Gravitational Lensing and dark matter characterization

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Beyond WIMPs workshop – October 1st, 2024 – LAPTh Annecy

James Webb Space Telescope (JWST) vs Hubble



https://www.spaceze.com/news/jwst-sees-the-same-galaxy-from-three-different-angles-thanks-to-a-gravitational-lens



https://www.iflscience.com/what-are-we-actually-seeing-in-jwsts-first-deep-field-image-64410

Dark Matter direct detection sensitivities

Experiments / prototypes in preparation at CPPM:



DarkSide-20k

- →TPC with noble liquid (Xe, Ar): best limits 1 GeV 100 TeV
- →Next decade decisive to probe WIMPs down to neutrino floor

→ R&D program to improve signal sensitivity

Dark matter in large scale structure context



Cold vs Warm dark matter observables



Sub-Halo mass function

Galaxy cluster profile in SIDM



In simulations, dark matter can partially be distinguished from baryons at scales R < 20 kpc

The central density profile slope of ETG

Bolton et al. 2008

<u>SLACS</u>: 58 elliptical galaxies with gravitational arcs detected in SDSS spectra

Combination of SL mass in Einstein radius, and velocity dispersion of the stars σ_0 in SDSS spectra (R_{fiber} = 3")

Confirmation that Early Type Galaxies (ETG) follow isothermal density profile $\gamma = 2$ on average







The Strong Lensing Legacy Survey (SL2S)

Combination of 25 lenses from SL2S, 53 from SLACS and 4 from Lenses Structure and Dynamics (LSD)

- Redshift range : 0.2 < z < 0.8
- Stellar mass range: log M* / M $_{\odot}$ = 11 12
- Galaxy size range: $R_{eff} = 1 20 \text{ kpc}$

=> Understand the DM profile slope γ' variation

$$\frac{\partial \gamma'}{\partial z} = \alpha = -0.31 \pm 0.10,$$

$$\partial \gamma' / \partial \log \Sigma_* = 0.38 \pm 0.07$$

Slope γ' unchanged

- => The slope is rather constant $\langle \gamma' \rangle$ = 2, but this hides degeneracies:
- Stellar mass increases on the edges
- DM infall in the center (+contraction)



Cabanac et al. 2007, Gavazzi et al. 2012

Same results found in Li, Shu & Wang 2018 9

Sonnenfeld et al. 2013

Stellar Initial Mass Function with MANGA

MANGA observed in IFU mode 17 galaxies on 7deg² of sky (1423 fibers total, Bundy et al. 2015)



Measurement of IMF mismatch

$$lpha_{
m IMF} \equiv \left(M^*/L
ight)_{
m JAM}^{
m nogas}/(M^*/L)_{
m SPS}$$

=> α_{IMF} increases with σ_e (±50% uncertainty)

Li, Ge, Mao et al. 2017



Strong-lensing, dynamics & weak-lensing

7 galaxy clusters selected w/ SL arcs

DM density profile gNFW: free inner slope $\boldsymbol{\beta}$ to account for adiabatic contraction

Stellar mass M* derived from Stellar Population Synthesis => IMF assumption (quoted factor ~2 uncertainty)

Stellar density profile adjusted to Surface Brightness of central BCG, and scaled to $\alpha_{SPS} \times M^*$





profile β = 0.5±0.13

Newman et al. 2014

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Averaging over more clusters and groups

Chunxiang Wang et al. 2024

Weak Lensing for the larger scales and stellar kinematics in the center with MANGA (IFU) data

Stellar density profile adjusted on r-band SB distribution scaled to $\alpha_{SPS} \times M^*$

=> The DM profile inner slope is $\gamma > 1$





Strong lensing in galaxy clusters

Better modelling thanks to

- More multiple images constraints with deep HST observations (HFF program, JWST)
- Integral field spectroscopy data to constrain galaxy kinematics (MUSE)
- Dark matter and stellar content decoupled from the cluster DM component



Limousin et al. 2017



Cored profile is favored R (kpc)

Limousin et al. 2022

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=> Hint for self-Interacting DM? Or systematic bias? => need bigger sample

Selection effect?

- Strong lensing lenses are biased objects (Foex et al. 2014, Sonnenfeld et al. 2024)
 - SL lenses are trixial objects
 - Elongated halos along the line of sight

- Big efforts to characterize the selection function
 - Analytic predictions : including instrumental effects, e.g. Euclid, LSST, etc
 - Full hydro-simulations (e.g. Xu, Springel et al. 2017, Despali et al. 2021)
 - Spectroscopic observations : characterize redshift distribution of lenses and arcs (e.g. VLT-Xshooter program, PI: Jullo; 4MOST proposal PI: Collett; DESI secondary program Huang et al. in prep)





Einstein rings by galaxies

- Flux or position perturbation in Einstein rings reveals low mass subhalos (see also Chan et al. 2020 with axion part.)
- With optical/NIR observations in spectroscopy (~4h Kband/Keck, 3h NICMOS) => ~10⁸ M_☉/h
- Around 10⁵ Einstein rings to be discovered with Euclid => good sample of « jackpot » candidates

Combined constraints Lya, lensing, MW satellites : => Lepton with asymmetries L6 > 10 and 7.1 keV sterile neutrinos are ruled out







Quadruply imaged quasars : flux anomalies

- More small-scale substructures produce more frequent flux ratio anomalies
- Require long term monitoring of QSO flux variations
- Impact of Line of Sight structures (He, Li et al. 2021)
- Dependence on the simulation details (e.g. tidal destruction severity)

=> Move from standard modeling to summary statistics techniques to simplify the analyses



2M1310-1714

Gilman et al. 2021

Combined constraints: Lya, SL, MW sat.



- VLBI + ELT will reach 0.2 to 5 mas resolution to probe halos $10^6\,M_\odot$ (Spingola et al. 2018 for VLBI)
- JWST will allow to maximize contribution from LOS haloes for High-z sources => tighter constraints
- Euclid & LSST will bring many candidates (~10⁵)
- High-resolution, realistic hydro simulations will yield better dark matter models

=> For lepton asymmetries L6 > 10 , 7.1 keV sterile neutrinos are ruled out

Preparing the future: ELT-HARMONI simulations

- ELT-HARMONI expected first light ~2030
 - 42m telescope with Laser Guide Stars Adaptive Optics
 - IFU in NIR with 4mas spaxel resolution
- Simulated observations
 - Background galaxy at z = 2 with star formation clumps
 - Lens galaxy in $10^{13}\,M_{\odot}$ halo
 - Perturbation $10^8\,M_\odot$
 - Observational setup: Total exptime 5h, K grism, 30x60mas spaxels, LTAO, no moon, airmass 1.3
- Perturbation on the arc : 0.2±2 pixels => detection limit



How to join effort?

1) Gravitational lenses

=> WIMP & axion: galactic scale CDM behavior => unable to distinguish WIMP & axion?

2) Detection of DM particles

> Sensitivity depends on the density model of the Galaxy and subhalos
 > use of simulations, observations (lensing, galaxy rotation curves, etc.)

In the 2 cases

• Use of hydrodynamical simulations

Much to gain by exchanging/joining efforts between communities 1) and 2), especially at the level of simulations

SL current constraints



Simulation (Springel et al. 2008)

Milky Way modelling for direct detection

- 1. Hydrodynamical N-body (zoom-in) simulations including subhalos
- 2. Connecting cosmo simulations with astroparticles and dark matter detection
- 3. Phase space distribution beyond the Maxwellian distribution of the Standard Halo Model

Dark Matter direct detection sensitivities

4×10⁻²

₩ 10-2

 4×10^{-3}

Sky Lines

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We assume a two-photon coupling to the axion (Ressell 1991, Bershady et al. 1991)

Two-photon coupling leads to monochromatic emission line

Gravitational lensing is used to determine the cluster density profile, and apply optimal weighting for emission line detection

VLT-VIMOS IFU observations image the core of the cluster $I_{\lambda_a} = 2.68 \times 10^{-18}$ $\times \frac{m_{\rm a,eV}^7 \xi^2 \Sigma_{12} \exp[-(\lambda_r - \lambda_{\rm a})^2 c^2 / (2\lambda_{\rm a}^2 \sigma^2)]}{\lambda_{\rm a}} = 24\ 800\ \text{\AA}/m_{\rm a,eV}^{000} (1 + z_{\rm cl})^4 S^2(z_{\rm cl})} \ \text{cgs,}$ (4)

Scalar Field Dark Matter (SFDM) at small scales

A slice of density field of ψ DM simulation on various scales at z=0.1

Schive, Chiueh, and Broadhurst (2014)

 10^8 10^7 10^6 10^6 10^6 10^6 10^6 10^6 10^7 10^6 10^6 10^6 10^6 10^6 10^6 10^7 10^6 10^7 10^7 10^7 10^7 10^7 10^7 10^7

Radial density profiles of haloes formed in the ψ DM model

SFDM model

DM is represented by a scalar field minimally coupled to gravity given by the Lagrangian:

$$\mathcal{L}_{\phi} = -\frac{1}{2}g^{\mu\nu}\partial_{\mu}\phi\partial_{\nu}\phi - V(\phi),$$

The scalar field potential $V(\phi)$ must have a **parabolic minimum**

$$V(\phi) = \frac{m^2}{2}\phi^2 + \frac{V_I(\phi)}{2}$$

Self-interacting soliton

Soliton: Hydrostatic equilibrium $\Phi_N+\Phi_I+\Phi_{
m Q}=lpha,$

Thomas-Fermi regime $\rightarrow \Phi_Q \ll \Phi_I$

Soliton TF limit
$$\Phi_N + \Phi_I = lpha,$$

In this approximation, the soliton density profile :

 $ho_{
m sol}(r) =
ho_{
m 0sol} rac{\sin(\pi r/R_{
m sol})}{\pi r/R_{
m sol}},$

$$R_{\rm sol} = \pi r_a$$
, with $r_a^2 = \frac{3\lambda_4}{16\pi \mathcal{G}_N m^4}$
 r_a sets Jeans length !

We consider the semi-classical limit, where λ_{dB} is smaller than both the core and halo radii.

Flat halo with r_a of the order of the system

R.Galazo-García et al. (2024) acknowledgements to Jean Charles Lambert

Total profile

• We choose the model to study $\rightarrow R_{sol} = \pi r_a$

$$\begin{split} r < r_t : \qquad \rho(r) = \rho_{0 \text{sol}} \frac{\sin(\pi r/R_{\text{sol}})}{\pi r/R_{\text{sol}}}, \\ r_t < r < R : \qquad \rho(r) = \frac{\rho_s}{\frac{r}{r_s} \left(1 + \frac{r}{r_s}\right)^2}. \end{split}$$

- We calculate the value of r_t and ρ_{0sol} such that $M_{sol}(< r_t) = \alpha M_{NFW}(< r_t)$ and the total mass of the system is conserved.
- We have slight flexibility in the choice of α as long as we are in the Newtonian regime $(M \sim 10^{17} M_{\odot})$ and the mass of the system varies minimally.

Soliton + nfw, M = 2e+15M_o, ρ_c = 3.64e+08 M_o/kpc³, r_t = 15 kpc, α = 3

Study case: Halo $M \sim 10^{15} M_{\odot}$

Comparison with Andrew B. Newman, Tommaso Treu, Richard S. Ellis, and David J. Sand, 2013

Comparison with Dark Energy Survey Year 1 Results: Weak Lensing Mass Calibration of redMaPPer Galaxy Clusters 2018

Another approach: FDM granules

In ψ DM simulations (~Mpc size boxes, Schive et al. 2014) Small halos have large granule size $l_{\sigma} \propto M^{-1/3}$ e.g. central soliton core r_c ~ 300 pc and mass ~ 10^{8.5} M_☉, granules of mass 10⁶ M_☉

Model granule size: $l_{\sigma} = \hbar/m_a \sigma_v$ Variance of the granule density field: $\langle \delta \Sigma^2(r_{\perp}) \rangle = \sqrt{\pi} \langle l_{\sigma} \rangle \int dz \langle \rho_{\psi}(r) \rangle$

Observations with VLBI interferometer in radio (Powell et al. 2023) => undistinguishable from CDM at $m_{\chi} > 4.4 \times 10^{-21} \text{ eV}$

Chan et al. 2020, Powell et al. 2023

Take home messages

- 1. Combination of WL+SL+Kinematics is used to measure the slope γ of the dark matter density profile from galaxies to clusters
- 2. Uncertain stellar masses still impede firm conclusions on γ
- 3. Wide imaging surveys (eg. Euclid, LSST) will provide large samples of galaxies and clusters for stacking => selection function!
- 4. Future observations (ELT) will constrain the subhalo mass function
- 5. Axion model is promising and compatible with cluster constraints so far

Some Definitions

Ferreira E. 2021

CDM

WDM

 10^{2}

- CMB/LSS Clusters Galactic Satellite galaxies/ Transfer function substructures 10^{3} Small Scales: unconstrained DM constrained to be CDM $T_{\text{matter}}(k)^2 = \frac{P_{\text{WDM}}(k)}{P_{\text{CDM}}(k)}$ 10^{1} non-linear linear $\stackrel{(\gamma)}{\nabla}{}^{10^{-1}}$ • Half-mode scale λ_{hm} 10^{-3} . $\frac{\rm Fuzzy\,DM}{10^{-22}\,\rm eV}$ WDM = where the TF = 1/2 10^{-5} 10^{-7} 10^{-2} 10^{-1} 10^{0} 10^{1} $k \, [\mathrm{Mpc}^{-1}]$
- Half-mode mass M_{hm}
- => the mass in a halo of scale λ_{hm}
- => M_{hm} = 0 means CDM model

Cluster profile ...

Grillo et al. 2015

Finney, E.Q. et al. 2018

=> Forward modeling can give insight on 1) DM smooth component, 2) substructures and 3) baryon components

Splashback radius in simulations

Stacked clusters at z=0 with $M_{vir} = 1 - 4 \times 10^{14} M_{\odot}/h$ ECOSMOG-V & ECOSMOG-fR N-body code (Li B., et al. 2011, 2013)

Equation of motion of a shell at radius r

$$\ddot{r} = -\frac{GM(r)}{r^2} - \frac{H_0^2}{2}\Omega_{DE}(1+3w)r^{-2-3w}$$

 \Rightarrow Dependency on concentration, galaxy bias...(More et al. 2016)

SB radius sensitive <10%

 \Rightarrow Precision achievable with Euclid / Roman / CSST?

Splashback radius in observations

 \Rightarrow WL stacking in Euclid / Roman / CSST?

WL stack of 13 clusters @ z \sim 0.3 M₂₀₀ > 1.6x10¹⁴ M_{\odot}

...And cluster shape

Self-interacting dark matter tends to produce rounder clusters

Peter et al. 2013

Simulations

Cluster ellipticity

But strong and weak lensing observations can be biased ⇒ WL shape noise and LSS contribute to make lensing estimates rounder ⇒ Hybrid-Lenstool joint SL+WL modeling can help

observations

And filaments

Jauzac et al. 2012

6x3 HST ACS mosaic, ~50 gal/arcmin²

 \Rightarrow WL stacking in Euclid / CSST?

 $Q \equiv \alpha_0 Q^{(0)} + \alpha_1 Q^{(1)} + \alpha_2 Q^{(2)},$

Aperture Multipole Moments (Schneider & Bartelmann 1997)

Cluster Substructures

=> Forward modeling can give insight on DM and baryon components

=> Degeneracy between subhalos and host

Cluster SubStructures

- Light 2.5keV WDM has the lowest number counts, because this model has less low mass subhalos
- σ/m = 1.0 cm²/g SIDM has low counts at small radius because of heat transfert between 'hot' host DM, and 'cool' subhalo DM + enhanced tidal stripping because of cored density profile of subhaloes
- f(R) cosmology could also impact the mass segregation function (Arnolds & Li, 2019), because f(R)-gravity increases the number of low-mass halos (not screened)

DM tidal stripping Strong-Lensing in galaxy clusters

- Modeling of DM distribution with strong-lensing constraints
- Comparison of subhalo mass function with hydrosimulations

Cluster Subtructures

- \Rightarrow Enhancement of substructures at small radius
- ⇒ Substructures are more compact (vdisp is larger than in simulations)
- \Rightarrow More tidal stripping? Cored profiles? SIDM?
- \Rightarrow SIDM produces less arcs but they are more magnified

DM tidal stripping & M-c relation Stacking of weak-lensing in galaxy-clusters

Credit: Supercluster Saraswati (DECaLS, Bagchi et al. 2017)

Strong Lensing

Principle of "inversion" of multiple images

Dark Matter mapping

Credit: Colley & Turner (Princeton), Tyson (Lucent Technologies), HST, NASA

A background galaxy appears multiple times

Matter in galaxy clusters is distributed with a density peak in the center → Dark matter + baryons

Credit: Kochanski, Dell'Antonio and Tyson (Bell Labs)

Galaxy clusters content

A galaxy cluster contains

- 80% dark matter
- 15% hot gas (~10⁷ K)
- 5% stars in galaxies

Observable SL/WL Xray/SZ Kinematics

Credit: X-ray: NASA/CXC/CfA/M.Markevitch et al.; Optical: NASA/STScI; Magellan/U.Arizona/D.Clowe et al. Lensing Map: NASA/STScI; ESO WFI; Magellan/U.Arizona/D.Clowe et al; Movie: KIPAC/J. White/M. Bradac

Properties of galaxies in clusters

Galaxies evolve in clusters in the same way They have similar formation history

Cluster galaxies have similar colors ⇒ color-magnitude diagram

Cluster galaxies follow:

- Faber-Jackson(1976) relation between velocity and luminosity
- Kormendy(1977) relation between size and luminosity

=> Fundamental plane (eg. Djorgovski & Davis, 1987)

Strong lensing modeling strategy

Observationally motivated models

- Decomposition into halos
- Few constraints (multiple image positions)

$$\phi_{tot} = \phi_{cluster} + \Sigma_i \phi^i_{halos}$$

Need to scale the galaxy halo components with galaxy luminosity to limit the number of free parameters:

$$\sigma = \overbrace{\sigma_*}^{L} \frac{L}{L_*})^{1/4} \qquad r_{cut} = \overbrace{r_{cut}^*}^{*} \frac{L}{L_*})^{\eta}$$
$$\frac{M}{L} \propto L^{\eta - 1/2} \qquad \eta = 1/2 \text{ Constant M/I}$$
$$\eta = 0.8 \text{ FP scaling}$$

usual matter density profileS

Isothermal sphere $= \rho_0/\tilde{R}$ ρ $\frac{\sigma^2}{2\pi G}$ = ρ_0 PIEMD (Kassiolo, 1993) $\frac{\rho_0}{(1+\frac{\tilde{R}^2}{r_c^2})(1+\frac{\tilde{R}^2}{r_{cut}^2})}$ ρ = $\frac{\sigma_{\infty}^2}{2\pi G r_c^2}$ ρ_0

Navarro, Frenk, White (1996)

$$\rho = \frac{\delta_c \rho_c}{\frac{\tilde{R}}{r_s} (1 + \frac{\tilde{R}}{r_s})^2}$$
$$\delta_c = \frac{200}{3} \frac{c^3}{\ln(1+c) - c/(1+c)}$$

Strong lensing fit of a single source

- The model is validated when predicted and observed images fall at the same location
- In practice, a RMS ~ 0.5" for about 20-100 multiple images is usual
- Source plane fit: a few computing days

$$\chi^2 = \sum \left(\frac{S_i - \bar{S}}{\sigma}\right)^2$$

• Image plane fit: ~ 1 week

$$\chi^2 = \sum \left(\frac{\theta_i^{obs} - \theta_i^{pred}}{\sigma} \right)^2$$

Credits: Joaquin Hernandez & Giuseppe D'Ago

=> Galaxies in clusters are self-similar also in terms of DM halo size?

How to best use data?

The Arctomo cluster sample

	Cluster	Redshift	$N_{ m spec}$	$N_{ m vdisp}$	$N_{ m src}$	$\mathrm{E}_{\mathrm{B}-\mathrm{V}}$
	SGAS J1226+2152	0.437	51	22		0.0185
]	PSZ1 G311.65-18.48	0.443	59	32		0.0926
]	RCS2 032727-132609	0.565	70	32		0.0672
	SGAS J211119.34-011423.5	0.638	38	29		0.0714

Strong lensing modeling alternative

Credits: Felipe Urcelay

- GIGA-Lens uses GPU and machine learning tools for modeling strong gravitational lensing by Bayesian inference
- The likelihood is given by pixel-to-pixel residual
- It is intended mainly for galaxy-galaxy lensing
- Does not include the redshift

Gu, A., et al. arXiv preprint arXiv:2202.07663 (2022).

SL modeling with alternative techniques

Credits: Felipe Urcelay

=> Well designed for groundbased observation (ex: LSST)

	System 1: 3 lenses		System 2: 10 lenses		
	Source plane positions	Pixels	Source plane positions	Pixels	
GIGA-Lens P100 GPU	40s	140s	120s	9min 20s	
Lenstool i7 CPU 8 threads	6s	230s	30s	1h	

Strong lensing recent progress

Better modelling thanks to

- More multiple images constraints with deep HST observations (HFF program, JWST)
- Integral field spectroscopy data to constrain galaxy kinematics (MUSE)

Limousin et al. 2017

Strong Lensing simulation challenge

Simulation setup:

- 100,000 candidates to classify
- 4-bands ground-based (GB) images (KiDS-like)
- Single band space-based (SB) images (Euclid-like)

Challenge details:

- 48h to upload classification of all candidates
- 24 participanting teams
- Mostly ML approach (e.g. SVM, CNN) but also human inspection (

Metcalf et al. 2016

Euclid SCIENTIFIC challenge 8 (SC8

Simulated area: 150deg²+5deg²

- 200 SL systems + galaxies + high-z QSO
 + LBG + stars
- Set of 31-bands + 2 grism images
- Simulation: 4435 CPU x 2 days, 46TB
- Reduction storage: TBD

Goal of SC8

- Run the data reduction pipeline to the end
 - Pipeline validation, computing & storage assessment
- Get noise properties to prepare next SL challenge
 - Include all instrument noise calibrated on lab measurements
 - Include all pipeline data reduction noise

Future plans: Add QSO lensing

Next STEP: spectroscopy

- DESI
 - Already discovered ~1500 SL systems in *grz* imaging survey (Huan XC et al. 2021)
 - Declination > -20, mag ~ 23.5
 - Expected ~400 new SL spectroscopic redshifts

- 4MOST proposal for SL systems
 - R < 24 & 10,000 spectroscopic redshifts
 - 5000 velocity dispersion measurements
 - Decision in Dec 2021

4MOST Proposal (PI: Collett)

Project objective (3 years)

Interdisciplinarity: Combining diverse scientific disciplines to foster creativity and achieve a common goal through different approaches

Common goal: to characterize the nature of dark matter

Approach 1: Gravitational lensing in cosmology

• Able to measure density profile and number of subhalos in galaxies

Approach 2: Direct and indirect detection in (astro-)particle physics

• Able to distinguish DM particles

Project Milestones

=> Show that the micro nature of DM (WIMP or axion) modifies macro observables (analogy with baryonic feedback)

Quality and Ambition of the project

- 2 well-established CPPM & LAM teams: Recognized for their analysis expertise
 - Tools: Lenstool Jullo et al. 2007, Clumpy Nezri et al. 2012, RAMSES Nuñez et al. 2021
- CPPM & LAM involved in international projects
 - WIMP: **KM3NET**: Extend ANTARES telescope 12 lines -> 115 lines (2021-)
 - **DarkSide**: proven technology with innovative design \rightarrow DS-20k (2025-)
 - Axion: MadMax: innovative concept \rightarrow prototyping phase for validation (2021-25)
 - Euclid, HST+JWST: High Resolution Detection and Imaging of Gravitational Lenses
 - VLT & ELT-HARMONI: Gravitational lens spectroscopy (redshifts)
- Ambition:
 - Challenge simulations: impact of micro DM physics at the macro level. Analogy with baryon physics (Nuñez et al. 2021)
 - Lens profile measurement (Limousin et al. 2016, 2022) + detection of substructures in gravitational lenses (Natarajan et al. 2017)

Implementation modality

- WP1: Common language for modeling DM halos, tidal effects, tidal streams
- => The halo of DM halos is the common object of the DM search & gravitational probe communities
 => Implementation of consistent models in Lenstool & Clumpy
 => Analysis of lens systems and measurement of density profiles and number of subhalos
- WP2: Impact of baryons+DM on the morphology and evolution of (sub)halos
- => Run of hydrodynamic cosmological simulations with the same properties of DM and baryon physics Challenge: Find consistent recipes despite the different simulation scales (Mpc \rightarrow sub-pc)
- WP3: Using WP1 and WP2 results to estimate uncertainties in detections
- => Prediction of direct & indirect detection rates from models (MD+baryons), simulation results, observational results