Jet definition and TMD factorisation in SIDIS

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in collaboration with E. Iancu, A. H. Mueller, and F. Yuan (arXiv:2408.03129)

Goals and outline

This talk, in a nutshell:

A small-x perspective on TMD factorisation for single-inclusive **jet** production in DIS.

- First calculation of Sudakov logs in SIDIS with jets and their dependence on the jet algorithm.
- New asymmetric jet distance measure which ensures TMD factorisation.
- Emergence of the DGLAP and CSS evolutions from the small x approach.
  $\Rightarrow$ combined small x, CSS and DGLAP evolution within the TMD formalism.

Single inclusive jet production in DIS

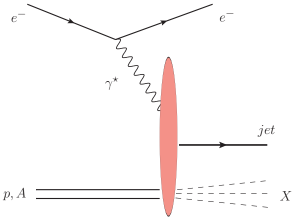
[PC, Iancu, Mueller, Yuan, 2408.03129]

⇒ Measure **jets** in DIS events and bin in terms of $P_\perp$ measured in Breit (or dipole) frame:

$$\frac{\mathrm{d}\sigma^{eA\to \mathrm{e'+jet}+X}}{\mathrm{d}x_{\mathrm{Bj}}\mathrm{d}Q^2\mathrm{d}P_\perp}$$

⇒ In the case of a hadron measurement, see Jamal Jalilian-Marian's talk just before.

⇒ Also accesses the sea quark TMD at small x in the limit $Q^2 \gg \boldsymbol{P}_\perp^2$.



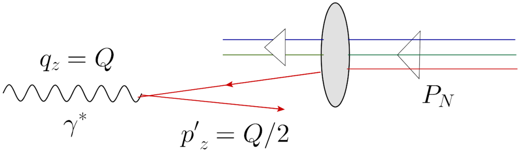
Breit frame and target picture at LO

- Breit frame: head-on $\gamma^* + A$ collision.
  $q^\mu = (0, 0, 0, q_z = Q)$

- Photon absorbed by the struck quark.

- Quark produced with $\boldsymbol{P}_\perp = \boldsymbol{0}_\perp$:

$$\left.\frac{\mathrm{d}\sigma^{\gamma^\star_{\mathrm{T}}+A\to q+X}}{\mathrm{d}^2\boldsymbol{P}_\perp}\right|_{\mathrm{LO}} = \frac{4\pi^2\alpha_{\mathrm{em}}e_f^2}{Q^2}\delta^{(2)}(\boldsymbol{P}_\perp)xf_q(x)$$



- Dominated by aligned jet configurations $z = \frac{k_{\mathrm{jet}}\cdot P}{P\cdot q} \sim 1$.

Breit frame and target picture at LO and small x

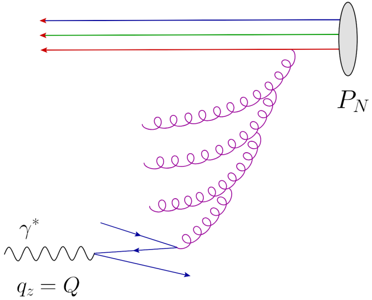
- At small x, rise of the gluon distribution ($\lambda \sim 0.2 - 0.3$)

$$Q_s^2(x) \sim \alpha_s \frac{xG(x, Q_s^2)}{\pi A^{2/3}} \sim \frac{A^{1/3}}{x^\lambda}$$

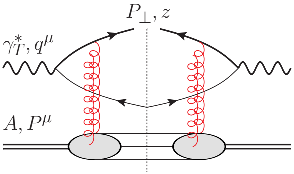
- **Sea** quark comes from a $g \to q\bar{q}$ splitting.

- For $Q^2 \gg P_\perp^2 \gg Q_s^2$, this splitting is DGLAP-like:

$$\left.\frac{\mathrm{d}\sigma^{\gamma_{\mathrm{T}}^\star + A \to q + X}}{\mathrm{d}^2 \boldsymbol{P}_\perp}\right|_{\mathrm{LO}} = \frac{8\pi^2 \alpha_{\mathrm{em}} e_f^2}{Q^2} \frac{\alpha_s}{2\pi^2} \frac{1}{P_\perp^2} \int_x^1 \mathrm{d}\xi P_{qg}(\xi) \frac{x}{\xi} G\left(\frac{x}{\xi}, P_\perp^2\right)$$



TMD factorisation in SIDIS at LO from the dipole picture



- Longitudinal boost to the dipole frame with $q^0 \sim q_z \gg Q$: $\gamma^* \to q\bar{q}$ splitting+interaction with the "shockwave" (CGC EFT). [Mueller (1990), Nikolaev and Zakharov (1991)]

- For $Q^2 \gg P_\perp^2, Q_s^2$, the CGC result factorises in terms of the (sea) quark TMD $x\mathcal{F}_q(x, P_\perp^2)$ [Marquet, Xiao, Yuan, PLB 682, 207 (2009)]

$$\left.\frac{\mathrm{d}\sigma^{\gamma_T^\star + A \to \mathrm{jet}+X}}{\mathrm{d}^2 \boldsymbol{P}_\perp}\right|_{\mathrm{LO}} = \frac{8\pi^2 \alpha_{\mathrm{em}} e_f^2}{Q^2} \times \underbrace{\frac{N_c}{\pi^2}\int_{\boldsymbol{b}_\perp}\int \frac{\mathrm{d}^2\boldsymbol{q}_\perp}{(2\pi)^2}\mathcal{D}(x,\boldsymbol{q}_\perp)\left[1 - \frac{\boldsymbol{P}_\perp \cdot (\boldsymbol{P}_\perp - \boldsymbol{q}_\perp)}{(P_\perp^2 - (\boldsymbol{P}_\perp - \boldsymbol{q}_\perp)^2)}\ln\frac{P_\perp^2}{(\boldsymbol{P}_\perp - \boldsymbol{q}_\perp)^2}\right]}_{\text{sea quark TMD}}$$

$$\underset{P_\perp \gg Q_s}{\longrightarrow} \frac{8\pi^2 \alpha_{\mathrm{em}} e_f^2}{Q^2}\frac{\alpha_s}{2\pi^2}\frac{1}{P_\perp^2}\int_x^1 \mathrm{d}\xi P_{qg}(\xi)\frac{x}{\xi} G\left(\frac{x}{\xi}, P_\perp^2\right)$$

Outline of the NLO computation in the dipole picture

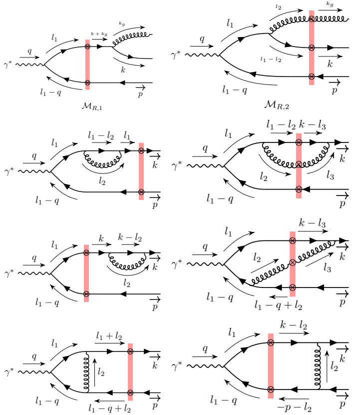
- NLO calculation at small x for general **jet** kinematics performed in [PC, Ferrand, Salazar, JHEP 05 (2024) 110].
  For single hadron, see [Bergabo, Jalilian-Marian, JHEP 01 (2023) 095 (inclusive), Fucilla, Grabovsky, Li, Szymanowski, Wallon, JHEP 02 (2024) 165 (diffractive)]
  See Michael Fucilla's talk after.

- Compute the NLO impact factor in the limit $Q^2 \gg P_\perp^2 \gg Q_s^2$.

$$\left.\frac{\mathrm{d}\sigma_{\mathrm{CGC}}^{\gamma_{\mathrm{T}}^\star + A \to q + X}}{\mathrm{d}^2 \boldsymbol{P}_\perp}\right|_{\mathrm{NLO}} = \left.\frac{\mathrm{d}\sigma_{\mathrm{CGC}}^{\gamma_{\mathrm{T}}^\star + A \to q + X}}{\mathrm{d}^2 \boldsymbol{P}_\perp}\right|_{\mathrm{LO}} \left[1 + \alpha_s \mathcal{I}(\boldsymbol{P}_\perp, Q, R, x_\star)\right]$$

- High energy factorisation with collinearly improved BK/BFKL. [Altinoluk, Jalilian-Marian, Marquet, 2406.08277]

- NLO impact factor depends on the jet definition.



NLO Feynman graphs

Jet definitions in DIS

- Jet definitions designed to ensure factorisation of inclusive jet cross sections in terms of universal pdf. Catani, Dokshitzer, Webber, PLB 285, 291 (1992), Webber, J. Phys. G 19, 1567 (1993)
- Longitudinally invariant "generalized-k_t" algorithms.

$$d_{ij} = \min(p_{t,i}^{2k}, p_{t,j}^{2k}) \frac{\Delta R_{ij}^2}{R^2}\,, \quad d_{iB} = p_{t,i}^{2k}$$

Catani, Dokshitzer, Seymour, Webber, NPB, 406 (1993), Cacciari, Salam, Soyez, JHEP 0804:063,2008

⇒ Many jet analysis at HERA chose longitudinally invariant k_t algorithm in the Breit frame. Ex: α_s extraction from jet cross-sections, ZEUS, PLB 547 (2002), H1 PLB 653, 134 (2007), ...

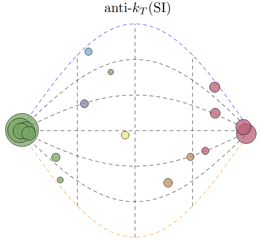
- e^+e^- spherically invariant jet definitions in the Breit frame.

$$d_{ij} = \min(E_i^{2k}, E_j^{2k}) \frac{1-\cos(\theta_{ij})}{1-\cos(R)}\,, \quad d_{iB} = E_i^{2k}$$

⇒ Recent studies on TMD factorisation with jets use this definition with WTA scheme. Gutierrez-Reyes, Scimemi, Waalewijn, Zoppi, PRL 121, (2018), JHEP 10, 031 (2019)

Known issues with previous options

- Spherically invariant jet definitions in the Breit frame are not boost invariant.
  $\Rightarrow$ Hard to distinguish beam remnant from forward jets.



- Longitudinally invariant jet definitions in Breit frame fail to cluster hadrons in the backward region.

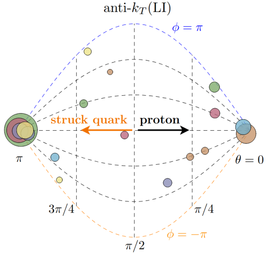


Fig. from Arratia, Makris, Neill, Ringer, Sato, 2006.10751

Jet definition in DIS: summary

- Not all jet definitions ensure factorisation of the fully inclusive **jet** cross section.

  [Catani, Dokshitzer, Webber, PLB 285, 291 (1992), Webber, J. Phys. G 19, 1567 (1993)]

- Does the same phenomenon arise for TMD factorisation?

Jet def	distance measure	dipole frame NLO clustering condition ($R \ll 1$)
LI C/A	$d_{ij} = \frac{\Delta R_{ij}^2}{R^2}$	$\frac{M_{ij}^2 z^2}{\boldsymbol{P}_\perp^2 R^2} \leq 1$
SI C/A	$d_{ij} = \frac{1-\cos(\theta_{ij})}{1-\cos(R)}$ **in Breit frame**	$\frac{M_{ij}^2 z^2}{Q^2 R^2} \leq 1$
new LI jet def	$d_{ij} = M_{ij}^2/(z_i z_j Q^2 R^2)$	$\frac{M_{ij}^2}{z_i z_j Q^2 R^2} \leq 1$

See also Centauro algorithm, Arratia, Makris, Neill, Ringer, Sato, 2006.10751

- $M_{ij}^2 = (k_i + k_j)^2$, $z_i = (k_i \cdot P)/(P \cdot q) = k_i^+/q^+$.
- Goal: find a jet definition which ensures TMD factorisation of the single inclusive jet cross-section.

Sudakov logarithms in the NLO impact factor

- NLO Sudakov logs $L=\ln(Q^2/\boldsymbol{P}_\perp^2)$ depend on the jet definition!

For LI C/A (or anti-k_t):

$$\left.\frac{\mathrm{d}\sigma^{\gamma_{\mathrm{T}}^*+A\to j+X}}{\mathrm{d}^2\boldsymbol{P}_\perp}\right|_{\mathrm{NLO}}=\left.\frac{\mathrm{d}\sigma^{\gamma_{\mathrm{T}}^*+A\to j+X}}{\mathrm{d}^2\boldsymbol{P}_\perp}\right|_{\mathrm{LO}}\times\frac{\alpha_s C_F}{\pi}\left[-\frac{3}{4}L^2+\left(\frac{3}{4}-\ln(R)\right)L+\mathcal{O}(1)\right]$$

while for SI C/A ($\beta=2$) and our new jet definition ($\beta=0$)

$$\left.\frac{\mathrm{d}\sigma^{\gamma_{\mathrm{T}}^*+A\to j+X}}{\mathrm{d}^2\boldsymbol{P}_\perp}\right|_{\mathrm{NLO}}=\left.\frac{\mathrm{d}\sigma^{\gamma_{\mathrm{T}}^*+A\to j+X}}{\mathrm{d}^2\boldsymbol{P}_\perp}\right|_{\mathrm{LO}}\times\frac{\alpha_s C_F}{\pi}\left[-\frac{1}{4}L^2+\left(\frac{3(1-\beta/2)}{4}+\ln(R)\right)L+\mathcal{O}(1)\right]$$

- From CSS evolution of the quark TMD alone, we expect the log structure

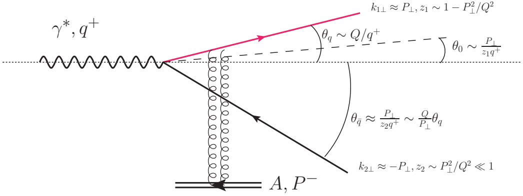
$$\frac{\alpha_s C_F}{\pi}\left[-\frac{1}{4}L^2+\frac{3}{4}L\right]$$

$\Rightarrow$ TMD factorisation implies $\beta=0$.
New LI jet definition in DIS suitable for TMD factorisation with jets.

- Sudakov DL for a jet measurement is half the DL for hadron measurement.

Physical interpretation

- Our new clustering condition equivalent to $\theta_{ij} \leq R\theta_{\rm jet}$ with $\theta_{\rm jet} \sim Q/q^+$.
- Angle of the jet set by its virtuality rather by its transverse momentum.
  (Naively, $\theta_{\rm jet} \sim \frac{P_\perp}{zq^+}$.)
- Soft gluons contributing to Sudakov must have $\theta_g \gg \theta_{\rm jet}$.
  $\Rightarrow$ stronger constraint than $\theta_g \gg \frac{P_\perp}{zq^+}$!
- Jet from the antiquark is backward in the Breit frame. Must be distinguished from the beam remnant.



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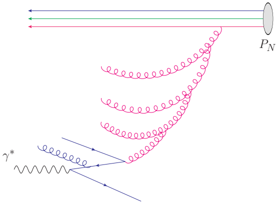
Aligned jet configuration in dipole frame.

Complete NLO result for $Q^2 \gg P_\perp^2 \gg Q_s^2$



- The complete NLO corrections are organized as

$$\left.\frac{\mathrm{d}\sigma^{\gamma_\mathrm{T}^*+A\to j+X}}{\mathrm{d}^2\boldsymbol{P}_\perp}\right|_{\mathrm{NLO}} = \left.\frac{\mathrm{d}\sigma^{\gamma_\mathrm{T}^*+A\to j+X}}{\mathrm{d}^2\boldsymbol{P}_\perp}\right|_{R} + \left.\frac{\mathrm{d}\sigma^{\gamma_\mathrm{T}^*+A\to j+X}}{\mathrm{d}^2\boldsymbol{P}_\perp}\right|_{V}$$

- Term where $P_\perp$ is given by a $q \to qg$ hard collinear splitting

$$\left.\frac{\mathrm{d}\sigma^{\gamma_\mathrm{T}^*+A\to j+X}}{\mathrm{d}^2\boldsymbol{P}_\perp}\right|_{R} = \frac{8\pi^2\alpha_{\mathrm{em}}e_f^2}{Q^2}\frac{\alpha_s}{2\pi^2}\frac{1}{P_\perp^2}\int_x^{1-\frac{RP_\perp}{Q}}\mathrm{d}\xi\, P_{qq}(\xi)\frac{x_{\mathrm{Bj}}}{\xi}f_q\left(\frac{x_{\mathrm{Bj}}}{\xi}, P_\perp^2\right)$$

- Sudakov logs and finite pieces, for our asymmetric jet clustering definition.

$$\left.\frac{\mathrm{d}\sigma^{\gamma_\mathrm{T}^*+A\to j+X}}{\mathrm{d}^2\boldsymbol{P}_\perp}\right|_{V} = \left.\frac{\mathrm{d}\sigma^{\gamma_\mathrm{T}^*+A\to j+X}}{\mathrm{d}^2\boldsymbol{P}_\perp}\right|_{\mathrm{LO}} \times \frac{\alpha_s C_F}{\pi}\left[-\frac{1}{4}\ln^2\left(\frac{Q^2}{P_\perp^2}\right) + \left(\frac{3}{4}+\ln(R)\right)\ln\left(\frac{Q^2}{P_\perp^2}\right)\right.$$
$$\left. -\frac{3}{2}\ln(R) + \frac{11}{4} - \frac{3\pi^2}{4} + \frac{3}{4}\ln^2(x_\star) + \frac{3}{8}\ln(x_\star) + \mathcal{O}(R^2)\right]$$

- $x_\star$ factorisation scale: gluons with $z_g \leq x_\star P_\perp^2/Q^2$ resummed with high energy evolution.

DGLAP+CSS resummation

- Beyond LO, the quark TMD also depends upon the photon virtuality Q^2.
- $x\mathcal{F}_q(x, P_\perp^2, Q^2)$ = number of quarks in the target with given x and $P_\perp$, as probed with a longitudinal resolution fixed by Q^2.
- Taking derivative w.r.t. $\ln(Q^2)$ and assuming Markovian evolution:

$$\frac{\partial \mathcal{F}_q(x, P_\perp^2, Q^2)}{\partial \ln Q^2} = \frac{C_F}{2\pi}\left\{\frac{\alpha_s(P_\perp^2)}{P_\perp^2}\int_{\Lambda^2}^{P_\perp^2} \mathrm{d}\ell_\perp^2\, \mathcal{F}_q(x, \ell_\perp^2, Q^2) - \int_{P_\perp^2}^{Q^2} \frac{\mathrm{d}\ell_\perp^2}{\ell_\perp^2}\alpha_s(\ell_\perp^2)\mathcal{F}_q(x, P_\perp^2, Q^2)\right\} + \frac{3}{2}\frac{\alpha_s(P_\perp^2)C_F}{\pi}\mathcal{F}_q(x, P_\perp^2, Q^2)$$

  $\Rightarrow$ "diagonal" version of the CSS equation for the quark TMD from the dipole picture.
- Integrating our 1-loop result up to Q^2 yields:

$$xf_q(x, Q^2) = xf_q^{(0)}(x, Q^2) + \int_{\Lambda^2}^{Q^2} \frac{\mathrm{d}P_\perp^2}{P_\perp^2}\frac{\alpha_s(P_\perp^2)}{2\pi}\int_x^1 \mathrm{d}\xi\, \mathcal{P}_{qq}(\xi)\,\frac{x}{\xi} f_q\left(\frac{x}{\xi}, P_\perp^2\right)$$

  $\Rightarrow$ DGLAP evolution of the quark pdf from the dipole picture.

Similar results obtained for the WW gluon TMD in PC, Iancu, 2406.04238

Conclusion

- Clarifying the jet definition (clustering algorithm) for jet production in SIDIS in the TMD limit $Q^2 \gg P_\perp^2$.
- Calculation of the Sudakov effect for jet production in SIDIS.
- NLO calculation in the high-energy formalism (dipole picture, CGC).
  Conclusions remain valid at moderate x
- Emergence of the DGLAP and CSS evolutions of the quark TMD from the small-x approach.
  See Edmond Iancu's talk tomorrow for similar results in the diffractive case.

THANK YOU!