(Progress-in) Unitarity triangles and CP violation

Amarjit Soni

(BNL-HET)

EW Moriond 2024

03/28/24

Valuable inputs from: Buras, Cirigliano, D'Ambrosio, Isidori, Martinelli, Pich…….

Citations Incomplete; apologies

Outline

- Motivation: It is exceedingly important to determine UTs as precisely as possible….
- Progress in lattice eps'….implications for both UTs though crucial for KUT
- K UT
- B UT: esp gamma
- Summary

Main: (Old) and new points

- **Naturalness assumed throughout:**
- **eps': Periodic Boundary Condition appear promising**
- **[with RBC-UKQCD]**
- **Improving LD contribution to K+ => pi+ nu nu [with Enrico Lunghi]**
- **K0=> pi0 l+ l-: should help significantly in constraining the extremely challenging gold plated mode: KL => pi0 nu nu. [with Stefan Schacht]**

Reg gamma : [ADS revisit] path involving one pi0 stressed esp. promising for Belle-II..May be also for LHCb.

Main points for 40+years on 'lattice' eps' effort

- **Calculational framework for K=> pi pi & eps'**
- **Obstacles aglore and major break-throughs**

- **Lattice chiral symmetry even for a finite non-vanishing lattice spacing! : DWQ**

  Kaplan; Shamir; Blum + AS

- **Direct K=> pi pi w/o ChPT using finite vol correlation functions**

  Lellouch-Luscher

- **Non-perurbative renormalization**

  Martinelli + Sachrajda... NPR

- **1st [prot-type] demonstration....~2015**
- **Difficulty therein : strong I=0 pi pi phase**
- **1st complete result with GPBC, 2020**

  GPBC, Chris Kelly

- **2nd independent method (PBC) developed, 2023**

  PBC, Masaaki Tomii

- **Lattice applications to K and B-UTs**

Recapitulate: Many fascinating aspects of kaons=> led to several profoundly important discoveries in Particle Physics

- I: $\Delta I = 1/2$ Rule / Puzzle

K^+: $\bar{s}$, u $\qquad$ K^0: $\bar{s}$, d $\quad K^0 \to \{K_S, K_L\}$ $\qquad I = 1/2$

$\to 2\pi \, (I=2, \Delta I = 3/2$ only$)$ $\qquad \to 2\pi \, (I = 0, 2; \Delta I = 1/2, 3/2)$

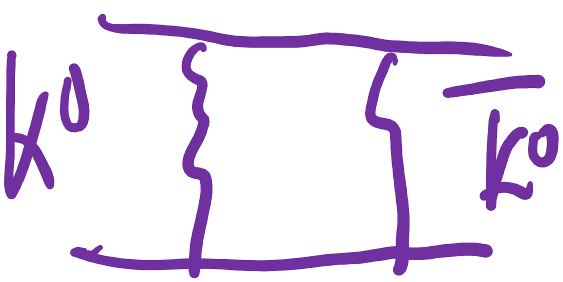
$\tau_{K^+}/\tau_{K_S} \sim 450! \gg 1 \Rightarrow \Delta I =$ Dominance/Rule/Puzzle

III Indirect CP violation

BNL 1964 Fitch, Cronin, Christensen + Turlay

$$\frac{A(K_L \to \pi\pi)}{A(K_S \to \pi\pi)} \neq 0 \; ! \quad \sim 2.23 \times 10^{-3} \equiv \epsilon_K$$

NOBEL PRIZE Cronin + Fitch



CPV in state mixing, $\Delta S=2$ H_{eff}

IV: ε′ / ε: Direct CPV

EXPERIMENTAL ROUTE

$$\eta_{+-} = |\eta_{+-}| e^{i\phi_{+-}} = \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)}$$

$$\eta_{00} = |\eta_{00}| e^{i\phi_{00}} = \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)}$$

$$\eta_{+-} = \epsilon + \epsilon', \qquad \eta_{00} = \epsilon - 2\epsilon'$$

$$\epsilon' \simeq \frac{1}{3}\left[\eta_{+-} - \eta_{00}\right] \Rightarrow O(10^{-3}) - O(10^{-3}) \Rightarrow 10^{-6}$$

$$\epsilon = \frac{1}{3}\left[2\eta_{+-} + \eta_{00}\right]$$

Conf on NP@LHC 02/29/16; A. Soni 10

K→2π

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = \mathrm{Re}\left\{\frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}\varepsilon}\left[\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right]\right\}$$

I=2 I=0

Use lattice to calculate 6 quantities:
ReA0, ReA2 known from expt; δ0,δ2 via ChPT etc..So very good checks;
ImA. ImA2 unknown

ω = ReA2/ReA0 ~ 0.045

Indirect CP

$$|\epsilon| = 2.228(11)\times 10^{-3},$$

$$\mathrm{Re}(\epsilon'/\epsilon) = 1.65(26)\times 10^{-3}.$$

DIRECT CP

$\epsilon' << \epsilon$

$\epsilon' \sim 10^{-6}$!

PHYSICAL REVIEW D VOLUME 56, NUMBER 1 1 JULY 1997

QCD with domain wall quarks

T. Blum* and A. Soni†
Department of Physics, Brookhaven National Laboratory, Upton, New York 11973
(Received 27 November 1996)

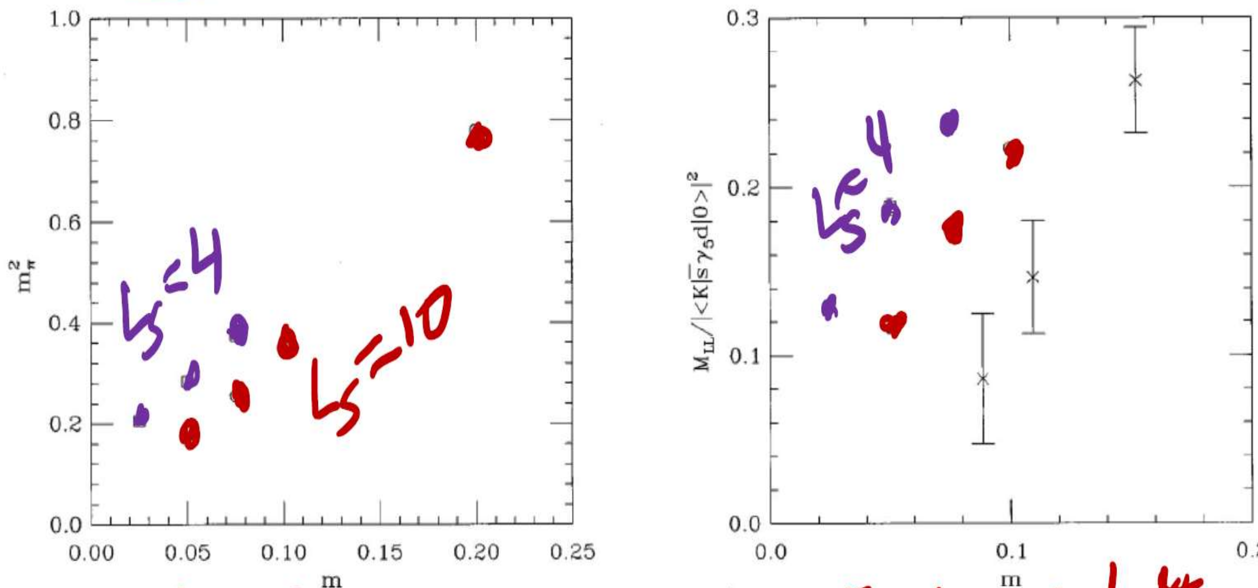
We present lattice calculations in QCD using Shamir's variant of Kaplan fermions which retain the continuum $SU(N)_L \times SU(N)_R$ chiral symmetry on the lattice in the limit of an infinite extra dimension. In particular, we show that the pion mass and the four quark matrix element related to K_0-$\bar{K}_0$ mixing have the expected behavior in the chiral limit, even on lattices with modest extent in the extra dimension, e.g., $N_s = 10$. [S0556-2821(97)00113-6]

Two key papers

Bypass Nielsen Ninomiya

1st Simulation with DWQ

Feb '97



excellent chiral symmetry with ~10 sites in 5th dim.

MAJOR BREAKTHROUGH FOR K→ππ Lattice Calculations

Commun. Math. Phys. 219, 31 – 44 (2001)

Communications in
Mathematical Physics

Weak Transition Matrix Elements from Finite-Volume Correlation Functions*

Laurent Lellouch[1,], Martin Lüscher[2,***]**

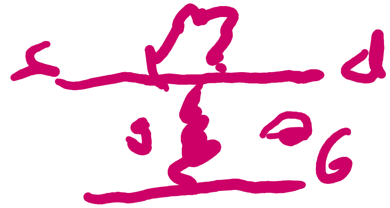
[1] LAPTH, Chemin de Bellevue, B.P. 110, 74941 Annecy-Le-Vieux Cedex, France
[2] CERN, Theory Division, 1211 Geneva 23, Switzerland

Received: 29 March 2000 / Accepted: 10 April 2000

Dedicated to the memory of Harry Lehmann

Abstract: The two-body decay rate of a weakly decaying particle (such as the kaon) is shown to be proportional to the square of a well-defined transition matrix element in finite volume. Contrary to the physical amplitude, the latter can be extracted from finite-volume correlation functions in euclidean space without analytic continuation. The $K \to \pi\pi$ transitions and other non-leptonic decays thus become accessible to established numerical techniques in lattice QCD.

Re A0

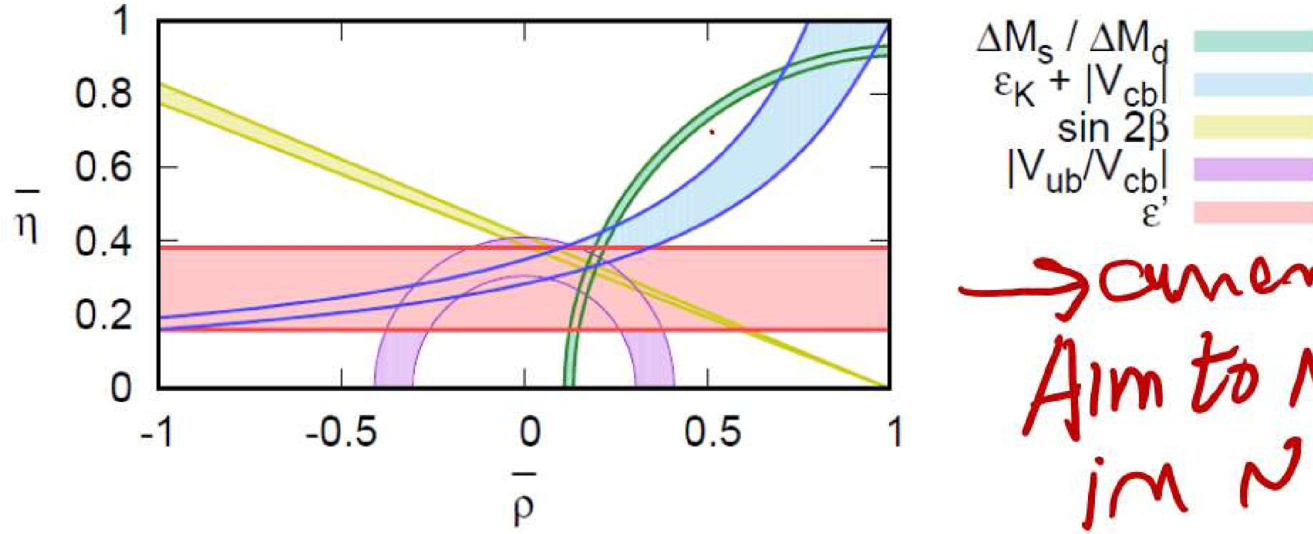


Im A0

	$\mathrm{Re}(A_0)$		$\mathrm{Im}(A_0)$	
i	$(\not{q},\not{q})$ ($\times 10^{-7}$ GeV)	(γ^μ,γ^μ) ($\times 10^{-7}$ GeV)	$(\not{q},\not{q})$ ($\times 10^{-11}$ GeV)	(γ^μ,γ^μ) ($\times 10^{-11}$ GeV)
1	0.383(77)	0.335(64)	0	0
2	2.89(30)	2.81(28)	0	0
3	0.0081(58)	0.0050(42)	0.20(14)	0.12(10)
4	0.081(23)	0.088(17)	1.24(35)	1.34(27)
5	0.0380(68)	0.0339(53)	0.552(99)	0.492(77)
6	−0.410(28)	−0.398(27)	−8.78(60)	−8.54(57)
7	0.001863(56)	0.001900(56)	0.02491(75)	0.02540(75)
8	−0.00726(14)	−0.00708(13)	−0.2111(40)	−0.2060(39)
9	$-8.7(1.5) \times 10^{-5}$	$-8.5(1.4) \times 10^{-5}$	−0.133(22)	−0.128(21)
10	$2.37(38) \times 10^{-4}$	$2.13(32) \times 10^{-4}$	−0.0304(49)	−0.0273(41)
Total	2.99(32)	2.86(31)	−7.15(66)	−6.93(64)

TABLE XVIII: The contributions of each of the ten four-quark operators to $\mathrm{Re}(A_0)$ and $\mathrm{Im}(A_0)$ for the two different RI-SMOM intermediate schemes. The scheme and units are listed in the column headers. The errors are statistical, only.

Chris Kelly et al PRD 2020



→ current syst ~35%. Aim to reduce this in ~3 y. to ~15%.

FIG. 12: The horizontal-band constraint on the CKM matrix unitarity triangle in the $\bar{\rho}-\bar{\eta}$ plane obtained from our calculation of ε', along with constraints obtained from other inputs [6, 70, 71]. The error bands represent the statistical and systematic errors combined in quadrature. Note that the band labeled ε' is historically (e.g. in Ref. [72]) labeled as ε'/ε, where ε is taken from experiment.

Motivations for independent calculation of eps' with PBC

- **For the first time RBC-UKQCD calculated eps' from 1st principles with a modest accuracy of ~35%. Because of naturalness reasoning, continuing to search for a BSM-CP odd phase with eps' is important and therefore continuing to calculate eps' with better accuracy is highly desirable.**
- **With GPBC configs have to be specially created making it very expensive to use multiple lattice spacings for taking a continuum limit.**
- **With PBC no need for special configs and in fact two different lattice spacings with ~physical pions already exist, so taking the continuum limit seems a lot more viable**
- **Given the importance of the result on eps' and the complexity of the calculation, an independent calculation of K=> 2 pion and epsilon' with possibly using PBC seems highly desirable**
- **With GPBC a lattice calculation of corrections on eps' due to EM+isospin appears very difficult, with PBC this may be less problematic**
- **Driving force behind current RBC/UKQCD-PBC effort is Masaaki Tomii**

a^2 [GeV^{-2}]

- <u>Ensembles already generated for periodic BC</u>
  - 24^3 x 64, a^{-1} = 1.0 GeV: measurements w 258 confs done → soon 440 confs
  - 32^3 x 64, a^{-1} = 1.4 GeV: measurements w 107 confs done → ~250 confs in a year
  - 48^3 x 96, a^{-1} = 1.7 GeV & 64^3 x 128, a^{-1} = 2.4 GeV: future work

Precision performance

Now ≈ 500

	32^3 G-parity BC (previous work)	24^3 Periodic BC	32^3 Periodic BC (w/o AMA correction)
# of configurations	741	258	107
$\Delta I = 1/2$ ME via Q_2^{lat}	10%	14%	14%
$\Delta I = 1/2$ ME via Q_6^{lat}	6.5%	8.9%	11%
Re A_0	11%	13%	14%

Error % (statistical) (applies to the last three rows)

Preliminary (applies to the 24^3 and 32^3 Periodic BC columns)

- Good precision performance of PBC (ME with excited-state ππ) compared to G-parity BC calculation (ME with ground-state ππ) arXiv: 2306.06781 exploratory 258 g.c

Masaaki, T et al

PBC

EXPLORATORY

Quantity	This work	Experiment
$\mathrm{Re}(A_2)$	$1.74(15)(48) \times 10^{-8}$ GeV	$1.479(4) \times 10^{-8}$ GeV
$\mathrm{Im}(A_2)$	$-5.91(13)(1.75) \times 10^{-13}$ GeV	...
$\mathrm{Re}(A_0)$	$3.13(69)(95) \times 10^{-7}$ GeV	$3.3201(18) \times 10^{-7}$ GeV
$\mathrm{Im}(A_0)$	$-9.3(1.5)(2.8) \times 10^{-11}$ GeV	...
$\mathrm{Re}(A_0)/\mathrm{Re}(A_2)$	18.0(4.4)(7.4)	22.45(6)
$\omega = \mathrm{Re}(A_2)/\mathrm{Re}(A_0)$	0.056(14)(23)	0.04454(12)
$\mathrm{Re}(\varepsilon'/\varepsilon)$	$31.8(6.3)(11.8)(5.0) \times 10^{-4}$	$16.6(2.3) \times 10^{-4}$

TABLE I. A summary of the primary results of this work shown in the middle column. The values in parentheses give the statistical and systematic errors, respectively. For the last entry the systematic error associated with electromagnetic and isospin breaking effects is listed separately as the third error, which we inherit from the estimation in Ref. [2] based on the large-N_c expansion of QCD and ChPT [49]. The corresponding experimental values are shown in the right column if applicable.

Key points (so far) on our PBC effort

Sommer et al JHEP '09

- **Demonstrated that with GEVP matrix elements of ground and 1st two excited states can be extracted quite well**
- **Good quality of signals with PBC obtained rather efficiently**
- **On our way to get results from 2 lattice spacings**
- **Optimistic that we can get epsilon' in the continuum limit (for the iso-symmetric) case in**

the next few months…That should appreciably reduce one of the major source of systematic errors.

- **PBC and other methods being studied to deal with EM+IB**

I=0, 2 strong phases already done : see M. Tomii et al PRD'23

K-UT..MANY REASONS TO GO FOR IT E.G. LONG-STANDING ISSUES INCLUSIVE VERSUS EXCLUSIVE TENSION IN VXB

K-UT: A dream for some

Blucher, Winstein and Yamanaka '09; see also Buras

Construction of a Kaon UT

- Click to add t

LITTENBERG PRD'89

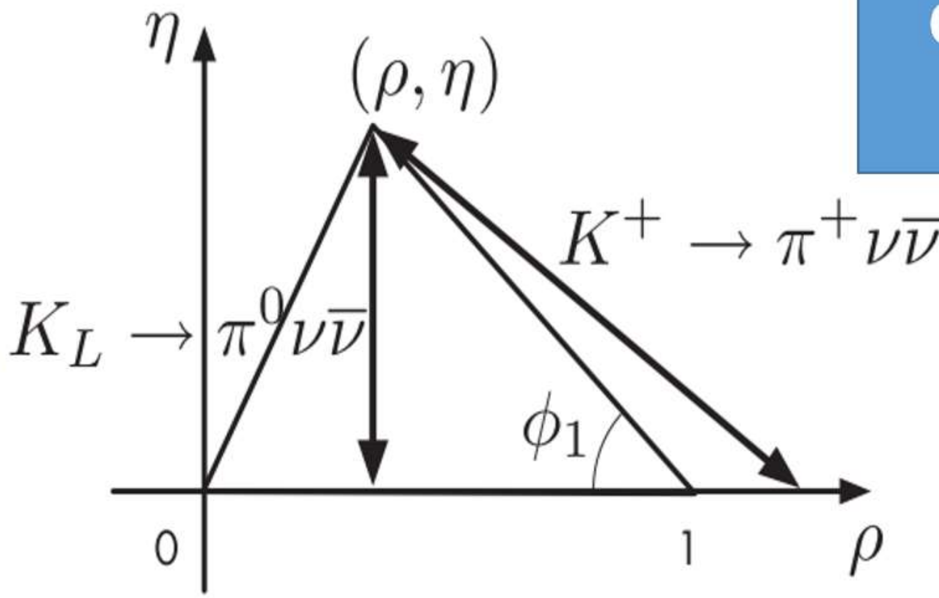


Fig. 14. Unitarity triangle.

Lehner+Lunghi+AS PLB'2016

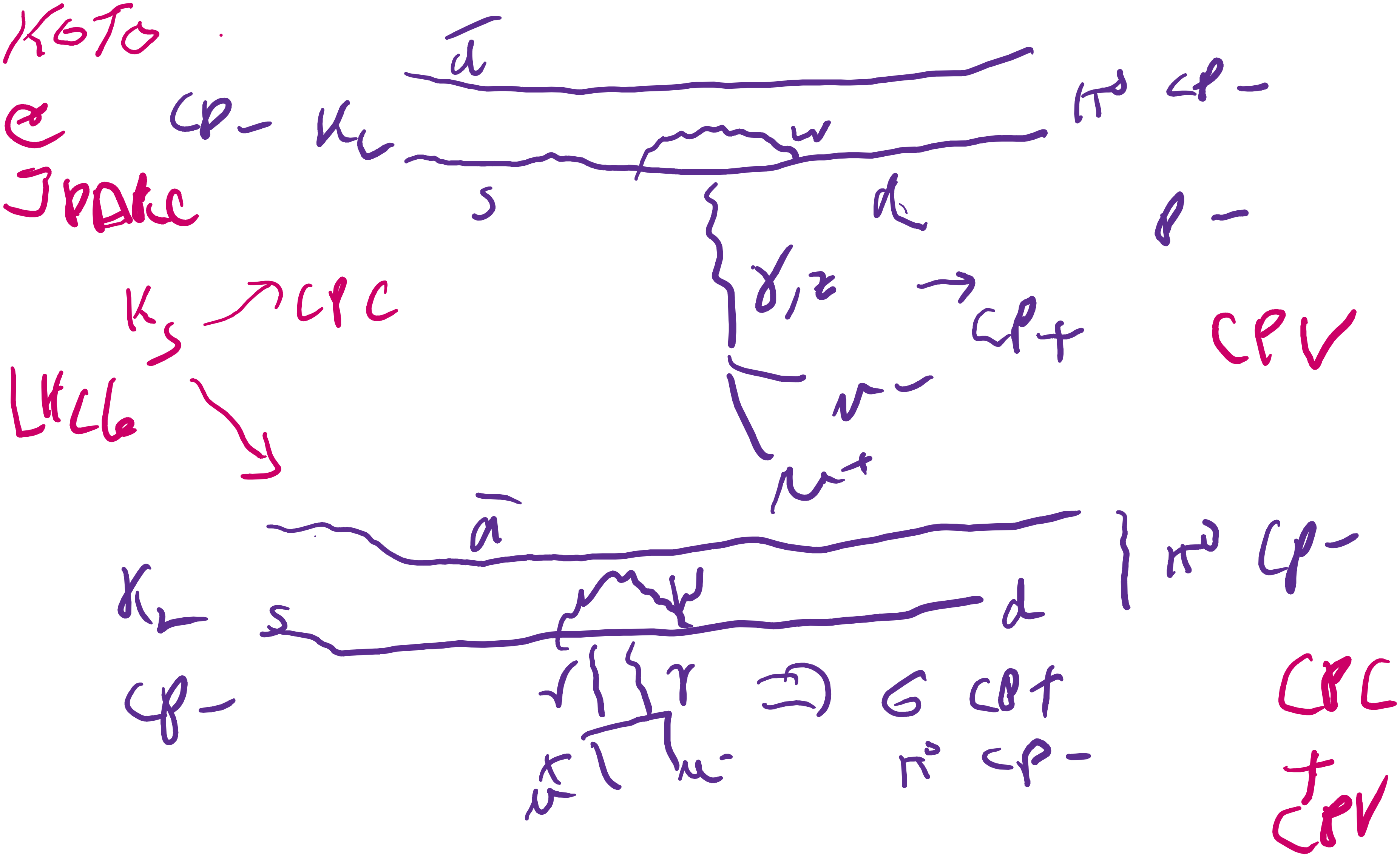
Instead of [or in addition to] $K_L \to \pi^0 \nu\bar{\nu}$ can now plan on using ϵ'/ϵ

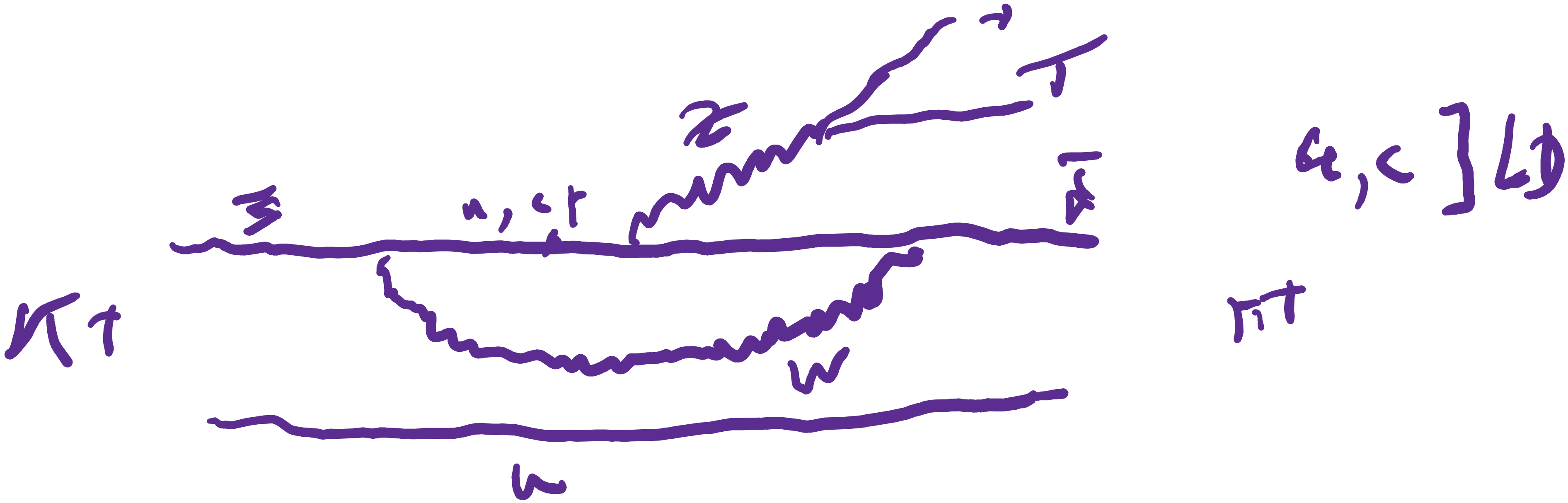
Also constrain KL=>pi0 nu nu via K0=>pi0 mu+mu- (c AS in Lat23)

K0=>pi0 mu+ mu- → see also ASoni LAT23

- LHCb: Ks
- JPARC:KL
- Pheno: Isidori et al…;D'Ambrosio et al;Schact + AS (WIP)
- Lattice: RBC+UKQCD many papers on

closely related rare K-decays → 1910.10644; 1806.11520; 1701.08258


KOTO
@
JPARC
CP-
K_L
$\bar{d}$
s
d
W
π^0 CP-
γ, Z
CP+
CPV
K_S → CPC
LHCb
μ^-
μ^+
K_L
CP-
$\bar{d}$
s
d
π^0 CP-
CP+
CPC
+
CPV



WITH ENRICO LUNGI

TRY Reduce LD uncertainty

WIP

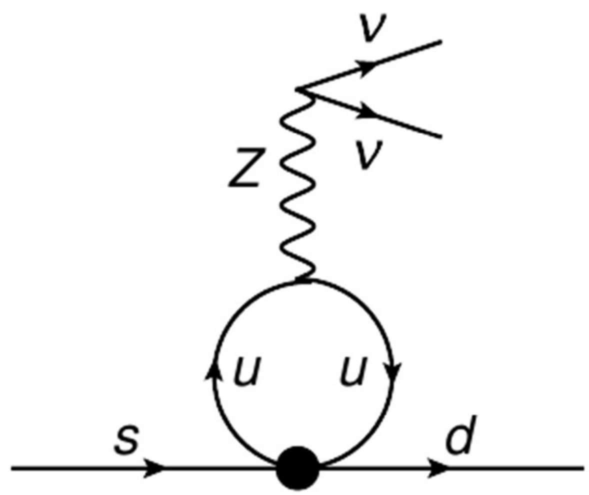


Figure 1. Long distance contributions to $K^+ \to \pi^+ \nu\bar{\nu}$ at the quark level.

CECCUCCI Rev.

$$B(K^+ \to \pi^+ \nu\bar{\nu}) = (8.39 \pm 0.30) \times 10^{-11} \left[\frac{|V_{cb}|}{40.7 \times 10^{-3}}\right]^{2.8} \left[\frac{\gamma}{73.2^\circ}\right]^{0.74}.$$

In the above formula, the explicit numerical uncertainty is the theoretical one originating from QCD and electroweak uncertainties, which amounts to 3.6%. Taking the latest values (28) for $|V_{cb}|_{\text{avg}} = (41.0 \pm 1.4) \times 10^{-3}$ and $\gamma = (72.1^{+4.1}_{-4.5})^\circ$, one finds the following:

$$B(K^+ \to \pi^+ \nu\bar{\nu})_{\text{SM}} = (8.5 \pm 1.0) \times 10^{-11}.$$

The predictions are currently dominated by the parametric uncertainty that will plausibly be reduced by new measurements of $|V_{cb}|$ and γ by LHCb and Belle II.

cannot be detected. A long series of decay-at-rest searches for $K^+ \to \pi^+ \nu\bar{\nu}$ have culminated with the final results of the BNL E787/E949 experiments, which found the following (50):

$$B(K^+ \to \pi^+ \nu\bar{\nu})_{\text{E787/E949}} = (17.3^{+11.5}_{-10.5}) \times 10^{-11}.$$

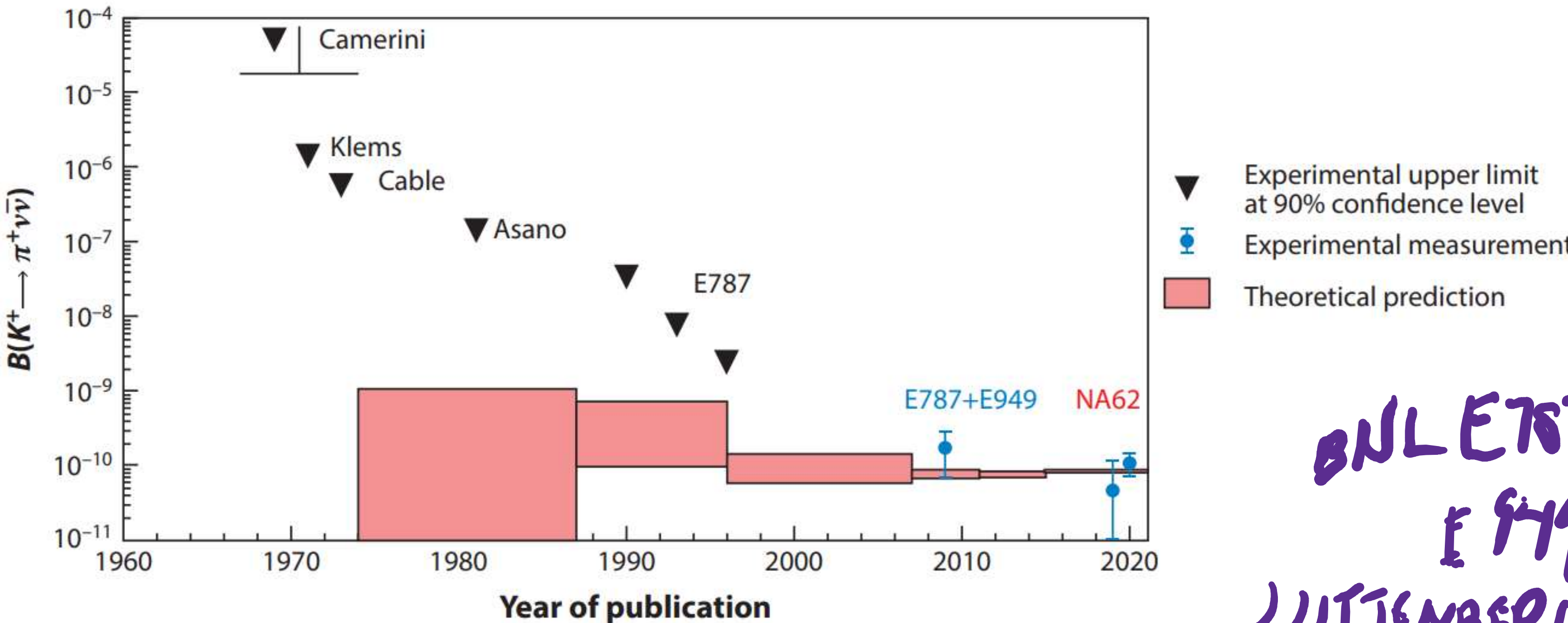
From these analyses, the best upper limit, at 90% confidence level (CL), has been obtained:

$$B(K^+ \to \pi^+ \nu\bar{\nu})_{\text{NA62(2016–2017)}} \leq 17.8 \times 10^{-11}.$$

The 2016–2017 data also allow one to set a 68% CL mean value for the branching ratio:

$$B(K^+ \to \pi^+ \nu\bar{\nu})_{\text{NA62(2016–2017)}} = (4.8^{+7.2}_{-4.8}) \times 10^{-11}.$$

CECCUCCI Rev.



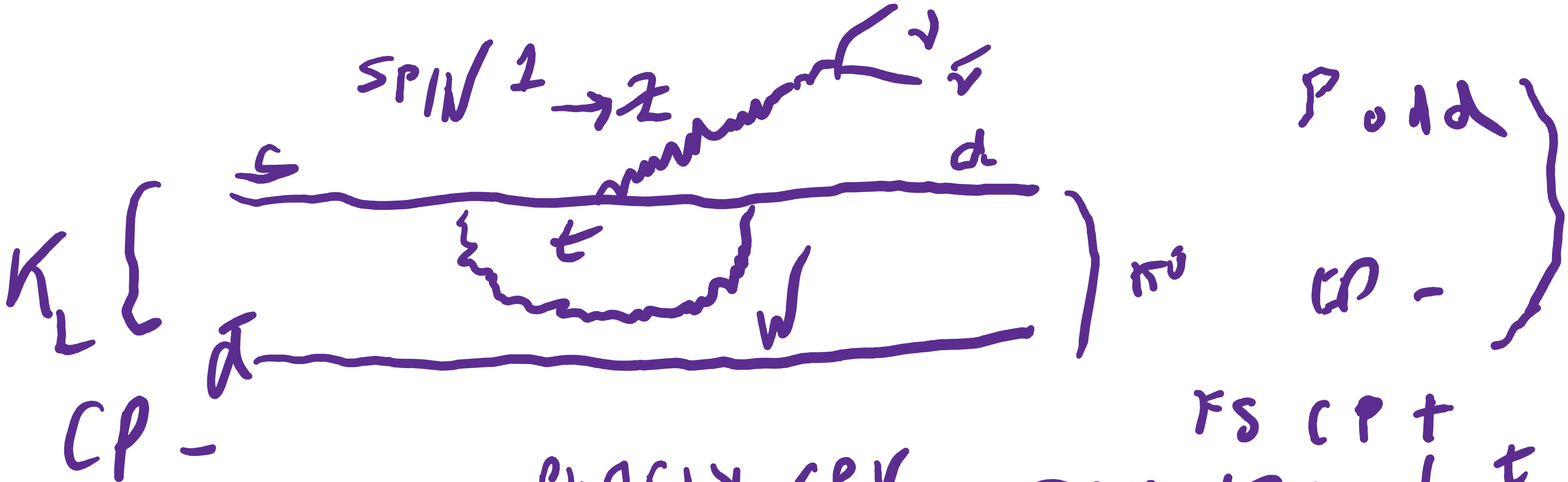
BNL E787 + E949 LITTENBERG PANOFSKY PRIZE

Figure 4

Timeline of theoretical predictions and experimental results for $K^+ \to \pi^+ \nu\bar{\nu}$ (10, 51, 57–64). Figure adapted with permission from Reference 58; copyright 2020 CERN for the benefit of the NA62 Collaboration.

the NA62 Collaboration reported the following:

$$B(K^+ \to \pi^+ \nu\bar{\nu})_{\mathrm{NA62(2016\text{–}2018)}} = (11.0^{+4.0}_{-3.5}{}_{\mathrm{stat}} \pm 0.3_{\mathrm{syst}}) \times 10^{-11},$$



PURELY CPV

LITTENBERG PRD 1989

GOLD PLATED

KOTO@JPARC

BUT EXPERIMENTALLY extremely challenging

CECCUCCI Rev.

$$B(K_L^0 \to \pi^0 \nu\bar{\nu}) = (3.36 \pm 0.05) \times 10^{-11} \left[\frac{|V_{ub}|}{3.88 \times 10^{-3}}\right]^2 \left[\frac{|V_{cb}|}{40.7 \times 10^{-3}}\right]^2 \left[\frac{\sin\gamma}{\sin 73.2^\circ}\right]^2,$$

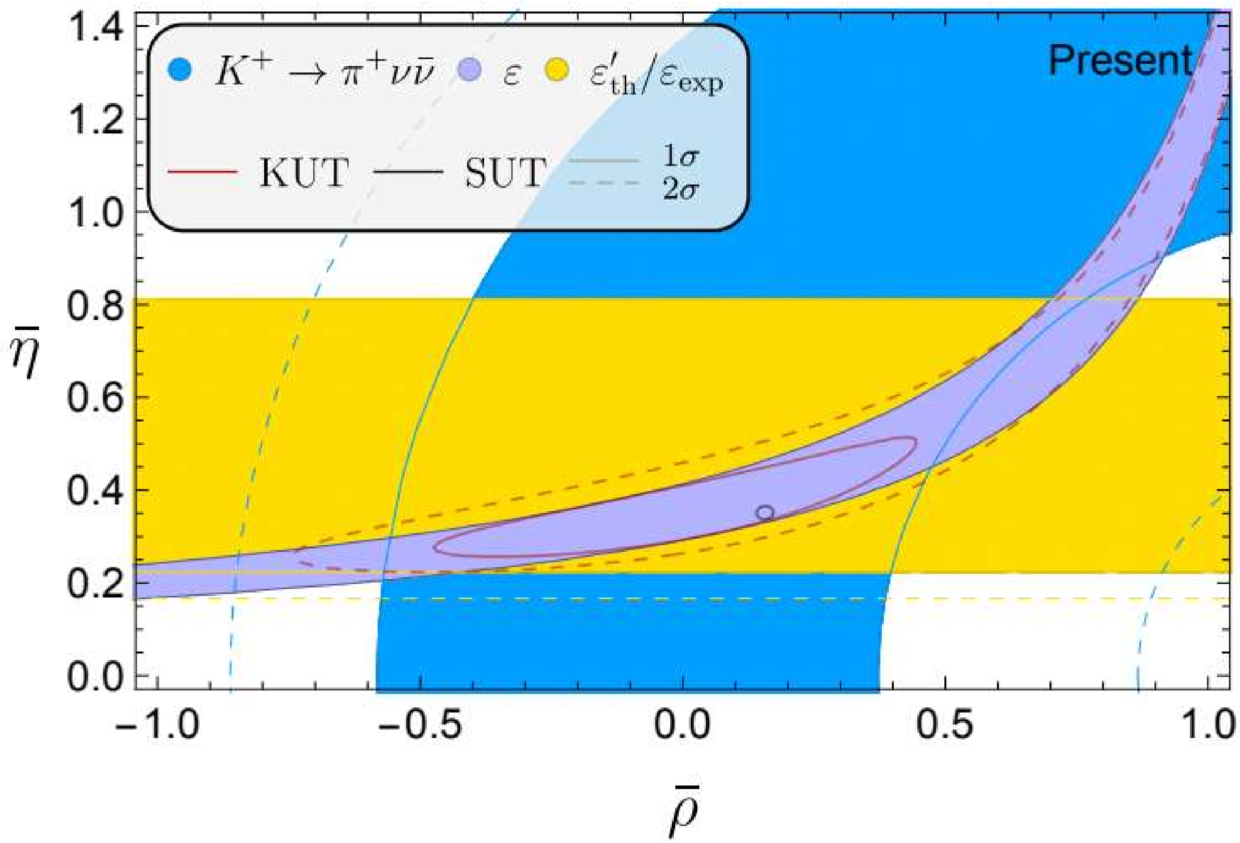
which, taking the latest values (28) for $|V_{cb}|_{avg} = (41.0 \pm 1.4) \times 10^{-3}$, $|V_{ub}|_{avg} = (3.82 \pm 0.24) \times 10^{-3}$, and $\gamma = (72.1^{+4.1}_{-4.5})^\circ$, leads to the following numerical prediction:

$$B(K_L^0 \to \pi^0 \nu\bar{\nu}) = (3.2 \pm 0.6) \times 10^{-11}.$$

While the experimental situation for $K^+ \to \pi^+ \nu\bar{\nu}$ shows that we have two independent experimental techniques that can reach SM sensitivities, with the NA62 experiment on the way to making a precise measurement, the situation for the neutral mode is more complex. Progress has been hampered by the lack of a clean experimental signature because no redundancy is available once the π^0 mass is used as a constraint to reconstruct the decay vertex. The KOTO experiment at J-PARC builds on the experience of the predecessor experiment E391a (67), which was performed at KEK. It is based on the technique of letting a well-collimated "pencil" beam enter the decay region surrounded by high-performance photon vetoes. By vetoing extra photons and applying a transverse momentum cut (150 MeV/c) to eliminate residual $\Lambda \to n\pi^0$ decays, KOTO is expected to reach SM sensitivities by the mid-2020s. The KOTO experiment has published the best upper limit (68):

$$B(K_L^0 \to \pi^0 \nu\bar{\nu})_{\text{KOTO}} < 3.0 \times 10^{-9} \text{ (90\% CL)}.$$

~ about 2 orders of magnitude to go



$K^+ \to \pi^+ \nu \bar{\nu}$
ε
$\varepsilon'_{\rm th}/\varepsilon_{\rm exp}$
KUT
SUT
1σ
2σ
Present
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
$\bar{\eta}$
−1.0
−0.5
0.0
0.5
1.0
$\bar{\rho}$

LLS PLB '16

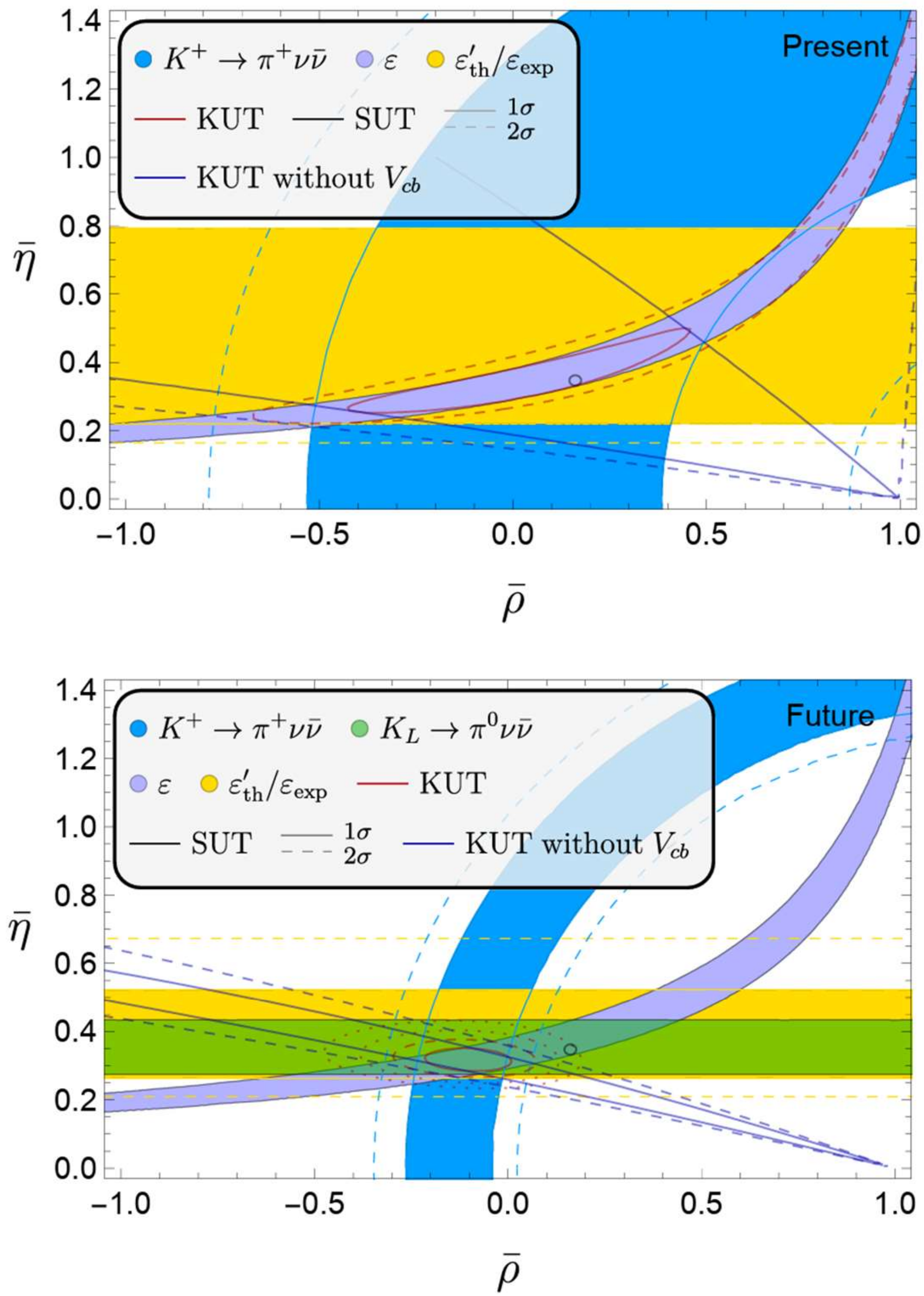


Figure 5. Top panel: current status of the Kaon Unitarity Triangle. Bottom panel: impact of improved calculations of $\mathrm{Im}A_{0,2}$ from lattice QCD and of expected measurements of charged (NA62) and neutral (KOTO) $K \to \pi\nu\bar{\nu}$ branching ratios on the Kaon Unitarity Triangle. The two dotted contours are the 3σ and 4σ KUT contours, respectively.

UT ANGLE GAMMA $\equiv \phi_3 \equiv -\arg\left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right]$

Dalitz analysis: Giri,Grossman, Soffer & Zupan PRD '03; Atwood, Dunietz + AS, PRD'01

ADS also PRL'97

- Both emphasize model independent (diff approaches) analysis

via the Dalitz plot

- Following the then existing experimental data from F 637 Collab … analysis us…

(though it d…

It should be realized that three body states $K^+\rho^-$, $K_s\rho^0$ and $K^{*+}\pi^-$ can all lead to the common final state $K_s\pi^+\pi^-$. If one examines the distribution in phase space,

1.Briefly ADS uses local regions of DP to look for minimum values of gamma; followed by searches globally
2. The crucial point is that it then uses A+S method of "optimized observables" (PRD92) and demonstrates that
solution to gamma thus obtained are just as good as the optimal construction gives

VERY HOPEFUL THAT BELLE-II (MAY BE EVEN LHCB?) WILL BE ABLE TO HANDLE FS WITH 1 PI0

Optimised observables (Atwood+AS, PRD 45,'92); see esp sec III

you are using expt'l data to determine λ

expand the total differential cross section in terms of λ we have

$$\Sigma = \Sigma_0 + \lambda \Sigma_1 \ . \qquad (6)$$

$$f = f_{\rm opt} = \frac{\Sigma_1}{\Sigma_0}$$

Construction is used extensively these days in ML applications

The simple proof is given in the paper

The ultimate theoretical error on γ from $B \to DK$ decays

→ Because of this utterly miniscule higher order correction γ is the "STANDARD CANDLE" in the SM-KM paradigm of CPV

Joachim Brod and Jure Zupan

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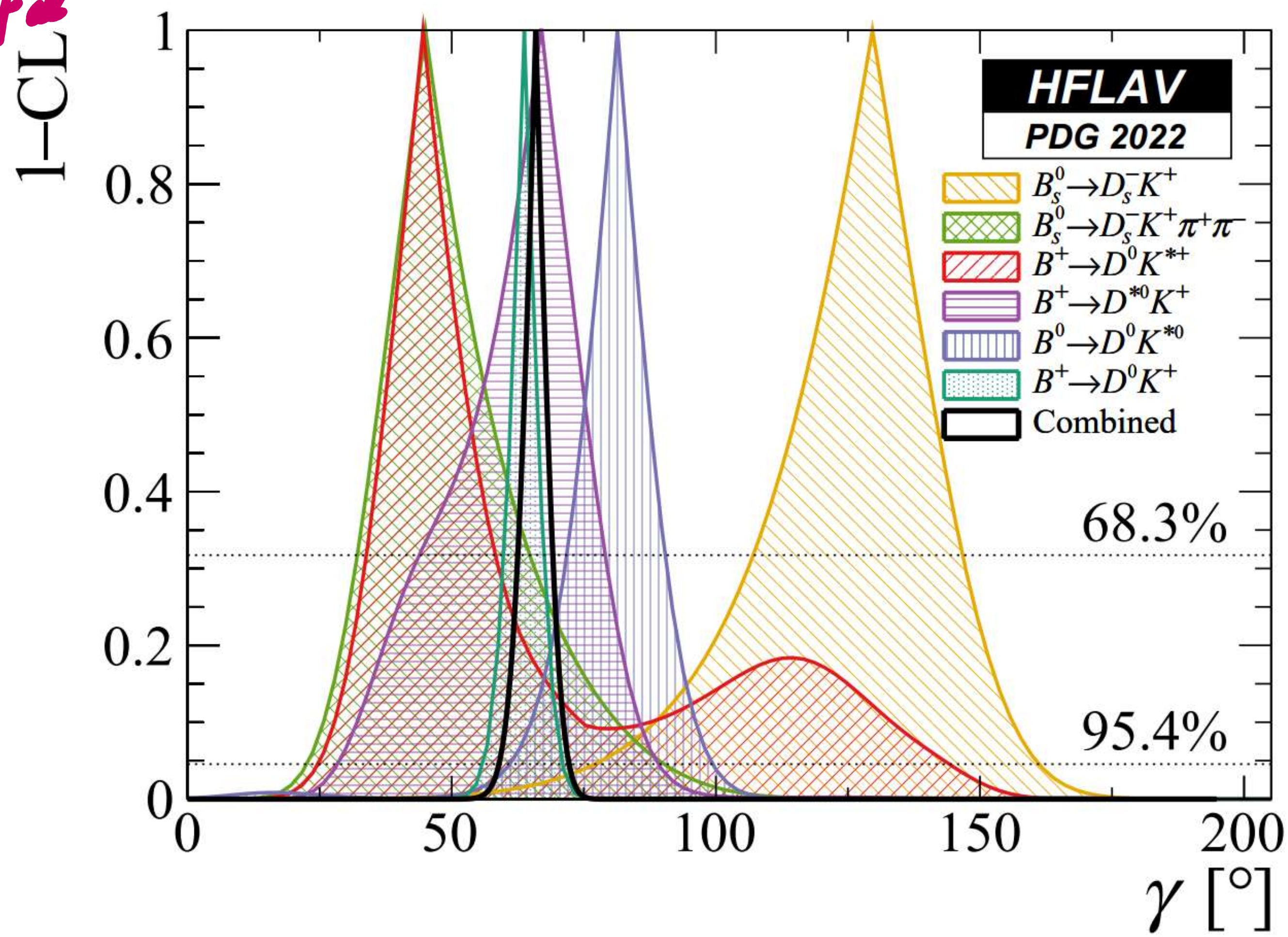
ABSTRACT: The angle γ of the standard CKM unitarity triangle can be determined from $B \to DK$ decays with a very small irreducible theoretical error, which is only due to second-order electroweak corrections. We study these contributions and estimate that their impact on the γ determination is to introduce a shift $|\delta\gamma| \lesssim \mathcal{O}(10^{-7})$, well below any present or planned future experiment.

JHEP01(2014)051

GERSHON et al

PDG2022



p. value ~36%.

Figure 77.3: World average of $\gamma \equiv \phi_3$, as well as contributions from individual modes, in terms of 1−CL.

Combined analysis

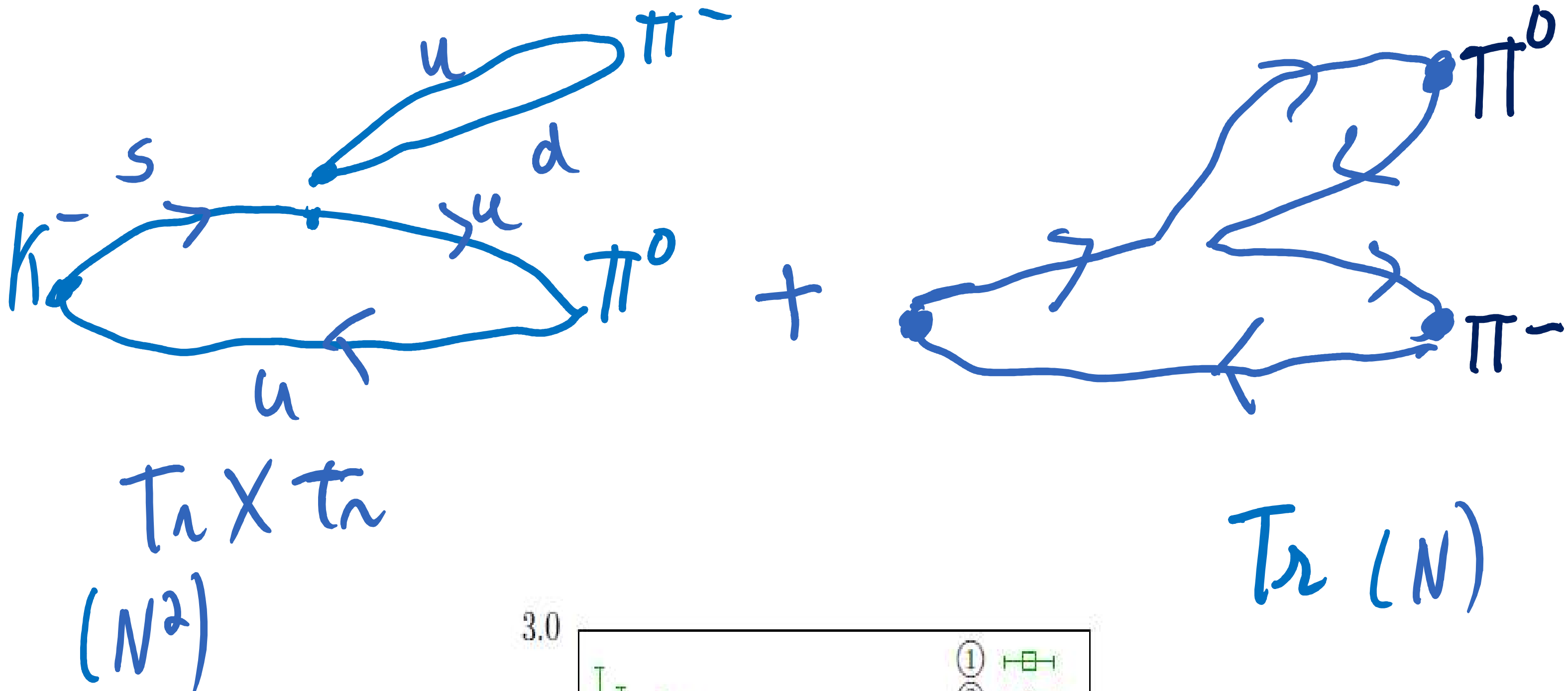
- B+, B0, Bs are all used to get the World Average:
- $\phi_3 \equiv \gamma = 65.9^{+3.3}_{-3.5}$ stat error dominates
- In particular higher order corrections are exceedingly small & will remain so for a very long time to come [BROD+Zupan '14]

Summary + Outlook

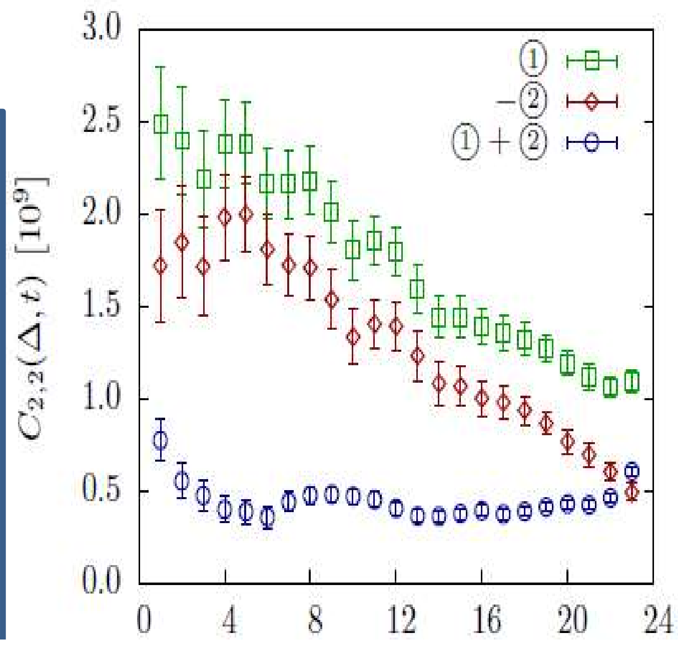
- **After decades of development and effort, using DWQ, and GPBC in 2020 completed the 1st calculation of eps' with a modest accuracy of 35% at a single lattice spacing~1.38 GeV; resulting eps' is compatible with experiment within 1 sigma [also attained qualitative and quantitative understanding of the Delta I=1/2 Rule]**
- **We are well on our way to get eps' along with scattering phases again, in a completely independent set up using PBC. Driving force for this effort is MASAAKI TOMII. With this method we are hopeful to get eps' for the 1st time in the continuum limit**
- **Showed how using eps' + eps + Br (K+ => pi+ nu nu) can construct the K-UT**
- **Also K0=>pi0 mu+ mu- input from LHCb, JPARC, pheno and lattice should provide important constraints for the gold plated KL=>pi0 nu nu mode being pursued by the KOTO expt @ JPARC**
- **UT gamma: D0 Dalitz decays with 1 pi0 in FS ….Belle-II, LHCb**
- **UT gamma: ADS PRD method should also be used => v likely get improve results**
- **It is exceedingly important to determine/constrain UTs as precisely as possible as it is highly unlikely to be just a triangle**

EXTRA'S

Dissecting (the much easier) ΔI=3/2 [I=2 ππ] Amp on the lattice: 2 contributing topologies only



Simplest basic step is Significantly different from phenomenological Expectations!



DRAMATIC CANCELLATION!
($m_\pi \sim 140$ MeV)

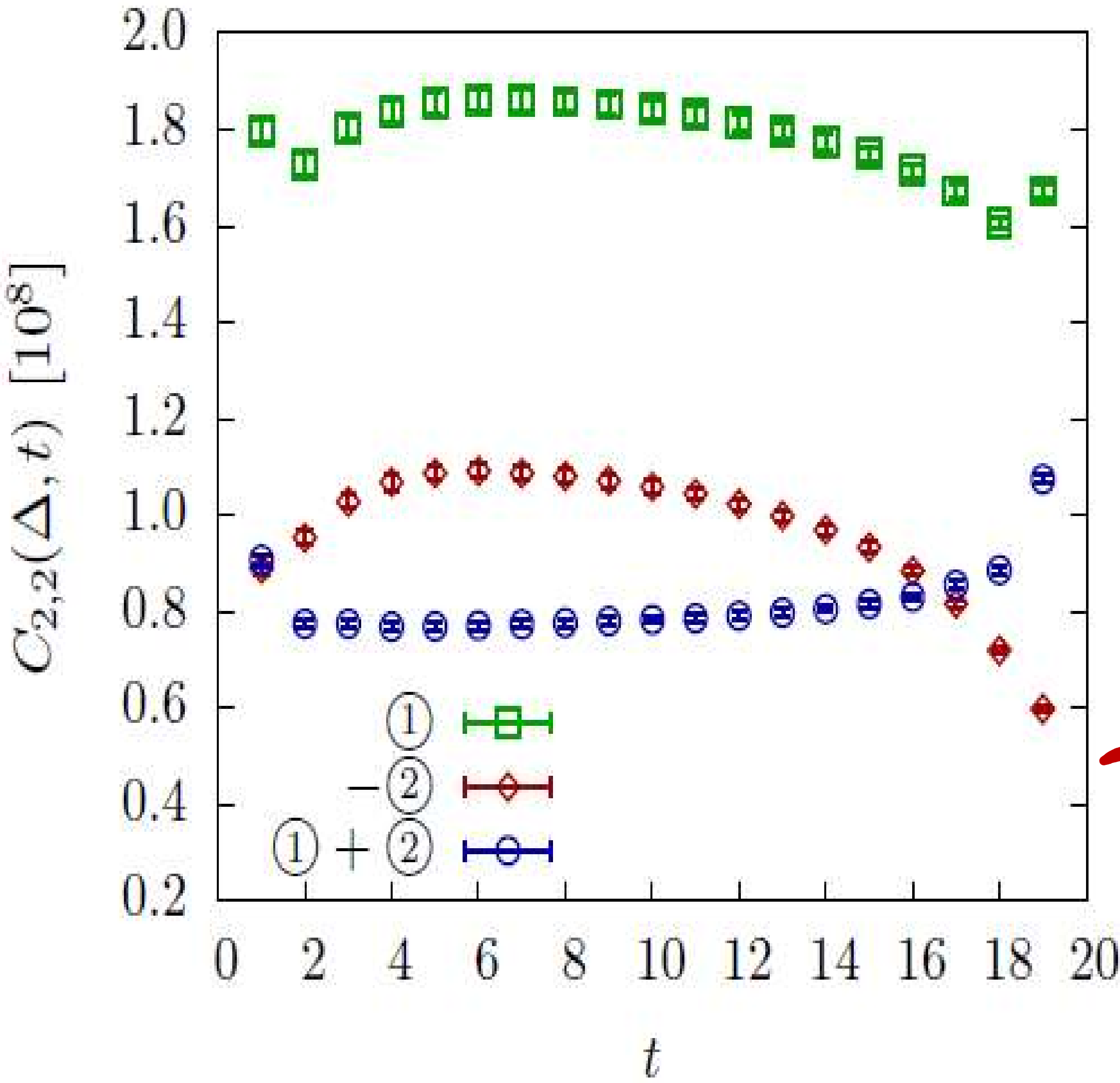


FIG. 3: Contractions ①, -② and ① + ② as functions of t from the simulation at threshold with $m_\pi \simeq 330\,\text{MeV}$ and $\Delta = 20$.

For heavier π, $m_\pi \sim 330$ MeV less cancellation bet. N^2 & N Large N begins to improve!

Ensemble

USED for A_0

- 32^3x64 Mobius DWF ensemble with IDSDR gauge action at β=1.75. Coarse lattice spacing (a^{-1}=1.378(7) GeV) but large, (4.6 fm)^3 box.
- Using Mobius params (b+c)=32/12 and L_s=12 obtain same explicit χSB as the L_s=32 Shamir DWF + IDSDR ens. used for ΔI=3/2 but at reduced cost.
- Utilized USQCD 512-node BG/Q machine at BNL, the DOE "Mira" BG/Q machines at ANL and the STFC BG/Q "DiRAC" machines at Edinburgh, UK.
- Performed 216 independent measurements (4 MDTU sep.).
- Cost is ~1 BG/Q rack-day per complete measurement (4 configs generated + 1 set of contractions).
- G-parity BCs in 3 spatial directions results in close matching of kaon and ππ energies:

$32^3 \times 64 \times 12$

$m_{res} = .0018$

$m_s = .045$

m_K=490.6(2.4) MeV

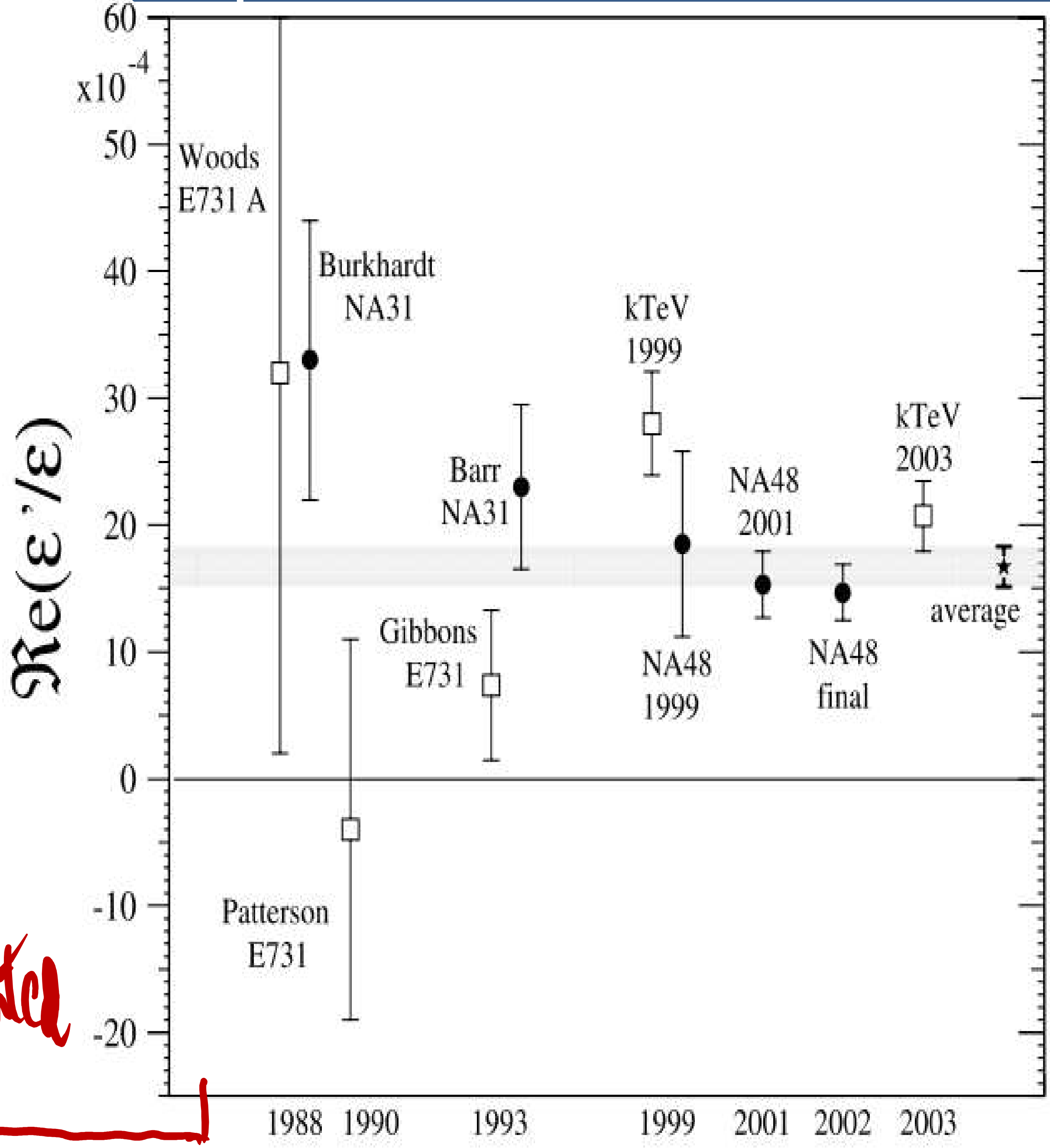
$E_{\pi\pi}$(I=0) = 498(11) MeV

$E_{\pi\pi}$(I=2) = 573.0(2.9) MeV

E_π=274.6(1.4) MeV   (m_π = 143.1(2.0) MeV)

PHYSICAL MASSES & Kinematics!

A monumental experimental achievement!



Konrad Kleinknecht "Uncovering CPV"

$16.6(2.3)\times10^{-4}$ PDG 2014

LATTICE work started

Systematic errors

Error source	Value
Excited state	-
Unphysical kinematics	5%
Finite lattice spacing	12%
Lellouch-Lüscher factor	1.5%
Finite-volume corrections	7%
Missing G_1 operator	3%
Renormalization	4%
Total	15.7%

TABLE XXV: Relative systematic errors on the infinite-volume matrix elements of $\overline{\text{MS}}$-renormalized four-quark operators Q'_j.

Re A_0 → ~20%

Error source	Value	
	Re(A_0)	Im(A_0)
Matrix elements	15.7%	15.7%
Parametric errors	0.3%	6%
Wilson coefficients	12%	12%
Total	19.8%	20.7%

TABLE XXVI: Relative systematic errors on Re(A_0) and Im(A_0).

Im A_0 → ~21%

EXPT

3.32 x10^{-7} GeV

0.00166

Quantity	Value
$\mathrm{Re}(A_0)$	$2.99(0.32)(0.59)\times10^{-7}$ GeV
$\mathrm{Im}(A_0)$	$-6.98(0.62)(1.44)\times10^{-11}$ GeV
$\mathrm{Re}(A_0)/\mathrm{Re}(A_2)$	19.9(2.3)(4.4)
$\mathrm{Re}(\varepsilon'/\varepsilon)$	0.00217(26)(62)(50)

→ due IB see foll pages

TABLE I: A summary of the primary results of this work. The values in parentheses give the statistical and systematic errors, respectively. For the last entry the systematic error associated with electromagnetism and isospin breaking is listed separately as a third error contribution.

IB based on CIRIGLIANO et al JHEP 2020

Results for ε'

Numerical results will be superseded by the higher stat improved calculation in a few months

- Using Re(A$_0$) and Re(A$_2$) from experiment and our lattice value for Im(A$_0$) and Im(A$_2$) and the phase shifts, (t or er

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = \mathrm{Re}\left\{\frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}\varepsilon}\left[\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right]\right\}$$

$$= 1.38(5.15)(4.43) \times 10^{-4},$$

$$16.6(2.3) \times 10^{-4}$$

EWP

QCDP ~ 80%

LARGE CANCELLATION!!

216 gauge configs

RBC-UKQCD PRL'15 EDITOR'S CHOICE

Bearing in mind the largish errors in this first calculation, we interpret that our result are consistent with experiment at ~2σ level

Computed ReA2 excellent agreement w

Computed ReA0 good agreement with expt

Offered an "explanation" of the Delta I=1/2 enhancement

$$\omega = \frac{\mathrm{Re}A_2}{\mathrm{Re}A_0} \sim 0.045$$

Systematic errors

Error source	Value
Excited state	-
Unphysical kinematics	5%
Finite lattice spacing	12%
Lellouch-Lüscher factor	1.5%
Finite-volume corrections	7%
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Im A_0 ↓ ~21%

TABLE XXVI: Relative systematic errors on $\mathrm{Re}(A_0)$ and $\mathrm{Im}(A_0)$.

Why EWK cannot be neglected : 3 Reasons

- Despite $\alpha_{QED,EWK}$ << α_{QCD} , EWK contributions are extremely important and CANNOT be neglected:
- EWK are (8,8) and QCD are (8,1), and (8,8) go to constant whereas (8,1) vanish in the chiral limit
- EWK, i.e. those due Z exch have Wilson coeff that go as mt^2/mW^2
- In Ɛ' they enter as

EWP → QCDP

$$\left[\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right]$$

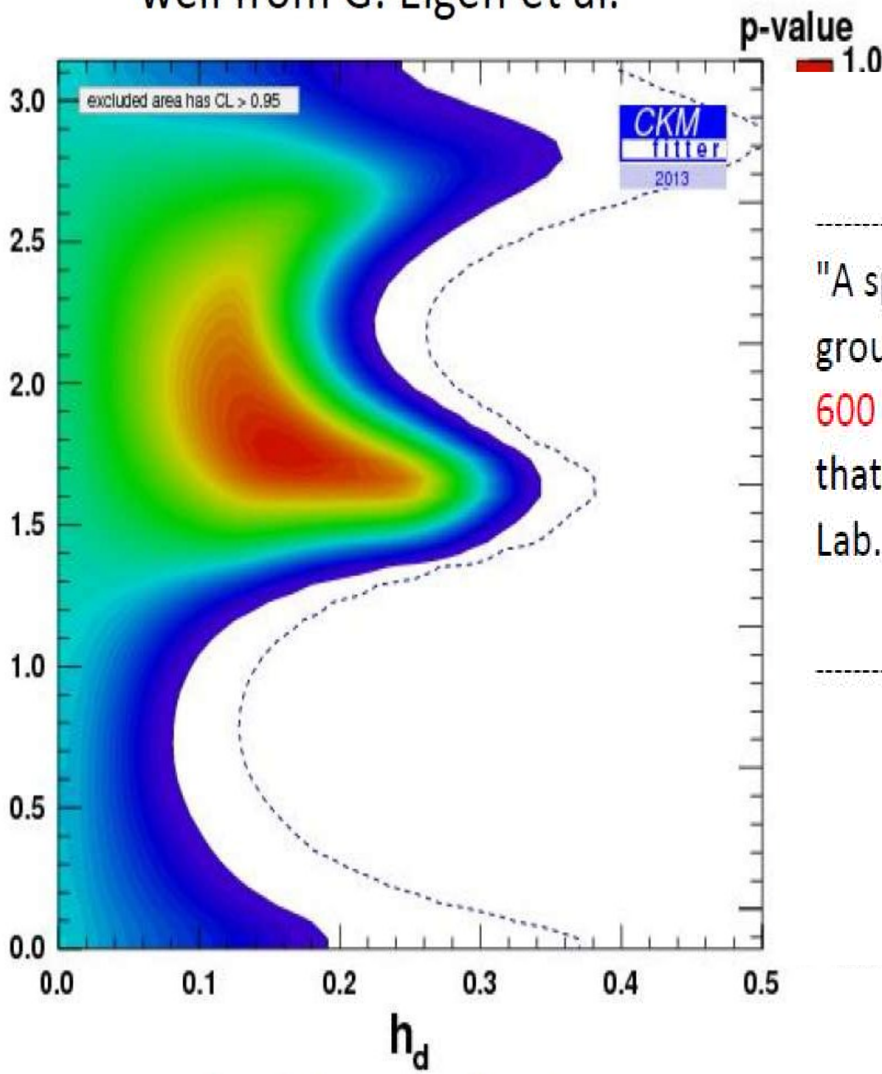
small ← → large

$\frac{ReA_0}{ReA_2} \sim 22$

ICHEP2014: Similar results from UTFIT (D. Derkach) as well from G. Eigen et al.

Current O(few%) tests are far away from O(0.1%) asymmetry in KL=>pi pi

NP Phase



NP/SM amplitude ratio

A lesson from history (I)

"A special search at Dubna was carried out by E. Okonov and his group. They did not find a single $K_L \rightarrow \pi^+ \pi^-$ event among 600 decays into charged particles [12] (Anikira et al., JETP 1962). At that stage the search was terminated by the administration of the Lab. The group was unlucky."

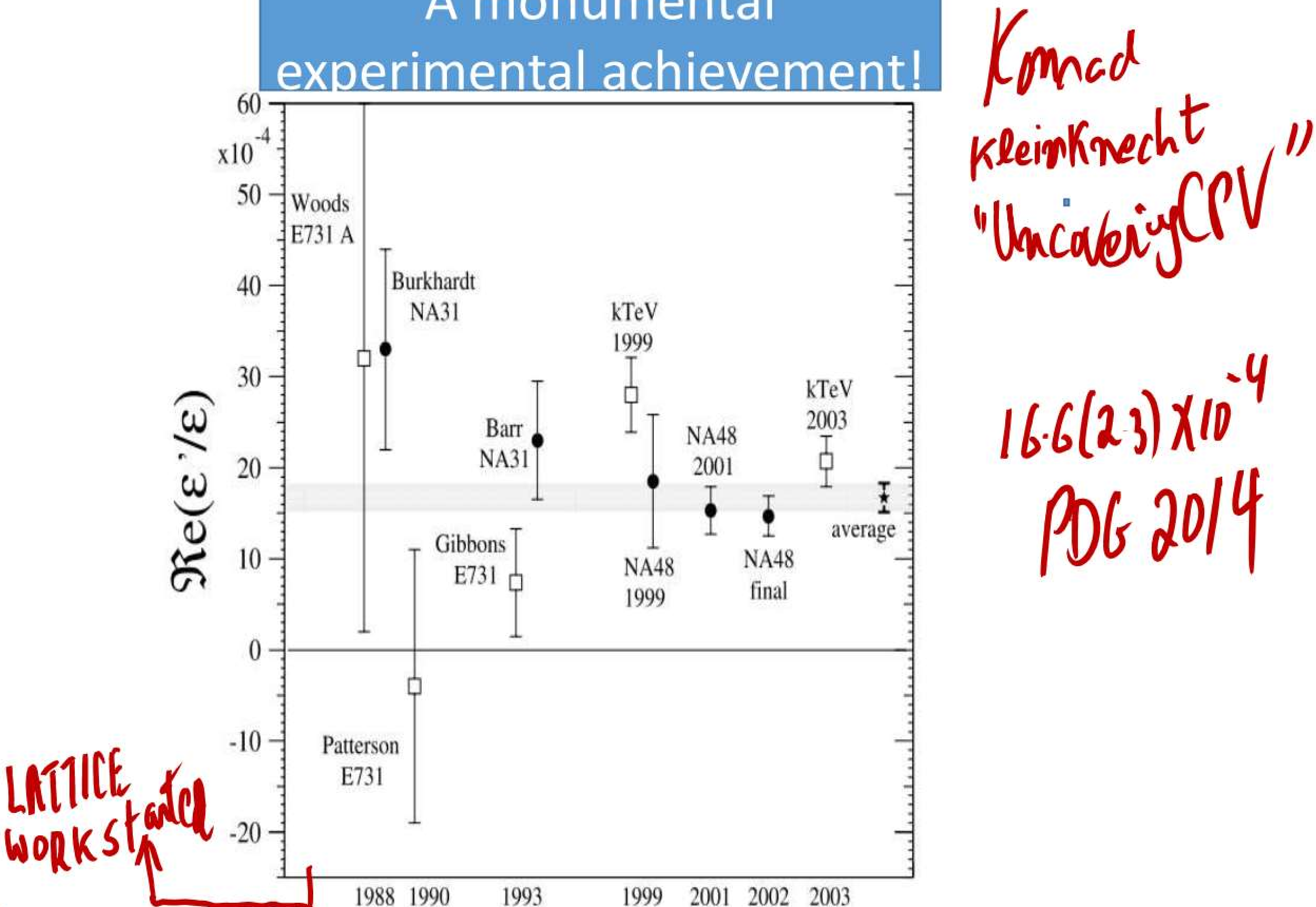
-Lev Okun, "The Vacuum as Seen from Moscow"

1964: BF= 2×10^{-3}

A failure of imagination ? Lack of patience ?

Had KL=>pi pi been abandoned, history of Particle Physics would have been significantly different!

A monumental experimental achievement!

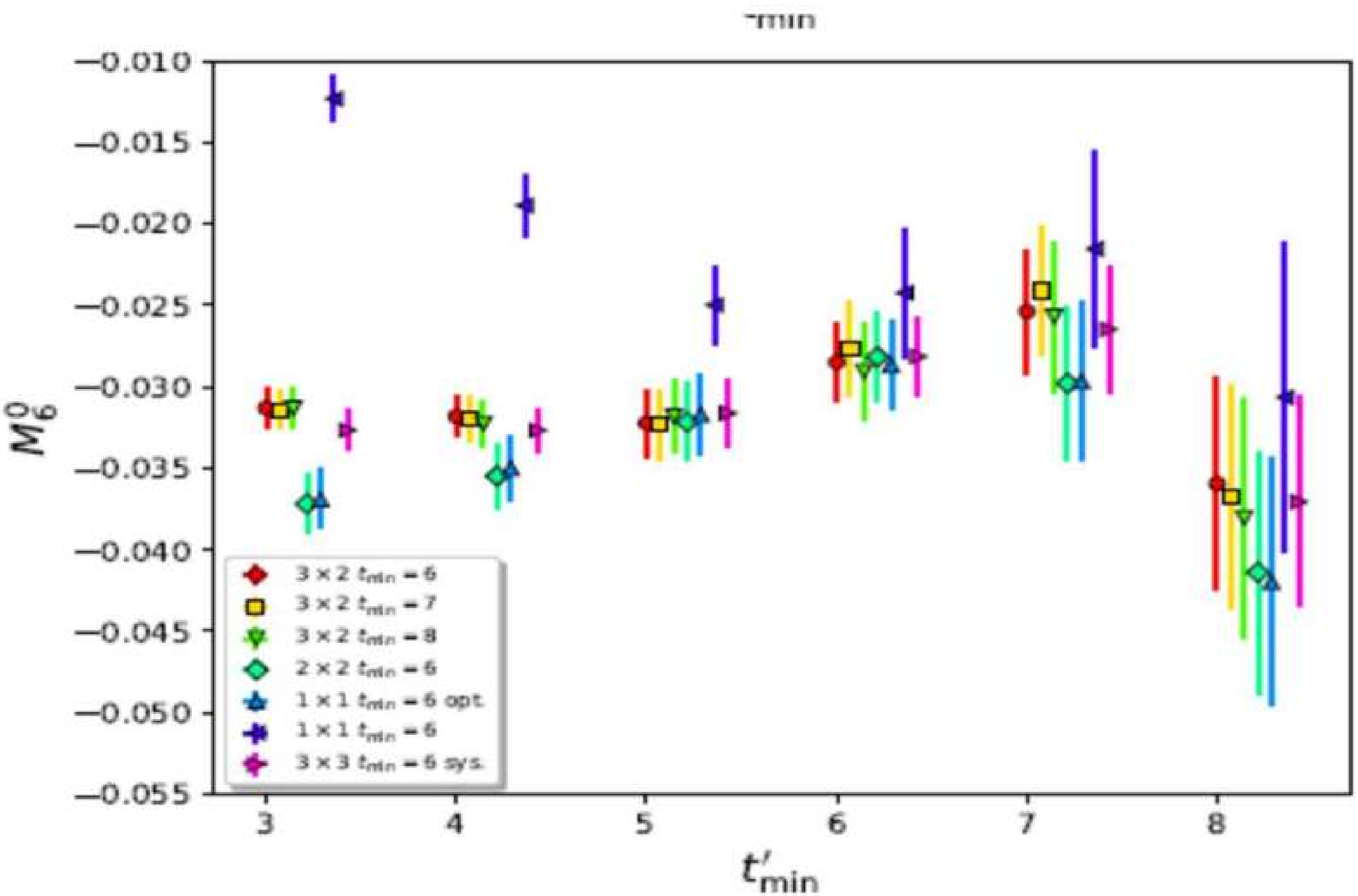


XTRAS

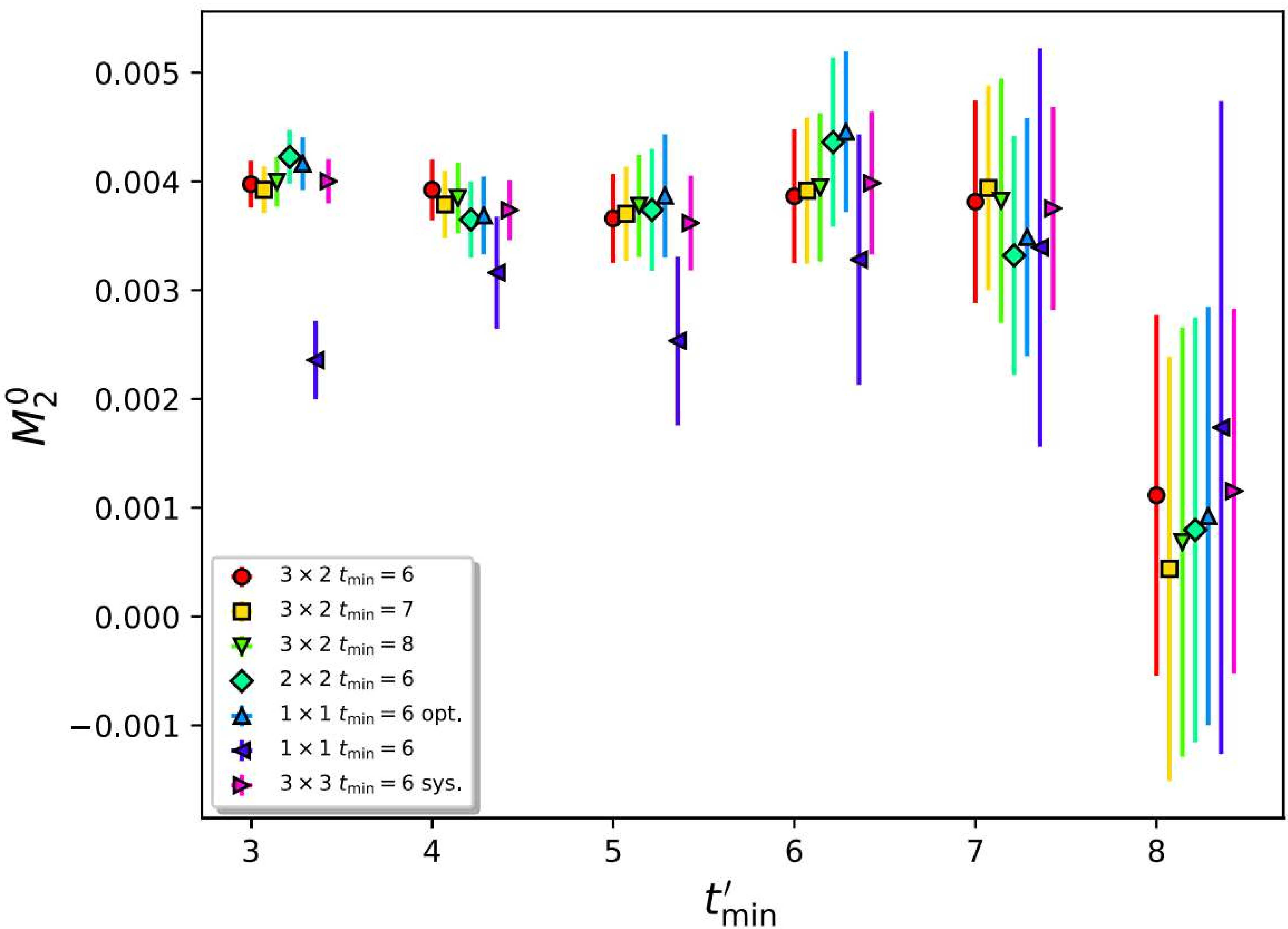
A.S. in Proceedings of Lattice '85 (FSU)..1st Lattice meeting ever attended

The matrix elements of some penguin operators control in the standard model another CP violation parameter, namely ϵ'/ϵ.[6,8) Indeed efforts are now underway for an improved measurement of this important parameter.[10)] In the absence of a reliable calculation for these parameters, the experimental measurements, often achieved at tremendous effort, cannot be used effectively for constraining the theory. It is therefore clearly important to see how far one can go with MC techniques in alleviating this old but very difficult

With C. Bernard [UCLA]



−0.010
−0.015
−0.020
−0.025
−0.030
−0.035
−0.040
−0.045
−0.050
−0.055
M_6^0
3
4
5
6
7
8
t'_{min}
3 × 2 t_{min} = 6
3 × 2 t_{min} = 7
3 × 2 t_{min} = 8
2 × 2 t_{min} = 6
1 × 1 t_{min} = 6 opt.
1 × 1 t_{min} = 6
3 × 3 t_{min} = 6 sys.



0.005
0.004
0.003
0.002
0.001
0.000
−0.001
M_2^0
3
4
5
6
7
8
$t'_{\rm min}$
3×2 $t_{\rm min} = 6$
3×2 $t_{\rm min} = 7$
3×2 $t_{\rm min} = 8$
2×2 $t_{\rm min} = 6$
1×1 $t_{\rm min} = 6$ opt.
1×1 $t_{\rm min} = 6$
3×3 $t_{\rm min} = 6$ sys.

Exploring excited-state signals

- ππ energies in PBC
  - ≈ $2m_\pi$ for ground st.
  - **Need excited-state signals to extract kinematics of K → ππ**

Picture in non-interacting 2-pion system with rest frame

	$\vec{p}$	$E = 2\sqrt{\lvert\vec{p}\rvert^2 + m_\pi^2}$
ground st.	(0,0,0)	$2m_\pi$
1st excited st.	2π/L x (1,0,0)	could be ≈ m_K
2nd excited st.	2π/L x (1,1,0)	

- Variational method useful [Lüscher, 1990]
  - Solving GEVP (Generalized Eigenvalue Problem)

$$C(t)v_n(t,t_0) = \lambda_n(t,t_0)C(t_0)v_n(t,t_0) \quad \begin{cases} C(t) \text{ : N x N correlator matrix} \\ C_{ab}(t) = \langle O_a(t)O_b(0)^\dagger\rangle \end{cases}$$

  - $O'_n = \sum_a v^*_{n,a} O_a$ couples with only n-th, N+1-th & higher states
  - $\lambda_n(t,t_0) = e^{-E_n(t-t_0)}$

- We employ 5 independent ππ operators
  - $O_a \in \pi_{p=(0,0,0)}\pi_{p=(0,0,0)},\ \pi_{p=(0,0,1)}\pi_{p=(0,0,-1)},\ \pi_{p=(0,1,1)}\pi_{p=(0,-1,-1)},\ \pi_{p=(1,1,1)}\pi_{p=(-1,-1,-1)}$ & σ

$$Q_1 = (\bar{s}_\alpha d_\alpha)_L (\bar{u}_\beta u_\beta)_L,$$

$$Q_2 = (\bar{s}_\alpha d_\beta)_L (\bar{u}_\beta u_\alpha)_L,$$

$$Q_3 = (\bar{s}_\alpha d_\alpha)_L \sum_{q=u,d,s} (\bar{q}_\beta q_\beta)_L,$$

$$Q_4 = (\bar{s}_\alpha d_\beta)_L \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_L,$$

$$Q_5 = (\bar{s}_\alpha d_\alpha)_L \sum_{q=u,d,s} (\bar{q}_\beta q_\beta)_R,$$

$$Q_6 = (\bar{s}_\alpha d_\beta)_L \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_R,$$

$$Q_7 = \frac{3}{2} (\bar{s}_\alpha d_\alpha)_L \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\beta)_R,$$

$$Q_8 = \frac{3}{2} (\bar{s}_\alpha d_\beta)_L \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\alpha)_R,$$

$$Q_9 = \frac{3}{2} (\bar{s}_\alpha d_\alpha)_L \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\beta)_L,$$

$$Q_{10} = \frac{3}{2} (\bar{s}_\alpha d_\beta)_L \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\alpha)_L,$$

Indirect CP violation in KL=>3 pi

The basic expression for ε is

$$\varepsilon = e^{i\phi_\varepsilon} \frac{G_F^2 m_W^2 f_K^2 m_K}{12\sqrt{2}\pi^2 \Delta m_K^{\exp}} \hat{B}_K \kappa_\varepsilon \,\mathrm{Im}\Big[\eta_1 S_0(x_c) \left(V_{cs} V_{cd}^*\right)^2 + \eta_2 S_0(x_t) \left(V_{ts} V_{td}^*\right)^2 + 2\eta_3 S_0(x_c, x_t) V_{cs} V_{cd}^* V_{ts} V_{td}^* \Big], \quad (41)$$

where the numerical inputs we use are summarized in Table 2. The quantity κ_ε summarizes the impact of long distance effects and can be extracted from the knowledge of Im A_0 and from an estimate of the long distance contributions to Δm_K. Following Ref. [76], we have:

$$\kappa_\varepsilon = \sqrt{2}\sin(\phi_\varepsilon) \left(1 + \frac{\rho}{\sqrt{2}\,|\varepsilon_{\exp}|} \frac{\mathrm{Im}(A_0)}{\mathrm{Re}(A_0)}\right) \quad (42)$$

where $\rho = 0.6 \pm 0.3$. Using the most recent RBC determination of $\mathrm{Im}(A_0)$ and ϕ_ε of Eq. (32), we obtain $\kappa_\varepsilon = 0.963 \pm 0.014$ (see also the analysis presented in Ref. [77]).

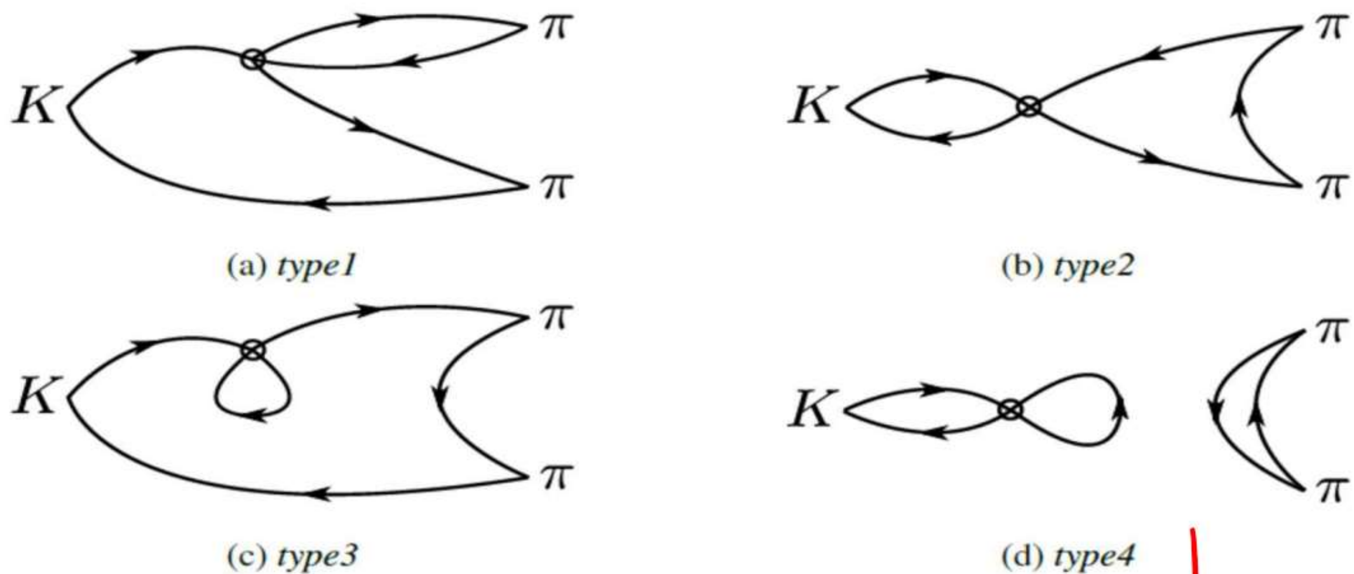


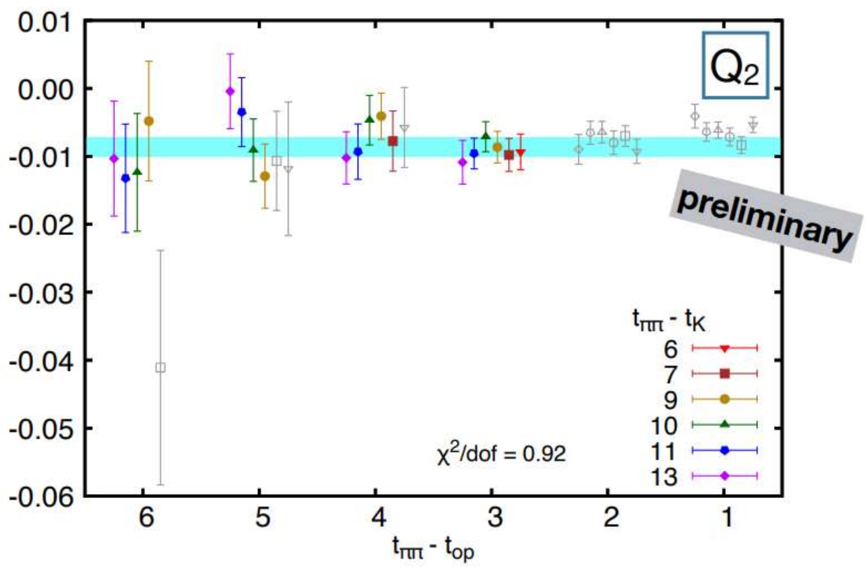
FIG. 2: The four classes of $K \to \pi\pi$ Wick contractions.

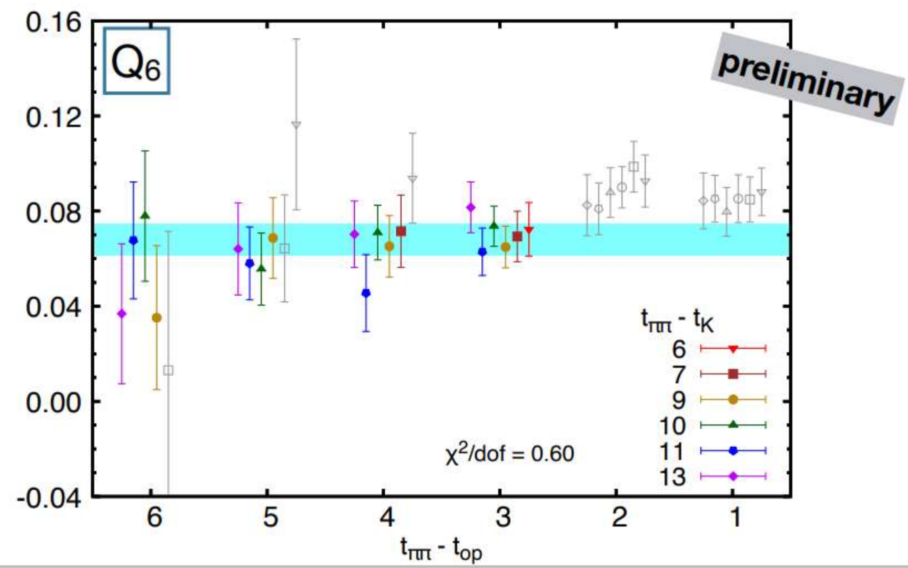
ALL ARE INCLUDED!

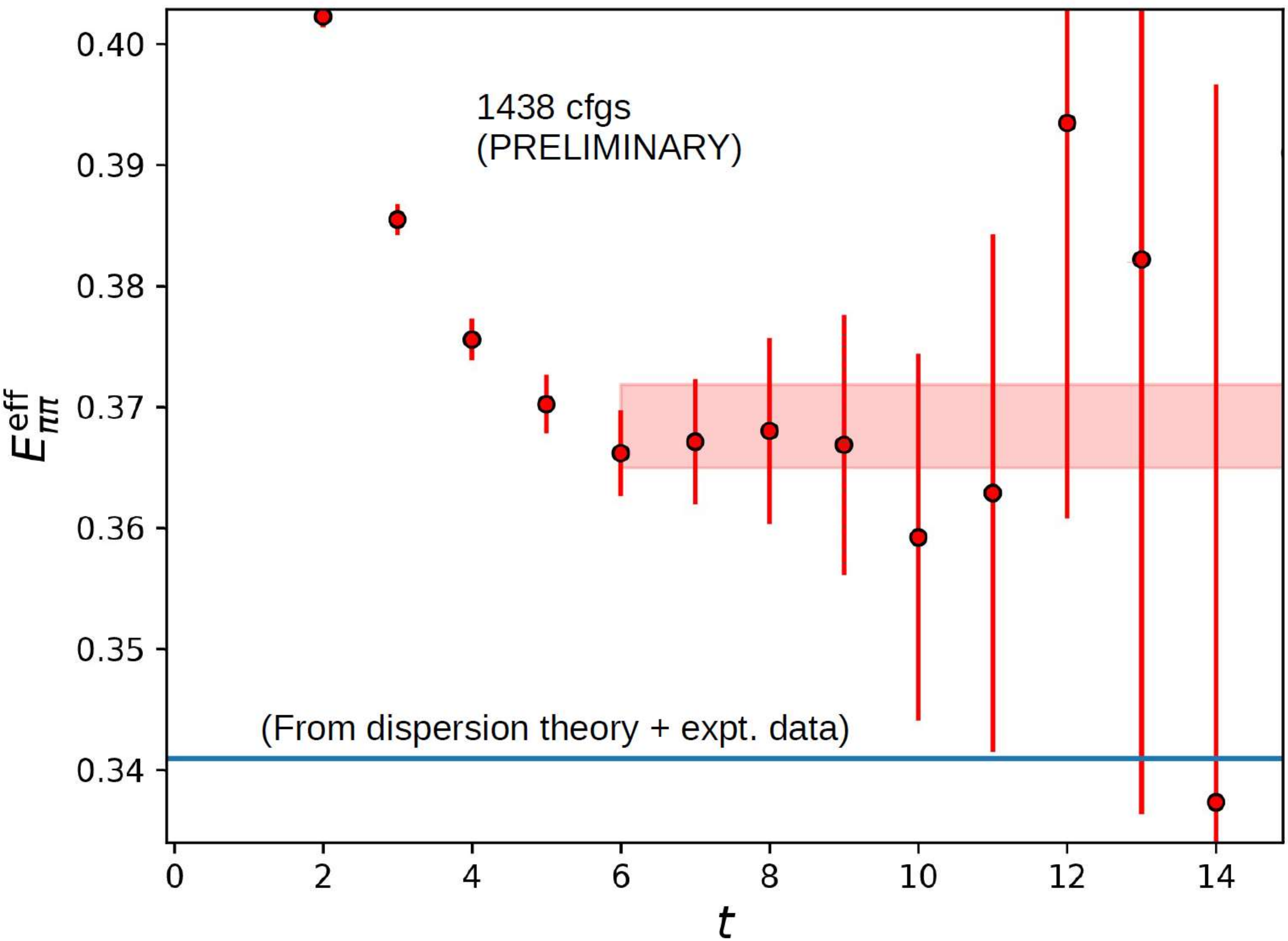
↑ "DISCONNECTED" very difficult

Effective matrix elements ($\Delta I = 1/2$)

- Plateau appears
- (cyan band) : Example of correlated fit result with $t_{op}-t_K \geq 3$ && $t_{\pi\pi}-t_{op} \geq 3$ (colored filled data points)







1438 cfgs
(PRELIMINARY)
(From dispersion theory + expt. data)
$E_{\pi\pi}^{\rm eff}$
t
0.40
0.39
0.38
0.37
0.36
0.35
0.34
0
2
4
6
8
10
12
14

Back to the core story. . . --Confronting χSymmetric lattice

A chance (crucial) meeting: Yigal Shamir visits me in Haifa ~94 summer

- For K=> pi pi project, way to overcome the fine-tuning problem of Wilson Fermions is to use a new formulation of

fermions on the lattice=> **DOMAIN WALL FERMIONS** [computationally much harder but are continuum -like possessing chiral symmetry]

- Furman + Shamir: hep-lat/9405004
- See also Yigal Shamir, hep-lat 9303005

WAY FORWARD: Adopt DWF for $K \to \pi\pi$ & ϵ'? 95-96?.

- ***As a result, the large accidental cancellations significantly enhances sensitivity of ε' to NP***

More demands on the calculation

- ~ The 1995 discovery of the huge top mass accentuated the cancellation of I=0 and I=2 contributions to ε' significantly, putting additional demands on the calculation but also enhancing the potential for discovery of new physics

$\epsilon_8 \propto m_t^2 / M_W^2$

We use

$$\frac{\varepsilon'}{\varepsilon} = \frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}\varepsilon}\left[\frac{\mathrm{Im}(A_2)}{\mathrm{Re}(A_2)} - \frac{\mathrm{Im}(A_0)}{\mathrm{Re}(A_0)}\right]$$

→ isospin sym formula.

$\sim (17 \pm 9.1) \times 10^{-2}$

IB+EM eff

⟹

$$\frac{\varepsilon'}{\varepsilon} = \frac{i\omega_+ e^{i(\delta_2-\delta_0)}}{\sqrt{2}\varepsilon}\left[\frac{\mathrm{Im}(A_2^{\mathrm{emp}})}{\mathrm{Re}(A_2^{(0)})} - \frac{\mathrm{Im}(A_0^{(0)})}{\mathrm{Re}(A_0^{(0)})}\left(1-\hat{\Omega}_{\mathrm{eff}}\right)\right].$$

See Cirigliano et al 1911.01359

THIS IS NOT our work

WE CHOOSE TO include THIS in our syst error

12/28/2022 HET-Lunch-071720 61

The ultimate theoretical error on γ from $B \to DK$ decays

Because of this ultra miniscule higher order correction γ is the "STANDARD CANDLE" in the SM-KM paradigm of CPV

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Abstract: The angle γ of the standard CKM unitarity triangle can be determined from $B \to DK$ decays with a very small irreducible theoretical error, which is only due to second-order electroweak corrections. We study these contributions and estimate that their impact on the γ determination is to introduce a shift $|\delta\gamma| \lesssim \mathcal{O}(10^{-7})$, well below any present or planned future experiment.

JHEP01(2014)051

A difficulty: strong phases

- The continuum and our lattice determinations of strong phase

differ

-

$$\phi_{\epsilon'} = \delta_2 - \delta_0 + \frac{\pi}{2} = \begin{cases} (42.3 \pm 1.5)^\circ & \text{[illegible] [2]} \\ (54.6 \pm 5.8)^\circ & \text{RBC [47, 48]} \end{cases}$$

Colangelo etal
ChPT etc

RBC-UKQCD

Challenges of physical K=>pi pi kinematics on the lattice

"at rest" unphysical kinematics; Qi Liu PhD (2012); RBC-UKQCD PRD 2011

- Primary challenge is to assure physical kinematics: For periodic BCs, amplitude with 2 stationary pions in final state dominates. However

$|\vec{P}_\pi| \sim 205$ MeV

$$2m_\pi \approx 200 \text{ MeV} \ll m_K \approx 500 \text{ MeV}$$

280

II
- Desired state with moving pions is next-to-leading term: require 2exp fits? ⇐ New

now under study... T Blum, D Hoying et al

I
- Avoid 2-exp fits by removing stationary pion state from system through manipulating lattice spatial boundary conditions:

C Kelly et al

  - Antiperiodic BCs on down-quark for A_2
  - G-parity BCs on both quarks for A_0

→ $p_\pi = 0 \rightarrow \pi/L$

tune L to match E_K and $E_{\pi\pi}$

underway for about 7 years

Resolving the [I=0] Energy & phase shift in the pi pi channel

- 2015 result has 2σ+ discrepancy between our I=0 ππ phase shift (δ_0=23.8(4.9)(1.2)°) and dispersion theory prediction (~34°).

[RBC&UKQCD PRL 115 (2015) 21, 212001]
[Colangelo *et al*, Nucl.Phys. B603 (2001) 125-179]

CKe CKM'18

- Observed discrepancy more significant (~5σ) with 6.5x stats.
- Most likely explanation is excited-state contamination.
- To address added scalar (σ=ūd) ππ operator to the 2-pt function calculation.
- Combined fits (or GEVP) to ππ→ππ, σ→ππ and σ→σ correlators result in considerably lower ground-state energy:

Fn GEVP see Sommer et al 1108.3774

508(5) MeV [1386 cfgs] from ππ→ππ alone
vs
483(1) MeV [501 cfgs] from sim. fit of all 3 correlators.

- New phase shift δ_0=30.9(1.5)(3.0)° [prelim] compatible with dispersive result.

Colangelo et al

- Strong evidence for nearby excited finite-volume ππ state.
Indeed such a state with E ~ 770 MeV is predicted by dispersion theory.

NOTE: $\delta_2 = -11.6 \pm 2.5 \pm 1.2^\circ$ & $E_{\pi\pi}(I=2) = 573.0 \pm 2.9$ MeV see PRL 2015 arXiv: 2004.09440

RBC-UKQCD 2020

Parameter	Value	
	2-state fit	3-state fit
Fit range	6-15	4-15
$A^0_{\pi\pi(111)}$	0.3682(31)	0.3718(22)
$A^0_{\pi\pi(311)}$	0.00380(32)	0.00333(27)
A^0_{σ}	−0.0004309(41)	−0.0004318(42)
E_0	0.3479(11)	0.35030(70)
$A^1_{\pi\pi(111)}$	0.1712(91)	0.1748(67)
$A^1_{\pi\pi(311)}$	−0.0513(27)	−0.0528(30)
A^1_{σ}	0.000314(17)	0.000358(13)
E_1	0.568(13)	0.5879(65)
$A^2_{\pi\pi(111)}$	—	0.116(29)
$A^2_{\pi\pi(311)}$	—	0.063(10)
A^2_{σ}	—	0.000377(94)
E_2	—	0.94(10)
p-value	0.314	0.092

TABLE III: Fit parameters in lattice units and the p-values for multi-operator fits to the $I = 0$ $\pi\pi$ two-point functions. Here E_i are the energies of the states and A^i_α represents the matrix element of the operator α between the state i and the vacuum, given in units of $\sqrt{1 \times 10^{13}}$. The second column gives the parameters for our primary fit which uses two-states and three operators. The third column shows a fit with the same three operators and one additional state that is used to probe the systematic effects of this third state on the $K \to \pi\pi$ matrix element fits.

PHYSICAL REVIEW D **102,** 054509 (2020)

Editors' Suggestion | Featured in Physics

Direct *CP* violation and the $\Delta I = 1/2$ rule in $K \to \pi\pi$ decay from the standard model

R. Abbott,[1] T. Blum,[2,3] P. A. Boyle,[4,5] M. Bruno,[6] N. H. Christ,[1] D. Hoying,[3,2] C. Jung,[4] C. Kelly,[4] C. Lehner,[7,4] R. D. Mawhinney,[1] D. J. Murphy,[8] C. T. Sachrajda,[9] A. Soni,[4] M. Tomii,[2] and T. Wang[1]

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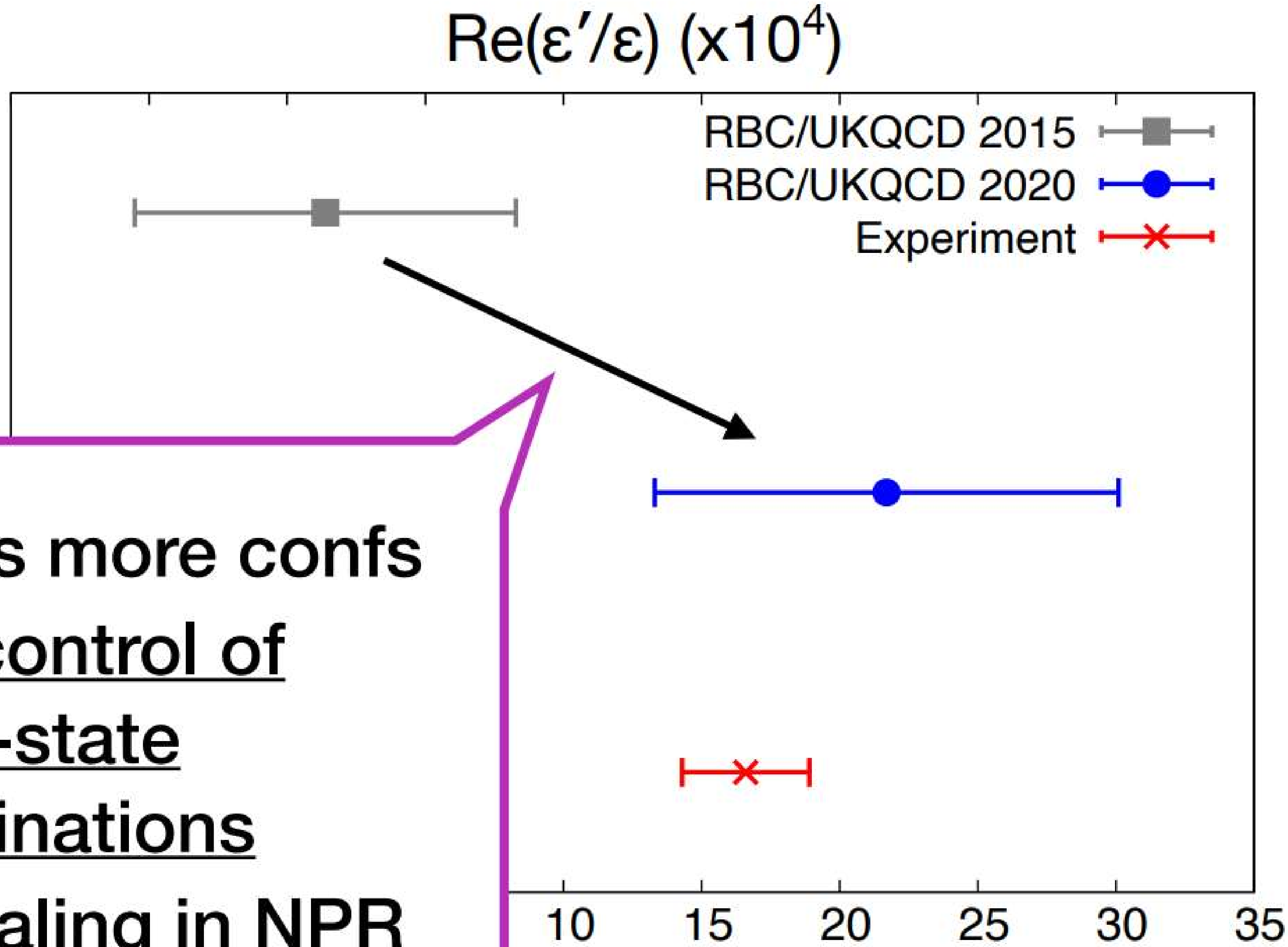
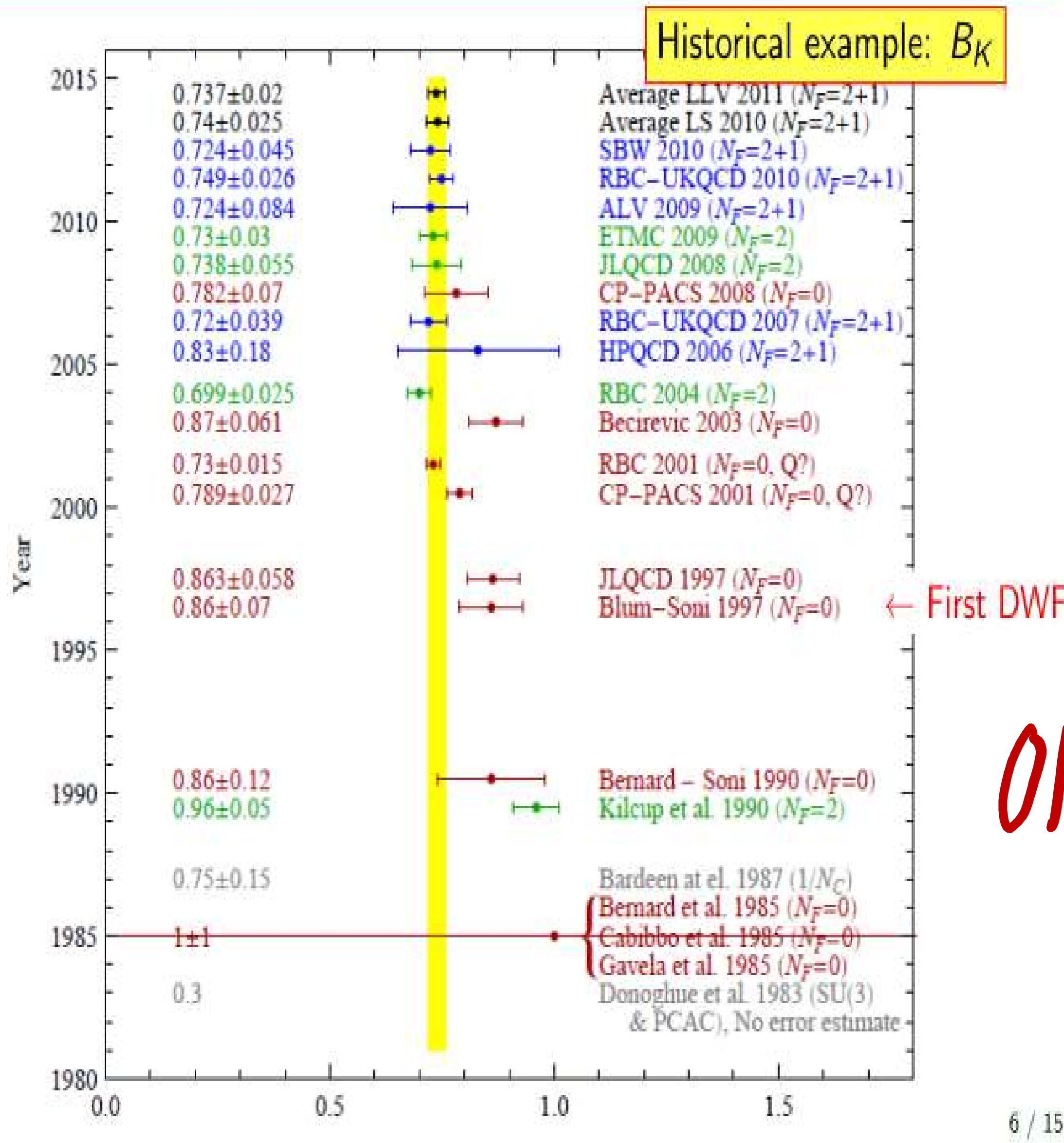
(Received 18 May 2020; accepted 13 August 2020; published 17 September 2020)



- 3+ times more confs
- <u>Better control of excited-state contaminations</u>
- Step scaling in NPR

Power of the lattice: Only method to systematically reduce the NP error!



AB-initio Calculat

$$B_K = \frac{\langle K|(\bar{s}d)^2|\bar{K}\rangle}{8/3\, f_K^2\, m_K^2}$$

ONE ILLUSTRATION

The RBC & UKQCD collaborations

UC Berkeley/LBNL
Aaron Meyer

BNL and BNL/RBRC
Yasumichi Aoki (KEK)
Peter Boyle (Edinburgh)
Taku Izubuchi
Chulwoo Jung
Christopher Kelly
Meifeng Lin
Nobuyuki Matsumoto
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Relating lattice ME to physical amplitudes

$$A_{2/0} = F\frac{G_F}{\sqrt{2}}V_{ud}V_{us}\sum_{i=1}^{10}\sum_{j=1}^{7}\left[\left(z_i(\mu)+\tau y_i(\mu)\right)Z_{ij}^{\mathrm{lat}\to\overline{\mathrm{MS}}}M_j^{\frac{3}{2}/\frac{1}{2},\mathrm{lat}}\right]$$

F is the Lellouch-Luscher factor which relates finite volume ME to the infinite volume

$$A = \frac{1}{\pi q}\sqrt{\frac{\partial\phi}{\partial q}+\frac{\partial\delta}{\partial q}}\sqrt{m_K}E_{\pi\pi}L^{2/3}M$$

→ Phase shift

A/M is LL factor F

$q = \frac{pL}{2\pi}$;

↳ $\sim \frac{\delta}{q}$ for small p

φ is a somewhat complicated function of q and boundary Conditions [See Daiqian Zhang thesis]