

# Axion dark matter

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Matière noire : candidats au-delà des WIMPs

Annecy

30 sept. / 1<sup>er</sup> oct. 2024

# Outline

## ★ Axions

- Motivations in brief
- Relevant mass range, an educated guess

## ★ Local axion field

## ★ Experimental strategies

- From narrow to broad band
- Ideas and prospects

Biased talk:  
personal interests  
photon coupling  
French connection

# General QCD lagrangian

$$\mathcal{L}_{\text{QCD}} = \sum_{\text{quarks } q} \bar{\psi}_q (i\mathcal{D} - m_q e^{i\theta_q}) \psi_q - \frac{1}{4} G_{\mu\nu,a} G_a^{\mu\nu} - \theta \frac{\alpha_s}{8\pi} G_{\mu\nu,a} \tilde{G}_a^{\mu\nu}$$

SU(3) covariant derivative  
 $\mathcal{D} = \gamma^\mu \partial_\mu - ig_3 \gamma^\mu G_\mu^a T^a$   
 $q$  quark matt, real

phase of Yukawa coupling  
 angle  $[-\pi, \pi]$   
 CP-odd quantity  
 $\propto \vec{E}_a \cdot \vec{B}_a$

- $\theta$ -factor :
- Contribution from QCD vacuum topology
  - EW contribution from phase of  $\mathcal{M}_q$

$\theta_q$  can be removed with chiral transformation  $\psi_q \rightarrow e^{-i\gamma_5 \theta_q/2} \psi_q$

$$\mathcal{L}_{\text{QCD}} = \sum_{\text{quarks } q} \bar{\psi}_q (i\mathcal{D} - m_q) \psi_q - \frac{1}{4} G^2 - \left( \theta - \arg \det \mathcal{M}_q \right) \frac{\alpha_s}{8\pi} G \tilde{G}$$

$$\theta = \theta_{\text{vacuum}} + \theta_{\text{weak}}$$

# Constraints on the $\theta$ factor

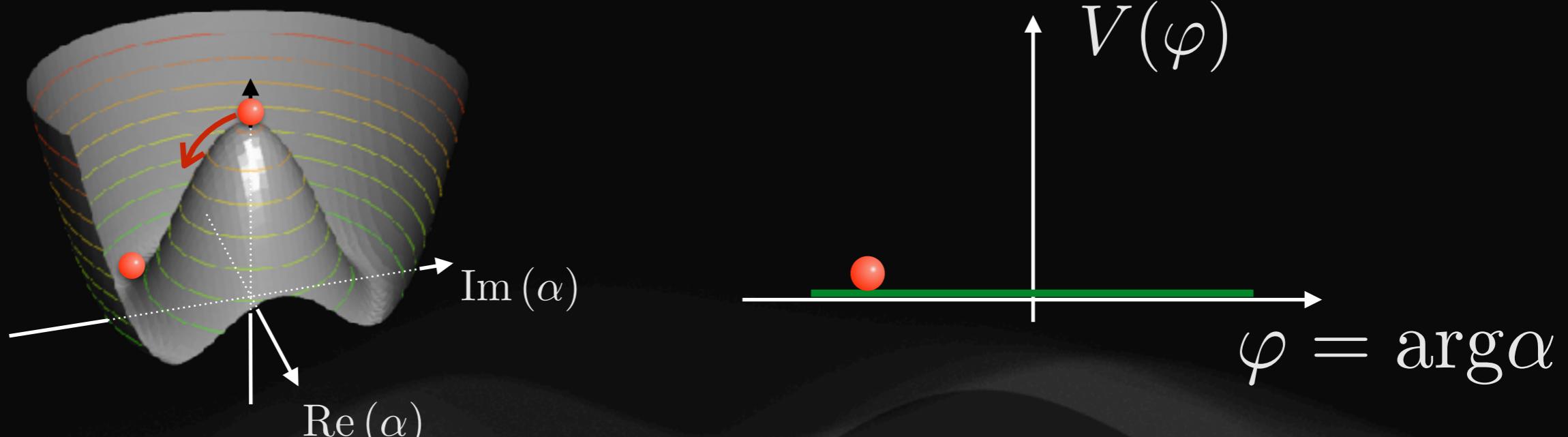
- ★  $\theta \neq 0 \Rightarrow$ 
  - $\eta \rightarrow \pi^+ \pi^-$
  - Neutron electric dipole moment  $d_n = 4.5 \cdot 10^{-5} \theta \text{ ecm}$
  
- ★ Measurements :
  - $BR(\eta \rightarrow \pi^+ \pi^-) < 1.5 \times 10^{-5}$
  - $|d_n| < 2.9 \times 10^{-26} \text{ ecm}$

$$\theta < 10^{-11}$$

Textbook-case of a fine-tuning problem

# Peccei-Quinn and the axion

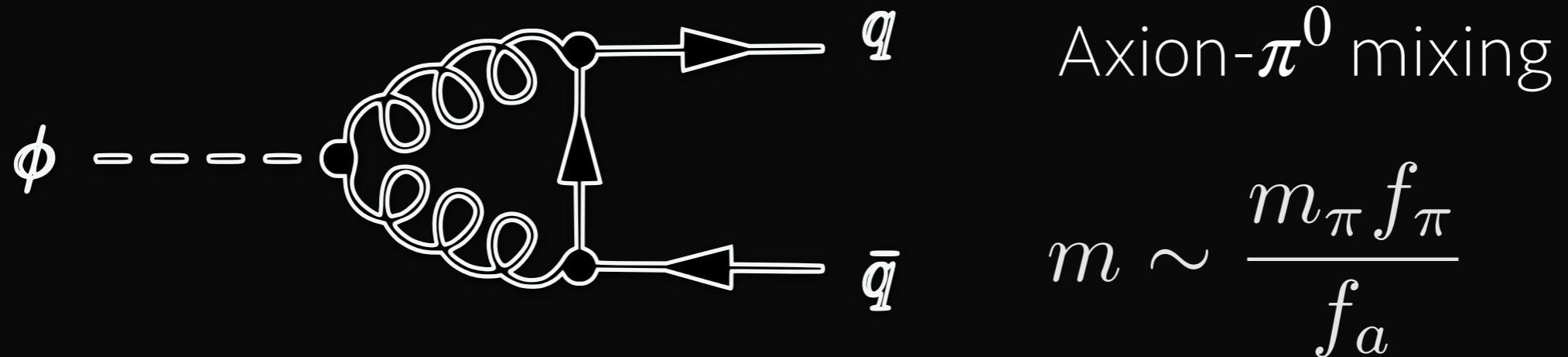
- ★ New symmetry: U(1) Peccei-Quinn
- ★  $U(1)_{\text{PQ}}$  spontaneously broken at some high scale  $f_a$



- ★ Goldstone boson: **the axion**
- ★ Axion  $\varphi$  is introduced massless

# Generating the axion mass

QCD phase transition :  $\langle \bar{\psi}_{q,L} \psi_{q,R} \rangle \neq 0$

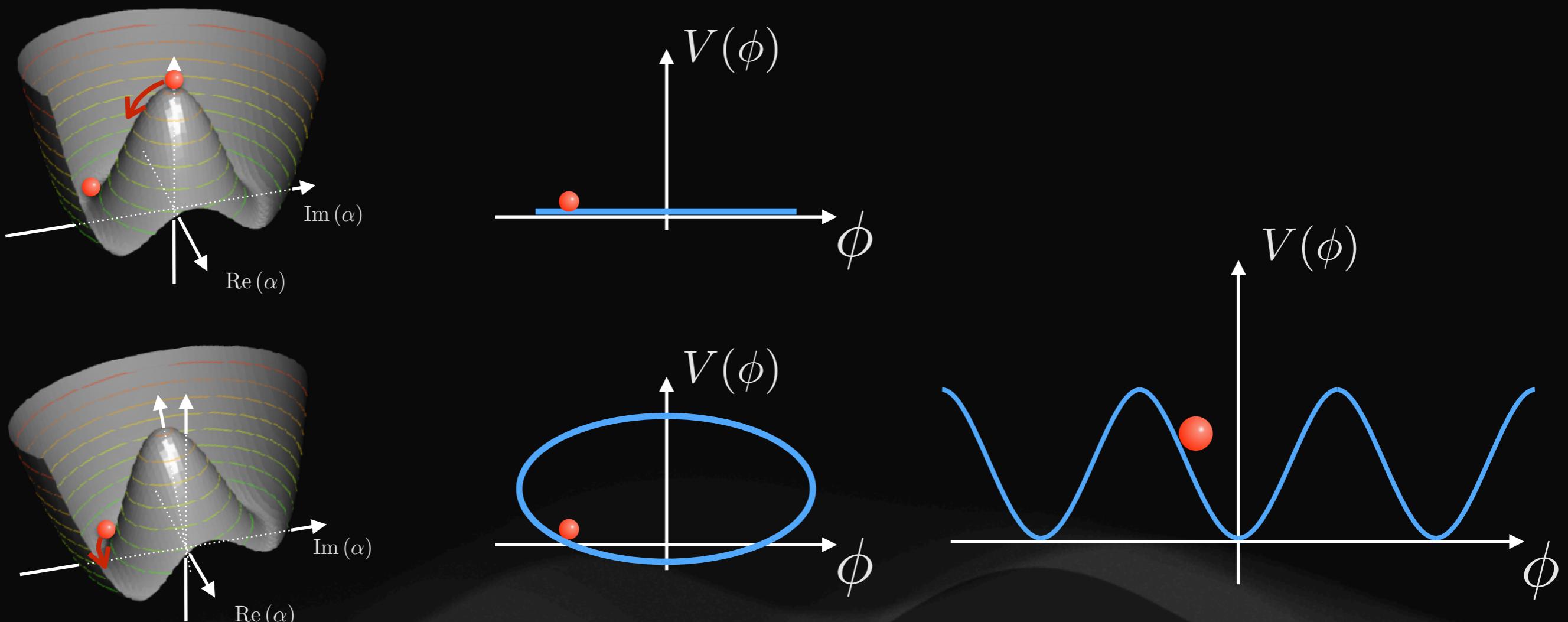


$$\mathcal{L}_{\text{axion}} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \phi \frac{1}{f_a} \frac{\alpha_s}{8\pi} G_{\mu\nu,a} \tilde{G}_a^{\mu\nu} - \frac{1}{2} m^2 \phi^2 - \frac{g_{a\gamma}}{4} \phi F_{\mu\nu} \tilde{F}^{\mu\nu}$$

(  $\phi$  is shifted  $\varphi$ )

lowest order

# Generating the axion mass

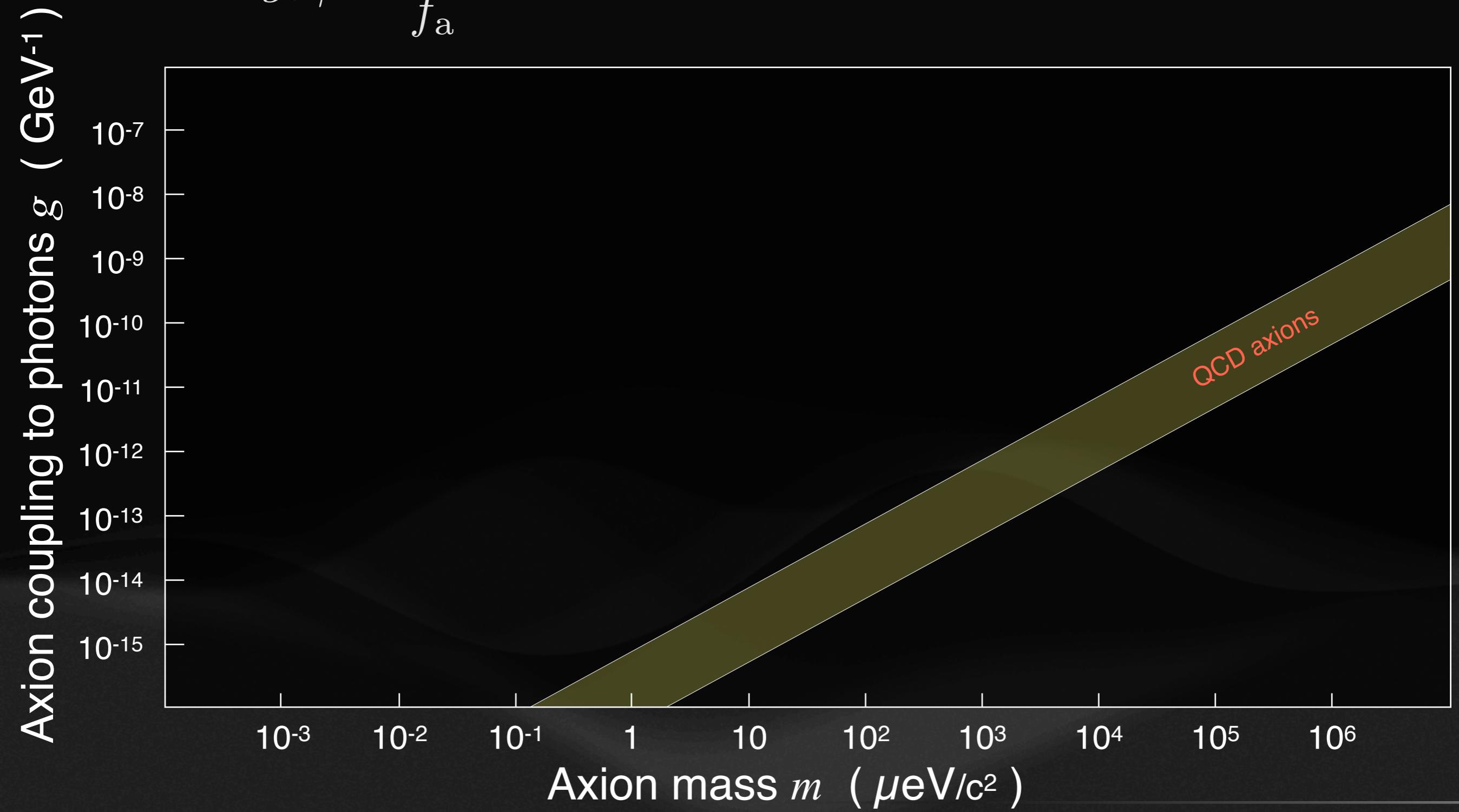


$$V_{QCD}(\theta) = \chi_0 (1 - \cos \theta)$$

$$\theta = \frac{\phi}{f_a}$$

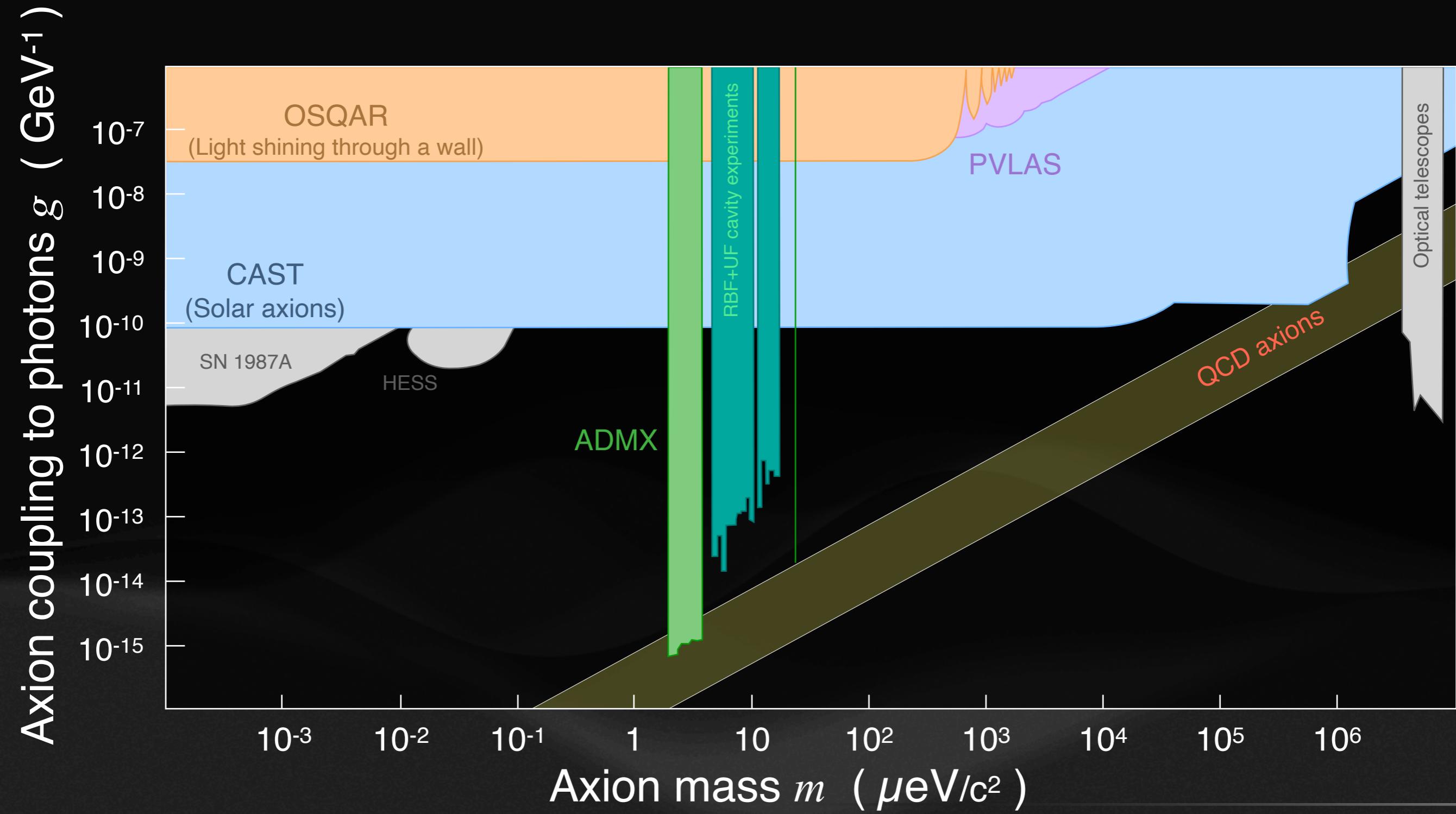
# Axion-photon parameter space

$$g_{a\gamma} \sim \frac{\alpha_s}{f_a} \times O(1) \text{ model-dependent factors}$$



# Examples of constraints

Astrophysics / dedicated observations / laboratory experiments



# Axion as dark matter

- ★ Energy density  $\rho = \frac{1}{2}\dot{\phi}^2 + \frac{m^2}{2}\phi^2$
- ★ Pressure  $p = \frac{1}{2}\dot{\phi}^2 - \frac{m^2}{2}\phi^2$
- ★ From equipartition  $\frac{1}{2}\langle\dot{\phi}^2\rangle = \frac{m^2}{2}\langle\phi^2\rangle$
- ★ Non relativistic matter equation of state  $p = 0$

# Evolution in FRLW

Metric:  $d\ell^2 = c^2 dt^2 - R^2(t) d\vec{u}^2$

Evolution of  $\theta$ , w/  $\phi = f_a \theta$ :

$$\ddot{\theta} + 3H\dot{\theta} - \frac{1}{R^2} \nabla^2 \theta + \frac{1}{f_a} \partial_\theta V_{\text{QCD}}(T, \theta) = 0$$

QCD vacuum susceptibility:  $m^2 f_a^2 = \frac{\partial^2 V_{\text{QCD}}}{\partial \phi^2} \equiv \chi$

$$V_{\text{QCD}}(T, \theta) \simeq \chi(T)(1 - \cos \theta)$$

gives the mass at lowest order in  $\theta$ :  
 $V_{\text{QCD}}(T, \theta \ll 1) = \chi(T) \frac{\theta^2}{2}$

$$\Rightarrow \frac{1}{f_a^2} \partial_\theta V_{\text{QCD}}(T, \theta) = \frac{\chi(T)}{f_a^2} \sin \theta = m^2(T) \sin \theta$$

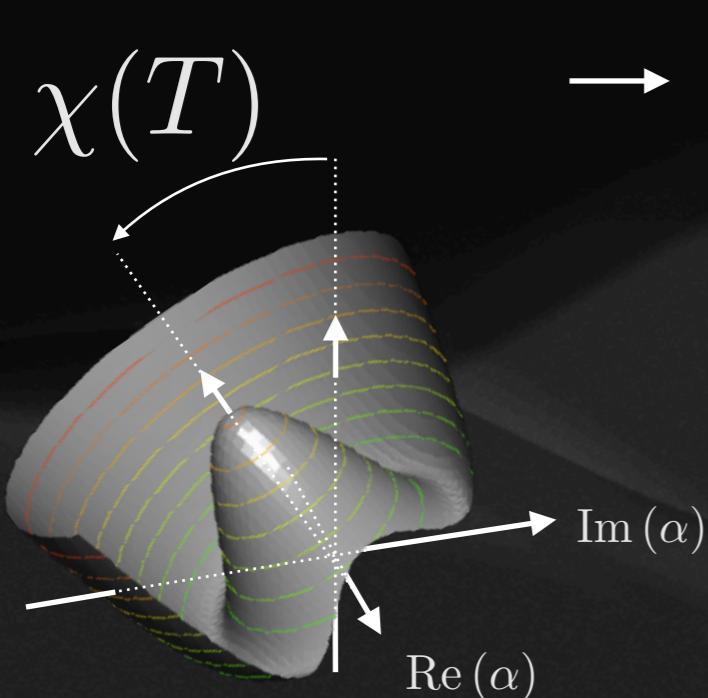
# Homogeneous evolution

For the  $k = 0$  mode :

$$\ddot{\theta} + 3H\dot{\theta} + m^2(T) \sin \theta = 0$$

depend on  $T$  (or  $t$ )

→ numerical resolution



→  $H(t)$  from cosmology

$$\rightarrow \chi(T) \equiv m^2(T)f_a^2$$

Many methods, approx., etc.

O. Wantz, E. Shellard, 2011

S. Borsanyi et al., 2016

# Mass: T-dependance

Parametrization of simulations:

$$\star T > \Lambda_{\text{QCD}} : m^2(T) = \frac{\alpha \Lambda_{\text{QCD}}^4}{f_a^2 \left( \frac{T}{\Lambda_{\text{QCD}}} \right)^n} \quad \begin{cases} \alpha = 1.68 \times 10^{-7} \\ n = 6.68 \end{cases}$$

$$\star T < \Lambda_{\text{QCD}} : \begin{cases} \chi(T=0) = (75.5 \text{ MeV})^4 \\ m(T=0) = 57 \text{ } \mu\text{eV} \left( \frac{10^{11} \text{ GeV}}{f_a} \right) \end{cases}$$

NB : same order of magnitude as KSVZ :

T=0 value similar from  
lattice QCD and NNLO

$$m^2 f_a^2 = m_\pi^2 f_\pi^2 \frac{m_u m_d}{(m_u + m_d)^2} \longrightarrow m \sim 75 \text{ } \mu\text{eV}$$

# Solution for $k = 0$ mode

$$\ddot{\theta} + 3H\dot{\theta} + m^2(T) \sin \theta = 0$$

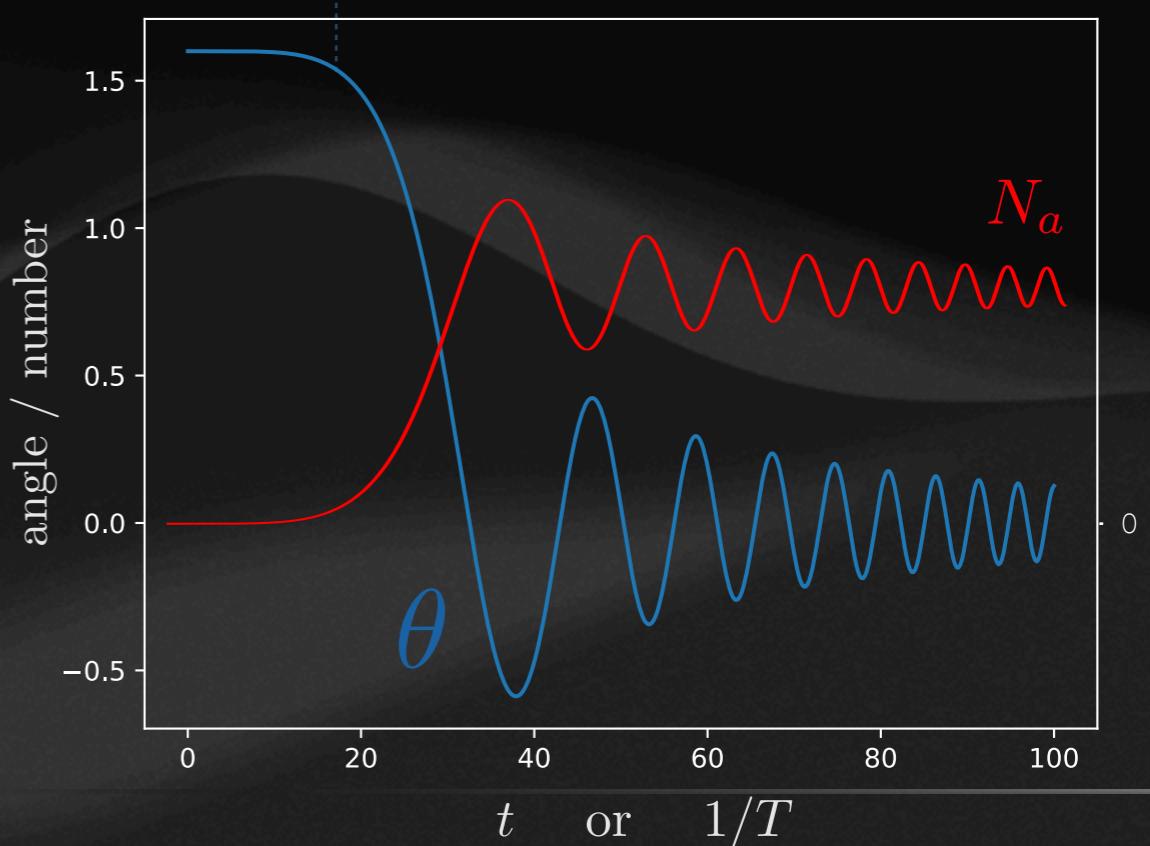
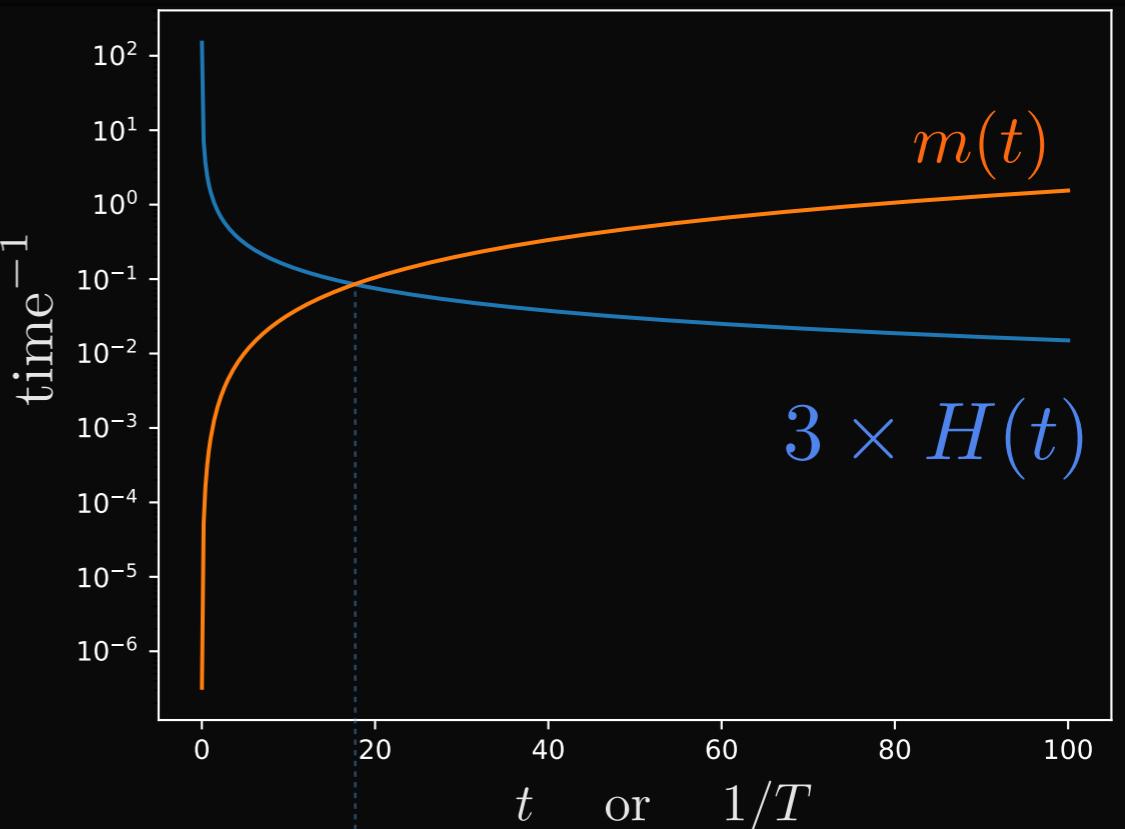
$$H = \frac{1}{2t}$$

Two regimes:

$$\frac{3\dot{\theta}}{2t} \gg m^2 \sin \theta : \text{axion frozen}$$

$$\frac{3\dot{\theta}}{2t} \ll m^2 \sin \theta :$$

- axion oscillates, small dissipation
- axion number conserved



# Initial oscillation temperature

- ★ Radiation era:  $H^2 = \frac{8\pi^3}{90m_{\text{Pl}}^2} T^4 g_\star(T)$

$$m_{\text{pl}} = \sqrt{\frac{1}{\mathcal{G}}} = 1.22 \times 10^{19} \text{ GeV}$$

- ★ Transition temperature:  $m \sim 3H$

$$m^2(T) = \frac{\alpha \Lambda_{\text{QCD}}^4}{f_a^2 \left( \frac{T}{\Lambda_{\text{QCD}}} \right)^n}$$



$$T_i = \left( 3.3 \times 10^{-31} \text{ GeV}^{-2} \right)^{\frac{-1}{n+4}} g(T_i)^{\frac{-1}{n+4}} f_a^{\frac{-2}{n+4}} \Lambda_{\text{QCD}}$$

# Axion number conservation

$\langle \cdot \rangle$ : average over a period

$$\ddot{\theta} + 3H\dot{\theta} + m^2(T) \sin \theta = 0 \quad \Rightarrow \quad \langle \dot{\rho} \rangle = \langle \rho \rangle \left( \frac{\dot{m}}{m} - 3 \frac{\dot{R}}{R} \right)$$

$$\langle \rho \rangle = f_a^2 m^2 \theta^2$$

$$\boxed{\frac{\langle \rho \rangle R^3}{m} = cte}$$

For  $k = 0, E = m$  et  $\frac{\langle \rho \rangle}{m} = N$  number of axions

- ★ Energy is not conserved
- ★ Number of axions is conserved

# Relic density

$$N = \frac{\rho R^3}{m} \quad \frac{\rho_a^0 R_0^3}{m} = \frac{\rho_i R_i^3}{m_i}$$

initial conditions  
when oscillations starts

$$\rho_a = \frac{\sqrt{\alpha} f_a m \theta_i^2}{T_0^3} \frac{g_{\star s}(T_i)}{g_{\star s}(T_0)} \Lambda_{\text{QCD}}^{2-n} T_i^{3-n}$$

$T_i = (3.3 \times 10^{-31} \text{GeV}^{-2})^{\frac{-1}{n+4}} g(T_i)^{\frac{-1}{n+4}} f_a^{\frac{-2}{n+4}} \Lambda_{\text{QCD}}$

$$\rho_a = (3.3 \times 10^{-31} \text{ GeV}^{-2})^{\frac{n-3}{n+4}} \frac{\sqrt{\alpha} f_a m \theta_i^2}{T_0^3 g_{\star s}(T_0)} g_{\star s}(T_i)^{\frac{n+3}{n+4}} f_a^{\frac{n+2}{n+4}} \Lambda_{\text{QCD}}^{3-n}$$

$$\langle \rho_a \rangle = \theta_i^2 \left( 5.79 \times 10^{-48} \text{ GeV}^4 \right) \left( \frac{10 \mu\text{eV}}{m} \right)^{1.19}$$

# Mass-initial $\theta$ relation

Critical density:  $\rho_c = \frac{3m_{\text{Pl}}^2}{8\pi} H^2$

Dark matter density:  $\rho_{\text{CDM}} \simeq 0.84 \times 0.3 \times \frac{3m_{\text{Pl}}^2 H_0^2}{8\pi}$  GeV<sup>4</sup>

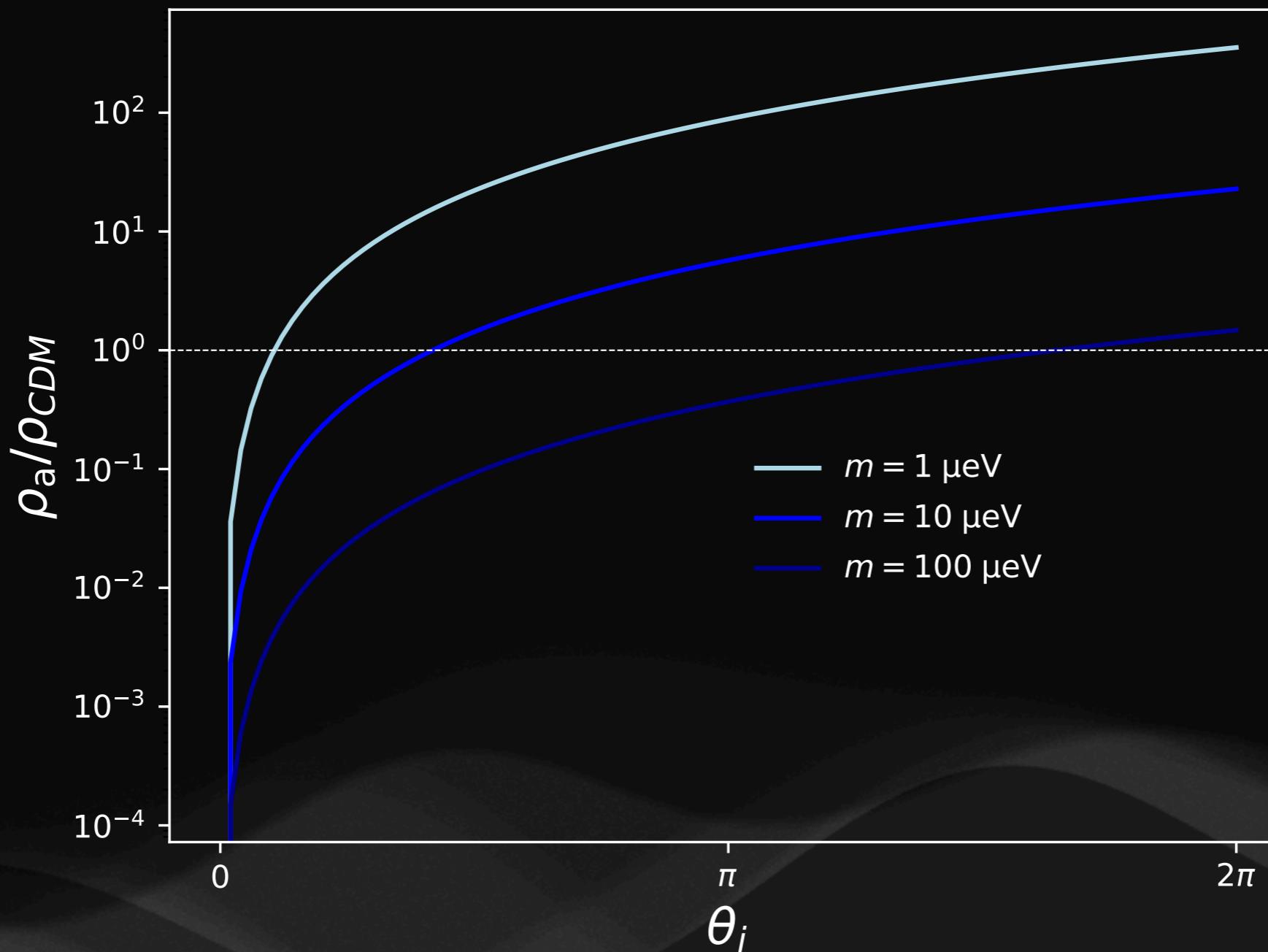
$$\rho_{\text{CDM}} = 1.3 \text{ GeV/m}^3 = 1.3 \times 10^{-6} \text{ GeV/cm}^{-3}$$

Imposing  $\rho_a = \rho_{\text{CDM}}$  gives:

$$\theta_i = 1.314 \left( \frac{m}{10 \text{ } \mu\text{eV}} \right)^{0.595}$$

Cosmological parameters

# Limit on the axion mass

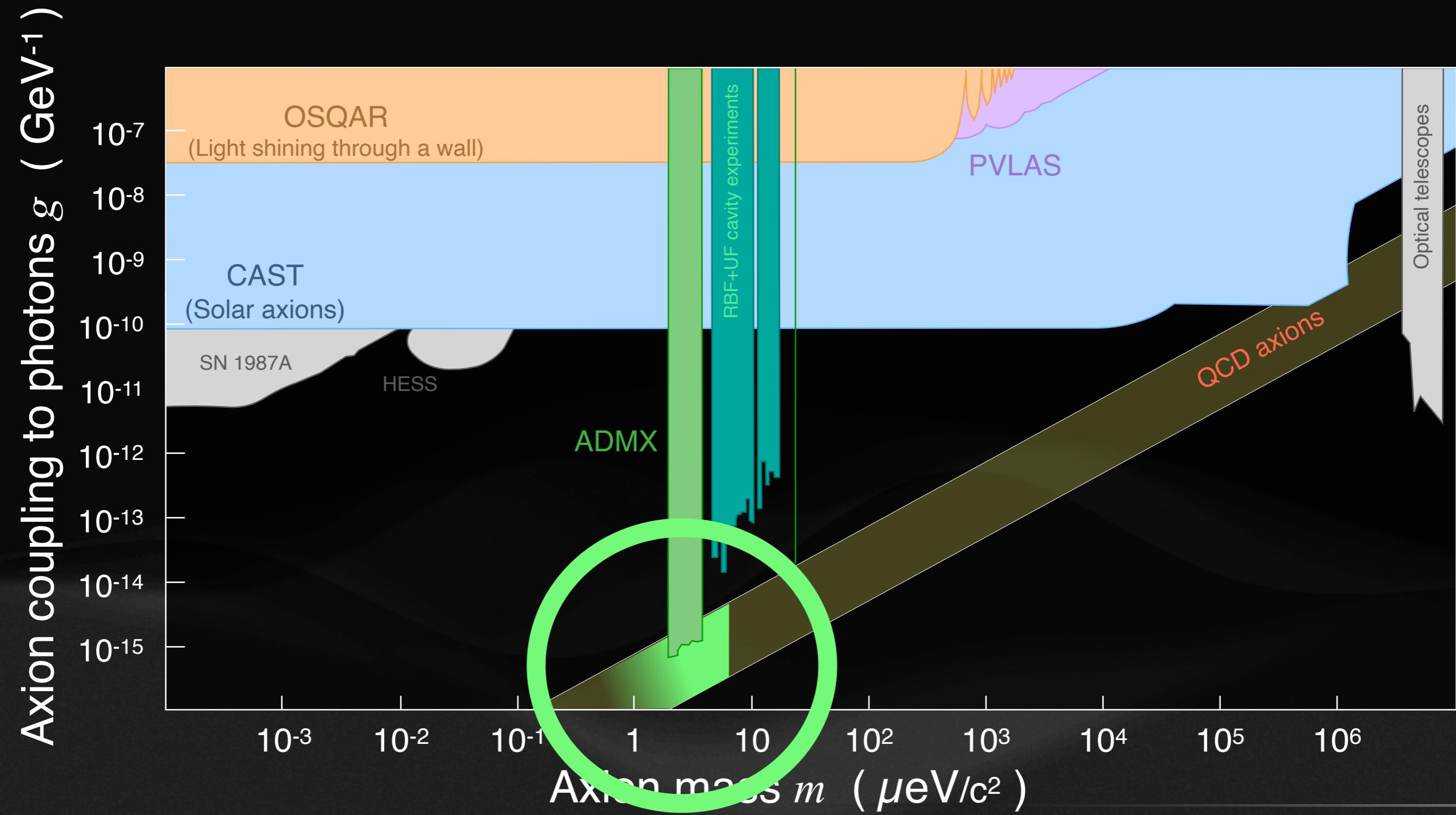


$\theta_i \sim 1$  gives an upper limit on the mass:  $m \lesssim 6 \mu\text{eV}$

NB :  $\theta_i = \pi \Rightarrow m = 43 \mu\text{eV}$

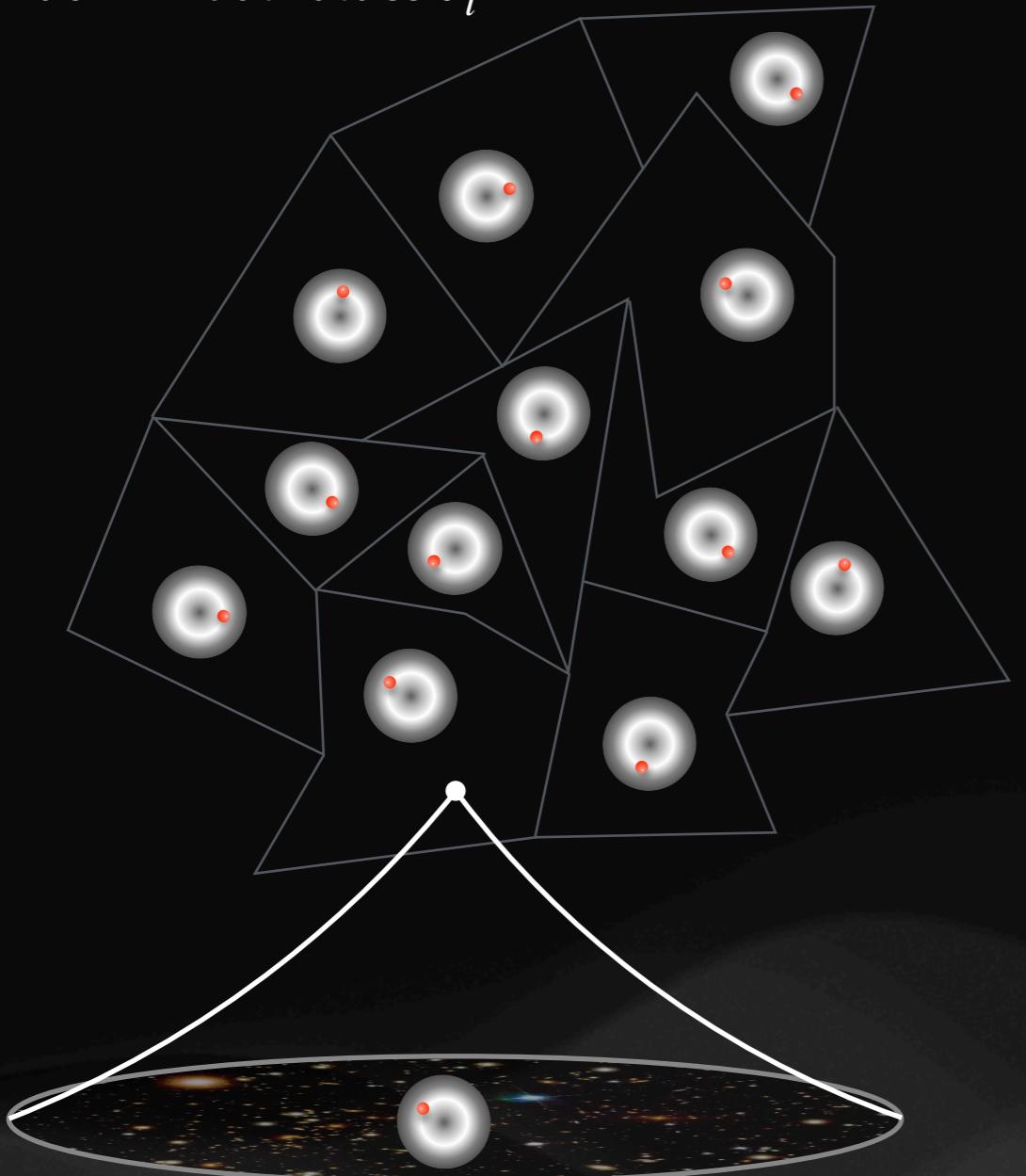
# Preferred mass range

« pre-inflation scenario »



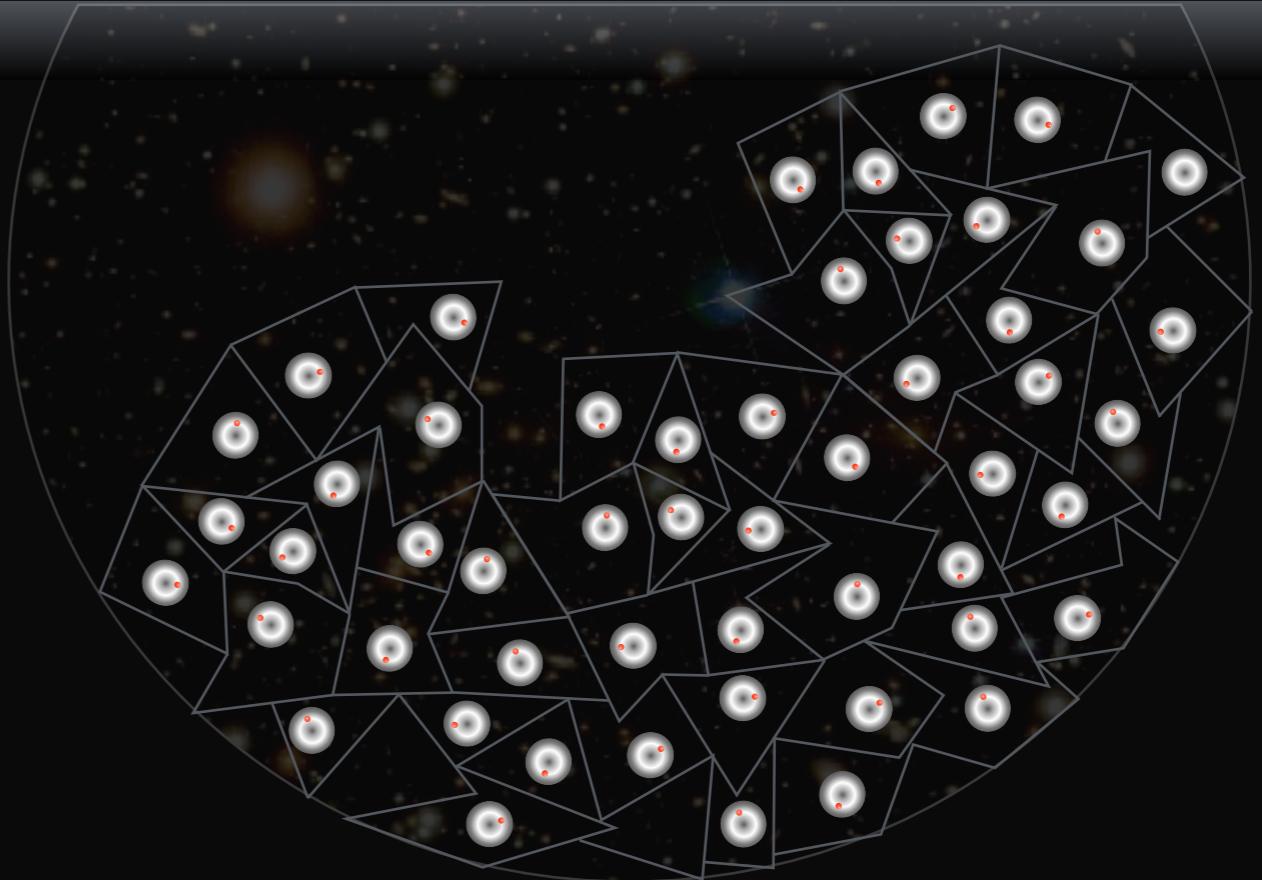
# How many initial conditions?

Random initial values  $\theta_i$



Low-energy inflation

- $\theta_i$  picked up *before* inflation
- Our Hubble radius contains one single  $\theta_i$



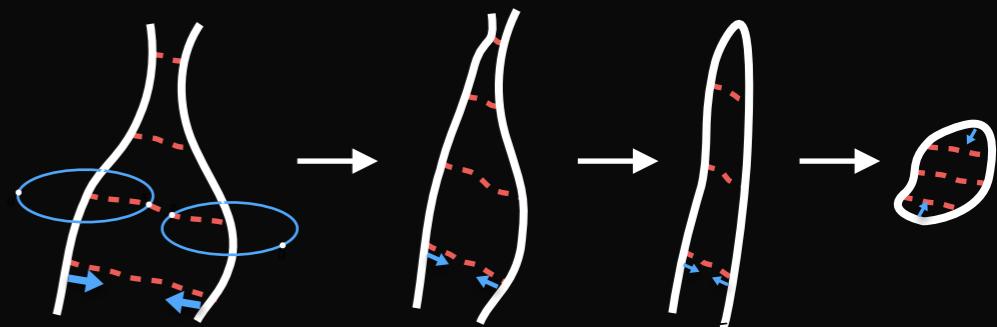
High-energy inflation

- $\theta_i$  picked up *after* inflation
- Our Hubble radius contains many  $\theta_i$

# Axions from topological defects

New contributions:

- ★ Misalignment (same as previous)
- ★ Cosmic strings rearrangements
- ★ decay of domain walls



$$\rho_a = \rho_{\text{mis.}} + \rho_{\text{strings}} + \rho_{\text{decay}}$$

3 same-order contributions

Examples of numerical predictions:

$$m = 115 \pm 25 \mu\text{eV}$$

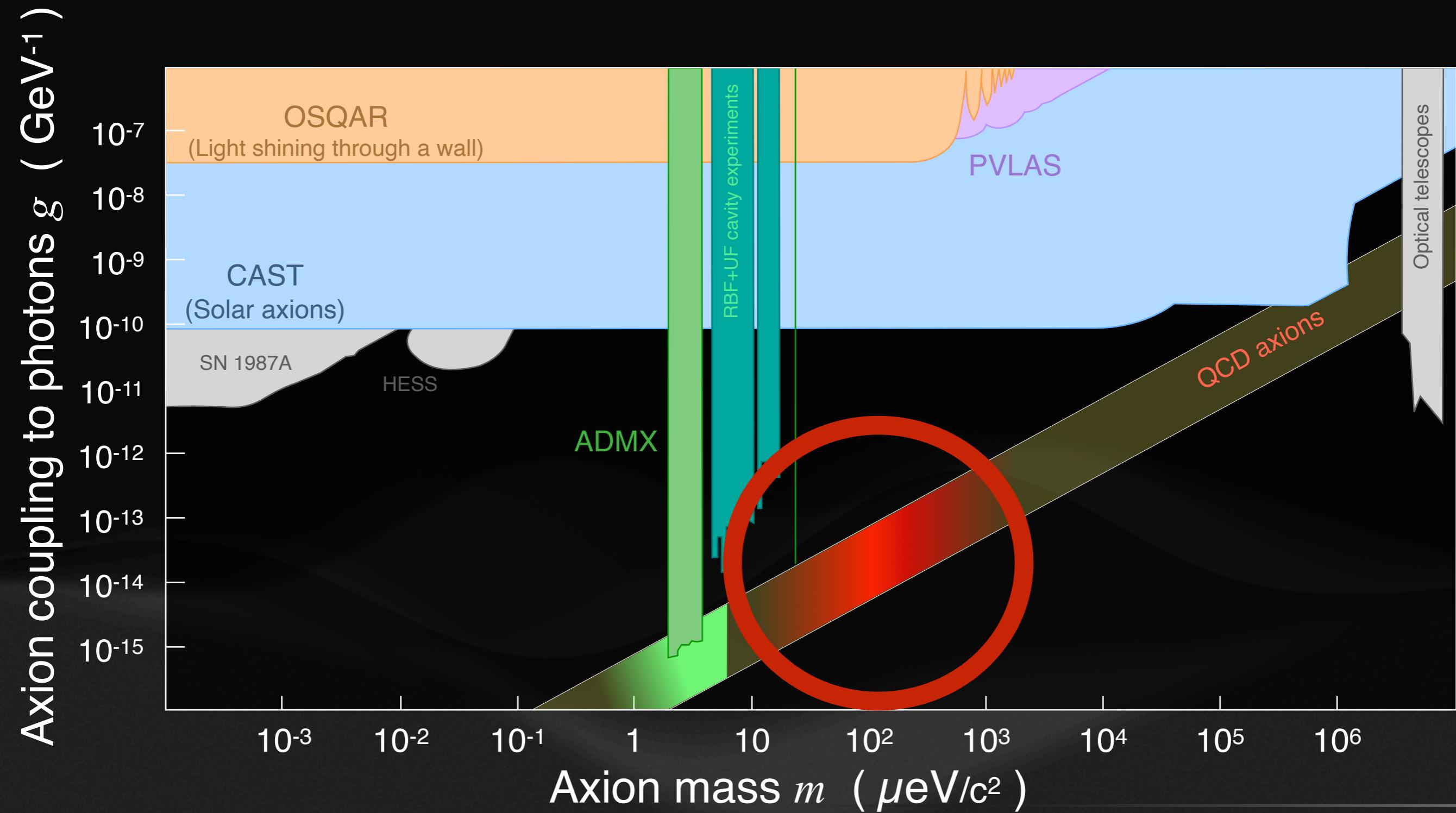
Kawasaki *et al.* 2015

$$m = 26.2 \pm 3.4 \mu\text{eV}$$

Klaer & Moore 2017

# Preferred mass range

« post-inflation scenario »



# A Galactic halo of axions

$$N_{\text{particles}} \sim \frac{10^{12} M_\odot}{m} \simeq 10^{83} \times \left( \frac{10 \text{ } \mu\text{eV}/c^2}{m} \right)$$

$$N_{\text{cells}} \sim \frac{\frac{4\pi}{3} p_{\text{max}}^3 \times \frac{4\pi}{3} R^3}{2\pi \hbar^3} \simeq 2 \times 10^{59} \times \left( \frac{m}{10 \text{ } \mu\text{eV}/c^2} \right)^3$$

$$p_{\text{max}} = m v_{\text{escape}} = m \times 550 \text{ km/s}$$

$$R \simeq 50 \text{kpc}$$

$$\frac{N_{\text{particles}}}{N_{\text{cells}}} \simeq 5 \times 10^{23} \times \left( \frac{10 \text{ } \mu\text{eV}/c^2}{m} \right)^4$$

→ Classical field

# Mass-frequency relation

- ★ Axion field oscillates on scale > solar system
- ★ Velocity of the lab is  $< 300 \text{ km/s} = 10^{-3} c$

$$v = \frac{k c^2}{\omega} \quad \omega = \frac{mc^2}{\hbar} \left( 1 + \frac{v^2}{2c^2} + \dots \right)$$

- ★ Oscillation frequency set by axion mass

$$k = 0 \Rightarrow \omega = \frac{mc^2}{\hbar} \quad \nu = \frac{mc^2}{2\pi\hbar}$$

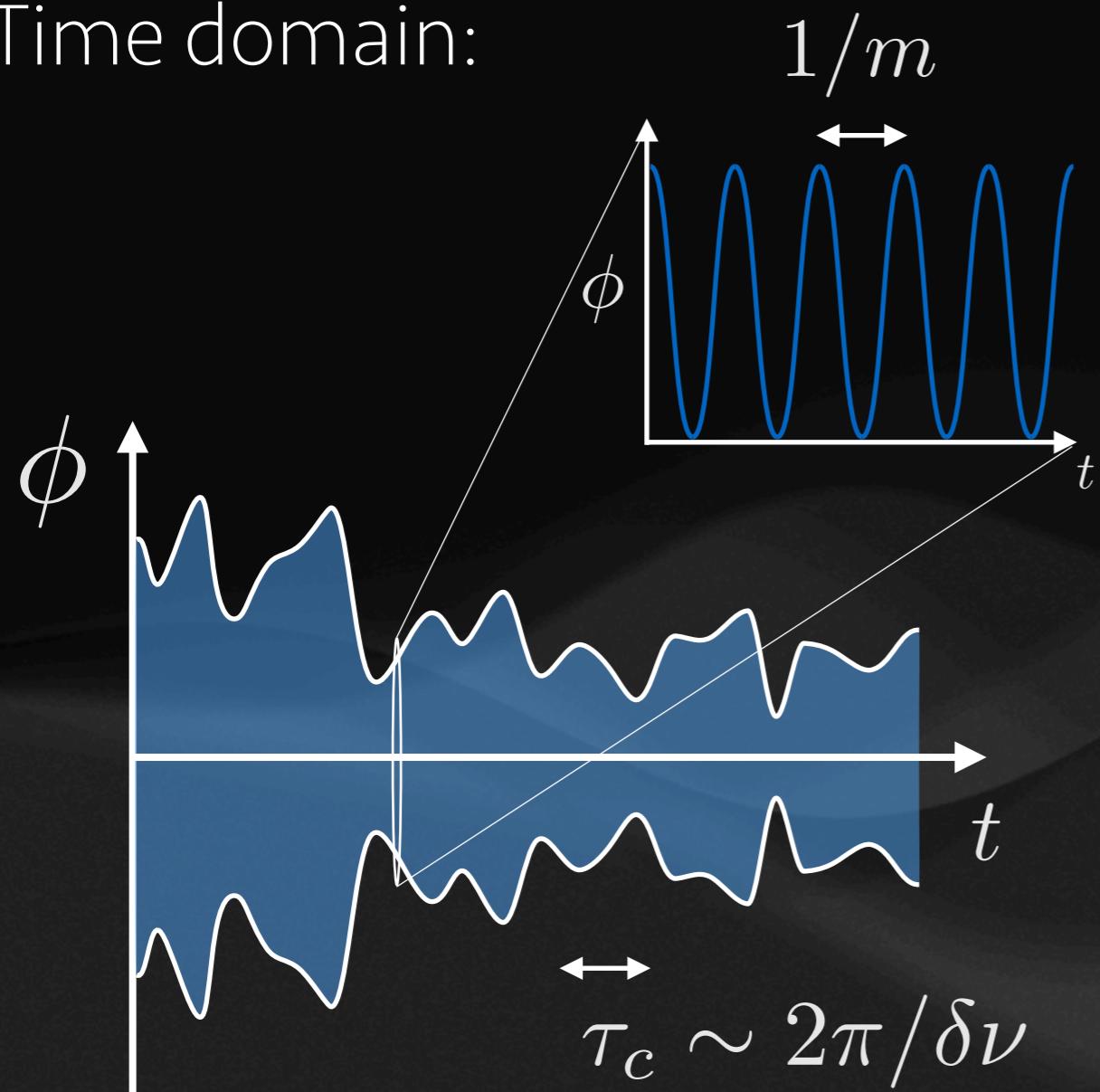
$$\nu = 2.4 \text{ GHz} \times \frac{m}{10 \mu\text{eV}}$$

- ★ Known signal dispersion  $\frac{\delta\nu}{\nu} = \frac{v^2}{c^2} < 10^{-6}$

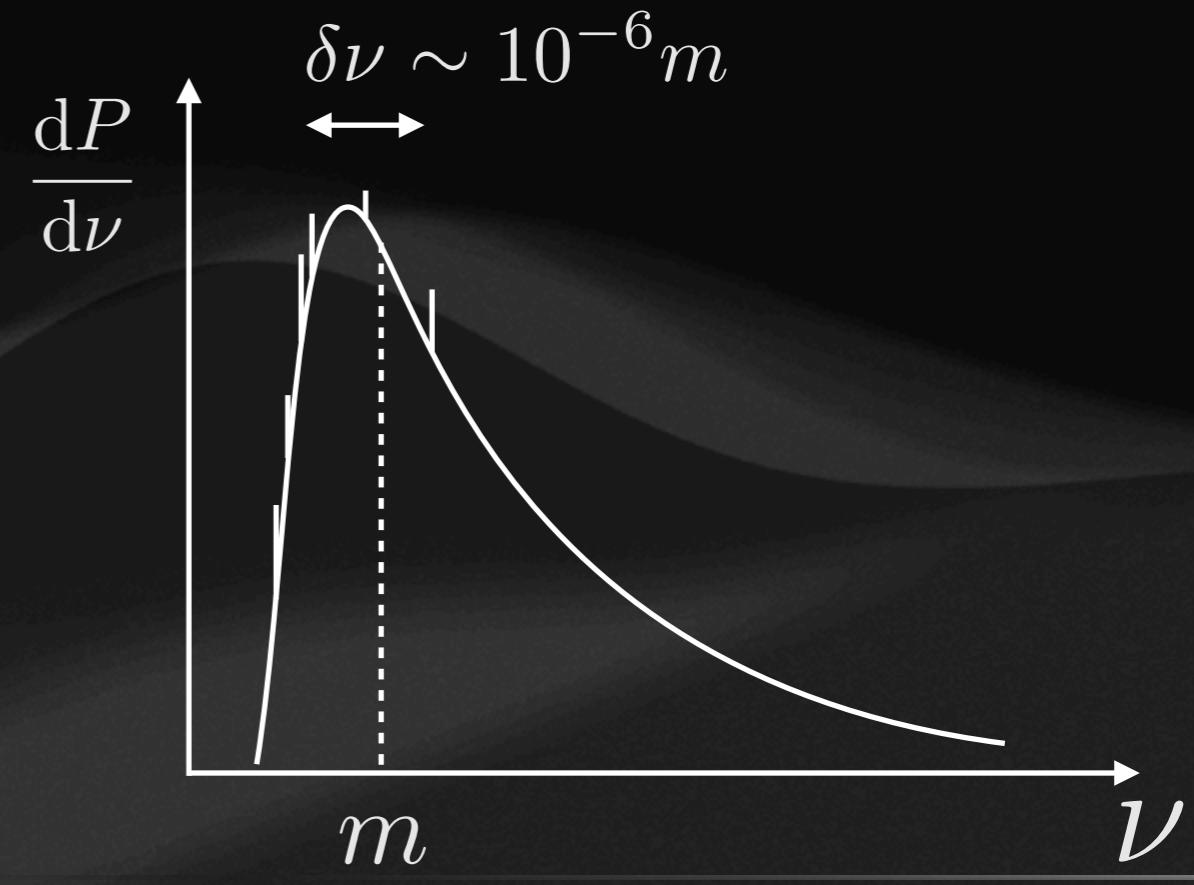
# Spectral content

Coherence length:  $\lambda_c \sim \frac{2\pi}{m \delta\nu} = 12 \text{ m} @ 100 \mu\text{eV}$  (  $\sqrt{\langle v^2 \rangle} \sim 10^{-3}c$  )

Time domain:



Frequency domain:



# Local phenomenology

Relevant part of the lagrangian:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}g_\gamma \phi F_{\mu\nu}\tilde{F}^{\mu\nu} = \frac{1}{2}\left(\vec{E}^2 - \vec{B}^2\right) - g_\gamma \phi \vec{E} \cdot \vec{B}$$

Use strong magnetic field...



...search for new signal

Modified Maxwell's equations:

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0} - g_\gamma \vec{\nabla} \phi \cdot \vec{B}$$

$$\partial_t \vec{B} + \vec{\nabla} \wedge \vec{E} = \vec{0}$$

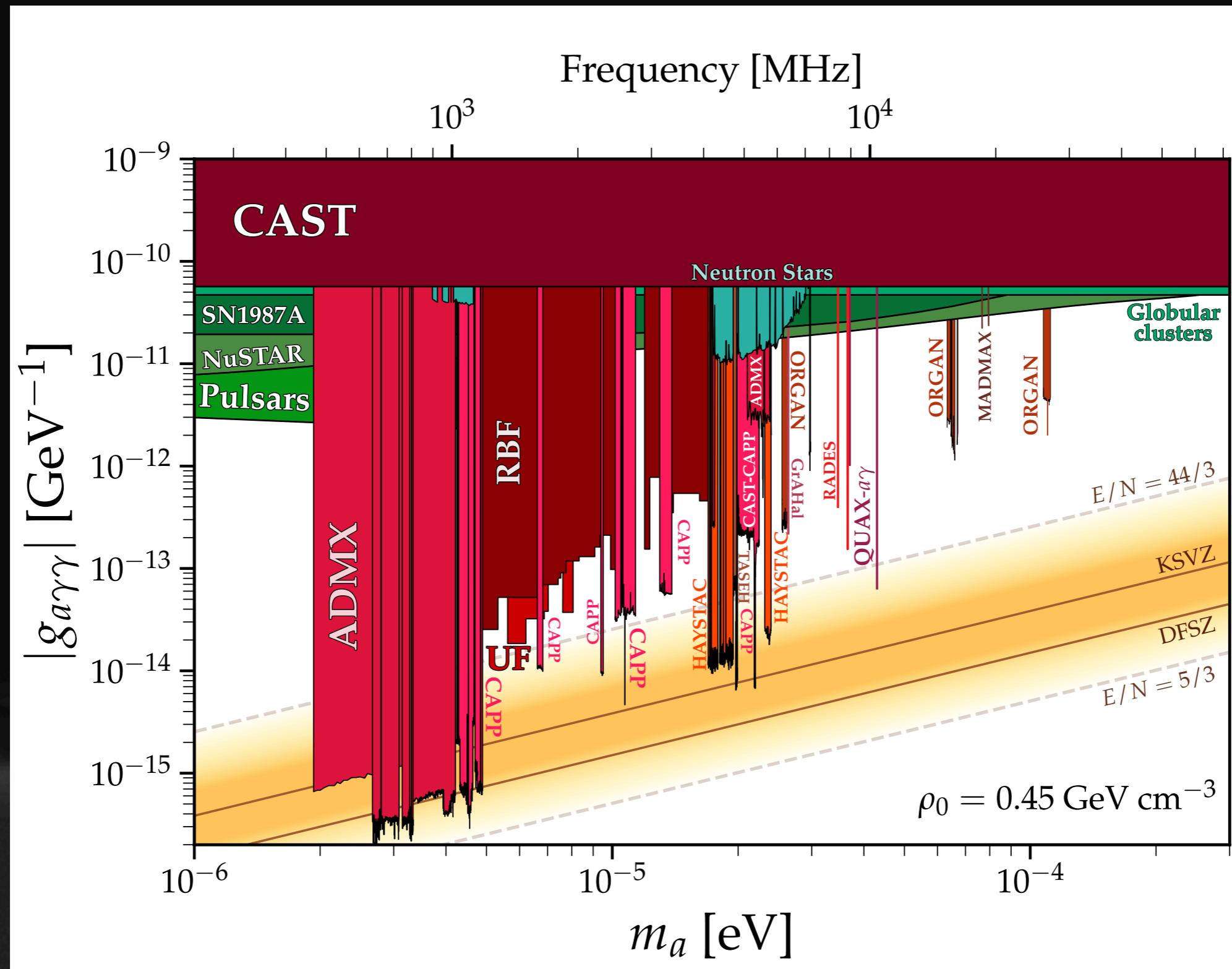
$$\vec{\nabla} \wedge \vec{B} - \partial_t \vec{E} = \mu_0 \vec{j} + g_\gamma \dot{\phi} \vec{B} + g_\gamma \vec{\nabla} \phi \wedge \vec{E}$$

Equations of motion in a  $\vec{B}$  background and homogeneous  $\phi$ :

$$\square \vec{e} = \frac{1}{f_a} \ddot{\phi} \left( \vec{B} + \vec{b} \right) + \frac{1}{f_a} \dot{\phi} \left( \dot{\vec{B}} + \dot{\vec{b}} \right)$$

$$\square \vec{b} = -\frac{1}{f_a} \dot{\phi} \vec{\nabla} \wedge \left( \vec{B} + \vec{b} \right)$$

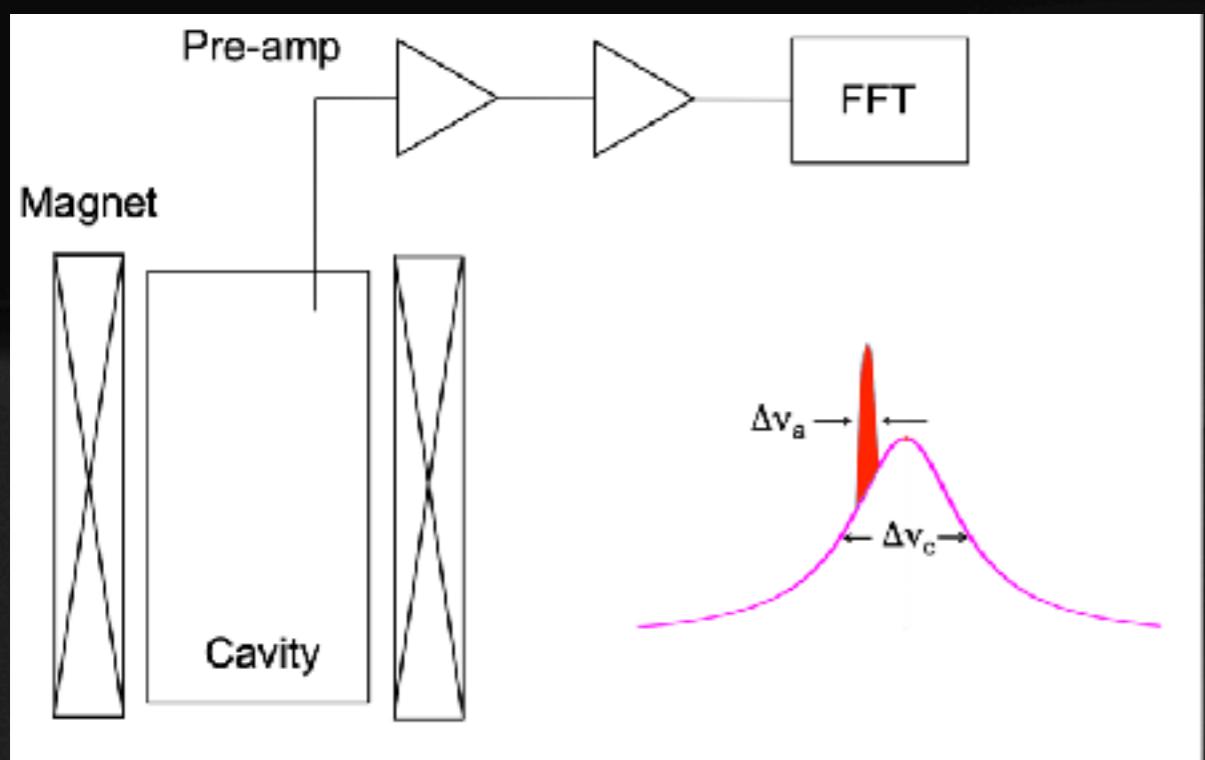
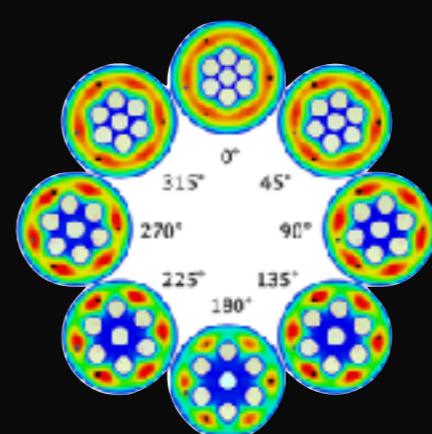
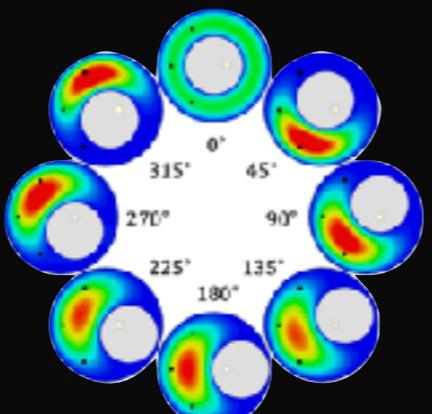
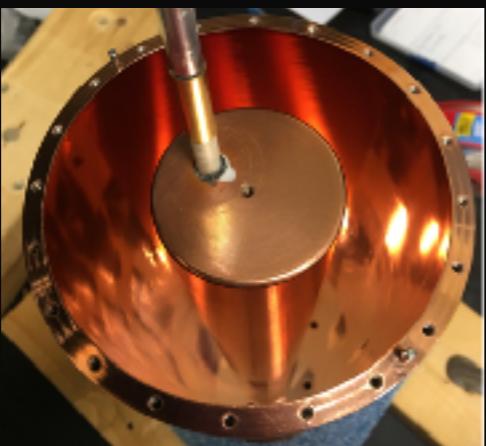
# Reduced mass range



# Resonant searches

ADMX, CAPP, HAYSTAC (...): use of resonant cavities w/ variable geometry

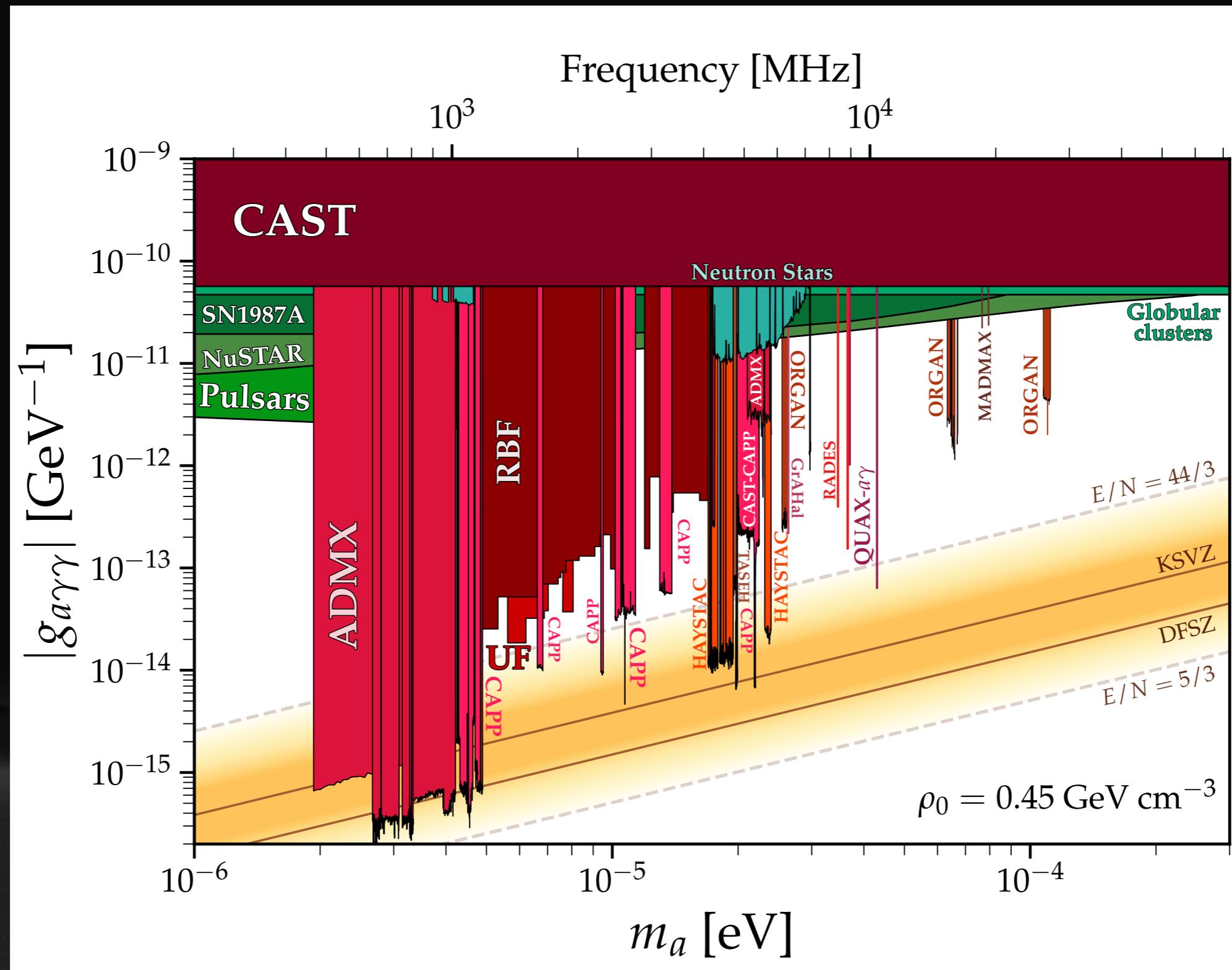
Example: HAYSTAC, 9.4 T



$$P_{\text{signal}} \propto g_\gamma^2 \frac{\rho_a}{m} B^2 V Q$$

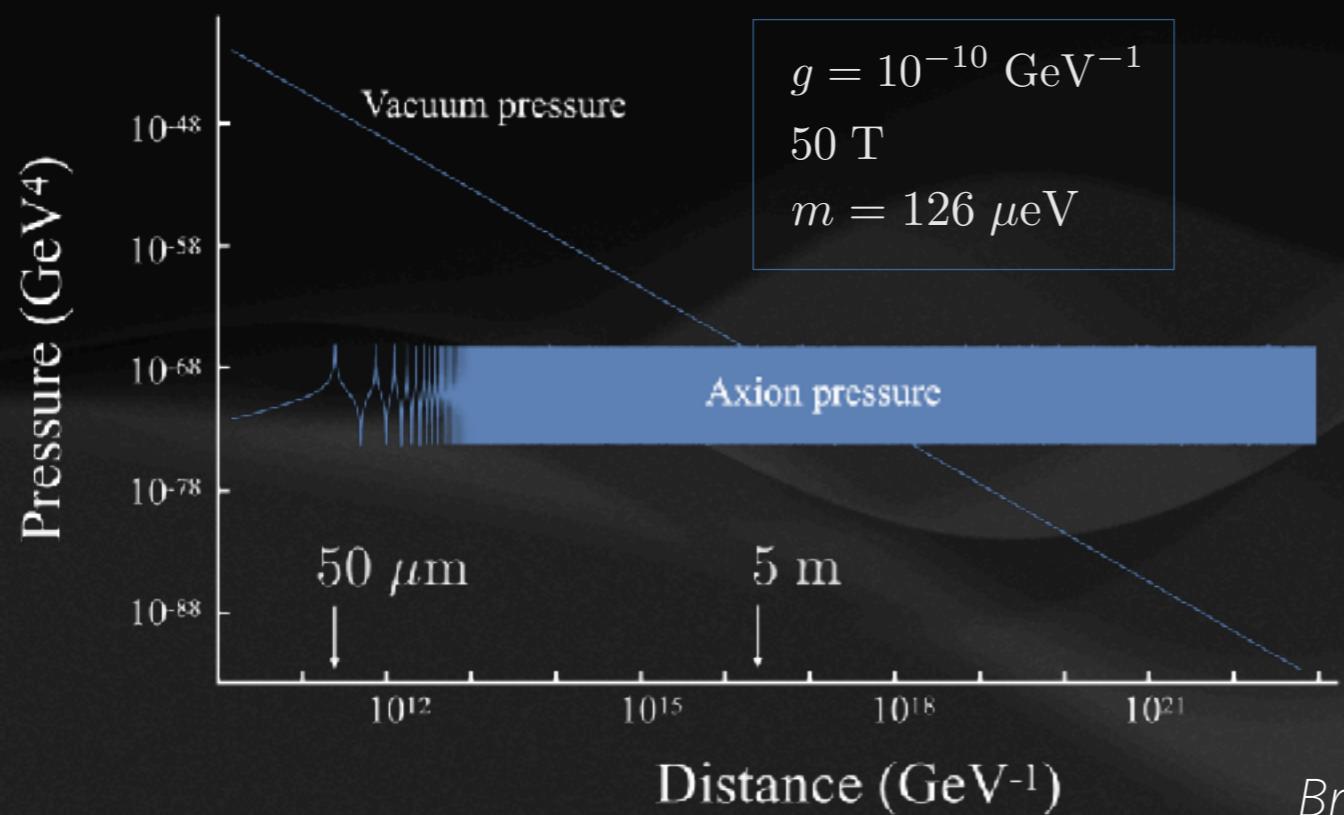
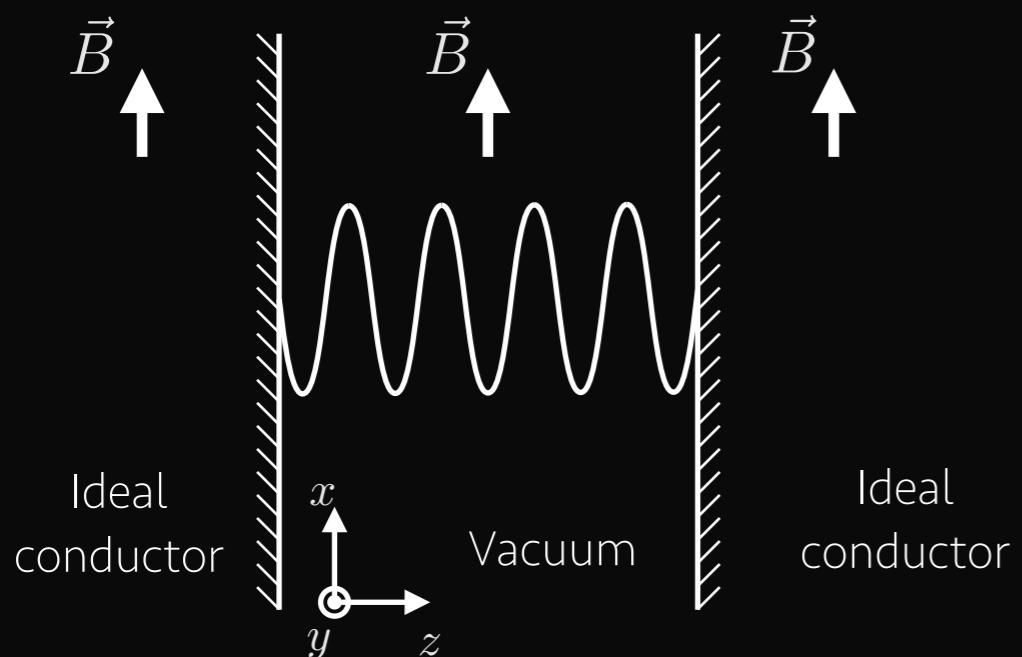
Mass scan, search for resonances  
Quality factor of the order of  $Q \sim 10^5$

# Reduced mass range



# A (narrow?) idea...

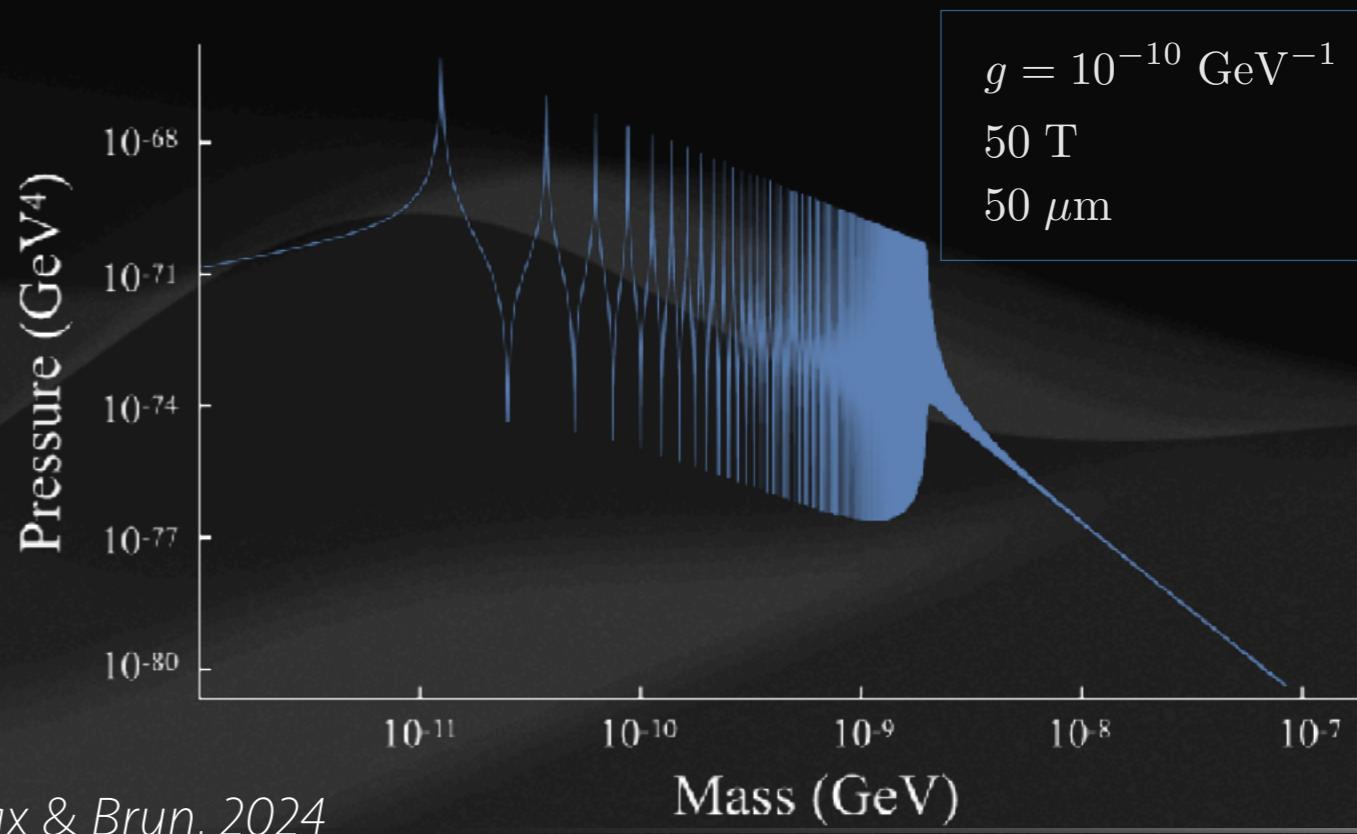
An empty cavity has Casimir pressure



Axion / vacuum modes coupling leads to resonances for distances:

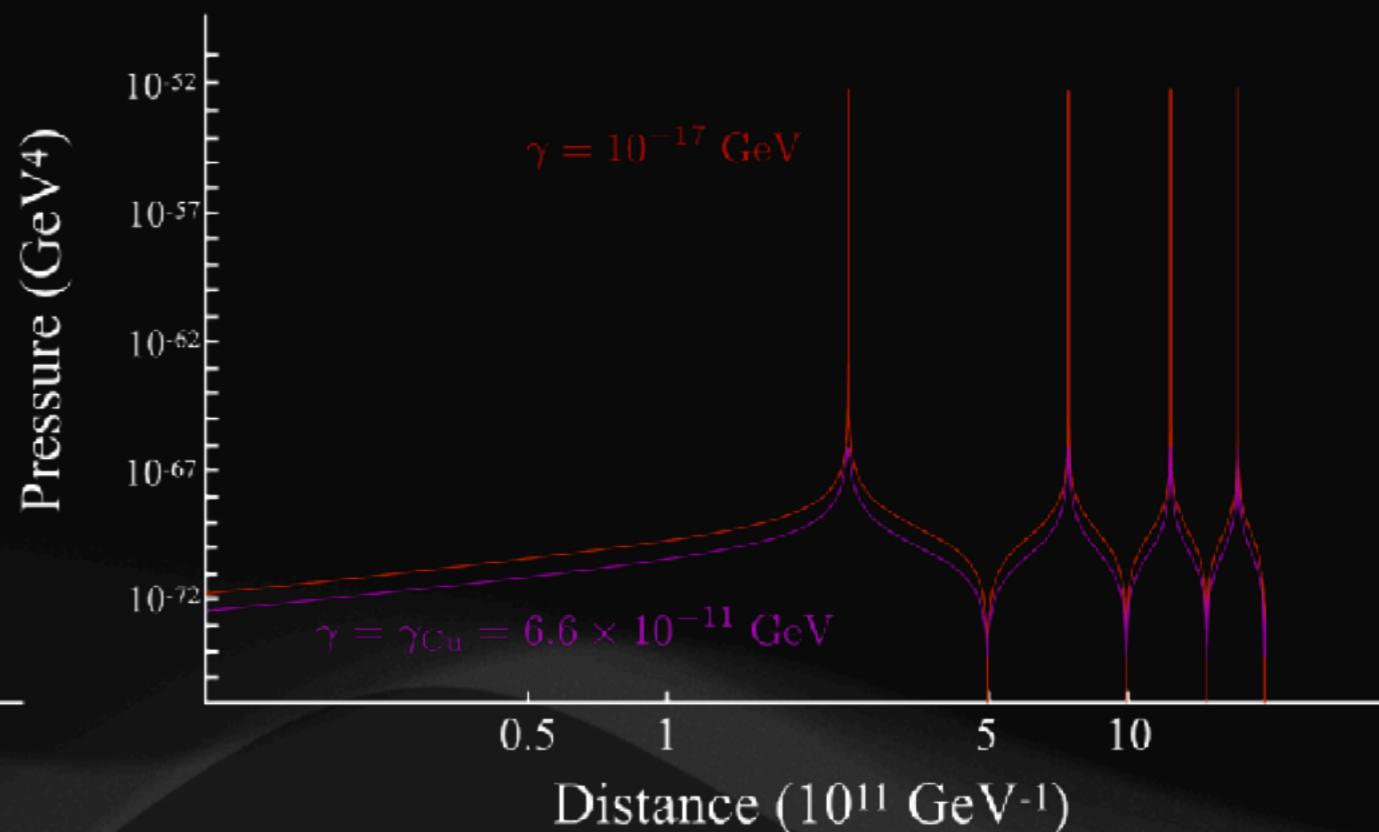
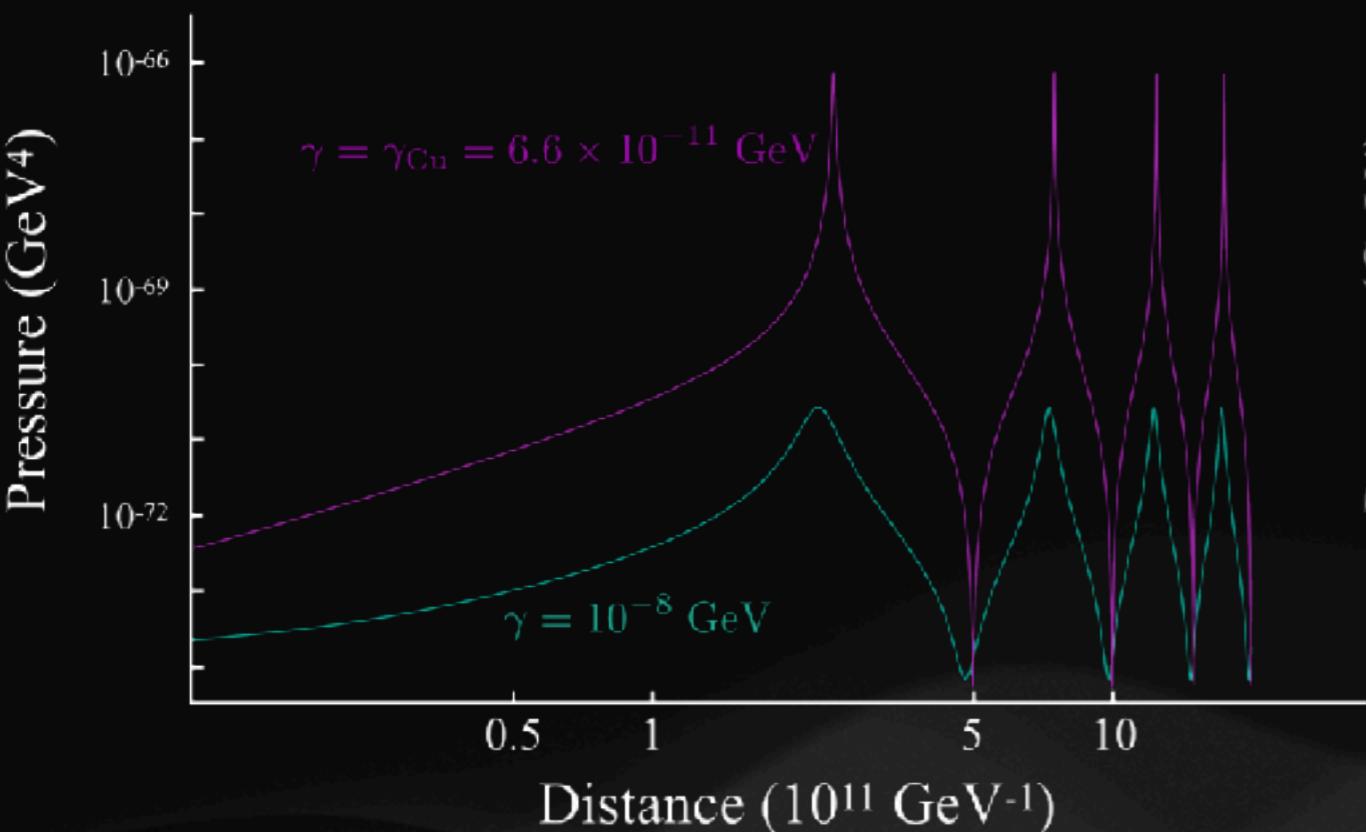
$$d_n = \frac{(2n + 1)\pi}{m}$$

Resonances regularized by dissipation



# Axions & Casimir pressure

Resonances can be made very narrow w/ right material

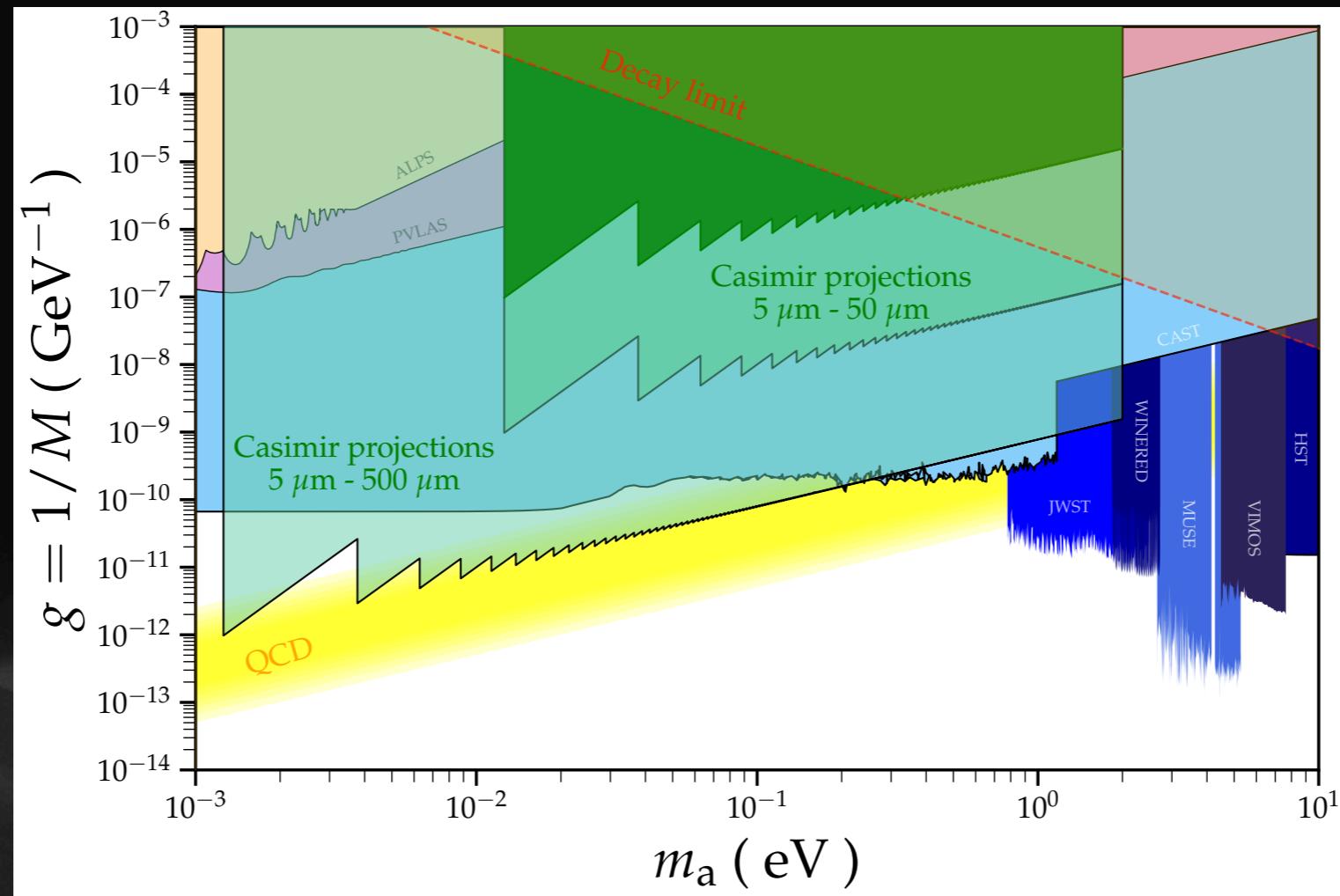


$$\gamma : \text{damping factor in the plaque} \quad \gamma = \frac{\omega_{\text{plasma}}^2}{\sigma}$$

# Axions & Casimir pressure

Some (very optimistic) prospects...

$$\frac{P_{\text{axion}}}{P_{\text{vacuum}}} = 1\%$$

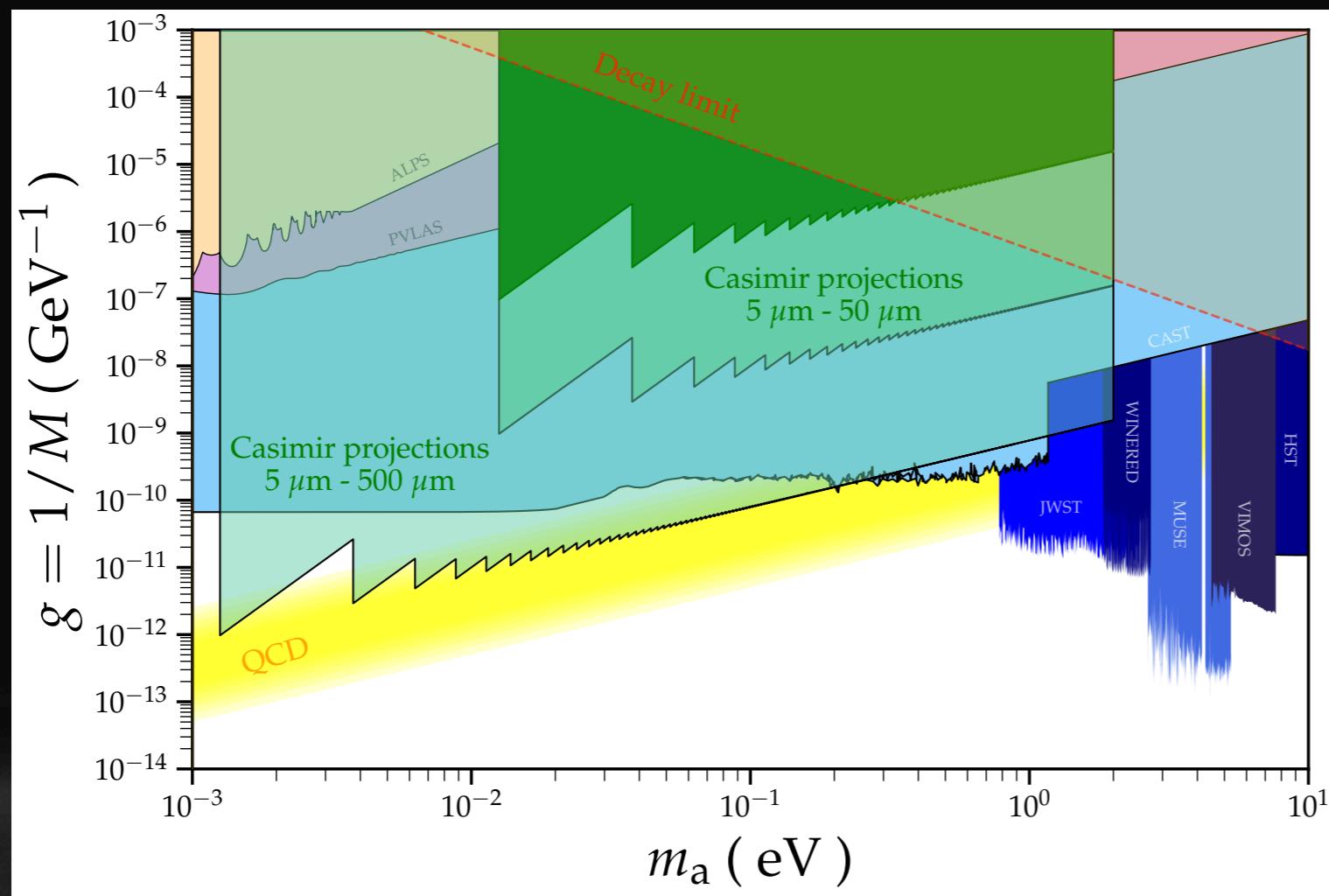


Brax & Brun, 2024

# Axions & Casimir pressure

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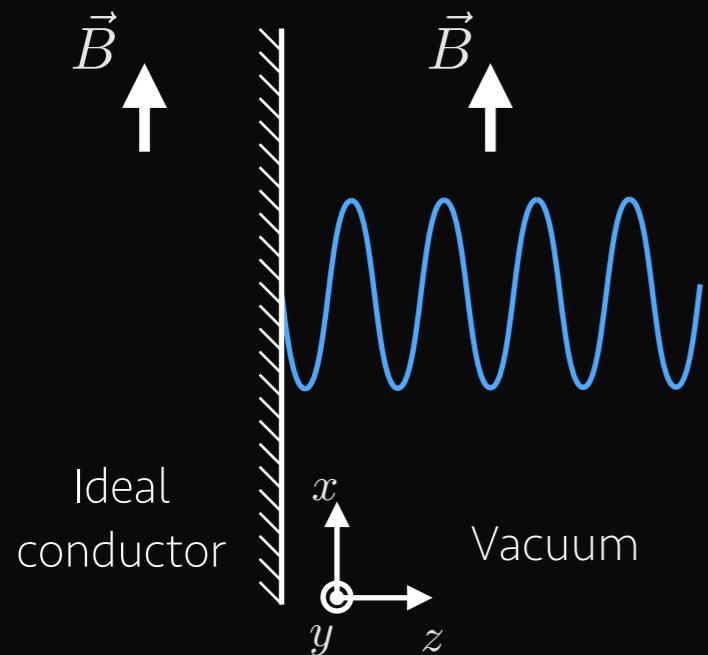
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Brax & Brun, 2024



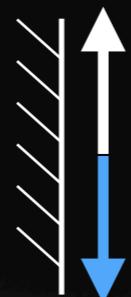
# Towards broadband searches



Axion = long wavelength ( $k \sim 0$ ) excitation

$$\vec{E} = i\phi_0 g_\gamma B_x e^{-imt} \vec{e}_x$$

Field must cancel on the interface:



$$\vec{E}_{\text{total}} = \vec{E} + \vec{E}_{\text{out}}$$

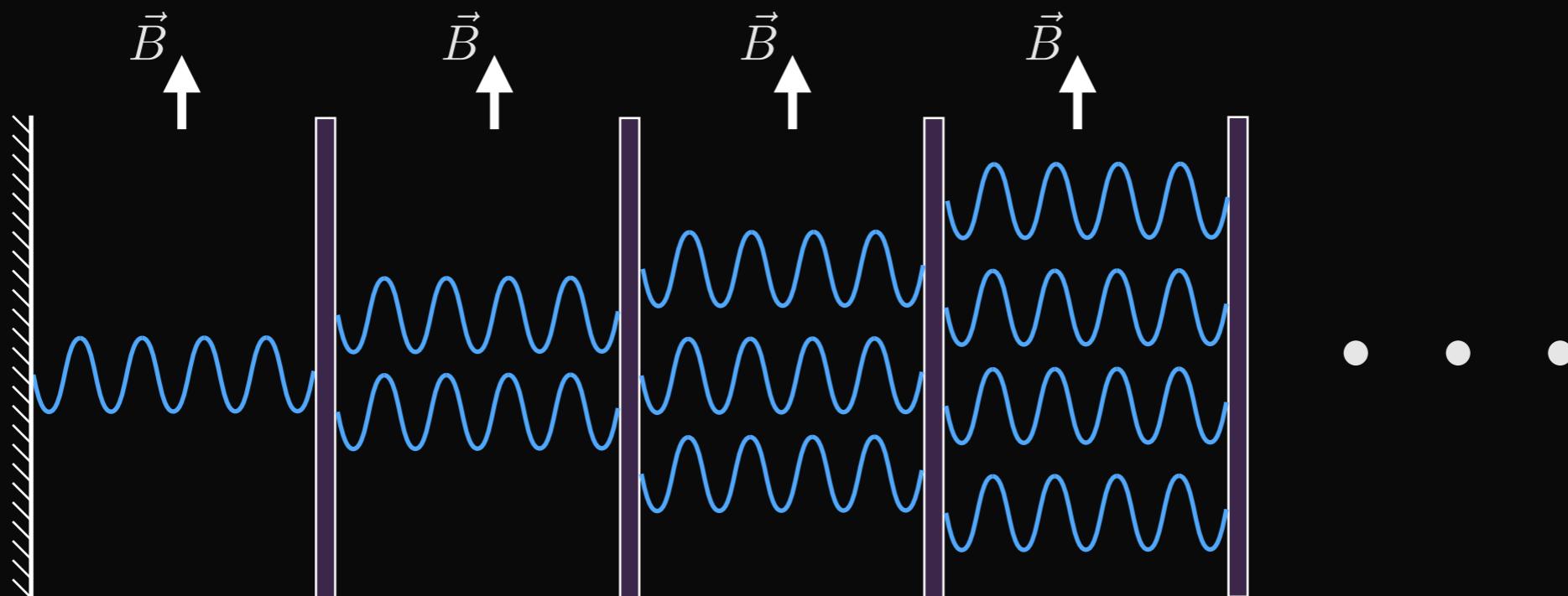
$$\Rightarrow \text{emitted signal: } \vec{E}_{\text{out}} = -i\phi_0 g_\gamma B_x e^{-i(mt-kx)} \vec{e}_x$$

Output power:

$$\Pi = 2.76 \times 10^{-30} \text{ W/m}^2 \left( \frac{\rho_{\text{CDM}}}{\text{GeV/cm}^3} \right) \left( \frac{\text{B}}{1 \text{ T}} \right)^2 \left( \frac{\text{m}}{100 \mu\text{eV}} \right)^{-2} \left( \frac{\text{g}}{10^{-14} \text{ GeV}^{-1}} \right)^{-2}$$

# MadMax

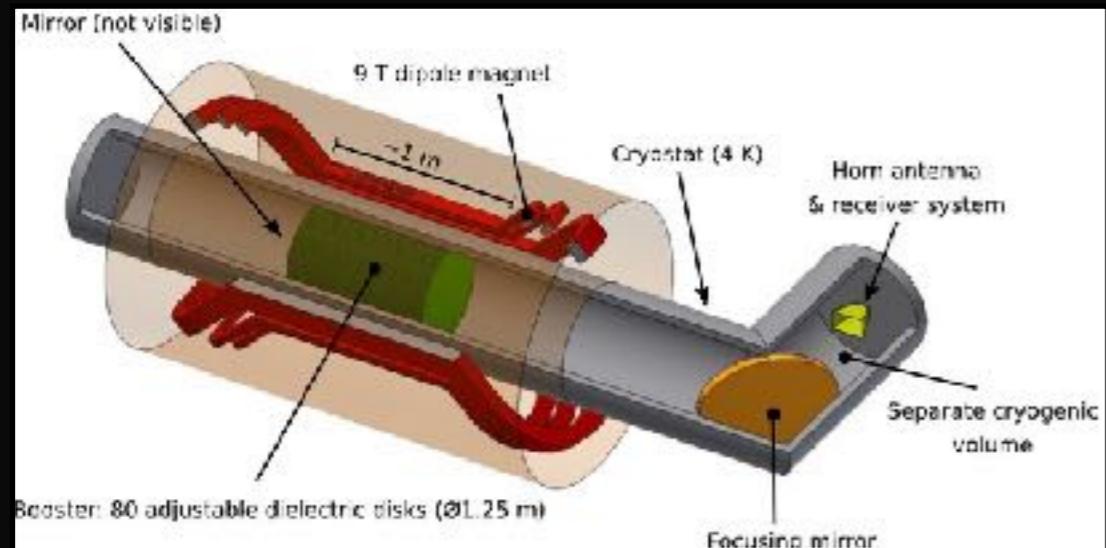
Principle: semi-transparent materials used to increase signal



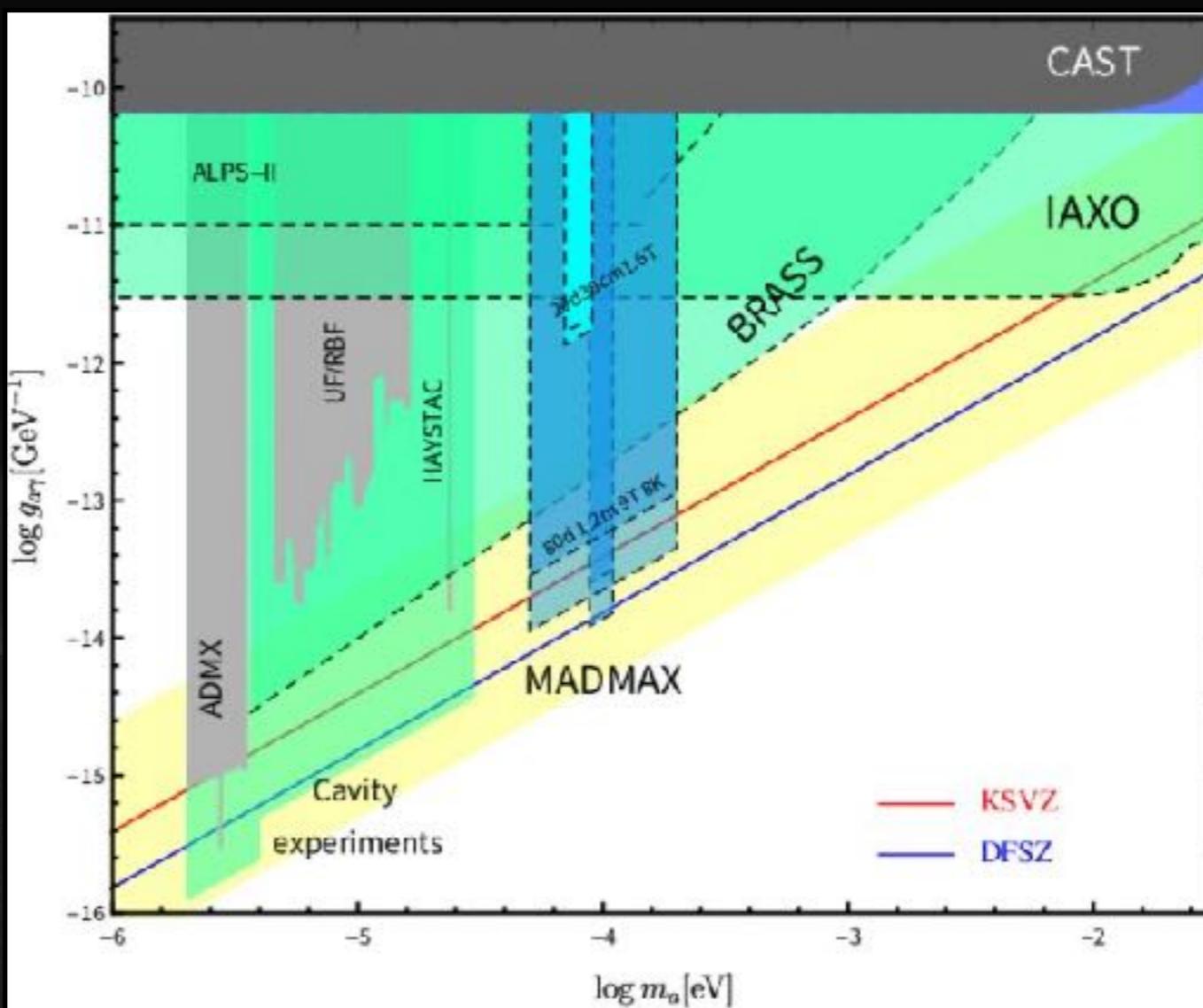
Boost depends on spacing: adjustable broadbandness

Expected boost factor up to  $10^5$

# MadMax setup & prospect

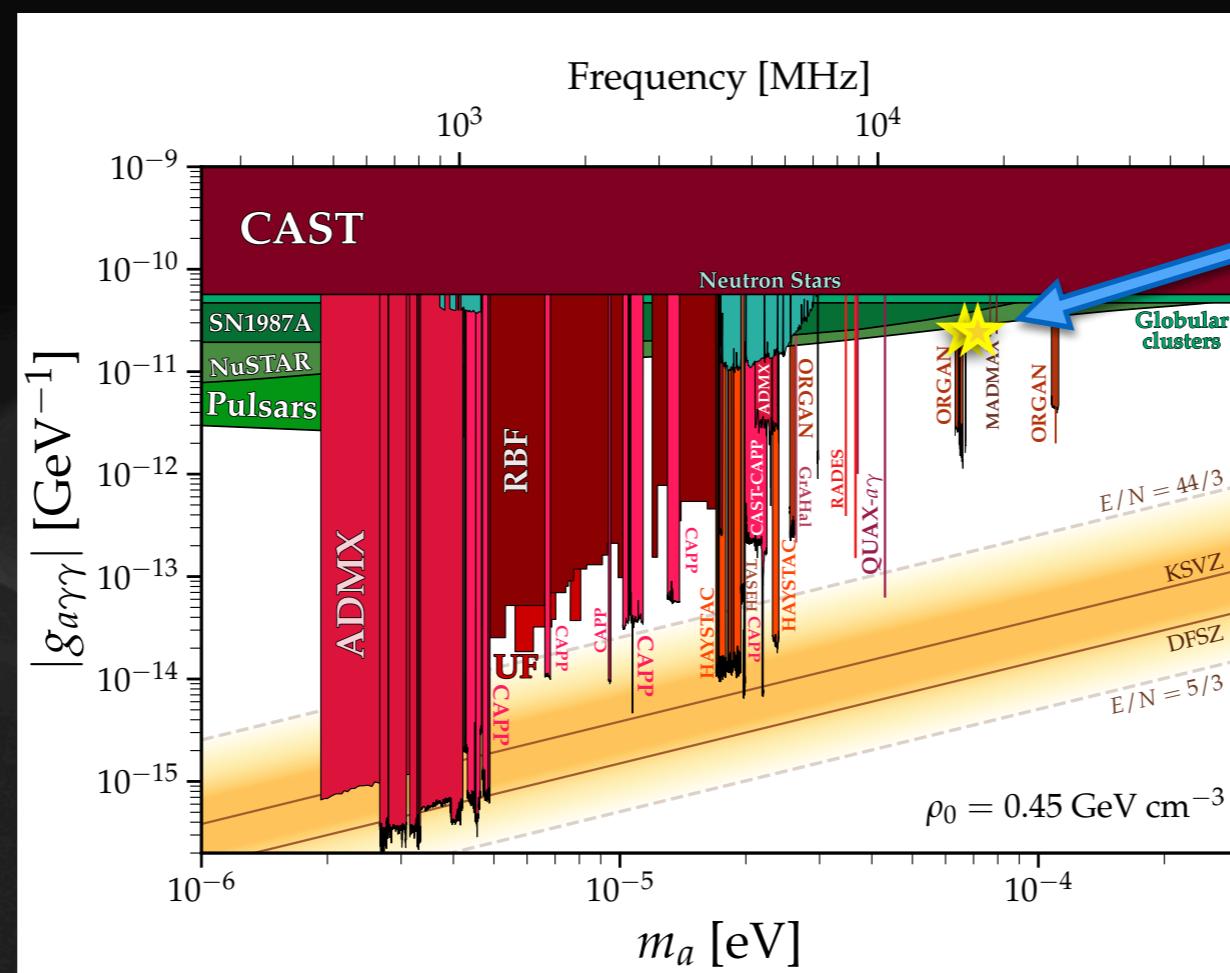
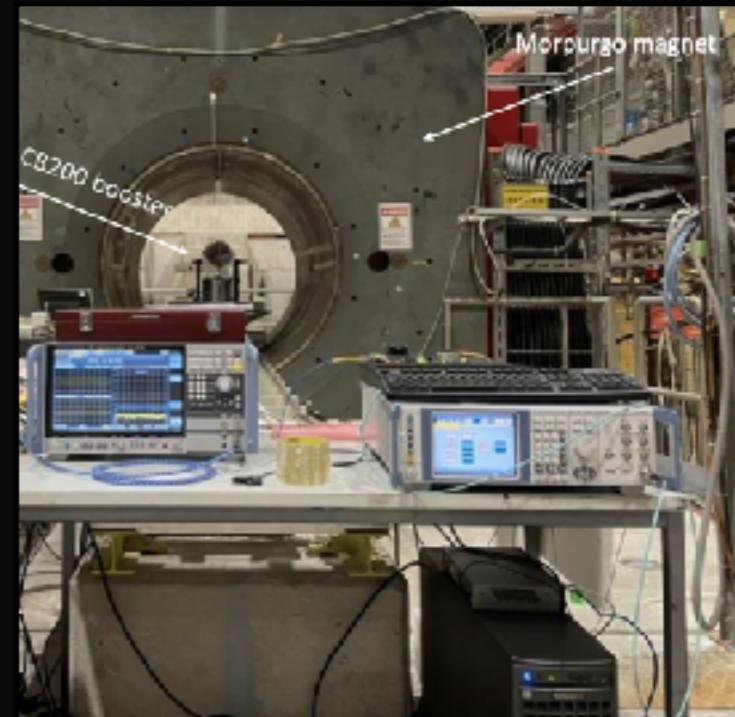
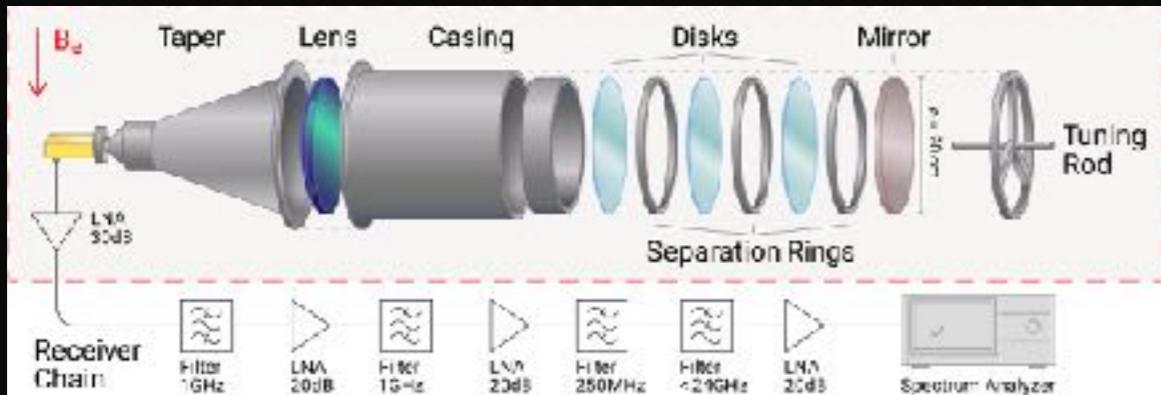


Large dipole magnet  
mirror / booster / focusing mirror



# Test run at CERN

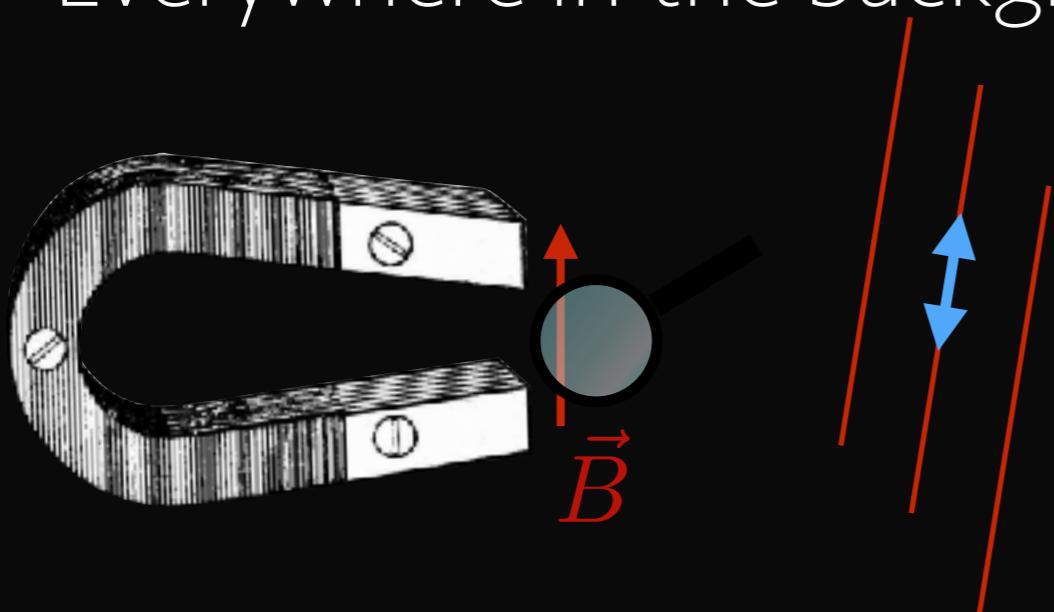
Full apparatus expected 2028  
Test run in a 1.6 T magnet at CERN



Expected reach

# Broadband concentrators

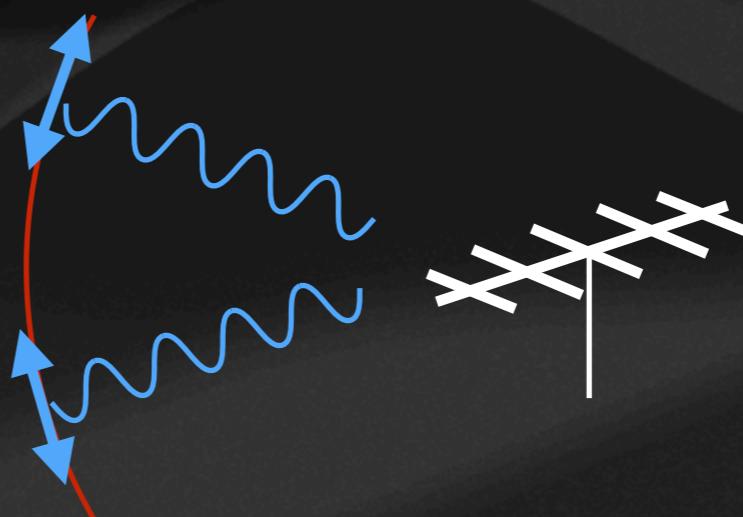
Everywhere in the background :  $\phi = \phi_0 e^{-imt}$



$$g_\gamma \phi F_{\mu\nu} \tilde{F}^{\mu\nu} \propto g_\gamma \phi \vec{E} \cdot \vec{B}$$

small oscillating electric field along each B-field line

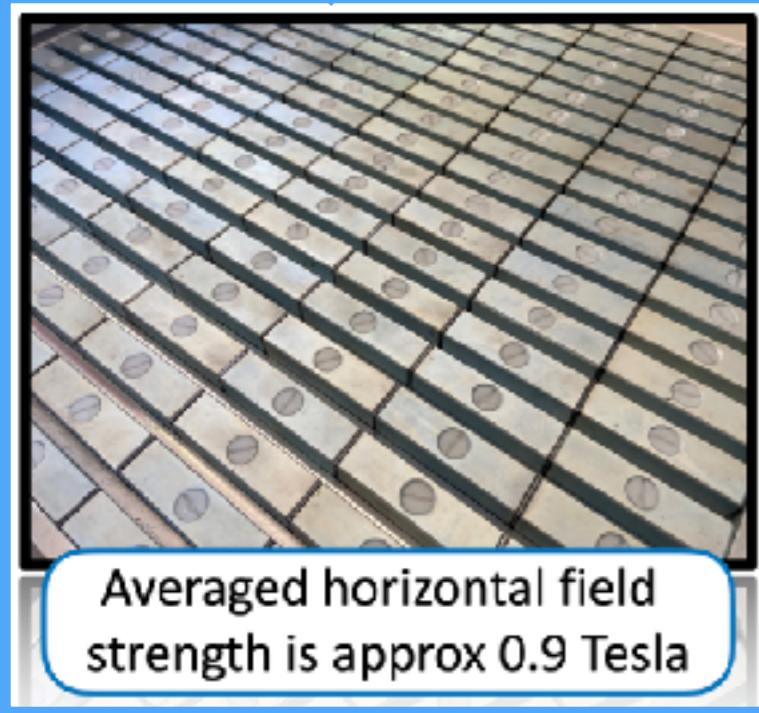
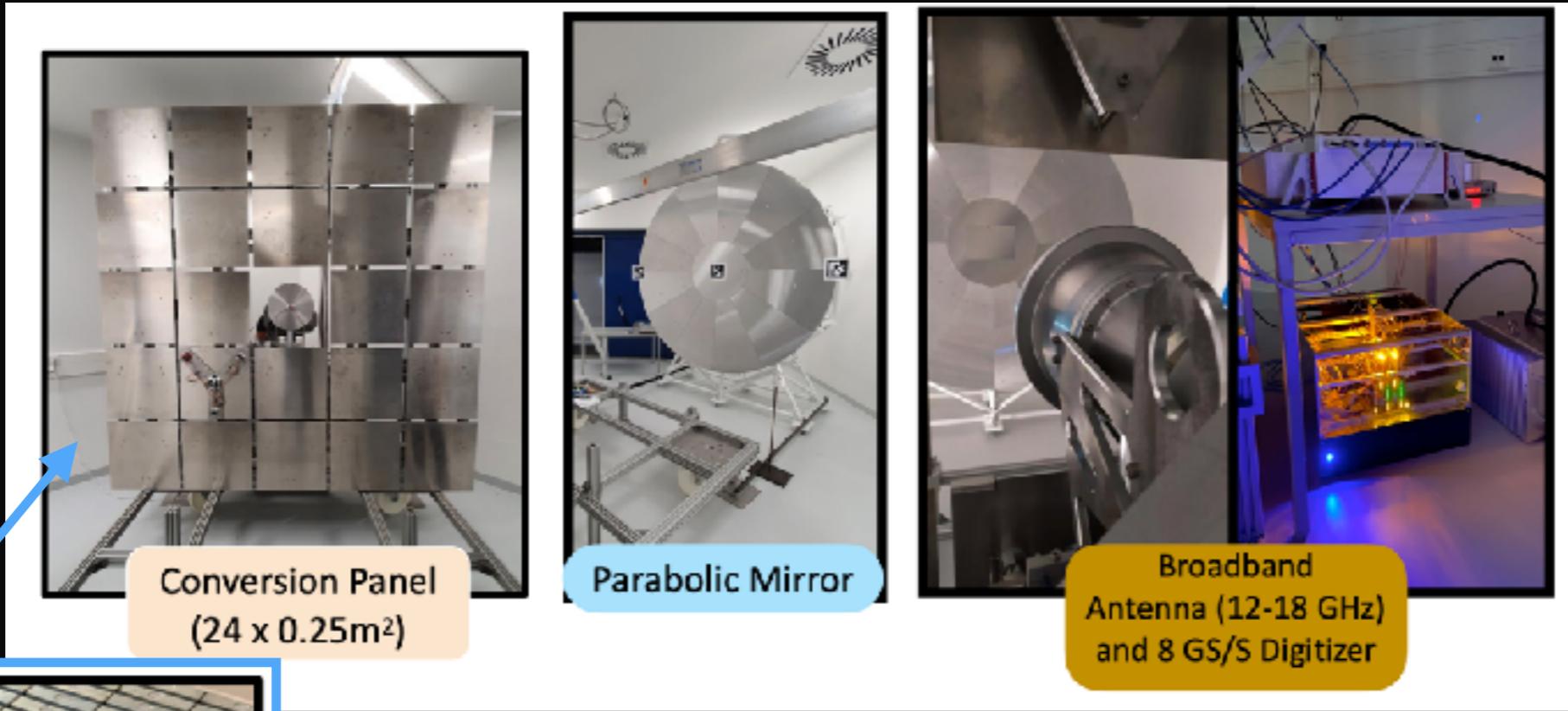
Possible lab experiment :  
bend field lines,  
search for excess power



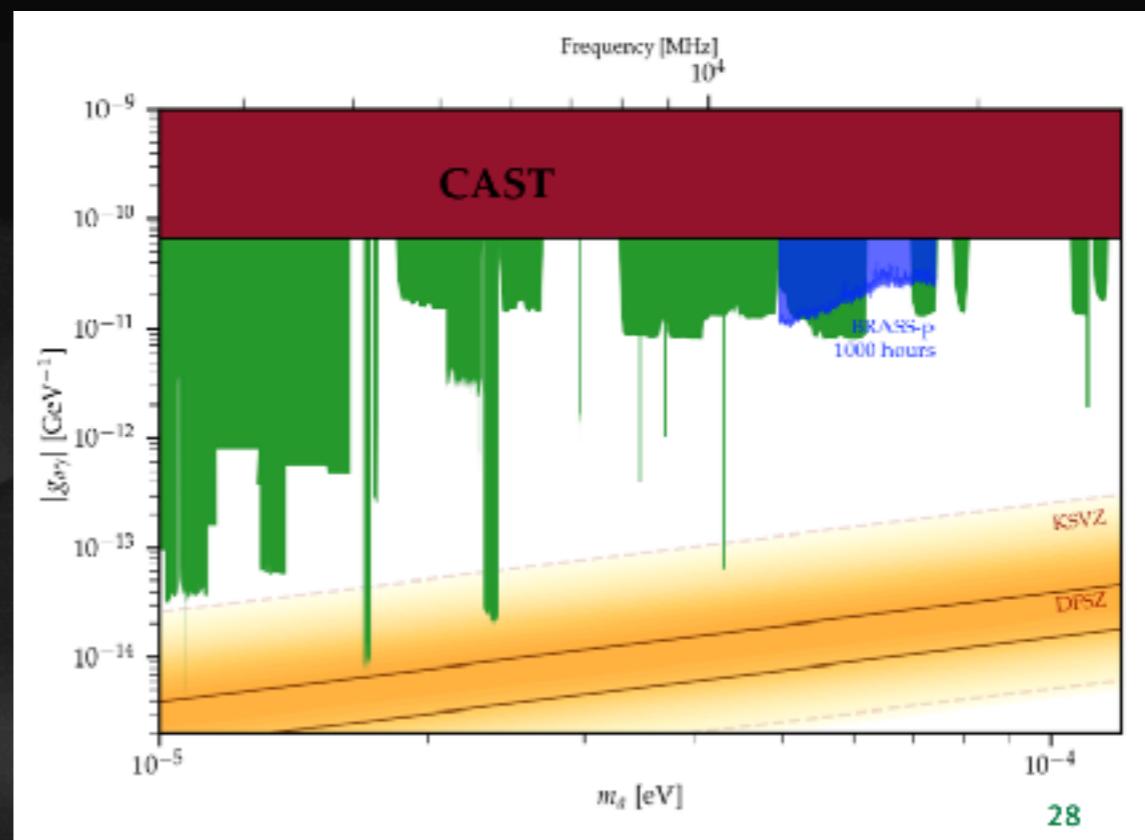
# BRASS

Broadband Radiometric Axion SearcheS

Nguyen, PATRAS 2022

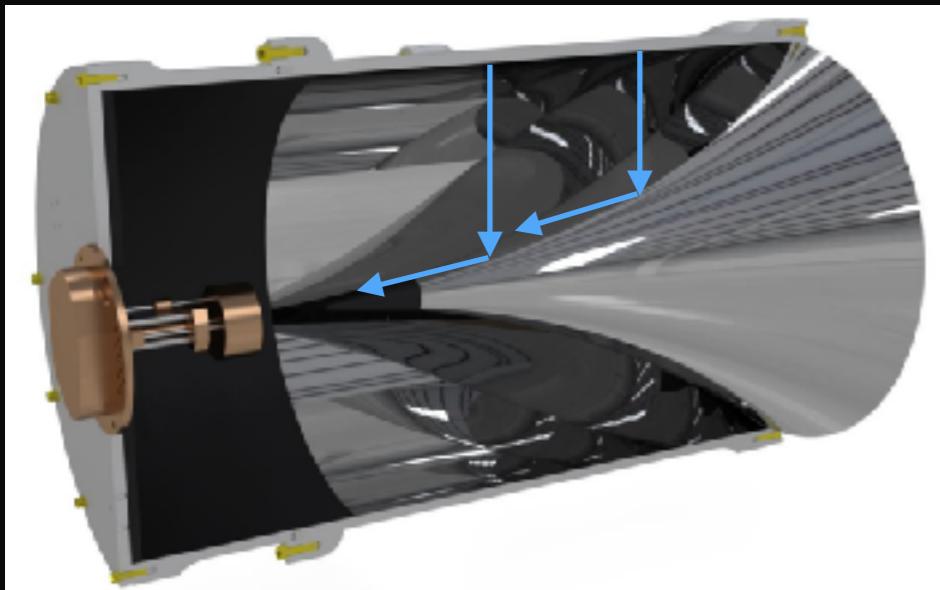


Array of permanent magnets



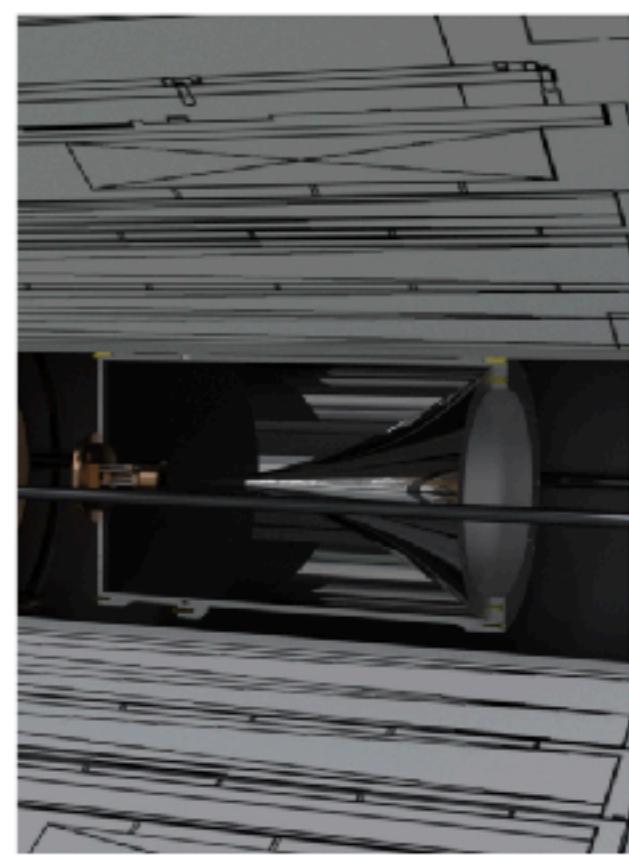
# BREAD

Broadband Reflector Experiment for Axion Detection



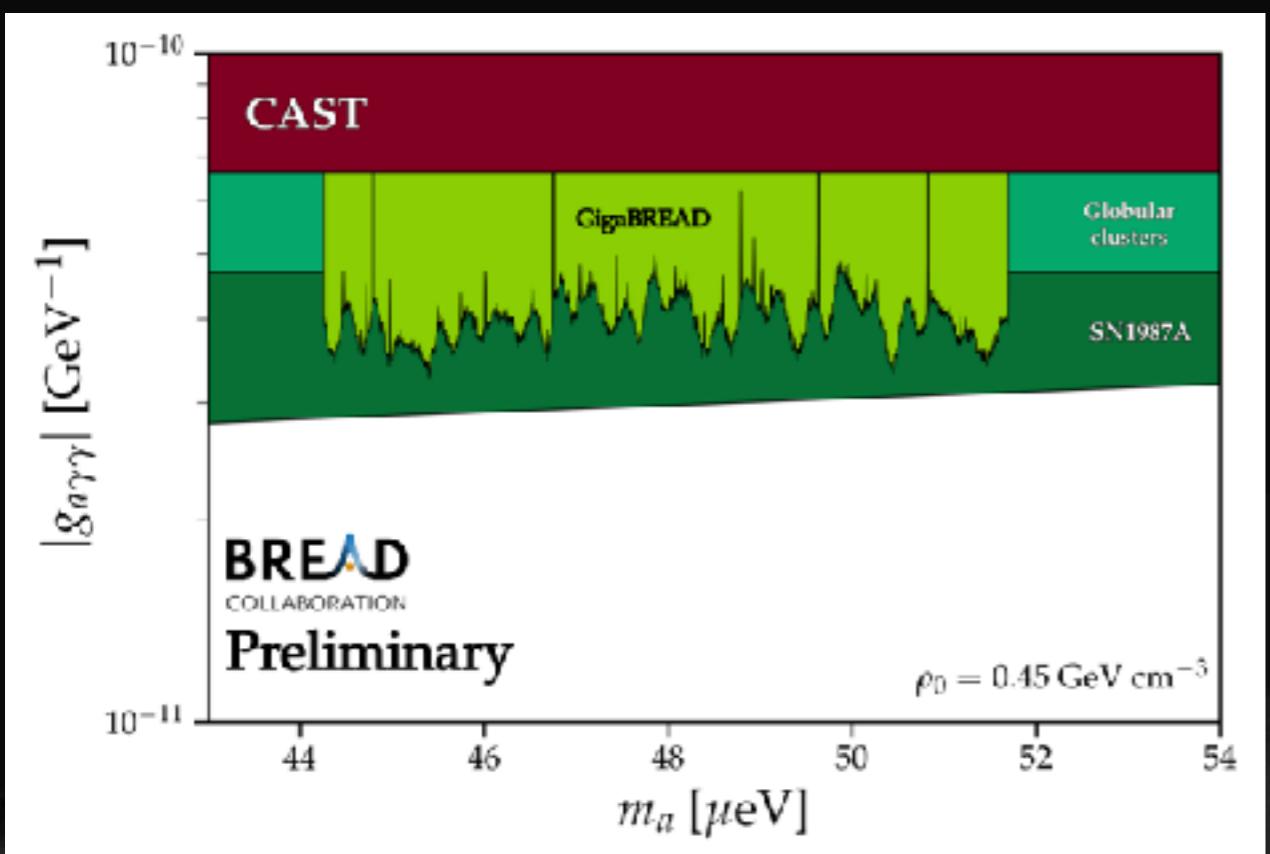
Use of a solenoid magnet

Custom-shaped reflector sends the signal to focus

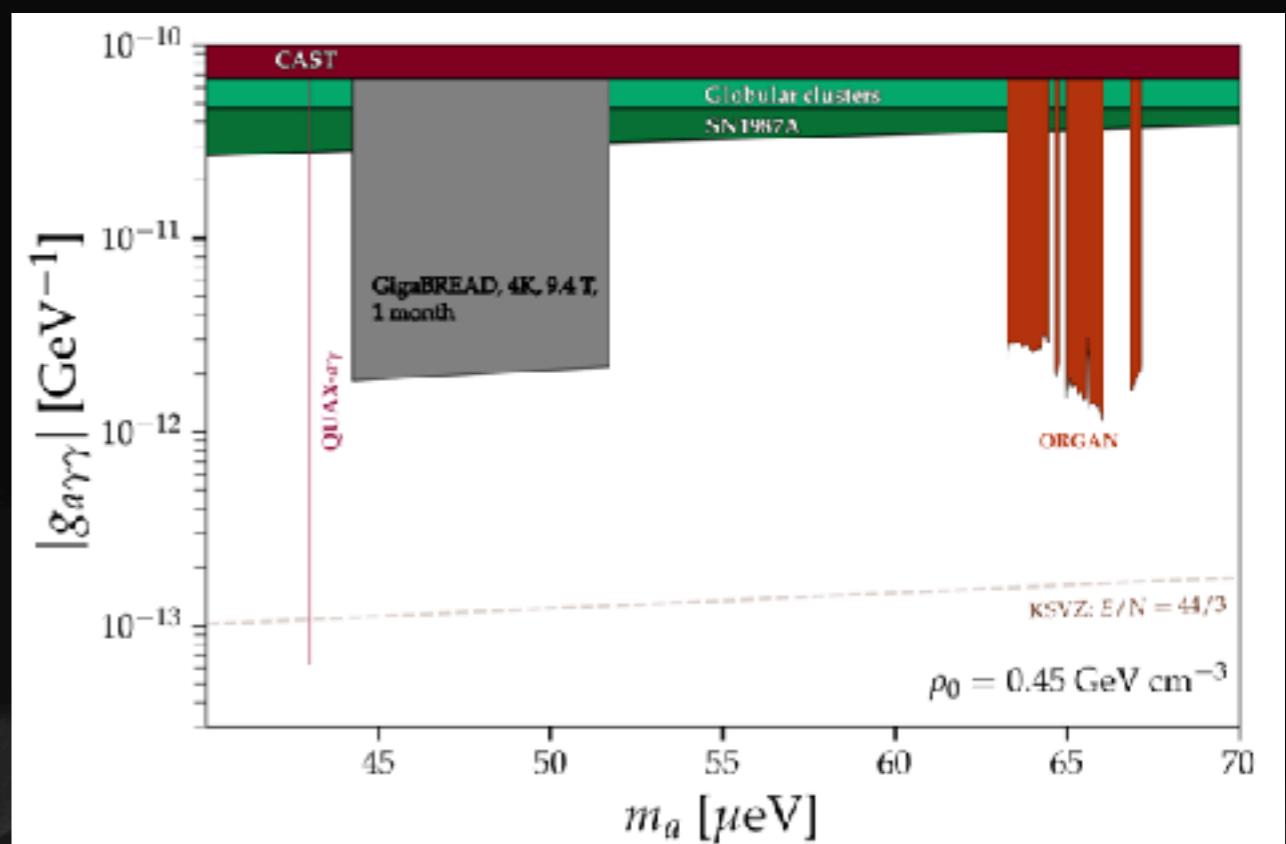


# BREAD

Test run in a 3.9 T MRI magnet

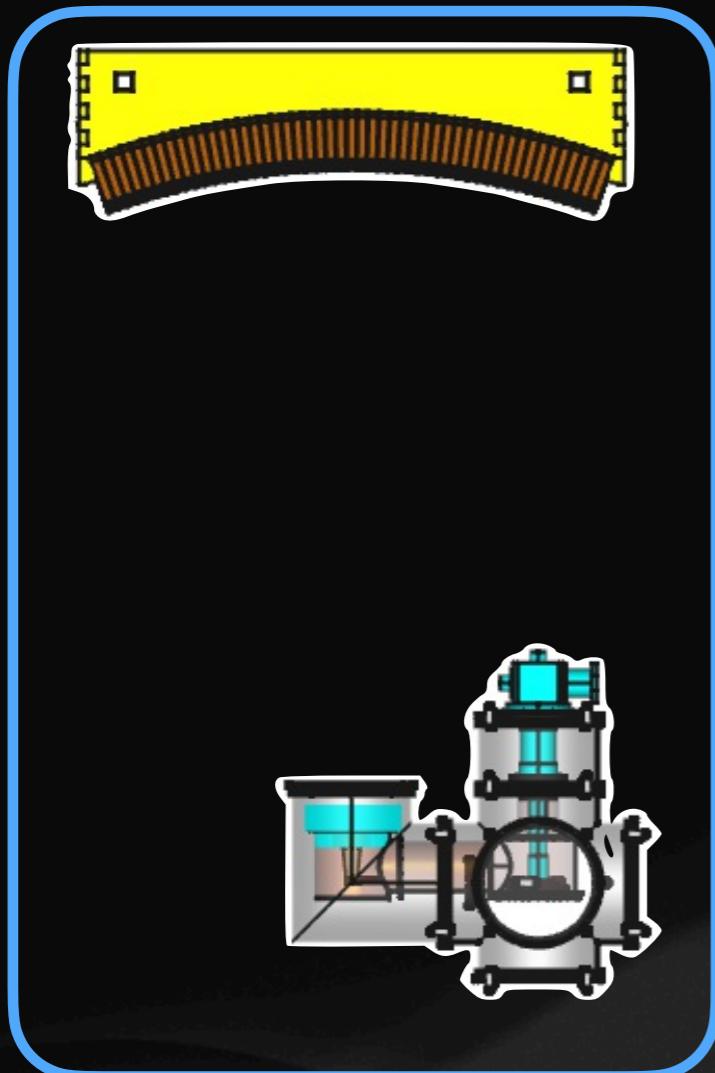


Plans to use a 9.4 T magnet



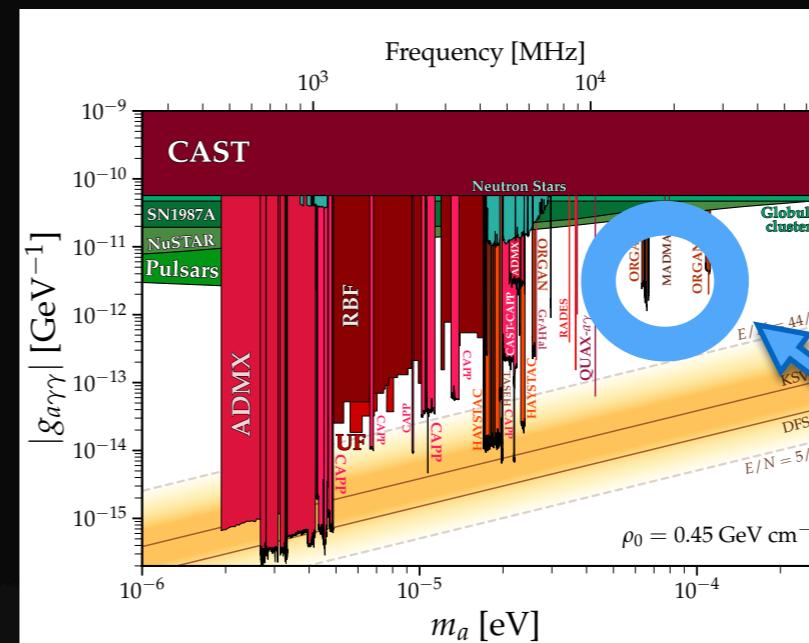
# DAWA

Dark Axion Wideband Approach



Receiver cooled down 4 K

Array of permanent magnets



Expected reach

- ★ First fully modular setup
- ★ First axion runs late 2024
- ★ Switch to superconducting magnets late 2025