Bootstrapping the LRI

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0. Motivation

SR models at criticality well studied

LR models remain more mysterious: nonlocal; no stress-tensor

- Line of FP $\to$ many nonlocal CFTs
- MC for 2d LRI $\to$ conflicting
- OPE relations $\to$ more nonlocal theories: defect CFTs, QFTs in AdS

1. The LRI

LRI in p dim $\quad 1 \leq p < 4$

$$H = -J \sum_{i,j} \frac{\sigma_i \sigma_j}{|x_i - x_j|^{p+s'}} \qquad J > 0$$

s': line of FPs

criticality for

$$\underset{\text{MFT end}}{p/2} \leq s' \leq \underset{\text{SR end}}{p - 2\Delta_\sigma^*}$$

[Fisher, Ma, Nickel '72] [Sak, '73]

MFT end

$$s = \frac{p+\epsilon}{2} \qquad \lambda \sim O(\epsilon)$$

$$S = \mathcal{N}_s \int d^p\tau_1 d^p\tau_2 \frac{\phi(\tau_1)\phi(\tau_2)}{|\tau_{12}|^{p+s'}} + \int d^p\tau \frac{\lambda}{\sqrt{\epsilon}} \phi^4$$

$$\Delta\phi = \frac{p-s}{2} \ ; \quad \Delta\phi^3 = \frac{p+s}{2}$$

$\Delta\phi + \Delta\phi^3 = p \to$ shadow pair

SR end [

$$s' = p - 2(\Delta_\sigma^* + \delta) \qquad \Delta_\sigma^* = \begin{cases} 1/8 & p=2 \\ 0.52.. & p=3 \end{cases}$$

[Behan, Rastelli, Rychkov, Zan '17]

$$S' = \mathcal{N}_s \int d^p\tau_1 d^p\tau_2 \frac{\chi(\tau_1)\chi(\tau_2)}{|\tau_{12}|^{p-s}} + S_{SRI} + g\int \sigma \cdot \chi \qquad g \sim O(\delta)$$

$\Delta_\chi = \frac{p+s}{2}$; $\Delta\sigma = \frac{p-s}{2}$ shadow pair

2. OPE relations

First derived from nonlocal eom [Behan '18]

LRI as defect [Paulos, v Rees, Rychkov, Zan '15]

Nonlocal kinetic term $\to$ local kin term of free field in higher d bulk

$d = p + \underline{q}$ non-integer , $x = (x_\perp, \hat{\tau})$

MFT and

$$S = \int d^d x\, (\partial\phi)^2 + \boxed{\int d^p\tau \frac{\lambda}{\sqrt{4!}} \phi^4}$$

localized on the **defect**

$SO(d+1,1) \to SO(p+1,1) \times SO(\underline{q})$

transverse spin j

$\boxed{j = 0}$

$q = 2 - s$

SR

$$S = \int d^d x\, (\partial\chi)^2 + \boxed{S_{SRI} + \int d^p\tau\, g \sigma \chi}$$

$q = 2 + s$

usual OPE : $O_i(x_1) \times O_j(x_2) \sim \sum_k \lambda_{ijk}\, C(x_{12}, \partial)\, O_k(x_2)$

DOE : $\Phi(x) \sim \sum_k b_0^{\Phi\psi}\, C(x_\perp)\, \hat{\varphi}_k(\tau)$

$\Box\Phi = 0$: $\hat{\varphi}_0^+ : \Delta_+ = \Delta_\Phi \to \phi, \chi$

$\hat{\varphi}_0^- : \Delta_- = \Delta_\Phi + 2 - q \to \phi^3, \sigma$

Now consider $\langle \Phi(x_1)\, \hat{\varphi}_0^{\pm}(\tau_2)\, \hat{\mathcal{O}}^{(\ell)}(0,\infty)\rangle$

$$\Big\downarrow \text{BOE}$$

$$\varphi_0^{\pm}(\tau_1)$$

For $\tau_1 = \tau_2$ discontinuity should vanish [Lauria, Liendo, van Rees, Zhao '21]

$$\lambda_{\varphi^+\varphi^+\hat{T}} = \kappa_1\, \lambda_{+-T} \quad ; \quad \lambda_{--\hat{T}} = \kappa_2 \quad \lambda_{+-\hat{T}}$$

$$\kappa_i = \kappa_i(\Delta_T, \Delta_\phi, p, q, \hat{b}_0^{\pm}) \to \text{product of } \Gamma\text{-fcns}$$

$$\hookrightarrow \frac{\hat{b}_0^+}{\hat{b}_0^-} \sim R(a_{\phi^2}) \; : \; \langle\langle \Phi^2(x)\rangle\rangle = \frac{a_{\phi^2}}{|x_\perp|^{\Delta_\Phi}}$$

equivalent relations MFT end and SR end ; related by
<u>Shadow trafo</u> : $\Delta_0^+ \leftrightarrow \Delta_0^-$ or $q \leftrightarrow 4-q$
$$s' \leftrightarrow -s'$$

- MFT : $\hat{b}_0^+ = 1$, $\hat{b}_0^- = 0$, $a_{\phi^2} = 0$
- SR : $\hat{b}^+ = 0$, $\hat{b}^- = 1$, $a_{\chi^2} = 0 \longleftrightarrow a_{\phi^2} = \begin{cases} -7.18 & p=2 \\ -0.58 & p=3 \end{cases}$

special case for odd spin:

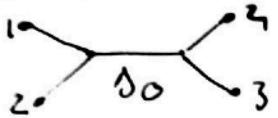
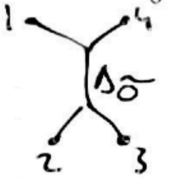
Bose symm : $\lambda_{--\hat{T}} = \lambda_{++\hat{T}} = 0$, $\lambda_{+-\hat{T}} \neq 0$

To get rid of disc, need $\Delta_{\mathcal{O}_\ell} = p + \ell + 2n$ ℓ odd

3. Numerical bootstrap

goal: implement OPE rels in bootstrap of 4-pt fcns of LRI

$\langle \varphi^\pm \varphi^\pm \varphi^\pm \varphi^\pm \rangle$: Decompose in CB, e.g. $\langle \varphi^+ \varphi^+ \varphi^+ \varphi^+ \rangle = \dfrac{g(u,v)}{|x_{12}|^{2\Delta_\varphi} |x_{34}|^{2\Delta_\varphi}}$ $\qquad u = \dfrac{x_{12} x_{34}}{x_{13} x_{24}}$ $\quad v = \dfrac{x_{14} x_{23}}{x_{13} x_{24}}$

Crossing: $\sum \lambda_O^2$  $- \sum \lambda_{\tilde{O}}^2$  $= 0$

Put bounds on lowest-lying Δ_O as fcns of $\Delta_{O'}$ using SDPB

- OPE relations $\longrightarrow$ use a_{φ^2} as parameter
+ protected sector
- $\ell = 1$ operator

plots

left most kink? $\to$ GFF + min model? [Behan, di Pietro, Lauria, v Rees '22]

1d LRI ? critical for $0 \leq s \leq 1$ $\qquad$ ope rels, bootstrap, pert. th for MFT end

conceptual issue: no phase transition for 1d SRI $\qquad$ WiP

what is SR end? $\to$ [Kosterlitz '76]

1. Further applications

LRI: defect is auxiliary (q noninteger), what about physical defect?

[Behan, di Pietro, Lauria, v Rees '20 '22]: bdy ($q=1$): OPE cels + bootstrap

shadow relation $q \longleftrightarrow 4-q$ $\Rightarrow$ $q=3$ kinematically related

$\boxed{q=2}$ is left: monodromy in free bulk $d=4, p=2$ W.i.P Behan, Lauria, PvV

Free bulk field Φ /w $\mathbb{Z}_2$ monodromy: $\Phi \to e^{i\pi}\Phi$

DOE: $\Phi(x) = \sum_{k,j} \hat{b}_{kj}\, C(x_\perp,\partial)\, \hat{\varphi}_{k,j}(\tau)$

com: $\Delta_\pm = \Delta_\phi \pm |j|$ unitarity $\to$ $j = \pm 1/2$

Correlators: all possible combi's of $\psi^\pm_{\pm 1/2}$ that are neutral under $SO(q)$

$\to$ 10 X-eqns

$\lambda_{\psi^+_{1/2}\psi^+_{1/2}\hat{O}}\, \lambda_{\hat{O}\psi^-_{-1/2}\psi^-_{-1/2}} = f(\tilde{\kappa}_1, \tilde{\kappa}_2)\, \lambda_{\psi^+_{1/2}\psi^-_{1/2}O}\, \lambda_{O\psi^+_{-1/2}\psi^-_{-1/2}} =$

$= \lambda_{\psi^+_{-1/2}\psi^+_{-1/2}O}\, \lambda_{O\psi^-_{+1/2}\psi^-_{+1/2}}$

\+ protected odd spin sector

\+ unprotected " " "