

# Direct bounds on Left-Right gauge boson masses

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for further details and references, see [JHEP02\(2024\)027](#),  
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# LRMs: gauge boson sector

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- **Enlarged gauge group**;  $X = (B-L)/2$
- A **discrete symmetry** (e.g. parity) can in principle be restored at energies  $\gg K_R$
- **LR-SSB**: SM at low energies; hypercharge:  $Y = T_R^3 + (B-L)/2$
- **New massive gauge bosons,  $W_R$  &  $Z_R$** ; LR corrections to the SM:  $\epsilon \sim K_{EW}/K_R \ll 1$
- Additional weak coupling constants; **perturbative region**  $g_R, g_X \sim < 1$

[Pati, Salam, Mohapatra, Senjanovic '70s ...]

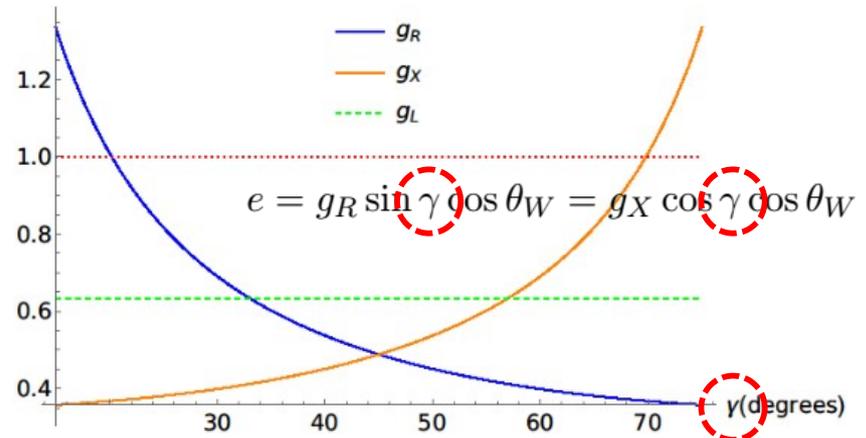
$$LRM = \underbrace{SU(3)_C}_{g_s} \times \underbrace{SU(2)_L}_{g_L} \times \underbrace{SU(2)_R}_{g_R} \times \underbrace{U(1)_X}_{g_X}$$

LR-SSB @ scale  $\kappa_R, \gamma$ : mixing  $W_R^3$  and  $W_X$

$$SM = SU(3)_C \times SU(2)_L \times U(1)_Y$$

EW-SSB @ scale  $\kappa_{EW}, \theta_W$ : mixing  $W_L^3$  and  $W_Y$

$$SU(3)_C \times U(1)_{EM}$$



# LRMs: scalar sector

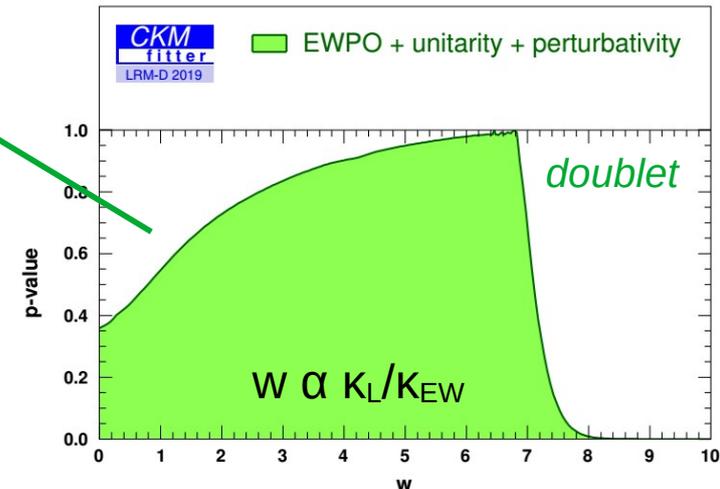
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- Multiple ways of implementing LR-SSB: (T) triplet, or (D) doublet under  $SU(2)_R$  are the most studied cases

Triplets:  $\Delta_R \sim (1, 1, 3)_1$ ,  $\Delta_L \sim (1, 3, 1)_1$       Doublets:  $\chi_R \sim (1, 1, 2)_{1/2}$ ,  $\chi_L \sim (1, 2, 1)_{1/2}$

[In (T), doubly charged scalars are present]

- In (D), the EW-SSB can be triggered by the  $SU(2)_L$  doublet VEV  $\kappa_L$ ; in (T), EWPOs limit the size of the  $SU(2)_L$  triplet VEV
- Additional scalar representations are considered for various reasons, e.g. EW-SSB in (T) scenario



[Bernard, Descotes-G., LVS '20]

# LRMs: fermionic sector

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- A bi-doublet  $\Phi \sim (\mathbf{1}, \mathbf{2}, \mathbf{2})_0$  also participates in the SSB, needed in the triplet case; Yukawa interactions: **Dirac masses** for  $(Q_L, \Phi Q_R)$  and  $(L_L, \Phi L_R)$
- (Eff) Non-ren. dim.-5 interactions with  $SU(2)_L$  and  $SU(2)_R$  doublets, e.g.  $(Q_L, \chi_L \chi_R^\dagger Q_R)/\Lambda$  or  $(L_R^c, \sigma_2 \chi_R \chi_R^\dagger \sigma_2 L_R)/\Lambda$

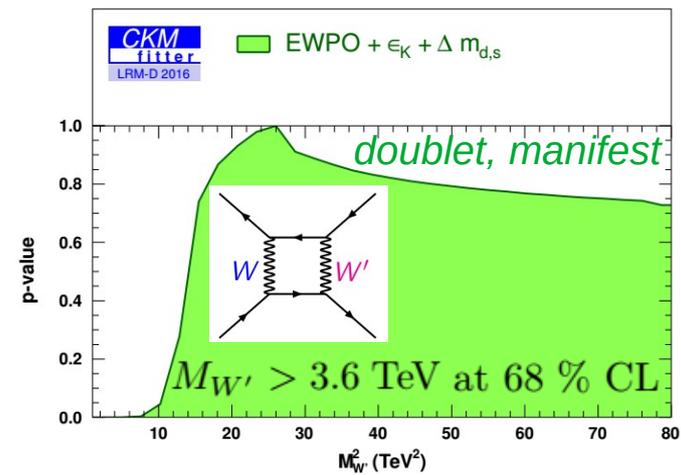
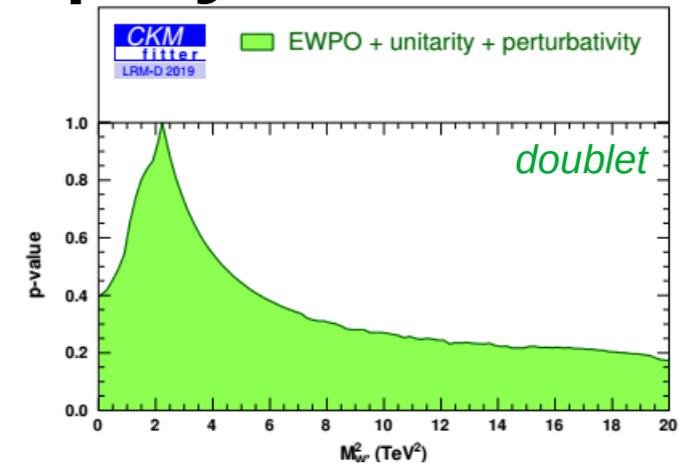
- RH neutrinos are introduced (B-L is anomaly free); no additional fermion in the simplest version
- In (D), **Dirac masses only**; in (T), also **Majorana masses**  $(L_R^c, \sigma_2 \Delta_R L_R)$  and  $(L_L^c, \sigma_2 \Delta_L L_L)$ : see-saw mechanism
- Extension of the PMNS matrix; RH unitary counterpart  $V^R$  of the CKM-like matrix  $V^L$  in the quark sector
- P:  $V^R \sim V^L$  (manifest); C:  $V^R \sim V^{L*}$  (pseudo-manifest)

	Left	Right
$SU(2)_L$	2	1
$SU(2)_R$	1	2
quarks :	$\begin{pmatrix} U_L \\ D_L \end{pmatrix}$	$\begin{pmatrix} U_R \\ D_R \end{pmatrix}$
$B = 1/3$		
leptons :	$\begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix}$	$\begin{pmatrix} \nu_R \\ \ell_R \end{pmatrix}$
$L = -1$		

# EWPOs & quark flavor physics

- LRM parameters **largely unconstrained**
- Beyond tree level: many parameters from the scalar potential intervene

- 
- $Z'$  (typically) does not introduce **FCNCs** at tree level, part of the scalar sector typically does
  - **RH charged weak currents**, possibly new sources of **CP violation**; observables: meson-mixing, etc.
  - Bounds **strongly depend on RH mixings**, e.g. when  $V^R \sim V^{L*}$ : no constraint from  $\epsilon_K$  (indirect CPV in kaons)



[Bernard, Descotes-G., LVS '16 '20]

# Collider limits on LRMs



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Various strategies used to look for  $W'$ ,  $Z'$ ; for instance (LHC only):

- $Z'$  to  $\ell\ell$  [1709.07242, 1903.06248, 2103.02708, etc.]
- Di-jet searches [1910.08447, 1911.03947, etc.]
- $W'$ : production and decay of massive RH neutrinos (includes LLPs)  
[1809.11105 ( $M(N_R) < 2 \cdot M(W_R)$ ), 1811.00806, 1904.12679, 2112.03949, 2304.09553, etc.]
- Leptonic decay of the  $W'$  with light (RH) neutrinos  
[1807.11421, 1906.05609, 2202.06075, 2402.16576, etc.]
- Search for  $W'$ ,  $Z'$  decaying to heavy quark flavors  
[tb unsuppressed for  $V^R \sim V^{L(*)}$ ; 1801.07893, 1807.10473, 2104.04831, 2308.08521, 2310.19893, etc.]
- Decays to gauge bosons ( $W'$  to  $WZ$ ,  $W'$  to  $Wh$ ,  $Z'$  to  $WW$ ,  $Z'$  to  $Zh$ )  
[1906.08589, 2004.14636, 2102.08198, 2109.06055, etc.]

**SUMMARY PLOTS EXOTICS: CMS, ATLAS**

# Model-independent strategy

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- Available bounds on  $W'$ ,  $Z'$  masses may correspond to **different models** (e.g. Sequential SM), or **specific realizations of LRMs** (e.g. having heavy RH neutrinos)
- **HERE:**
  - Consider **general  $g_R$ ,  $g_X$**  (perturbative); move **beyond the (pseudo-)manifest** case  $V^R \sim V^{L(*)}$  in the quark sector
  - Focus on **fermionic decay modes**: more restrictive bounds
  - Impact of the scalar sector: **total width, neutrino sector**

[see e.g. Langacker, Uma Sankar '89; Frank, Ozdal, Poullose '18]

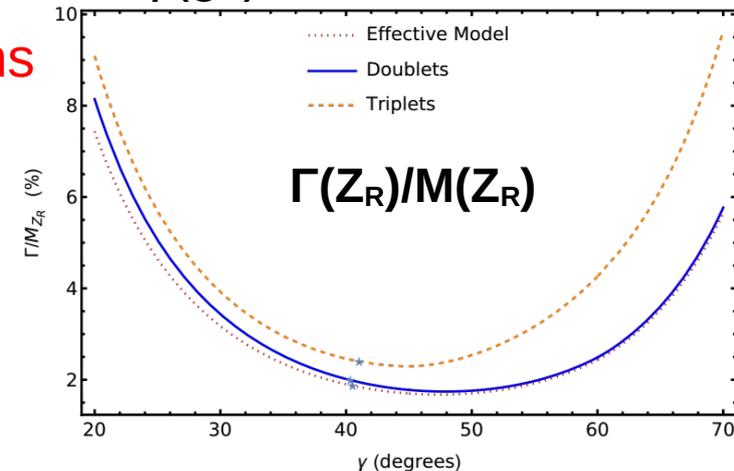
# Total widths of the $W_R, Z_R$



- Usually, only fermion sector considered when calculating  $\Gamma$
- Non-fermionic width depends on specific scenario (scalar potential)
- To be conservative, **maximize the total width** ( $g_{R,X}$  fixed) => **minimize BR**; consider limit where the full scalar sector is accessible in decays, and exploit equivalence theorem to simplify the expression of  $\Gamma$
- $\Gamma(W_R)/M(W_R)$  [below  $\sim 10\%$ ] prop to  $g_R^2$ , decreases with  $\gamma(g_R)$
- **Collider searches available for different total widths**

$$W_R^\pm \rightarrow f\bar{f}', W_L^\pm Z_L, W_L^\pm h^0, W_L^\pm H^0, H^\pm Z_L, H^\pm h^0, W_L^\mp H^{\pm\pm}, H_1 H_2$$

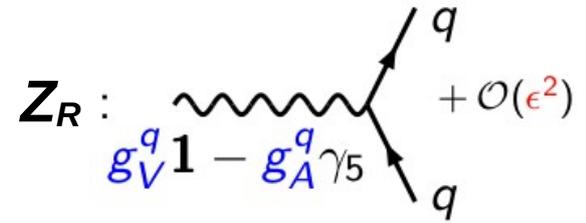
$$Z_R \rightarrow f\bar{f}, W_L^+ W_L^-, Z_L h^0, Z_L H^0, W_L^\pm H^\mp, h^0 H^0, W_L^\pm W_R^\mp, H_1 H_2, W_R^\pm H^\mp, W_R^+ W_R^-$$



# RH sector parameters: $\gamma(g_R)$



- “Effective couplings”  $c_q^f$ : also functions of  $\gamma(g_R)$  only
- $Z_R$  to  $\ell\ell$ :  $M(Z_R) > 4.2$  TeV (for the  $\gamma(g_R)$  that minimizes  $\sigma$ )  
 $> 4.8$  TeV for  $g_R = g_L$  &  $\Gamma(\text{fermions})$



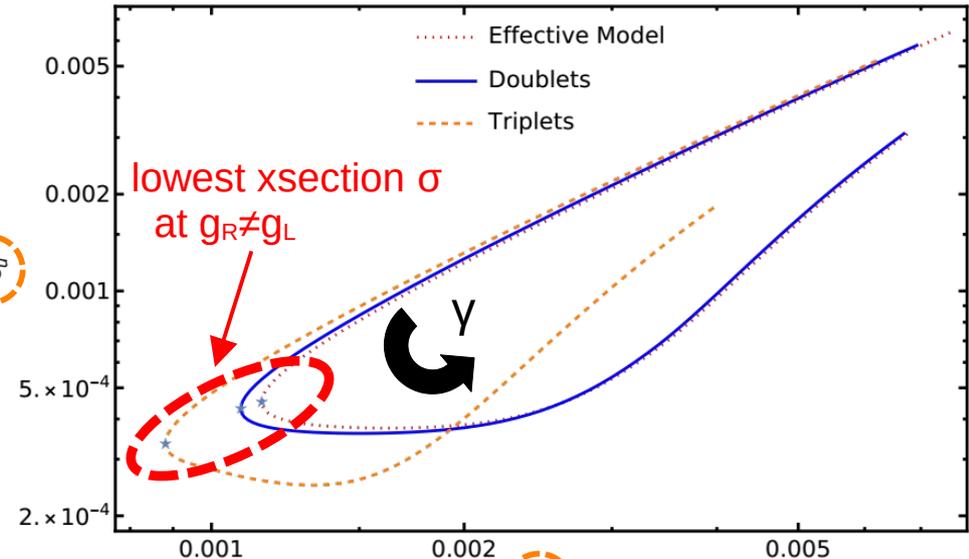
[e,  $\mu$ : ATLAS  $\sqrt{s}=13$  TeV, 139/fb, CMS  $\sqrt{s}=13$  TeV,  $\sim 140$ /fb;  
 tau: ATLAS  $\sqrt{s}=13$  TeV, 36.1/fb]

$$\sigma(pp \rightarrow Z_R X \rightarrow f\bar{f}X) \approx \frac{\pi}{6s} \sum_q c_q^f \omega_q(s, M_{Z_R}^2)$$

PDFs

$$c_q^f = \frac{1}{2} \underbrace{((g_V^q)^2 + (g_A^q)^2)}_{\text{production}} \underbrace{\text{Br}(Z_R \rightarrow f\bar{f})}_{\text{decay}}$$

$c_d$



$c_d$

# RH sector parameters: $V^R$



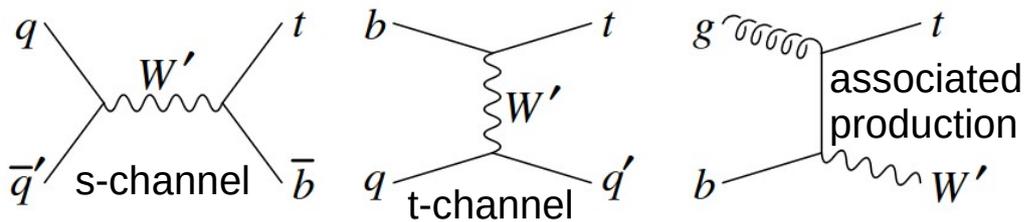
- Production of  $W_R$  affected by  $V^R$  texture: relax by  $\sim 1$  TeV bounds on  $M(W_R)$

$$\sigma(pp \rightarrow W_R^\pm X) \approx \frac{2\pi^2 g_R^2}{3s} \frac{1}{4\pi} \sum_{ij} |(V^R)_{ij}|^2 \underbrace{\omega_{ij}(M_{W_R}^2/s, M_{W_R})}_{\text{PDFs}}$$

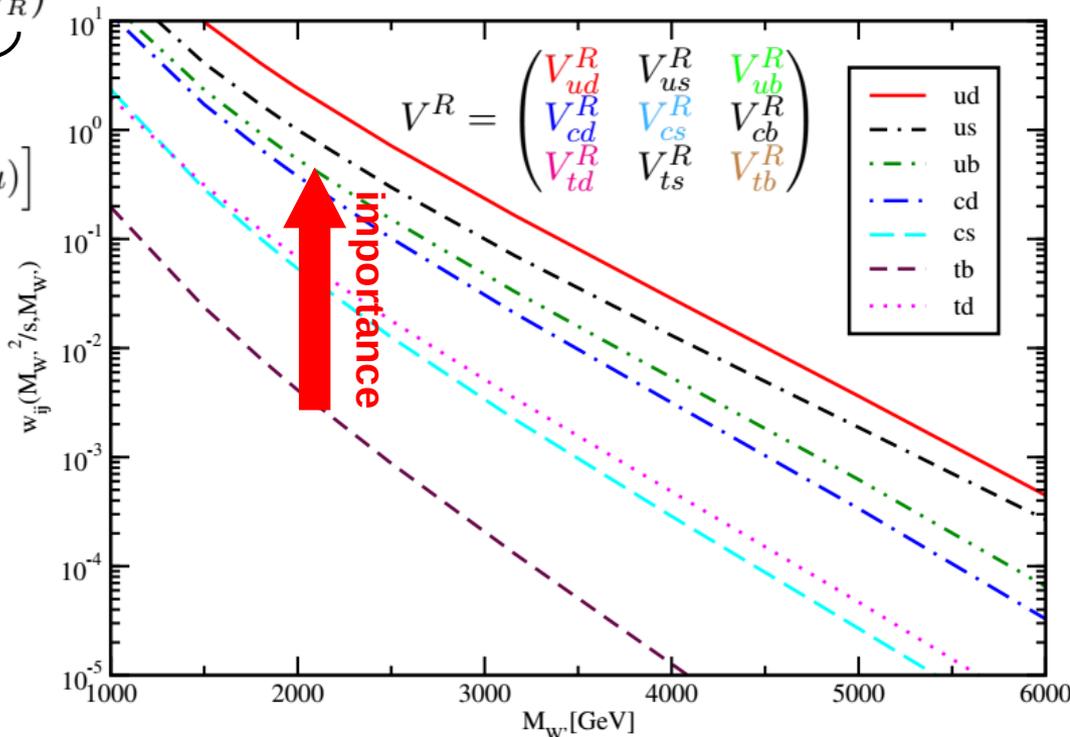
PDFs

$$\omega_{ij}(z, \mu) = \int_z^1 \frac{dx}{x} [u_i(x, \mu) \bar{d}_j(z/x, \mu) + \bar{u}_i(x, \mu) d_j(z/x, \mu)]$$

[NLO: Sullivan '02; t-channel & assoc. prod.: small]



[ $W_R$ : Frank, Hayreter, Turan '10;  
 $W'$ : Abdullah, Calle, Dutta, Florez, Restrepo '18, etc.]



[NNPDF '12; Bernard, Descotes-G., LVS '20]

# Neutrino sector



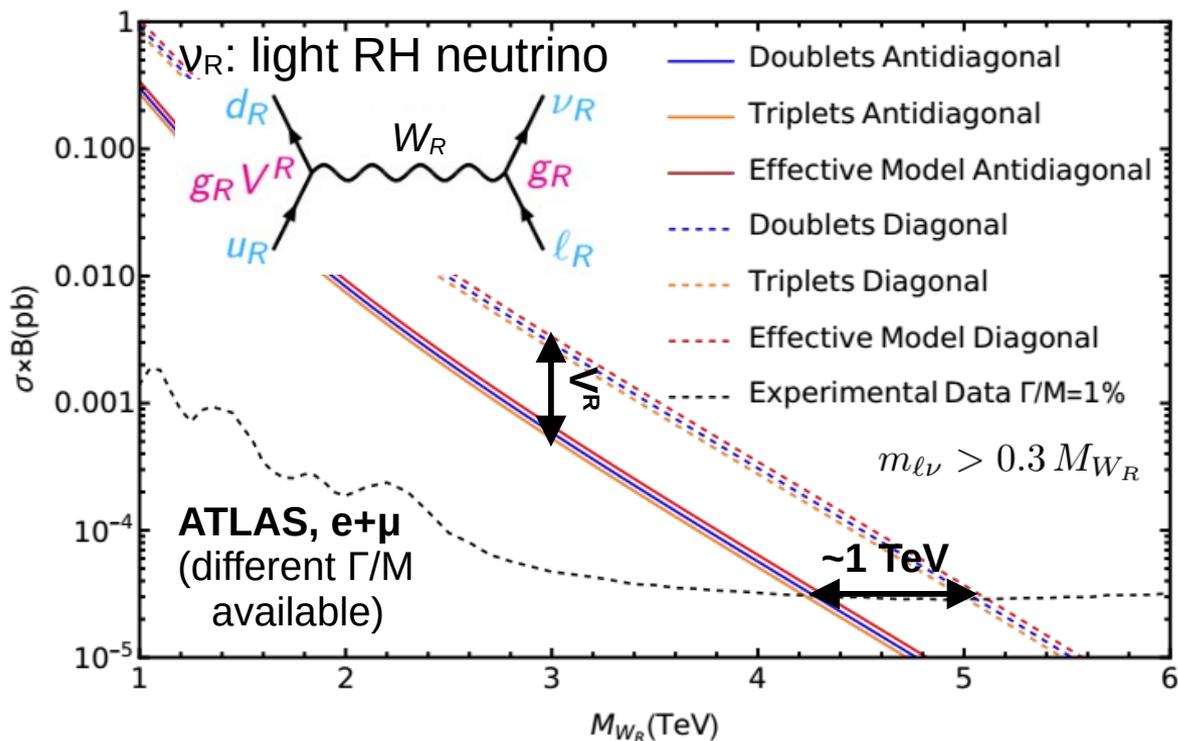
- Typical bounds for heavy RH neutrinos:  $M(W_R) > \mathcal{O}(5) \text{ TeV}$  [for  $M(N_R) < M(W_R)$ ]
- Searches of  $W_R$  based on massive RH neutrinos depend on **leptonic RH mixings**

• **Light RH neutrino case applies in the doublet LR scenario; it can also apply in the other cases**

- Sum over neutrino species:  

$$\sum_i |U^{R_{i\ell}}|^2 = 1$$

[e,  $\mu$ : ATLAS  $\sqrt{s}=13 \text{ TeV}$ , 139/fb,  
 CMS  $\sqrt{s}=13 \text{ TeV}$ , 138/fb]  
 [tau: ATLAS  $\sqrt{s}=13 \text{ TeV}$ , 139/fb,  
 CMS  $\sqrt{s}=13 \text{ TeV}$ , 36/fb]



# Summary of bounds



- More conservative bounds: minimize w.r.t.  $\gamma(g_R)$  inside perturbative region
- Bounds can be relaxed by about **~1 TeV**

significant impact of the  $\nu_R$  texture, decay mode, and neutrino sector

Channel		$\Phi + \chi_{L,R}$ (D)	$\Phi + \Delta_{L,R}$ (T)	$\chi_{L,R}$ (Eff)
$Z_R \rightarrow l_i \bar{l}_i$	$M_{Z_R}$	4.3	4.2	4.3
$W_R \rightarrow jj$ , anti-diag.	$M_{W_R}$	2.1	2.0	2.1
$W_R \rightarrow l_i \bar{\nu}_R$ , anti-diag.		4.3	4.2	4.3
$W_R \rightarrow jj$ , diag.	$M_{W_R}$	2.9	2.7	3.0
$W_R \rightarrow l_i \bar{\nu}_R$ , diag.		5.1	5.0	5.1

not significant impact of the scalar realization

masses in TeV, about 95%CL

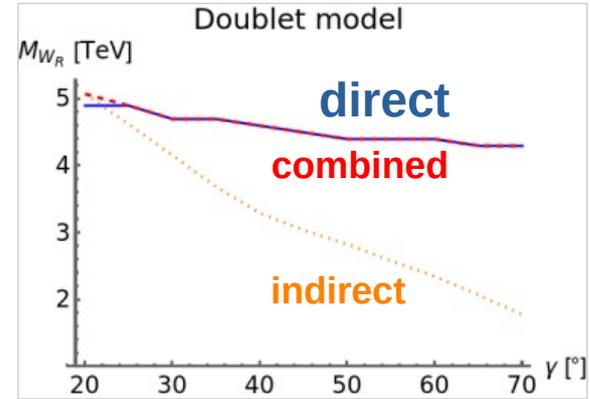
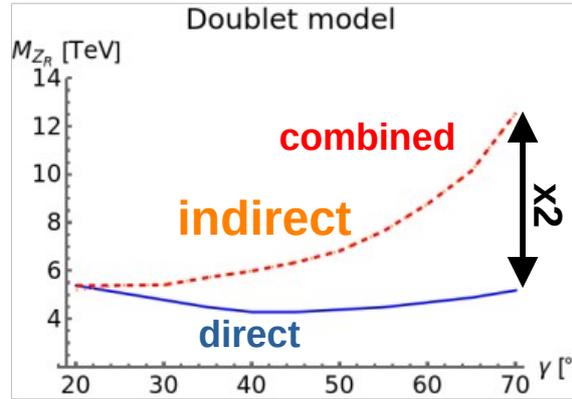
$\nu_R$ : light RH neutrino

# Synergy of $W_R$ , $Z_R$ searches

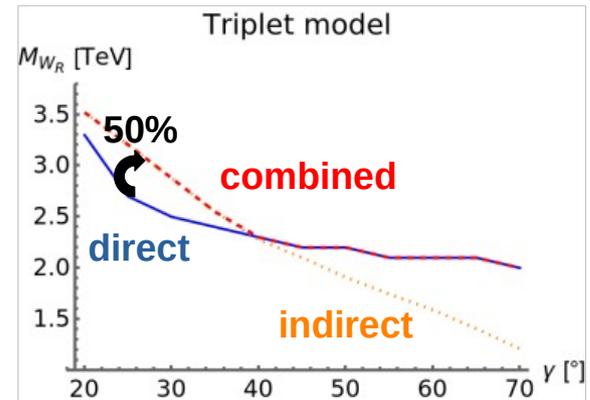
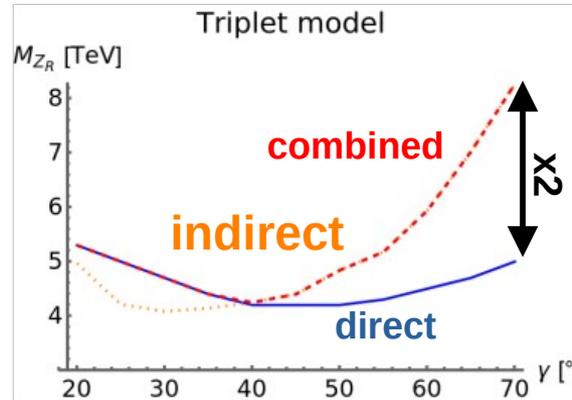
Doublet:  $Z_R$  to  $\ell\ell$ ,  $W_R$  to  $\ell\nu_R$

- In typical LRMs,  $M(W_R)$  and  $M(Z_R)$  are deeply connected
- From doublet to triplet,  $\sqrt{2}$  due to LR-SSM
- For higher  $\gamma$  [ $\cos(\gamma) \rightarrow 0$ ], bound on  $M(W_R)$  tends to **dominate**; for lower  $\gamma$ , the inverse happens

[see also Araz, Frank, Fuks, Moretti, Ozdal '21 (in the limit  $g_R=g_L$ , etc.)]



$$M_{W_R} = \cos \gamma M_{Z_R}$$



$$\sqrt{2} M_{W_R} = \cos \gamma M_{Z_R}$$

Triplet:  $Z_R$  to  $\ell\ell$ ,  $W_R$  to  $jj$  (avoid  $U^R(\nu_R)$ )

# Conclusions



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- LRMs: different realizations, e.g. a LR discrete symmetry can be pushed to higher energies
- Mostly impacted by  $g_R \neq g_L$ ,  $V^R \neq V^{L(*)}$  parametric LR asymmetries
- After relaxing bounds, still **sensitive to multi-TeV** gauge bosons
- **Complementarity of  $W_R$  and  $Z_R$  searches**
- Collider constraints provide a **powerful way to constrain LRMs**; very competitive w.r.t. flavor physics

**Thanks!**

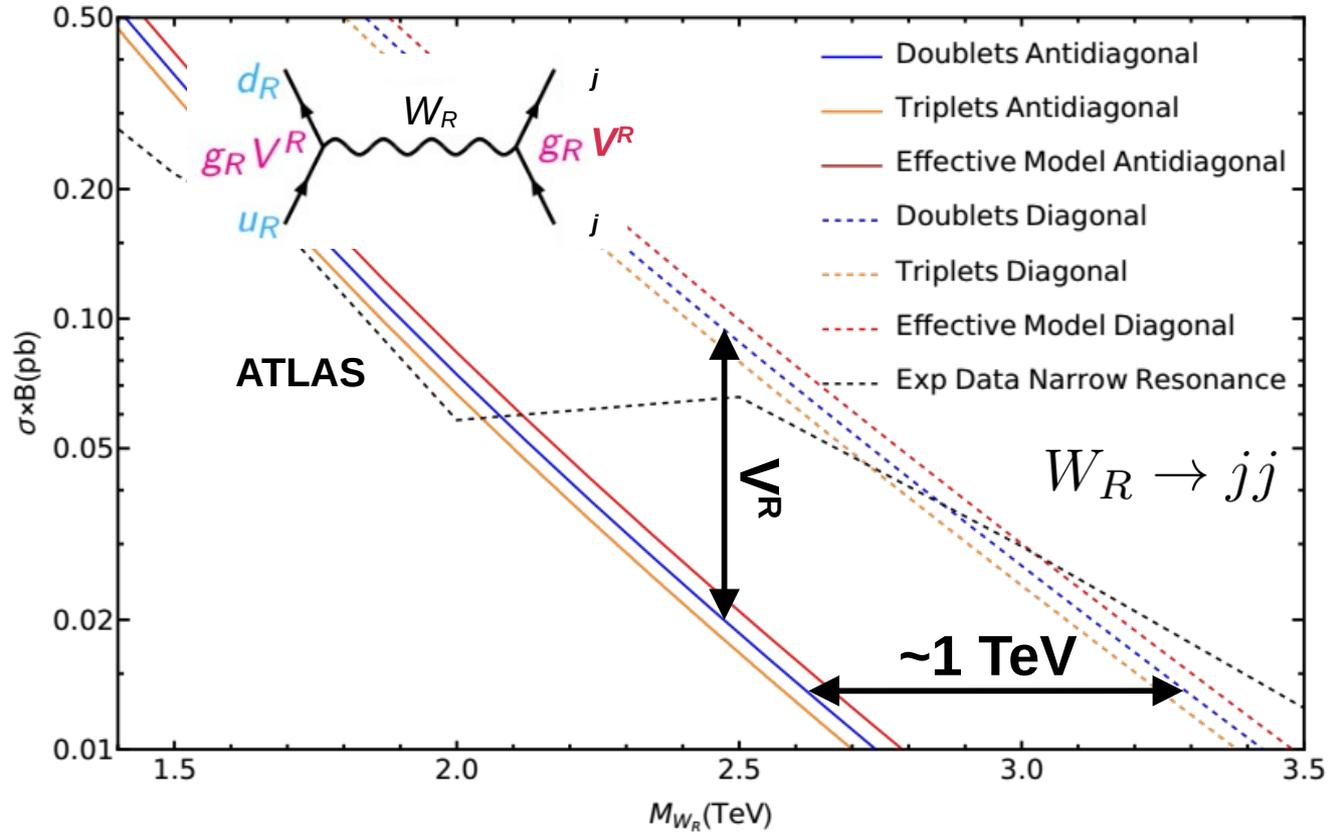
BACK UP

# Di-jet searches

- $W_R$  to  $jj$ : independent of the neutrino sector
- $jj$ : top-quark considered as a possible final state flavor (i.e. inclusive in the flavor)
- Similar bounds achieved with  $W_R$  to  $tb$

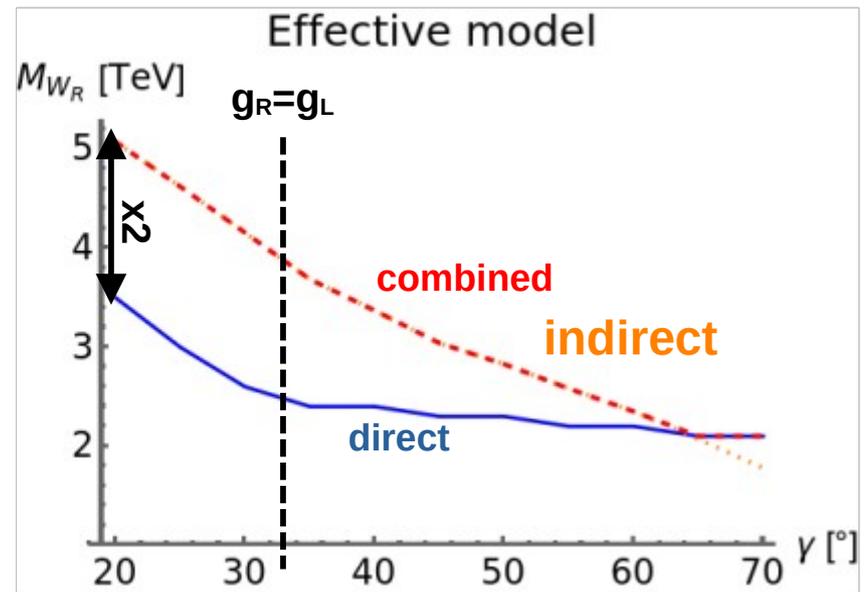
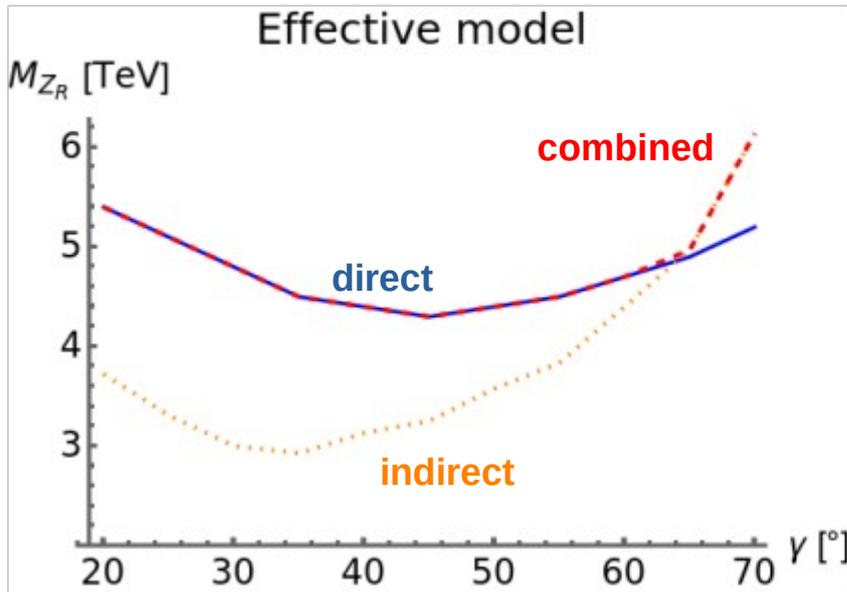
[ $W_R$ : ATLAS  $\sqrt{s}=13$  TeV, 139/fb; CMS  $\sqrt{s}=13$  TeV, 139/fb, with different  $\Gamma/M$  available, also  $Z_R$ ;  $Z_R$ : ATLAS  $\sqrt{s}=13$  TeV, 139/fb; CMS  $\sqrt{s}=13$  TeV, 35.9/fb]

[ $tb$ , had: ATLAS  $\sqrt{s}=13$  TeV, 139/fb; CMS  $\sqrt{s}=13$  TeV, 137/fb][ $tb$ , lept: ATLAS  $\sqrt{s}=13$  TeV, 139/fb; CMS  $\sqrt{s}=13$  TeV, 138/fb, with different  $\Gamma/M$  available]



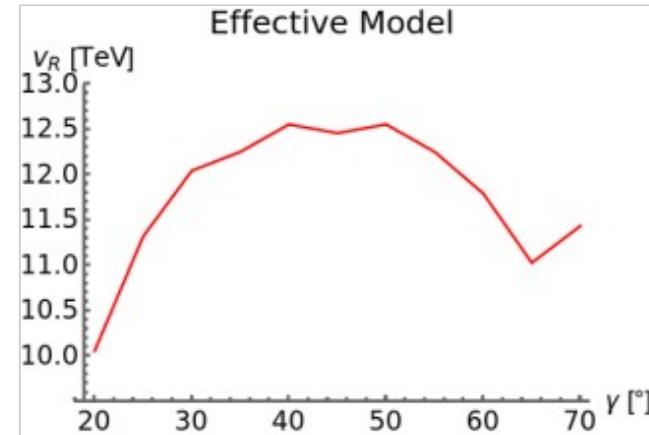
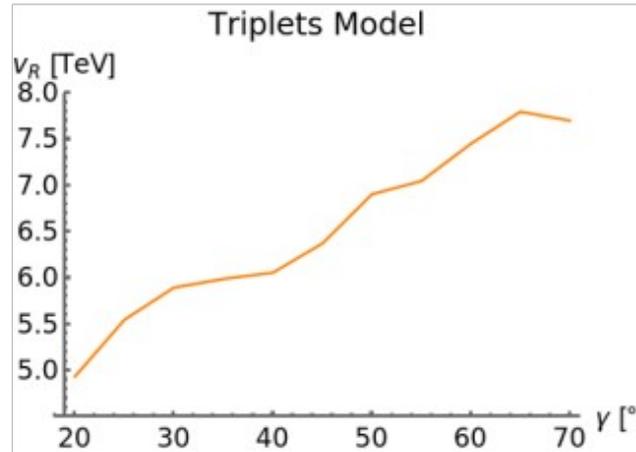
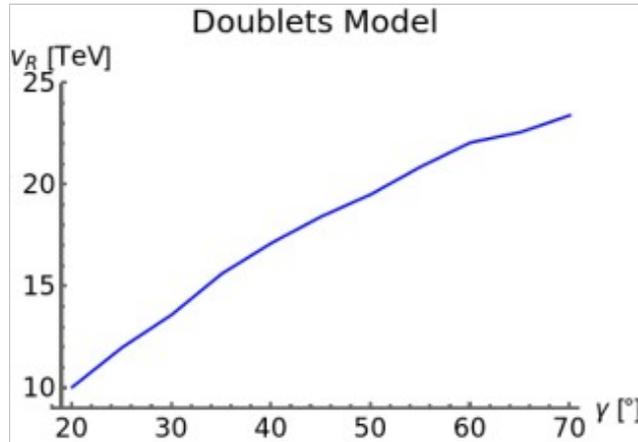
# Synergy of $W_R$ , $Z_R$ searches

Effective:  $Z_R$  to  $\ell\ell$ ,  $W_R$  to  $jj$  (avoid  $U^R(v_R)$ )



$$M_{W_R} = \cos \gamma M_{Z_R}$$

# LR-SSB scale



$$|v_R|_D \gtrsim 10 \text{ TeV}, \quad |v_R|_T \gtrsim 4.9 \text{ TeV}, \quad |v_R|_{\text{Eff}} \gtrsim 10 \text{ TeV}.$$

# The $\chi_L + \chi_R$ Effective LR Model

- The scalar sector is very simple. We only have two physical degrees of freedom:

$$\chi_{L,R} := \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_{L,R} + \chi_{L,R}^{0r} \end{pmatrix} \quad \begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \chi_R^{0r} \\ \chi_L^{0r} \end{pmatrix}$$

(Unitary Gauge)  $v_R \gg v_L = v_{EW}$

- Only 5 free parameters in the scalar potential

$$V = -\mu_L^2 \chi_L^\dagger \chi_L - \mu_R^2 \chi_R^\dagger \chi_R + \lambda_L (\chi_L^\dagger \chi_L)^2 + \lambda_R (\chi_R^\dagger \chi_R)^2 + \lambda_{LR} (\chi_L^\dagger \chi_L) (\chi_R^\dagger \chi_R).$$

$$M_H^2 \approx 2\lambda_R v_R^2, \quad M_h^2 \approx \frac{4\lambda_L \lambda_R - \lambda_{LR}^2}{2\lambda_R} v_L^2$$

$$\tan \theta = \frac{\lambda_{LR} v_L v_R}{\lambda_L v_L^2 - \lambda_R v_R^2}$$

# The $\chi_L + \chi_R$ Effective LR Model

- No  $W_L - W_R$  Mixing

$$M_{W_L} = \frac{1}{2} g_L v_L, \quad M_{W_R} = \frac{1}{2} g_R v_R, \quad M_{Z_L} \approx \frac{M_{W_L}}{\cos \theta_W}, \quad M_{Z_R} \approx \frac{M_{W_R}}{\cos \gamma}$$

- We need Effective Operators to produce Fermion Masses

$$\mathcal{L}_Y = -\frac{1}{\Lambda} \left\{ C_d^{ij} \bar{q}_L^i \chi_L \chi_R^\dagger q_R^j + C_u^{ij} \bar{q}_L^i \tilde{\chi}_L \tilde{\chi}_R^\dagger q_R^j + C_e^{ij} \bar{l}_L^i \chi_L \chi_R^\dagger l_R^j + C_{\nu D}^{ij} \bar{l}_L^i \tilde{\chi}_L \tilde{\chi}_R^\dagger l_R^j \right\} \quad (\text{Dirac Masses})$$

$$+ C_{\nu L, M}^{ij} \bar{l}_L^i \tilde{\chi}_L \tilde{\chi}_L^{\dagger T} l_L^j + C_{\nu R, M}^{ij} \bar{l}_R^i \tilde{\chi}_R^* \tilde{\chi}_R^\dagger l_R^j \left. \right\}$$

(Majorana Masses)

$$\tilde{\chi}_{L,R} := i\sigma^2 \chi_{L,R}^*$$

# The $\chi_L + \chi_R$ Effective LR Model

- Dirac Masses for Quarks and Charged Leptons and Majorana Masses for Neutrinos

$$m_{q,l^\pm} \propto \frac{v_L v_R}{\Lambda}, \quad m_{\nu_h} \propto \frac{v_R^2}{\Lambda}, \quad m_{\nu_l} \propto \frac{v_L^2}{\Lambda}$$

- No FCNCs in the Hadronic Sector

$$\mathcal{L}_{u,d,e}^Y = - \left(1 + \frac{\chi_L^{0r}}{v_L}\right) \left(1 + \frac{\chi_R^{0r}}{v_R}\right) \sum_{f=u,d,e} \bar{f} \mathcal{M}_f f.$$

# Sources of **CP** violation in the quark sector

→  $\Phi$  is included for a mass generation mechanism for fermions:

$$\bar{Q}_L Y \Phi Q_R + \bar{Q}_L \tilde{Y} \tilde{\Phi} Q_R + h.c.$$

$$\Rightarrow M_u = \kappa_1 Y + \kappa_2 \tilde{Y} \text{ and } M_d = \kappa_1 \tilde{Y} + \kappa_2 Y$$

→ Bi-diagonalization: mixing matrices  $V^L$  and  $V^R$

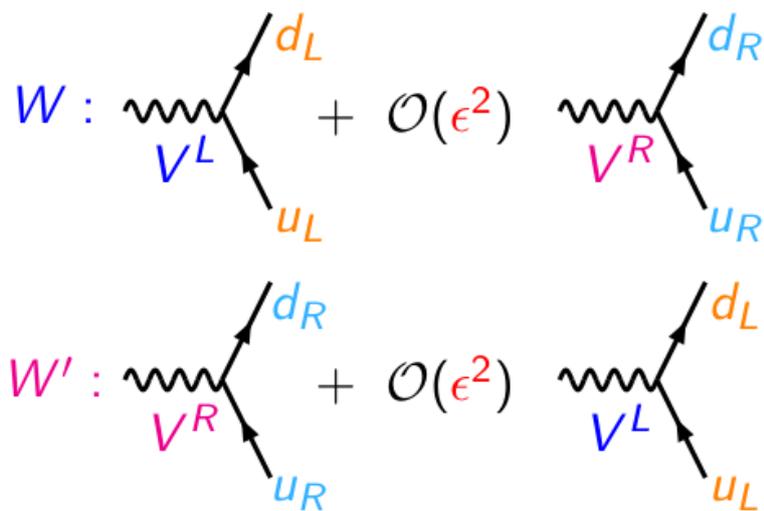
→ Mixing of gauge bosons:

$V^{CKM}$  interpreted as  $V^L + \mathcal{O}(\epsilon^2) V^R$

→  $V^R$ : new currents violating **CP**

→ **P** relates  $V^R$  to  $V^L$  [Senjanovic, Tello '15]

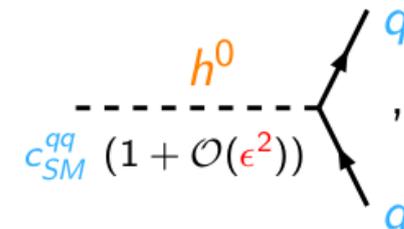
→ If the unification of *couplings* happens above  $\kappa_R$ , the structure of  $V^R$  is left unknown



# Light SM-like Higgs

$$h^0 = \frac{1}{\sqrt{\kappa_1^2 + \kappa_2^2 + \kappa_L^2}} \left( \underbrace{\kappa_1 \varphi_1^{0r} + \kappa_2 \varphi_2^{0r}}_{bi\text{-doublet}} + \underbrace{\kappa_L \chi_L^{0r}}_{SU(2)_L\text{-doublet}} \right) + \underbrace{\mathcal{O}(\epsilon) \chi_R^{0r}}_{SU(2)_R\text{-doublet}}$$

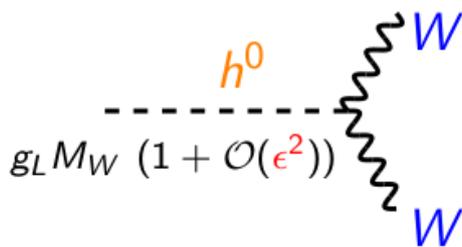
Couplings to fermions similar to the SM:

$$- \left( \frac{m_d^i}{\sqrt{\kappa_1^2 + \kappa_2^2 + \kappa_L^2}} \bar{d}_L^i d_R^i + \frac{m_u^j}{\sqrt{\kappa_1^2 + \kappa_2^2 + \kappa_L^2}} \bar{u}_L^j u_R^j \right) h^0 + \mathcal{O}(\epsilon^2) + h.c.$$


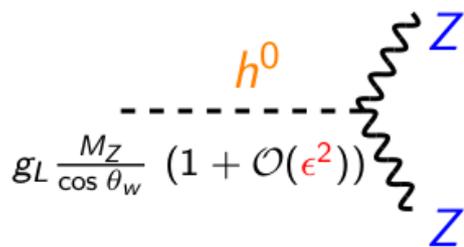
The diagram shows a dashed line representing the Higgs boson  $h^0$  on the left, which splits into two fermion lines labeled  $q$  on the right. The vertex is labeled with the SM coupling  $c_{SM}^{qq} (1 + \mathcal{O}(\epsilon^2))$ .

and in particular  $h^0$  is flavour diagonal up to  $\mathcal{O}(\epsilon^2)$

Couplings to gauge bosons corrected at  $\mathcal{O}(\epsilon^2)$



The diagram shows a dashed line representing the Higgs boson  $h^0$  on the left, which splits into two  $W$  boson lines on the right. The vertex is labeled with the coupling  $g_L M_W (1 + \mathcal{O}(\epsilon^2))$ .



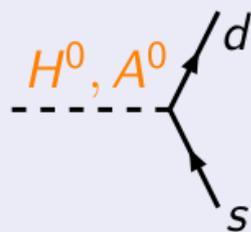
The diagram shows a dashed line representing the Higgs boson  $h^0$  on the left, which splits into two  $Z$  boson lines on the right. The vertex is labeled with the coupling  $g_L \frac{M_Z}{\cos \theta_w} (1 + \mathcal{O}(\epsilon^2))$ .

# Physical scalars and FCNC

- Scalar content:
- $1 \times$  light SM-like Higgs  $h^0$
  - $3 \times$   $\mathcal{CP}$ -even  $H_{1,2,3}^0$ ,  $2 \times$   $\mathcal{CP}$ -odd  $A_{1,2}^0$ , of mass  $\sim \kappa_R$
  - $2 \times$  singly charged  $H_{1,2}^\pm$ , of mass  $\sim \kappa_R$
  - $2 \times$  doubly charged  $H_{1,2}^{\pm\pm}$ , of mass  $\sim \kappa_R$  (triplet only)

## Flavour Changing Neutral Currents (FCNC) at tree level

$\bar{Q}_L Y \Phi Q_R + \bar{Q}_L \tilde{Y} \tilde{\Phi} Q_R + h.c. \Rightarrow M_u = \kappa_1 Y + \kappa_2 \tilde{Y}$  and  $M_d = \kappa_1 \tilde{Y} + \kappa_2 Y$ , since couplings are not diagonalized simultaneously: FCN couplings



Higgses	d.o.f.	FCNC for $sd$
$H_1^0, A_1^0$	$f_1 \Phi + g_1 \chi_L$	$f_C \sum_a m_u^a V_L^{as*} V_R^{ad}$
$H_2^0, A_2^0$	$f_2 \chi_L + g_2 \Phi$	$g_C \sum_a m_u^a V_L^{as*} V_R^{ad}$
$f_{1,2,C} \rightarrow 1, g_{1,2,C} \rightarrow 0, \text{ when } \kappa_L \rightarrow 0$		

FCNC: **triplets**  $\{H_1^0, A_1^0\}$ , and **doublets**  $\{H_1^0, A_1^0, H_2^0, A_2^0\}$

# Higgs potential, differences triplet and doublet

**P symmetric case** (parameters in total: 15 + 1 complex phase):

$$\begin{aligned} V = & -\mu_1^2 \text{tr}(\Phi^\dagger \Phi) - \mu_2^2 \text{tr}(\tilde{\Phi}^\dagger \Phi + \tilde{\Phi} \Phi^\dagger) - \mu_3^2 (\chi_L^\dagger \chi_L + \chi_R^\dagger \chi_R) \\ & + \mu'_1 (\chi_L^\dagger \Phi \chi_R + \chi_R^\dagger \Phi^\dagger \chi_L) + \mu'_2 (\chi_L^\dagger \tilde{\Phi} \chi_R + \chi_R^\dagger \tilde{\Phi}^\dagger \chi_L) + \lambda_1 [\text{tr}(\Phi^\dagger \Phi)]^2 \\ & + \lambda_2 \left( [\text{tr}(\tilde{\Phi}^\dagger \Phi)]^2 + [\text{tr}(\tilde{\Phi} \Phi^\dagger)]^2 \right) + \lambda_3 \text{tr}(\tilde{\Phi}^\dagger \Phi) \text{tr}(\tilde{\Phi} \Phi^\dagger) \\ & + \lambda_4 \text{tr}(\Phi^\dagger \Phi) \text{tr}(\tilde{\Phi}^\dagger \Phi + \tilde{\Phi} \Phi^\dagger) + \rho_1 [(\chi_L^\dagger \chi_L)^2 + (\chi_R^\dagger \chi_R)^2] \\ & + \rho_3 (\chi_L^\dagger \chi_L)(\chi_R^\dagger \chi_R) + \alpha_1 (\chi_L^\dagger \chi_L + \chi_R^\dagger \chi_R) \text{tr}(\Phi^\dagger \Phi) \\ & + \frac{\alpha_2}{2} \left\{ e^{i\delta_2} [\chi_L^\dagger \chi_L \text{tr}(\tilde{\Phi} \Phi^\dagger) + \chi_R^\dagger \chi_R \text{tr}(\tilde{\Phi}^\dagger \Phi)] \right. \\ & \quad \left. + e^{-i\delta_2} [\chi_L^\dagger \chi_L \text{tr}(\tilde{\Phi}^\dagger \Phi) + \chi_R^\dagger \chi_R \text{tr}(\tilde{\Phi} \Phi^\dagger)] \right\} \\ & + \alpha_3 (\chi_L^\dagger \Phi \Phi^\dagger \chi_L + \chi_R^\dagger \Phi^\dagger \Phi \chi_R) + \alpha_4 (\chi_L^\dagger \tilde{\Phi} \tilde{\Phi}^\dagger \chi_L + \chi_R^\dagger \tilde{\Phi}^\dagger \tilde{\Phi} \chi_R), \end{aligned} \tag{1}$$

**Triplet case:** no  $\mu'_1, \mu'_2$ , but other terms are present (parameters in total: 17 + 1 complex phase)

**C symmetric case:** further phases are present beyond  $\delta_2$