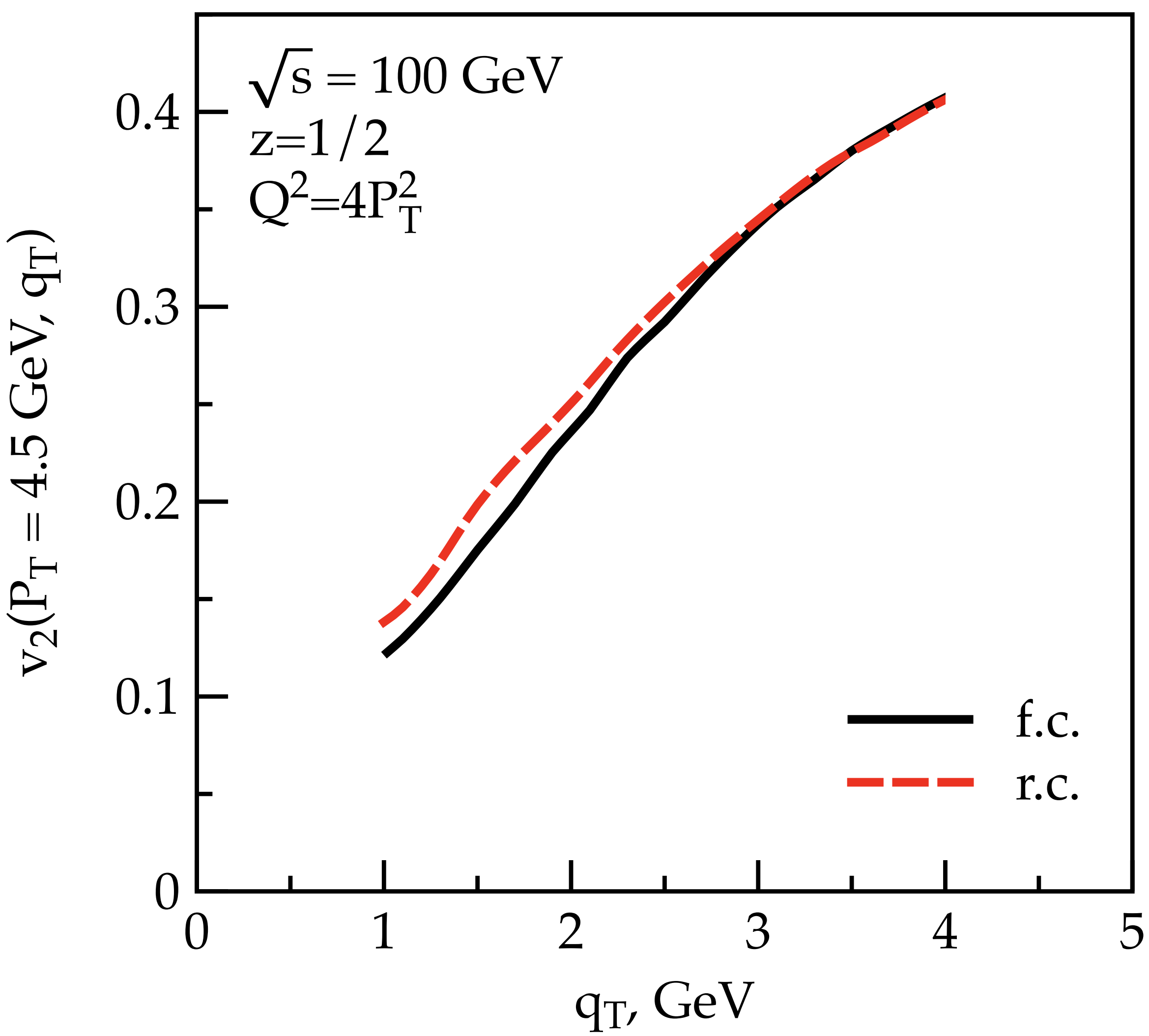
However, this does not include TMD or x-evolution

Inclusive dijet production at EIC

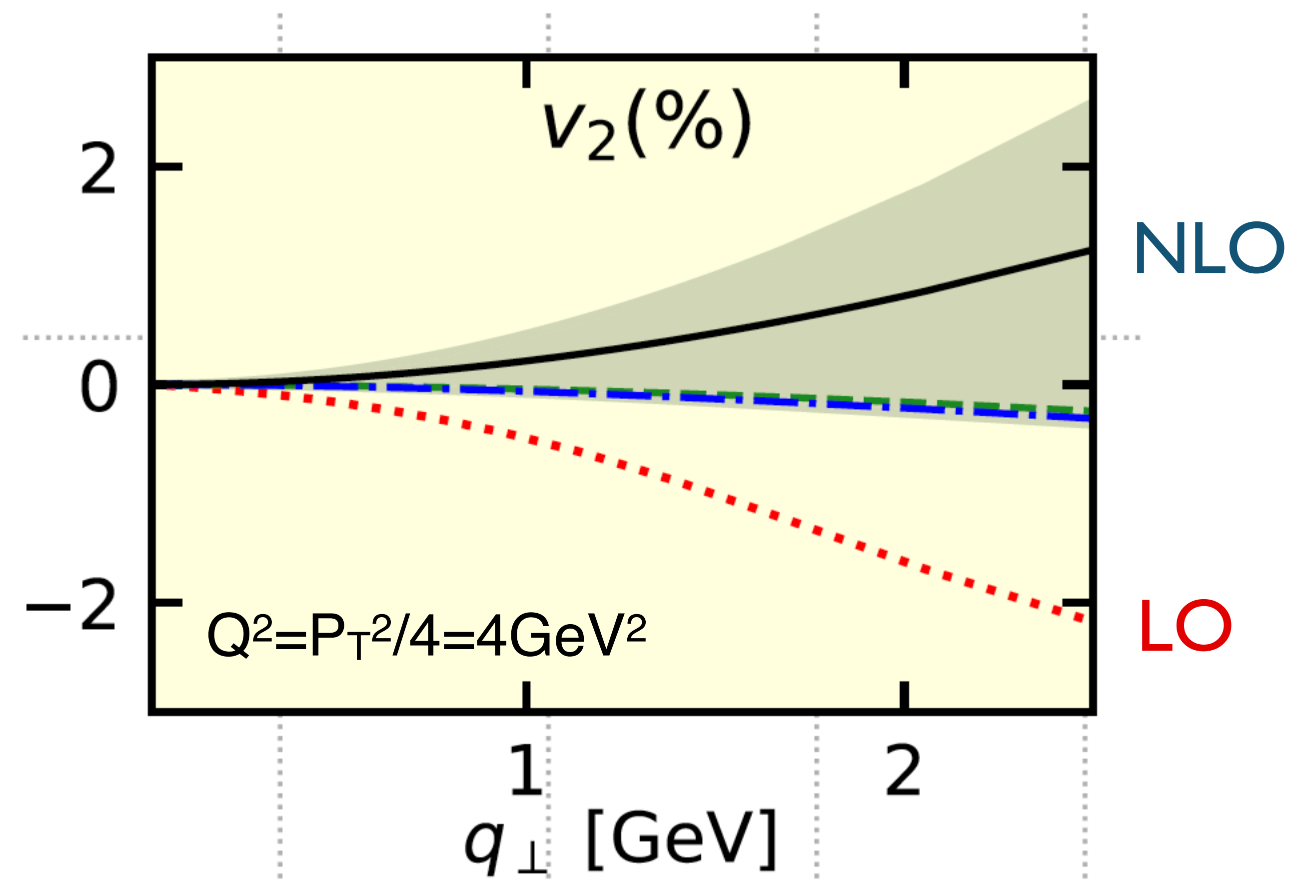
$\phi = \phi_T - \phi_\perp$

WW linear gluon polarization shows itself through a cos2ϕ distribution ("v_2")



Large effects are found

Dumitru, Lappi, Skokov, 2015



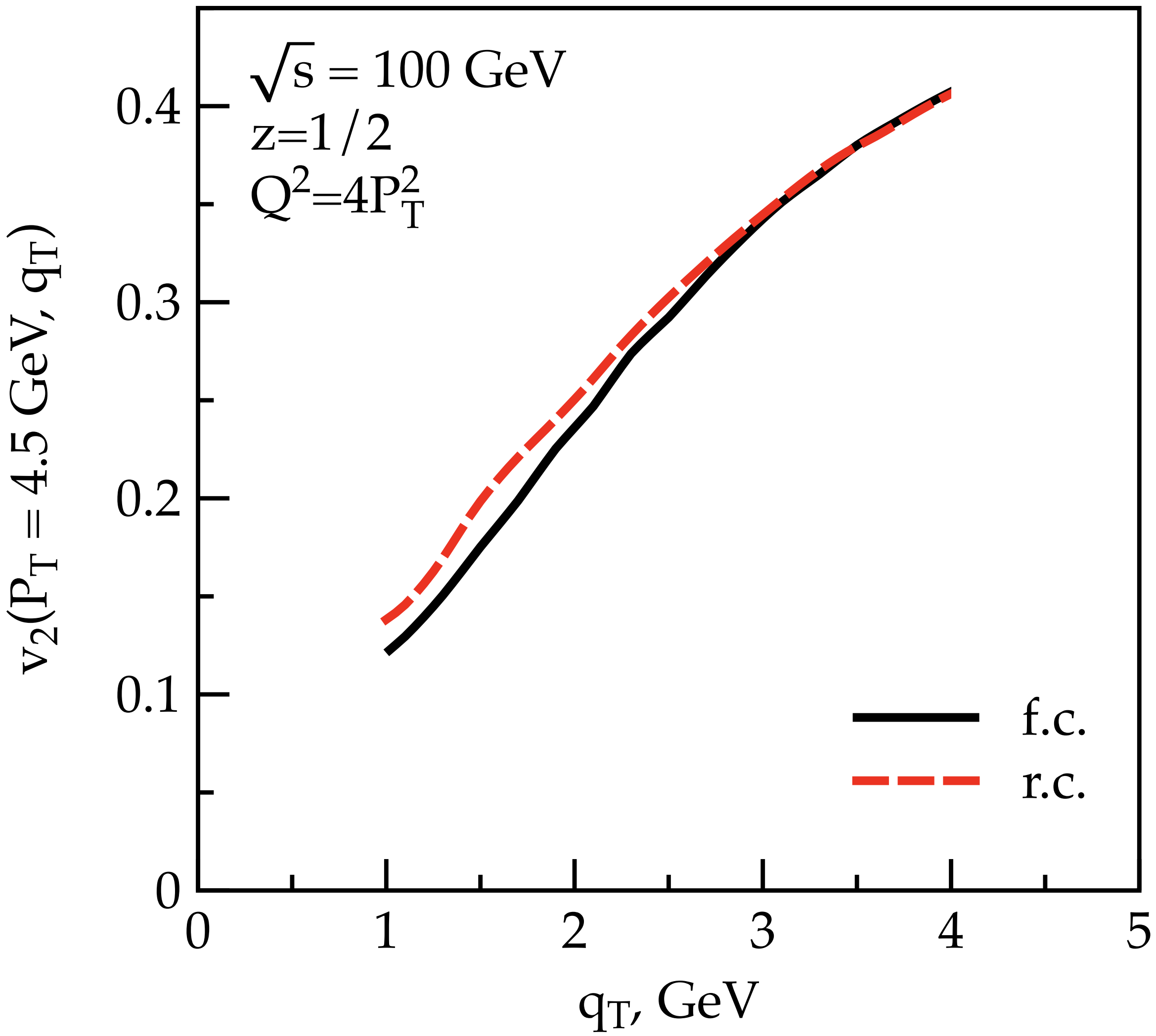
Effect of including NLO corrections

Caucal, Salazar, Schenke, Stebel, Venugopalan, 2024

Inclusive dijet production at EIC

$\phi=\phi_T-\phi_\perp$

WW linear gluon polarization shows itself through a $\cos 2\phi$ distribution ("v_2")



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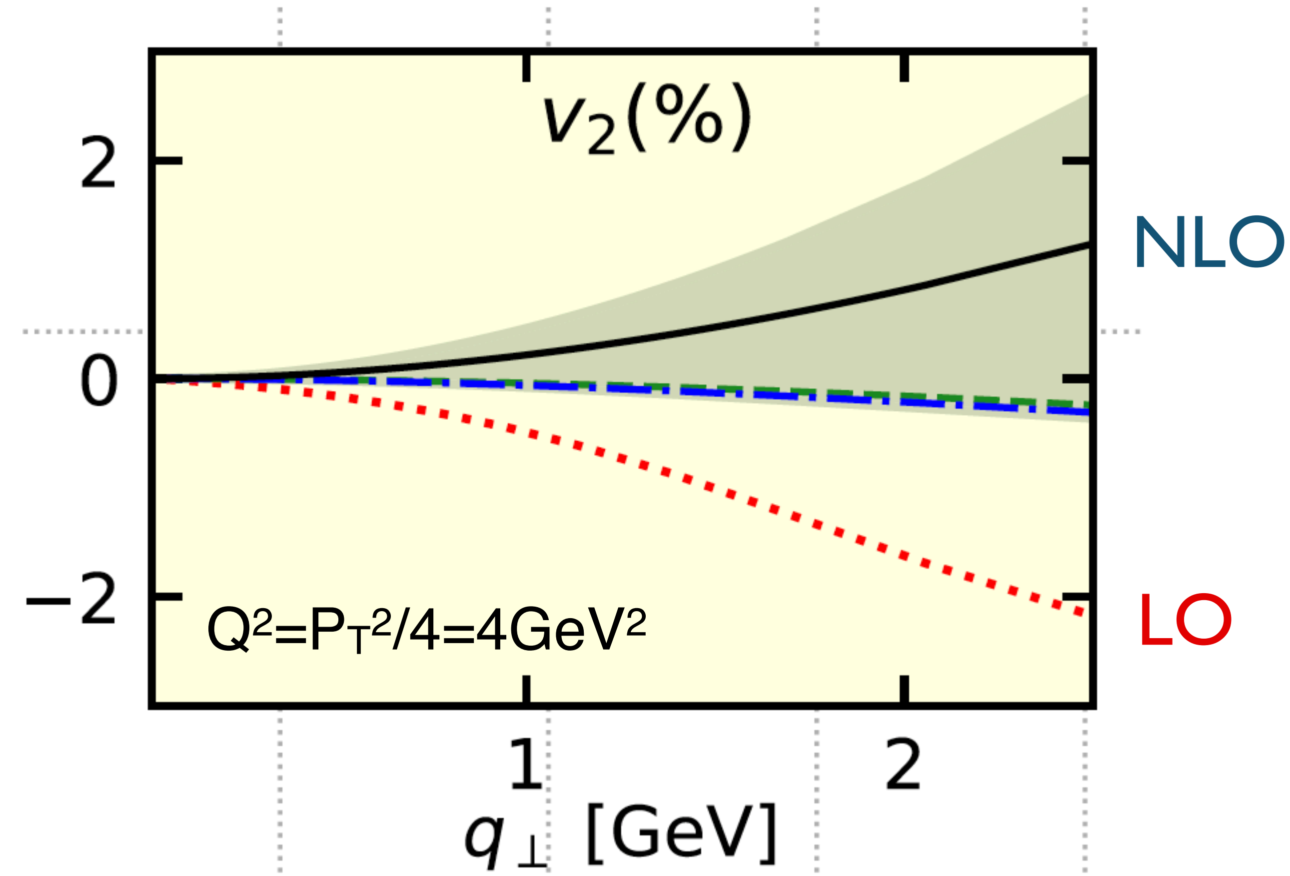
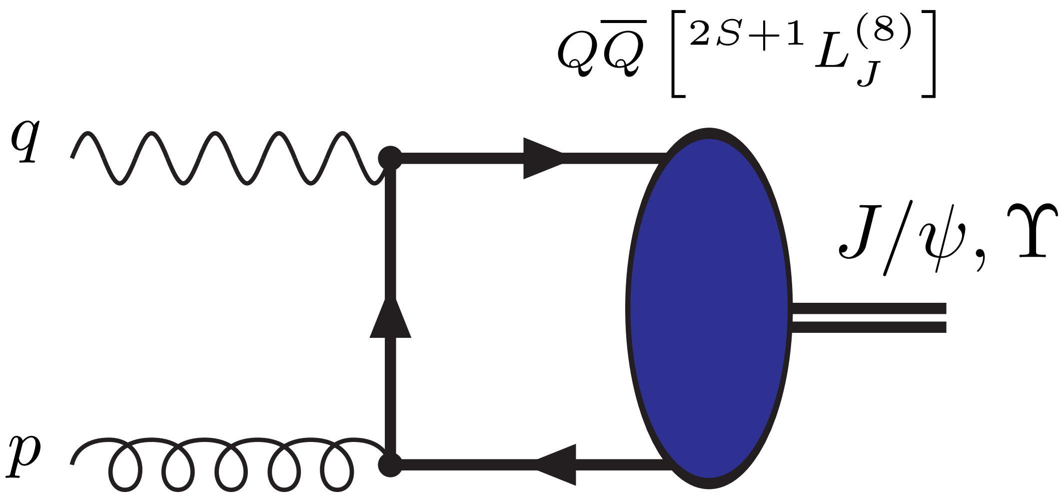


Effect of including NLO corrections

Caucal, Salazar, Schenke, Stebel, Venugopalan, 2024

Sign of v_2 is matter of definition and depends on sign of $h_1^\perp$, but it can apparently flip due to HO corrections; note that WW distribution does not satisfy same BK eq as f_1

Quarkonium production in ep



A $\cos(2\phi_T)$ asymmetry probes $h_1^{\perp\, g}$

$$\langle \cos 2\phi_T \rangle = \frac{(1-y)\,\mathcal{B}_T^{\gamma^* g\to \mathcal{Q}}}{[1+(1-y)^2]\mathcal{A}_{U+L}^{\gamma^* g\to \mathcal{Q}} - y^2 \mathcal{A}_L^{\gamma^* g\to \mathcal{Q}}} \times \frac{\boldsymbol{q}_T^2}{2M_p^2}\, \frac{h_1^{\perp\, g}(x, \boldsymbol{q}_T^2)}{f_1^g(x, \boldsymbol{q}_T^2)}.$$

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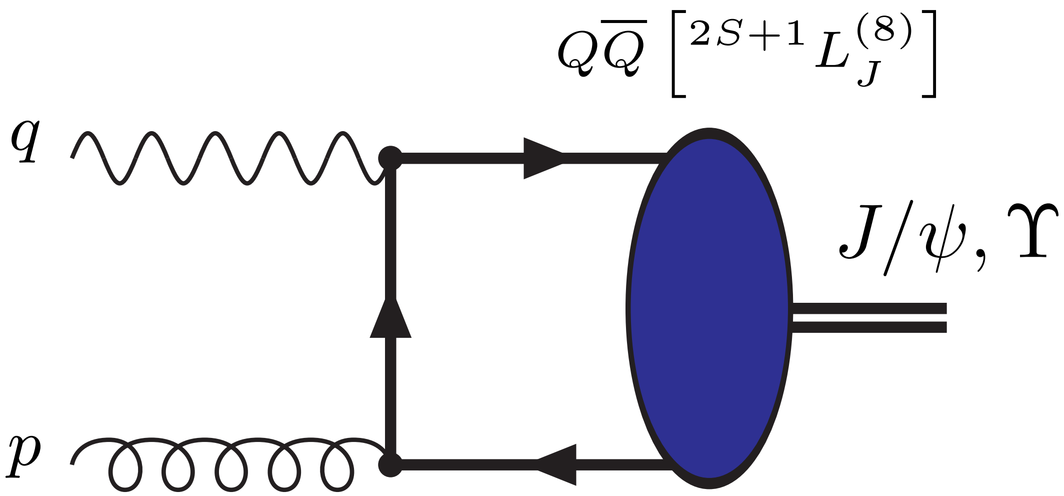

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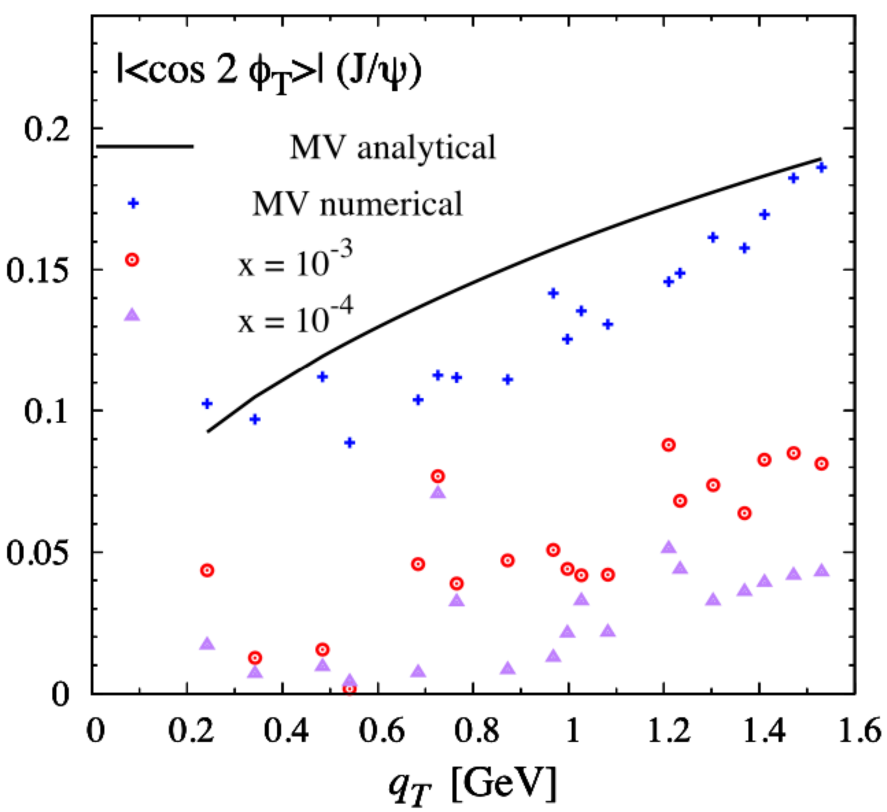
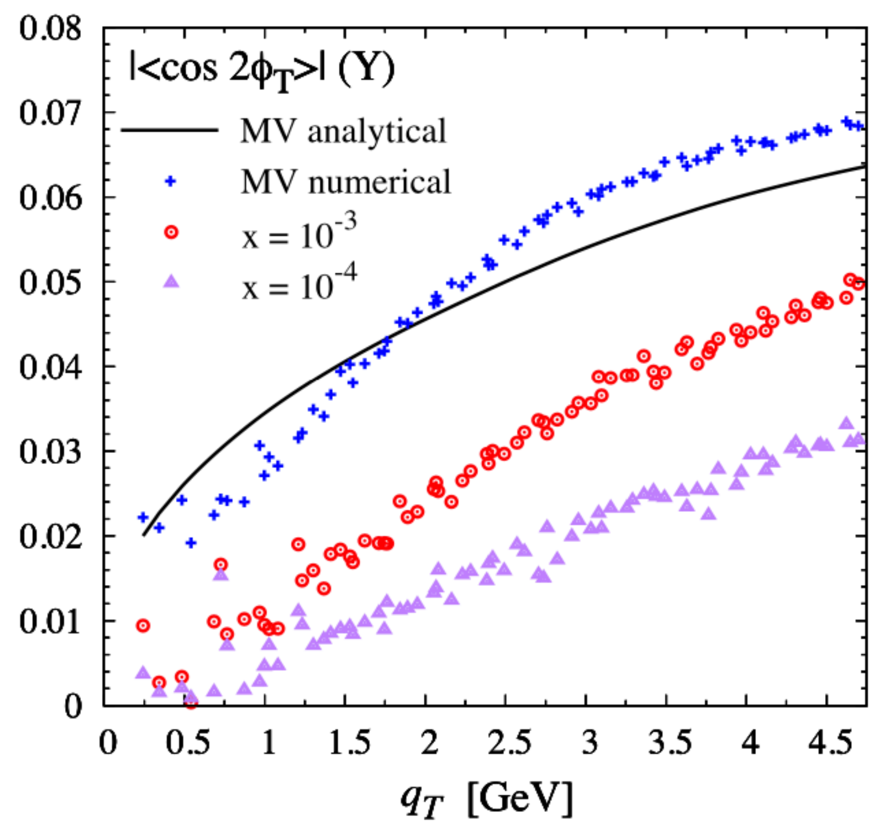




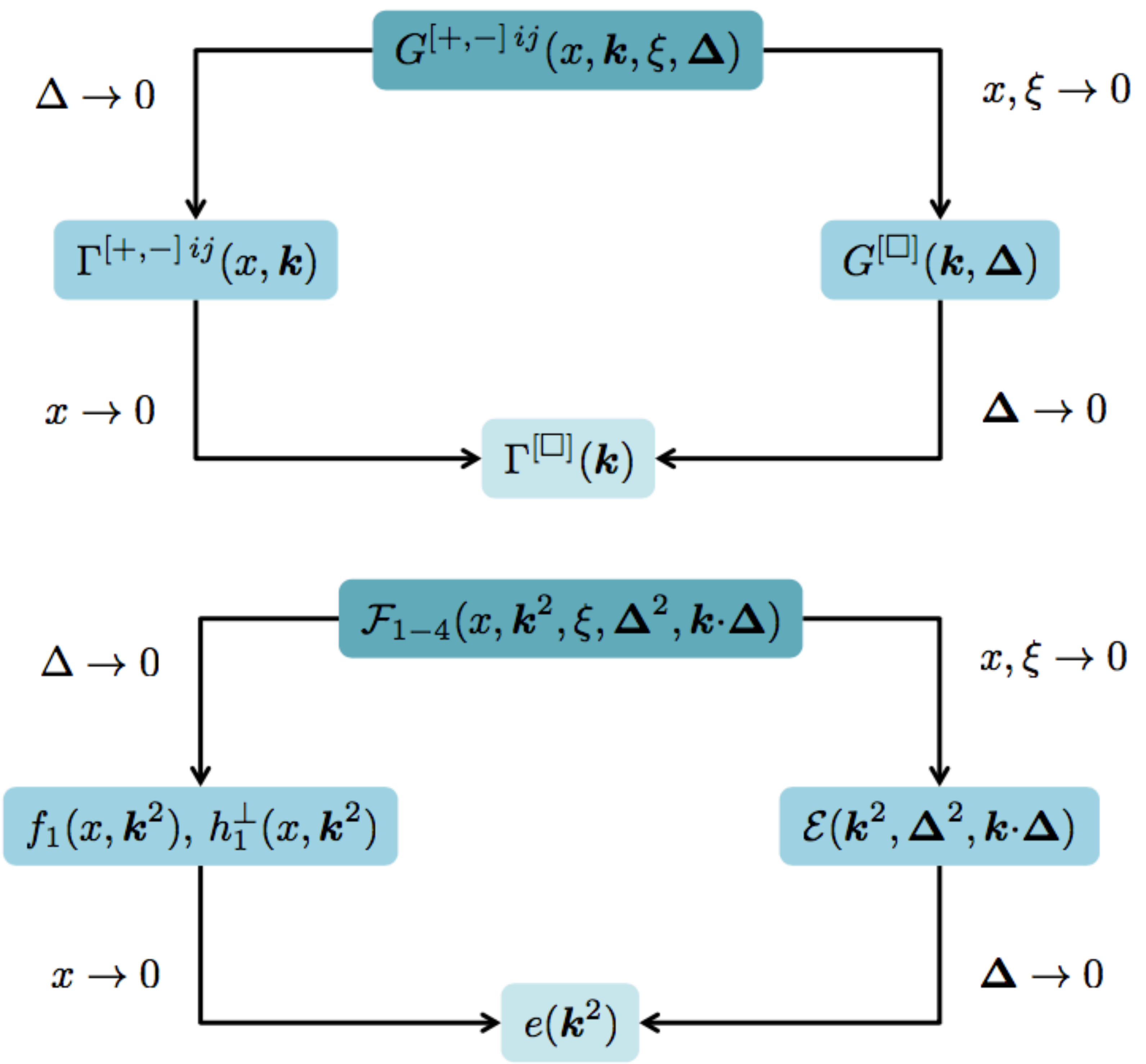
Asymmetries for $Q=M_Q$ & $y=0.1$

in the MV model and including nonlinear evolution on a 2D lattice

$\cos 2\phi_T$ asymmetry decreases towards small x

Bacchetta, Boer, Pisano, Taels, 2018

Dipole gluon GTMDs



From PhD thesis by Tom van Daal, 2018

T-odd gluon TMDs at small x

The spin-dependent odderon

J. Zhou, 2013

$$\Gamma^{\mu\nu}_{\text{T-odd}}(x, k_T; S_T) = \frac{k_T^\mu k_T^\nu N_c}{2\pi^2 \alpha_s x} \frac{\epsilon_T^{\alpha\beta} S_{T\alpha} k_{T\beta}}{M} O_{1T}^{\perp}(x, k_T^2)$$

Implies

$$x f_{1T}^{\perp g} = x h_{1T}^{g} = x h_{1T}^{\perp g} = \frac{-k_T^2 N_c}{4\pi^2 \alpha_s} O_{1T}^{\perp}(x, k_T^2)$$

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$$\Delta\Gamma^{ij}(x, \boldsymbol{k}_T) = \frac{x}{2}\Bigg[\frac{g_T^{ij}\,\epsilon_T^{k_T S_T}}{M} f_{1T}^{\perp}(x, \boldsymbol{k}_T^2) + i\epsilon_T^{ij}\, g_{1s}(x, \boldsymbol{k}_T^2)$$
$$- \frac{\epsilon_T^{k_T\{i} S_T^{j\}} + \epsilon_T^{S_T\{i} k_T^{j\}}}{4M} h_{1T}(x, \boldsymbol{k}_T^2) - \frac{\epsilon_T^{k_T\{i} k_T^{j\}}}{2M^2} h_{1s}^{\perp}(x, \boldsymbol{k}_T^2)\Bigg]$$

$$\lim_{x\to 0} x f_{1T}^{\perp}(x, \boldsymbol{k}_T^2) = \lim_{x\to 0} x h_1(x, \boldsymbol{k}_T^2) = -\frac{\boldsymbol{k}_T^2}{2M^2} \lim_{x\to 0} x h_{1T}^{\perp}(x, \boldsymbol{k}_T^2) = \frac{1}{2} \lim_{x\to 0} x h_{1T}(x, \boldsymbol{k}_T^2)$$

$$h_1(x, \boldsymbol{k}_T^2) \equiv h_{1T}(x, \boldsymbol{k}_T^2) + \frac{\boldsymbol{k}_T^2}{2M^2} h_{1T}^{\perp}(x, \boldsymbol{k}_T^2)$$

Similar relations hold for spin-1 hadrons as well

DB, Cotogno, van Daal, Mulders, Signori & Ya-Jin Zhou, 2016

MV-like model

We consider the MV-like model:

$$\mathcal{F}^{[\Box]}(\boldsymbol{k}_\perp, \boldsymbol{\Delta}_\perp) = 4N_c \int \frac{d^2\boldsymbol{r}_\perp d^2\boldsymbol{b}_\perp}{(2\pi)^2}\, e^{-i\boldsymbol{k}_\perp\cdot\boldsymbol{r}_\perp} e^{i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp} e^{-\epsilon_r r_\perp^2} \left[1 - \exp\left(-\frac{1}{4} r_\perp^2 \chi Q_s^2(b_\perp) \ln\left[\frac{1}{r_\perp^2\Lambda^2} + e\right]\right)\right]$$

Similar expression as considered by Hagiwara, Hatta, Pasechnik, Tasevsky & Teryaev, 2017; Salazar, Schenke, 2019

ϵ_r cuts out the region where the dipole size becomes large compared to the target size, where the model should not be applicable

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χ sets the normalization of Q_s and is x dependent (of GBW form)

$$\chi(x) = \bar{\chi}\left(\frac{x_0}{x}\right)^\lambda \qquad x_0 = 3\times 10^{-4} \quad \lambda = 0.29$$

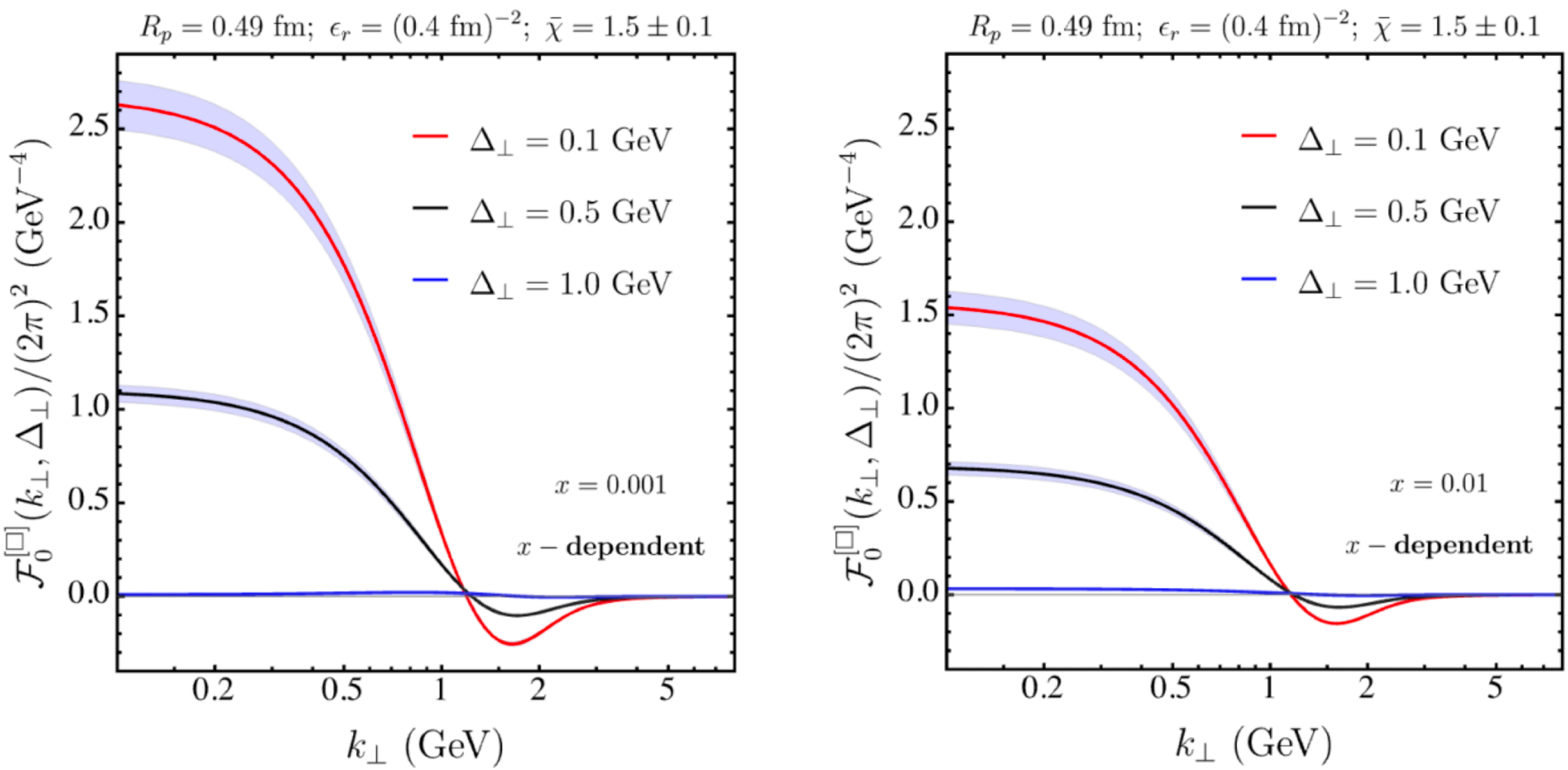
Q_s is proportional to the proton (Gaussian) or nuclear (Woods-Saxon) profile

For details see DB, Setyadi, 2023

x-dependent gluon GTMD model

$$\mathcal{F}^{[\Box]}(\boldsymbol{k}_\perp, \boldsymbol{\Delta}_\perp) = \mathcal{F}_0^{[\Box]}(k_\perp, \Delta_\perp) + 2\mathcal{F}_2^{[\Box]}(k_\perp, \Delta_\perp)\cos 2\theta_{k\Delta} + \ldots$$

The data is angle integrated, therefore we restrict to $\mathcal{F}_0$



Dominant contribution from: $\Delta_\perp \ll K_\perp$ or M_V

Dihadron production through DPS

$\mathcal{W}_1$ does lead to odd harmonics in dihadron production through double parton scattering in pA collisions (not exclusive DPS in this case)

$$\frac{d\sigma_{\rm DPS}^{pA\to h_1 h_2 X}}{dy_1 dy_2\, d^2\boldsymbol{k}_1 d^2\boldsymbol{k}_2} \propto \int d^2\boldsymbol{b}_1\, d^2\boldsymbol{b}_2\, F_p(x_1, x_2, \boldsymbol{b}_1 - \boldsymbol{b}_2) \int \frac{d^2\boldsymbol{r}_1 d^2\boldsymbol{r}_2}{(2\pi)^4}\, e^{-i\boldsymbol{k}_1\cdot\boldsymbol{r}_1 - i\boldsymbol{k}_2\cdot\boldsymbol{r}_2}$$
$$\times \left\langle S\left(\boldsymbol{b}_1 + \tfrac{\boldsymbol{r}_1}{2}, \boldsymbol{b}_1 - \tfrac{\boldsymbol{r}_1}{2}\right)\, S\left(\boldsymbol{b}_2 + \tfrac{\boldsymbol{r}_2}{2}, \boldsymbol{b}_2 - \tfrac{\boldsymbol{r}_2}{2}\right)\right\rangle$$

Lappi, Schenke, Schlichting, Venugopalan, 2016

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Lappi, Schenke, Schlichting, Venugopalan, 2016

In the large N_c limit this factorizes further:

$$\frac{d\sigma_{\rm DPS}^{pA\to h_1 h_2 X}}{dy_1 dy_2\, d^2\boldsymbol{k}_1 d^2\boldsymbol{k}_2} \propto \int d^2\boldsymbol{b}_1\, d^2\boldsymbol{b}_2\, F_p(x_1, x_2, \boldsymbol{b}_1 - \boldsymbol{b}_2)\, xW(x, \boldsymbol{b}_1, \boldsymbol{k}_1)\, xW(x, \boldsymbol{b}_2, \boldsymbol{k}_2)$$

Simplify further assuming a Gaussian form for the double quark distribution:

$$F_p(x_1, x_2, \boldsymbol{b}_1 - \boldsymbol{b}_2) = f_p(x_1, x_2)\, \frac{1}{4\pi R_N^2}\, e^{-\frac{(\boldsymbol{b}_1 - \boldsymbol{b}_2)^2}{4R_N^2}}$$

Directed flow

$$\frac{d\sigma_{\rm DPS}^{pA\to h_1 h_2 X}}{dy_1 dy_2\, d^2\boldsymbol{k}_1 d^2\boldsymbol{k}_2} \propto \frac{\pi}{8R_N^2}\, f_p(x_1,x_2) \int db_1^2\, db_2^2\; e^{-\frac{b_1^2+b_2^2}{4R_N^2}}$$
$$\times \Bigg[2I_0\left(\frac{b_1 b_2}{2R_N^2}\right) x\mathcal{W}_0(x,\boldsymbol{b}_1^2,\boldsymbol{k}_1^2)\, x\mathcal{W}_0(x,\boldsymbol{b}_2^2,\boldsymbol{k}_2^2)$$
$$+ 4\cos(\phi_{k_1}-\phi_{k_2})\, I_1\left(\frac{b_1 b_2}{2R_N^2}\right) x\mathcal{W}_1(x,\boldsymbol{b}_1^2,\boldsymbol{k}_1^2)\, x\mathcal{W}_1(x,\boldsymbol{b}_2^2,\boldsymbol{k}_2^2)$$
$$+ 4\cos 2(\phi_{k_1}-\phi_{k_2})\, I_2\left(\frac{b_1 b_2}{2R_N^2}\right) x\mathcal{W}_2(x,\boldsymbol{b}_1^2,\boldsymbol{k}_1^2)\, x\mathcal{W}_2(x,\boldsymbol{b}_2^2,\boldsymbol{k}_2^2)\Bigg]$$
$$+ \ldots$$

This shows the cross section displays directed flow (v_1)

This can arise from azimuthal anisotropy (rotationally non-invariant targets)

Dumitru, Giannini 2015; Dumitru, Skokov, 2015; Lappi, Schenke, Schlichting, Venugopalan, 2016

but also without breaking of rotational symmetry (C-odd squared effect)

Boer, van Daal, Mulders, Petreska, 2018