JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Electroweak Precision Running of $\hat{\alpha}$ and $\sin^2\hat{\theta}$

Moriond 2024

Rodolfo Ferro Hernandez

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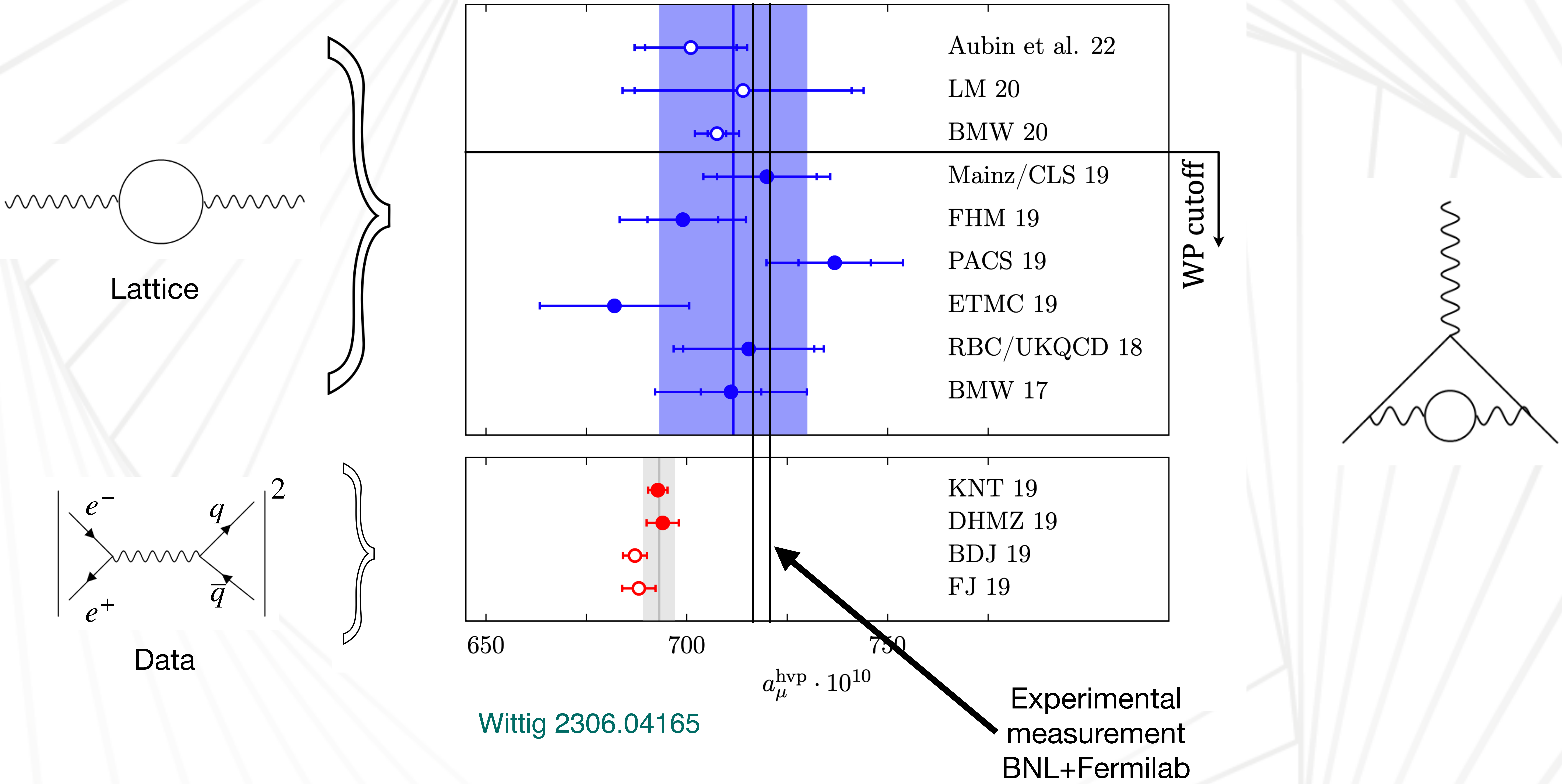
g-2 tension



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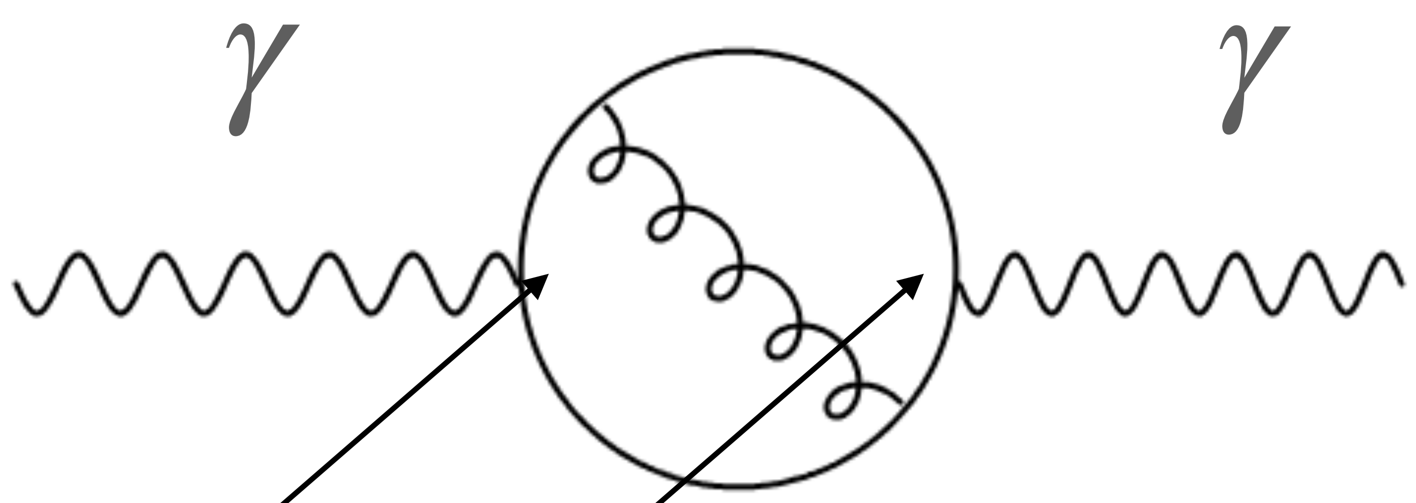
Quick basics

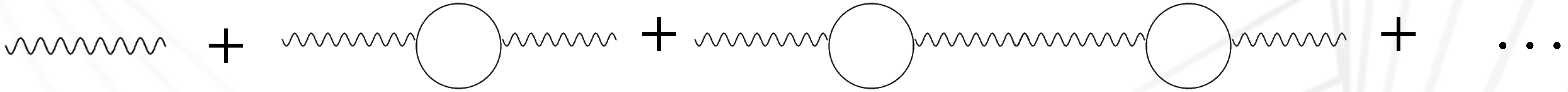


The vacuum polarization function is defined as

$$\left(-q^2\eta^{\mu\nu} + q^\mu q^\nu\right)\hat{\Pi}(q^2,\mu^2) = i\int d^4x e^{iqx}\langle 0\,|\,TJ^\mu_{em}(x)J^\nu_{em}(0)\,|\,0\rangle$$

The running coupling $\hat{\alpha}(\mu^2)$ is constructed to absorb the large logarithms that appear in this expression. It is given by:

$$\hat{\alpha}(\mu^2) = \frac{\alpha}{1-\Delta\hat{\alpha}(\mu^2)} \qquad \Delta\hat{\alpha}(\mu^2) \equiv 4\pi\alpha\hat{\Pi}(0,\mu^2)$$



$\hat{\alpha}$: a key parameter of the SM

Rel error $\sim 10^{-10}$

$$\hat{\alpha}(0)$$

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$e^+e^- \to had$ **o lattice + pQCD**

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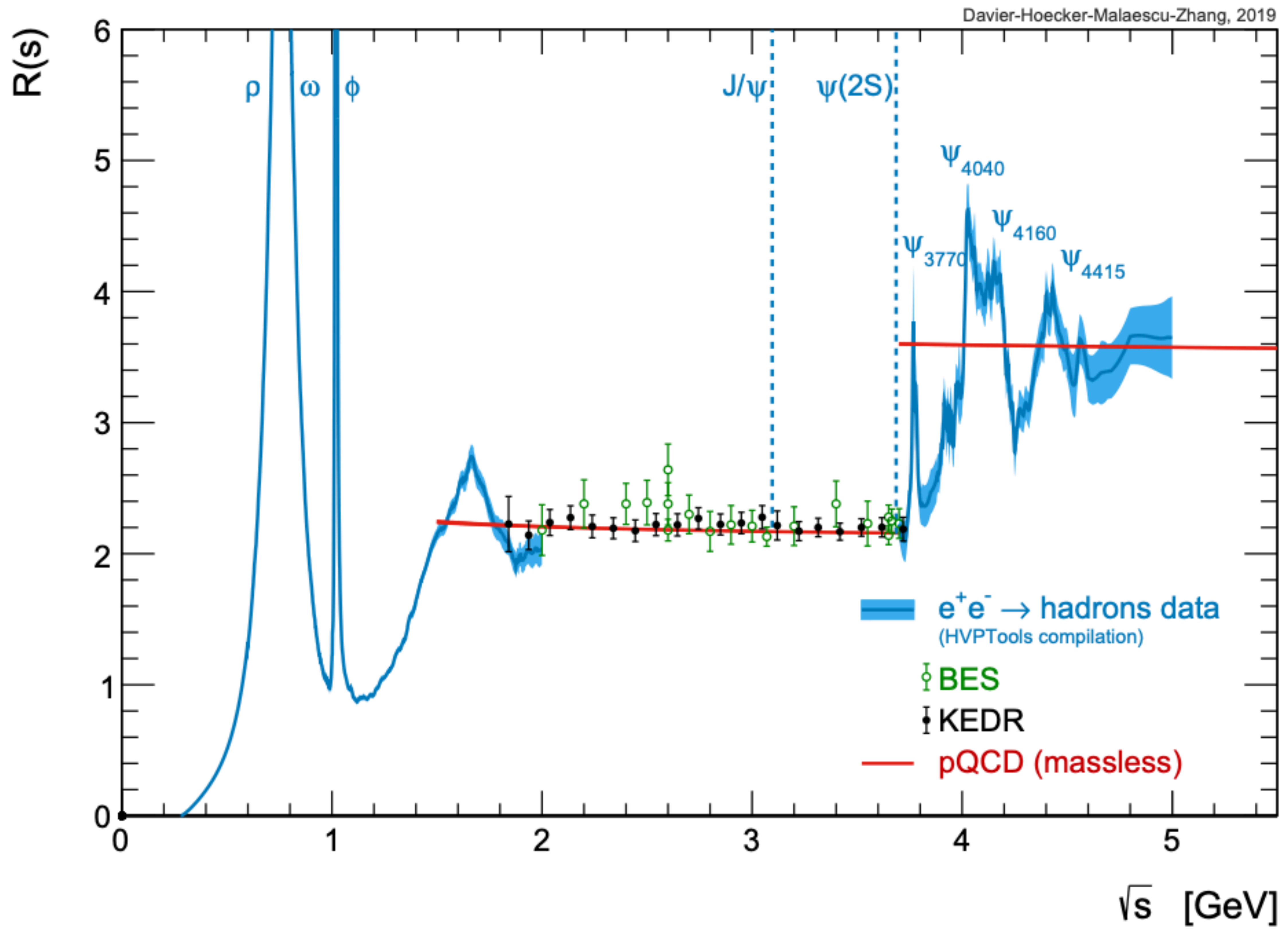

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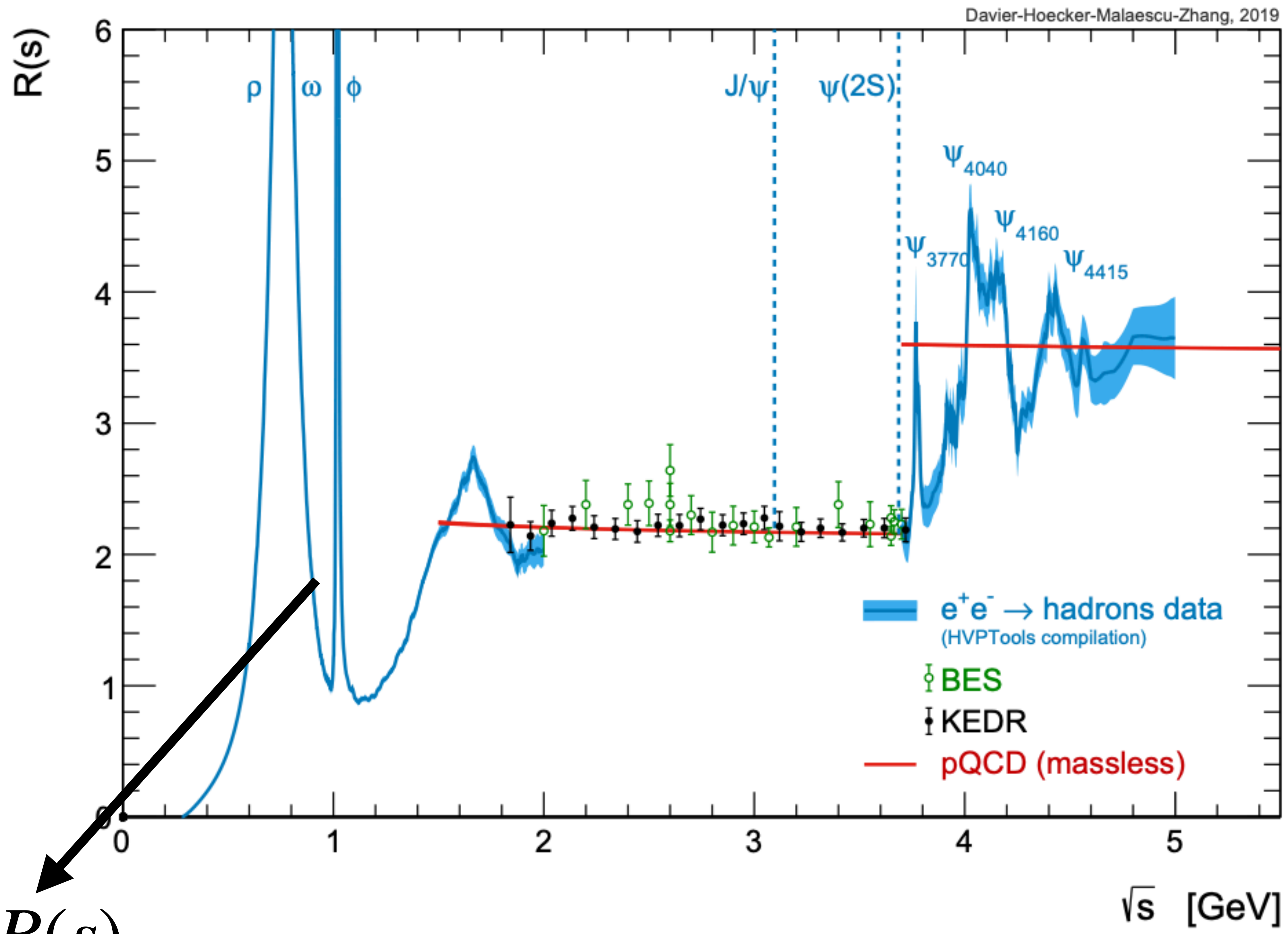

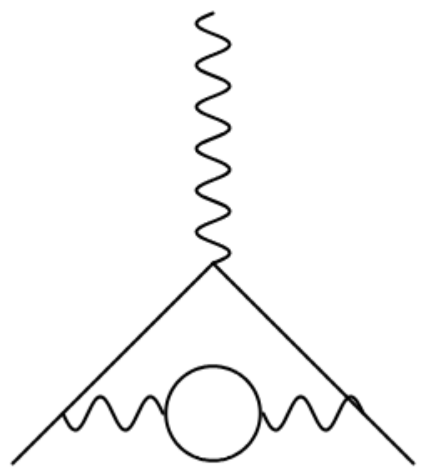

$$a_\mu^{\text{had}} = \frac{\alpha^2}{3\pi^2}\int \frac{ds}{s} K(s) R(s)$$

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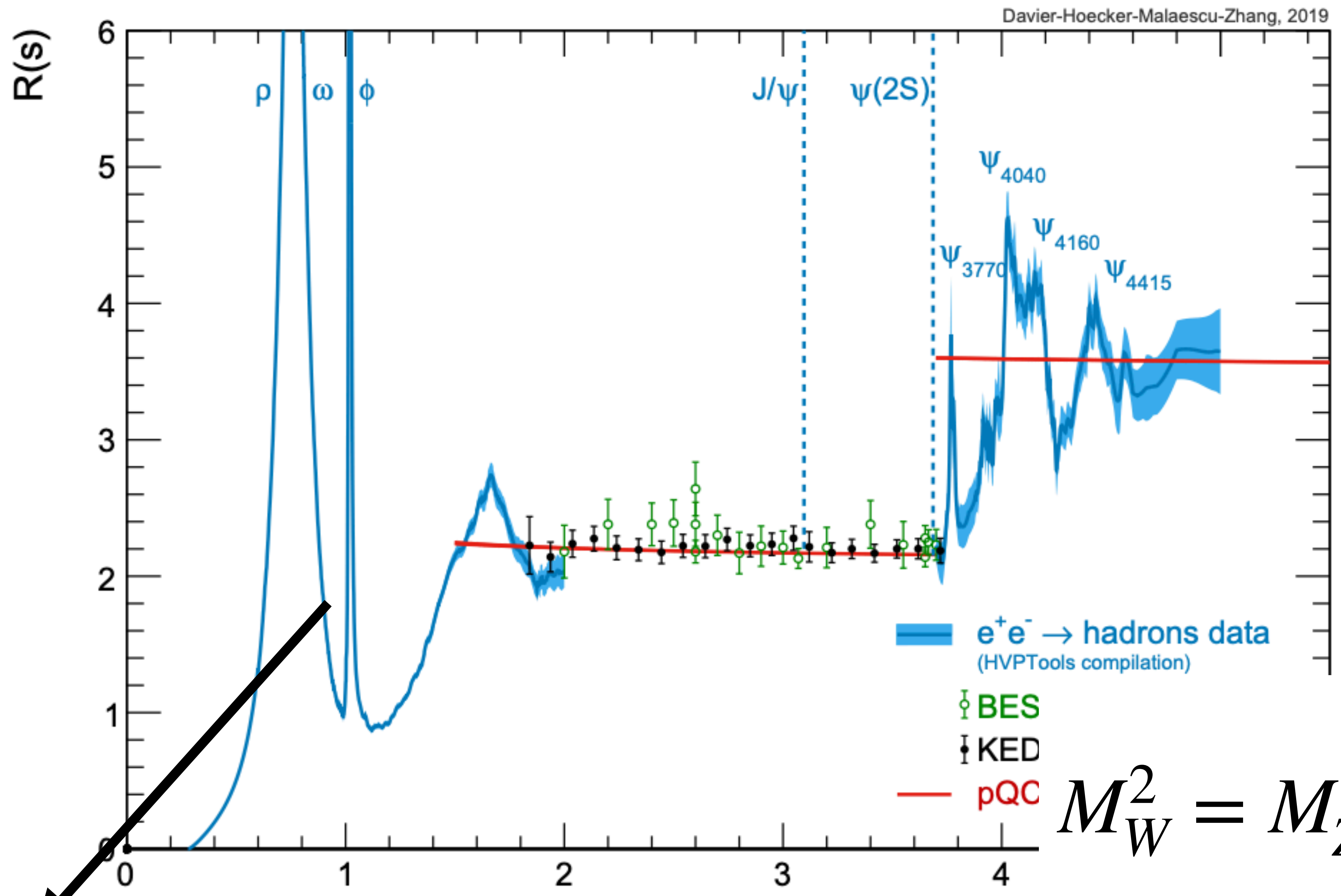
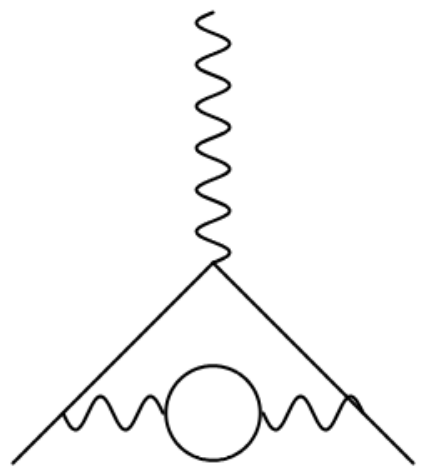
enters in EW relations



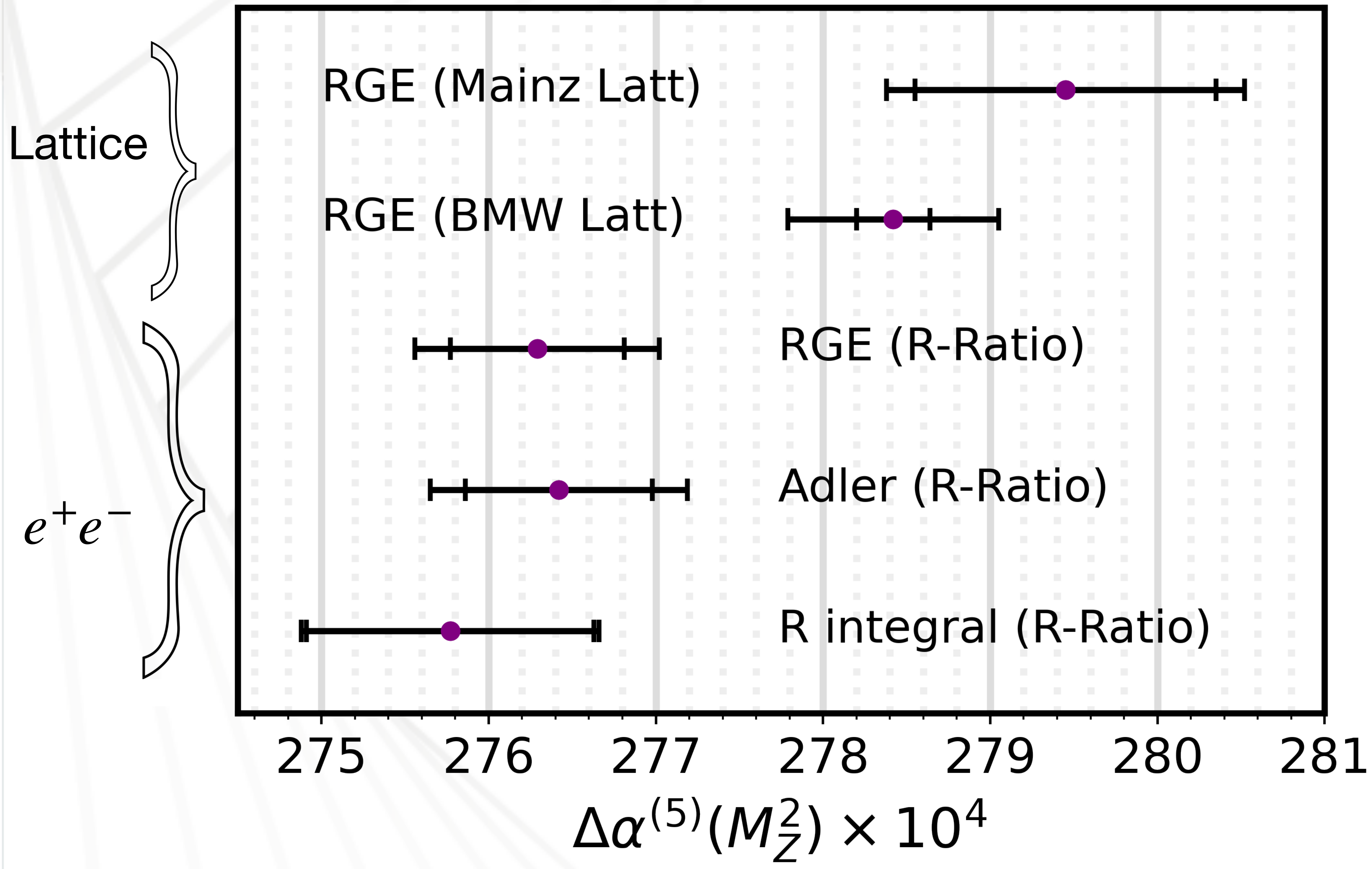


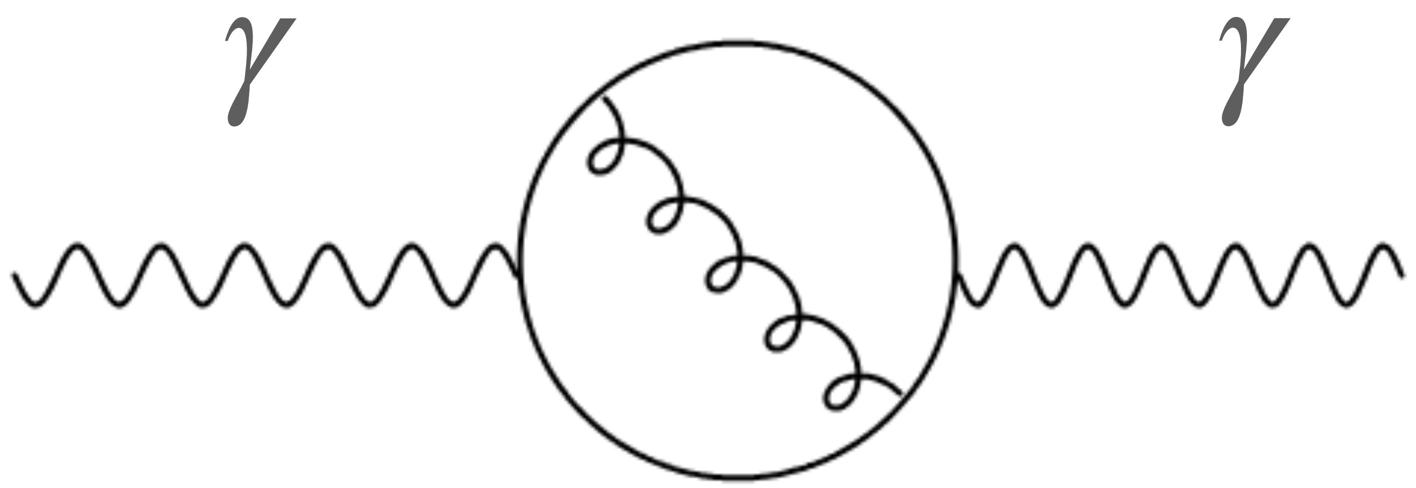
$$a_\mu^{\text{had}} = \frac{\alpha^2}{3\pi^2}\int \frac{ds}{s} K(s) R(s)$$

$$M_W^2 = M_Z^2 \hat{\rho} \left(1 - \frac{\pi \hat{\alpha}(M_Z^2)}{\sqrt{2} G_F M_W^2} \left[1 + \Delta \hat{r}_W \right] \right)$$

Running $\hat{\alpha}$ comparison

Erler, Ferro-Hernandez, 10.1007/JHEP12(2023)131





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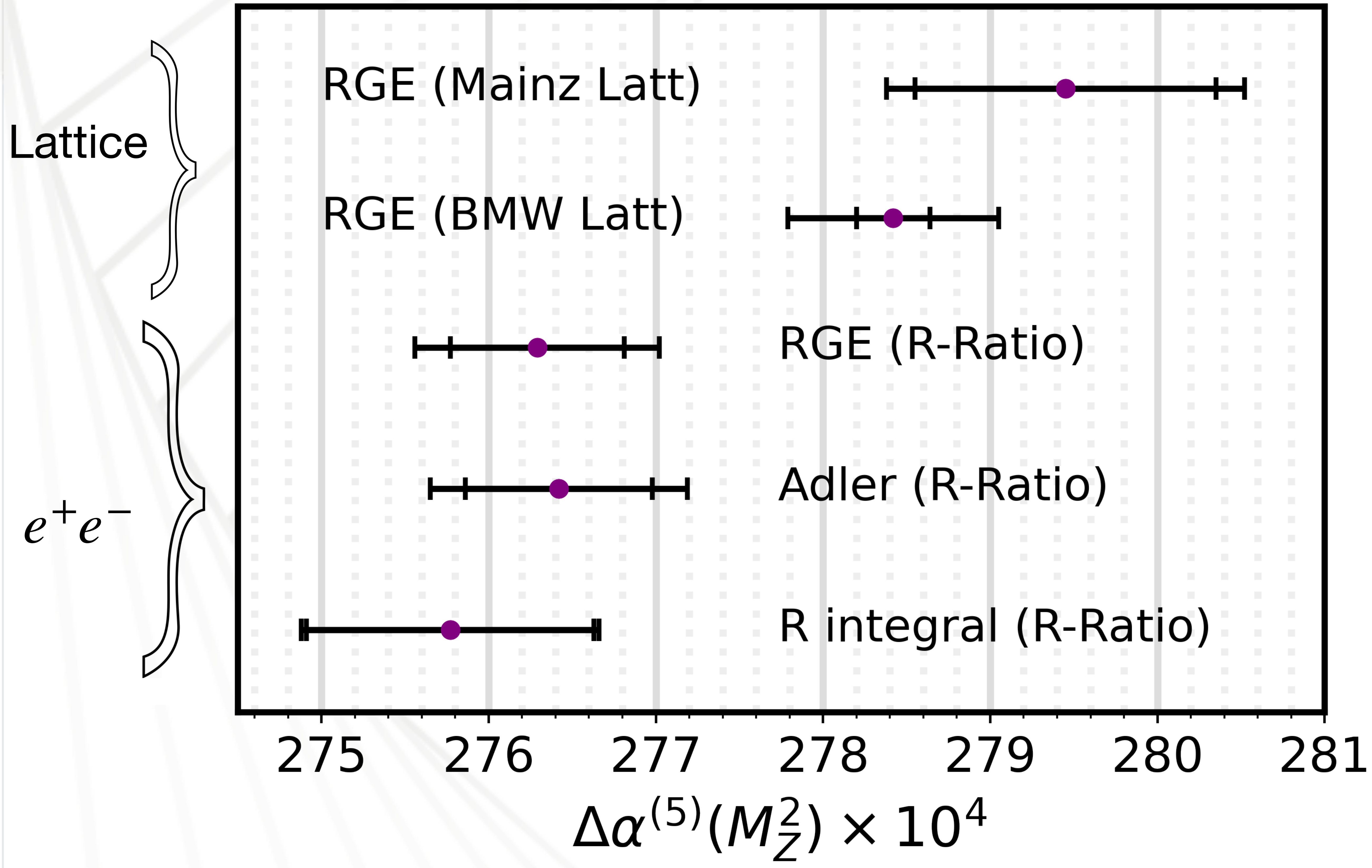
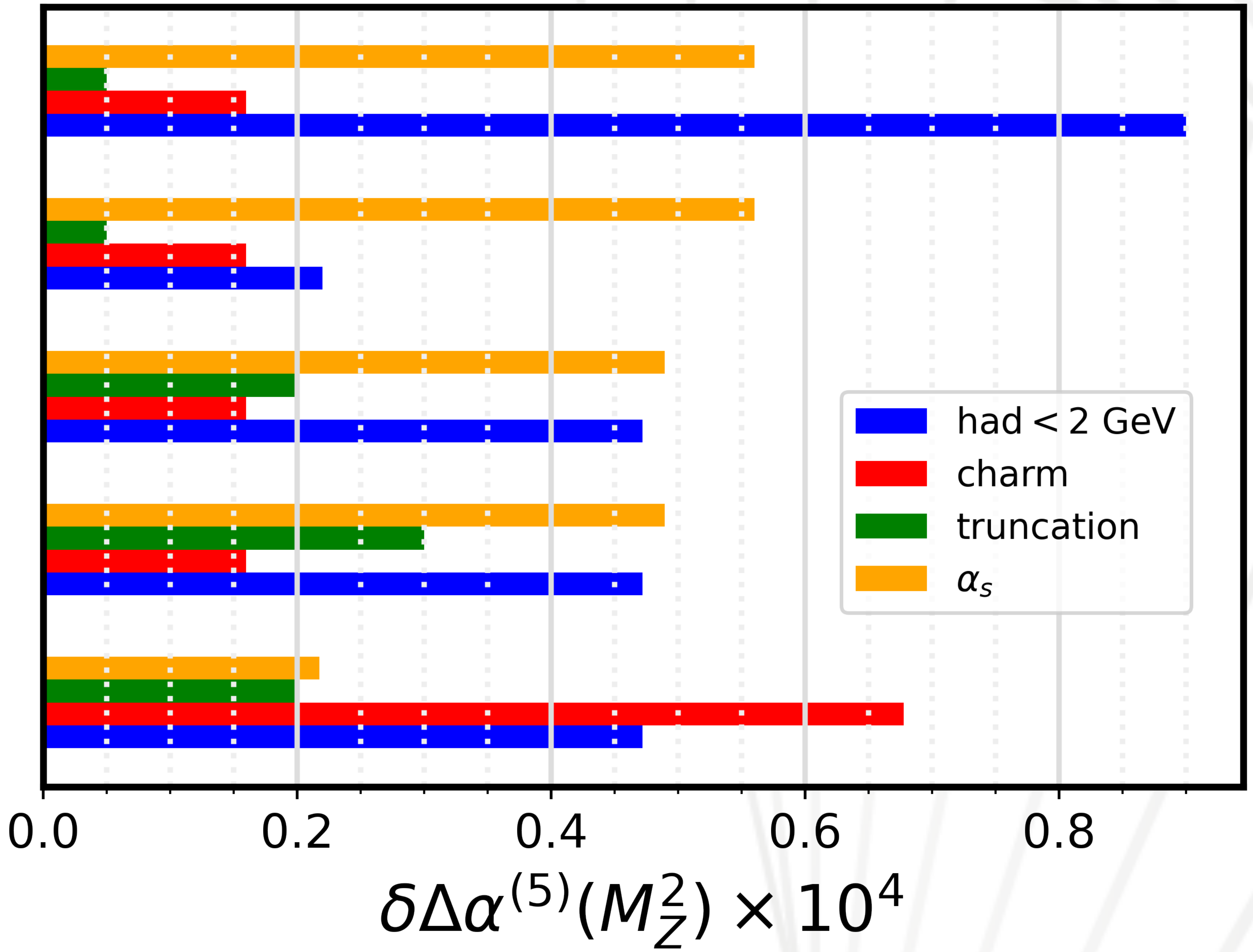
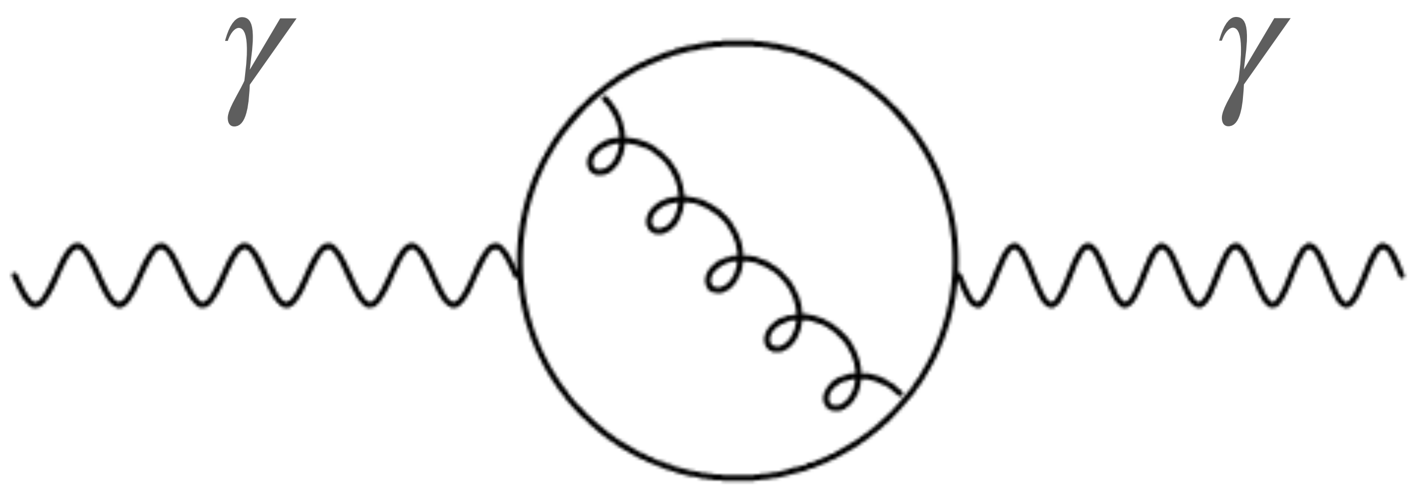


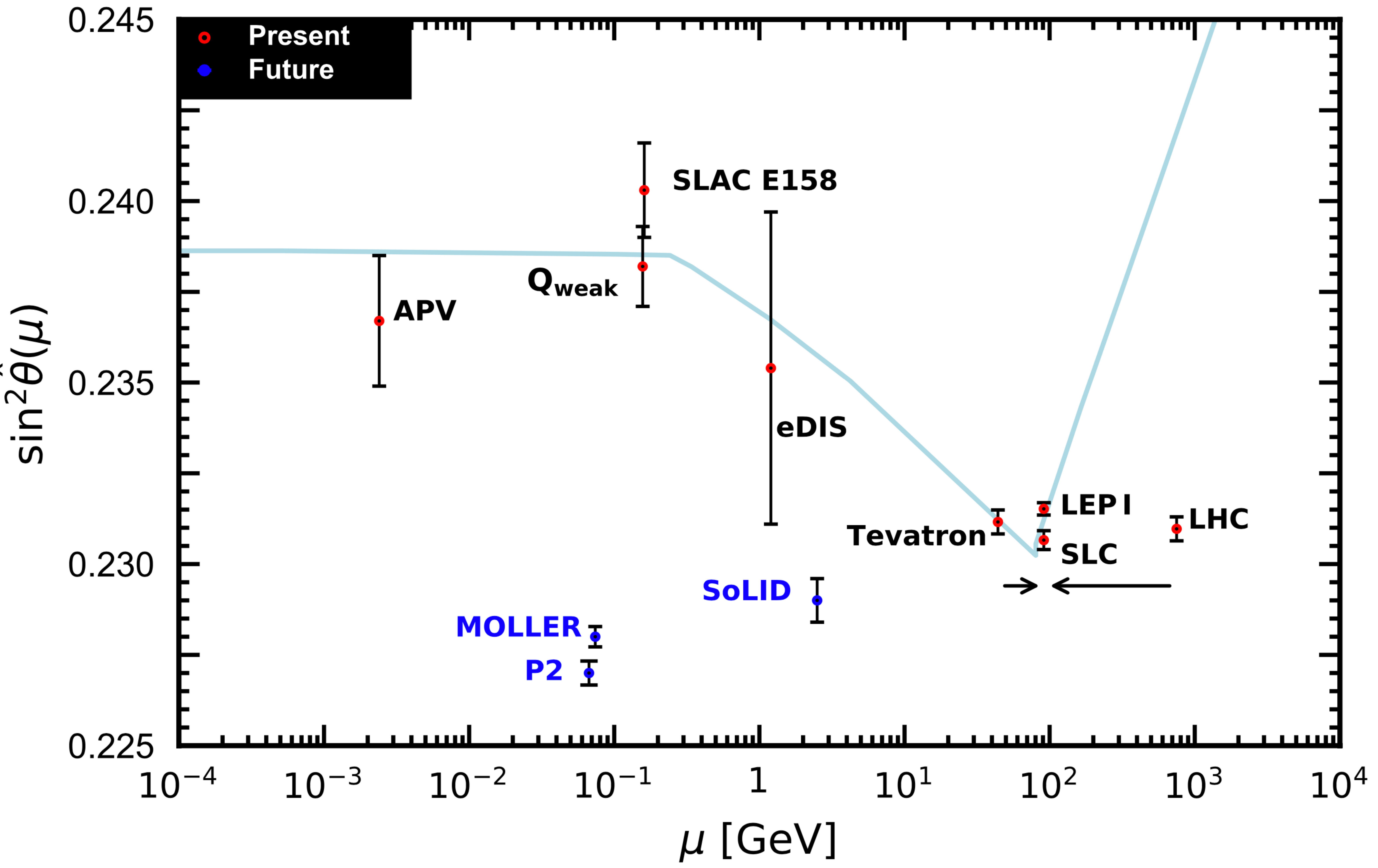


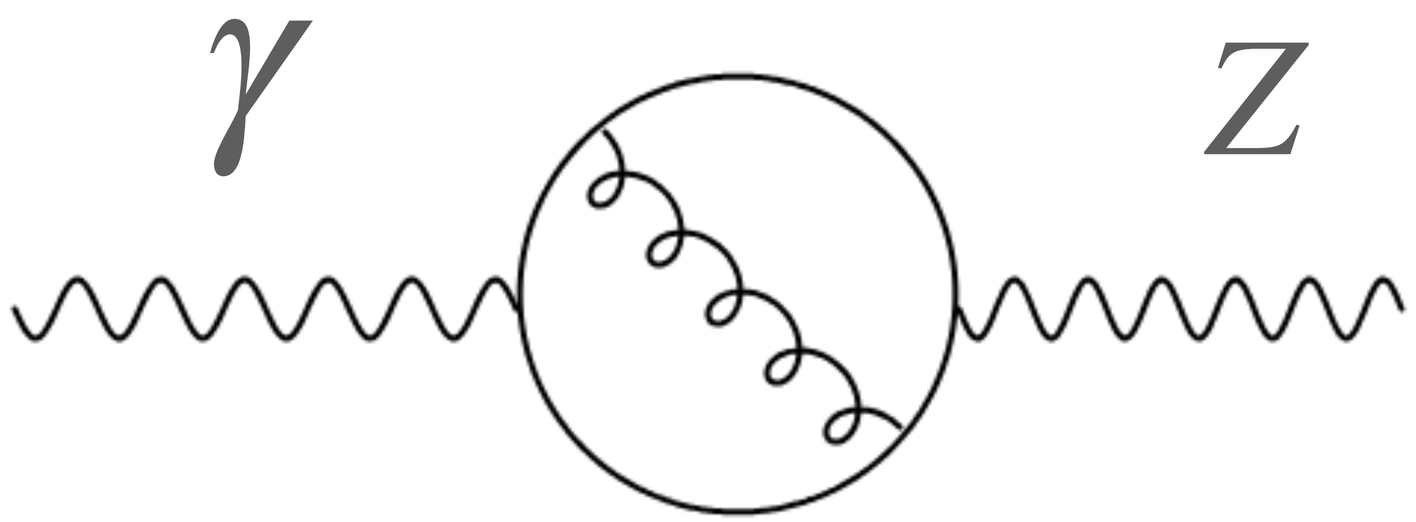


$\delta\hat{\alpha}_s = 0.0016$

$\delta\hat{m}_c = 0.008\,\mathrm{GeV}$

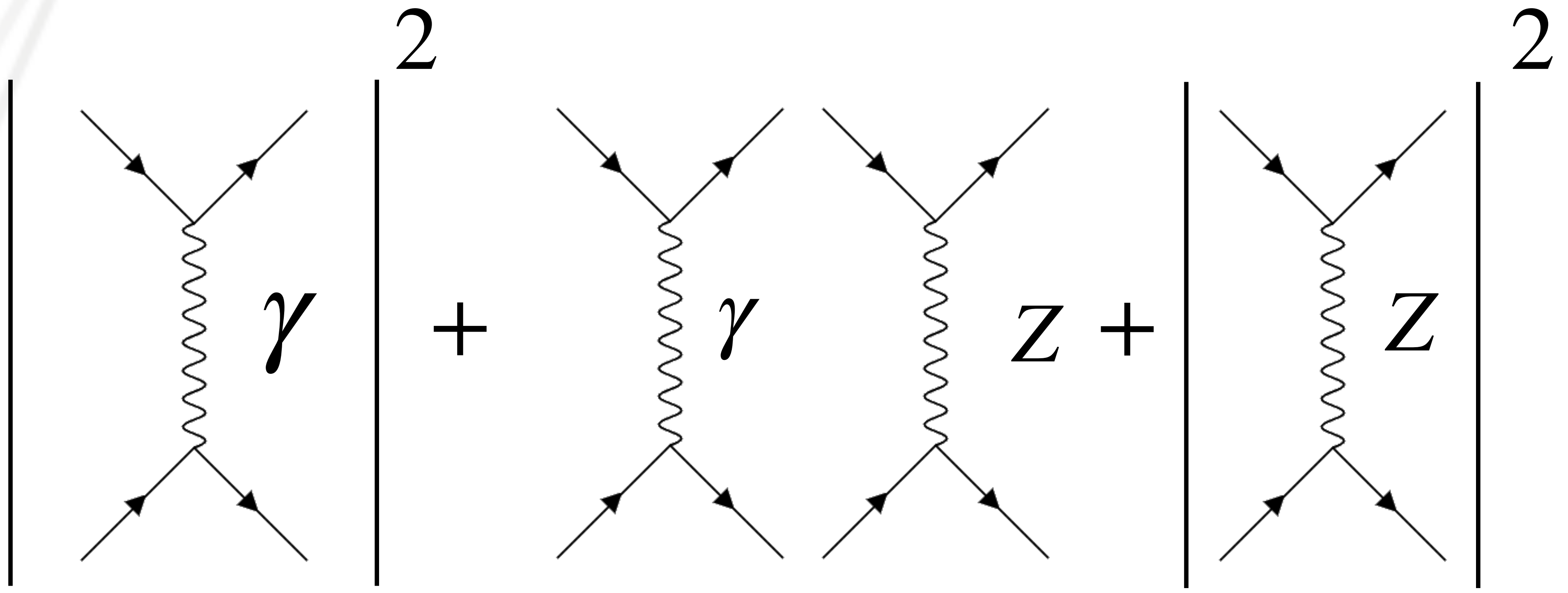
Low energy Parity Violation





PV asymmetry example: ep scattering

$$A_{PV} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} =$$



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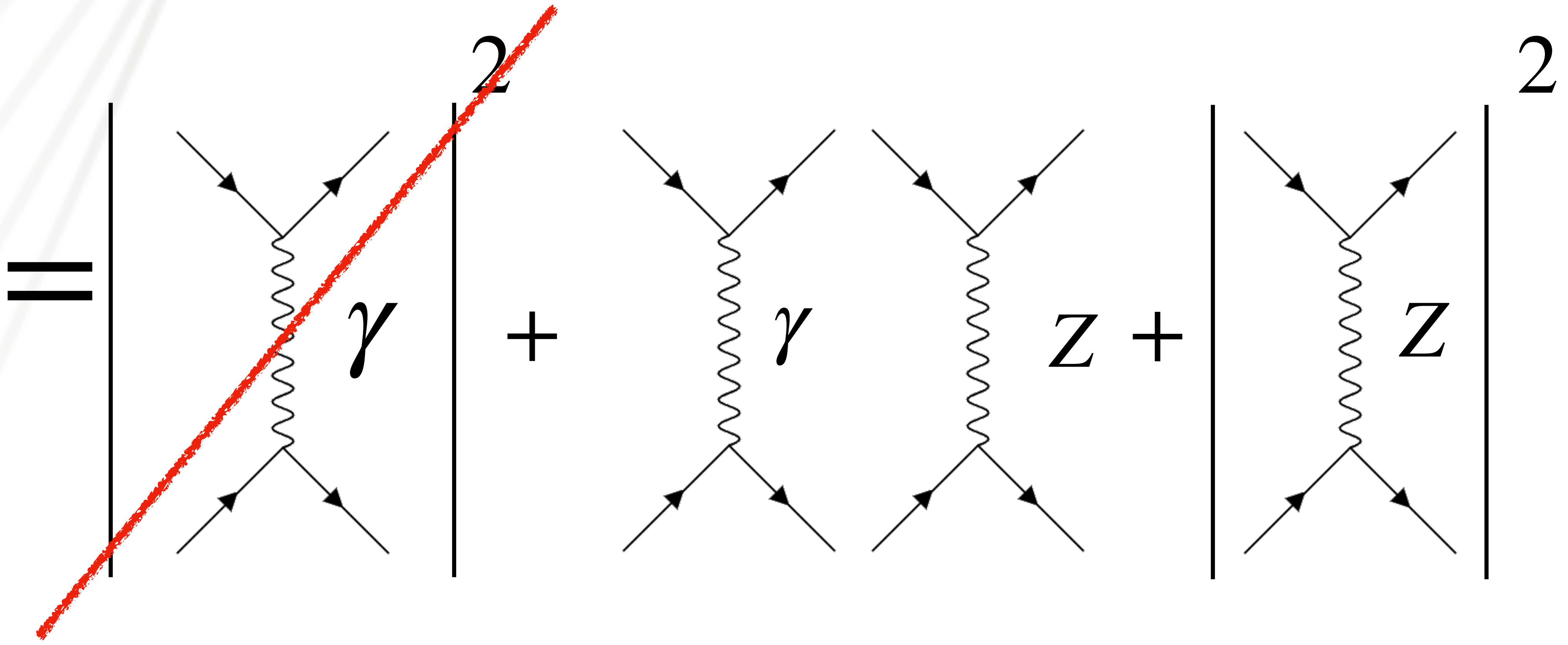

Parity
Conserving

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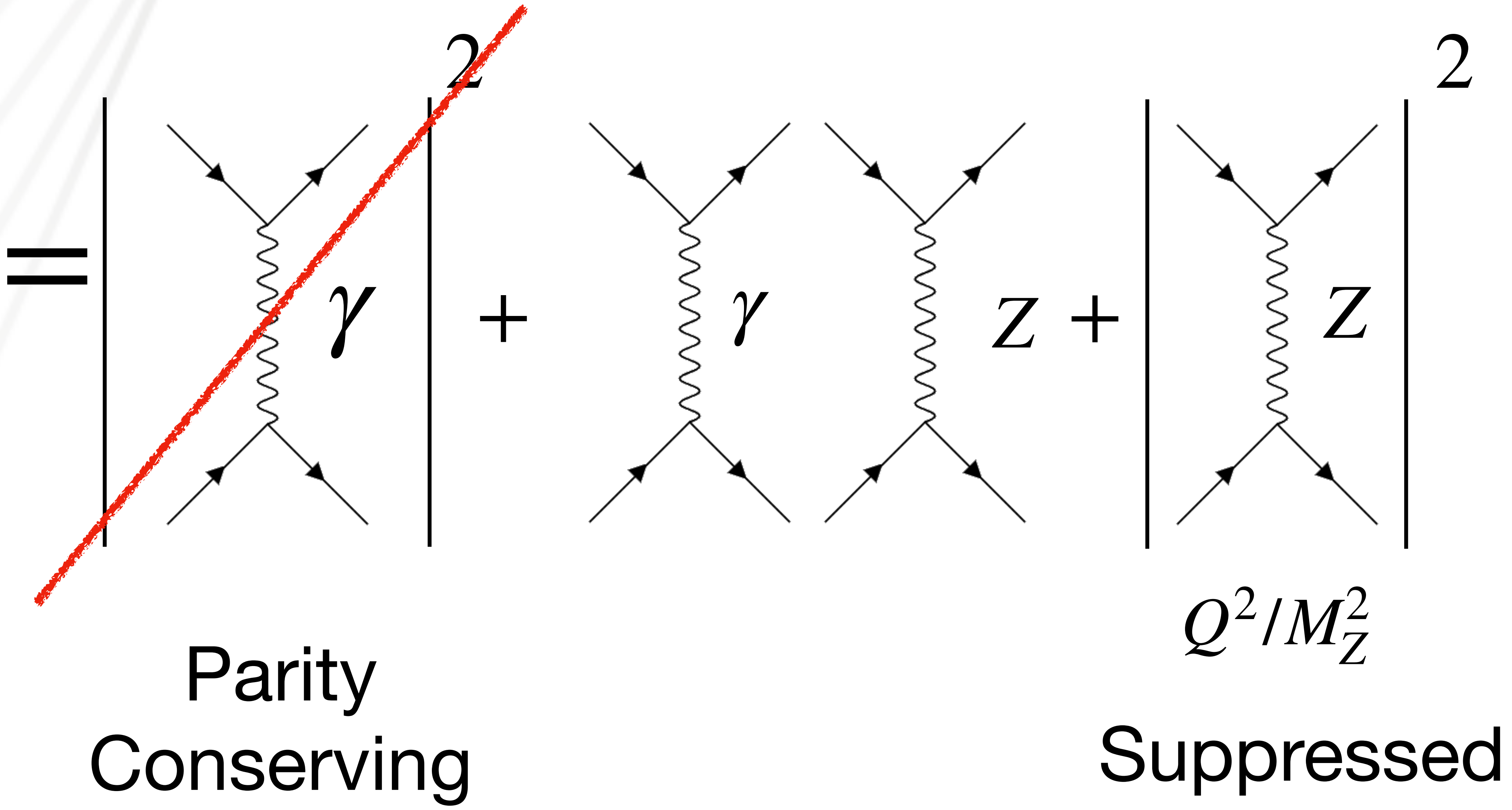

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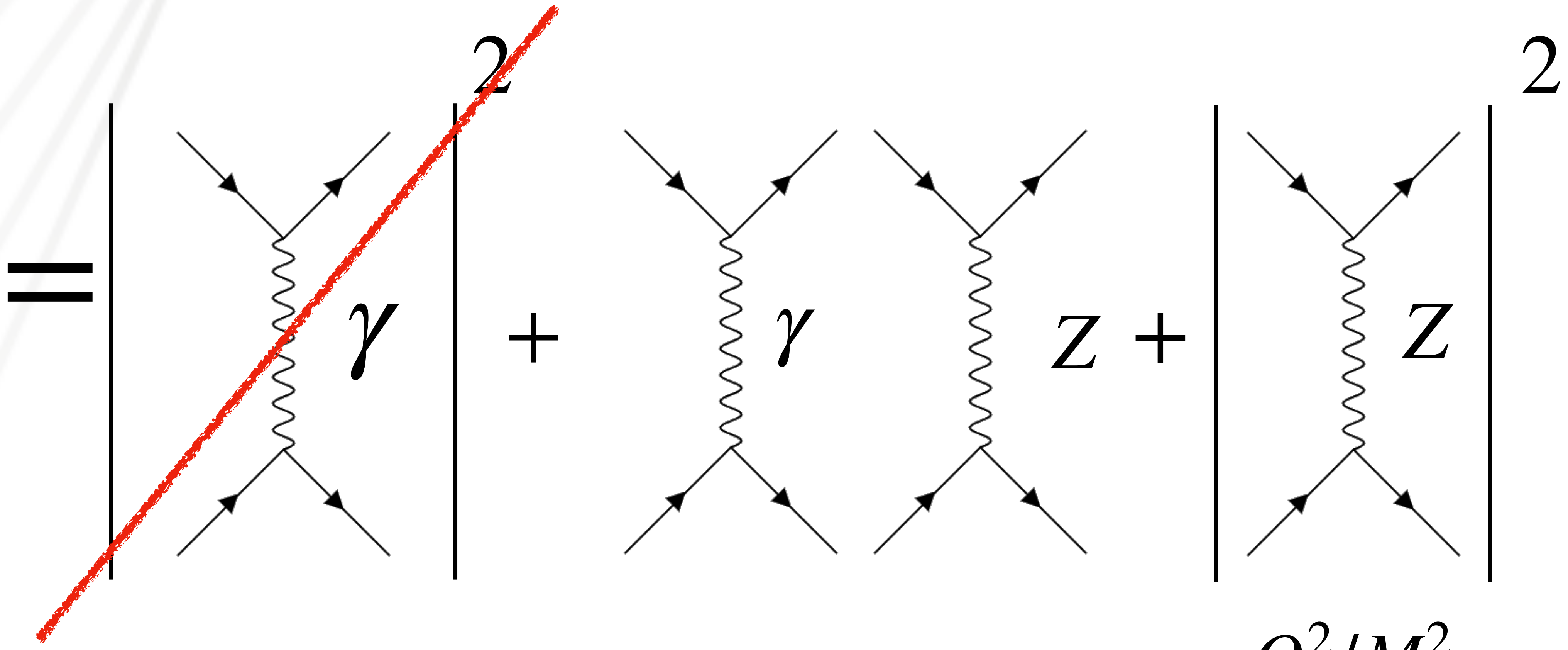

$$= \frac{G_F Q^2}{4\pi\alpha\sqrt{2}} \left[Q_W^p - F(E_i, Q^2) \right]$$

Q^2/M_Z^2

Parity Conserving

Suppressed

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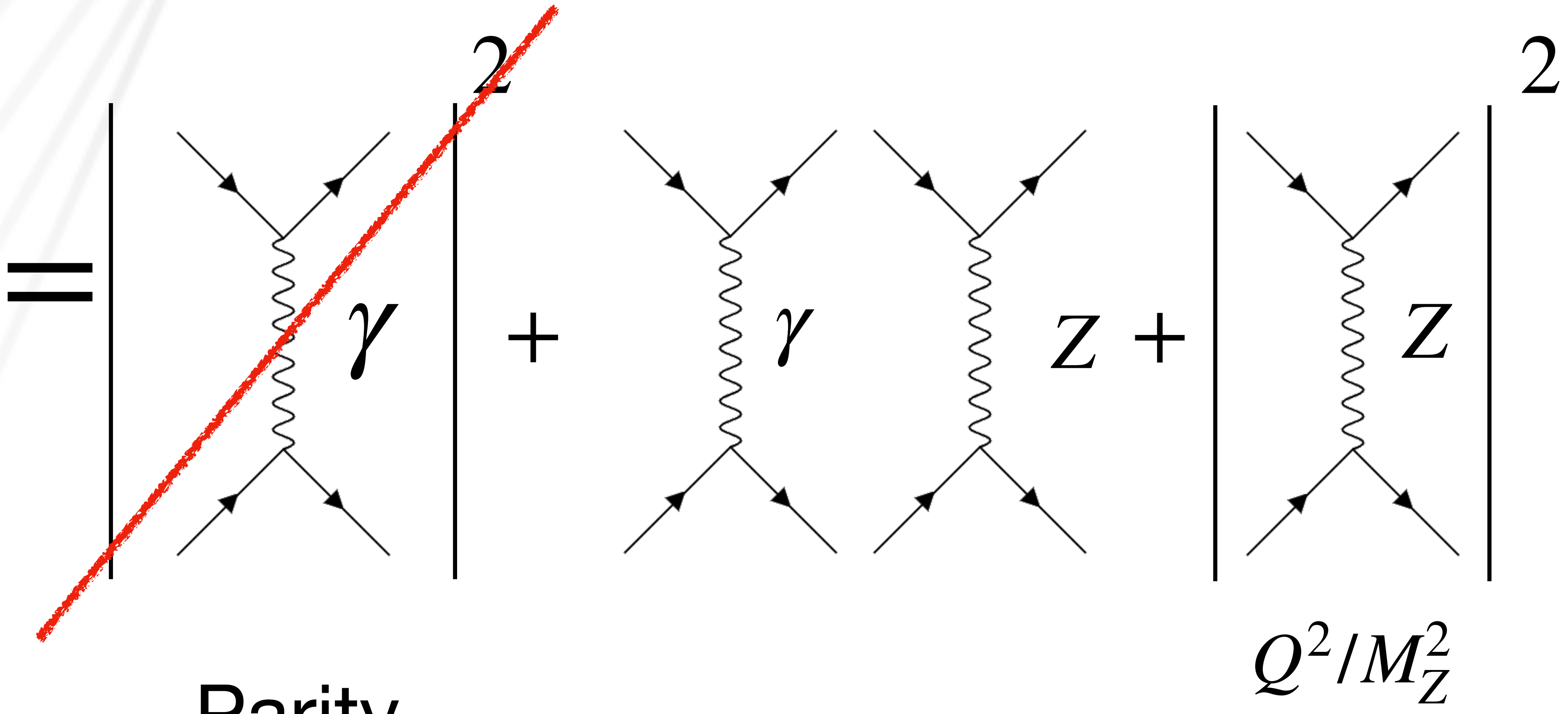

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Form factors

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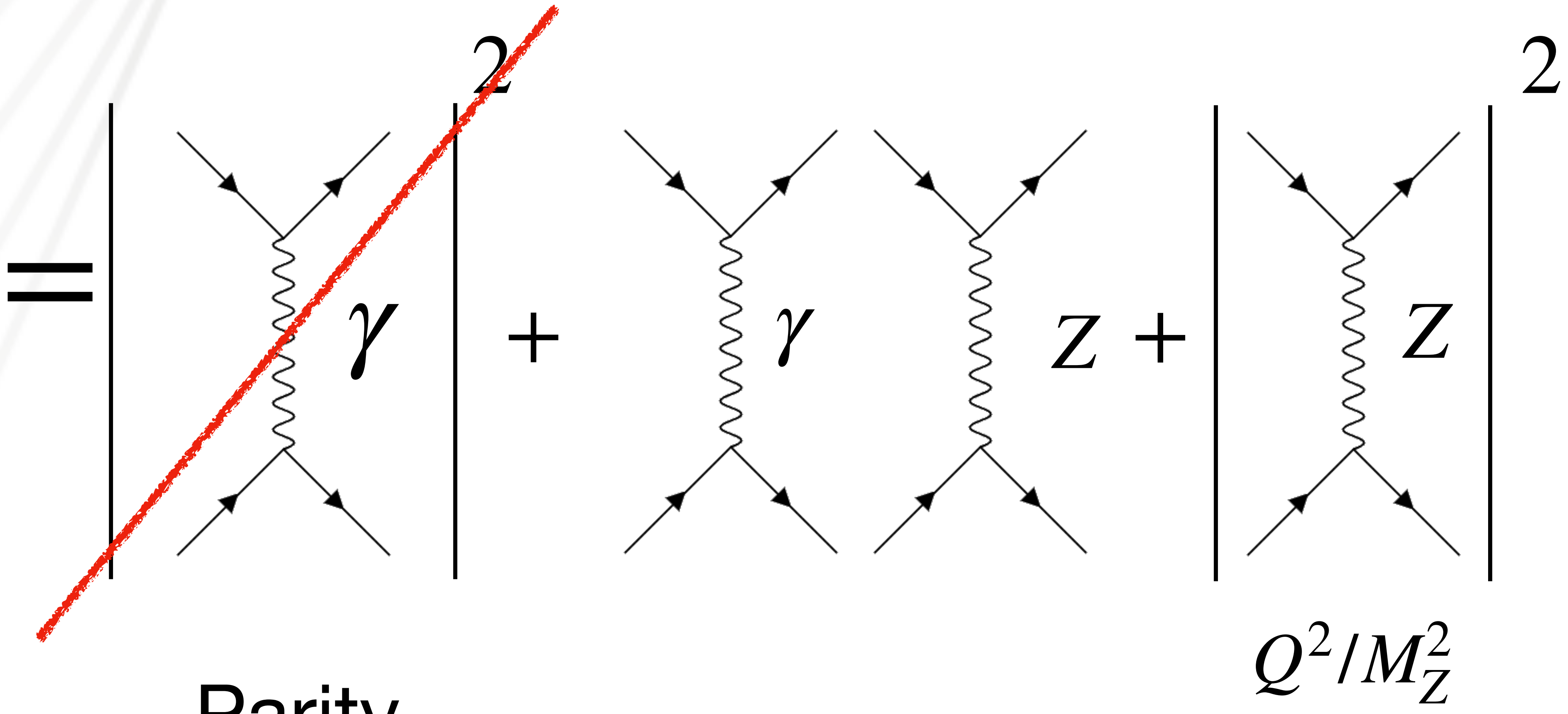

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At higher orders….

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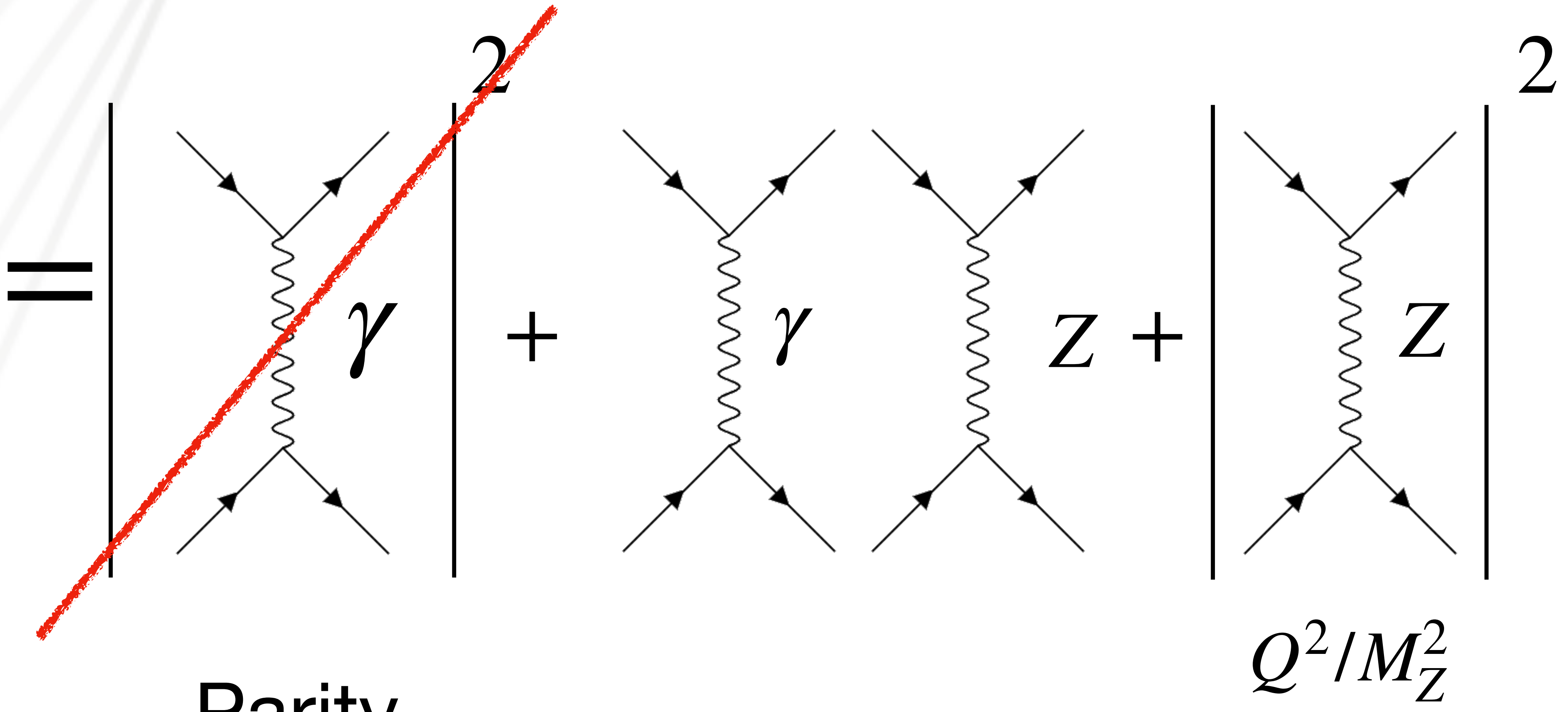

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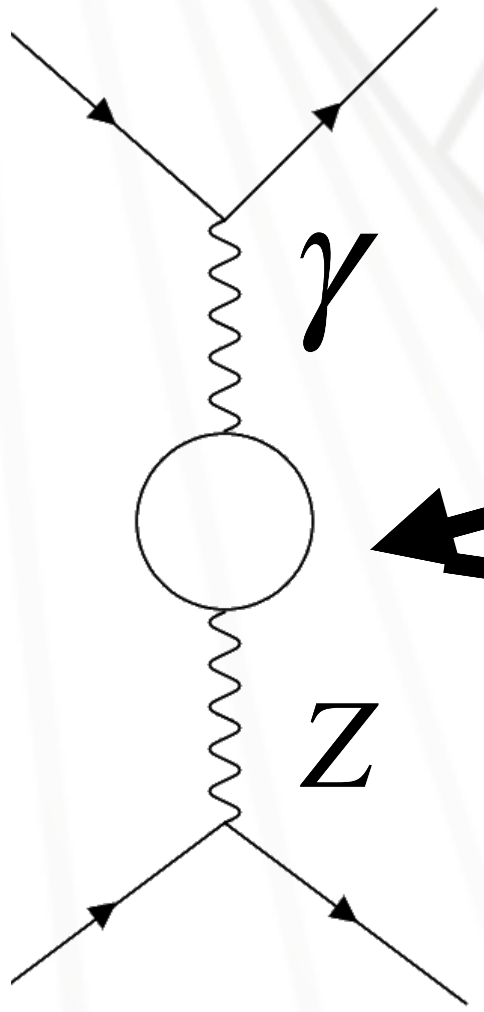

$$Q_W^p = 1 - 4\sin^2\theta_W(0) + \text{vacuum} + \text{vertex} + \text{box} + \text{higher order}$$

$$g_{Vf} = T_f - 2Q_f^2 \sin^2\hat{\theta}$$

Huge effect, 40% reduction of the asymmetry!
Important to resum large logs

Use $\hat{\alpha}$ to compute $\sin^2\hat{\theta}_W \equiv \hat{s}^2$ ($\overline{\text{MS}}$ scheme)

Erler, *Phys.Rev.D* 72 (2005) 073003

$$\hat{s}^2(\mu) = \hat{s}^2(\mu_0)\frac{\hat{\alpha}(\mu)}{\hat{\alpha}(\mu_0)} + \lambda_1\left[1 - \frac{\hat{\alpha}(\mu)}{\hat{\alpha}(\mu_0)}\right] + \frac{\hat{\alpha}(\mu)}{\pi}\left[\frac{\lambda_2}{3}\ln\frac{\mu^2}{\mu_0^2} + \frac{3\lambda_3}{4}\ln\frac{\hat{\alpha}(\mu)}{\hat{\alpha}(\mu_0)} + \tilde{\sigma}(\mu_0) - \tilde{\sigma}(\mu)\right]$$

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Requirements

$\hat{\alpha}(M_Z)$ from α

$\sin^2\hat{\theta}(0)$ from $\sin^2\hat{\theta}(M_Z)$

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pQCD	✓	✓

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pQCD	✓	✓
Total HVP	✓	✓

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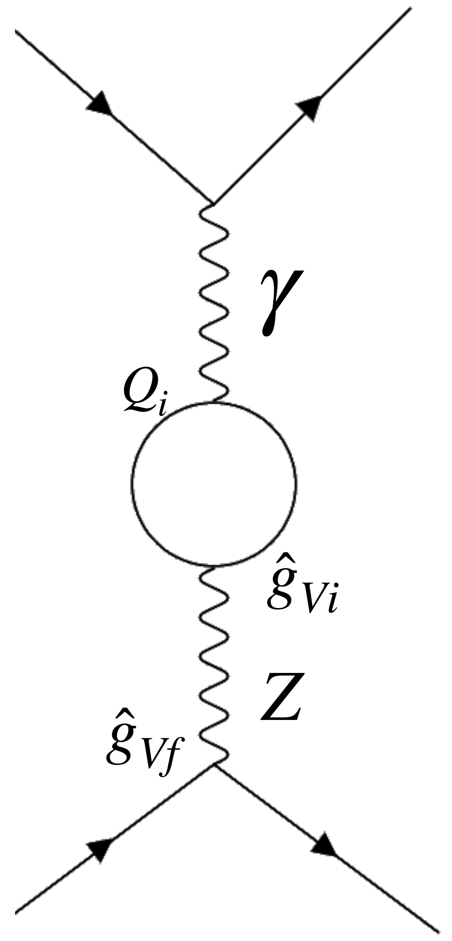
pQCD	✓	✓
Total HVP	✓	✓
Flavor Separation	✗	✓



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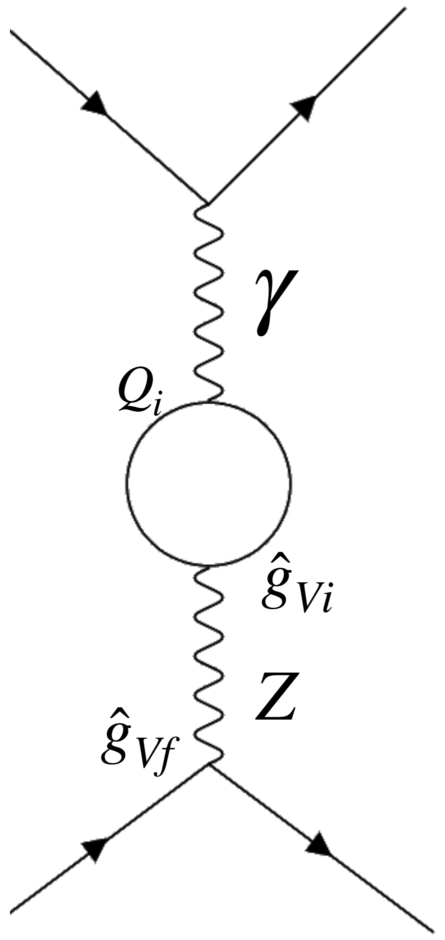
pQCD	✓	✓
Total HVP	✓	✓
Flavor Separation	✗	✓ Inferred from "s channels" like the ϕ
Using cross section data	Erler, Ferro-Hernandez, 10.1007/JHEP03(2018)196	Erler, Ferro-Hernandez, 10.1007/JHEP03(2018)196



Theory error is negligible compared with future experiments

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pQCD	✓	✓
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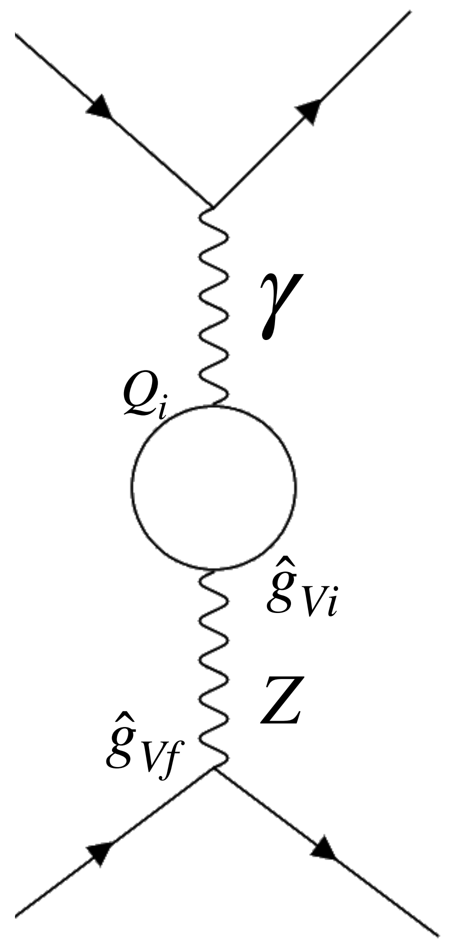
Flavor Separation	✗	✓
Using cross section data	Erler, Ferro-Hernandez, 10.1007/JHEP03(2018)196	Erler, Ferro-Hernandez, 10.1007/JHEP03(2018)196
Using Lattice	Erler, Ferro-Hernandez, 10.1007/JHEP12(2023)131	To be published soon



Lattice can compute the contribution of each flavor

Lattice flavor separation

Parameter	Value and error	$\Pi_{\rm disc}$	Π_s	Π_{ud}
$\Pi_{\rm disc}$	$(-3.7 \pm 1.0) \times 10^{-4}$	1.0	−0.5	−0.6
Π_s	$(83.0 \pm 1.3) \times 10^{-4}$	−0.5	1.0	0.9
Π_{ud}	$(587.8 \pm 8.3) \times 10^{-4}$	−0.6	0.9	1.0

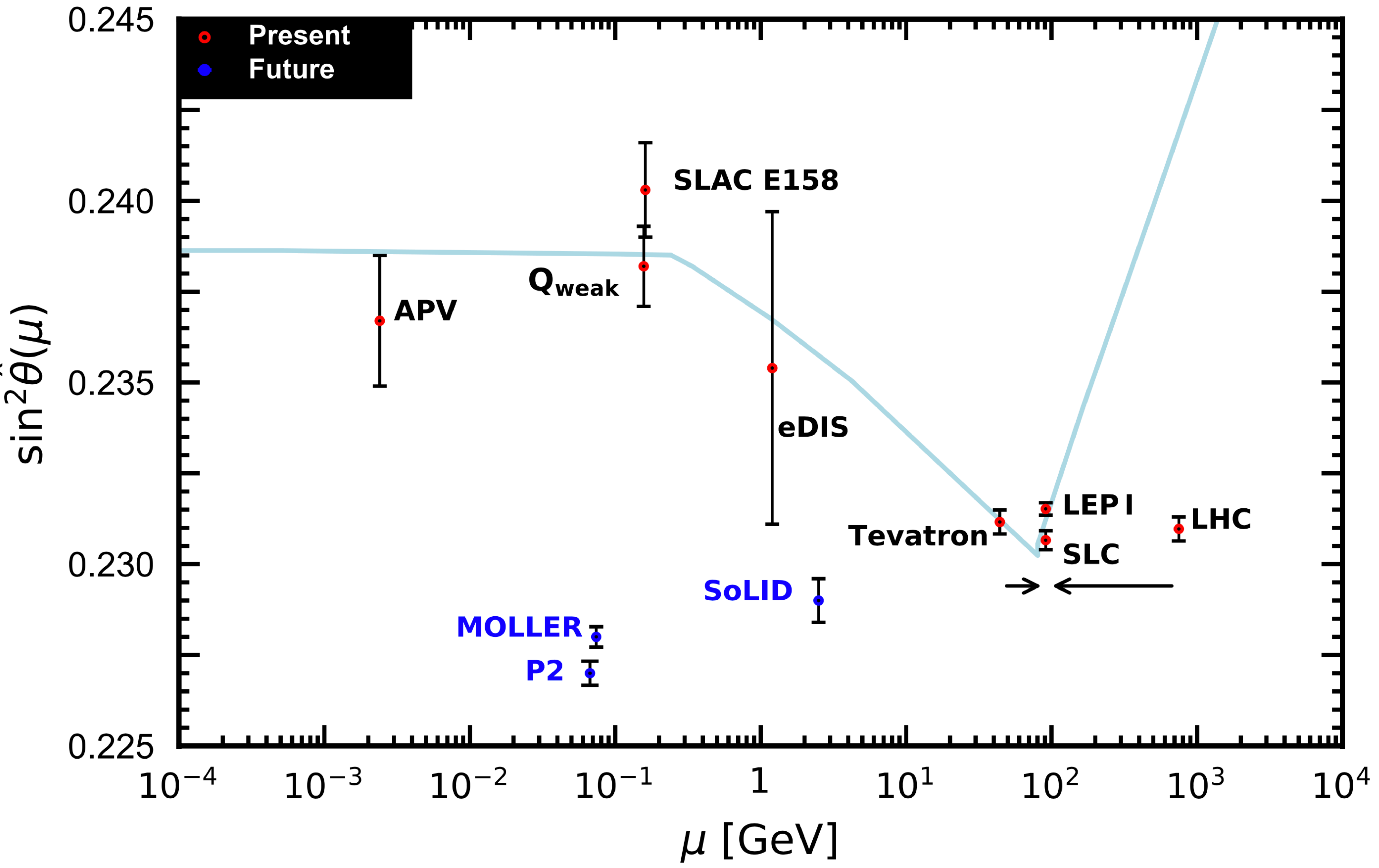
Ce et al 10.1007/JHEP08(2022)220

$$\Pi_f(-Q^2) = \hat{\Pi}_f(0,\mu^2) - \hat{\Pi}_f(-Q^2,\mu^2)$$

$$\hat{\Pi}_f(-Q^2,Q^2) = \frac{Q_f^2}{4\pi^2}\sum_{n=0}^{3} c_n \left(\frac{\hat{\alpha}_s(Q^2)}{\pi}\right)^n$$

pQCD

Flavor separation



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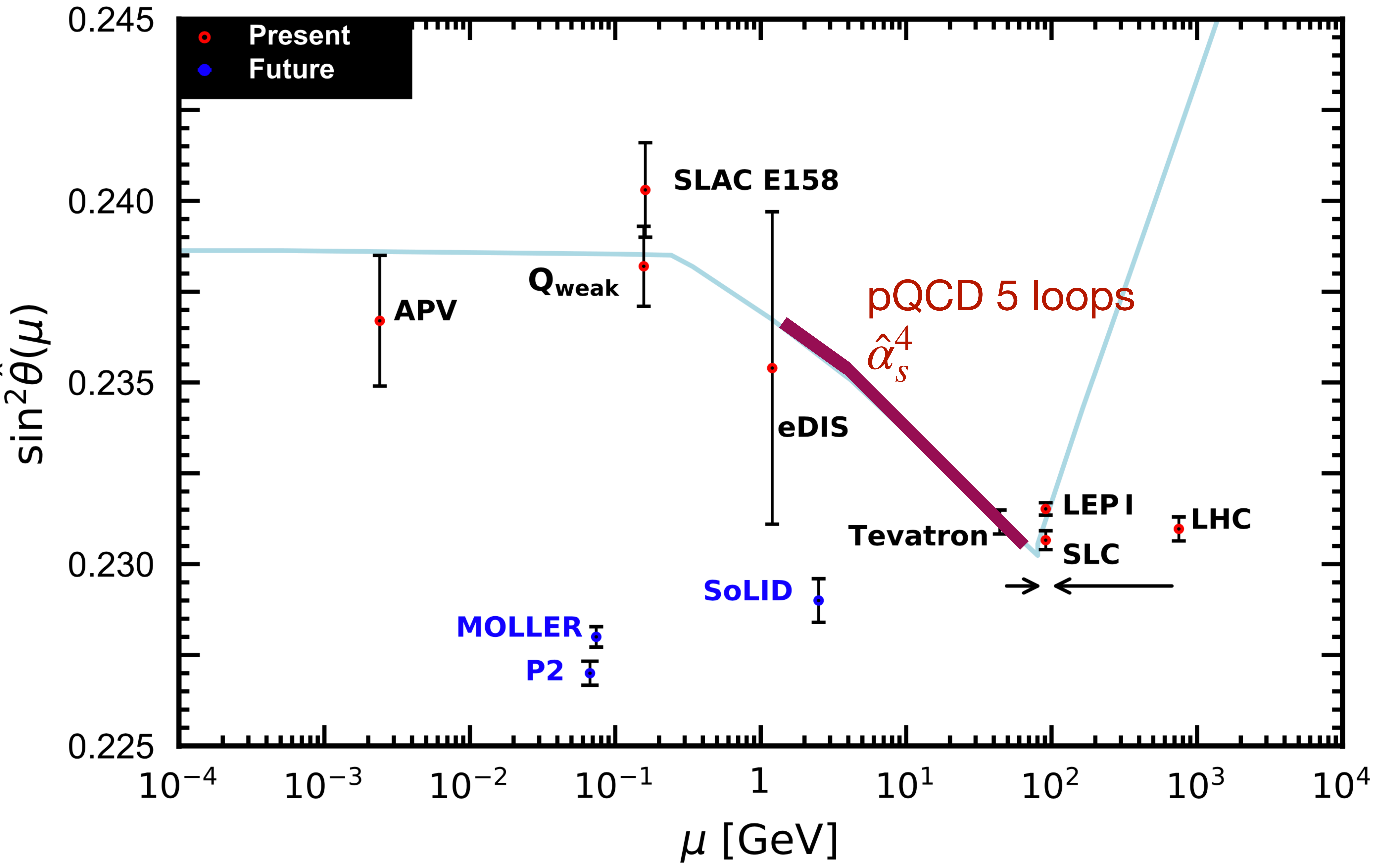

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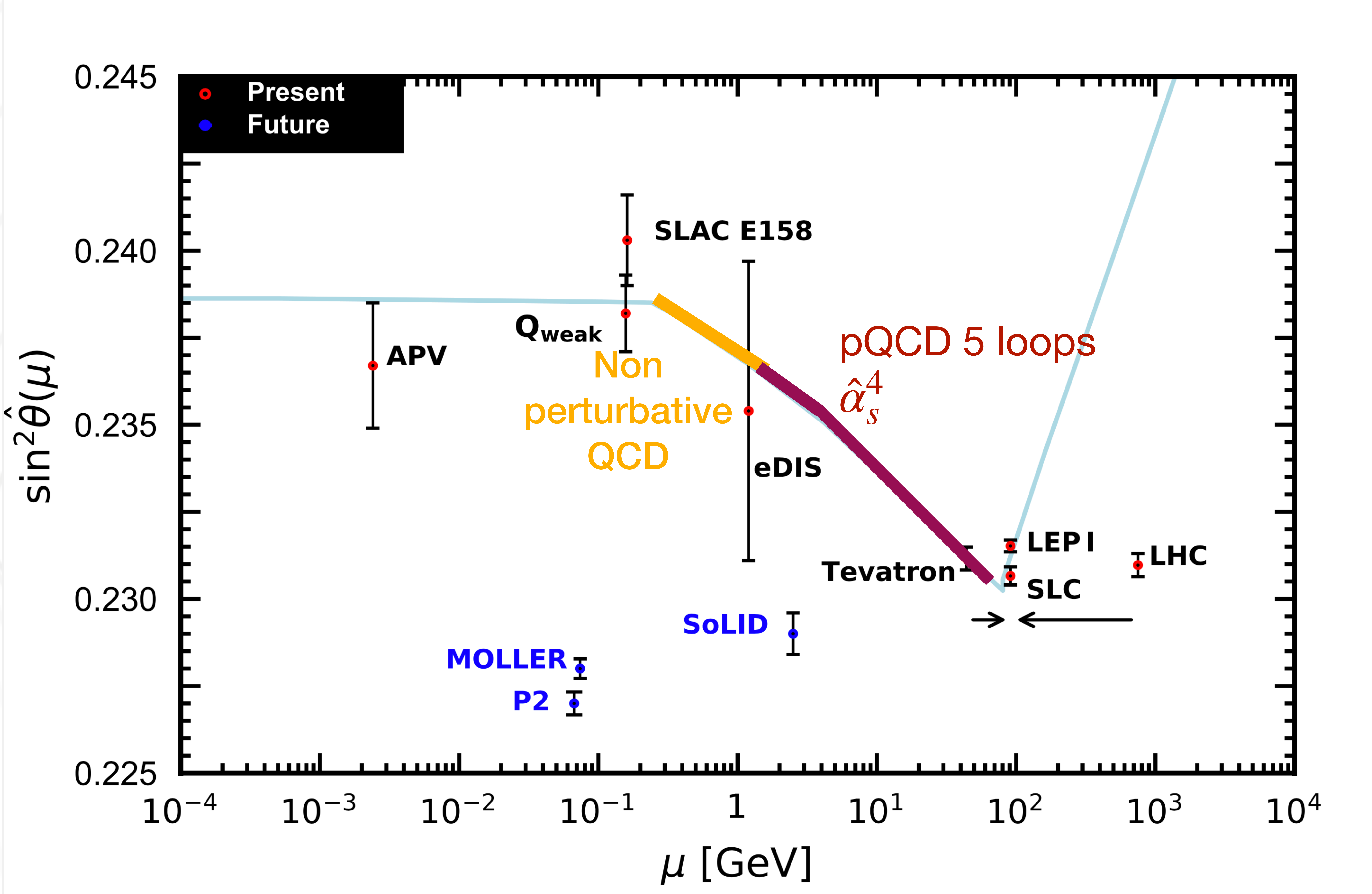

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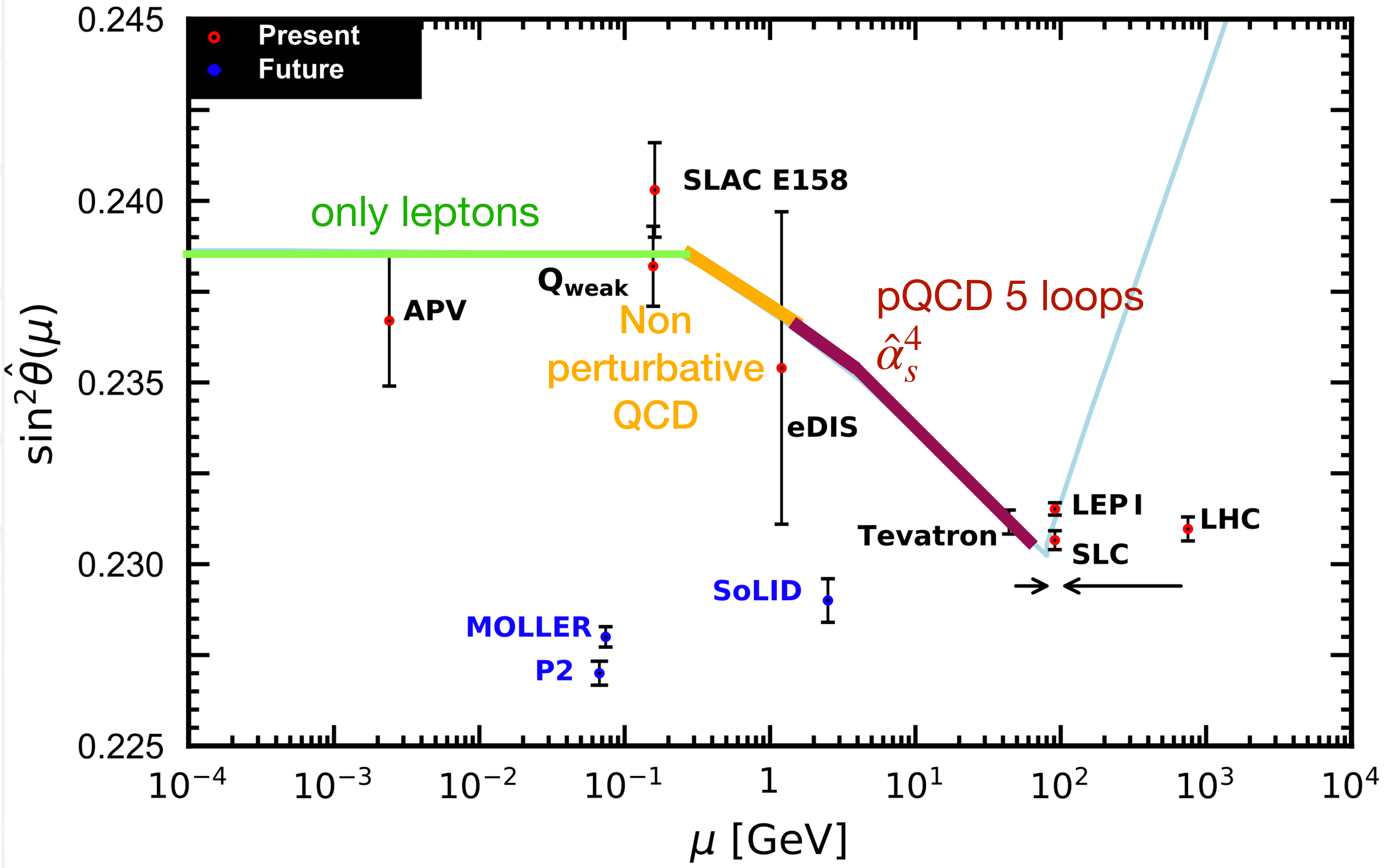

Results

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Defining

$$\hat{\kappa}(0) = \frac{\sin^2 \hat{\theta}_W(0)}{\sin^2 \hat{\theta}_W(M_Z)}$$

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We obtain

$$\hat{\kappa}(0)_{\text{lat}} = 1.03233 - 0.42\Delta\hat{s}_Z^2 + 0.030\Delta\hat{\alpha}_s - 0.0012\Delta\hat{m}_c - 0.0003\Delta\hat{m}_b \pm 0.00010,$$

$$\Delta\hat{\alpha}_s \equiv \hat{\alpha}_s(M_Z) - 0.1185\,\text{GeV} \qquad \Delta\hat{m}_b \equiv \hat{m}_b(\hat{m}_b) - 4.18\,\text{GeV} \qquad \Delta\hat{m}_c \equiv \hat{m}_c(\hat{m}_c) - 1.274\,\text{GeV}$$

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While from cross section data the result is: $\hat{\kappa}(0)_{e^+e^-} = 1.03200 \pm 0.00008,$

Erler, Ferro-Hernandez,10.1007/JHEP03(2018)196

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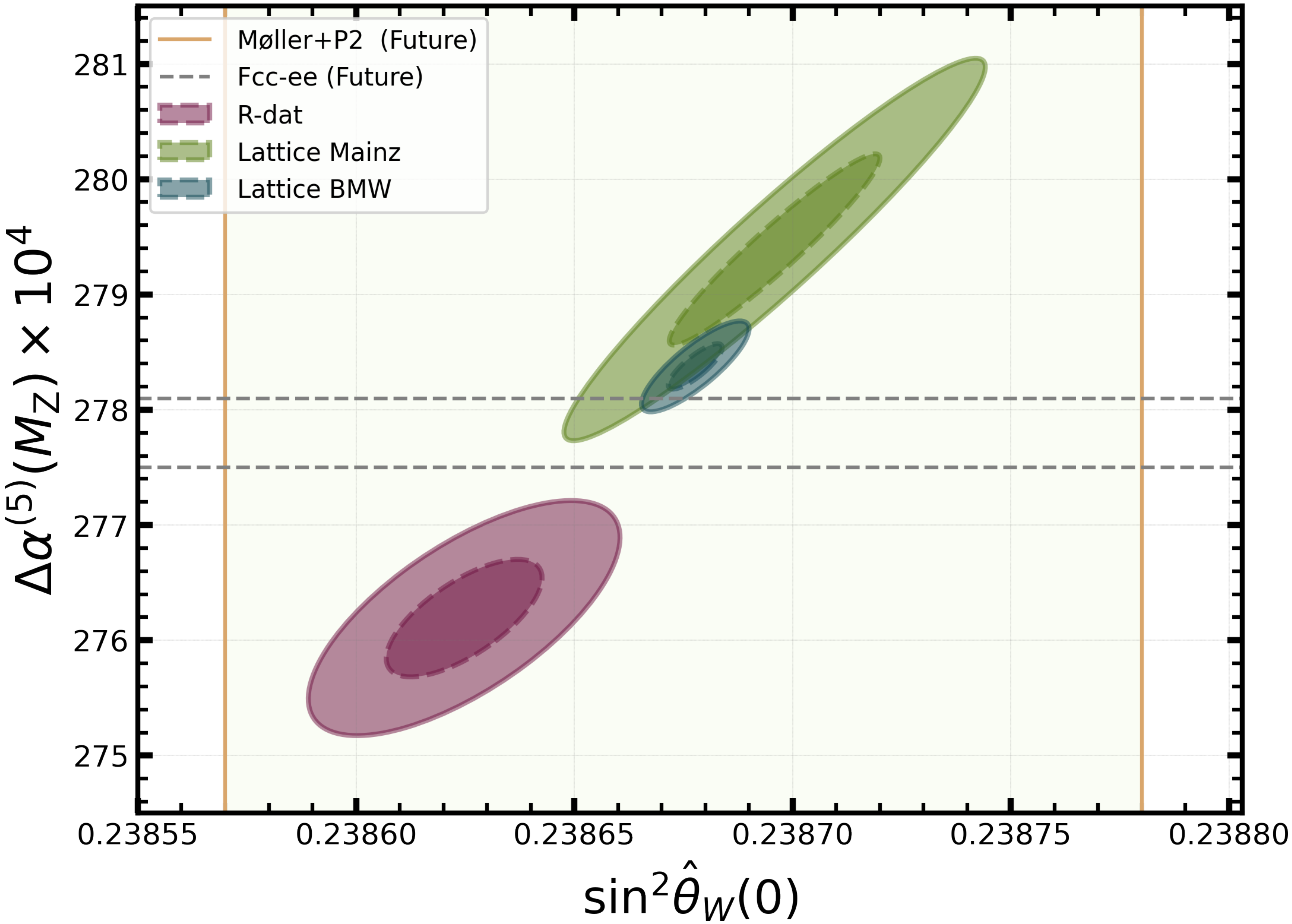
$$\hat{\kappa}(0)_{\text{lat}} - \hat{\kappa}(0)_{e^+e^-} = 0.00033 \pm 0.00013$$

Results



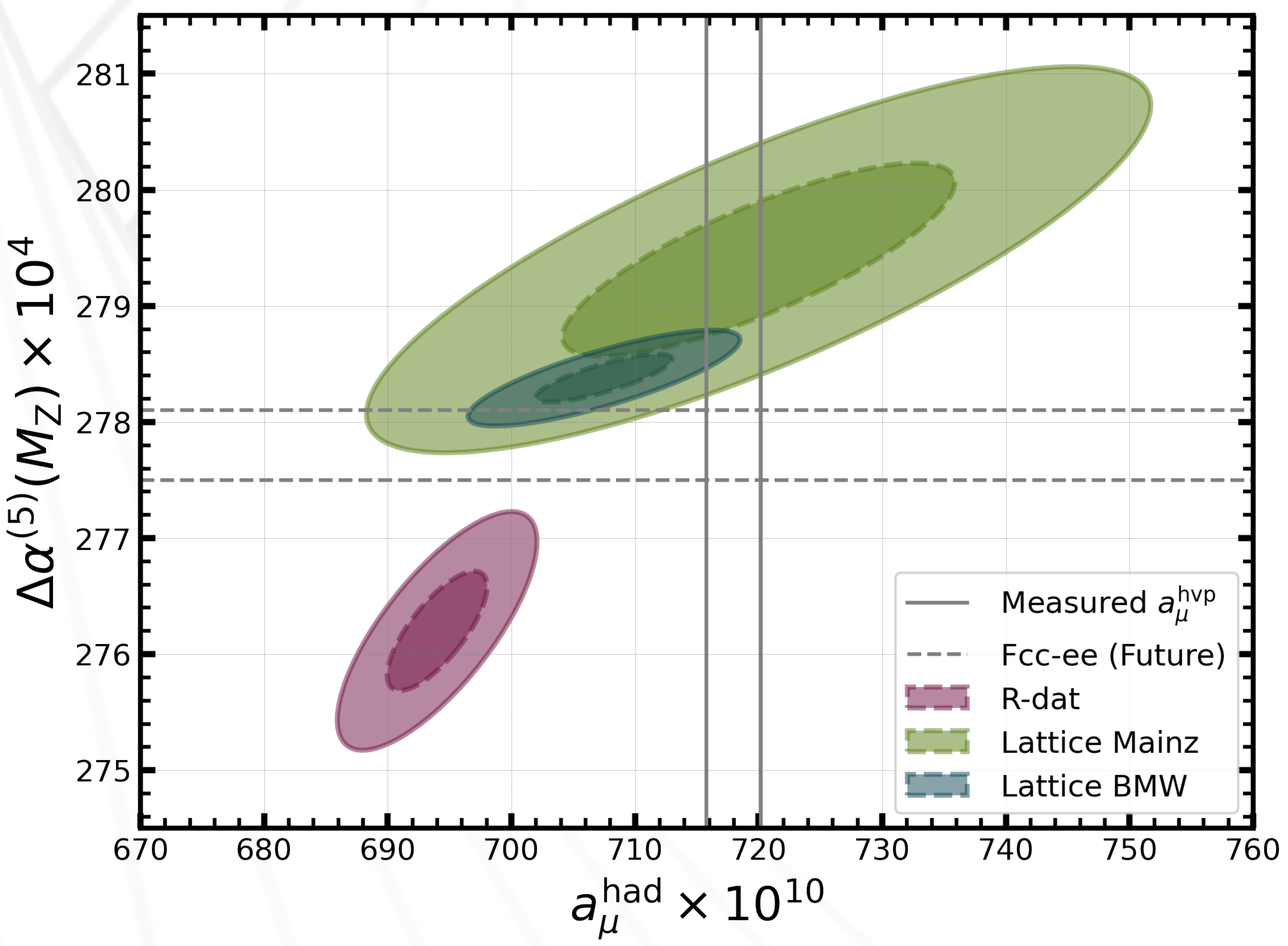
Møller+P2 (Future)
Fcc-ee (Future)
R-dat
Lattice Mainz
Lattice BMW
281
280
279
278
277
276
275
$\Delta\alpha^{(5)}(M_Z) \times 10^4$
0.23855
0.23860
0.23865
0.23870
0.23875
0.23880
$\sin^2\hat{\theta}_W(0)$

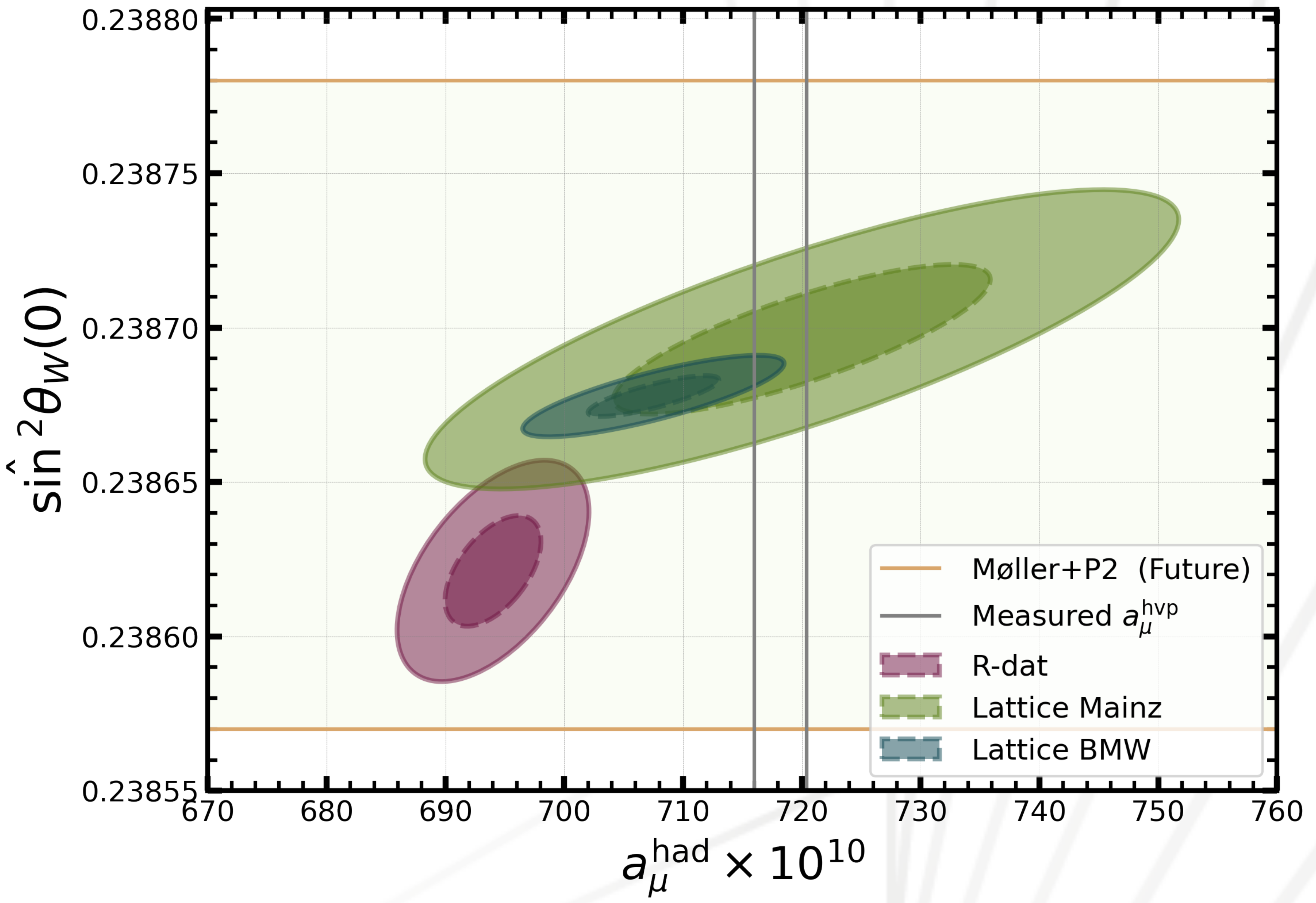
Results

$$f(K^2) = \frac{m_\mu^2 K^2 Z^3 \left(1 - K^2 Z\right)}{1 + m_\mu^2 K^2 Z^2}$$

$$Z = -[K^2 - (K^4 + 4m_\mu^2 K^2)^{1/2}]/2m_\mu^2 K^2$$

$$a_\mu^{hvp} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dK^2 f(K^2)\Pi(K^2)$$





Summary

Summary

1. We computed $\sin^2\hat{\theta}_W(0)$ using lattice QCD as input.

2. We found a $\sim 3\sigma$ tension when compared to the result using e^+e^- cross section data.

3. As expected the tension is in the same direction as the tension in α.

4. Tension smaller than the precision expected in future PV experiments.

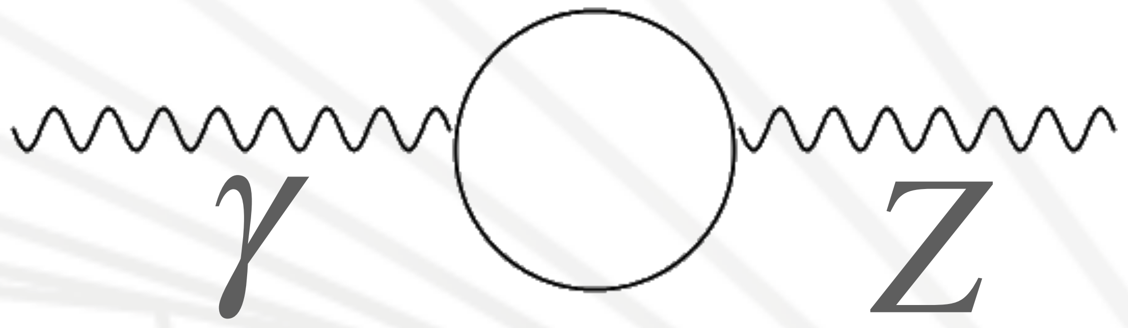
5. We computed the correlation of a_μ^{hvp} with both $\hat{\alpha}$ and $\sin^2\hat{\theta}_W(0)$.

6. There is consistency between the SM prediction and the experimental average of M_W.

Thank you

Backups

$\sin^2\hat{\theta}$ is analogous to α



The weak mixing angle is also a key parameter in the Standard Model.

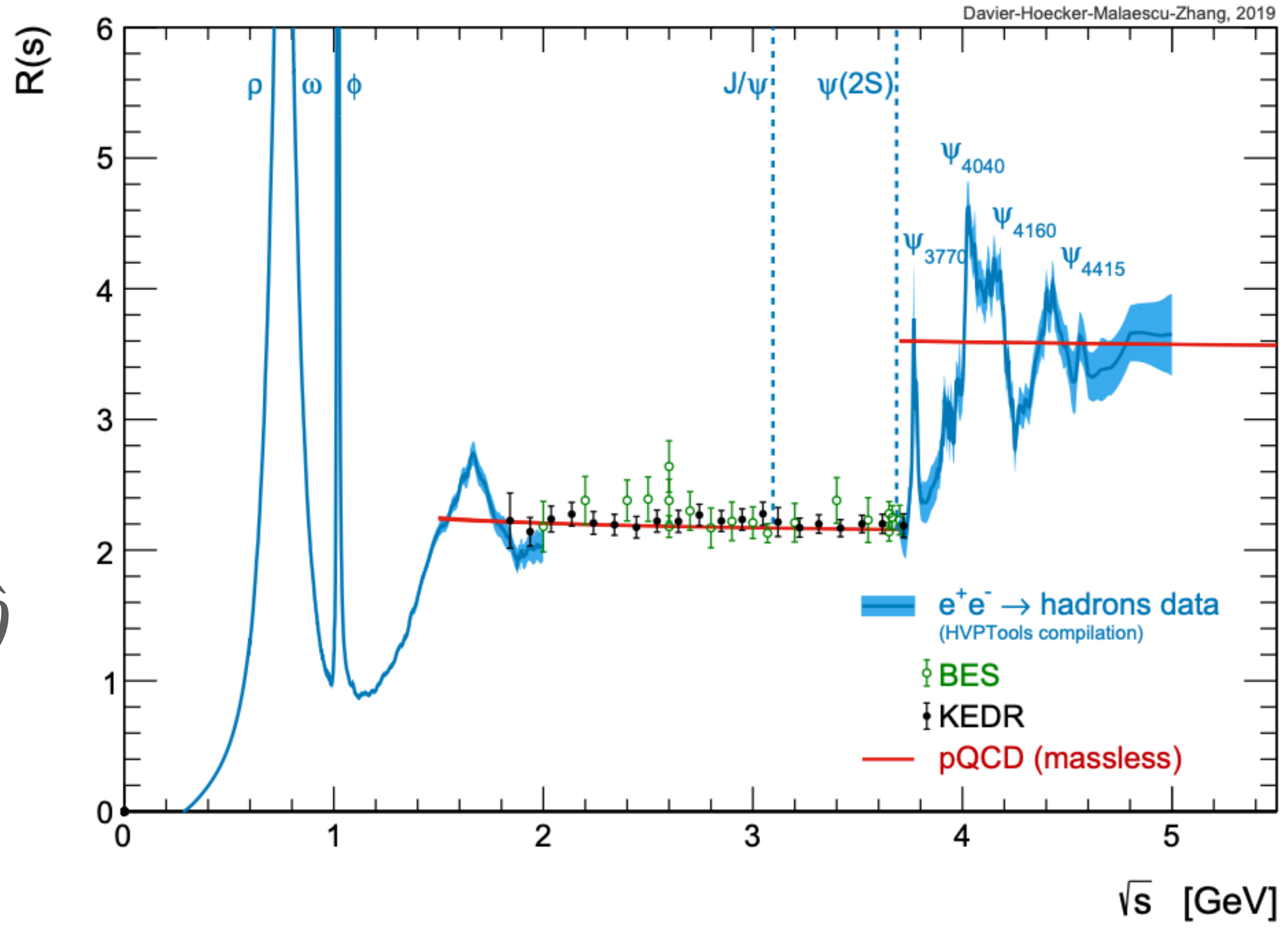
Rel error $\sim 10^{-3\,(2)}$

$\sin^2\hat{\theta}(0)$

$e^+e^- \rightarrow had$ **o lattice + pQCD**

Rel error $\sim 10^{-3}$

$\sin^2\hat{\theta}(M_Z)$



Low energy Parity Violation Experiments

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} \propto 1 - 4\sin^2\hat{\theta}$$

Measured at the Z pole

$$A_f = 2\frac{g_{Vf}g_{Af}}{g_{Vf}^2 + g_{Af}^2}$$

$$g_{Vf} = T_f - 2Q_f^2\sin^2\hat{\theta}$$

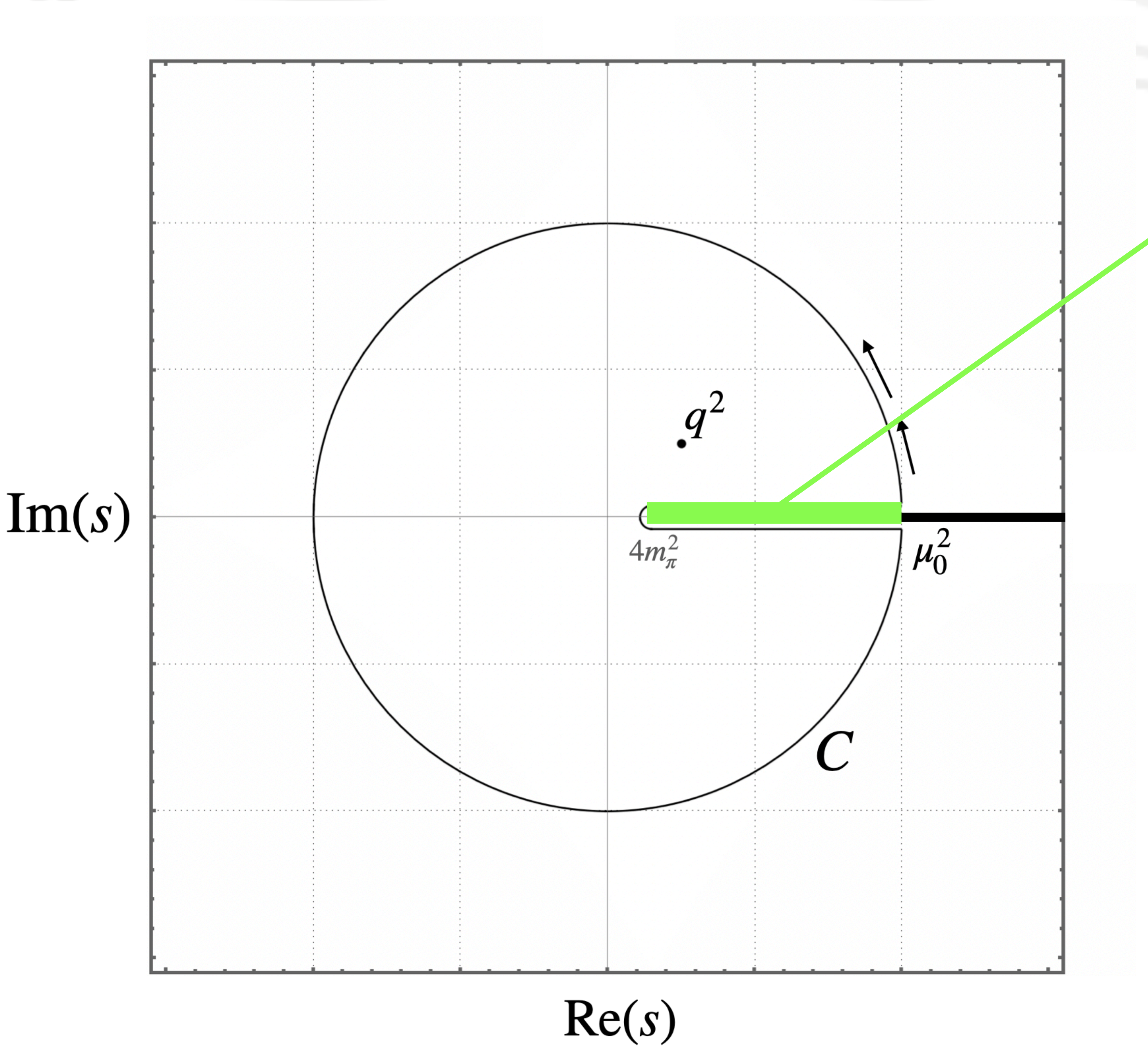
$$g_{Af} = T_f$$

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{3}{4}A_eA_f$$

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = A_e$$

Explicit integration over R

Relation with cross section



Picks up $\mathrm{Im}\,\hat{\Pi}_{\mathrm{had}}(s,\mu^2)$

Use optical theorem

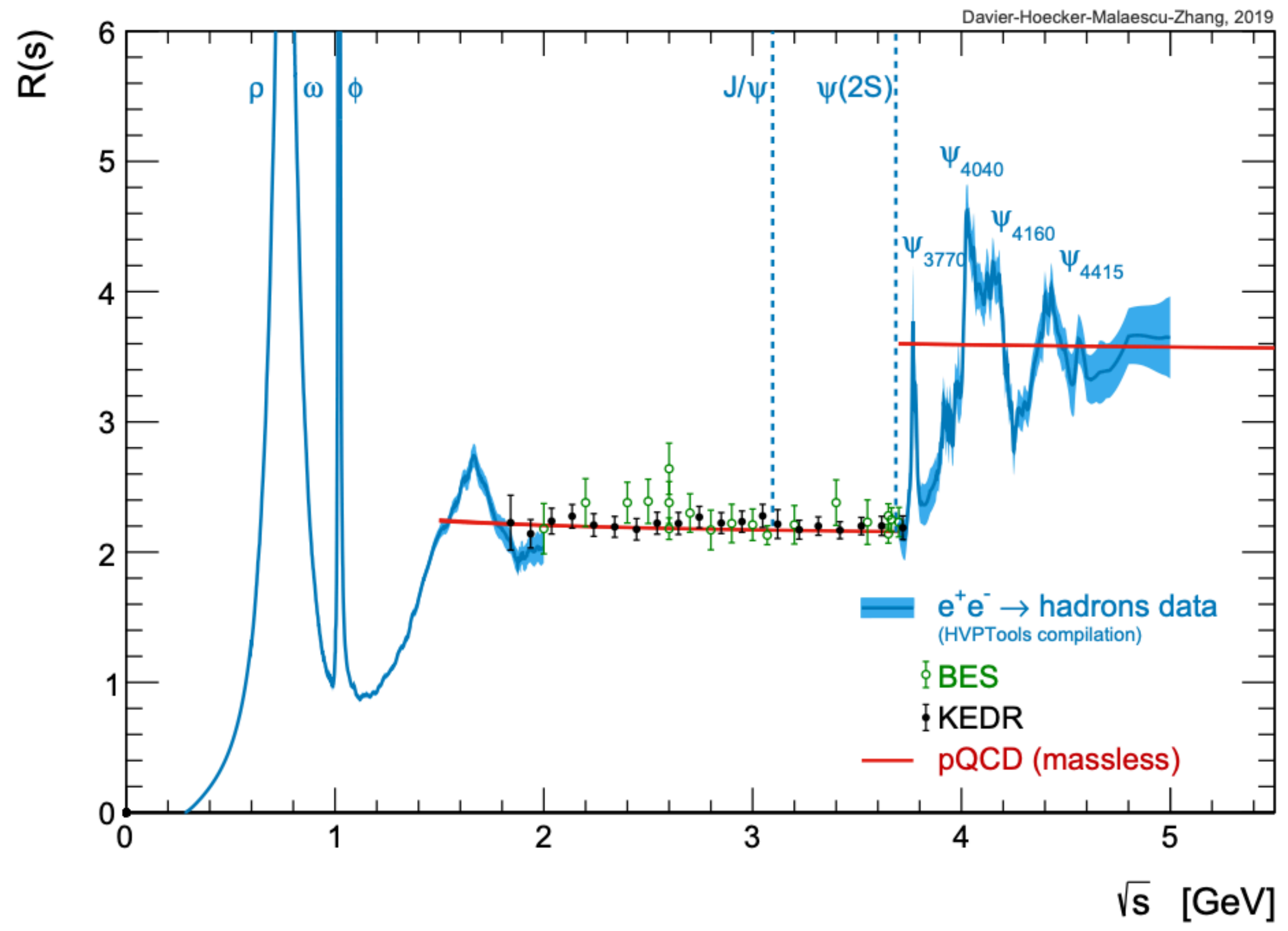
$R = 12\pi \mathrm{Im}\,\Pi_{\mathrm{had}}(s)$

$$R = \frac{\sigma(e^+e^- \to \mathrm{had})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

$\hat{\Pi}_{\mathrm{had}}(s,\mu^2)$ is analytic in the complex plane of s, except for poles and bra

$$\hat{\Pi}_{\mathrm{had}}(q^2,\mu^2) = \frac{1}{2\pi i}\oint_C \frac{\hat{\Pi}_{\mathrm{had}}(s,\mu^2)}{s-q^2}ds$$

$$\hat{\Pi}_{\mathrm{had}}(q^2,\mu^2) = \frac{1}{12\pi^2}\int_{4m_\pi^2}^{\mu_0^2}\frac{R(s)}{s-q^2}ds + \frac{1}{2\pi i}\int_{|s|=\mu_0^2}\frac{\hat{\Pi}_{\mathrm{had}}(s,\mu^2)}{s-q^2}ds$$



Explicit integral over R

Compute $\hat{\Pi}_{\rm had}(q^2, \mu^2) - \hat{\Pi}_{\rm had}(0, \mu^2)$ directly

$$\Delta\alpha_{\rm had}(q^2) = -\,\mathrm{Re}\left[\frac{\alpha q^2}{3\pi}\int_{4m_\pi^2}^{\infty}\frac{R(s)}{s(s-q^2)}ds\right]$$

Very similar to the expression used to compute a_μ!
Collaborations usually quote both results

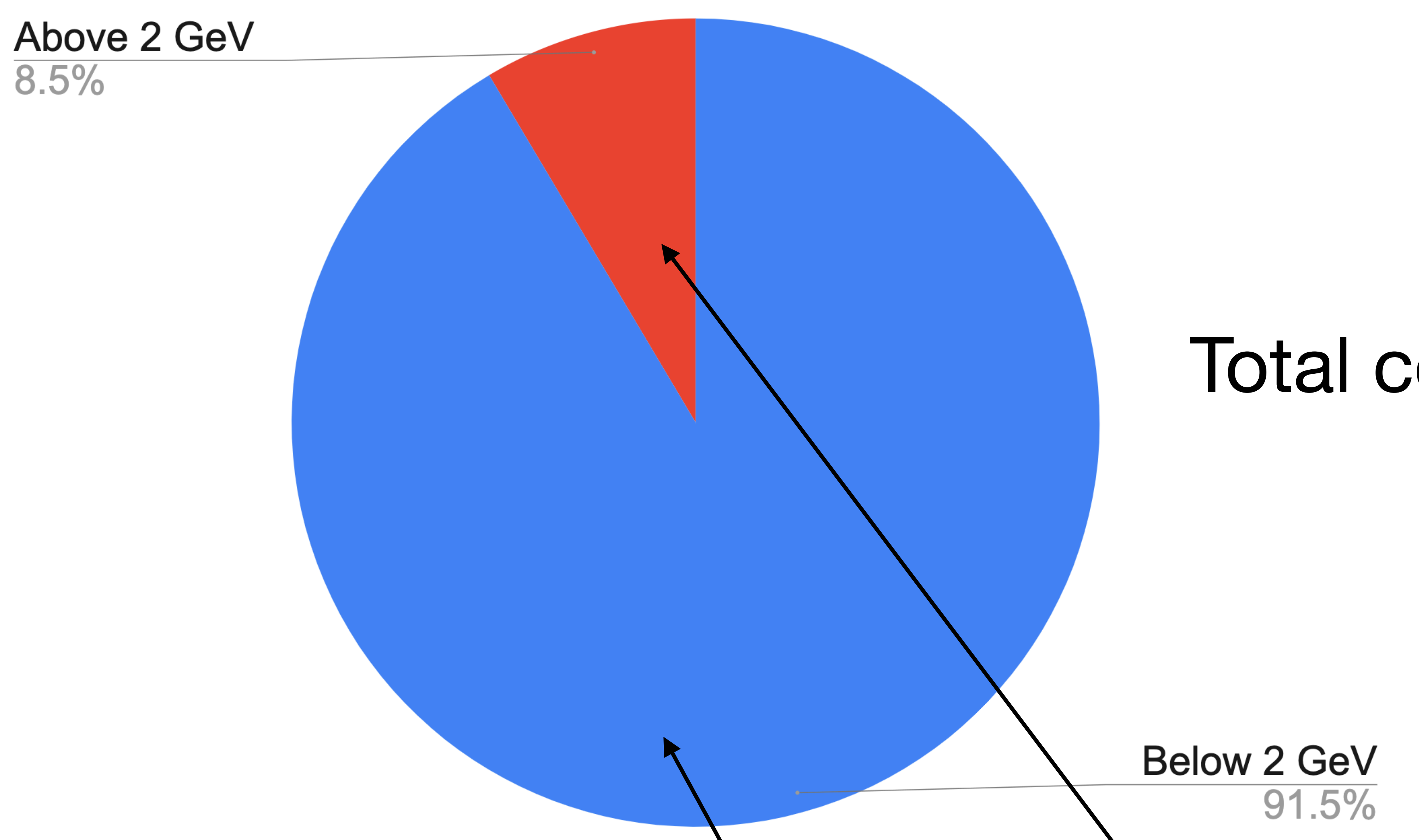
$$a_\mu^{\rm had} = \frac{\alpha^2}{3\pi^2}\int\frac{ds}{s}K(s)R(s) \qquad K(s) \sim \frac{1}{s}$$

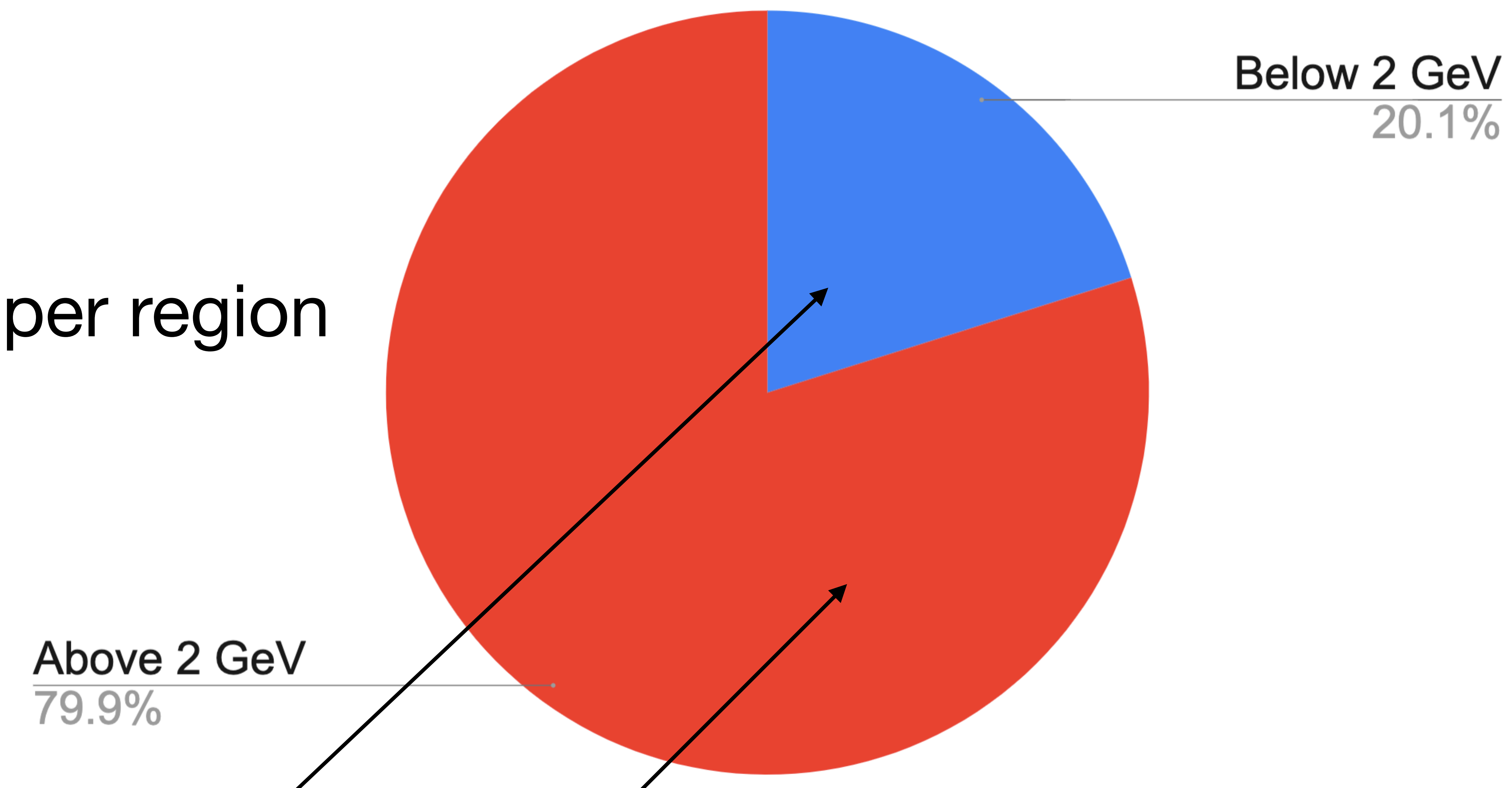
(F. Jegerlehner :arXiv:1905.05078)
(M. Davier, A.Hoecker, B.Malaescu, Z. Zhang: arXiv:1908.00921)
(A. Keshavarzi, D.Nomura and Thomas Teubner: arXiv:1911.00367)

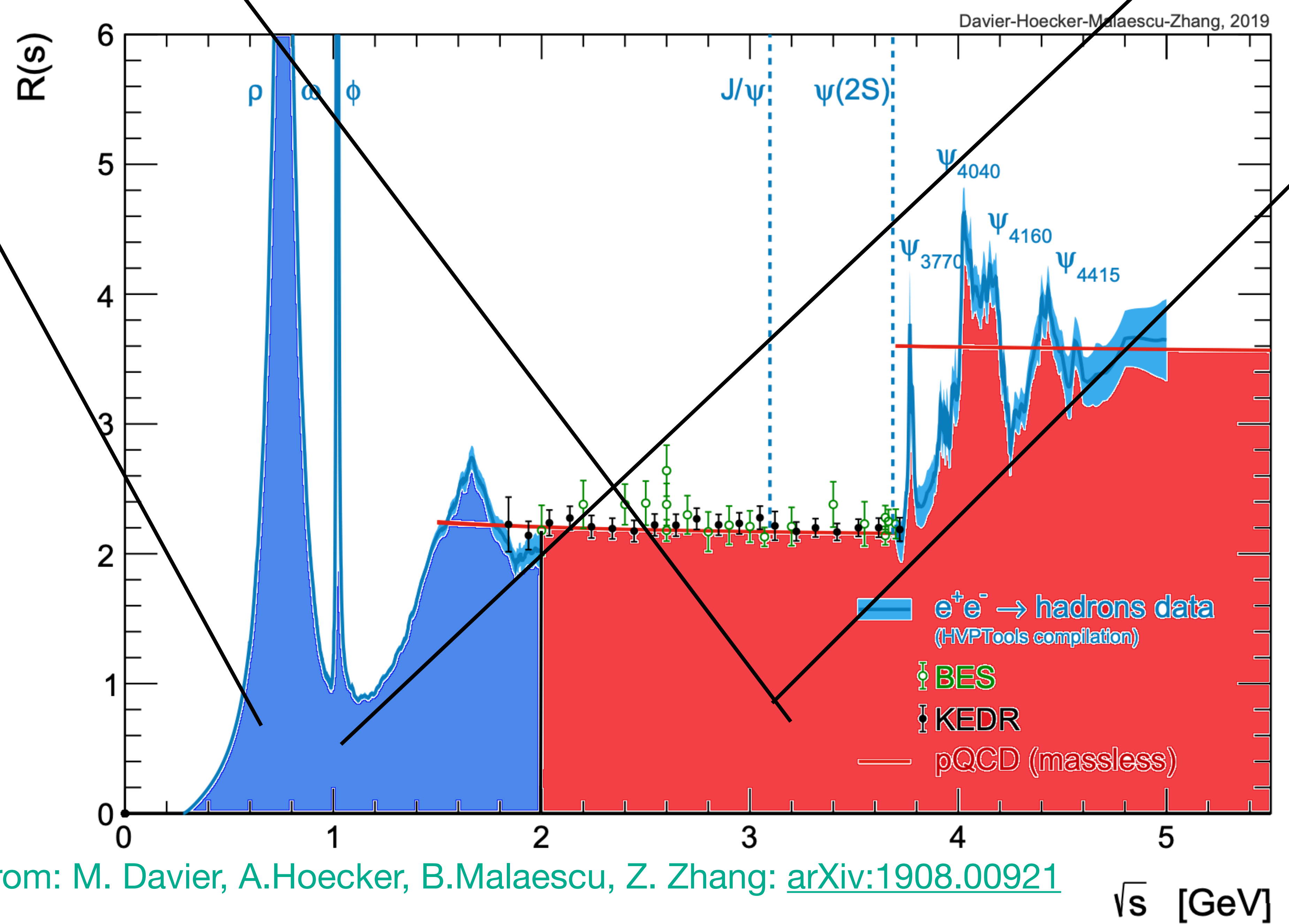
But, sensitivity to different regions of the integral is different……

a_μ^{had} $\Delta\alpha(M_Z^2)$

Total contribution per region

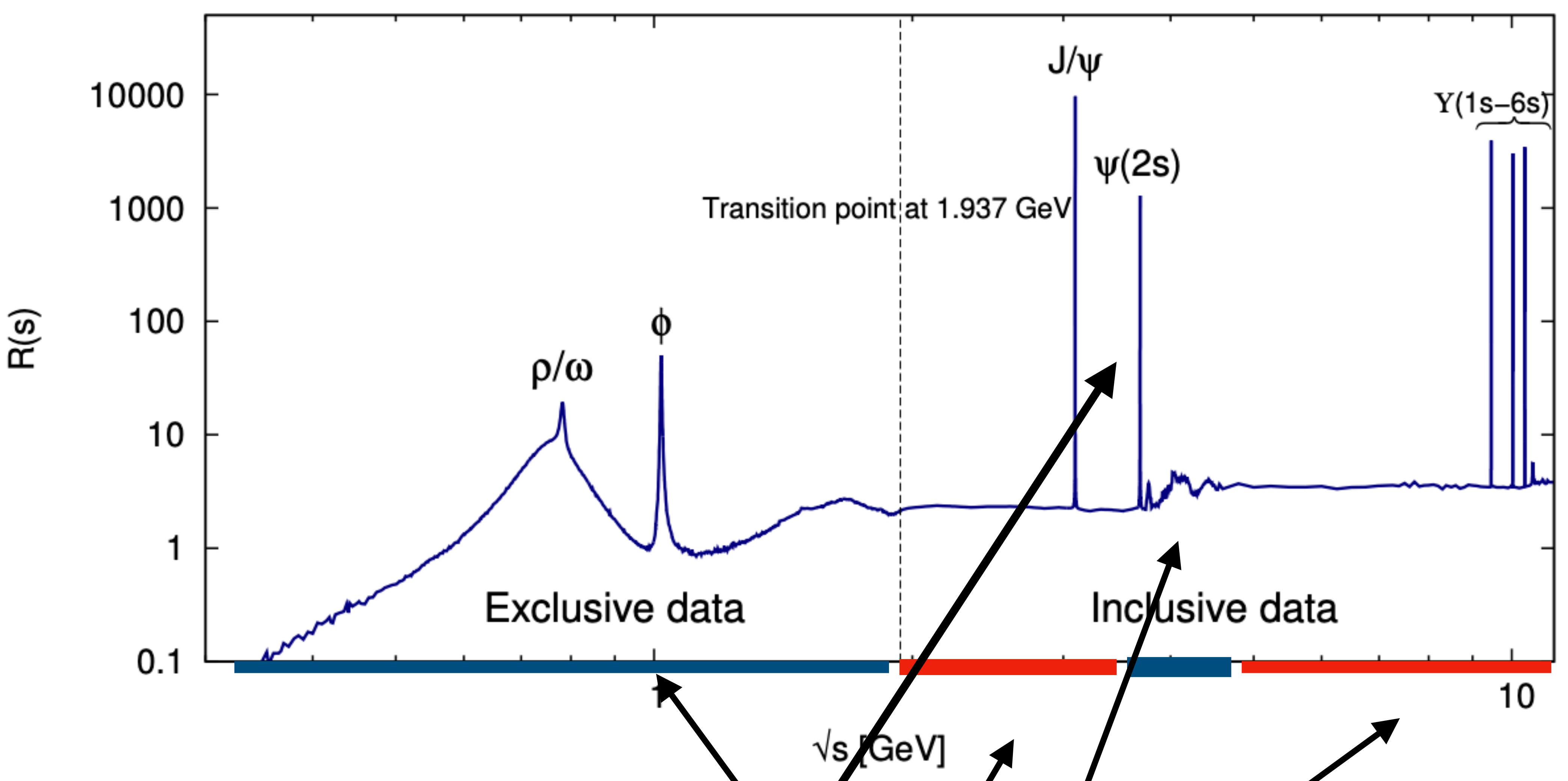






Plot modified from: M. Davier, A.Hoecker, B.Malaescu, Z. Zhang: arXiv:1908.00921

Integral over R



Region where pQCD is used

$$R(s) = 3\sum_n Q_n^2 \left[1 + \frac{\alpha_s(s)}{\pi} + \ldots\right] + \text{mass corrections}$$

Chetyrkin et al. hep-ph/9606230
Chetyrkin et al. hep-ph/0005139,
Harlander et al. hep-ph/0212294
Baikov et al. hep-ph/0801.1821
Maier et al. hep-ph/1110.5581
Baikov et al. hep-ph/1501.06739

Region data is used

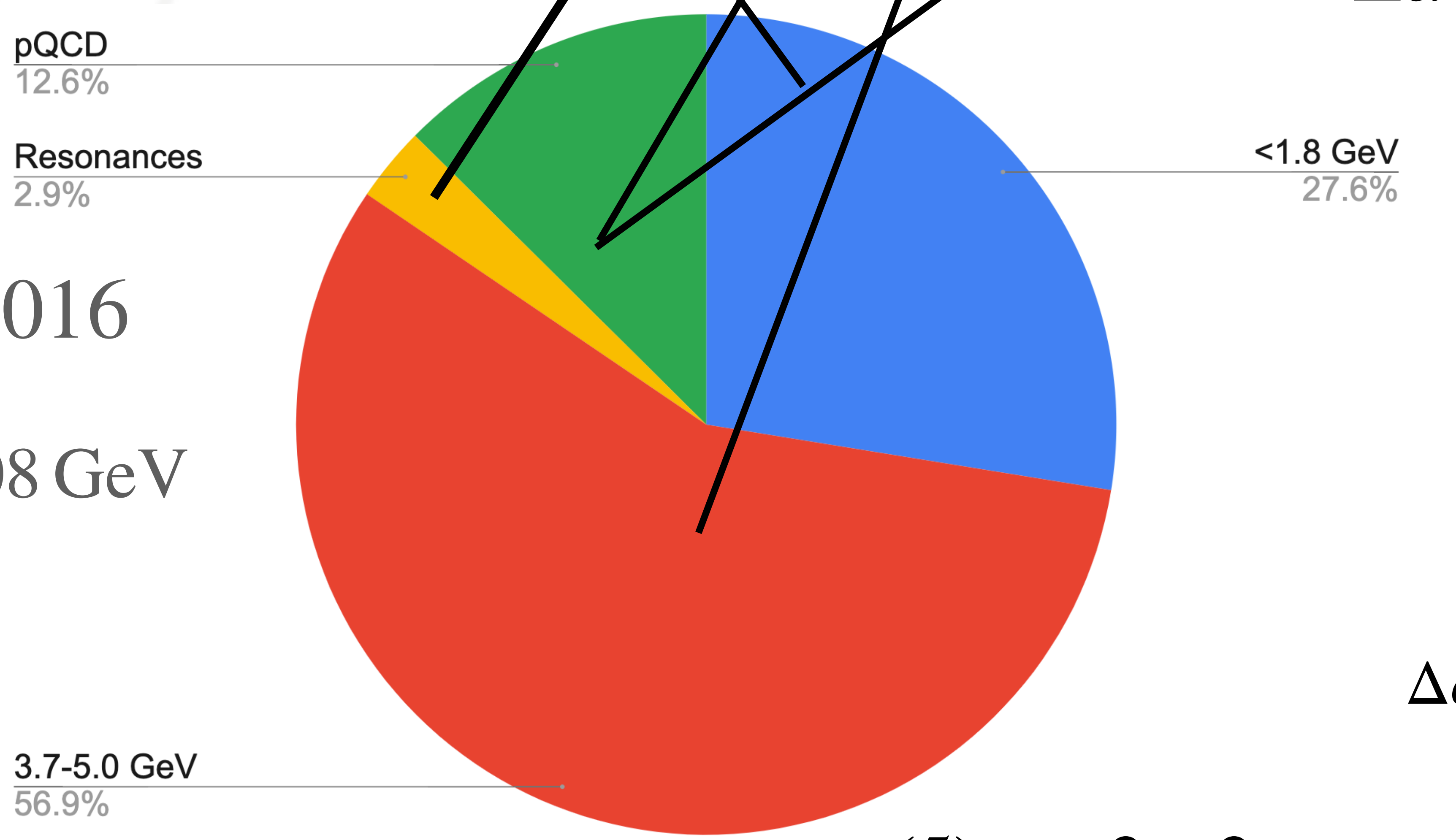
(A. Keshavarzi, et al uses data up to 11 GeV (plot taken from them)

$$\Delta\alpha^{(5)}(M_Z) = \Big[275.77 + 141\,\delta\hat{\alpha}_s + 0.7\,\delta\hat{m}_c - 1.3\,\delta\hat{m}_b \pm 0.67_{\text{c-thr}} \pm 0.19_{\text{trunc}} \pm 0.28_{\text{dual}}$$

$$\pm 0.38_{\text{dat}<1.8\,\text{GeV}} \pm 0.15_{J/\psi}\Big] \times 10^{-4} \qquad \text{(Timelike method, R input)}$$

$\delta\hat{\alpha}_s = 0.0016$

$\delta\hat{m}_c = 0.008\,\text{GeV}$



Distribution of $(\delta\Delta\alpha^{(5)}(M_Z^2))^2$

Main error associated to the charm quark

$$\Delta\alpha^{(c)}(M_Z^2) = \Big[78.72 + 27\,\delta\hat{\alpha}_s + 0.7\,\delta\hat{m}_c \pm 0.02_{\text{trunc}} \pm 0.67_{\text{c-thr}} \pm 0.13_{J/\psi} \pm 0.08_{\psi}\Big] \times 10^4 .$$

RGE method

RGE method

Steps:

1. Determine light quark contributions to $\hat{\Pi}(0,\mu^2)$ at a low energy scale.
2. Match the charm quark contribution.
3. Run to bottom quark, match it and run.
4. Convert back to the on-shell scheme (effective coupling).

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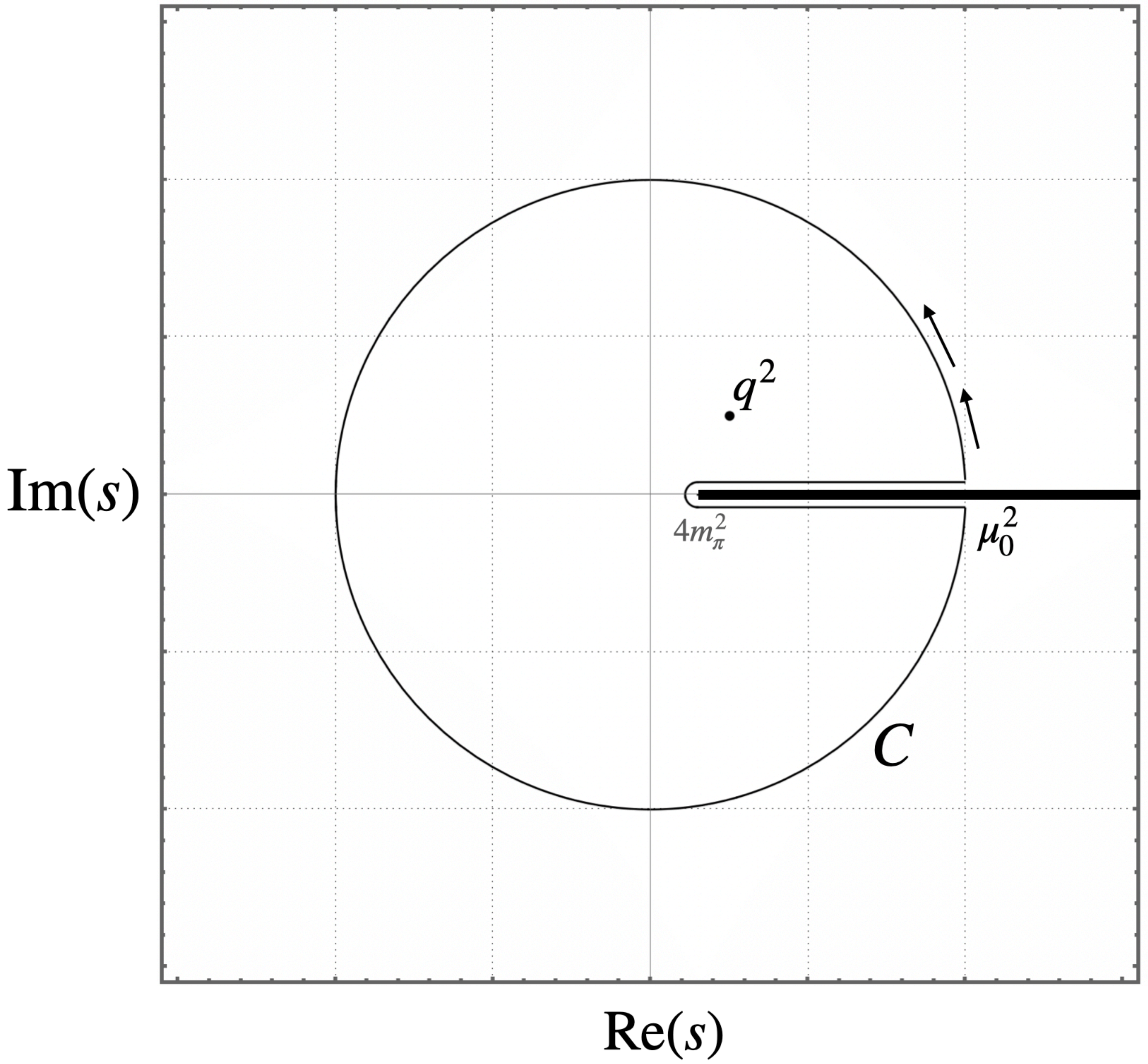
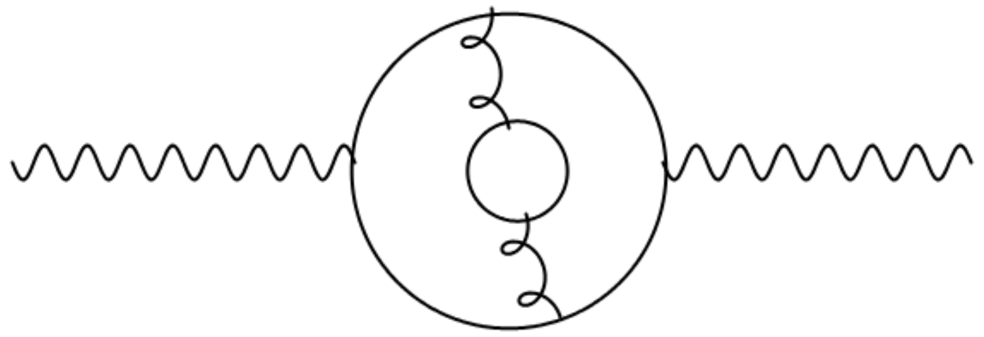
RGE method: light quark contribution



When need



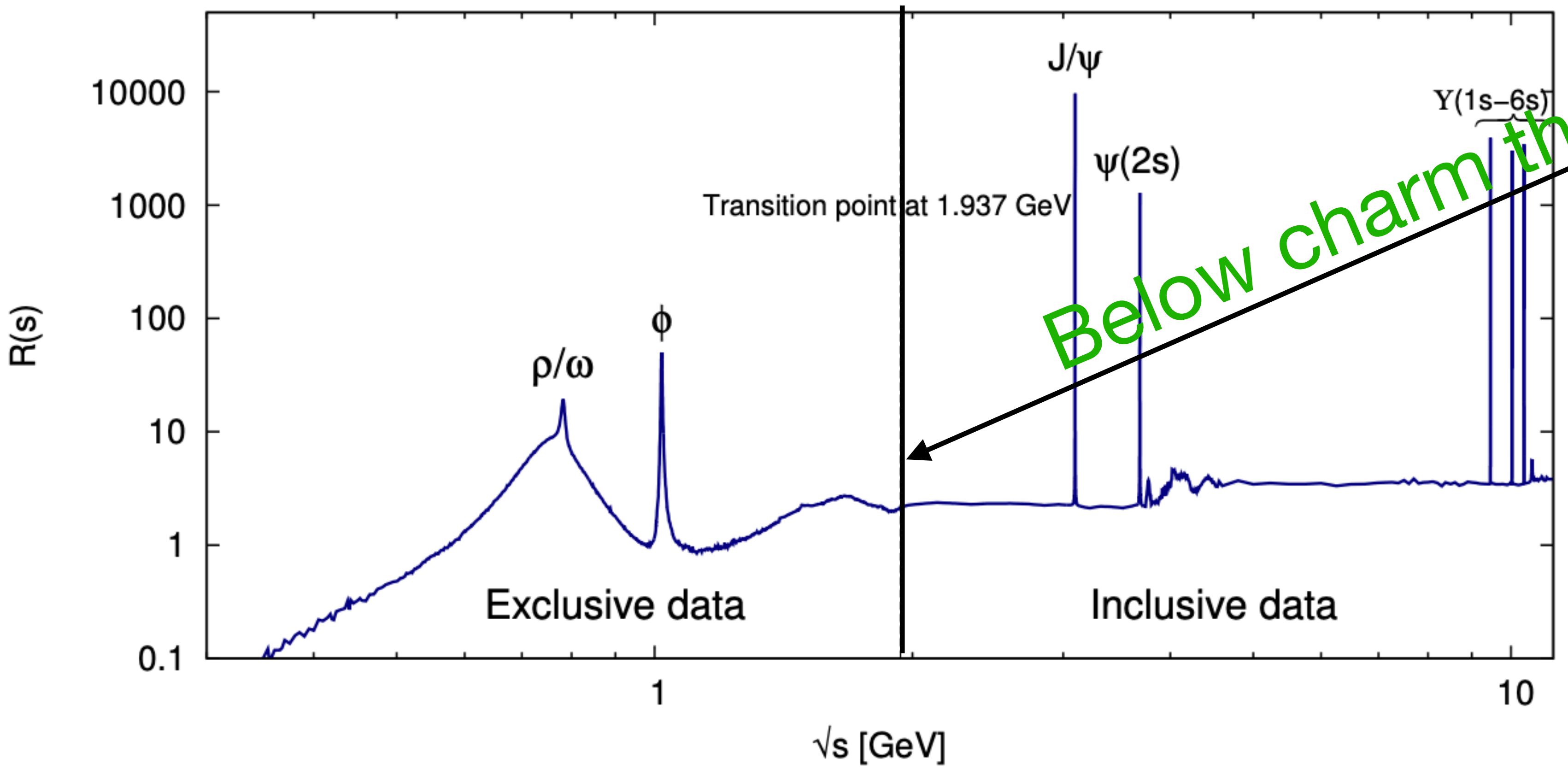
$$\hat{\alpha}(\mu^2) = \frac{\alpha}{1 - \Delta\hat{\alpha}(\mu^2)} \qquad \Delta\hat{\alpha}(\mu^2) \equiv 4\pi\alpha\hat{\Pi}(0,\mu^2)$$

So lets go back to our contour and set $q^2 = 0$ (J. Erler. hep-ph/9803453)

$$\hat{\Pi}_{\rm had}(0,\mu^2) = \frac{1}{12\pi^2}\int_{4m_\pi^2}^{\mu_0^2} \frac{R(s)}{s}ds + \frac{1}{2\pi i}\oint_{|s|=\mu_0^2} \frac{\hat{\Pi}^{(3)}_{\rm had}(s,\mu^2)}{s}ds$$

Gives 3 light quarks contribution

Below charm threshold!



Use data for the low part, and pQCD for the circle integral

$\hat{\Pi}_{\rm had}(s,\mu^2)$ is known up to order $\hat{\alpha}_s^3$ for massless quarks

Condensate effects are suppressed by two powers of $\hat{\alpha}_s$

RGE method

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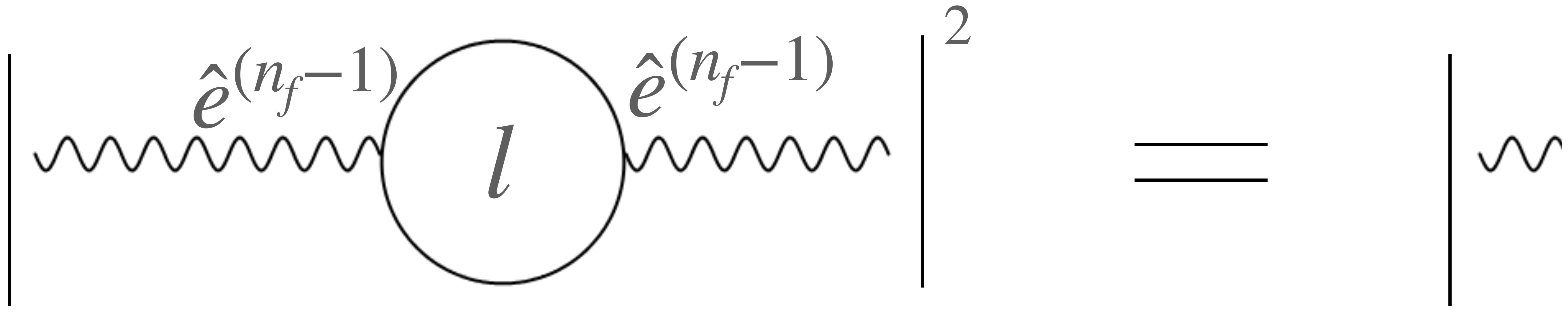
RGE method: matching

Go from a theory with n_l quark

Cross sections at low q^2 must be equiva

$$\hat{\alpha}^{(n_f-1)} = \xi\hat{\alpha}^{(n$$



$$\Delta\hat{\alpha}^{(n_f-1)}(\hat{m}_f^2) = \Delta\hat{\alpha}^{(n_f)}(\hat{m}_f^2) - \frac{15}{16}N_c Q_f^4 a^2\left(1 + \Delta\hat{\alpha}^{(n_f)}(\hat{m}_f)\right) - aQ_f^2\left\{\hat{a}_s^{(n_f)}\frac{13}{12}\right.$$

$$+ \hat{a}_s^{(n_f)\,2}\left[\frac{361}{1296}n_f + \frac{655}{144}\zeta_3 - \frac{3847}{864}\right]$$

$$+ \hat{a}_s^{(n_f)\,3}\left[-\frac{85637a_4}{1620} - \frac{656a_5}{27} - \frac{928399\zeta_2^2}{129600} - \frac{1289}{135}\zeta_2^2 l_2 - \frac{164}{81}\zeta_2 l_2^3\right.$$

$$+ \frac{85637\zeta_2 l_2^2}{6480} - \frac{49\zeta_5}{32} + \frac{42223463\zeta_3}{604800} - \frac{321165301}{21772800} + \frac{82 l_2^5}{405} - \frac{85637 l_2^4}{38880}$$

$$+ n_f\left(-\frac{17a_4}{27} + \frac{4487\zeta_2^2}{2160} + \frac{17}{108}\zeta_2 l_2^2 - \frac{21379\zeta(3)}{5184} - \frac{86101}{62208} - \frac{17 l_2^4}{648}\right)$$

$$\left.\left. + n_f^2\left(\frac{17897}{93312} - \frac{31}{216}\zeta_3\right)\right]\right\} - a\sum_{(l\neq f)} Q_l^2\left\{\hat{a}_s^{(n_f)\,2}\frac{295}{1296}\right.$$

$$+ \hat{a}_s^{(n_f)\,3}\left[\frac{67}{360}\zeta_2^2 + \frac{1}{9}\zeta_2 l_2^2 + \frac{163}{162}\zeta_3 - \frac{86369}{186624} - \frac{l_2^4}{54} - \frac{4a_4}{9}\right.$$

$$\left.\left. + \left(\frac{6625}{46656} - \frac{11\zeta_3}{108}\right)n_f\right]\right\} - aQ_f^2\hat{a}_s^{(n_f)\,3}\left\{\frac{2411}{6048} - \frac{365a_4}{36}\right.$$

$$\left. + \frac{2189\zeta_2^2}{576} + \frac{365}{144}\zeta_2 l_2^2 - \frac{25\zeta_5}{72} - \frac{6779\zeta_3}{1344} - \frac{365 l_2^4}{864}\right\}$$

$$- a\sum_{l\neq f} Q_f Q_l \hat{a}_s^{(n_f)\,3}\left\{-\frac{\zeta_2^2}{6} - \frac{25\zeta_5}{36} + \frac{655\zeta_3}{432} + \frac{515}{1296}\right\},$$

The matching conditions are known to order $\hat{\alpha}_s^3$.

Chetyrkin et al. hep-ph/9708255
Sturm hep-ph/1404.3433

RGE method

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RGE method: Running

$$\mu^2 \frac{d\hat{\alpha}}{d\mu^2} = \frac{\hat{\alpha}^2}{\pi}\beta \longrightarrow \mu^2 \frac{d}{d\mu^2}\Delta\hat{\alpha} = \frac{\alpha}{\pi}\beta$$

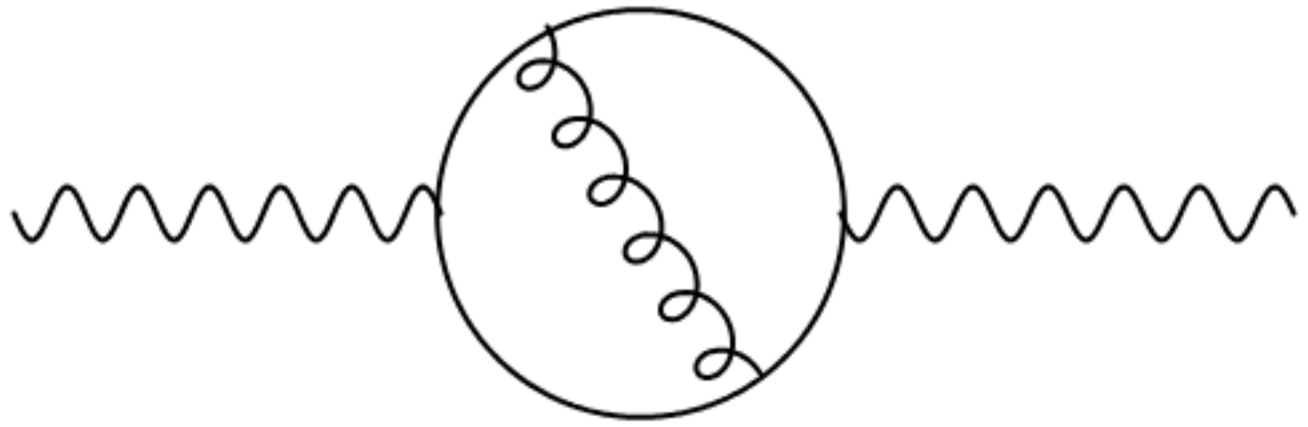
β known to 5 loops

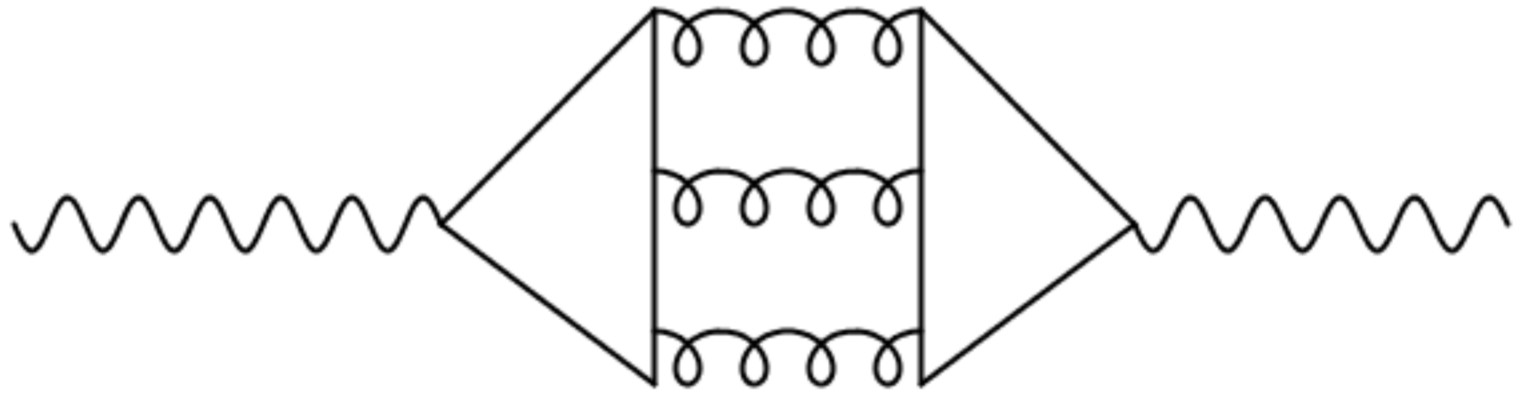
(Bikov et al. arXiv:1206.1284)

$$\beta = \left[\frac{1}{3}\sum_q K_q Q_q^2 + \sigma\left(\sum_q Q_q\right)^2\right],$$

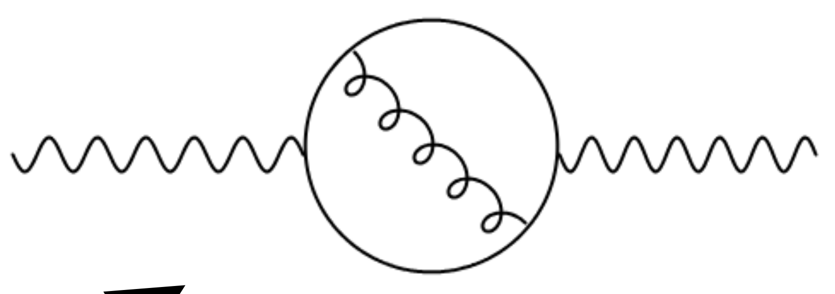
$$K_q = N_c\left[1 + \frac{\hat{\alpha}_s}{\pi} + \ldots\right]$$

$$\sigma = \left(\frac{\hat{\alpha}_s}{\pi}\right)^3\left(\frac{55}{216} - \frac{5}{9}\zeta_3\right) + \ldots$$

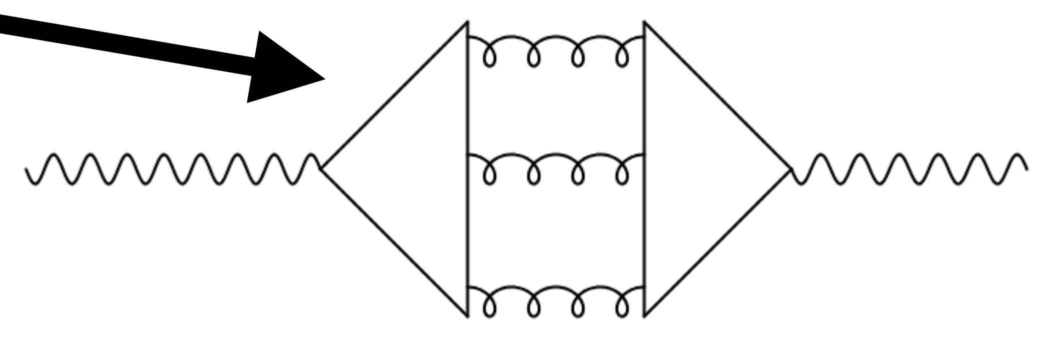




Renormalization group equation

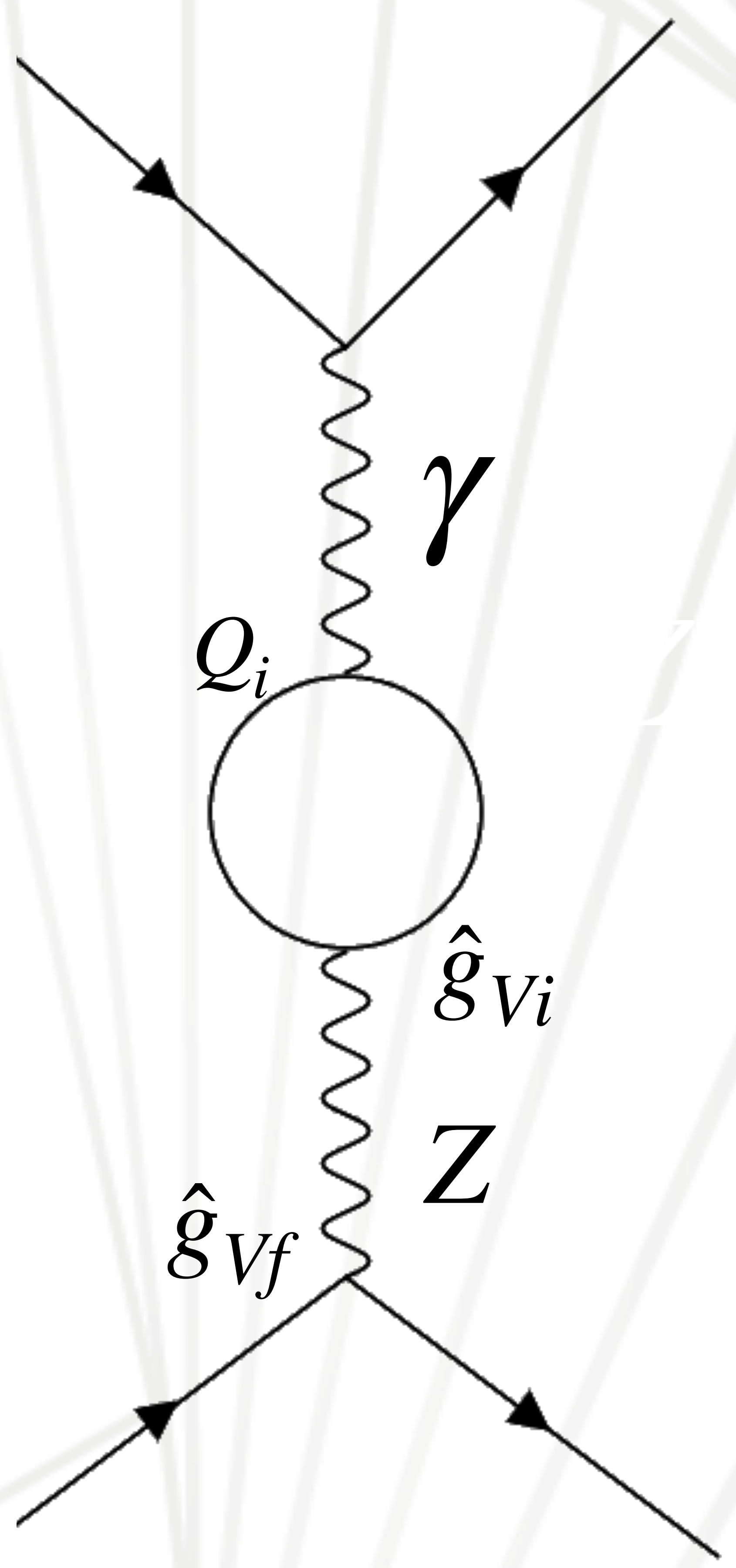


$$\frac{d\hat{\alpha}}{d\ln\mu^2} = \frac{\hat{\alpha}^2}{\pi}\left[\frac{1}{24}\sum_i K_i\gamma_i Q_i^2 + \sigma\left(\sum_Q Q_q^2\right)^2\right]$$



$$\hat{g}_{Vf} = T_f - 2Q_f^2 \sin^2\hat{\theta}$$

$$\frac{d\hat{g}_{Vf}}{d\ln\mu^2} = \frac{\hat{\alpha}^2}{\pi}\left[\frac{1}{24}\sum_i K_i\gamma_i \hat{g}_{V\,i} Q_i + 12\sigma\left(\sum_Q Q_q\right)\left(\sum_Q \hat{g}_{V\,q}\right)\right]$$



RGE method

Steps:

1. Determine light quark contributions to $\hat{\Pi}(0, \mu^2)$ at a low energy scale.
2. Match the charm quark contribution.
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4. Convert back to the on-shell scheme (effective coupling).

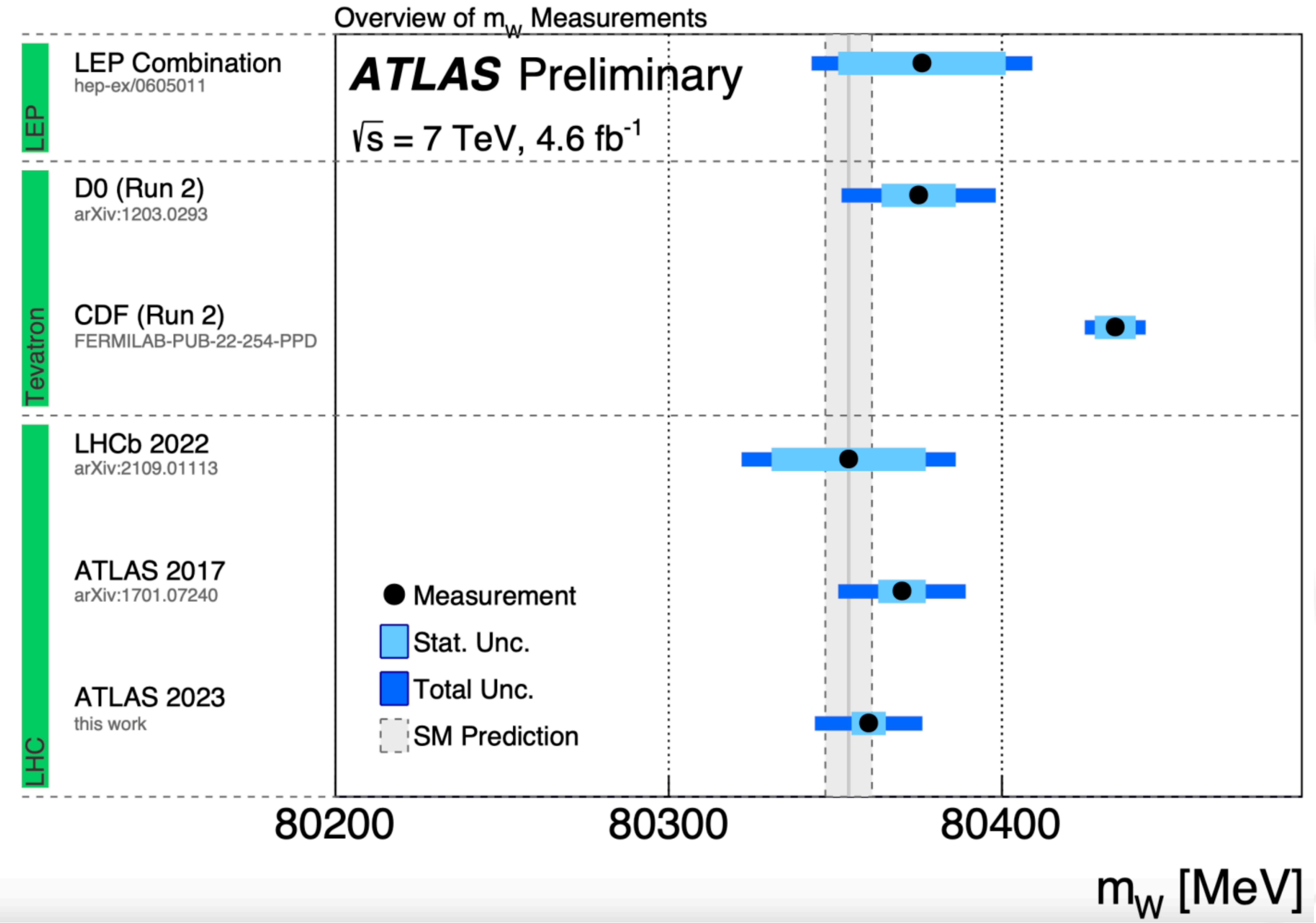
Use again

$$\Delta\alpha^{(5)}(M_Z^2) = \Delta\hat{\alpha}^{(5)}(\mu^2 = M_Z^2) - 4\pi\alpha \mathrm{Re}\left[\hat{\Pi}^{(5)}(M_Z^2, \mu^2 = M_Z^2)\right]$$

From the running

Scheme conversion

Some anomalies in the SM



Plot taken from: ATLAS collaboration, March 2023

More back ups

Atomic spectroscopy $R_\infty = \frac{\alpha^2 m_e c}{4\pi\hbar}$

1) Extract R_∞ from the data $\nu_{ij} = \varepsilon_j - \varepsilon_i$

$$\varepsilon_i = -\frac{R_\infty c}{n_i^2}(1+\delta_i)$$

2) Determine $\frac{\hbar}{m_e}$

From the bound g factor of the electron

$$\frac{\hbar}{m_e} = \frac{u}{m_e}\frac{M_X}{u}\frac{\hbar}{M_X}$$

Recoil velocity

cyclotron frequency of an ion in the constant magnetic field

From Robert Szafron (2019 Mainz Talk)