$pp \to \ell^+\ell^- + X$ via $\gamma\gamma$ and γZ

S. I. Godunov, E. K. Karkaryan, V. A. Novikov,
A. N. Rozanov, M. I. Vysotsky, E. V. Zhemchugov

based on
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Electroweak Interactions & Unified Theories

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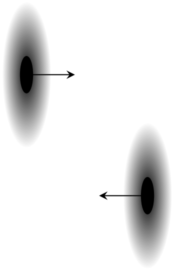
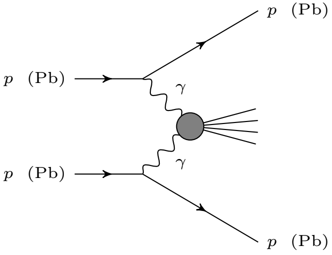
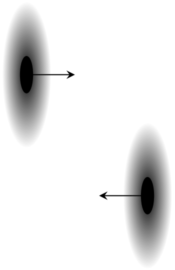
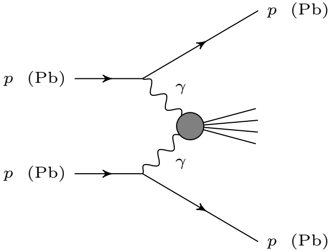
Ultraperipheral collisions (UPC) at the LHC





- It is possible to detect protons in forward detectors to reconstruct full kinematics.
- Accessible analytically with equivalent photons approximation (EPA).
- Formulae can be easily adopted for new particles (γ couples to electric charge).





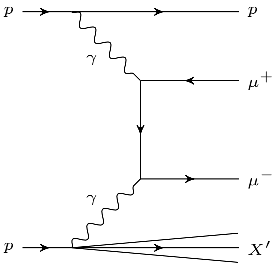
- It is possible to detect protons in forward detectors to reconstruct full kinematics.
- Accessible analytically with equivalent photons approximation (EPA).
- Formulae can be easily adopted for new particles (γ couples to electric charge).

$$Q^2 \lesssim (200 \text{ MeV})^2 \quad \Rightarrow \quad \frac{Q^2}{M_Z^2 + Q^2} \sim 10^{-5}$$

Weak interaction contribution is negligible

Semi-inclusive processes with proton(s) in forward detector

ATLAS [PRL 125, 261801 (2020)]



Experimental selections:

- $p_T > 15$ GeV.
- $|\eta| < 2.4$.
- $p_T^{\mu\mu} < 5$ GeV.
- 20 GeV $< m_{\mu\mu} <$ 70 GeV or $m_{\mu\mu} >$ 105 GeV.
- At least one proton hits a forward detector.

$\gamma\gamma$ and γZ fusion are not the only diagrams! (bremsstrahlung-like: production of real Z)

See our recent results (Eur. Phys. J. C82, 1055 (2022));

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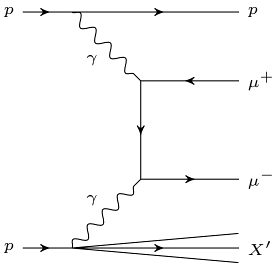

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For $(p_T^{\mu\mu})^2 \gg 1$ GeV^2 we have $Q_2^2 \approx (p_T^{\mu\mu})^2$ and

$$\frac{Q^2}{M_Z^2 + Q^2} \approx \frac{(p_T^{\mu\mu})^2}{M_Z^2 + (p_T^{\mu\mu})^2} \sim 10^{-3}$$

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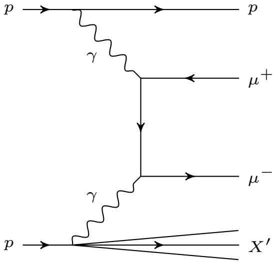

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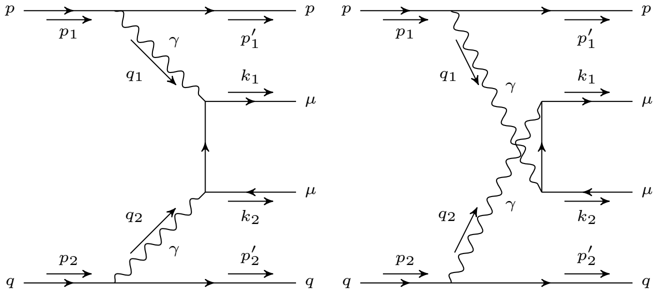
But what if we remove cuts?

Inelastic part

Inelastic part dominates when no cuts applied, see our paper JETP Lett. 115, 59 (2022).

Within the parton approximation:

$$\sigma_{\text{inelastic}}(pp \to p\mu^+\mu^- X) = \sum_q \sigma(pq \to p\mu^+\mu^- q)$$



$Q_2^2 \equiv -q_2^2$ is not necessarily much smaller than W^2; special consideration required.

We closely follow review Budnev *et al*, Phys. Rep. 15, 181 (1975).

$\gamma\gamma$ fusion cross section

$$\mathrm{d}\sigma_{pq\to p\mu^+\mu^- q} = 2\cdot\frac{Q_q^2(4\pi\alpha)^2}{(q_1^2)^2(q_2^2)^2}\left(q_1^2\rho^{(1)}_{\mu\nu}\right)\left(q_2^2\rho^{(2)}_{\alpha\beta}\right)M_{\mu\alpha}M^*_{\nu\beta}\times$$
$$\times\frac{(2\pi)^4\delta^{(4)}(q_1+q_2-k_1-k_2)\,\mathrm{d}\Gamma}{4\sqrt{(p_1p_2)^2-m_p^4}}\frac{\mathrm{d}^3p_1'}{(2\pi)^3 2E_1'}\frac{\mathrm{d}^3p_2'}{(2\pi)^3 2E_2'}f_q(x,Q_2^2)\,\mathrm{d}x$$

$$\rho^{(1)}_{\mu\nu} = -\left(g_{\mu\nu}-\frac{q_{1\mu}q_{1\nu}}{q_1^2}\right)G_M^2(Q_1^2)-\frac{(2p_1-q_1)_\mu(2p_1-q_1)_\nu}{q_1^2}D(Q_1^2),$$
$$D(Q_1^2) = \frac{G_E^2(Q_1^2)+(Q_1^2/4m_p^2)G_M^2(Q_1^2)}{1+Q_1^2/4m_p^2}.$$

Here $Q_1^2=-q_1^2$, and $G_E(Q_1^2)$, $G_M(Q_1^2)$ are the Sachs form factors of the proton. For the latter we use the dipole approximation:

$$G_E(Q^2)=\frac{1}{(1+Q^2/\Lambda^2)^2},\quad G_M(Q^2)=\frac{\mu_p}{(1+Q^2/\Lambda^2)^2},\quad \Lambda^2=\frac{12}{r_p^2}=0.66\ \mathrm{GeV}^2,$$

where $\mu_p = 2.79$ and $r_p = 0.84$ fm.

$$\rho^{(2)}_{\alpha\beta} = -\frac{1}{2q_2^2}\,\mathrm{Tr}\{\hat{p}_2'\gamma_\alpha\hat{p}_2\gamma_\beta\} = -\left(g_{\alpha\beta}-\frac{q_{2\alpha}q_{2\beta}}{q_2^2}\right)-\frac{(2p_2-q_2)_\alpha(2p_2-q_2)_\beta}{q_2^2}.$$

Helicity representation

Calculations are performed in the c.m.s of the colliding photons.

$$e_1^+ = \frac{1}{\sqrt{2}}(0,-1,-i,0), \quad e_1^- = \frac{1}{\sqrt{2}}(0,1,-i,0), \quad e_1^0 = \frac{i}{\sqrt{-q_1^2}}(\tilde{q},0,0,\tilde{\omega}_1),$$
$$e_2^+ = \frac{1}{\sqrt{2}}(0,1,-i,0), \quad e_2^- = \frac{1}{\sqrt{2}}(0,-1,-i,0), \quad e_2^0 = \frac{i}{\sqrt{-q_2^2}}(-\tilde{q},0,0,\tilde{\omega}_2).$$

$$\rho_i^{\mu\nu} = \sum_{a,b} (e_i^{a\mu})^* e_i^{b\nu} \rho_i^{ab},$$

$$\rho_i^{ab} = (-1)^{a+b} e_i^{a\mu} \left(e_i^{b\nu}\right)^* \rho_i^{\mu\nu},$$

$$\begin{aligned}\rho_1^{\mu\nu}\rho_2^{\alpha\beta} M_{\mu\alpha} M^*_{\nu\beta} &= (-1)^{a+b+c+d}\rho_1^{ab}\rho_2^{cd} M_{ac} M^*_{bd} = \\ &= \rho_{++}^{(1)}\rho_{++}^{(2)}|M_{++}|^2 + \rho_{++}^{(1)}\rho_{--}^{(2)}|M_{+-}|^2 + \rho_{++}^{(1)}\rho_{00}^{(2)}|M_{+0}|^2 + \\ &+ \rho_{--}^{(1)}\rho_{++}^{(2)}|M_{-+}|^2 + \rho_{--}^{(1)}\rho_{00}^{(2)}|M_{-0}|^2 + \rho_{--}^{(1)}\rho_{--}^{(2)}|M_{--}|^2.\end{aligned}$$

M_{++} and M_{--} are not vanishing in the limit $m \to 0$ due to chiral anomaly!

$$\rho_{++}^{(1)} = \rho_{--}^{(1)} \approx D\left(Q_1^2\right)\frac{2E^2 q_{1\perp}^2}{\omega_1^2 Q_1^2},$$

$$\rho_{++}^{(2)} = \rho_{--}^{(2)} \approx \frac{2x^2 E^2 q_{2\perp}^2}{\omega_2^2 Q_2^2}, \quad \rho_{00}^{(2)} \approx \frac{4x^2 E^2 q_{2\perp}^2}{\omega_2^2 Q_2^2}.$$

$\gamma\gamma$ fusion

$$\frac{\mathrm{d}\sigma_{pq\to p\mu^+\mu^- q}}{\mathrm{d}W} = W \int\limits_{\frac{W^4}{36\gamma^2 s}}^{s-W^2} \sigma_{\gamma\gamma^*\to\mu^+\mu^-}\left(W^2, Q_2^2\right)\mathrm{d}Q_2^2 \int\limits_{\frac{1}{2}\ln\left(\frac{W^2+Q_2^2}{s}\cdot\max\left(1,\frac{m_p^2}{9Q_2^2}\right)\right)}^{\frac{1}{2}\ln\frac{s}{W^2+Q_2^2}} n_p(\omega_1)\frac{\mathrm{d}n_q(\omega_2)}{\mathrm{d}Q_2^2}\mathrm{d}y,$$

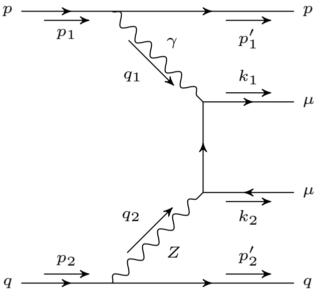
$$\sigma_{\gamma\gamma^*\to\mu^+\mu^-} = \sigma_{TT} + \sigma_{TS},$$

$$\sigma_{TS} \approx \frac{16\pi\alpha^2 W^2 Q_2^2}{(W^2+Q_2^2)^3}, \quad \sigma_{TT} \approx \frac{4\pi\alpha^2}{W^2}\left[\frac{1+Q_2^4/W^4}{(1+Q_2^2/W^2)^3}\ln\frac{W^2}{m^2} - \frac{(1-Q_2^2/W^2)^2}{(1+Q_2^2/W^2)^3}\right],$$

$$\omega_1 = \sqrt{W^2+Q_2^2}\cdot e^{y}/2, \quad \omega_2 = \sqrt{W^2+Q_2^2}\cdot e^{-y}/2,$$

$$n_p(\omega_1) = \frac{\alpha}{\pi\omega_1}\int\limits_0^\infty \frac{D\left(Q_1^2\right)q_{1\perp}^2\,\mathrm{d}q_{1\perp}^2}{Q_1^4},$$ (can be integrated analytically; see arXiv:2308.01169)

$$\frac{\mathrm{d}n_q(\omega_2)}{\mathrm{d}Q_2^2} = \frac{\alpha Q_q^2}{\pi\omega_2}\int\limits_{\sqrt{\frac{W^2+Q_2^2}{s}}\cdot e^{-y}\cdot\max\left\{1,\frac{m_p}{3\sqrt{Q_2^2}}\right\}}^{1} \frac{Q_2^2-(\omega_2/3x\gamma)^2}{Q_2^4} f_q\left(x,Q_2^2\right)\mathrm{d}x.$$



$$\tilde{\rho}^{(2)}_{\alpha\beta} = -\frac{1}{2q_2^2}\left[\frac{g_V^q}{2}\,\mathrm{Tr}\{\hat{p}_2^{'}\gamma_\alpha\hat{p}_2\gamma_\beta\} + \frac{g_A^q}{2}\,\mathrm{Tr}\{\hat{p}_2^{'}\gamma_\alpha\hat{p}_2\gamma_\beta\gamma_5\}\right],$$

$$\tilde{\tilde{\rho}}^{(2)}_{\alpha\beta} = -\frac{1}{2q_2^2}\,\mathrm{Tr}\left\{\hat{p}_2^{'}\left(\frac{g_V^q}{2}\gamma_\alpha + \frac{g_A^q}{2}\gamma_\alpha\gamma_5\right)\hat{p}_2\left(\frac{g_V^q}{2}\gamma_\beta + \frac{g_A^q}{2}\gamma_\beta\gamma_5\right)\right\}.$$

$$\tilde{\rho}^{(2)}_{ab} \approx \frac{g_V^q}{2}\rho^{(2)}_{ab}, \quad \tilde{\tilde{\rho}}^{(2)}_{ab} \approx \frac{(g_V^q)^2 + (g_A^q)^2}{4}\rho^{(2)}_{ab}.$$

Amplitudes

$$\mathcal{A} = A_\mu \cdot \bar{p}'\gamma_\mu p/q_1^2, \; A_\mu = \frac{eQ_q}{q_2^2}\bar{q}'\gamma_\alpha q M^\gamma_{\mu\alpha} + \frac{e}{s_W c_W (q_2^2 - M_Z^2)}\bar{q}'\left[\frac{g_V^q}{2}\gamma_\alpha + \frac{g_A^q}{2}\gamma_\alpha\gamma_5\right] q M^Z_{\mu\alpha}.$$

For the $\gamma\gamma \to \mu^+\mu^-$ and $\gamma Z \to \mu^+\mu^-$ amplitudes we have:

$$M^\gamma_{\mu\alpha} = Q_\mu^2 e^2 \left[\bar{\mu}\gamma_\mu \frac{1}{\hat{k}_1 - \hat{q}_1 - m}\gamma_\alpha \mu + \bar{\mu}\gamma_\alpha \frac{1}{\hat{q}_1 - \hat{k}_2 - m}\gamma_\mu \mu\right],$$

$$M^Z_{\mu\alpha} = \frac{Q_\mu e^2}{s_W c_W}\left\{\frac{g_V^\mu}{2}\left[\bar{\mu}\gamma_\mu \frac{1}{\hat{k}_1 - \hat{q}_1 - m}\gamma_\alpha \mu + \bar{\mu}\gamma_\alpha \frac{1}{\hat{q}_1 - \hat{k}_2 - m}\gamma_\mu \mu\right] + \frac{g_A^\mu}{2}[\gamma_\alpha \to \gamma_\alpha\gamma_5]\right\} =$$

$$= \frac{1}{s_W c_W}\frac{g_V^\mu}{2Q_\mu}M^\gamma_{\mu\alpha} + \frac{Q_\mu e^2}{s_W c_W}\frac{g_A^\mu}{2}\left[\bar{\mu}\gamma_\mu \frac{1}{\hat{k}_1 - \hat{q}_1 - m}\gamma_\alpha\gamma_5 \mu + \bar{\mu}\gamma_\alpha\gamma_5 \frac{1}{\hat{q}_1 - \hat{k}_2 - m}\gamma_\mu \mu\right].$$

$$|\mathcal{A}|^2 \equiv \varkappa |\mathcal{A}_{\gamma\gamma}|^2, \qquad \varkappa\left(Q_2^2\right) = 1 + 2\cdot\frac{g_V^\mu}{Q_\mu}\cdot\frac{g_V^q}{Q_q}\cdot\lambda + \frac{(g_V^\mu)^2 + (g_A^\mu)^2}{Q_\mu^2}\cdot\frac{(g_V^q)^2 + (g_A^q)^2}{Q_q^2}\cdot\lambda^2,$$

$$\lambda \equiv \frac{1}{(2 s_W c_W)^2 (1 + M_Z^2/Q_2^2)}.$$

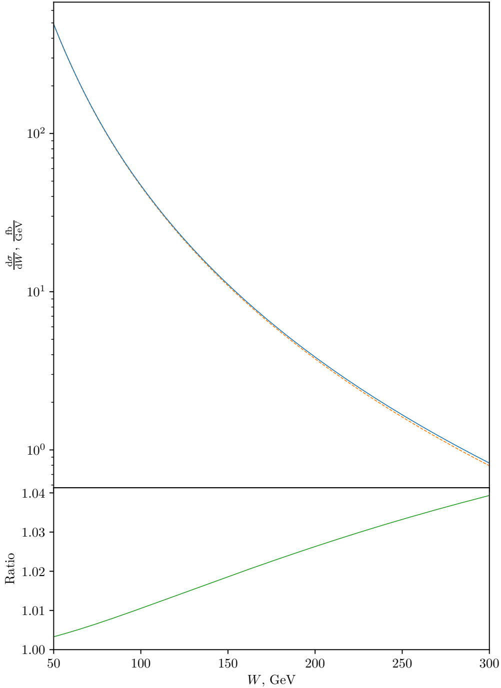
Cross section

$$\frac{\mathrm{d}\sigma_{pp\to p\mu^+\mu^- X}}{\mathrm{d}W} = \frac{4\alpha W}{\pi}\sum_q Q_q^2 \int\limits_{\frac{W^4}{36\gamma^2 s}}^{s} \frac{\sigma_{\gamma\gamma^*\to\mu^+\mu^-}\left(W^2, Q_2^2\right)}{(W^2+Q_2^2)Q_2^4} \cdot \varkappa\left(Q_2^2\right) \cdot \mathrm{d}Q_2^2 \times$$

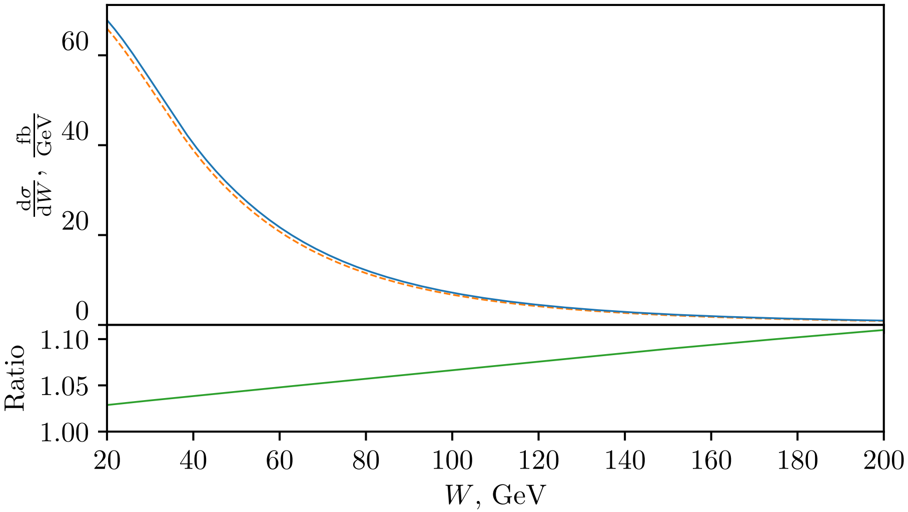
$$\times \int\limits_{\frac{W^2+Q_2^2}{s}\cdot\max\left(1,\frac{m_p}{3\sqrt{Q_2^2}}\right)}^{1} \mathrm{d}x f_q(x, Q_2^2) \int\limits_{\frac{1}{2}\ln\left(\frac{W^2+Q_2^2}{x^2 s}\cdot\max\left(1,\frac{m_p^2}{9Q_2^2}\right)\right)}^{\frac{1}{2}\ln\frac{s}{W^2+Q_2^2}} \omega_1 n_p(\omega_1)\left[Q_2^2 - (\omega_2/3x\gamma)^2\right]\mathrm{d}y,$$

$$\varkappa\left(Q_2^2\right) = 1 + 2\cdot\frac{g_V^\mu}{Q_\mu}\cdot\frac{g_V^q}{Q_q}\cdot\lambda + \frac{(g_V^\mu)^2+(g_A^\mu)^2}{Q_\mu^2}\cdot\frac{(g_V^q)^2+(g_A^q)^2}{Q_q^2}\cdot\lambda^2,$$

$$\lambda \equiv \frac{1}{(2s_W c_W)^2\left(1+M_Z^2/Q_2^2\right)}.$$

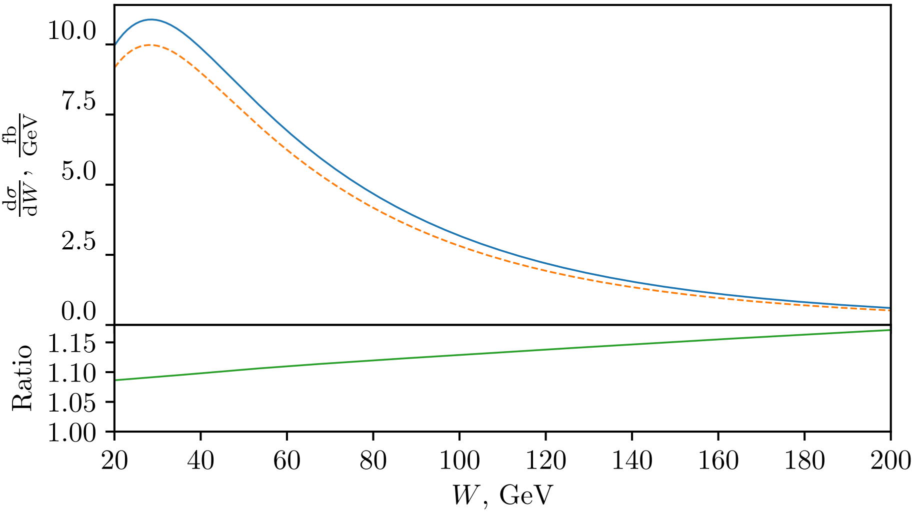

$\frac{d\sigma}{dW}$, $\frac{fb}{GeV}$
10^2
10^1
10^0
Ratio
1.04
1.03
1.02
1.01
1.00
50
100
150
200
250
300
W, GeV

$\hat{Q}_2 = 30$ GeV



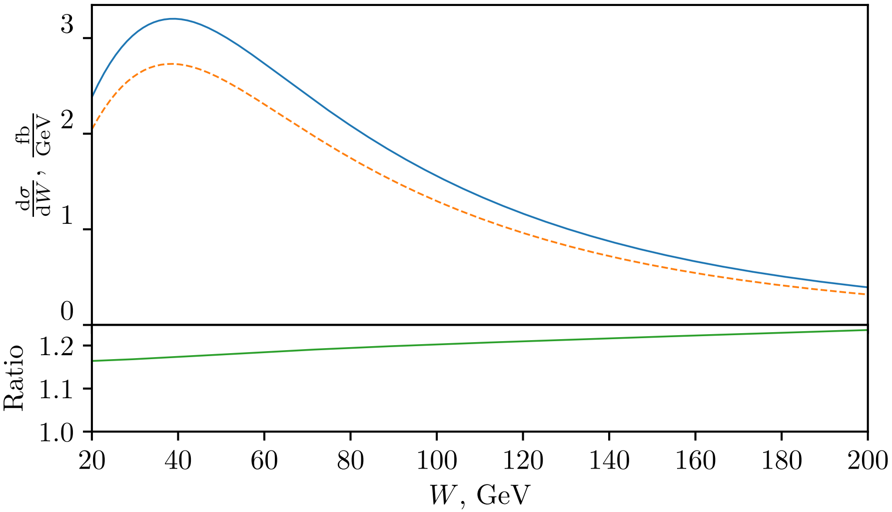
60
40
20
0
$\frac{d\sigma}{dW}$, $\frac{fb}{GeV}$
1.10
1.05
1.00
Ratio
20
40
60
80
100
120
140
160
180
200
W, GeV

$\hat{Q}_2 = 50$ GeV



$\frac{d\sigma}{dW}$, $\frac{fb}{GeV}$
10.0
7.5
5.0
2.5
0.0
Ratio
1.15
1.10
1.05
1.00
20
40
60
80
100
120
140
160
180
200
W, GeV

$\hat{Q}_2 = 70$ GeV


$\frac{d\sigma}{dW}$, $\frac{fb}{GeV}$
0
1
2
3
Ratio
1.0
1.1
1.2
20
40
60
80
100
120
140
160
180
200
W, GeV

$Q_2^2 \gg M_Z^2$

In the limit $Q_2^2 \gg M_Z^2$ the function $\varkappa(Q_2^2)$ reaches its maximum value:

$$\varkappa\left(Q_2^2\right) \approx 1 + 2 \cdot \frac{g_V^\mu}{Q_\mu} \cdot \frac{g_V^q}{Q_q} \cdot \frac{1}{\left(2 s_W c_W\right)^2} + \frac{\left(g_V^\mu\right)^2 + \left(g_A^\mu\right)^2}{Q_\mu^2} \cdot \frac{\left(g_V^q\right)^2 + \left(g_A^q\right)^2}{Q_q^2} \cdot \frac{1}{\left(2 s_W c_W\right)^4} \approx$$

$$\approx \begin{cases} 1.35 & \text{for} \quad q = u,\ \bar{u},\ c,\ \bar{c}; \\ 2.76 & \text{for} \quad q = d,\ \bar{d},\ s,\ \bar{s},\ b,\ \bar{b}. \end{cases}$$

However, to get an idea of what the correction in principle might be, one should know how much up- and down-type quarks contribute to the cross section in photon–photon fusion. For $\hat{Q}_2 = 70$ GeV and $W = 20$ GeV, u quark gives 41 % of the cross section, $\bar{u}$ gives 16 %, d — 7 %, $\bar{d}$ — 4 %, $s + \bar{s}$ — 7 %, $c + \bar{c}$ — 21 %, $b + \bar{b}$ — 4 %. Up-type quarks combined $(u + \bar{u} + c + \bar{c})$ give 78 %. Consequently, the asymptotic value of the enhancement is $0.78 \cdot 1.35 + 0.22 \cdot 2.76 \approx 1.6$.

Conclusions

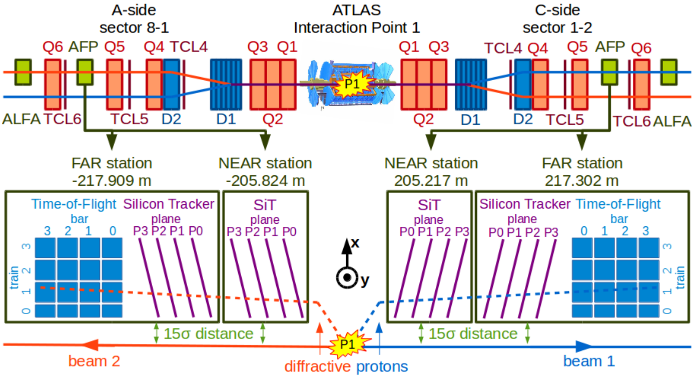
- For ultraperipheral collisions weak interaction correction is negligible.
- Weak interaction correction to the lepton pair production in semi-exclusive process gives few percent increase of the production cross section.
- When the lower limit on the net transverse momentum of the produced pair is set, the correction goes up and can reach 20 %.
- Numerical calculations were performed with the help of `libepa` (https://github.com/jini-zh/libepa) — a library for calculations of cross sections of ultraperipheral collisions (and beyond!) under the equivalent photons approximation.

  Now with description of included physics, arXiv:2311.01353

Backup slides

Forward detectors

[1909.10827]



Distance from the IP, m	200	420
ξ range	0.015–0.15	0.002–0.02
6.5 TeV p energy loss, GeV	97.5–975	13–130
in the center-of-mass frame, MeV	14–141	1.9–19
0.5 PeV ^{208}Pb energy loss, TeV	7.8–78	1.0–10
in the center-of-mass frame, GeV	2.9–29	0.37–3.7

Semi-inclusive cross section

- Experiment: 7.2 ± 1.6 (stat.) $\pm\, 0.9$ (syst.) $\pm\, 0.2$ (lumi.) fb.

- Exclusive process ($pp \to pp\mu^+\mu^-$): 8.6 fb.
- Inclusive process ($pp \to pX\mu^+\mu^-$): 9.2 fb.

Survival factor should reduce the cross section by up to $\sim 30\%$
(10% for the elastic cross section;
$\sim 50\%$ for the inelastic cross section according to MC simulations).