# **Some gravitational aspects of scalar field dark matter**

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Annecy - October 1st, 2024

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arXiv: 2204.09401, 2304.10221, 2305.18540, 2307.15391

**Galaxy-scale dynamics: Formation of SFDM halos with a flat core**

**Dark Matter** I- Evidence

- 
- II- Ultra-Light Dark Matter
- III- Scalar-Field Dark Matter models (SFDM)
- IV- Quartic self-interaction

- I- Non-relativistic regime
- II- Soliton
- III- Soliton formation

#### **Black Hole dynamics inside DM solitons / Accretion and Dynamical friction** III- Subsonic regime IV- Supersonic regime

- I- Radial infall onto a BH
- II- BH moving inside a SFDM cloud (soliton)
	-
	-
	- I- Additional forces on the BHs due to the dark matter environment
	- II- Decay of the orbital orbit

#### **Gravitational Waves emitted by a BH binary inside a SFDM soliton**

- 
- III- Phase of the GW waveform
- IV- Fisher matrix analysis
- 

V- Region in the parameter space that can be detected

#### **Impact of the time-dependent DM gravitational potential on GW**

#### **I- DARK MATTER**



#### Rich evidence for Dark Matter through its gravitational effects, from galactic to cosmological scales.

- 1933, Zwicky: motion of galaxies in the Coma cluster
- 1970s, Bosma, Rubin: rotation curves of spiral galaxies
- 1970s, Ostriker, Peebles: stability of disks in spiral galaxies
- 



Rotational velocities for seven galaxies as a function of distance from nucleus. Rubin et al. (1978).



Gravitational lensing in Webb's First Deep Field taken by JWST (2022). Galaxy cluster SMACS 0723 Credit edit: NASA, ESA, CSA, and STScl

- 1980s, Peebles, Primack, Bond, White, …: Cosmic Microwave Background, Gravitational lensing, mass in X-ray clusters, …



CMB temperature fluctuations at different angular scales on the sky. **Credit: ESA and the Planck Collaboration** 



*Bullet cluster (Clown et al. 2006): colors=X-ray gas, green isocontours=projected density measured by gravitational lensing*









- 27% of the energy density of the universe • **27% of the energy density of the universe.**
- Cold (non-relativistic)
- Dark: small electromagnetic interactions • **Dark** (transparent): no/weakly electromagnetic interactions. • **27% of the energy density of the universe.** • **Collisionless**: no/weakly self-interaction or interaction with baryons
- Collisionless / pressureless: small self-interactions or interactions with baryons • **Cold** (non-relativistic): moves much slower than c. • **Dark** (transparent): no/weakly electromagnetic interactions.



#### Bertone and Tait, 1810.01668

### **II- Ultra-Light Dark Matter**

Renewed interest in recent years (Hui, Ostriker, Tremaine, Witten 2017), especially since WIMPs have not been detected yet and ULDM might alleviate some small-scale tensions of LCDM.

# Pawlowski/Bullock/Boylan-Kolchin

#### **Missing satellite problem**

**Predicted ACDM substructure** 

Known Milky Way satellites

Simulation by V. Robles and T. Kelley and collaborators.

James S. Bullock, M. Boylan-Kolchin, M. Pawlowski

These problems may be solved by a proper account of baryonic physics (feedback from Supernovae and AGN), but ULDM remains an interesting candidate on its own.



#### Core/cusp problem

Density profiles observations and simulations

Antonino Popolo, Morgan Le Delliou (2017)

#### Fuzzy Dark Matter







The DM density field behaves like CDM on large scales but structures are suppressed below  $\,\,\lambda_{\text{dB}}$ 

In particular, hydrostatic flat cores (« solitons ») can form at the center of DM halos.

For Fuzzy Dark Matter: *m* 

$$
uv \rangle \simeq \left(\frac{m}{10^{-22} \,\mathrm{eV}}\right)^{-1} \left(\frac{v}{100 \,\mathrm{km/s}}\right)^{-1} \,\mathrm{kpc}
$$

$$
\sim 10^{-22} \text{eV} \qquad \lambda_{\text{dB}} \sim 1 \,\text{kpc}
$$



Radial density profiles of haloes formed in the  $\psi$ DM model

However, this model already seems ruled out by Lyman-alpha forest power spectra (because of this suppression of small-scale power).



A slice of density field of  $\psi$ DM simulation on various scales at  $z=0.1$ Schive, Chiueh, and Broadhurst (2014)

In the FDM model, the wavelike dynamics below  $\,\lambda_{\text{dB}}$  , which leads to the suppression of small-scale power, appears as an effective « quantum pressure » in the hydrodynamical regime.

Instead of relying on this quantum pressure (large  $\lambda_{\text{dB}}$  ), we can also suppress small-scale structures through self-interactions.

This also generates an effective pressure, which is now due to the self-interactions.

Scalar field Dark Matter with self-interactions

#### **III- Scalar-field models**  $\alpha$ -field medels  $\alpha = \int d^4x$   $\sqrt{a}$   $\left[1 - \frac{1}{2}a^4x + \frac{1}{2}a^2x + \frac{1}{2}a^2x + \frac{1}{2}a^2x + \frac{1}{2}a^2x + \frac{1}{2}a^2x + \frac{1}{2}a^$ <u>the complex scalar field with the complex</u> terms of the scalar field. We again describe the perturbative  $S_A =$ fluid. Then, in a fashion similar to primordial black holes, i.e.,  $\mathbf{r}$  and  $\mathbf{r}$ We present our main conclusions in Sec. V. We find the second of <u>complete our discussion with discussion with discussion with discussion with discussion with discussion and the </u>

#### tound:<br>the density production of the density production of the clumps, reflecting to the clumps, reflection of the clu space of this second scenario and the size of the scalar clumps. Again, we check they do

Brax et al. 2019 \_ t al.  $2019$ 

$$
S_{\phi} = \int d^4x \sqrt{-g} \left[ -\frac{1}{2} g^{\mu\nu} \partial_{\mu} \phi \partial_{\nu} \phi - V(\phi) \right].
$$

**Background:**  
\n
$$
V
$$
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$$
V
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$$
V
$$
\n
$$
e.g., no self-interactions: V = \frac{1}{2}m^2\phi^2
$$
\n
$$
V = \frac{1}{2}m^2\phi^2
$$



**they are governed by the set of the scalar state of the scalar set of the scalar state of the scalar state**  $\rho \propto a^{-3}$  $\delta \propto a^{-3}$  $a^{-3}/$ 



$$
V(\phi) = \frac{1}{2}m^2\phi^2 + V_I(\phi)
$$
 
$$
V_I \ll \frac{1}{2}m^2\phi^2
$$

the scalar field oscillates with frequency m, and a slow decay of the amplitude: #

$$
\bar{\phi}(t) = \bar{\phi}(t) \cos(mt - \bar{S}(t))
$$

 $S_{\phi} =$ 

$$
\phi = \phi_0 (a/a_0)^{-3/2} \cos(mt)
$$

k matter: 
$$
\rho \propto a^{-3}
$$
  $V \propto \phi^n$   $\Rightarrow$   $w = \frac{\langle p_{\phi} \rangle}{\langle \rho_{\phi} \rangle} = \frac{n-2}{n+2}$   
Brax et al. 2019

$$
\bar{\varphi} = \bar{\varphi}_0 a^{-3/2} \qquad \qquad \bar{S}(t) = \bar{S}_0 - \int_{t_0}^t dt m \Phi_I \left( \frac{m^2 \bar{\varphi}_0^2}{2a^3} \right)
$$

 $\sqrt{2}$  $B_{\rm{max}}$ 



For a mostly quadratic potential with small self-interactions: → Sinan → Sin−miceraction>.<br>
→ Sinan → Son−miceraction>. complex scalar field  $\sim$  The amplitude of the scalar field  $\sim$ 

 $\overline{\phantom{a}}$ 

$$
V(\phi) = \frac{1}{2}m^2\phi^2 + V_1(\phi)
$$
\n
$$
V_1 \ll \frac{1}{2}m^2\phi^2
$$

mation of the harmonic oscillator,

 $\overline{a}$ 

#### <u>C</u> Se  $\overline{\mathbf{r}}$ <u>−interact</u> 2 **IV- Quartic self-interaction**

Fuzzy Dark Matter (FDM) + self-interactions Fuzzy Dark Matter (FDM) + self-interactions are negligible. This regime regim<br>- This regime regim

ions 
$$
S_{\phi} = \int d^{4}x \sqrt{-g} \left[ -\frac{1}{2} g^{\mu\nu} \partial_{\mu} \phi \partial_{\nu} \phi - V(\phi) \right]
$$
  
\n $\phi^{4}$ ,  $\lambda_{4} > 0$   
\nRenuisive self-interaction  $\longrightarrow$  Effective pressure



**One characteristic density / length-scale:** One characteristic density / length-scale:  $\rho_a = \frac{\rho_a}{2}$ 

 $\frac{1}{4}$ 

 $\int$ 

 $\overline{\phantom{a}}$ 

Rel

Very large occupation numbers:  $I_{\rm V} \sim \frac{1}{m n^3} \gg 1$  $\frac{1}{\sqrt{2}}$ For y large occupation numbers.

 $\mathbf{v}'_{\phi} =$ 

z<br>Z

De Broglie wavelength:  $\lambda_{\text{dB}} = \frac{2}{3}$ De Broglie wavelength:  $\lambda_{\text{dB}} = \frac{2\pi}{mv} \lesssim 1 \text{ kpc}$  $\theta$  ie wavelength:  $\lambda_{d1}$ De Broglie wavelength:  $\lambda_{\text{dB}} =$ 

massive black hole (BH) at the center of the center of the halo is taken black hole is taken black ho

Also, k-essence models:  $S_{\phi} = \int d^4x \sqrt{-g} \left[ \Lambda^4 K(X) - \frac{m^2}{2} \phi^2 \right]$ sonic and supersonic regimes and calculates the large-Also, k-essence models:  $S_\phi =$ Also, k-essence models: with a positive  $\int_{A} dA = \int_{A} dA$ Also, K-essence models and  $\Delta_{\phi} = \int d^{3}x \sqrt{-g} \, |\Lambda^{3}K(x)|$ 

$$
_{\rm dB} =
$$

$$
V(\phi) = \frac{m^2}{2}\phi^2 + V_1(\phi)
$$
 with  $V_1(\phi) = \frac{\lambda_4}{4}\phi^4$ ,  $\lambda_4 > 0$   
\n $\phi \propto a^{-3}$  Repulsive self-interaction  $\longrightarrow$  Effective pressure

The parameters *m* and <sup>4</sup> determine the characteristic

One characteristic density / length-scale: 
$$
\rho_a = \frac{4m^4}{3\lambda_1}, r_a = \frac{1}{\sqrt{4\pi G \rho_a}}
$$
  
\nRelativistic regime -  
\nSłong self-interaction  
\nVery large occupation numbers:  $N \sim \frac{\rho}{mp^3} \gg 1$   $m \ll 1$  eV  
\nDe Broglie wavelength:  $\lambda_{dB} = \frac{2\pi}{mv} \lesssim 1$  kpc  $m \gtrsim 10^{-22}$  eV  
\nAlso, k-essence models:  $S_{\phi} = \int d^4x \sqrt{-g} \left[ \Lambda^4 K(X) - \frac{m^2}{2} \phi^2 \right]$   $X = -\frac{1}{2\Lambda^4} g^{\mu\nu} \partial_{\mu} \phi \partial_{\nu} \phi$   $K(X) = X + K_1(X)$ 



- 
- 
- 
- 



# **Galaxy-scale dynamics:**

# **Formation of DM halos with a flat core**

#### **THAT IN A SET AND INCORDINAL INC. A SET AND INCLOSURATIVISTIC REGIME** The latter is then governed by the Schrödinger equation. ρ\_ þ ∇ · ðρv unity now. As we are interested in the classical behavior of  $\mathcal{L}$ the field  $\overline{\phantom{a}}$  in the nonrelativistic limit, it is convenient to interval the nonrelativistic limit, it is convenient to  $\overline{\phantom{a}}$

On the scale of the galactic halo we are in the nonrelativistic regime: the frequencies and wave numbers of interest are much smaller than  $\ m$ and the metric fluctuations are small. Thus, in the nonrelativistic regime, we can go from the  $\ddot{\phantom{a}}$  $\tilde{=}$  $2n \cos \alpha$ i  $\cos \alpha$  music sinanci chan  $\cos \alpha$ st ai e much smaller than  $\; m$ 

$$
\phi = \frac{1}{\sqrt{2m}} (e^{-imt}\psi + e^{imt}\psi^*)
$$
 factori

 $\gamma$  and  $\gamma$  and  $\gamma$ 

 $\psi(x,t)$ 

#### B. Nonrelativistic regime In the nonrelativistic weak-gravition weak-gravitativity regime, it is in the convenience of the convenience o **A) From Klein-Gordon eq. to Schrödinger eq.:**

 $Dosemness$  the real scalar field  $\phi$  in terms of t Decompose the real scalar field  $\,\phi\,$  in terms of the complex scalar field  $\,\psi$ 

Instead of the Klein-Gordon eq., it obeys a (non-linear) Schrödinger eq.:

terms in the scalar field action (5) dominate, following (7).

It is interesting the Madelüng terms)  $\left($  keep only even terms)



$$
i\left(\dot{\psi} + \frac{3}{2}H\psi\right) = -\frac{\nabla^2\psi}{2ma^2} + m\Phi_N\psi + \frac{\partial V_I}{\partial\psi^*}
$$
  
Newtonian  
gravitational potential self-interactions

- $\mathcal P$
- factorizes (removes) the fast oscillations of frequency *m* i  $\ddot{\phantom{0}}$ υι π  $\overline{5}$ lneı quency  $m$  $\overline{a}$  $m$
- $\sum_{i}$  is  $\ll m n!$  $\gamma$  as the condition (27) below, where the self-the self-t  $\psi \ll m\psi, \quad \nabla \psi \ll m\psi$
- ives slowly on astrophysical or cosmological scales was sidivity, on as a ophysical of cosmological scales.  $\psi(x,t)$  evolves slowly, on astrophysical or cosmological scales. here we internative nonaction produced potential via the visit of the visit or cosmological scales. averaging over the leading over the leading over the leading of  $\mathcal{L}$

 $V_{\rm I}(\phi)=\Lambda^4\sum$  $p\geq 3$  $\overline{\lambda}_{p}$ p  $\int \phi$ Λ  $\lambda_p$   $\left(\phi\right)^p$  $V_I(\psi, \psi) - \Lambda \sum_{p \geq 2} \overline{2p} \, \overline{(p!)^2} \, \Big( \overline{2p} \Big)$  $\frac{1}{\sqrt{2}}$ m<sub>2</sub>m<sub>2</sub>m<sub>2</sub>  $\mathcal{L}$  $\frac{r}{2}$ p one obtains [33]  $\mathcal{V}_\mathrm{I}(\psi,\psi^\star)=\Lambda^4{\sum}$  $p\geq 2$  $\lambda_{2p}$  $2p$  $(2p)!$  $(p!)^2$  $\int \psi \psi^{\star}$  $2m\Lambda^2$ 

term are required to be dominant, leading to an upper I he real and imaginary parts of the Schrod r eq. lead to the continuity and Euler eqs.: The real and imaginary parts of the Schrödinger eq. lead to the the nonlinear Klein-Gordon equation between the fast of the fa<br>The fast of the fast of th ary parts of the Schrödinger eq. lead to the continuity and Euler eqs.: The latter is then governed by the Schrödinger equation. The real and imaginary parts of the Schrödinger eq. lead to the conservation of th ⃗ of the Schrödinger eg. lead to the continuity and Euler egs.:  $T_{\text{max}}$  the interaction term is a small perturbation that  $\frac{1}{2}$ The real and imaginary parts of the Schrödinger eq. lead to the continuity and Euler eqs.:

Inside aalactic halos we neglect th ne<br>Inside galactic halos, we neglect the Hubble expansion:

> $i\dot{\psi} = -\frac{r}{2m} + m(\frac{r}{2m})$  $i\dot{\psi} = -\frac{\nabla^2 \psi}{2m}$ 2m

One can man the Schrödinger eq to hydr  $t_{\rm max}$  oscillations of the scalar field due to the  $\frac{1}{2}$ One can map the Schrödinger eq. to hydrodynamical eqs.: oscillations at frequency m and a slowly varying envelope  $\log$  cq. to hydrodynamical cqs.. self-interaction is the Schröde attractive self-interactions of the series of the self-interaction of the pressure pressure pressure pressure p ρa

#### B) From Schrödinger eg, to Hydrody cosmological background cease to grow and oscillate, From Schrodinger eq. to Hydrodynamical eqs ( large scales in the late Universe and to astrophysical scales  $\frac{1}{2}$  from BH  $\frac{1}{2}$ Equation is the usual continuity and equations, which is a probability of the usual continuity  $\mathcal{L}$ **B) From Schrödinger eq. to Hydrodynamical eqs (Madelung transformation):** <sup>2</sup> <sup>ϕ</sup><sup>2</sup> <sup>þ</sup>VIðϕ<sup>Þ</sup> with <sup>V</sup>IðϕÞ ¼λ<sup>4</sup>  $\frac{4}{\pi}$ The coupling coupling constant  $\frac{1}{\sqrt{2}}$ al ed.<br>Die beste staat beste stad beste stad beste staat beste staat beste staat beste staat beste staat beste staat <br>Die beste stad beste staat beste stad the following.

$$
\Phi_{\rm I}=\frac{\rho}{\rho_a}
$$

per de se de la poste de l<br>La poste de la  $\Gamma$ ions, we can go from the non-regime, we can go from the non-regime, we can get  $\Gamma$ er<br>S  $\frac{1}{\sqrt{2}}$ Self-interactions

ρ and the phase s by the Madelung transform [39], Defining the curl-free velocity field v with ρ<sup>a</sup> 4m<sup>4</sup> On the cosmological background or on galactic scales,

One can map the Schrödinger eq. to hydrodynamical eqs.: 
$$
\psi = \sqrt{\frac{\rho}{m}}e^{is} \qquad \vec{v} = \frac{\nabla s}{m}
$$

 $s$ ervation of matter for  $\rho$ for  $\sqrt{2}$  $\psi$  conservance  $s$  is in the harmonic order that  $s$  is the scalar of the scalar o  $\frac{1}{2}$  racion of probability for  $\frac{1}{2}$  existed values of a conservation of probability for  $\psi$  and  $\Longrightarrow$  conservation of matter for  $\rho$ 

 $m$  is the Newtonian gravity and  $\mathcal{S}_{\text{a}}$  is the  $n$  is the  $n$ the non-the non-the self-interaction potential. For the self-interaction potential. For the quarticity of the  $\sim$  $\nabla^2 \psi$  is ready deviations from the CDM scenario on small scales. In  $\mu\psi = -\frac{2m}{2m} + m(\Phi_N + \Psi_I)\psi$ <br> $\Phi_I = \frac{m}{2m}$ Self-interactions  $+m(\Phi_{\rm N}+\Phi_{\rm I})\psi$ SUBSONIC ACCRETION AND DEVICION AND DEVICE AND DEVICE AND DEVICE A LATE OF A LATE OF A LATE OF A LATE OF A LAT where we include a quartic self-interaction and the self-interaction of the self-interaction,  $\frac{1}{2}$ <sup>V</sup>ðϕÞ ¼m<sup>2</sup> <sup>2</sup> <sup>ϕ</sup><sup>2</sup> <sup>þ</sup>VIðϕ<sup>Þ</sup> with <sup>V</sup>IðϕÞ ¼λ<sup>4</sup> 4 ϕ4  $2m$ Newtonian gravity

$$
\text{effective pressure} \quad P_{\text{eff}} \propto \rho^2
$$

 $\gamma=2$  $\gamma = 2$ 

 $M<sub>2</sub>$ Madelung 1927, Chavanis 2012, ....

$$
\vec{\rho} + \nabla \cdot (\rho \vec{v}) = 0
$$
\nconservation of probability for  $\psi$  conservation of  
\n
$$
\vec{v} + (\vec{v} \cdot \nabla)\vec{v} = -\nabla(\Phi_Q + \Phi_N + \Phi_I)
$$
\nSelf-interactions  
\n
$$
\Phi_I = \frac{\rho}{\rho_a}
$$
\nSelf-interactions  
\neffective pressure  $P_c$   
\ncomes from part of the kinetic terms in  $\psi$ 

In the following, we neglect the « quantum prest ιsι ¼  $\frac{1}{2}$ (which dominates for FDN ient to write the real scalar field  $\alpha$  in terms of a complex of ch dominates fo associated with the following we nealect the  $\kappa$  and interaction dominates over the quantum pressure. Then, In the following, we neglect the « quantum pressure » (which dominates for FDM) s\_ m <mark>glect the</mark> E ng, we neglect the « quantum pressure » (which dominat and the size of hydrostatic equilibria (solitons) that can be calculated that can be calculated to  $\mathcal{A}$ 

$$
\nabla^2 \Phi_{\rm N} = 4\pi \mathcal{G} \rho \qquad \qquad \rho = m |\psi|^2
$$



 $\propto \rho^2$ 

ions 
$$
V_{\rm I}(\phi) = \frac{\lambda_4}{4} \phi^4
$$

$$
\Phi_{\rm I}=\frac{m|\psi|^2}{\rho_a}
$$

Defining the curl-free velocity field v

#### <u>II- SOLITON (ground state): HYDROSTATIC EQUILIBRIUM</u> stars. They are again bound ground states of the  $\frac{1}{2}$ interaction allows for the formation of static equipped of  $\mathbf{a}$ configurations with a configurations with  $\frac{1}{2}$ , which are configurated as a configuration of the configuration  $\frac{1}{2}$ **N (ground state): HYDROSTATIC EQUILIBRIUM** IMPACT OF KINETIC AND POTENTIAL SELF-INTERACTIONS … PHYS. REV. D 100, 023526 (2019). PHYS. REV. D 100, 023526<br>Rev. 2019). Phys. Rev. 2019, phys. Rev. 2019, phys. Rev. 2019, phys. Rev. 2019. Phys. Rev. 2019, phys. Rev. 20 black hole moving within a self-interacting scalar dark <u>dund state): HYDROSTATIC EQU</u> Here *m* is the mass of the scalar field and <sup>4</sup> its coupling



 $\overline{P}$ ;  $\overline{P}$ 

 $\frac{1}{\sqrt{2}}$ 

 $0.0$   $0.2$   $0.4$   $0.0$   $0.0$   $1.0$ 

 $\mathsf{U}.\mathsf{L} \qquad \mathsf{U}.\mathsf{U} \qquad \mathsf{U}.\mathsf{U}$ 



 $m\sim 10^{-21}{\rm eV}$  : Fuzzy Dark Matter (de Broglie wavelength of galactic size): galactic soliton governed by the balance between

 $m \gg 10^{-18} \text{eV}$  : galactic soliton governed by the balance between the repulsive self-interaction and self-gravity.

the quantum pressure and self-gravity.

Numerical simulations of FDM indeed find that solitons form, from gravitational collapse, within an extended NFW-like out-of-equilibrium halo. externed in v<sub>ertik</sub>e

#### **A) Numerical simulations** the semiclassical regions of  $\mathbb{R}$ . The semicidae  $\mathbb{R}$ <u>is component states of the incoherent stock stochastic fluctuations states and stochastic fluctuations and si</u> A) Numerical simulations  $\alpha$  take the assumed the assumed the assumed the assumed the assumed the dimensionless variables  $\psi_{\rm initi}$ **Social Simulations** neglect the central soliton, we define our system by *RS* and the central soliton, and the central soliton, we define the contract of the central soliton, we define the contract of the central solution of the central solut

<sup>2</sup> *,* ⇢0sol form *x* (31) **.** (31) . *a*<sup>1</sup> *a*<sub>2</sub> *a*<sup>*n*</sup></sup> *a*<sup>*m*</sup> *a*<sup>*n*</sup> *a*<sup>*n*</sup>



sol*.* (30) Stochastic halo: sum over eigenmodes of the target gravitational potential with random coefficients asht halo. Suill over eigerindues of th *<sup>n</sup>*`*m*(~ *,* (37) is governed by the balance between gravity and selfinteractions in the conditions in the condition  $\sim$  $\frac{1}{2}$ 

#### **III- SOLITON FORMATION IN THE THOMAS-FERMI REGIME** <sup>p</sup>*G<sup>N</sup>* ⇢?*mL*? Thus, the limit ✏ ! 0 corresponds to the semiclassical **N THE T** *dr r*<sup>2</sup> *<sup>R</sup><sup>n</sup>*1`*R<sup>n</sup>*2` <sup>=</sup> *<sup>n</sup>*1*,n*<sup>2</sup> *.* (35) the scalar field behaves like cold dark matter. <u>III- SULI IUN FUKMAI IUN</u> ˆ*n*`*m*(~ *<sup>x</sup>*) = *<sup>R</sup>n*`(*r*)*<sup>Y</sup> <sup>m</sup>* ` (✓*,* ')*,* (33) THE SOLITON FORMATION <u>quantum contractors in other in the independent of</u> the independent of the independent of the independent of the<br>Independent of the independent o neglect the self-interactions and the central soliton, former case the soliton is always dominated by the self-<u>in ine inumas-fermi regim</u>i

4*R*<sup>2</sup>

Initial conditions: halo (+ central soliton):  $\sin(\pi r/R_{\rm sol})$  and  $\sin(\pi r/R_{\rm sol})$  and  $\sqrt{2\pi r^2/R_{\rm sol}}$ dB, with a velocity dispersion set by the virial velocsociated target gravitational potential ¯ *<sup>N</sup>* (*r*), where we Initial conditions: halo (+ central soliton): sol(~*x, t*) = *eiE*sol*t/*✏ ˆsol(*r*)*.* (28) sol*/*⇡. As the size of the halo is *R*halo = 1, we consider *n*`*m*

#### witational potential with random coefficients te comparation is a compact of the distribution of the distribution of the distribution of the distribution of equation, are solved in terms of the contractors or an area of the energy contracts eigenmodes *eiEt/*✏ ˆ*E*(~

$$
\psi_{\rm initial} = \psi_{\rm sol} + \psi_{\rm halo}
$$

contains: halo (+ central solution):

\n
$$
\rho_{sol}(r) = \rho_{0sol} \frac{\sin(\pi r/R_{sol})}{\pi r/R_{sol}}, \quad \hat{\psi}_{sol}(r) = \sqrt{\rho_{sol}(r)}.
$$

$$
\psi_{\rm halo}(\vec{x},t) = \sum_{n\ell m} a_{n\ell m} \hat{\psi}_{n\ell m}(\vec{x}) e^{-iE_{n\ell}t/\epsilon}
$$

#### signal (Self-interactions dominate over the quare ˆ (Self-interactions dominate over the quantum pressur interactions dominate over the quantum pressure for ⇢ & dom realizations, that is, over the uncorrelated phases <u>ssure in the soliton)</u> (Self-interactions dominate over the quantum pressure in the soliton)

phases ⇥*n*`*m*. Defining the average h*...*i over these ran-

 $a(E)^2 = (2\pi\epsilon)^3 f(E)$   $f(E) = \frac{1}{\epsilon(E)} \int_0^0$  $u(E)$  = (27(e)  $J(E)$  is satisfied. The condition  $J(E)$  is  $\sqrt{2\pi^2}$   $\overline{dE}$   $\int_E$   $\overline{\sqrt{d}}$ classical regime ✏ = 0*.*01 ⌧ 1. The central soliton  $\alpha(L)$  -  $(2nC)$   $J(L)$  $a(E)^2 - (2\pi\epsilon)^3 f(E)$   $f(E) = \frac{1}{\epsilon(E)} \frac{d}{E} \int_0^0 \frac{d}{E}$  $g_{\mu\nu}(E) = (2\pi\epsilon)^{-1}(E)$  and  $g_{\mu\nu}(E) = \frac{1}{2\sqrt{2}\pi^2}\frac{1}{dE}\int_E \sqrt{\Phi}$  $a(E)^2=(2\pi\epsilon)$  ${}^{3}f(E)$   $f(E) = \frac{1}{2\sqrt{5}}$  $a(E)$  $\lambda^2$  – (2)  $(\epsilon)^{\mathsf{d}} f(E)$  $\overline{2\sqrt{2}\pi^2}$ 

$$
e_m(\vec{x})e^{-iE_{n\ell}t/\epsilon} - \frac{\epsilon^2}{2}\nabla^2\hat{\psi}_E + \bar{\Phi}\hat{\psi}_E = E\hat{\psi}_E
$$
\nandom phase\n
$$
\bar{\Phi}(r) = \bar{\Phi}_N(r), \quad \nabla^2\bar{\Phi}_N = 4\pi\bar{\rho}.
$$
\nandom phase\n
$$
a(E)
$$

$$
\frac{d}{dE} \int_E^0 \frac{d\Phi_N}{\sqrt{\Phi_N - E}} \frac{d\rho_{\rm classical}}{d\Phi_N}
$$

$$
\langle \rho_{\rm halo} \rangle = \sum_{n \ell m} a (E_{n\ell})^2 |\hat{\psi}_{n\ell m}|^2
$$

$$
a_{n\ell m} = a(E_{n\ell})e^{i\Theta_{n\ell m}}
$$
 random phase

*a*(*En*`)

 $\frac{1}{2}$ 

<sup>2</sup> *<sup>R</sup>*<sup>2</sup>

#### 1) Soliton radius of the same order as the halo size



- Afterwards, the soliton slowly grows.

#### 2) Soliton radius much smaller than the halo size



- 
- At  $t \sim 180$ , FDM peak.
- At t  $\sim$  200, self-interacting soliton forms,  $R_{sol} = 0.1$ .

**Transition from a FDM phase to a self-interacting phase.**



#### 3) Dependence of the soliton mass on the formation history

Growth with time of the soliton mass



- The soliton always forms and grows, with a growth rate that decreases with time.

- Its mass can reach 50% of the total mass of the system.
- 





- There is no sign of a scaling regime, where the growth rate would be independent of initial conditions.

Probably no well-defined halo-mass/soliton mass relation

#### **B) Kinetic theory**

To understand the growth of the soliton, we develop a kinetic theory:

Instead of following the wave function, we try to follow the evolution of the **occupation numbers** of the various eigenmodes of the Schrödinger eq. in a reference potential

Non-linear Schrödinger eq. (Gross-Pitaevskii):

Chan et al. (2022) considered the case of FDM over a flat background, expanding over plane waves. Here we consider a non-flat background (self-gravity of the halo), with possibly a soliton in the initial conditions.



We follow the evolution of the **occupation numbers** of the eigenmodes We cannot use Fourier analysis<br>in the reference potential



$$
i\epsilon \frac{\partial \psi}{\partial t} = -\frac{\epsilon^2}{2} \nabla^2 \psi + \Phi \psi \qquad \Phi = (4\pi \nabla^{-2} + \lambda) \psi \psi^* \qquad \epsilon = \frac{\lambda_{\text{dB}}}{2\pi L_{\star}} \ll 1
$$

If  $\Phi$  is fixed,  $\psi$  can be decomposed over the eigenmodes with the simple time dependence  $e^{-iE_n t/\epsilon}$ and there is no secular growth or evolution of the system. However the fluctuations (or interference terms) induce a time-dependent potential and drive the evolution of the system.

 $\bar{\Phi} = (4\pi\nabla^{-2} + \lambda)\sum$ *j*  $M_j\psi$  $\hat{b}^2$ *j*



$$
I_j \dot{\theta}_j = 2M_j E_j + \sum_{j'} 2\sqrt{M_j M_{j'}} e^{i(\theta_j - \theta_{j'})/\epsilon} \int d\vec{x} \,\hat{\psi}_j \delta \Phi \hat{\psi}_j.
$$

$$
\delta \Phi = (4\pi \nabla^{-2} + \lambda) \sum_{j \neq j'} \sqrt{M_j M_{j'}} e^{i(\theta_j - \theta_{j'})/\epsilon} \hat{\psi}_j \hat{\psi}_j
$$
 initially rand

 $y^{(0)} + M^{(1)}_j + M^{(2)}_j + \ldots$ 

 $V_{13;24} =$ z<br>Z  $d\vec{x}\,\psi$  $\hat{j}$  $_{1}\psi$  $\hat{\psi}_3 (4\pi \nabla^{-2} + \lambda) \hat{\psi}_3$ 4-leg vertices:  $V_{13;24} = \int d\vec{x} \, \psi_1 \psi_3 (4\pi \nabla^{-2} + \lambda) \psi_2 \psi_4$ 

- $i \epsilon M_j + 2M_j$ - We substitute into the Schrödinger eq.:
- We define the reference potential as the sum of the diagonal terms: and the remainder is given by the off-diagonal terms:
- We perform a perturbative expansion (over powers of  $\delta\Phi$ ): and we average over the random initial phases  $\,\theta$ (0) *j*

$$
(\vec{x},t)
$$
\n\nfluctuating  
\npart  
\n
$$
M_j(t)e^{-i\theta_j(t)/\epsilon}\hat{\psi}_j(\vec{x})
$$
\n\nmass contained in the eigenmode *j*  
\nInitial mass  $M_j$  is fixed, initial phase  $\theta_j$  is random  
\n
$$
M_i\dot{\theta}_i = 2M_iE_i + \sum 2\sqrt{M_iM_{ii}}e^{i(\theta_j-\theta_{j'})/\epsilon} \int d\vec{x} \hat{\psi}_i \delta \Phi \hat{\psi}_{ij}
$$

initially deterministic



$$
\bar{\omega}_j = E_j + \sum_{j'}^{j' \neq j} M_{j'} V_{jj';j'j} \qquad \text{renormalised free}
$$
  

$$
\omega_{12}^{34} = \omega_1 + \omega_2 - \omega_1
$$

 $j \neq 0: \omega_j > \omega_0$ 

 $\overline{\phantom{a}}$ This is somewhat similar to four-wave systems (e.g., weak wave turbulence) over an homogeneous background, where we would have:

- Zeroth order:
- First order:
- Second order:

$$
M_j^{(0)}(t) = \bar{M}_j, \quad \theta_j^{(0)}(t) = \bar{\theta}_j + \bar{\omega}_j t \qquad \qquad \bar{\omega}_j = E_j + \sum_{j'}^{J \neq j} M_{j'} V_{jj';j'j} \qquad \text{for } j \leq 1, \ldots, j \leq N-1, \quad \mu_{j'}^{34} = \mu_{j'}^{34}.
$$
\n
$$
\langle \dot{M}_j^{(1)} \rangle = 0
$$
\n
$$
\langle \dot{M}_j^{(2)} \rangle = \frac{2}{\epsilon} \sum_{234} \bar{M}_1 \bar{M}_2 \bar{M}_3 \bar{M}_4 \left\{ \frac{\sin(\bar{\omega}_{12}^{34} t/\epsilon)}{\bar{\omega}_{12}^{34}} \hat{V}_{13;24} \left[ \frac{\hat{V}_{13;24} + \hat{V}_{14;23}}{\bar{M}_1} + \frac{\hat{V}_{23;14} + \hat{V}_{24;13}}{\bar{M}_2} \right] - \frac{\hat{V}_{31;42} + \hat{V}_{32;41}}{\bar{M}_3} - \frac{\hat{V}_{41;32} + \hat{V}_{42;31}}{\bar{M}_4} \right\} + \frac{\sin(\bar{\omega}_1^{3} t/\epsilon)}{\bar{\omega}_1^{3}} \hat{V}_{12;23} \left[ \frac{\hat{V}_{14;43}}{\bar{M}_1} - \frac{\hat{V}_{34;41}}{\bar{M}_3} \right] + \frac{\sin(\bar{\omega}_2^4 t/\epsilon)}{\bar{\omega}_2^4} \hat{V}_{23;34} \frac{\hat{V}_{14;21} - \hat{V}_{12;41}}{\bar{M}_2} \right\},
$$





$$
\langle \dot{M}_1^{(2)} \rangle = \frac{2}{\epsilon} \sum_{234} \bar{M}_1 \bar{M}_2 \bar{M}_3 \bar{M}_4 \frac{\sin(\bar{\omega}_{12}^{34} t/\epsilon)}{\bar{\omega}_{12}^{34}} 2 \hat{V}_{1234}^2 \left[ \frac{1}{\bar{M}_1} + \frac{1}{\bar{M}_2} - \frac{1}{\bar{M}_3} - \frac{1}{\bar{M}_4} \right]
$$

For the soliton, ground state *j*=0, some of the Dirac factors (resonances) vanish and the equation simplifies:

$$
\dot{M}_0 = \frac{\pi}{\epsilon} \sum_{123} M_0 M_1 M_2 M_3 \, \delta_D(\omega_{01}^{23}) \left(V_{02;13} + V_{03;12}\right)^2 \left(\frac{1}{M_0} + \frac{1}{M_1} - \frac{1}{M_2} - \frac{1}{M_3}\right).
$$

 $M_0 \to 0:$   $\dot{M}_0 =$  $2\pi$  $\epsilon$  $\sum$ 123  $\textsf{Small} \text{~solutions} \text{~grow:~} \quad M_0 \to 0: \quad \dot{M}_0 = \frac{2}{\epsilon} \sum M_1 M_2 M_3 \, \delta_D(\omega_{01}^{23}) \, (V_{02;13} + V_{03;12})^2 > 0$  We make a simple approximation for the occupation numbers of the excited modes:

$$
\sum_{j}^{E_j < E_{\text{coll}}} \left(2\ell + 1\right) M_{n\ell}(0) =
$$



we assume that the increase of mass of the soliton comes from the lowest energy modes, which are depleted up to some energy threshold while the occupation numbers of higher energy levels are not modified

> Formation of a frequency gap, which prevents resonances and decreases the soliton growth rate.





With this approximation, we recover

- the positivity of the growth rate
- its initial order of magnitude
- qualitatively its fast falloff with time.

However, we underestimate the growth rate at late times: the low energy modes are probably partly replenished and we should improve the treatment of their occupation numbers.





# **BH dynamics inside DM solitons -**

# **Accretion and Dynamical friction**

(Schwarzschild BH)

Classical Bondi problem: steady-state spherical accretion of gas onto a central BH hence, we expect supermassive BHs to form as well in these l<del>c</del>auy-sid ly-state spherical accretion of gas onto a cent

Here:  $\gamma=2$  in the Newtonian regime, and we perform a relativistic analysis: In particular, the BH event horizon (Schwarzschild radius) tonian regime, and we

 $p_{\text{in}}$  $\frac{1}{2}$  is the set of the close end close black hole  $\frac{1}{2}$  is the black hole  $\frac{1}{2}$ - metric deviations from Minkowski are large close to the BH horizon

static spherical symmetry:

hild metric close to the BH:  $\frac{r_s}{4} < r < r_{\text{NL}}$ :  $f(r) = (r_{\text{NL}} + r_{\text{NL}})$ IC close to the  $B$ H: \* Schwarzschild metric close to the BH: ischild metric close to the BH:  $\frac{1}{4} < r < p$ 

ic fluctuations and self-gravity far from the BH, in the galaction  $\mathsf{L}_{\mathsf{c}}$  and then, then, then, then, then, then, the galactic solitons of  $\mathsf{L}_{\mathsf{c}}$ are due to the balance between gravity and the balance between gravity and the repulsive and the and solf gravity far from the RH in the galactic ton:  $\frac{1}{2}$  $^\ast$  small metric fluctuations and self-gravity far from the BH, in the galactic-scale soliton:  $r = \frac{1}{2}$  $\ast$  small metric fluctuations and self-gravity far from the BH, in the galactic-scale soliton:

- 1. Strong-gravity regime dominated by the BH GRUIVISTIC dialysis, below a radius relativistic relativistic relativistic relativistic relativistic relativistic r
- Minkowski metric, dominated by the BH gravity. There, we 4 າດ zon<br>

#### Again,

**1** symmetry: 
$$
ds^2 = -f(r)dt^2 + h(r)(dr^2 + r^2d\vec{\Omega}^2).
$$
 (isotropic coordinates)



#### **I- RADIAL INFALL ONTO A BH** initial seeds could result from the remnants of massive stars <u>Father collapse of the collapse of stellar clusters.</u> The collapse of  $47<sup>o</sup>$  $\blacksquare$  paniai ine <u>l Inf</u> ;

#### **A) Spherically symmetric relativistic and nonlinear system** processes should also be processes should also be present in scalar DM cosmologies; and the present in scalar<br>DM cosmologies; and the processes should be presented in scalar DM cosmologies; and the processes of the proce

form (10), where  $\mathcal{L}$  is now given by the scalar-field Poisson  $\mathcal{L}$  $\mathbf{a}$ are dominated by the BH gravity up to  $\mathcal{A}$ :ropic coordinates) fh<sup>3</sup>

annroach to the K G eq the central BH [90,91] are easily met. **PERIMPREDIES** and how approach to the K.G. eq. strong-gravity regime, with nonlinear deviations from the

O A BH Again, 
$$
V_I(\phi) = \frac{\lambda_4}{4} \phi^4
$$
.

gravity can be further split over the strong-gravity regime,  $\epsilon$  $\overline{\text{se}}$ 

Then,  $1 < \gamma < 5/3$  and  $P \propto \rho^{\gamma}$ strong-gravity regime, with nonlinear deviations from the  $\frac{1}{2}$  $:$  onto a

$$
\text{ischild metric close to the BH:} \qquad \qquad \frac{r_s}{4} < r < r_{\text{NL}} \colon f(r) = \left(\frac{1 - r_s/(4r)}{1 + r_s/(4r)}\right)^2, \qquad \qquad h(r) = (1 + r_s/(4r))^4,
$$

$$
\Phi \ll 1
$$
,  $f = 1 + 2\Phi$ ,  $h = 1 - 2\Phi$   $r \gg r_{sg}: \nabla^2 \Phi = 4\pi \mathcal{G} \rho_{\phi}$ ,

of and the cosine is significantly defermed by the self interac  $S<sub>o</sub>$  and the cosme is significantly deformed by the seif interaction expressions for the scalar-field profile and its inflow onto the scalar-field profile and its inflow on the sc<br>Its inflow on the scalar-field profile and its inflow on the scalar-field profile and its inflow on the scalar s: annarmonic osciliations  $r = \frac{1}{2} \int_0^1 \frac{1}{2} \, dt$ e cosine is significantly deformed by the self-interactions:  $\mathcal{L}$  becomes the motion of motion of motion of motion of  $\mathcal{L}$ To summarize, at all radii the metric is given by Eq. (6), at all radii the metric is given by Eq. (6), at all  $I$  eformed by the self-interactions; anharmonic  $\alpha$ reads in these coordinates as recapper in the coordinate contractions  $\mathbf{F}$ large and the cosine is significantly deformed by the the BH: d oscillations are large and the cosine is significantly deformed by the self-interactions: anharmonic oscillat where  $\rho$  is the scalar-field energy density. This is the scalar-field energy density. This in turn is in turn of  $\eta$ ned by the self-interactions: annarmonic osciliations - field oscillations are large and the cosine is significantly deformed by the self-interactions: anharmonic oscillations

2r<sup>2</sup>h ffiffiffiffiffiffiffiffiffiffiffi <sup>1</sup> <sup>−</sup> <sup>f</sup> <sup>p</sup> <sup>f</sup> <sup>ð</sup><sup>1</sup> <sup>−</sup> <sup>f</sup> <sup>−</sup> ffiffiffiffiffiffiffiffiffiffiffi  $\overline{\phantom{a}}$ In the large-mass limit, use a nonlinear local approximation:  $\phi = \phi_0(r) \text{cn} [\omega(r)t - \mathbf{K}(r) \beta]$ Fs  $\ddot{\phantom{a}}$ In the *large-mass limit*, use a nonlinear local approximation:

 $\text{cn}(u, k)$  is a generalization of the cos  $\alpha$  , the magnetic density of the scalar-field energy density density of the scalar-field energy density of  $\alpha$ remains finite at the Scheme<br>Velocity by F (1999) co are normical (easily oscinator.  $\partial u^2$  $\mathbf{a}$  and the modulus  $\mathbf{b}$  and the modulus  $\mathbf{b}$  $\tan(u,k)$  is a generalization of the cosine to the nonlinear (cubic) oscillator:  $\frac{1}{\partial u^2} = (2k^2 - 1)$ cn –  $2k^2$ cn<sup>3</sup>,  $r_{\rm s}$  , shows that as expected the associated the set of the set (Jacobi elliptic function)  $\mathbf{S}^{\text{max}}$  parameters m and to the mass m and to the ma eneralization of the cosine to the <mark>non</mark>li (Jacobi elliptic function)  $k = 0$  :  $cn(u, k = 0) = cos(u)$ ∂ϕ ∂r  $\frac{1}{2}$  $\frac{1}{1}$  $\mathrm{cn}(u,k)$  is a generalization of the cosine to the nonlinear (cubic) oscillator:

$$
\frac{\partial^2 \phi}{\partial t^2} - \sqrt{\frac{f}{h^3}} \frac{1}{r^2} \frac{\partial}{\partial r} \left[ \sqrt{f h} r^2 \frac{\partial \phi}{\partial r} \right] + f m^2 \phi + f \lambda_4 \phi^3 = 0.
$$

is limit, use a nonlinear local approximation: 
$$
\phi = \phi_0(r) \text{cn}[\omega(r)t - \mathbf{K}(r)\beta(r), k(r)],
$$
<sup>1.0</sup>



er Pront wavelengen snotter than the benwar Eschild radius ر<br>Ah n the 2<br>12k2 ichwarzschild radius



#### <u>Þjus</u> O I : ð49Þ **rand is very regared with the component value of the component value of the component value of the component v**  $\mathbb{R}^n$ <u>Dy Normineal Oschia</u> **B) Nonlinear oscillator**

Nonlinear Klein-Gordon eq. of motion:  $\frac{\partial^2 f}{\partial t^2}$  $\partial^2 \phi$ "∂<sup>ϕ</sup>  $\overline{\phantom{a}}$ Nonlinear Klein-Gordon eq. of motion:  $\frac{\partial \Psi}{\partial t^2} - \sqrt{\frac{J}{h^3} \frac{d}{r^2} \frac{\partial}{\partial r}}$ 

$$
\phi_0(r)
$$
,  $\omega(r)$ ,  $\beta(r)$ ,  $\mathbf{K}(r)$ ,  $k(r)$  are slow functions of  $r$   
\n $\omega \sim \beta \sim m$   $\nabla_r \ll m$   $1/m \ll r_s$  Compton wavelength shorter than the Schwarzschild radius

Substituting into the Klein-Gordon eg. → Φ ⇒ <del>αποσταστηγο</del> πιο στο στοπείο στο στοποιό της στοποιός στο στοποιός στο στοποιός στο στοποιός στο στοποιός στο σ  $k(r)$  is determined by a self-consister  $\mathcal{S}$  r+3. The density point  $\mathcal{S}$  grows linearly with jFj, as the density  $\mathcal{S}$ ermines all para Taking into account the radial dependence, we can look account the radial dependence, we can look a look a loo<br>Taking into account the radial dependence, we can look a look a look account the radial dependence, we can loo  $k(\mathcal{r})$  is determined by a self-consistency constraint: the mean flux (averaged over the fast oscillations) must be cor Substituting into the Kloin Gordon es determing radial derivatives of the phase s and the wave functions wave functions was wave functions was wave functions w parameters  $\int_{\phi}$  is  $\mathbb{K}$  interms of  $\int_{\phi}$  $\beta$  purantecers ( $\gamma$ 0, w,  $\beta$ ,  $\pm$ ) in certify or  $\beta$  ( $\gamma$ ) lon eq. detei mines all parametei<sup>.</sup> This synchronous or the seed from the seed of the second term of the second term in the se  $\phi_0$ ,  $\omega$ ,  $\beta$ ,  $\mathbf{K}$  in terms of  $k(r)$ Substituting into the Klein-Gordon eq. determines all parameters  $\{\varphi_0, \; \omega, \; \beta, \; \mathbf{R}\}$  in terms of  $\; \kappa(\mathit{r}) \;$ (averaged over the fast oscillations) must be con  $\big)$  is determined by a self-consistency constraint: the mean flux (averaged over the fast oscillations) must be cor  $k(r)$ to the day of consistency constraint; the mean flux (overaged over the fast <sup>3</sup>k<sup>2</sup> <sup>ð</sup>2ð2k<sup>2</sup> <sup>−</sup> <sup>1</sup>ÞC<sup>2</sup> <sup>þ</sup> <sup>1</sup> <sup>−</sup> <sup>k</sup><sup>2</sup>Þ; <sup>ð</sup>93<sup>Þ</sup> onsistency constraint: the  $\frac{1}{\sqrt{2}}$ fi<br>fi tl  $\ddot{\cdot}$ e<br>sti1 oscillation period of period of period does not do not do not depend on the second one of the se leading orders do not show secular terms that grow with Gordon eq. determines all parameters  $\,\{\phi_0,\;\;\omega,\;\;\beta,\;\;\mathbf{k}\}$ the consistency constraint. The mean nux (avera Substituting into the Klein-Gordon eq. determines all parameters  $\{\phi_0, \;\;\omega,\;\;\beta,\;\;\mathbf{K}\}$  in terms of  $\;k(r)$   $\qquad \qquad$  (at leading order)  $k(r)\,$  is determined by a self-consistency constraint: the mean flux (averaged over the fast oscillations) must be constant over radius: <code>steady state</code>

$$
\nabla_{\mu} T_0^{\mu} = 0, \qquad F = -\sqrt{f h^3} r^2 \langle T_0^r \rangle = \sqrt{f h} r^2 \phi_0^2 \omega \mathbf{K} \beta' \left\langle \left(\frac{\partial \mathbf{c} \mathbf{n}}{\partial u}\right)^2 \right\rangle, \qquad \text{is a constant.}
$$

 $f$ <sub>1</sub> nonlinear cubic term due to the self-interactions

<sup>4</sup> <sup>ð</sup>89<sup>Þ</sup>

sm<sup>4</sup>ð<sup>1</sup> <sup>þ</sup> <sup>α</sup>Þ<sup>2</sup>

#### $\frac{1}{2}$  . Fig. , we can see that if  $\frac{1}{2}$  must be seen that if  $\frac{1}{2}$  $0.66$  of the peak,  $r_{\rm eff}$ **C) Critical flux: unique transsonic solution**

Schwarzschild radius, and ρ ∼ m<sup>4</sup> from dimensional s qualitatively similar to the classical Bondi prok x⋆. If jFj < jFcj, there exist two distinct solutions k1ðxÞ < The behaviour is qualitatively similar to the classical Bondi problem:

- en flux  $F < F_\star \,$  there are 2 solutions: a fully sul  $\lambda$  the peak throughout. It is only for the condition  $\lambda$ - For a given flux  $F < F_\star \;$  there are 2 solutions: a fully subsonic and a fully supersonic solution.
- tical flux  $F_{\star}$  these 2 branches join at a critical r  $\mathcal{L}^2$  where  $\mathcal{L}^2$  where  $\mathcal{L}^2$  where  $\mathcal{L}^2$  where  $\mathcal{L}^2$  with the radius coincide with the radius  $\mathcal{L}^2$
- The boundary conditions select the unique transsonic aary conditions coloct the unique transcenties



- At the critical flux  $F_\star$  these 2 branches join at a critical radius  $\,r_\star\,$  , which allows 2 unique transsonic solutions.

- The boundary conditions select the unique transsonic solution that is subsonic at large radii and supersonic at small radii.

matching to the hydrostatic soliton free fall at the BH horizon





Characteristic density: teristic den: ρ<br>Γ ity:  $\rho_a \equiv$ 

 $\mathbf{r}$  calar cloud to reach an hydrostatic equilibrium, where  $\mathbf{r}$  $\blacktriangleright$  greater repulsive self-interactions decrease the scalar-field energy density and flux. greater repulsive self-interactions decrease the scalar-field energy density and flux. metric potentials are still dominated by the central BH. We show ractions decrease the scalar-field energy density and flu ctions decrease the scalar-field energy density and flu

3λ<sup>4</sup>

a<mark>ctio</mark>

:<br>:<br>:

ðΦ þ ΦIÞ ¼ 0, and we have

Critical flux:

\n
$$
F_c = F_{\star} F_s \qquad \text{with} \qquad F_{\star} \sim 0.7 \qquad F_s = \frac{r_s^2 m^4}{\lambda_4}
$$
\n
$$
r \sim r_s: \quad \rho \sim \rho_a, \quad v \sim c
$$

ra ¼  $\mathsf{n}$ dominated by the scalar-field soil - large radii (weak gravity dominated by the scalar-field soliton self-gravity):  $\sqrt{2}$ FIG. 4. Scalar-field energy density computed in the Eddington

> $\overline{\phantom{a}}$ 1<br>1 prosessor  $\sim$  relativiation much emaller the intervention of the intervention of the contract of the intervention of the contract of the intervention of the contract of the intervention of the intervention of the intervention of th : öldur í staðar er einnig er<br>Staðar er einnig er  $r_{\star}\sim 2.4r_s$  in the relativistic regime ↑ relativistic, much smaller than Bondi Some scalar-field dark matter models can be constrained by constraints can be constrained by constrained by co by the measurement of stellar dynamics at small  $\mathcal{B}$  at small  $\mathcal{B}$ ∼ ra, we find that rsg < mass to the halo dark matter mass is of the order of 10<sup>−</sup><sup>5</sup> −  $r_{\star}\sim 2.4r_s$  in the relativistic regime relativistic, much smaller than Bondi

$$
\dot{M}_{\text{Bondi}} = \frac{2\pi\rho_0 \mathcal{G}^2 M_{\text{BH}}^2}{c_s^3}
$$
\n
$$
\dot{M}_{\text{SFDM}} = \frac{12\pi F_{\star} \rho_0 \mathcal{G}^2 M_{\text{BH}}^2}{c_s^2 c}
$$



BH

 $c_s^2$  $\frac{2}{s}c$ 

- interme sinta radii  $rel$ ra (weak gravity dominate  $\frac{1}{2}$ by the R  $\gamma$  ure ;  $\mathbf{u}$ 

 $4m^4$ 

#### **II- BH MOVING INSIDE A SFDM CLOUD**



#### **A) Soliton and BH frames**

#### **B) Large-distance domain** <u>arge-urs</u>

where  $\mathcal{L}_{\mathcal{A}}$  and  $\mathcal{L$ 

 $\Gamma$  antiquity on  $\Gamma$  Fulor on the bulk is also related to the transition radius  $\alpha$ Continuity eq. + Euler eq.  $\qquad \qquad$  Potential flow  $\qquad \bar{v}$ 





mass and momentum flux through any arbitrarily distant surface: Conservation of mass and momentum allow us to obtain the



2πρ0G<sup>2</sup>M<sup>2</sup>

Isentropic potential flow eq.:

$$
\hat{\nabla}(\hat{\rho}\vec{v}) = 0
$$

 $\hat{\nabla} \cdot$ 



Allows us to obtain analytical results from large-distance expansions

**Example:**<br>The interpretation of a product the adiab<br>and is controlled as a control of a product the set of a set of a<br>diab. Far from the BH: hydrodynamical equations of an isentropic gas of effective adiabatic index  $\;\gamma=2$ 



$$
\frac{\binom{2}{0}}{\binom{2}{0}} \frac{\partial^2 \delta \beta}{\partial z^2} = \frac{v_0 z}{\rho_0 r^3}
$$

 $v_0 < c_{s0}$ : subsonic BH velocity, elliptic eq., boundary-value problem, smooth

 $v_0 > c_{s0}$ : hypersonic BH velocity, hyperbolic eq., Cauchy problem, shock





#### **C) Subsonic and supersonic regimes**

#### **III- SUBSONIC REGIME** 2v<sup>0</sup> # <u>I- SUBSONI</u> e de <u>de</u>  $\ddot{ }$ **b** nonlinearity in (67) generates nonzero contributions to all converges. The first term in Eq. (131) is the first term in Eq. (13

Exact analytical results using a large-distance expansion:  $T_{\rm eff}$  is the difference with the linear flow (85) is that the cubic non linearity in (67) generates nonzero contributions to all exact and  $\mathsf{L}$ 



Con:

Conse





ˆð1<sup>Þ</sup> evenðθÞ ¼ <sup>X</sup> ∞ b2<sup>l</sup>P2<sup>l</sup>ðcos θÞ; ð107Þ

alytical results using a large-distance expansion:  $\hat{\beta}$  =  $\hat{\beta} = \hat{\beta}_{-1} + \hat{\beta}_0 + \hat{\beta}_1 + \dots, \text{ with } \hat{\beta}_n$  $\hat{\beta}$  $= \beta$  $\hat{\beta}_{-1}+\hat{\beta}_{0}$  $\beta$  $\hat{\beta}$  $\beta_1 + \ldots, \quad \text{with} \quad \beta$  $\hat{\beta}$ ictore overseign:  $\hat{\beta} = \hat{\beta}$  $\hat{\beta}_n \sim \hat{r}^-$ 

 $\mathsf{d}^2\mathsf{A}$ 

$$
\frac{V_{\text{out}}}{\
$$

#### $\mathbf{w}$  and  $\mathbf{w}$  decomponents not obtained and even components of  $\mathbf{w}$ Velocity field (v)



# Velocity and Density Fields in Subsonic Regime

(Supersonic Regime Up-Coming!)







#### **IV- SUPERSONIC REGIME** DEDCANIC DECIME The moderate and high-velocity be-

#### However, for moderate Mach numbers this accretion rate **A) Moderate Mach number**





Max. radial accretion rate

#### **is order of the B) High Mach number** higher than the radial accretion rate (29). The latter is





Hoyle-Lyttleton accretion mode Edgar (2004) HL accretion rate

Most of the accretion occurs through a narrow accretion column at the rear.



In the bulk, upstream:

In the bulk, downstream:

In the boundary layers:  $\beta$ 

$$
\hat{\beta}=v_0\hat{r}u-\frac{1}{2v_0}\, \mathrm{l}
$$

 $\beta$  $\hat{\hat{\beta}}$ 

#### <u>s using large-distance expansions and asyn</u> ˆ  $\mathbf{u}$  $\mathbf{A} = \mathbf{A} \mathbf{A} \mathbf{A}$  for the matching of the matching and asymptotic matering  $\mathcal{L} = \mathcal{L} \times \mathcal{L}$  that the large-distance expansions of the large-distance expansions of the large-distance expansions of the largena asymptotic matching **C) Analytical results using large-distance expansions and asymptotic matching**



han h min and eat hy the ealf-interaction Thus, the scalar-field self-interactions are much more UV cutoff greater than b\_min and set by the self-interactions:  $\sim$  times from small scales to the dynamical friction and in the dynam and a series of the series o<br>East of the series of the {3 greater than b\_min and set by the self-interactions:  $r_{\text{UV}} = 64$ *r*UV ◆

$$
r_{\rm UV} \simeq \sqrt{\frac{18}{e}} r_{\rm sg} \mathcal{M}_0^{-3/2}
$$
  $r_{\rm sg} = \frac{r_s}{c_{s0}^2}, \quad c_{s0}^2 = \frac{\rho_0}{\rho_a}$ 

of mass and mo Again, use conservation of mass and momentum: *<sup>z</sup>* <sup>=</sup> ˙ B. Dynamical friction *r r*sg : ⇢(*r*) = ⇢<sup>0</sup>



2/3 smaller than Chandrasekhar's expression converted into a radial pattern at small radial pattern at small radial pattern at small radii, but now by a r<br>Into a radii, but now by a rad  $\overline{\phantom{a}}$ 2/3 smaller than Chandrase 12c*h*<br>BHz 2000 College College<br>BHz 2000 College Coll :xpr B. Dynamical friction in Chandrasakhar's avnrassion also associated with the self-interaction of the self-interaction and contained t a communication speed of the second speed of the spee

$$
t_0^{-3/2} \t r_{\rm sg} = \frac{r_s}{c_{s0}^2}, \t c_{s0}^2 = \frac{\rho_0}{\rho_a}
$$

 $t$  $\rho_0$ 

## **D) Dynamical friction**

*.*

$$
\text{s:} \qquad r_{\text{UV}} = 6\sqrt{\frac{2}{e}} \frac{\mathcal{G}m_{\text{BH}}}{c_s^2} \left(\frac{c_s}{v_{\text{BH}}}\right)^{3/2}
$$



# **Gravitational Waves emitted by a BH binary inside a SFDM soliton**

$$
m_{\rm BH} \dot{\mathbf{v}}_{\rm BH}|_{\rm halo} = -\frac{4\pi}{3} \mathcal{G} m_{\rm BH} \rho_0 (\mathbf{x} - \mathbf{x}_0)
$$

Accretion drag:

 $m_{\rm BH}$  $\dot{v}_{\text{BH}}|_{\text{acc}} = -\dot{m}_{\text{BH}}v_{\text{BH}}$ 

#### **I- Additional forces on the BHs due to the dark matter environment**

#### momentum dark matter does not change the BH momentum Gravity of the dark matter cloud: accretion and *accretion* Gravity of the dark matter cloud:

 $m_{\rm BH}$ <br>Dynamical friction:<br> $m_{\rm BH}$ AQd Vee KRZ, MXVW beORZ WKaW, ´LQcUeaVLQg eQeUg\µ LV OabeOed aV



corretion drag:

\n
$$
m_{\text{BH}} \dot{\mathbf{v}}_{\text{BH}}|_{\text{acc}} = -\dot{m}_{\text{BH}} \mathbf{v}_{\text{BH}}
$$
\nynamical friction:

\n
$$
m_{\text{BH}} \dot{\mathbf{v}}_{\text{BH}}|_{\text{df}} = -\frac{8\pi \mathcal{G}^2 m_{\text{BH}}^2 \rho_0}{3v_{\text{BH}}^3} \ln\left(\frac{r_{\text{IR}}}{r_{\text{UV}}}\right) \mathbf{v}_{\text{H}}
$$



#### accretion of dark matter, the dynamical friction and the emission of Gws. This gives the total drift of the total drift of the orbital drift of the orbital drift of th<br>This gives the orbital drift of the orbital drift of the orbital drift of the orbital drift of the orbital dri where *P* is the rate of energy loss by gravitational waves and **THE DECAY of the orbital radius**

*.* (3.32) Dynamical friction **Dynamical** frequency and the energy reading

 $\mathsf{r}_i$  and  $\mathsf{r}_i$  and  $\mathsf{r}_i$  motion, we obtain  $\mathsf{r}_i$ circular orbits of radius 0 the velocity of radius 0 the velocity of radius 0 the velocity of radius 0 the vel<br>The velocity of radius 0 the veloci

$$
\langle \dot{a} \rangle = \langle \dot{a} \rangle_{\text{acc}} + \langle \dot{a} \rangle_{\text{df}} + \langle \dot{a} \rangle_{\text{gw}}
$$

is negligible. Although the additional halo gravity increases  $\sim$ **2** Correction due to the halo **k** Correction due to the halo bulk gravity

 $B_{\rm acc}$ 

$$
\langle \dot{a} \rangle_{\text{gw}} = -\frac{64\nu \mathcal{G}^3 m^3}{5c^5 a^3} \left( 1 - \frac{4\pi \rho_0 a^3}{3m} \right)
$$



$$
\langle \dot{a} \rangle_{\text{acc}} = -a A_{\text{acc}} - a \left( \frac{a}{\mathcal{G}m} \right)^{3/2} B_{\text{acc}}
$$
 Acc

$$
\langle \dot{a} \rangle_{\text{df}} = -a \left( \frac{a}{\mathcal{G}m} \right)^{3/2} \left[ B_{\text{df}} + C_{\text{df}} \ln \left( \sqrt{\frac{\mathcal{G}m}{a}} \frac{1}{c_s} \right) \right]
$$

Accretion drag

#### **III- Phase of the GW waveform** f 2/3 if we need the dark matter corrections in the amplitude of the <u>त ।</u> e GV<br>C 5 **III- Phase of the GW waveform**

transform variable used below in the Fourier-space analysis of

matter detection the control of the<br>The control of the c

3.3 FIN  $/$  -5.5 PN *,* halo = 2c *<sup>M</sup>* <sup>=</sup> ⌫3/5<*,* (4.17)

 $\overline{\phantom{a}}$   $\overline{\phantom{a}}$ 

 $\Psi_{\mathrm{df}}$  -5.5 PN  $\overline{1}$ *,...*  $\mathbb{F}_{\mathbb{F}_{q}}$  $\overline{\mathsf{p}}$ 

 $\textrm{ctions}$   $\textrm{S}$ *1. Dark matter halo gravity*



$$
\text{In this, } \mathbf{r} = \frac{1}{\pi} \sqrt{\frac{\mathcal{G}m}{a^3}} \left( 1 + \frac{2\pi \rho_0 a^3}{3m} \right)
$$
\n
$$
\text{Hint: } \mathbf{\hat{f}} = \frac{1}{\pi} \sqrt{\frac{\mathcal{G}m}{a^3}} \left( \frac{\dot{m}}{2m} - \frac{3\dot{a}}{2a} \right) + \mathcal{G}\rho_0 \left( \frac{a^3}{\mathcal{G}m} \right)^{1/2} \frac{\dot{a}}{a}
$$
\n
$$
\text{(} t \text{)} = 2\pi \int d\mathbf{f} \left( \mathbf{f} / \mathbf{\hat{f}} \right) \qquad \text{Time: } t = \int d\mathbf{f} \left( 1 / \mathbf{\hat{f}} \right)
$$
\n
$$
\text{From of the GW signal: } \frac{\mathbf{\hat{h}}(f)}{\mathbf{\hat{h}}(f)} = \mathcal{A}(f)e^{i\Psi(f)}
$$
\n
$$
\text{hase: } \frac{\Psi(f)}{\Psi(f)} = 2\pi f t_c - \Phi_c - \frac{\pi}{4} + \Psi_{\text{gw}} + \Psi_{\text{halo}} + \Psi_{\text{acc}} + \Psi_{\text{df}}
$$
\n
$$
\text{hase: } \frac{3}{\Psi} \left[ 1 + \frac{20}{9} \left( \frac{743}{336} + \frac{11}{4} \nu \right) \left( \frac{\pi \mathcal{G}m f}{c^3} \right)^{2/3} \right] \qquad \text{O} + 1 \text{ PN}
$$

Cgw =

$$
\Psi_{\rm halo} = \frac{25\pi}{924} \frac{\rho_0 G^3 \mathcal{M}^2}{c^6} (\pi \mathcal{G} \mathcal{M} f/c^3)^{-11/3}
$$

$$
\Psi_{\text{acc}} = -\frac{25\pi \mathcal{G}^3 \mathcal{M}^2 \rho_0}{38912c^6} \left(\frac{\pi \mathcal{G} \mathcal{M}f}{c^3}\right)^{-16/3} \sum_{i=1}^2 \Theta(f > f_{\text{acc},i}) \frac{m_i^3}{\mu^2 m} \left(3 + 2\frac{m_i^2}{m\mu}\right)
$$
\n
$$
-\frac{75\pi F_\star \nu^{2/5} \mathcal{G}^3 \mathcal{M}^2 \rho_a}{26624c^6} \left(\frac{\pi \mathcal{G} \mathcal{M}f}{c^3}\right)^{-13/3} \sum_{i=1}^2 \Theta(f < f_{\text{acc},i}) \left(3 + 2\frac{m_i^2}{m\mu}\right) \left[1 - \left(\frac{f}{f_{\text{acc},i}}\right)^{13/3} + \frac{13}{19} \left(\frac{f}{f_{\text{acc},i}}\right)^{16/3}\right]
$$

$$
\Psi_{\mathrm{df}} = \frac{875\pi\mathcal{G}^3\mathcal{M}^2\rho_0}{11829248c^6} \left(\frac{\pi\mathcal{G}\mathcal{M}f}{c^3}\right)^{-16/3} \sum_{i=1}^2 \frac{m_i^3}{\mu^2 m} \Theta(f_{\mathrm{df},i}^- < f_{\mathrm{df},i}^+) \Bigg\{ \Theta(f_{\mathrm{df},i}^- < f < f_{\mathrm{df},i}^+) \left[1 + \frac{304}{105} \ln \frac{f}{f_{\mathrm{df},i}^+} - \frac{361}{105} \left(\frac{f}{f_{\mathrm{df},i}^+}\right)^{16/3} + \frac{256}{105} \left(\frac{f}{f_{\mathrm{df},i}^+}\right)^{19/3} \right] \Bigg\}
$$
\n
$$
+ \Theta(f < f_{\mathrm{df},i}^-) \left[ -\frac{361}{105} \left(\frac{f}{f_{\mathrm{df},i}^+}\right)^{16/3} + \frac{361}{105} \left(\frac{f}{f_{\mathrm{df},i}^-}\right)^{16/3} + \frac{5776}{315} \left(\frac{f}{f_{\mathrm{df},i}^-}\right)^{16/3} \ln \frac{f_{\mathrm{df},i}^-}{f_{\mathrm{df},i}^+} + \frac{256}{105} \left(\frac{f}{f_{\mathrm{df},i}^+}\right)^{19/3} - \frac{4864}{105} \left(\frac{f}{f_{\mathrm{df},i}^-}\right)^{19/3} \ln \frac{f_{\mathrm{df},i}^-}{f_{\mathrm{df},i}^+} \Bigg\}
$$

#### **IV- Fisher matrix analysis**  are significant and the wavelength  $\alpha$  of the wavelength of the scalar field  $\alpha$ <u>are negligible. This regime re</u>

$$
\Gamma_{ij} = \frac{(\text{SNR})^2}{\int_{f_{\min}}^{f_{\max}} \frac{df}{S_n(f)} f^{-7/3} \int_{f_{\min}}^{f_{\max}} \frac{df}{S_n(f)} f^{-7/3} \frac{\partial \Psi}{\partial \theta_i} \frac{\partial \Psi}{\partial \theta_j}
$$

 $\varphi_0$  Parameters:  $\{\theta_i\} = \{t_c, \Phi_c, \ln(m_1), \ln(m_2), \rho_0, \rho_a\}$   $\rho_0$  $\begin{pmatrix} 0 & 1 \end{pmatrix}$  upcoming gravitational wave detectors such as  $\begin{pmatrix} 0 & 1 \end{pmatrix}$ ars:  $\{\theta_i\} = \{t_c, \Psi_c, \text{Im}(m_1), \text{Im}(m_2), \rho_0, \rho_a\}$  (b) halo b

 $\partial \Psi$  $\partial \theta_j$ 

 $\rho_0$  halo bulk density



$$
\rho_a=\frac{4m^4}{3\lambda_4}
$$

#### previous in the narameter space that <u>in the grounding pour online to ongoing research on a group on the </u> V- Region in the parameter space that can be detected through the phases (4.18)-(4.18)-(4.20). The phases of **V- Region in the parameter space that can be detected**





 $\rho_0$ The parameters *m* and <sup>4</sup> determine the characteristic halo bulk density **B. Events**  $\rho_0$  halo bulk density

$$
\frac{\rho_a}{\rho_0} = \frac{c^2}{c_s^2} \ge 1
$$



$$
1 M_{\odot}/\text{pc}^3 = 6.7 \times 10^{-23} \text{ g/cm}^3
$$

$$
\cdot \mathrm{m}^3
$$





 $\rho_0$  $\theta$  and  $\theta$  and  $\theta$  and  $\theta$  and  $\theta$  $\overline{\rho}_0$  is an approximate and radius  $\overline{\rho}_0$ halo bulk density

$$
\rho_a=\frac{4m^4}{3\lambda_4}
$$

**Critical density:**  $v^2$  masses. Below the equality recall  $\rho_c \sim 10$  g/cm  $\sim 10$   $M_{\odot}/pc$  $\rho_c \sim 10^{-29} \text{g/cm}^3 \sim 10^{-7} M_{\odot}/\text{pc}^3$ 

Solar neighborhood:

 $\rho_{\rm DM} \sim 1 \ M_\odot /{\rm pc}^3 \sim 7 \times 10^{-23} \ {\rm g/cm}^3$ 

Baryonic density in thick disks:

 $\alpha \leq 10^{-7} \frac{\text{m}}{\text{s}}^3$  $\frac{1}{\sqrt{2}}$  as compared with the data mass  $\frac{1}{\sqrt{2}}$  $\rho_{\rm b} \lesssim 10^{-7} {\rm g/cm}^3$ 









#### *Plane*  $(m_{\text{DM}}, \lambda_4)$



Radius of the scalar cloud (soliton)



$$
R_{\rm sol} = \pi \sqrt{\frac{3 \lambda_4}{2}} \frac{M_{\rm Pl}}{m^2}
$$

$$
R_{\rm sol} = \sqrt{\frac{\pi}{4 \mathcal{G} \rho_a}}
$$





 $Plane$   $(m<sub>DM</sub>, R<sub>sol</sub>)$ 



# **Impact of the time-dependent DM gravitational potential on GW**

#### **A) Frequency shift**

The density field has a subleading oscillatory component:  $\rho_D$ 

$$
T_{\mu\nu} = \partial_{\mu}\phi\,\partial_{\nu}\phi - \frac{1}{2}g_{\mu\nu}\left((\partial\phi)^2 - m^2\phi^2\right)
$$





in the non-relativistic limit equals *E* ' *m*+*mv*<sup>2</sup>*/*2. Since the characteristic time scale  $\phi(\vec{x}, t) = A(\vec{x}, t) \cos(mt + \alpha(\vec{x}, t)]$ slow variations on astrophysical scales fast oscillations

$$
\rho_0 = \frac{1}{2} m^2 A^2 \qquad \qquad \rho_{\text{osc}} \sim (\nabla \phi)^2 \sim k^2 \phi^2 \sim \frac{k^2}{m^2} \rho_0 < v^2 \rho_0 \qquad \qquad \lambda_{\text{dB}} = \frac{2\pi}{m v}, \quad k < \frac{2\pi}{\lambda_{\text{dB}}}
$$

The gravitational potential also has a subleading oscillatory component:

Khmelnitsky & Rubakov. 2013

$$
\Psi_N(\vec{x},t) = \Psi_0(\vec{x}) + \Psi_{\text{osc}}(\vec{x})\cos[\omega t + 2\alpha(\vec{x})]
$$

$$
\omega = 2m
$$

$$
\nabla^2 \Psi_0 = 4\pi \mathcal{G}\rho_0 \qquad \qquad \Psi_{\rm osc} = \pi \frac{\mathcal{G}\rho}{m^2}
$$

(shift of PTA time delays)

$$
\phi(\vec{x},t) =
$$

by their velocity dispersion (as for CDM). 233

 $\overline{\psi(f)}$ 





#### $1865$  where  $\rho$  is the DM density averaged over the fast over the fast over the fast oscil- $\rho$ **187 GW phase shift** and subject on whereas the substitution of the substitution o  $194$  typical wave numbers k of the DM density field verify field <u>nden and be much smaller in the much smaller in the much smaller in the much smaller in the smaller i</u> **208 the solar neighborhood estimate. The solar neighborhood estimate. The solar neighborhood estimate. The solar neighborhood estimate. The solar neighborhood estimate shift** 206 degenerate with binary parameters. We shall find below that mass m<sup>1</sup> and m2, and <sup>245</sup> lating component of the gravitational potential *<sup>N</sup>* , as in

Phase and time related to the frequency drift: 
$$
\Phi = 2\pi \int df \frac{f}{f}
$$
,  $t = \int df \frac{1}{f}$ 

$$
\frac{\Delta f}{f} = \Psi_N(\vec{x}_e, t_e) - \Psi_N(\vec{x}, t)
$$
\n
$$
f \ge \omega \quad \text{whence } m_{\phi} < \left(\frac{f_{\text{min}}}{1 \text{ Hz}}\right) 3 \times 10^{-16} \text{ eV}
$$
\nemission

\nrecaption (negligible)

ected (many oscillations along the l.o.s.): 
$$
\lambda = \frac{c}{f} \ll \frac{2\pi}{k}
$$

l ti<mark>ı</mark> : öldur er en stærke er en stærk<br>Det er en stærke er Phase and time related to the frequency drift: ia<br>Me related to the frequenc Phase and time related to the frequency drift:  $\Phi = 2\pi \int$ underly the field of the field. The field of the field of the Field of the Field of the Einestein of the Eines

GW signal: 
$$
h(t) = A(t) \cos[\Phi(t)]
$$
 Phase and time related to the frequency of

Going to Fourier space:  $\hat{h}(f) =$ Going to Fourier space:  $\hat{h}(f) = 0$ 

Going to Fourier space: 
$$
\tilde{h}(f) = \int dt \, e^{i2\pi f t} h(t) = A(f) e^{i\psi(f)}
$$

Saddle-point approximation: 214 system in vacuum, and Δf is the frequency shift due to the Saddle-point approximation:  $\mathsf{Gold}$ -noint annrovimation:  $A(\mathsf{C})$ Saddio political provintation.

on: 
$$
A(f) \propto f^{-7/6}
$$
,  $\psi(f) = 2\pi ft_x - \Phi(t_x) - \pi/4$ ,  $f(t_x) = f$ .

In the optical approximation, as for the Sachs-Wolfe effect for CMB photons, the gravitational potential along the line of sight leads to a frequency shift of the GW signal:  $\mathbb{R}^n$  $\overline{\mathbf{A}}$  $\mathbf{f}$ er and we redefined We redefined Western We<br>Program Western We I potential along the line of sight  $\frac{1}{2}$ of the GW signal: the fast of the GM density averaged over the fast over the fast oscilrestricted to the clouds as solitons with solitons in Fuzzy 227 In the optical approximation, as leads to a frequency shift of the za za zapostani za postani za zapostani za predstavanje za postani za zapostani za postani za zapostani za zap<br>Za zapostani za zap d<br>Galeria In the ontical annroximation, as for the Sachs-Wolfe effect for CMR photons, the gravitational po  $\overline{a}$  $A \sim \frac{1}{\sqrt{2}}$  *eads to a frequency shift of the GW s* a drift of the filling of the emitted and the emitted GMS, the gravital control of the emitted GMS, and the emit<br>GMS - We define the emitted and the emitted GMS - and the emitted of the emitted of the emitted gravitations



emissid  $\mathbf{u}$ simosion readquare reading sion<br> einssion reception (riegigioie)

The integrated Sachs-Wolfe effect is neglected (many oscillations along the l.o.s.):  $\lambda =$ The integrated Sache Welfe effect is n the megrated eache wore enect to h

At leading order, the frequency drift is due to the emission of GW: tc − t¯ ne trequency drift is due to the to the treat the sensitivity of the top in the to the the the the the the the<br>
<u>Exerces</u> ondy anne id dad to the diniod nission  $\overline{\phantom{a}}$ frequency drift is due to the emission of GW:  $\qquad \bar{\psi}(f) = 2\pi$ ie emission of GW:

 $2221$ The DM gravitational potential gives a correction: ction:  $\Delta \psi(f) = 2\pi \int_{\tau}^{f_c} dt \bar{f} \Psi.$ The DM gravitational potential gives a correction:  $\Delta \psi(f) = 2\pi \int_{\bar{t}_0}^\infty dt f \Psi(f)$ 272 order by  $\overline{\phantom{a}}$  included the first post-Newtonian correction correction

 $27$  using the constant part is dequality in the  $27$  $\sim$   $\sim$ The contribution from the constant part is degenerate Frie contribution from the constant<br>with the leading GW contribution: onstant part i<br>»ufion:  $\begin{array}{l} \text{The contribution from the constant part is do not be}\end{array}$  $\frac{1}{\sqrt{2}}$ ading ame ontribution from the constant pai Δψosch της Ευρώπης της Γεννής της<br>Δυνατολίτες προϊόντας της Γεννής with the leading GW contribution:<br>2006 with the leading GW contribution:  $Jt$ <sub>3</sub> The contribution from the constant part is degenerate THE CONTINUTION HOLD THE CONStant part is degenerate

 $\mathbf{F}$ contribution from the oscilla<br>
1  $\sum_{i=1}^{n}$ The contribution from the oscillatory part reads: "  $\sum_{i=1}^{n}$  The contribution from the oscillatory part reads:  $\Delta$ The contrib 13.



259 coalescence time. This gives the standard result for the Low scalar mass, degeneracy with leadli  $\overline{a}$ : <u>the best est and</u> eracy with leading GW term  $\qquad \qquad$   $\qquad$  large scala 265 (1-PN order). This gives two terms, which behave as f<sup>−</sup>5=<sup>3</sup> Low scalar mass, degeneracy with leading GW term

"−5=<sup>3</sup> Prope Scalar Indest<br>Endet Scalar Indes 268 order probably probably masses in the magnetic contribution of the magnetic contributions in the magnetic contribution of the magnetic contribution of the magnetic contribution of the magnetic contribution of the magne 269 between the spins of the compact of th  $\mathbf{B} = \mathbf{B} \mathbf$ smaller than for the constant potential contribution of 302  $\mu$ calar masses  $\overline{\phantom{a}}$ 

128 !

πGMf

!πGMω

267 m<sup>2</sup> from the observations [50]. We do not consider higher





e DM gravitational potential gives a correction: 
$$
\Delta \psi(f) = 2\pi \int_{\bar{t}_\star}^{t_c} dt \bar{f} \Psi.
$$

constant part is degenerate  
bution: 
$$
\Delta \psi_0(f) = \frac{\Psi_0}{16} \left( \frac{\pi \mathcal{G} \mathcal{M} f}{c^3} \right)^{-5/3}
$$

frequency drift is due to the emission of GW: 
$$
\bar{\psi}(f) = 2\pi f t_c - \Phi_c - \frac{\pi}{4} + \psi_{\text{GW}}(f),
$$
  
\n
$$
\psi_{\text{GW}}(f) = \frac{3}{128} \left( \frac{\pi \mathcal{G} M f}{c^3} \right)^{-5/3} \left[ 1 + \left( \frac{3715}{756} + \frac{55\nu}{9} \right) \left( \frac{\pi \mathcal{G} M f}{c^3} \right)^{2/3} \right]
$$
\n
$$
M = m_1 + m_2, \quad \nu = m_1 m_2 / M^2, \quad \mathcal{M} = \nu^{3/5} M
$$

$$
\text{reads:} \qquad \Delta \psi_{\text{osc}}(f) = \Psi_{\text{osc}} 2\pi \left(\frac{5}{256\pi}\right)^{3/8} \left(\frac{\pi \mathcal{G} \mathcal{M}\omega}{c^3}\right)^{-5/8} \text{Re}[e^{i(5\pi/16 + \theta - \omega t_c)} \gamma (5/8, -iy)]
$$
\n
$$
y = \omega(t_c - \bar{t}_\star) = \frac{m_\phi}{m_\star}, \qquad m_\star = f \frac{128\pi}{5} \left(\frac{\pi \mathcal{G} \mathcal{M} f}{c^3}\right)^{5/3}
$$

contribution (23). Thus Ψ0 cannot be discriminated from 290 cannot be discriminated from 290 cannot be discrimin<br>Professional cannot be discriminated from 290 cannot be discriminated from 290 cannot be discriminated from 2 Large scalar mass, degeneracy with cor  $\overline{\phantom{a}}$ cy with constant factor  $\,\Phi_c^{}\,$ where  $\chi$  is the incomplete gamma function and  $2965$  is the incomplete gamma function and  $2965$ ass, degeneracy wit witl i constant factor r<br>T e267 m2 from the observations in the observations in the observations in the observations in the observations o  $2688$  order post-Newtonian contributions in this paper, which can be approximately defined by  $\sim$ Large scalar mass, degeneracy with constant factor  $\, \Phi_{c} \,$ ا<br>أ ar mass, degeneracy with constant factor  $\Phi_c$ 

$$
m_{\phi} \ll m_{\star} \colon \Delta \psi_{\rm osc}(f) = \frac{\Psi_{\rm osc}}{16} \left( \frac{\pi \mathcal{G} \mathcal{M} f}{c^3} \right)^{-5/3} \cos(\omega t_c - \theta) \qquad m_{\phi} \gg m_{\star} \colon \Delta \psi_{\rm osc}(f)
$$

<sup>266</sup> and f<sup>−</sup><sup>1</sup>, that allow us to constrain both binary masses m<sup>1</sup> and

$$
u_{c} = \theta
$$
\n
$$
u_{c} = \theta
$$
\n
$$
m_{\phi} \gg m_{\star} \colon \Delta \psi_{osc}(f) = \Psi_{osc} \Gamma(5/8) 2\pi \left(\frac{5}{256\pi}\right)^{3/8} \left(\frac{\pi \mathcal{G}M\omega}{c^{3}}\right)^{-5/8} \cos(\omega t_{c} - \theta - 5\pi/16)
$$

 $\sim m_\star$  ,  $m_\star \ll 1$ for  $\frac{1}{2}$  $Mf/c^3$  $\checkmark$  $\log_{2b} \ll \lambda$  $2, 1, 10$ Sch  $\infty$   $\Lambda$  $\Gamma$  mass,  $\ell$  for  $(\ell M f/s^3) \approx 1 \quad D \approx 7$ Probe scalar masses  $\qquad \qquad m \sim m_\star, \quad m_\star \ll f \;\; {\rm for} \;\; \mathcal{G}$ 271 point time the state of the<br>271 point time the state of the<br>27  $\boxed{m \sim m_\star, m_\star \ll f \text{ for } (\mathcal{G} \mathcal{M} f / c^3) \ll 1, R_{\text{Sch}} \ll \lambda}$  $10$ f¯0 ðt¯  $\Gamma$  (*YNI]*  $\ll 1, \quad R_{\rm Sch} \ll \lambda$ smaller than for the constant potential contribution of 302  $\sum_{i=1}^{n} \frac{1}{2\pi} \sum_{i=1}^{n} \$  $) \ll$  $\log_{\text{C}} \sim$  $\leftarrow, \quad m_{\star} \ll f \;\;$  tor  $\Lambda d f / 3$   $\sim 1$   $D$   $\sim$  1  $m_{\star} \ll J$  for  $(g/WJ/C) \ll 1$ ,  $K_{\text{Sch}} \ll \lambda$  $\mathbf{r}$  $\ddot{\phantom{1}}$  $R_{\text{col}} \ll \lambda$  $\textsf{Probe scalar masses} \qquad m \sim m_\star, \quad m_\star \ll f \;\; \text{for} \;\; (\mathcal{G} \mathcal{M} f / c^3) \ll 1, \quad R_\text{Sch} \ll \lambda$ 

Eq. (23), and it is again degenerate with the GW phase 303

$$
m\thicksim m
$$

dtf

277 waveform,

277 waveform,

In many cases (CDM, supersonic motion in fluids or SFDM), the drag force on a BH moving within a medium takes the form of the Chandrasekhar result: ersonic motion in fluids or SFDM In many cases (CDM, supersonic motion in fluids or SFDM), the drag force on a BH moving within a medium<br>takes the farm of the Charakter soldbar was the various parameters as  $\overline{D}$ l securitario  $\overline{D}$  and  $\overline{D}$ ir SFDM), the drag force on a BH moving within a medium  $\,$  $t<sub>0</sub>$ 326 and the matter day of the day of the day of the day of the day in the bind by days of the bind by days of the bind by days of the bind by days and bind by days and bind b s or SFDM), the drag force on a BH moving within a medium  $\frac{1}{2}$ 1 IN TIUIAS OF SFL  $\mathsf{e}$ emission of Gw $\mathsf{e}$ 

 $m_i$  $\frac{1}{\boldsymbol{\eta}}$  $\vec{v}$ ⃗  $i = -\frac{4\pi G^2 m_i^2}{v^3}$  $\partial \frac{\partial^2 u}{\partial t^2} = 4\pi \mathcal{G}^2 m_i^2 \rho \Delta$  $i^{U}i = \frac{v_i^3}{v_i^3}$  is in vacuum.  $m_i\vec{v}_i = -\frac{\Delta v_i}{\Delta t} \Lambda \vec{v}_i,$ 

 $\frac{244}{100}$  explaining the collision in Eq. (289) derived for collision of  $\frac{289}{100}$ 344 such as CDM, also applies to Fuzzy DM or scenarios to Fuzzy DM or scenarios with the scenarios with the sc<br>344 such as CDM or scenarios with the scenarios with the scenarios with the scenarios with the scenarios with<br>3 This gives a correction to the frequency drift and to the GW phase, which is independent of the scalar mass:  $\overline{3}$  8.88  $\overline{3}$  the frequency drift and to the CIM phase 3 d concentrion to the negacity and and to the GM phase, **3220 However, for the Scalar Hidsen** Williams **Water** This gives a correction to the frequency drift and to the GW phase,  $\Delta w_{\rm df} = -\frac{15}{2.0016} \frac{\pi G^2 M \rho}{\rho} \left( \frac{\pi G M f}{2} \right)^{-10/3}$ which is independent of the scalar mass:

 $\sum_{i=1}^{\infty}$  Giehor matrix analysis  $\sum_{i=1}^{\infty}$ <u>27 than 10 minutes</u> **D) Fisher matrix analysis** <u>ποι πια</u>  $3388$  but the dynamical friction of the form of the usual friction 339 Chandrasekhar result <mark>in</mark>

 $\Gamma_{ij} =$  $\int_{c}^{f}$ max

#### **C) Comparison with dynamical friction** 336 compact objects, and the dynamical friction. The rate of <mark>3011 with a fightlear includit</mark>  $\bullet$  345 non-negligible self-interactions in the supersonic regime,  $\bullet$  $\overline{3}$  and the model. Therefore, it is interested on the model. The model is interest-**Friction** the Fisher matrix  $\mathbf{r}$  we obtain the covariance matrix  $\mathbf{r}$  and  $\mathbf{r}$  and  $\mathbf{r}$  are covariance matrix  $\mathbf{r}$  and  $\mathbf{r}$  are covariance matrix  $\mathbf{r}$  and  $\mathbf{r}$  are covariance matrix  $\mathbf{r$  $\overline{\phantom{a}}$  , which gives the standard deviation on the standard deviation on the standard deviation on the standard deviation on the standard deviation of  $\overline{\phantom{a}}$  $323$  compact the impact of the shift (23) and the parameter  $23$ <u>C) Comparison with dyn</u> **i** <u>ynamical friction</u> and assumed that 363 the Fisher matrix and assumed in Sec. III below. For a sec. III below. For 323 neglect the impact of the shift (23) and the parameter Ψ<sup>0</sup> in

$$
m_i \dot{\vec{v}}_i = -\frac{4\pi \mathcal{G}^2 m_i^2 \rho}{v_i^3} \Lambda \vec{v}_i,
$$

**D) Fisher matrix analysis** 
$$
\Gamma_{ij} = \frac{(SNR)^2}{\int_{f_{\min}}^{f_{\max}} \frac{df}{S_n(f)} f^{-7/3} \int_{f_{\min}}^{f_{\max}} \frac{df}{S_n(f)} f^{-7/3} \frac{\partial \psi}{\partial \theta_i} \frac{\partial \psi}{\partial \theta_j} \frac{\partial \psi}{\partial \theta_j}}{\{\theta_i\}} = \{t \in \Phi \mid n(m_1) \mid n(m_2) \mid \Psi_{\text{max}}\}
$$

 $\left\{ A\right\} = \left\{ A, \Phi, \Phi \right\}$  is  $\left( m \right)$  in  $\left( m \right)$  $\mathcal{O}_i f - \mathcal{O}_c, \Psi_c, \mathbf{m}(m_1), \mathbf{m}(m_2), \mathbf{1}_{\text{osc}}$  $\{\theta_i\} = \{t_c, \Phi_c, \ln(m_1), \ln(m_2), \Psi_{\text{osc}}\}$  $\{\theta_i\}$ wish to measure. In this paper we consider fθig ¼ <sup>377</sup>  $\{t_c, \Phi_c, \ln(m_1), \ln(m_2), \Psi_{\text{osc}}\}$ 



the frequency drift and to the GW phase,  
\n
$$
\Delta \psi_{\text{df}} = -\frac{75}{38912} \frac{\pi \mathcal{G}^3 \mathcal{M} \rho}{c^6} \left( \frac{\pi \mathcal{G} \mathcal{M} f}{c^3} \right)^{-16/3} \frac{\Lambda (m_1^3 + m_2^3)}{\nu^{1/5} \mathcal{M}^3}
$$



Detection thresholds for 1 event (comparison of various binary systems)  $\epsilon$  the upper boundary,  $\epsilon$ the Compton and de Broglie wavelengths, λdB ¼ λC=v, <sup>478</sup>



is the dynamical friction thresholds on the upper boundary  $\frac{1}{2}$  dynamical friction  $\epsilon$  $\mathfrak a$  includities to more general cases, such as solitons  $\mathfrak a$ dynamical friction

$$
2m < f_{\min} \text{ for MBBH}
$$

#### associated with the dynamical friction, and the dynamical friction,  $EM$  gravitational potential 429 LISA is guaranteed to observed many WD binaries **EXAMPLE 130 INCREASE EXAMPLE IN AN INCREASE ASSESSED. Note that the detection of the detection of the detectio**<br>Include increase in the detection of the suggests that clouds with R ∼ λ<sup>C</sup> would also be relativistic. 479



**FIG. 1. Throw SIG. 1. Throw SIG. 1. Throw SIG. 1. SIG. 1. Amplitude Excess of the Amplitude Excess of the F1:1.** oscillation  $R^3$  and  $\lambda_c^3$  and  $\lambda_c^1$ For  $m_\phi \gtrsim 10^{-21} \; \text{eV}$  dynamical friction is more important than the oscillations of the DM potential. For  $m_{\phi} \lesssim 10^{-21}$  eV aynamical friction is more in  $\mathcal{A}^{\text{max}}_{\text{max}}$  the order of the order of the phase shift of the  $M_{\rm cloud}$  associated with the DM environment, through the impact of impact of  $M_{\rm cloud}$  and  $N_{\rm cloud}$ **NON-TERRIVISTIC DIVI CIOUGE**  $\rho = \frac{p}{R^3}$  $\leq$   $\frac{1}{\sqrt{2}}$  $\mathbf{A}^{\mathbf{A}}$  $\frac{1}{2}$ vnamical friction is more important than the oscillations 405 the oscillation of the oscillation<br>« As we take Ψoscillation of the oscillation of the oscillation of the oscillation of the oscillation of t  $M_{\text{u}} = M_{\text{u}} = M_{\text{u}} / m$ ,  $\sqrt{3}$ **Hurting**  $\rho = \frac{M \cos \theta}{R^3} < \frac{M \cos \theta}{\lambda^3} = \frac{M \cos \theta}{1 M} \left(\frac{m \phi}{1 \text{ eV}}\right) 10^{45} \text{ g}$  $R^3$   $\lambda_c^3$   $10\%$   $(1eV)$ For  $m_\phi \gtrsim 10^{-21} \text{ eV}$  dynamical friction is more important than the oscillations of the DM potential.  $\overline{M}$  observed by the LISA interference between  $\overline{M}$  interference binary black bla  $\mathcal{A}^{\text{eff}}_{\text{eff}}$  dynamical friction is more important than than the oscillatory  $\mathcal{A}^{\text{eff}}_{\text{eff}}$  $N$ an-relativictic DM  $\frac{1}{2}$  and  $\frac{1}{2}$  compared with the density production interference interference with the DECIGO interference with the DECIGO interfere- 4988 and<br>The DECIGO interference with the DECIGO interference with the DECIGO interference with the DECIGO interference  $M_{\rm cloud}$   $M_{\rm cloud}$   $M_{\rm cloud}$   $m_{\phi}$   $\frac{3}{10^{45}}$   $\frac{3}{40^{45}}$  $\mu = \frac{M \text{ cloud}}{R^3} < \frac{M \text{ cloud}}{\lambda_c^3} = \frac{M \text{ cloud}}{1 M_{\odot}} \left(\frac{m \phi}{1 \text{ eV}}\right) 10^{45} \text{ g/cm}^3$ For  $m_{\phi} \gtrsim 10^{-21}$  eV dynamical friction is more important than the oscillations of the DM potential. For a given mass Mcloud of the DM cloud, the inequality 480  $\,$   $\,$  tion is more important than the oscillations of the DM poten  $\overline{M}_{\rm cloud}$  $rac{\text{cloud}}{R^3}$  <  $\overline{M}_{\rm cloud}$  $\lambda_c^3$  $\equiv$  $\overline{M}_{\rm cloud}$  $1M_{\odot}$ Non-relativistic DM cloud:

$$
\Delta \psi_{\rm osc}(f) \sim \Psi_{\rm osc} 2\pi \left(\frac{5}{256\pi}\right)^{3/8} \left(\frac{\pi \mathcal{GM} 2m_{\phi}}{c^3}\right)^{-5/8} \left|\gamma \left(\frac{5}{8}, -i\frac{m_{\phi}}{m_{\star}(f)}\right)\right|
$$
\n
$$
\Psi_{\rm osc} = \pi \frac{\mathcal{G}}{n}
$$

'D have smaller mass, which improves the detection threshold.  $\overline{\mathbf{r}}$ # ۱,  $\overline{a}$ nad<br>mar  $\mathsf{is},\mathsf{wh}$ .<br>. I  $\frac{1}{2}$  $\frac{1}{2}$ which improves the detection threshold.  $\frac{1}{2}$ WD have smaller mass, which improves the detection threshold. The density of the densit

$$
\Psi_{\rm osc} = \pi \frac{\mathcal{G}\rho}{m_{\phi}^2} \qquad \sigma_{\rho} \propto m^2 \sigma_{\Psi_{\rm osc}}
$$

threshold. The density threshold increases with the scalar mass. is the exclusion region associated with the upper bound (37) F1:6 the DM oscillation potential is definite at 4411 and 2411 and 2411 and 2411 and 2411 and 2411 and 2411 and 241 The density threshold increases with the scalar mass. 467 high redshifts, z ∼ 104. This would correspond to the matter somewhat more favorable for DECIGO. In particular, the 503 re scalar mass.

at than the oscillations of the DM potential.  $\theta$  not consider masses below 10  $\theta$  because they cannot be calculated the  $\theta$ constitute a large fraction of the DM (the DM (the de Broglie wave-192 DM cloud. The effective quantum pressure smoothes out ations of the DM potential.  $\mathcal{A}$  rapid instability, e.g. tachyonic  $\mathcal{B}$ . The instability, the instability of  $\mathcal{B}$ . is of the DIVI potential.

$$
\rho = \frac{M_{\text{cloud}}}{R^3} < \frac{M_{\text{cloud}}}{\lambda_c^3} = \frac{M_{\text{cloud}}}{1M_{\odot}} \left(\frac{m_{\phi}}{1 \text{ eV}}\right)^3 10^{45} \text{ g/cm}^3
$$
\n
$$
\lambda_c = \frac{2\pi}{m_{\phi}} = \left(\frac{m_{\phi}}{1 \text{ eV}}\right)^{-1} 4 \times 10^{-23} \text{ pc.} \qquad \text{Compton wavelength}
$$

#### **F) DECIGO**





The detection thresholds are of the same order as for LISA, but somewhat better.



LISA and DECIGO could provide us with a window on the 557

non-standard formation mechanism at  $\ z\sim10^4$  $\frac{1}{10^4}$ non-standard formation mechanism at  $\ z\sim 10^4$ 

#### **G) Conclusion** 524 with more direct observations of DM substructures. For

This probe is unlikely to be competitive with other more direct observations of DM substructures.

For  $m_{\odot} > 10^{-21} \; \text{eV}$  standard effects such  $\frac{1}{\sqrt{25}}$  gravitational potential conditional potential  $\frac{1}{\sqrt{25}}$ 

For  $m_\phi < 10^{-23}~\rm{eV}$  the clouds that could be detected would have a Compton wavelength greater than 1 pc. 533 wavelengths greater than the parsec scale. This implies DM

For  $m_{\phi} \sim 10^{-22}$  eV the clouds that could be detected by LISA would have a density that is greater than in the solar 532 10−23 neighbourhood by a factor of  $10^5$  , a mass above  $10^5 M_{\odot}$  and a radius above  $0.4 \,\mathrm{pc}$ For  $m_{\phi} \sim 10^{-22}$  eV the clouds that could be detected by LISA would have a density that is greater than in  $\blacksquare$  are only the orighbourhood by a neighbourhood by a factor of  $~10^5$  , a mass above  $~10^5\,M_\odot~$  and a radius above  $~0.4\,\mathrm{pc}$ the clouds that could be detected by LISA would have a density that is greater than in the solar

■ Except for a small region of the DM parameter space, standard analysis where such an effect is neglected are just  $54.54$  matrices as  $54.44$  matrices of the DM EXCEPT TOT & STIMM TEGION OF THE DIVI proter engee standerd englysis where such an effect is neale different opace, standard analysis where such an effect is hegie Except for a small region of the DM parameter space, standard analysis where such an effect is neglected are justified.

For  $m_{\phi} > 10^{-21}$  eV standard effects such as dynamical friction (accretion, gravitational pull) are expected to dominate. effects: dynamical friction, accretion, accretion, accretion, accretion, accretion, accretion, accretion, accr<br>Accretion, accretion, accretion, accretion, accretion, accretion, accretion, accretion, accretion, accretion, dupomical friction (coordion arovitational pull) are expected to a partical motion (accident, gravitational pail) are expected to



# **CONCLUSIONS**

- Scalar dark matter models with self-interactions allow detailed analysis in the large scalar-mass limit - Hydrodynamical picture in the non-relativistic regime (but does not always hold: mapping can be singular)

- Radial accretion onto a BH similar to Bondi problem, with unique transsonic solution,

but with a much smaller accretion rate, self-regulated by a bottleneck in the relativistic regime

- Such a dark matter environment could be detected by LISA and B-DECIGO, if it contains BH binaries.
- They would see scalar clouds that are smaller than 0.1 pc: difficult to detect by other probes

#### **THANK YOU FOR YOUR ATTENTION !**

- 
- 
- Solitons (flat cores) appear at the center of virialized halos
- They do not seem to converge to a scaling regime expect a large diversity of profiles
- Transitions between different regimes could take place for some models

Other topics: vorticity, gravitational atoms (superradiance),

$$
S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} F^2 g^{\mu\nu} \partial_\mu a \partial_\nu a - \mu^4 (1 - \cos a) \right]
$$
  
\n
$$
S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} (\partial \phi)^2 - \frac{\mu^4}{2F^2} \phi^2 + \frac{\mu^4}{4!F^4} \phi^4 + \dots \right]
$$
  
\n
$$
S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} (\partial \phi)^2 - \frac{\mu^4}{2F^2} \phi^2 + \frac{\mu^4}{4!F^4} \phi^4 + \dots \right]
$$
  
\n
$$
m = \frac{\mu^2}{F}, \quad \lambda_4 = -\frac{\mu^4}{6F^4}
$$
  
\nThe field starts oscillating when  $m \sim H$   
\nAt this time the DM density is  $\rho_{\Phi} \sim \mu^4$   
\n
$$
\phi \sim F, \quad \dot{\phi} \simeq 0
$$
  
\nAfterwards:  $\rho_{\phi} \propto a^{-3} \propto T^3$   
\n
$$
\frac{\Omega_{\phi} = 5 \times 10^{10} \left( \frac{F}{10^{17} \text{GeV}} \right)^2 \left( \frac{m}{1 \text{eV}} \right)^{1/2}}{\left( \frac{m}{10^{17} \text{GeV}} \right)^{1/2}}
$$

$$
S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} F^2 g^{\mu\nu} \partial_\mu a \partial_\nu a - \mu^4 (1 - \cos a) \right]
$$
  
\n
$$
F: \text{ axion decay constant}
$$
  
\n
$$
\alpha \to a + 2\pi
$$
  
\n
$$
\phi = Fa
$$
  
\n
$$
S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} (\partial \phi)^2 - \frac{\mu^4}{2F^2} \phi^2 + \frac{\mu^4}{4!F^4} \phi^4 + \dots \right]
$$
  
\n
$$
m = \frac{\mu^2}{F}, \quad \lambda_4 = -\frac{\mu^4}{6F^4}
$$
  
\nThe field starts oscillating when  $m \sim H$   
\nAt this time the DM density is  $\rho_{\Phi} \sim \mu^4$   
\n
$$
\phi \sim F, \quad \dot{\phi} \simeq 0
$$
  
\nAfterwards:  $\rho_{\phi} \propto a^{-3} \propto T^3$   
\n
$$
\Omega_{\phi} = 5 \times 10^{10} \left( \frac{F}{10^{17} \text{GeV}} \right)^2 \left( \frac{m}{1 \text{eV}} \right)^{1/2}
$$

Parameter space Hui et al. 2017

Approximate shift symmetry, broken by a periodic term

If:  $\Omega_{\Phi} \simeq 0.3$  then:  $F \propto m^{-1/4}$  $m = 1 \text{ eV}, \quad F = 2.5 \times 10^{11} \text{ GeV}, \quad \mu = 16 \text{ GeV}, \quad \lambda_4 = -3 \times 10^{-42}$  $m = 10^{-15} \text{ eV}, \quad F = 1.4 \times 10^{15} \text{ GeV}, \quad \mu = 3.7 \times 10^{-5} \text{ GeV}, \quad \lambda_4 = -9 \times 10^{-80}$ 

Same order of magnitude as the couplings that we consider.



#### Non-perturbative instanton effects

 $\mu^4 \sim M_{\rm Pl}^2 \Lambda^2 e^{-S}$  $m = 1 \,\text{eV}, \ \Lambda = 10^{18} \,\text{GeV}, \ \ S = 157$ 

instanton action



 $m = 10^{-15} \text{ eV}, \ \Lambda = 10^{18} \text{ GeV}, \ \ S = 208$ 



Near the BH horizon:

$$
\phi \sim \frac{m}{\sqrt{\lambda_4}}, \ \ \rho_{\phi} \sim \rho_a \sim \frac{m^4}{\lambda_4}
$$

$$
\rho_a \sim \mu^4, \quad \phi \sim \frac{m}{\sqrt{\lambda_4}} \sim F
$$

**Figure 12** OK: perturbative regime down to the BH horizon.