



Facultad  
de Ciencias



Universidad Autónoma  
de Madrid



Instituto de  
Física  
Teórica  
UAM-CSIC



# PROBING THE HEAVY NEUTRINO HYPOTHESIS

**Xabier Marciano**



Funded by the  
European Union



# MOTIVATION



# BSM BEHIND NEUTRINO MASSES



**electron  
neutrino**



**muon  
neutrino**



**tau  
neutrino**



# BSM BEHIND NEUTRINO MASSES



**electron  
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**muon  
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**tau  
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# BSM BEHIND NEUTRINO MASSES



**electron  
neutrino**



**muon  
neutrino**



**tau  
neutrino**



**Heavy  
neutrino**



Curriculum Vitae

# Heavy Neutrino

Right-handed/sterile neutrino, heavy neutral lepton (HNL)

**Spin**

1/2

**Work experience**

- ◆ Neutrino masses

**Color**

single

**Other skills**

- ◆ Osc. anomalies

**Isospin**

single

- ◆ Dark matter

**Charge**

0

- ◆ Baryogenesis

**Mass**

how dare you!?

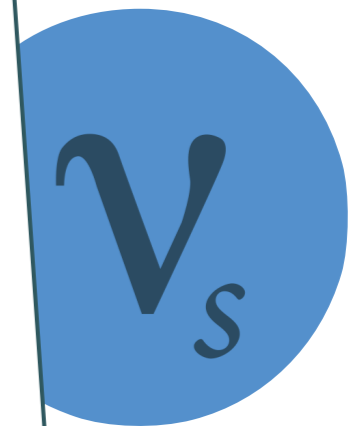
- ◆ Cooking

**Social life**

not much



**electron  
neutrino**

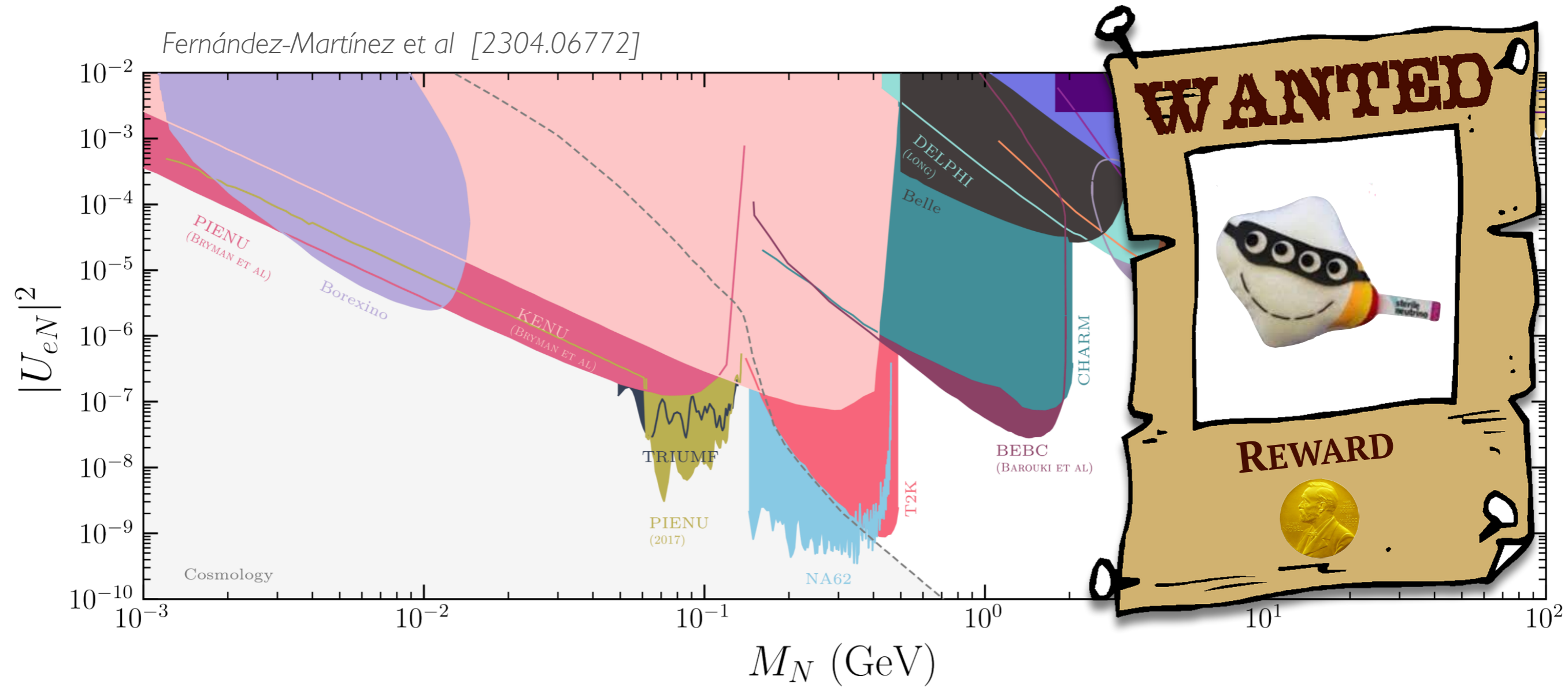


**heavy  
neutrino**

# SEARCHES FOR HEAVY NEUTRINOS



# SEARCHES FOR HEAVY NEUTRINOS





# WHICH TRACK?

Energy Frontier

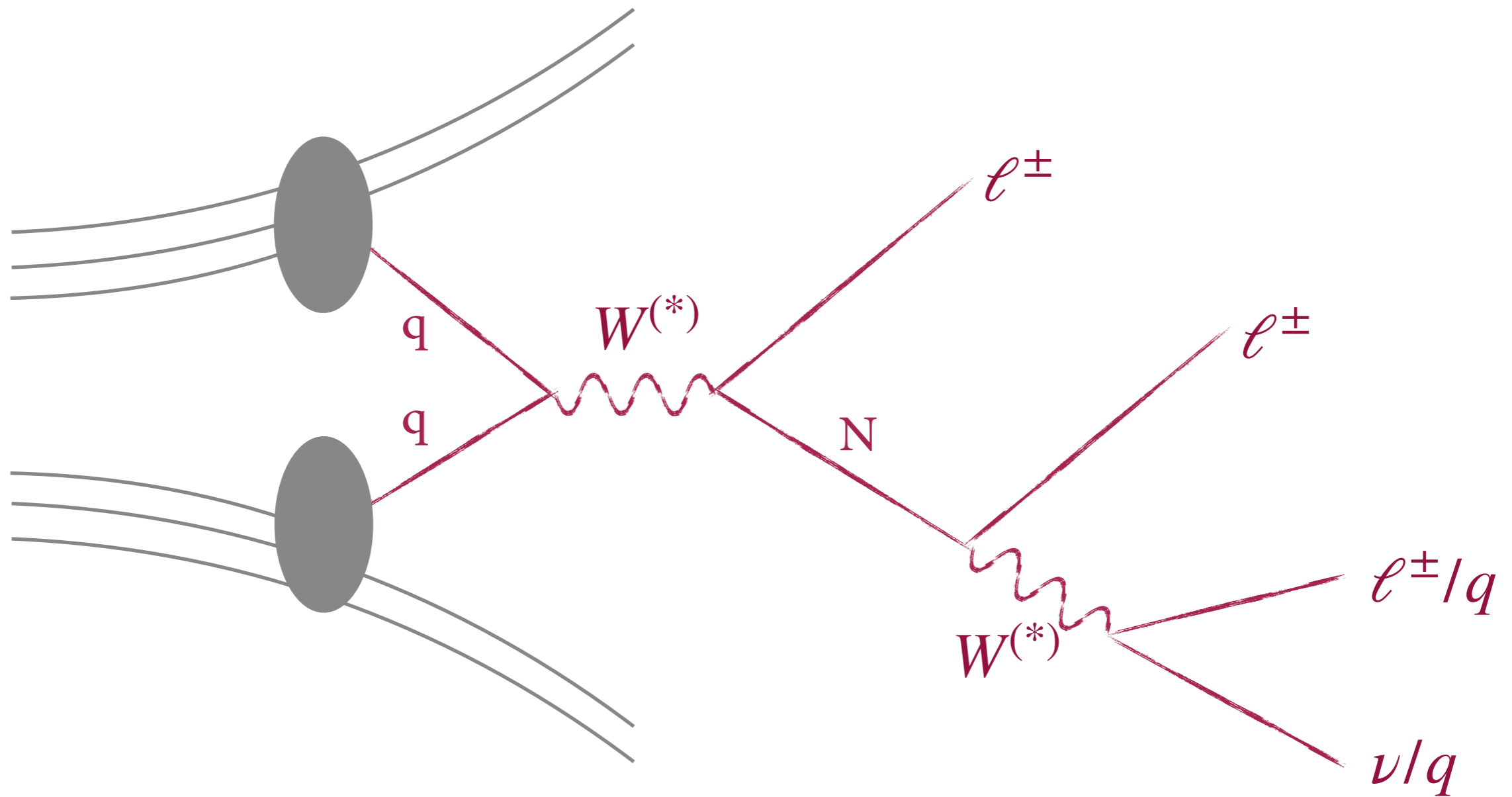
Intensity Frontier



## Heavy Neutrino (HNL) Searches at LHC

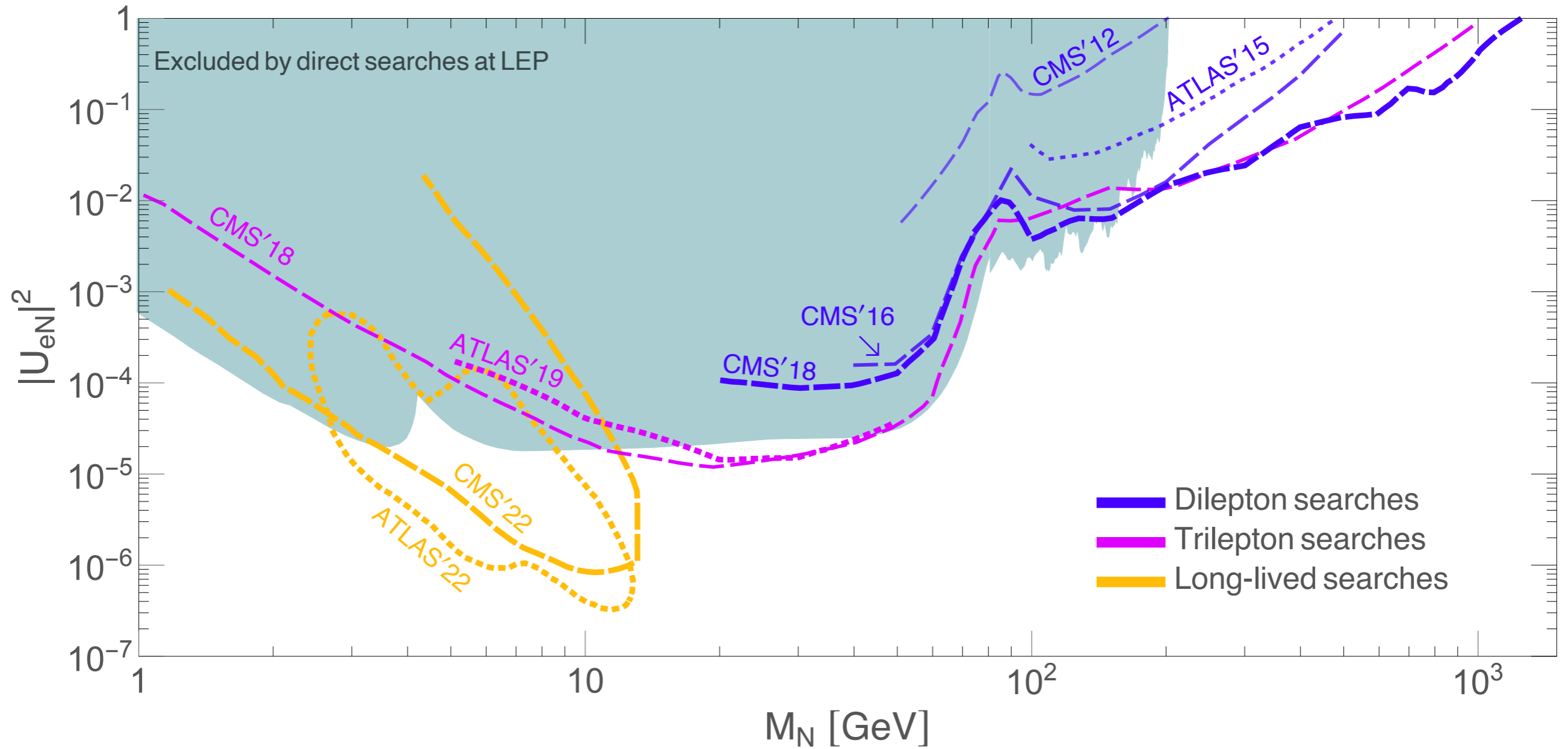
— see talk by Sergio Grancagnolo —

# PRODUCTION AND DECAY AT LHC



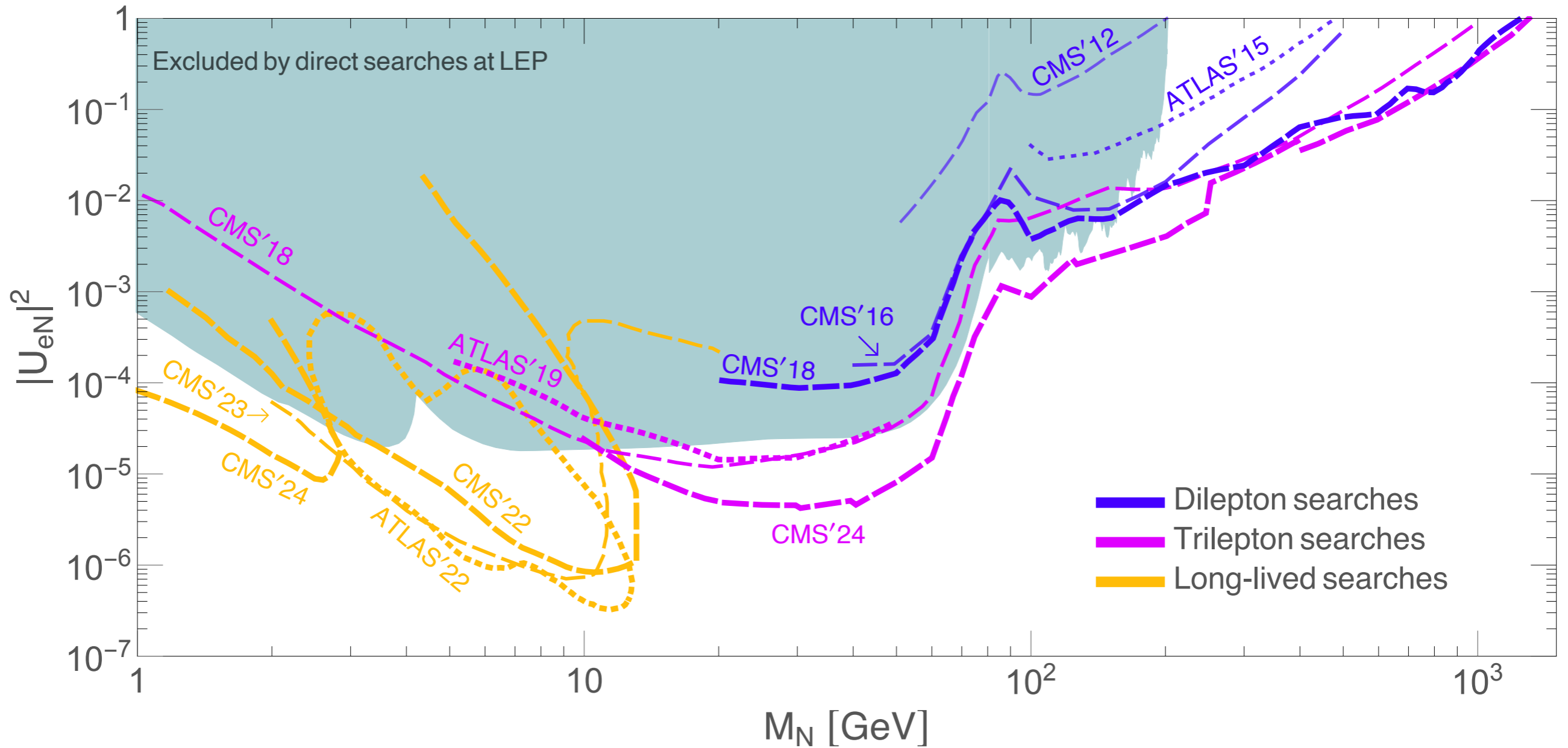
# CURRENT STATUS: ELECTRON

Abada, Escribano, XM, Piazza [2208.13882]



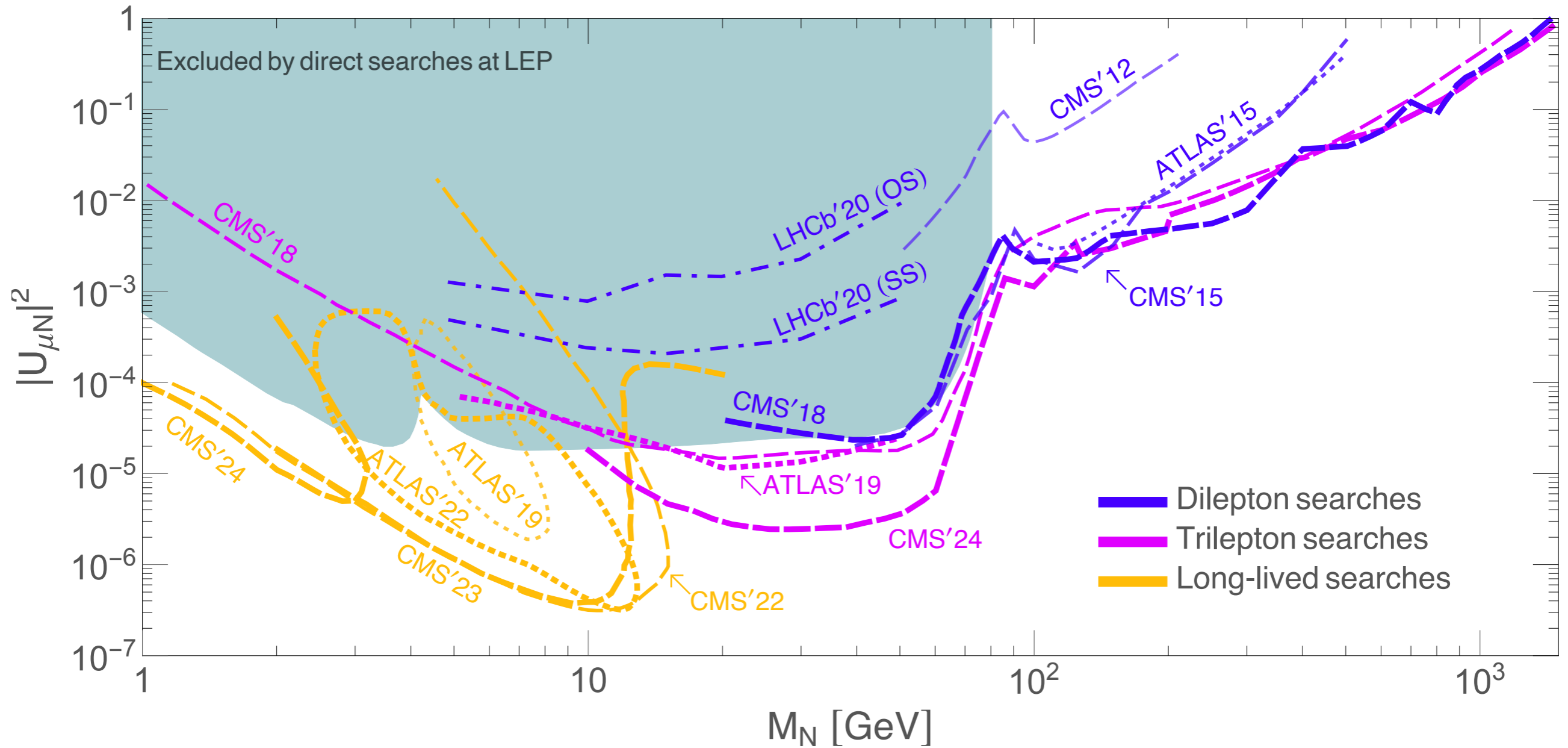
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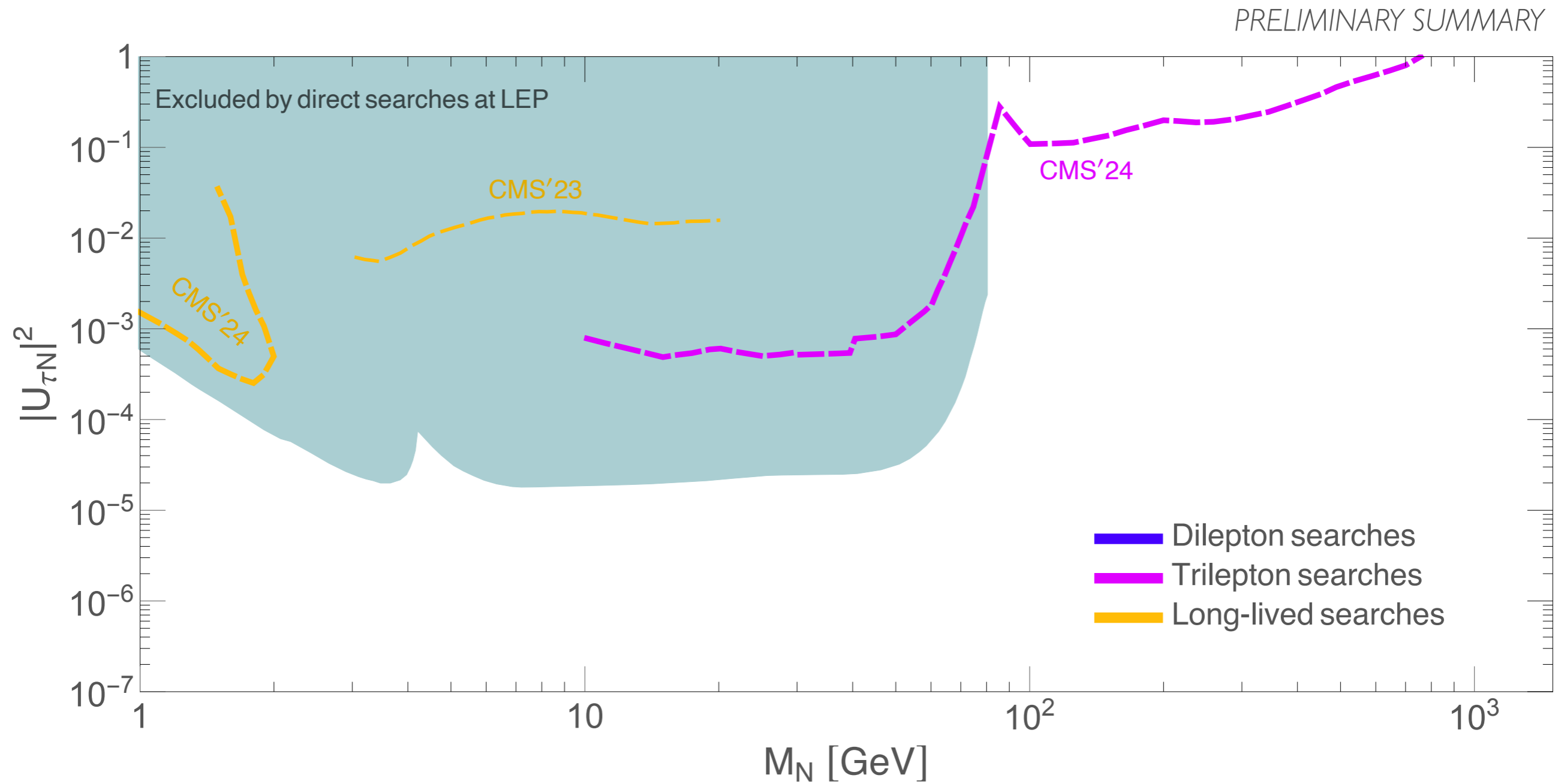


# CURRENT STATUS: MUON

updated from Abada, Escribano, XM, Piazza [2208.13882]



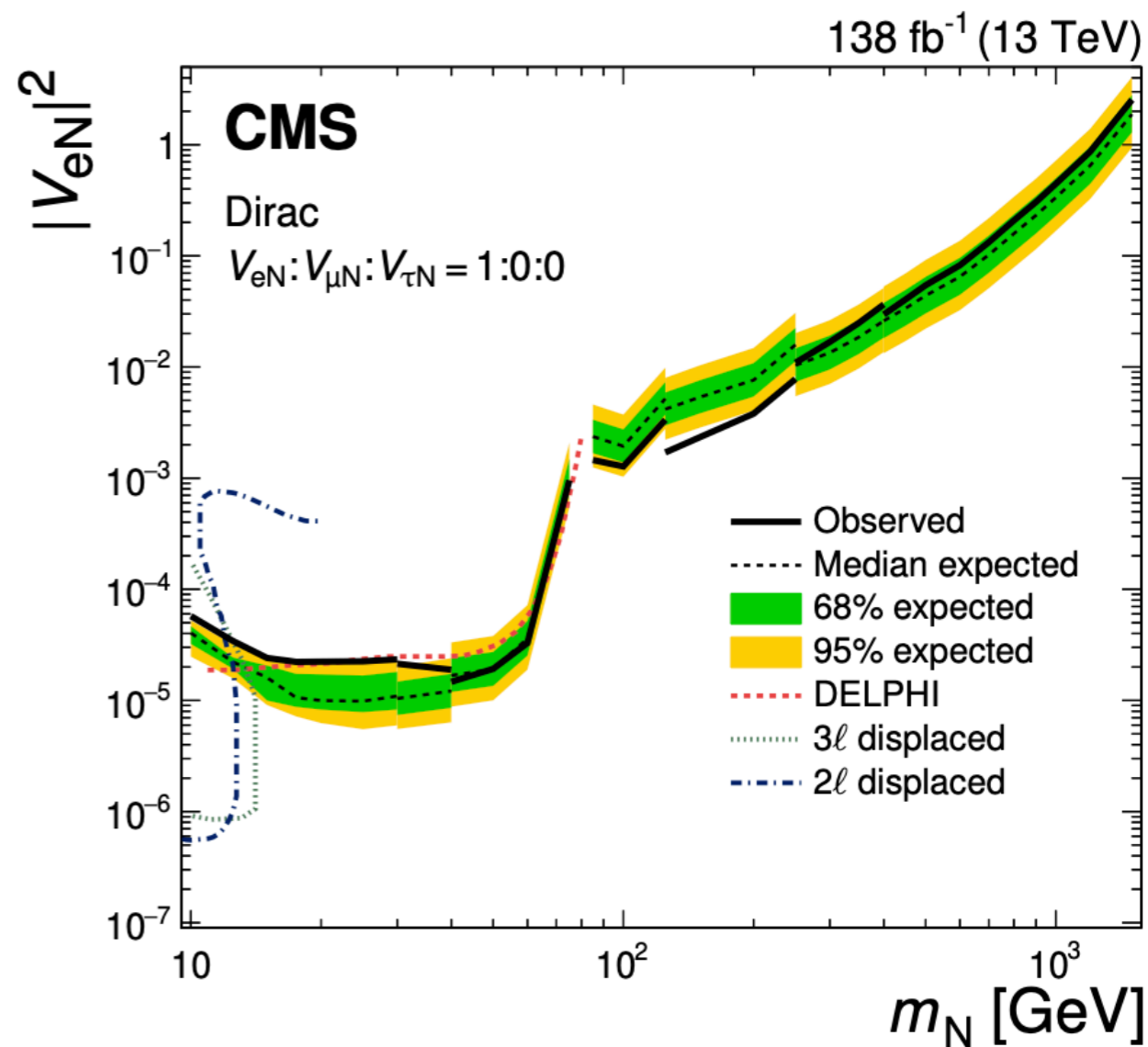
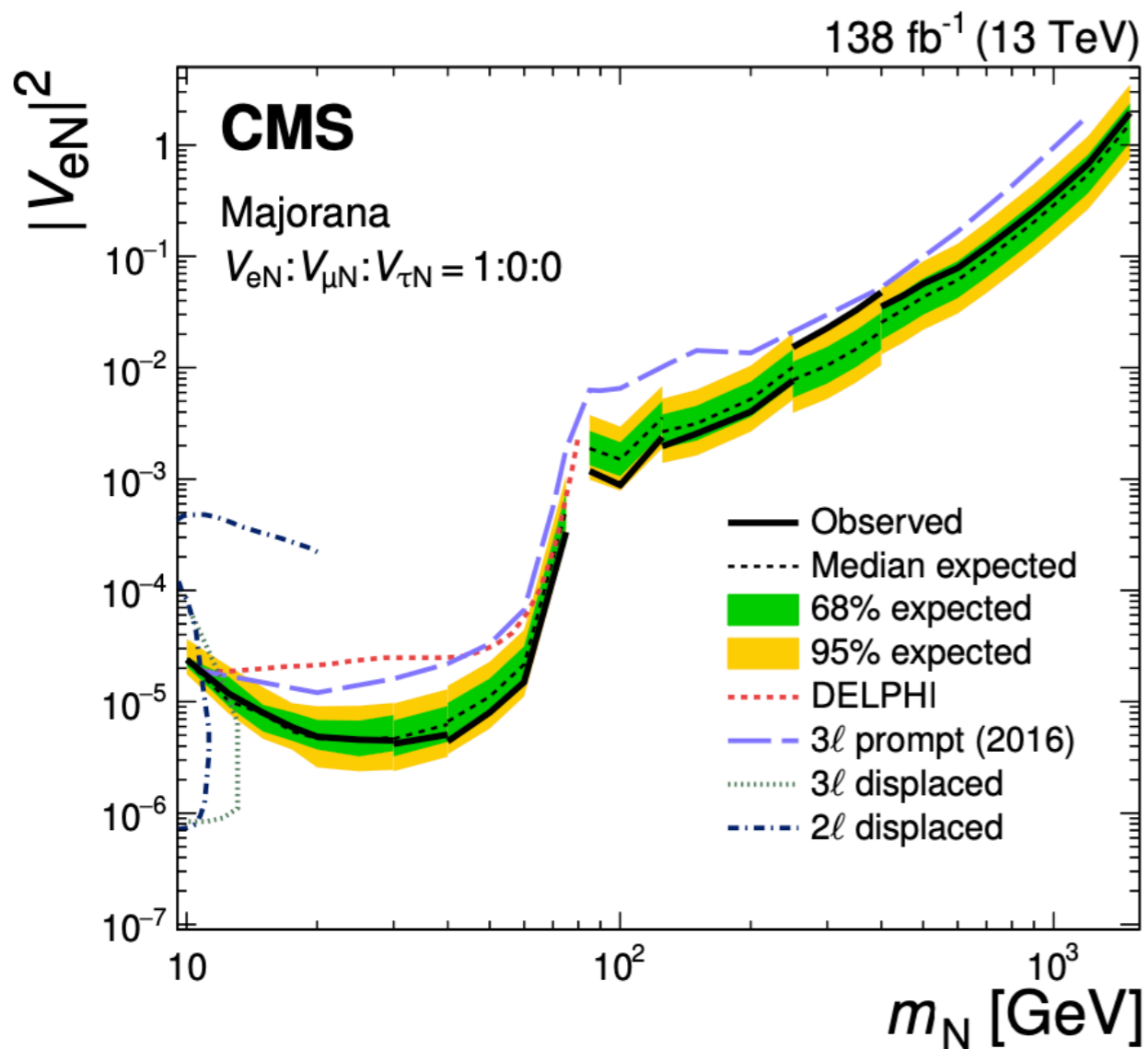
# CURRENT STATUS: TAU



# DIRAC VS MAJORANA

■ Search for both Dirac ( $LNV=0$ ) and Majorana ( $LNV=LNC$ ) hypotheses

CMS trilepton [2403.00100]

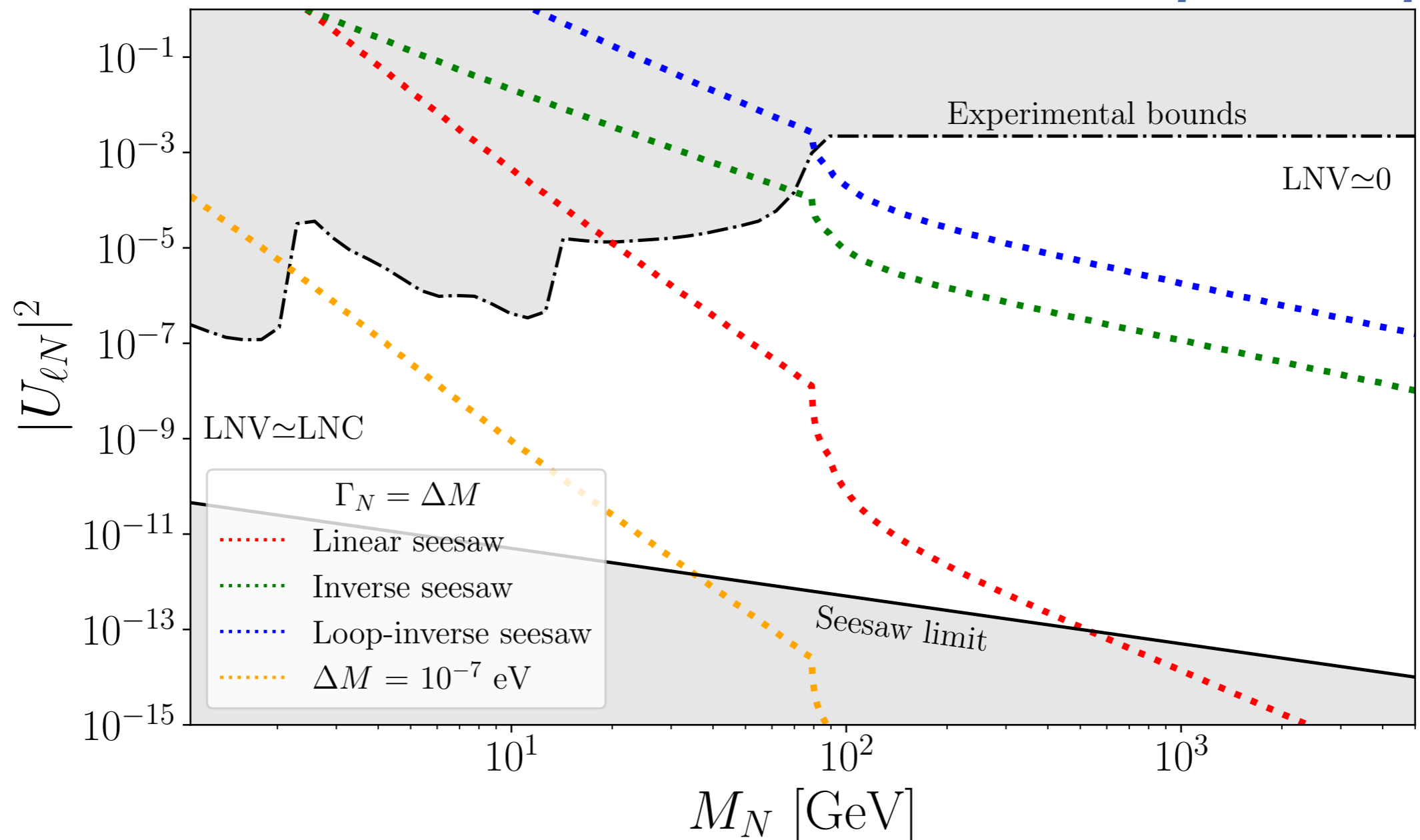




# DIRAC VS MAJORANA

- Search for both Dirac ( $LNV=0$ ) and Majorana ( $LNV=LNC$ ) hypotheses
- Discriminate between neutrino mass models (low-scale seesaws)

Fernández-Martínez, XM, Naredo-Tuero [2209.04461]



# GOING BEYOND THE SINGLE MIXING

*— Realistic Models usually predict mixings to all flavor —*

# GOING BEYOND THE SINGLE MIXING

– *Realistic Models usually predict mixings to all flavor* –

■ *Results in the HEPData record* ✓

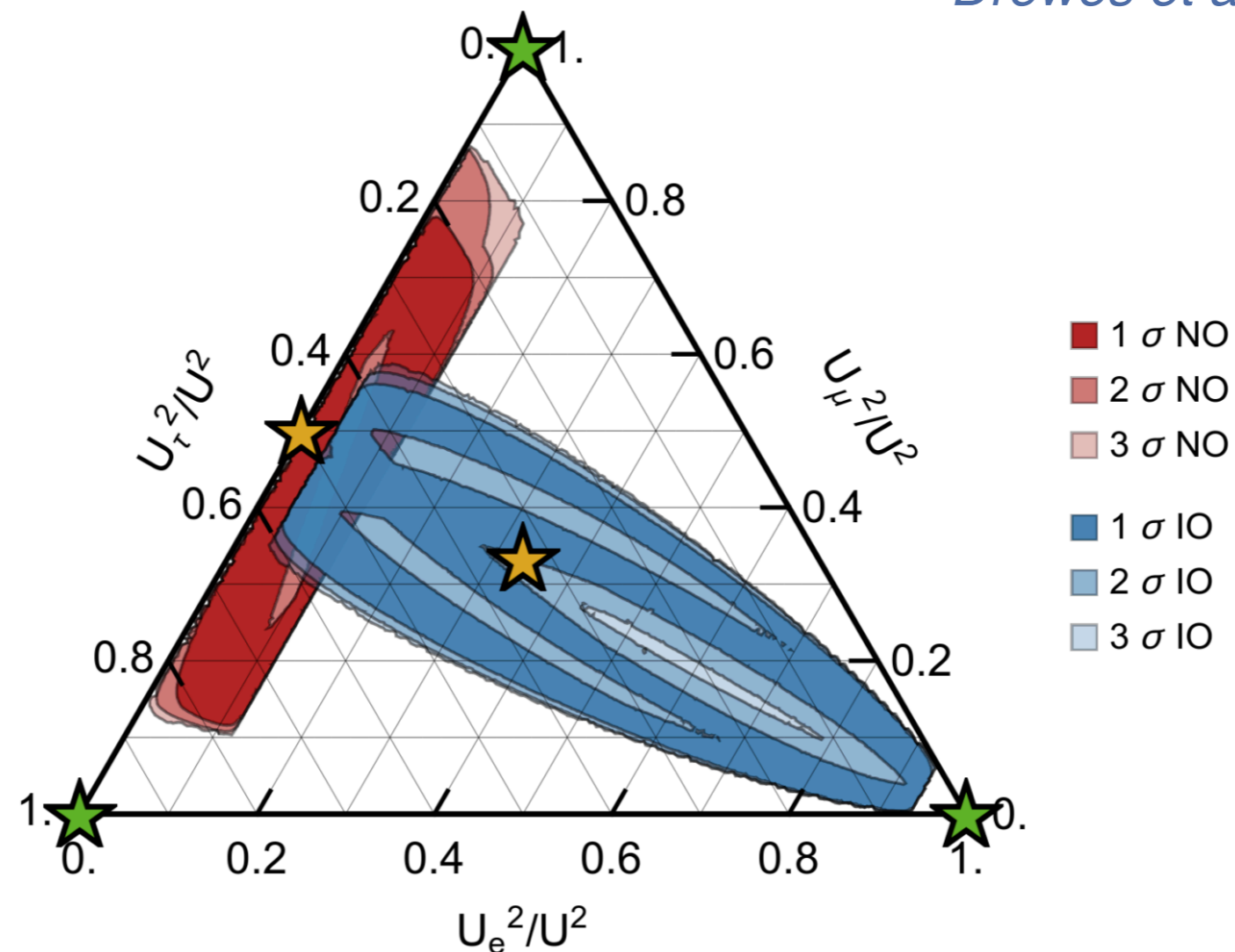
# GOING BEYOND THE SINGLE MIXING

— *Realistic Models usually predict mixings to all flavor* —

■ *Results in the HEPData record* ✓

■ *Use motivated Benchmarks from flavor patterns in minimal models*

— *Drewes et al [2207.02742]* —



# GOING BEYOND THE SINGLE MIXING

— Abada, Escribano, XM, Piazza [2208.13882]—

■ Simple recasting, e.g.  $pp \rightarrow \mu\mu jj$

$$\sigma(pp \rightarrow \mu N) \times \text{BR}(N \rightarrow \mu jj)$$



$$|U_{\mu N}|^2 \text{BR}(N \rightarrow \mu jj)$$



$$|U_{\mu N}|^2$$

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*hep-ex* →  $\sigma(pp \rightarrow \mu N) \times \text{BR}(N \rightarrow \mu jj)$



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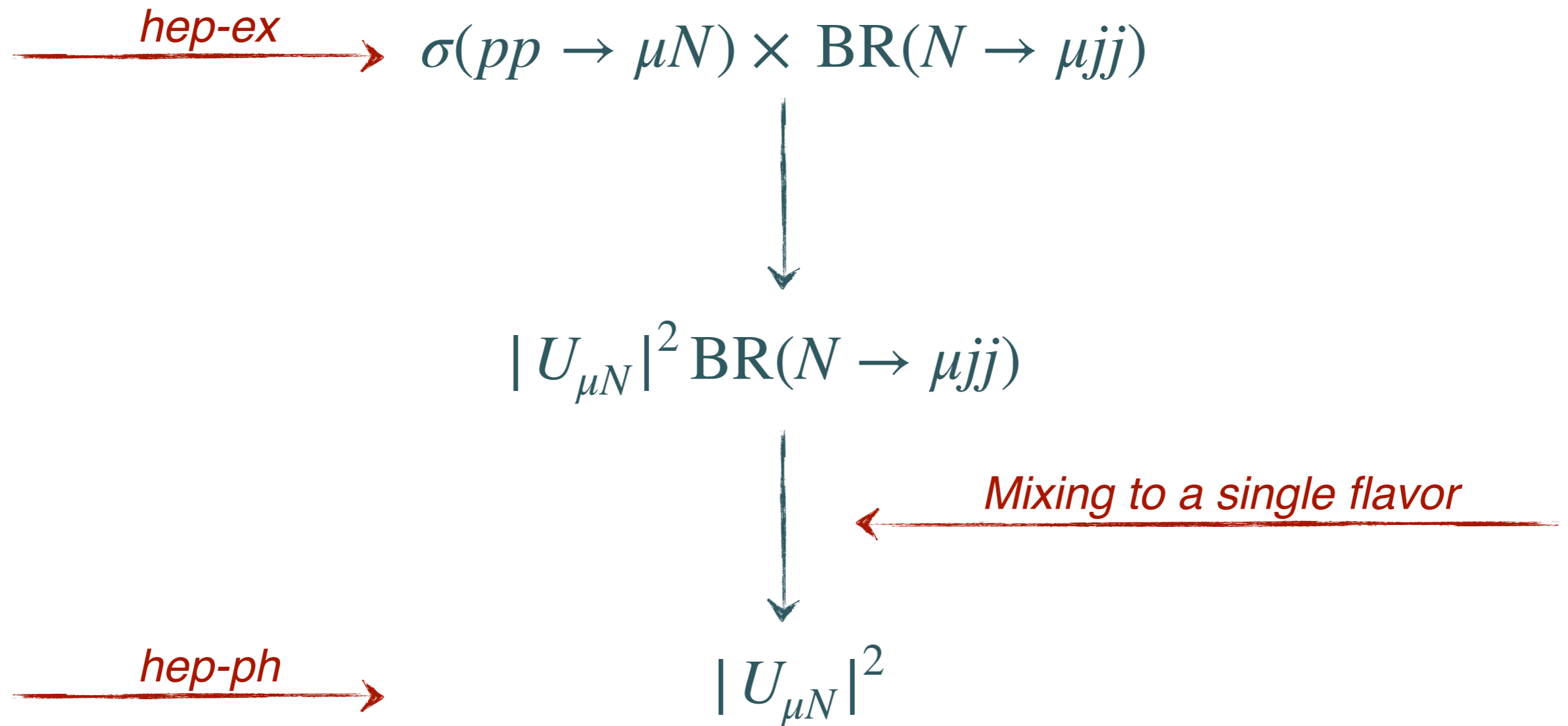
*hep-ph* →

$$|U_{\mu N}|^2$$

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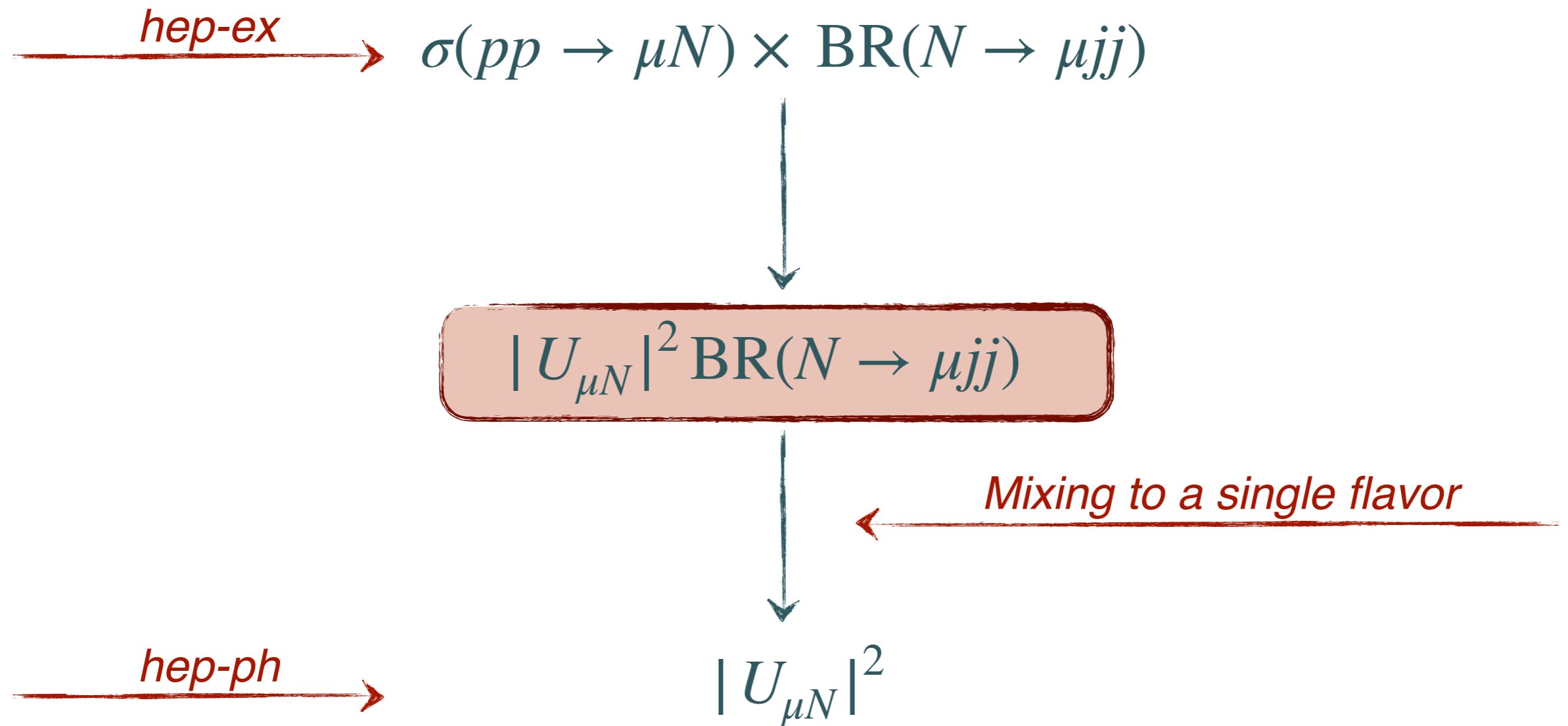
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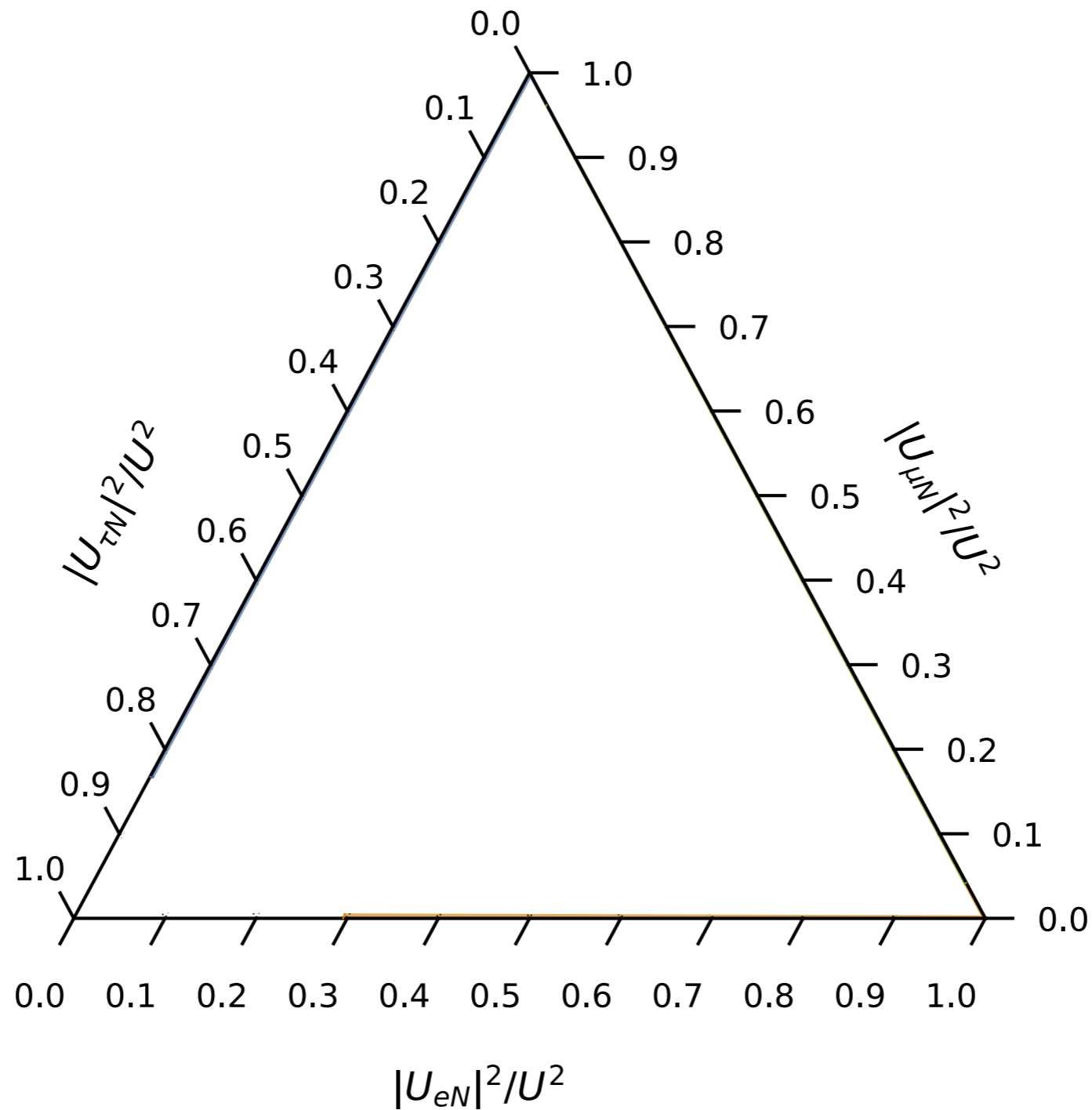
# RECAST — DILEPTONS

— Abada, Escribano, XM, Piazza [2208.13882]—

For a fixed mass and  $U^2$

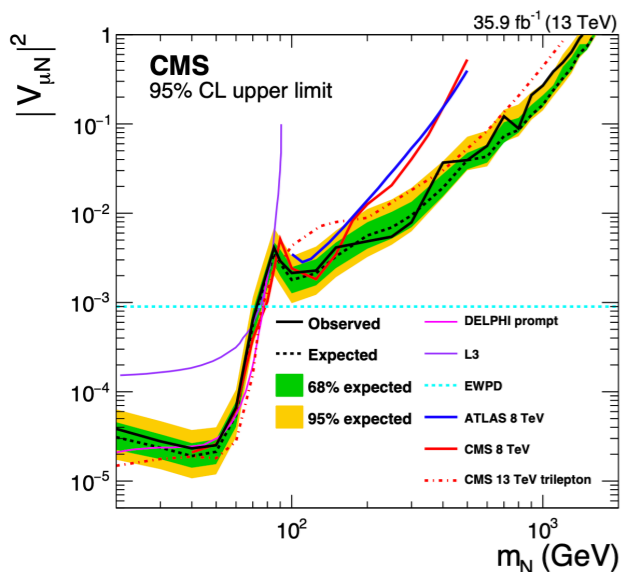
$M_N = 30 \text{ GeV}, U^2 = 10^{-3}$

- $\mu^\pm \mu^\pm$ , CMS '18
- $e^\pm e^\pm$ , CMS '18
- $e^\pm \mu^\pm$ , CMS '18



# RECAST — DILEPTONS

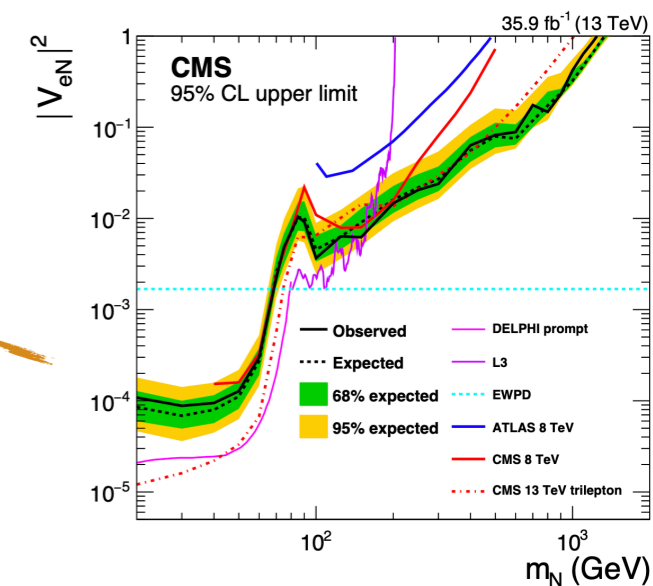
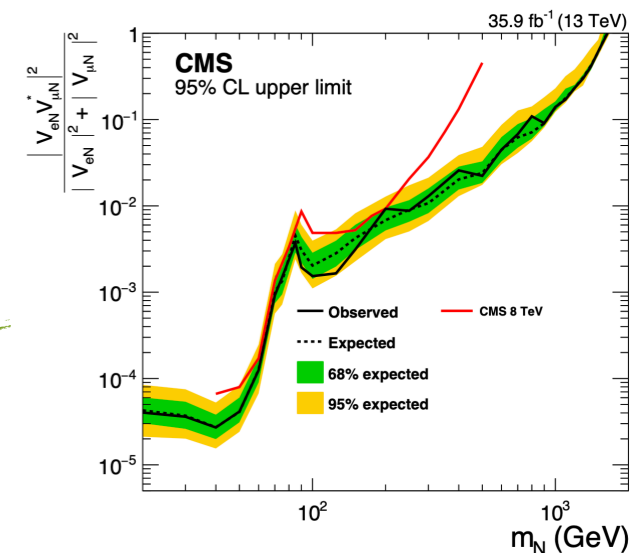
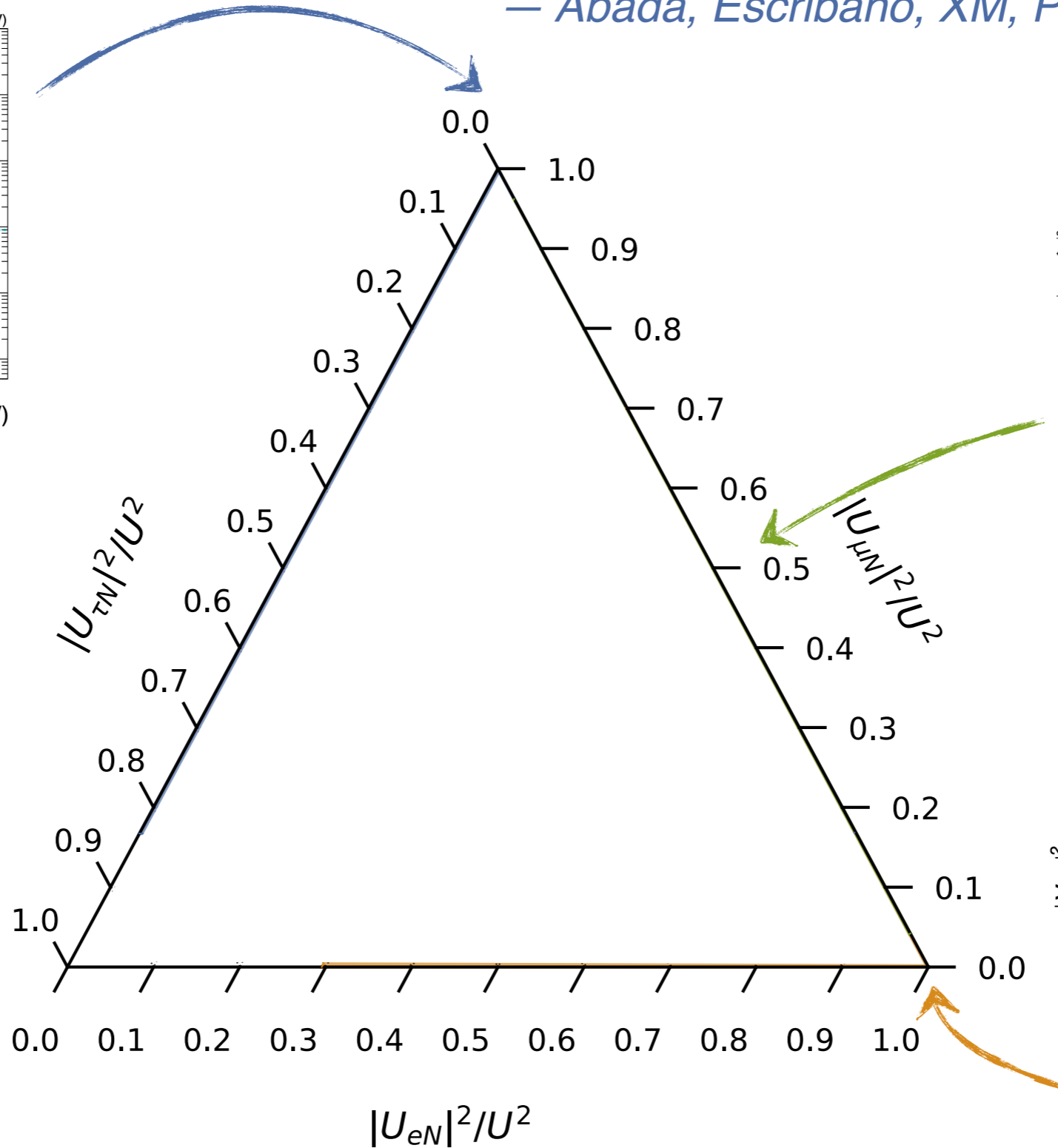
— Abada, Escribano, XM, Piazza [2208.13882] —



For a fixed mass and  $U^2$

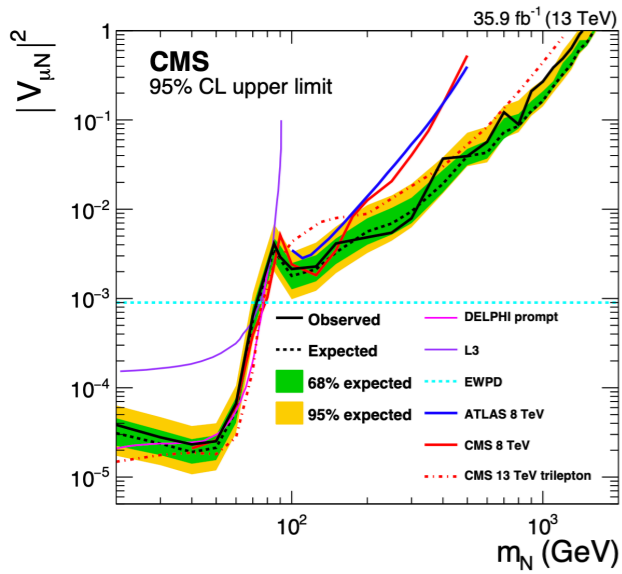
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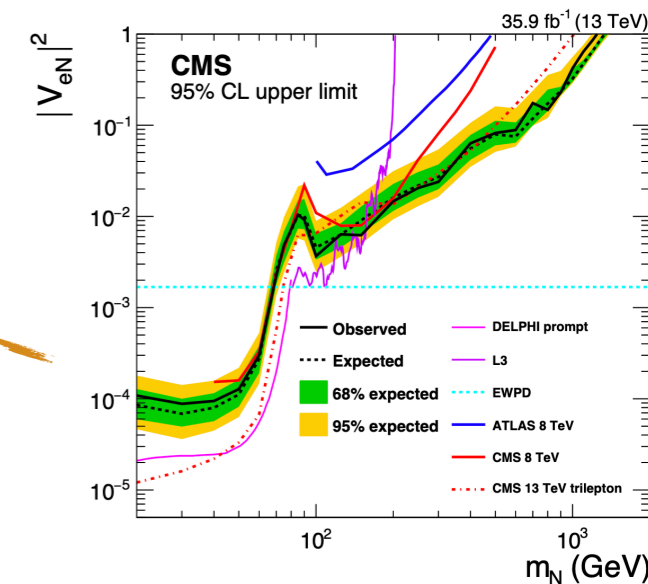
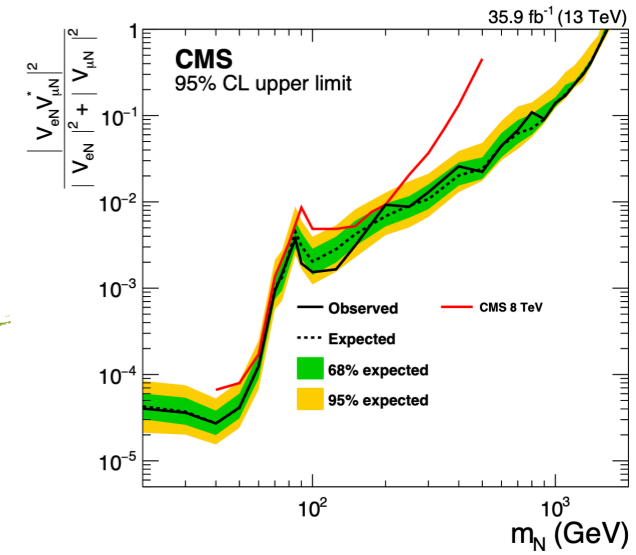
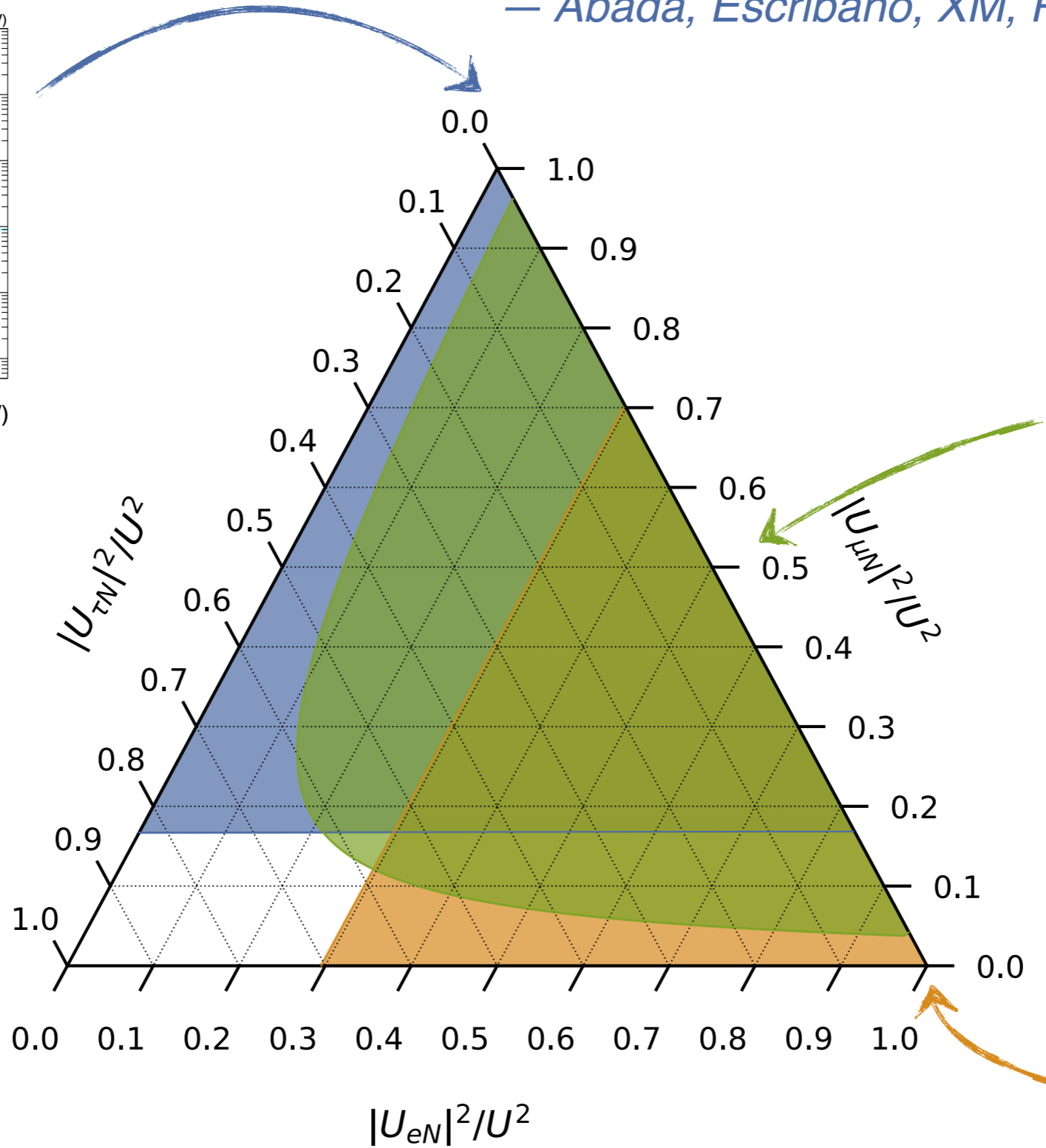
— Abada, Escribano, XM, Piazza [2208.13882] —



For a fixed mass and  $U^2$

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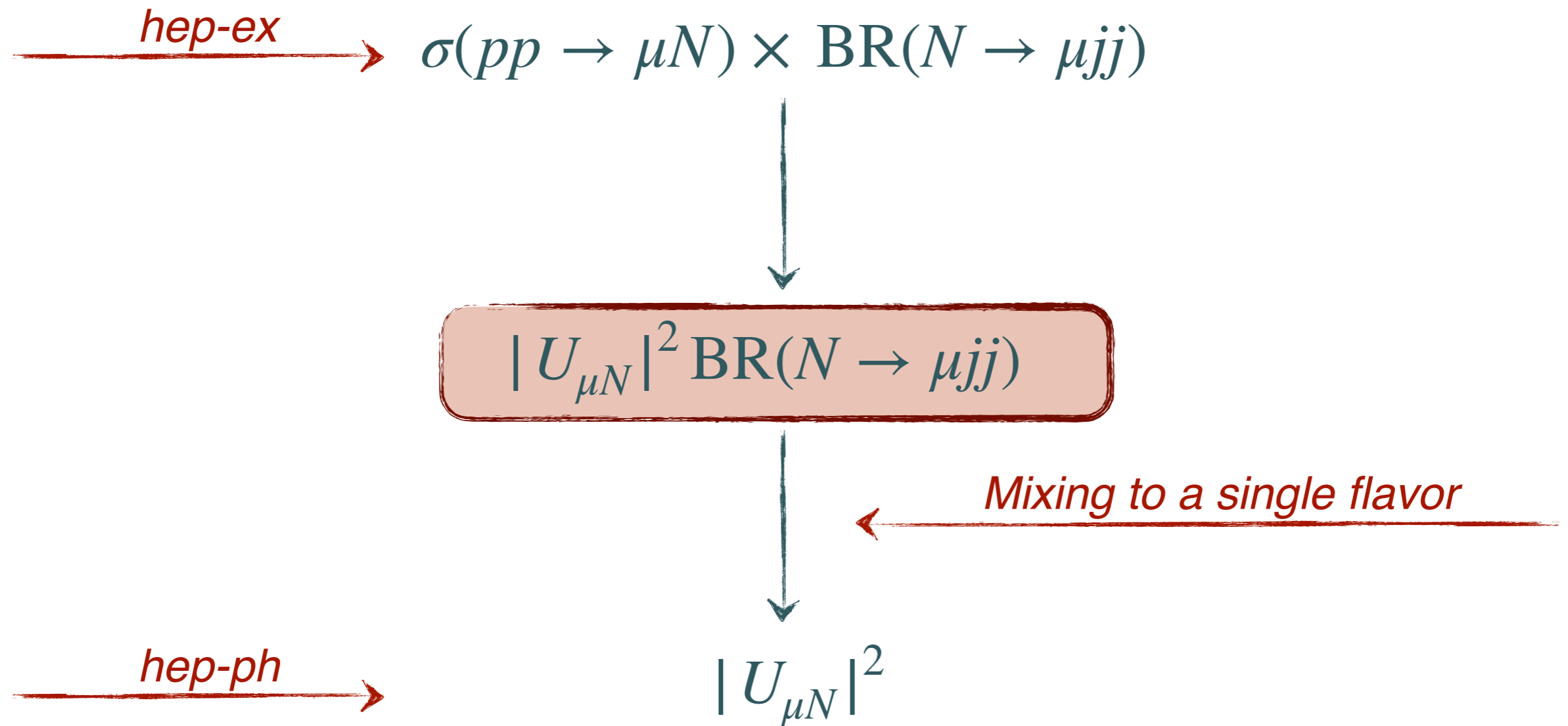
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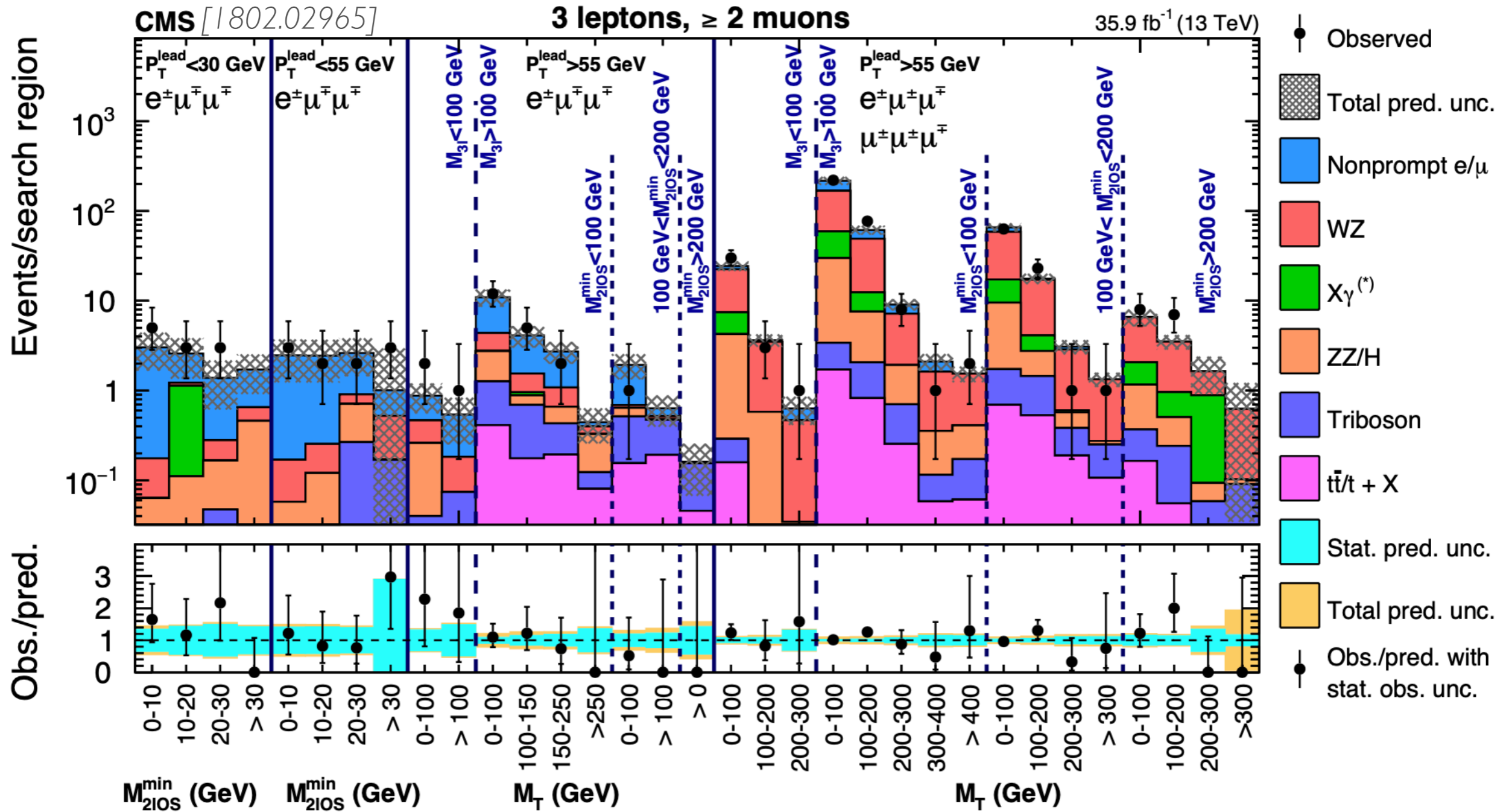
# GOING BEYOND THE SINGLE MIXING

— Abada, Escribano, XM, Piazza [2208.13882]—

- Simple recasting, e.g.  $pp \rightarrow \mu\mu jj$



# RECAST — TRILEPTONS



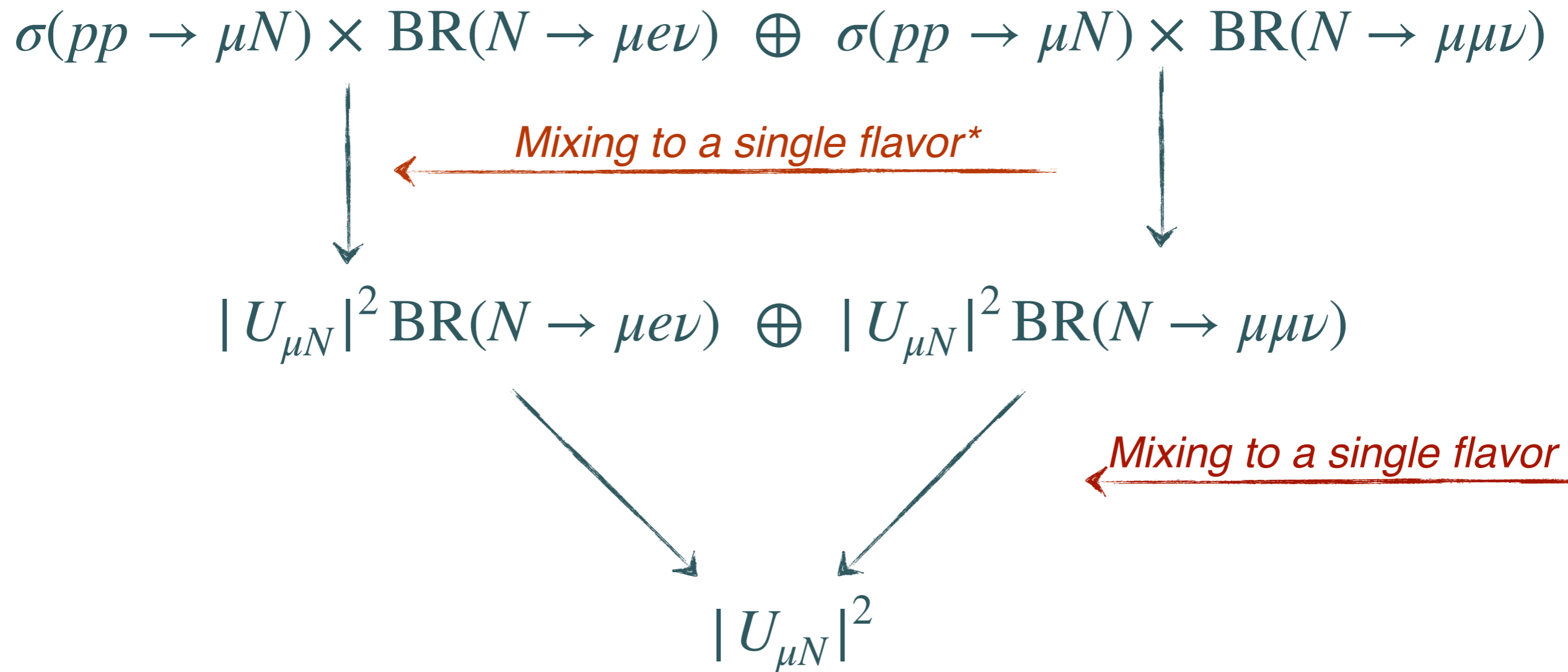
# RECAST — TRILEPTONS

■ Simple recasting, e.g.  $pp \rightarrow \mu\mu e\nu \oplus \mu\mu\mu\nu$

$$\begin{array}{ccc} \sigma(pp \rightarrow \mu N) \times \text{BR}(N \rightarrow \mu e \nu) \oplus \sigma(pp \rightarrow \mu N) \times \text{BR}(N \rightarrow \mu\mu\nu) & & \\ \downarrow & & \downarrow \\ |U_{\mu N}|^2 \text{BR}(N \rightarrow \mu e \nu) \oplus |U_{\mu N}|^2 \text{BR}(N \rightarrow \mu\mu\nu) & & \\ \swarrow & & \searrow \\ |U_{\mu N}|^2 & & \end{array}$$

# RECAST — TRILEPTONS

- Simple recasting, e.g.  $pp \rightarrow \mu\mu\nu \oplus \mu\mu\mu\nu$



— \*see also Tastet et al [2107.12980] —

Dear ~~Santa~~

LHC experimentalists

Please provide sensitivities on  $|U_{\alpha N}|^2 \times BR$   
(also) for each flavor channel individually

E.g. in trilepton searches for  $U_{\mu N}$ :

- Limits only from  $\mu\mu\mu$  channel (to-do)
- Limits only from  $e\mu\mu$  channel (to-do)
- Combined results (done)





## Low-energy effects in precision physics



— see talk by **Daniel Naredo** —

# NON-UNITARITY AND HEAVY NEUTRINOS

*If we integrate out the Heavy Neutrinos...*

- dim-5: neutrino masses (Weinberg operator)

$$m_\nu = - \Theta M_M \Theta^T$$

active-steriles mixings

heavy mass scale

- dim-6: **non-unitarity** of leptonic mixing matrix

$$N = (1 - \eta)U \quad \text{with} \quad \eta = \begin{pmatrix} \eta_{ee} & \eta_{e\mu} & \eta_{e\tau} \\ \cdot & \eta_{\mu\mu} & \eta_{\mu\tau} \\ \cdot & \cdot & \eta_{\tau\tau} \end{pmatrix} = \frac{1}{2} \Theta \Theta^\dagger$$

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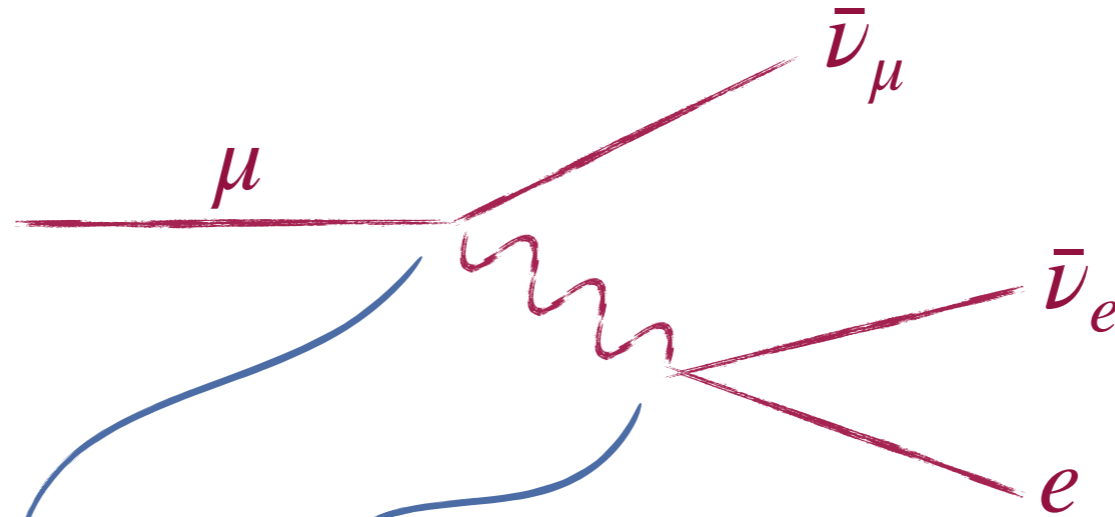
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→ **Modifies precision and flavor observables**

$$\sum_{i=1}^3 N_{\alpha i} N_{i\beta}^\dagger = \delta_{\alpha\beta} - 2\eta_{\alpha\beta} + \mathcal{O}(\eta_{\alpha\beta}^2)$$

# NON-UNITARITY EFFECTS

- Modifies the muon decay



$$\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3} \sum_{i=1}^3 |N_{\mu i}|^2 \sum_{j=1}^3 |N_{ej}|^2 \simeq \frac{G_F^2 m_\mu^5}{192\pi^3} (1 - 2\eta_{ee} - 2\eta_{\mu\mu}) \equiv \frac{G_\mu^2 m_\mu^5}{192\pi^3}$$

$$G_F \simeq G_\mu (1 + \eta_{ee} + \eta_{\mu\mu})$$

**Modifies all EWPO observables !!!**

# NON-UNITARITY EFFECTS

- Modifies  $G_F$

$$G_F \simeq G_\mu \left( 1 + \eta_{ee} + \eta_{\mu\mu} \right)$$

- Modifies  $M_W$  and  $s_W$

$$M_W = M_Z \sqrt{\frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\pi\alpha (1 + \eta_{ee} + \eta_{\mu\mu})}{\sqrt{2}G_\mu M_Z^2 (1 - \Delta r)}}}$$
$$s_W^2 = \frac{1}{2} \left( 1 - \sqrt{1 - \frac{2\sqrt{2}\pi\alpha (1 + \eta_{ee} + \eta_{\mu\mu})}{G_\mu M_Z^2 (1 - \Delta r)}} \right)$$

- Modifies Z invisible decay width

$$\Gamma_{\text{inv}} \simeq \frac{G_\mu M_Z^3}{12\sqrt{2}\pi} \left( 3 - (\eta_{ee} + \eta_{\mu\mu} + 4\eta_{\tau\tau}) \right)$$

- See **talk by Daniel Naredo** for more observables and details

# GLOBAL FIT TO PRECISION AND FLAVOR

— M. Blennow, E. Fernández Martínez, J. Hernández-García, J. López-Pavón, XM, D. Naredo-Tuero [2306.01040]—

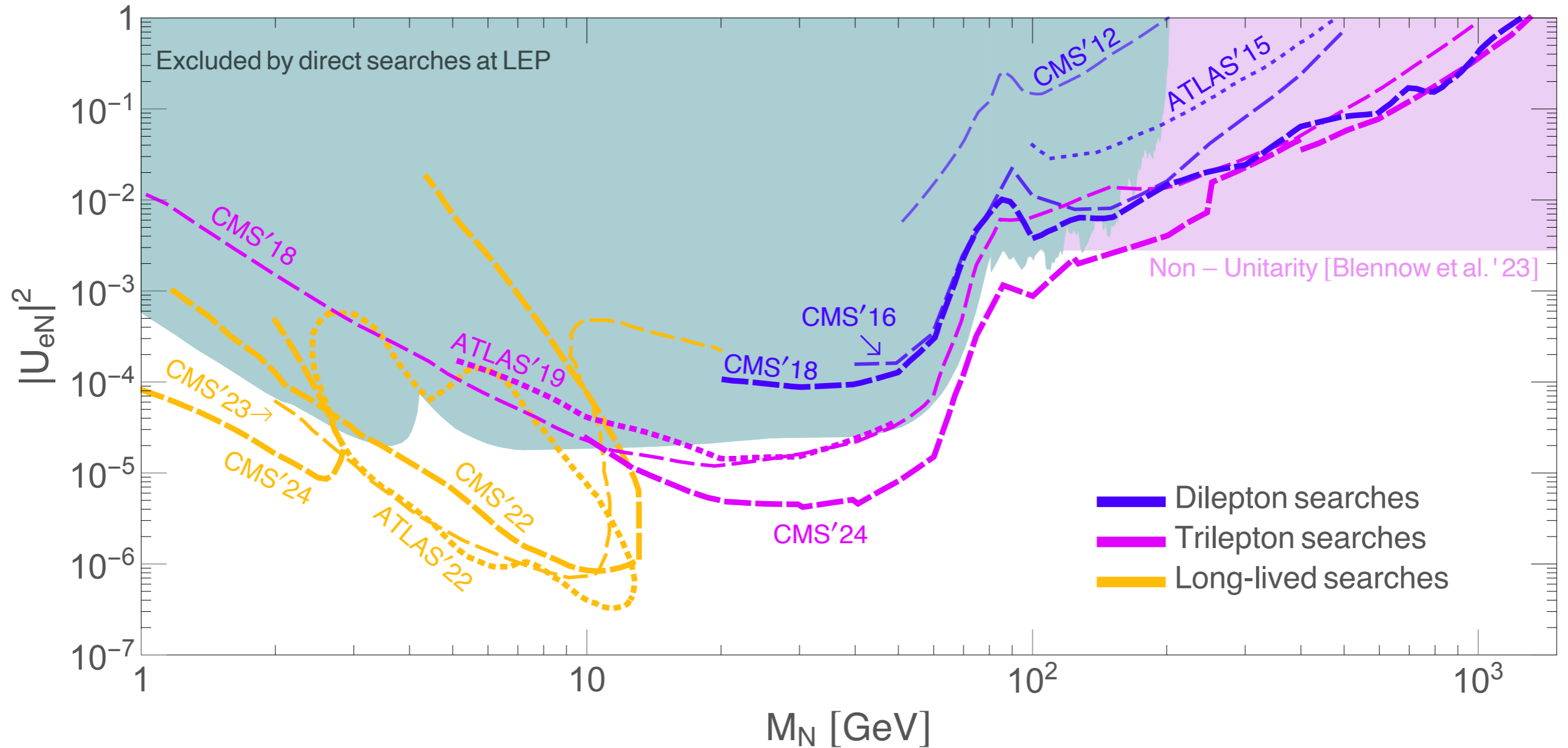
General Seesaw scenario

G-SS	LFC Bound		LFV Bound	
	68%CL	95%CL	68%CL	95%CL
$\eta_{ee}$	$[0.33, 1.0] \cdot 10^{-3}$	$[0.081, 1.4] \cdot 10^{-3}$	-	-
$\eta_{\mu\mu}$	$1.5 \cdot 10^{-5}$	$1.4 \cdot 10^{-4}$	-	-
$\eta_{\tau\tau}$	$1.6 \cdot 10^{-4}$	$8.9 \cdot 10^{-4}$	-	-
Tr $[\eta]$	$[0.28, 1.2] \cdot 10^{-3}$	$2.1 \cdot 10^{-3}$	-	-
$ \eta_{e\mu} $	$1.4 \cdot 10^{-4}$	$3.4 \cdot 10^{-4}$	<b><math>8.4 \cdot 10^{-6}</math></b>	<b><math>1.2 \cdot 10^{-5}</math></b>
$ \eta_{e\tau} $	<b><math>4.2 \cdot 10^{-4}</math></b>	<b><math>8.8 \cdot 10^{-4}</math></b>	$5.7 \cdot 10^{-3}$	$8.1 \cdot 10^{-3}$
$ \eta_{\mu\tau} $	<b><math>9.4 \cdot 10^{-6}</math></b>	<b><math>1.8 \cdot 10^{-4}</math></b>	$6.6 \cdot 10^{-3}$	$9.4 \cdot 10^{-3}$

— See talk by **Daniel Naredo** for details and more scenarios —

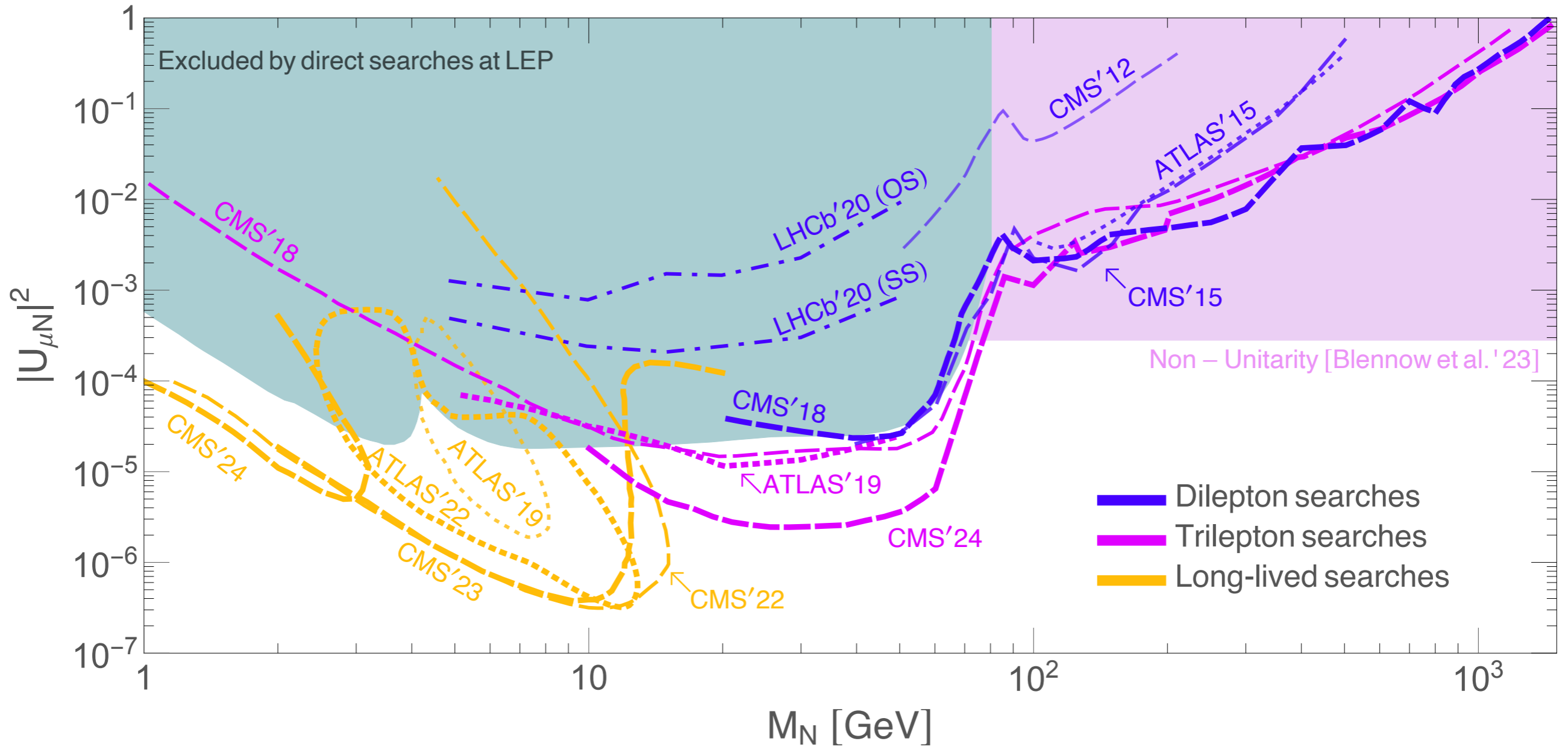
# CURRENT STATUS: ELECTRON

updated from Abada, Escribano, XM, Piazza [2208.13882]



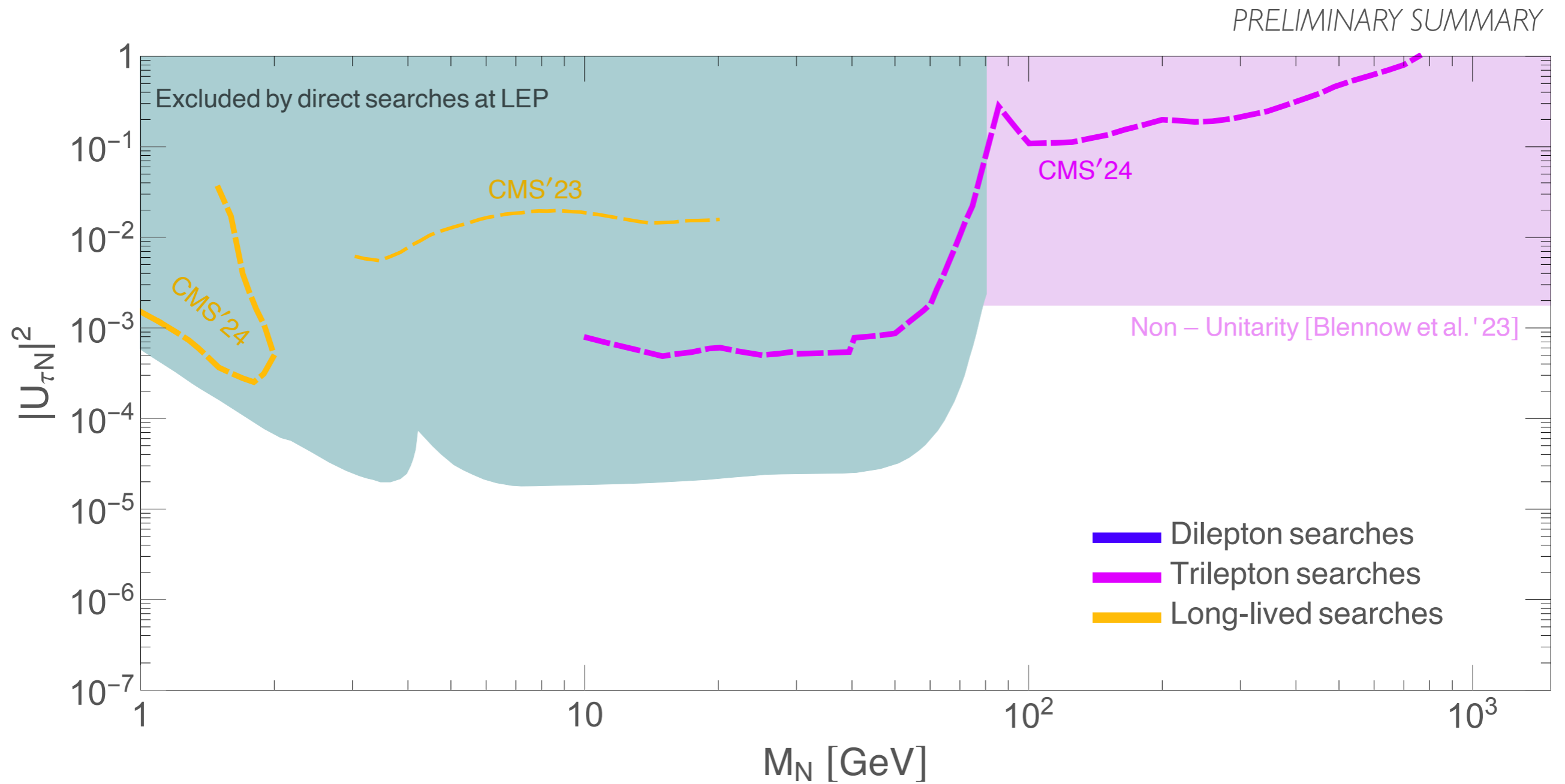
# CURRENT STATUS: MUON

updated from Abada, Escribano, XM, Piazza [2208.13882]






# CURRENT STATUS: TAU



# CONCLUSIONS

## *Heavy Neutrinos probes at both energy and intensity frontiers*



**Energy  
Frontier**

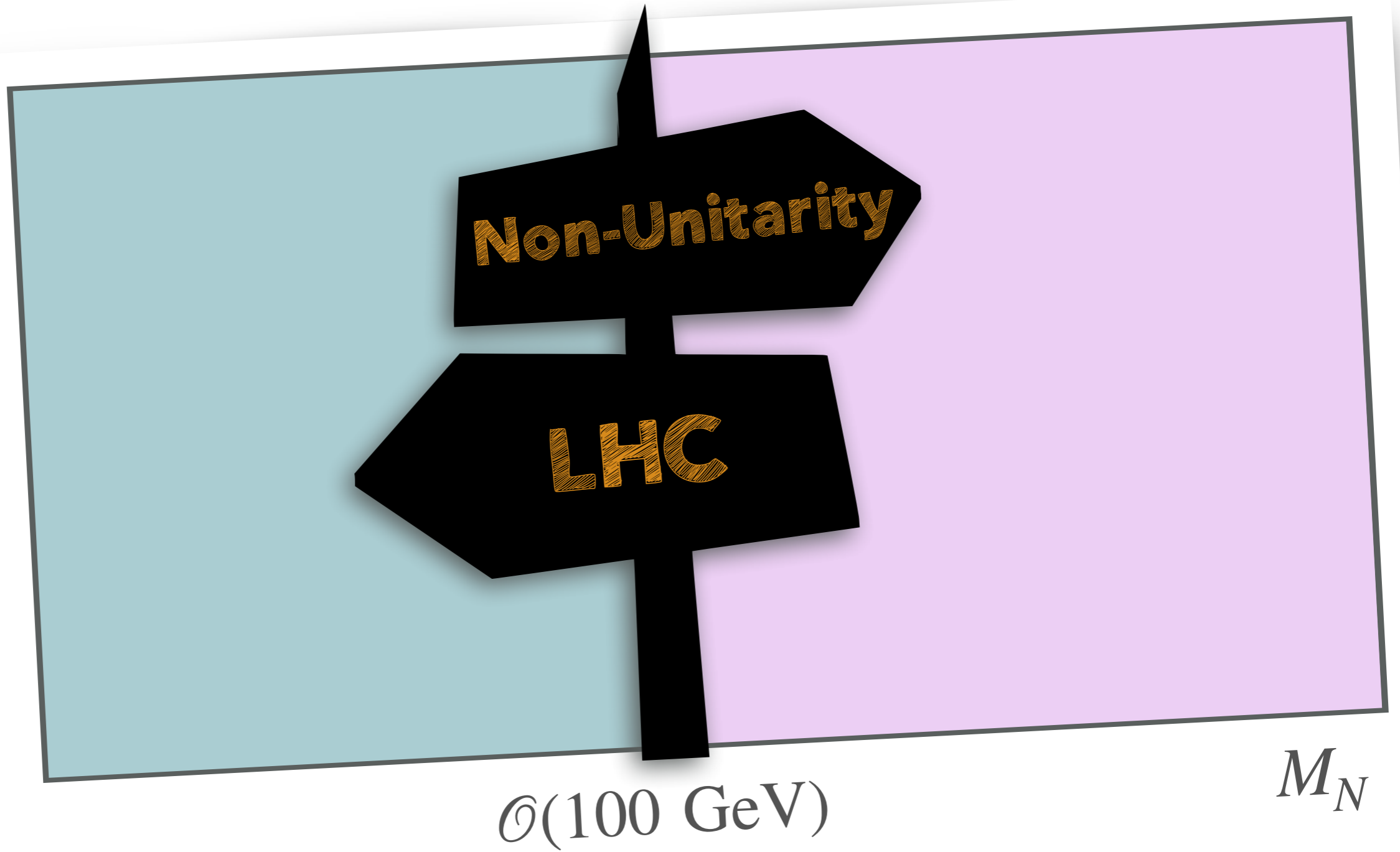
**Intensity  
Frontier**

- LHC improved LEP bounds for  $U_{eN}$  and  $U_{\mu N}$
  - LNV vs LNC help selecting neutrino mass models
  - Easy recasts w/ individual bounds on  $|U_{\alpha N}|^2 \times \text{BR}$
- 
- Heavy Neutrinos induce Non-Unitarity
  - New Global bounds from EWPO and flavor  
— talk by Daniel Naredo —
  - Strongest bounds for heaviest masses

# CONCLUSIONS

*Heavy Neutrinos probes at both energy and intensity frontiers*

$$|U_{\alpha N}|^2$$



# Thank you!

*Funded by the European Union's Horizon Europe Programme under the Marie Skłodowska-Curie grant agreement no. 101066105-PheNУmenal. Views and opinions expressed are however those of the author(s) only and do not necessarily reflect those of the European Union or the European Research Executive Agency (REA). Neither the European Union nor the granting authority can be held responsible for them.*



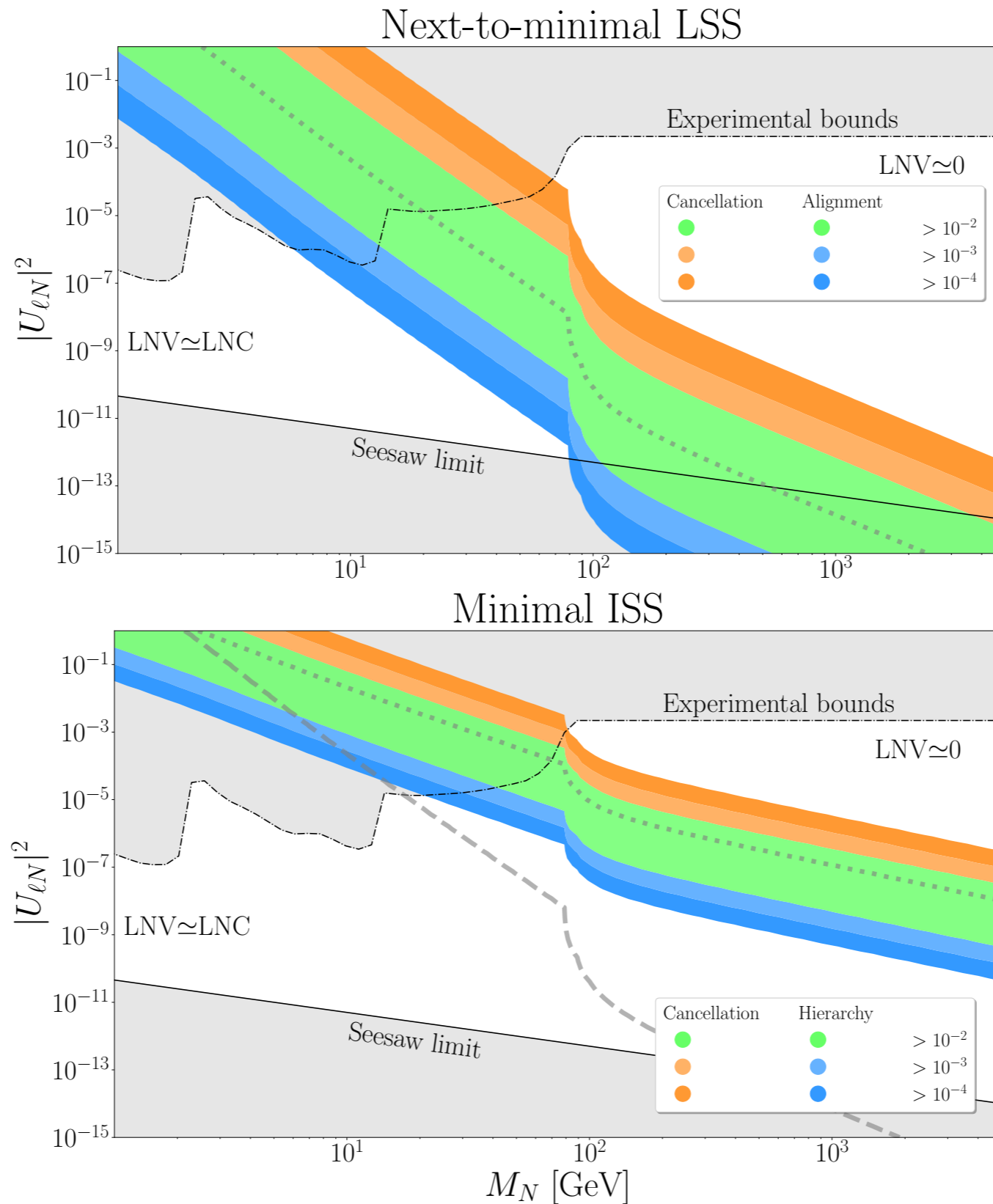
Funded by the  
European Union



**BACK UP**

# DISCRIMINATE BETWEEN LOW-SCALE SEESAWS

Fernández-Martínez, XM, Naredo-Tuero [2209.04461]



# ABOUT CDF-II

— M. Blennow, E. Fernández Martínez, J. Hernández-García, J. López-Pavón, XM, D. Naredo-Tuero [2306.01040]—

Set of observables	p-g.o.f.	tension	p-value
CDF-II vs $M_W/s_{\text{eff}}^2$	21.75/1	$4.7\sigma$	$3.1 \cdot 10^{-6}$
CDF-II vs Z-pole	21.48/1	$4.6\sigma$	$3.6 \cdot 10^{-6}$
CDF-II vs $M_W/s_{\text{eff}}^2$ and Z-pole	27.30/1	$5.2\sigma$	$1.7 \cdot 10^{-7}$

■ *Non-unitarity could push  $M_W$  towards CDF-II*

*Blennow et al [2204.04559]*

$$M_W \simeq M_W^{\text{SM}} \left( 1 + 0.20 (\eta_{ee} + \eta_{\mu\mu}) \right)$$

■ *But  $\eta_{ee} + \eta_{\mu\mu}$  also modifies  $s_w$  and Z-pole obs, which agree with SM*

■ ***CDF-II in strong tension, can't be explained just with non-unitarity***

— *not included in our GF* —

# HAND-WAVING ESTIMATES

**Observables at the per-mille precision**

$$\eta \lesssim \mathcal{O}(10^{-3})$$

**Few non-trivial tendencies:**

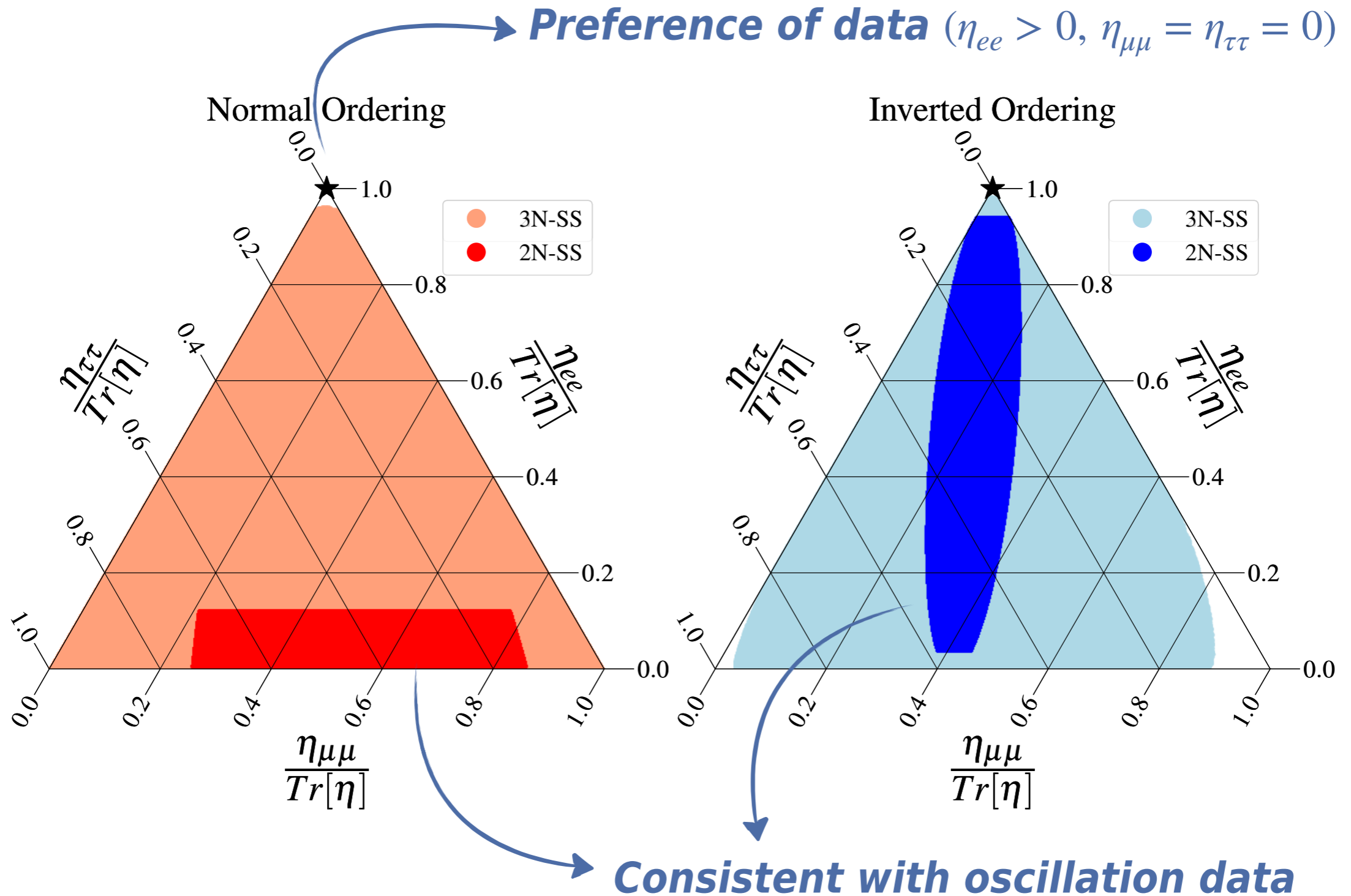
- ▶  $M_W, S_W$   $\longrightarrow$  mild preference for  $\eta_{ee} + \eta_{\mu\mu} > 0$
- ▶  $V_{ud}$   $\longrightarrow$  disfavors  $\eta_{\mu\mu} > 0$
- ▶ LFU  $\longrightarrow$  mild preference for  $\eta_{ee} > \eta_{\mu\mu}$
- ▶ Others just SM-like

**mild preference (1-2 $\sigma$ ) for**

$$\eta_{ee} > 0, \quad \eta_{\mu\mu} = 0, \quad \eta_{\tau\tau} = 0$$



# IMPORTANCE OF CORRELATIONS DUE TO $m_\nu$



— M. Blennow, E. Fernández Martínez, J. Hernández-García, J. López-Pavón, XM, D. Naredo-Tuero [2306.01040]—

# QUALITATIVE RESULTS

**95%CL, just order of magnitude**

	<b>2N-SS</b>		<b>3N-SS</b>		<b>G-SS</b>
	<b>NO</b>	<b>IO</b>	<b>NO</b>	<b>IO</b>	
$\eta_{ee}$	$10^{-5}$	$10^{-4}$	$10^{-3}$	$10^{-3}$	$[0.1,1] \cdot 10^{-3}$
$\eta_{\mu\mu}$	$10^{-4}$	$10^{-5}$	$10^{-5}$	$10^{-5}$	$10^{-4}$
$\eta_{\tau\tau}$	$10^{-4}$	$10^{-5}$	$10^{-3}$	$10^{-3}$	$10^{-3}$
$ \eta_{e\mu} $	$10^{-5}$	$10^{-5}$	$10^{-5}$	$10^{-5}$	$10^{-5}$
$ \eta_{e\tau} $	$10^{-5}$	$10^{-4}$	$10^{-3}$	$10^{-3}$	$10^{-3}$
$ \eta_{\mu\tau} $	$10^{-4}$	$10^{-5}$	$10^{-5}$	$10^{-5}$	$10^{-4}$

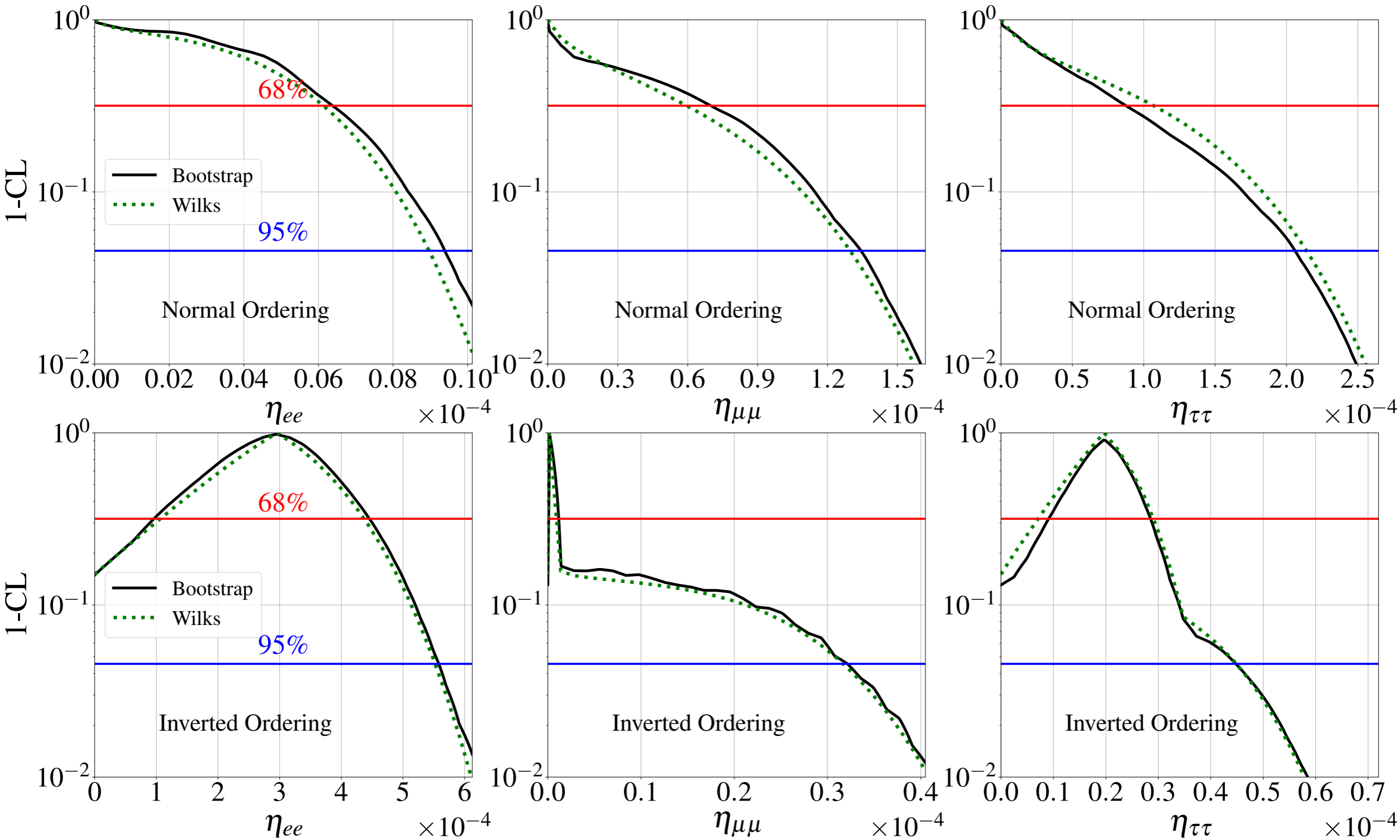
# RESULTS FOR THE MINIMAL SCENARIO

2N-SS	Normal Ordering		Inverted Ordering	
	68%CL	95%CL	68%CL	95%CL
$\eta_{ee} = \frac{ \theta_e ^2}{2}$	$6.4 \cdot 10^{-6}$	$9.4 \cdot 10^{-6}$	$[0.98, 4.4] \cdot 10^{-4}$	$5.5 \cdot 10^{-4}$
$\eta_{\mu\mu} = \frac{ \theta_\mu ^2}{2}$	$6.9 \cdot 10^{-5}$	$1.3 \cdot 10^{-4}$	$[0.20, 1.0] \cdot 10^{-6}$	$3.2 \cdot 10^{-5}$
$\eta_{\tau\tau} = \frac{ \theta_\tau ^2}{2}$	$8.6 \cdot 10^{-5}$	$2.1 \cdot 10^{-4}$	$[0.94, 2.8] \cdot 10^{-5}$	$4.5 \cdot 10^{-5}$
$\text{Tr} [\eta] = \frac{ \theta ^2}{2}$	$1.6 \cdot 10^{-4}$	$2.9 \cdot 10^{-4}$	$[1.1, 4.8] \cdot 10^{-4}$	$6.0 \cdot 10^{-4}$
$ \eta_{e\mu}  = \frac{ \theta_e \theta_\mu^* }{2}$	$8.3 \cdot 10^{-6}$	$1.2 \cdot 10^{-5}$	$[0.37, 1.0] \cdot 10^{-5}$	$1.3 \cdot 10^{-5}$
$ \eta_{e\tau}  = \frac{ \theta_e \theta_\tau^* }{2}$	$1.5 \cdot 10^{-5}$	$2.2 \cdot 10^{-5}$	$[0.25, 1.2] \cdot 10^{-4}$	$1.4 \cdot 10^{-4}$
$ \eta_{\mu\tau}  = \frac{ \theta_\mu \theta_\tau^* }{2}$	$7.2 \cdot 10^{-5}$	$1.3 \cdot 10^{-4}$	$[0.38, 3.0] \cdot 10^{-6}$	$3.5 \cdot 10^{-5}$

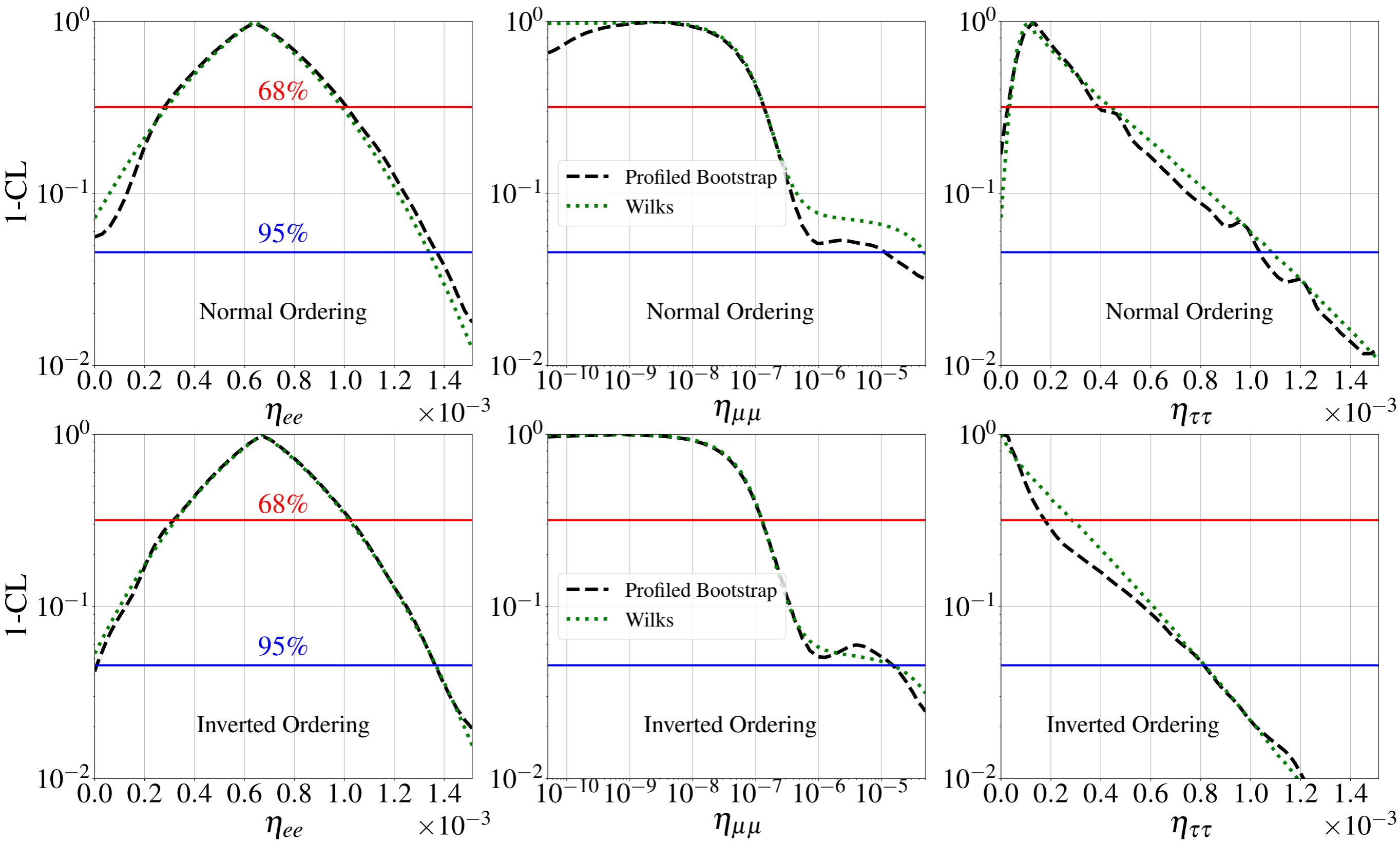
# RESULTS FOR THE NEXT-TO-MINIMAL SCENARIO

3N-SS	Normal Ordering		Inverted Ordering	
	68%CL	95%CL	68%CL	95%CL
$\eta_{ee} = \frac{ \theta_e ^2}{2}$	$[0.28, 0.99] \cdot 10^{-3}$	$1.3 \cdot 10^{-3}$	$[0.31, 1.0] \cdot 10^{-3}$	$1.4 \cdot 10^{-3}$
$\eta_{\mu\mu} = \frac{ \theta_\mu ^2}{2}$	$1.3 \cdot 10^{-7}$	$1.1 \cdot 10^{-5}$	$1.2 \cdot 10^{-7}$	$1.0 \cdot 10^{-5}$
$\eta_{\tau\tau} = \frac{ \theta_\tau ^2}{2}$	$[0.3, 3.9] \cdot 10^{-4}$	$1.0 \cdot 10^{-3}$	$1.7 \cdot 10^{-4}$	$8.1 \cdot 10^{-4}$
$\text{Tr} [\eta] = \frac{ \theta ^2}{2}$	$[0.35, 1.3] \cdot 10^{-3}$	$1.9 \cdot 10^{-3}$	$[0.33, 1.0] \cdot 10^{-3}$	$1.5 \cdot 10^{-3}$
$ \eta_{e\mu}  = \frac{ \theta_e \theta_\mu^* }{2}$	$8.5 \cdot 10^{-6}$	$1.2 \cdot 10^{-5}$	$8.5 \cdot 10^{-6}$	$1.2 \cdot 10^{-5}$
$ \eta_{e\tau}  = \frac{ \theta_e \theta_\tau^* }{2}$	$[1.3, 5.1] \cdot 10^{-4}$	$9.0 \cdot 10^{-4}$	$3.3 \cdot 10^{-4}$	$8.0 \cdot 10^{-4}$
$ \eta_{\mu\tau}  = \frac{ \theta_\mu \theta_\tau^* }{2}$	$5.0 \cdot 10^{-6}$	$5.7 \cdot 10^{-5}$	$3.8 \cdot 10^{-6}$	$1.8 \cdot 10^{-5}$

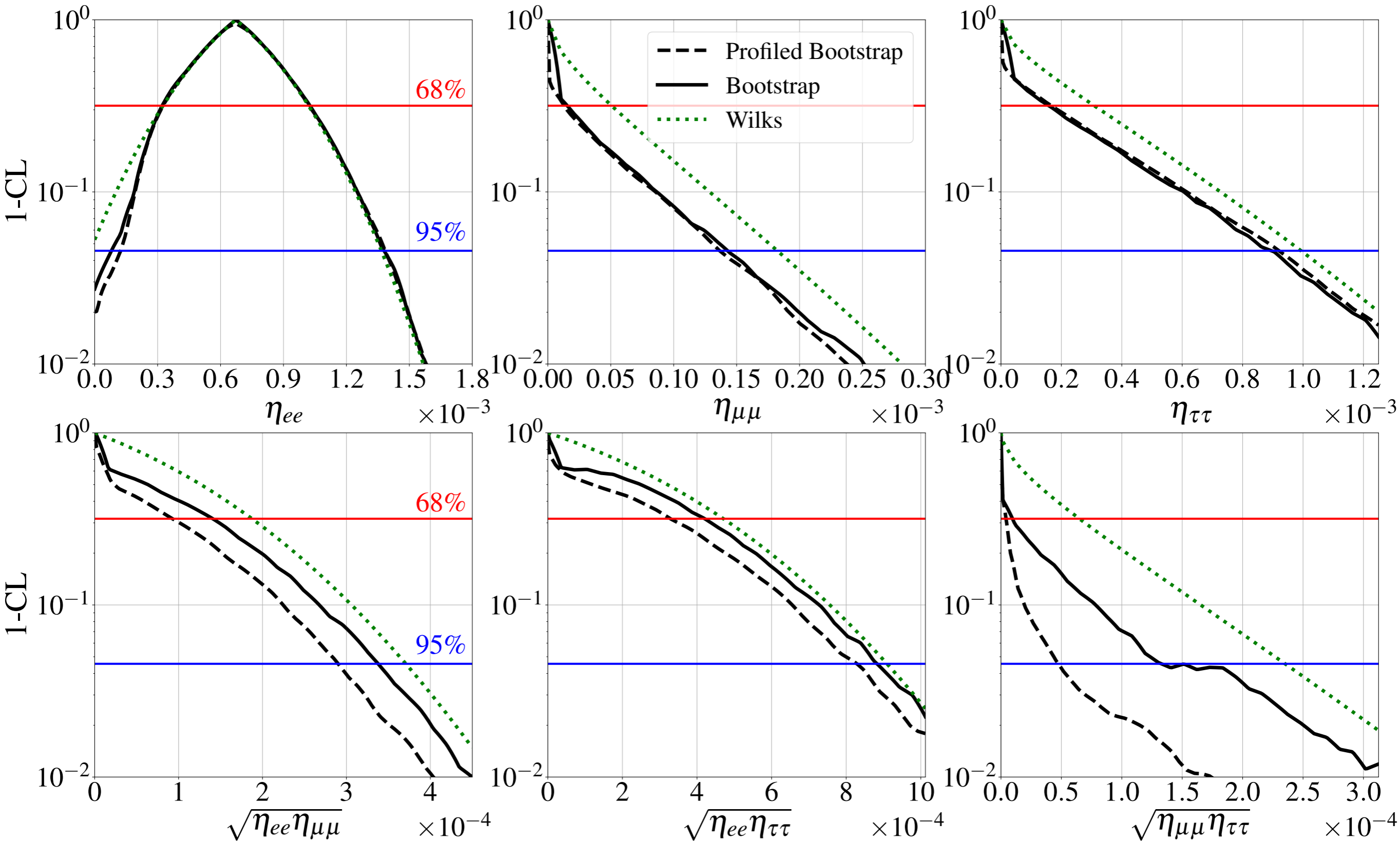
# PROFILES FOR THE 2N-SS



# PROFILES FOR THE 3N-SS



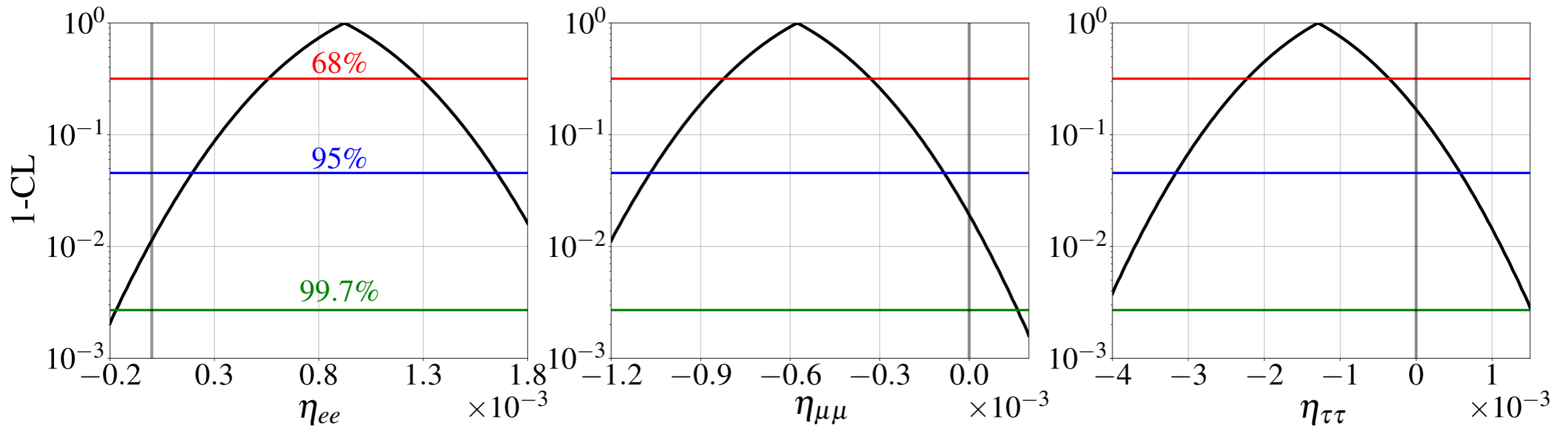
# PROFILES FOR THE G-SS



# GENERIC UNITARITY VIOLATION

— M. Blennow, E. Fernández Martínez, J. Hernández-García, J. López-Pavón, XM, D. Naredo-Tuero [2306.01040]—

■ Not imposing  $\eta$  to be definite positive (**beyond just heavy neutrinos**)

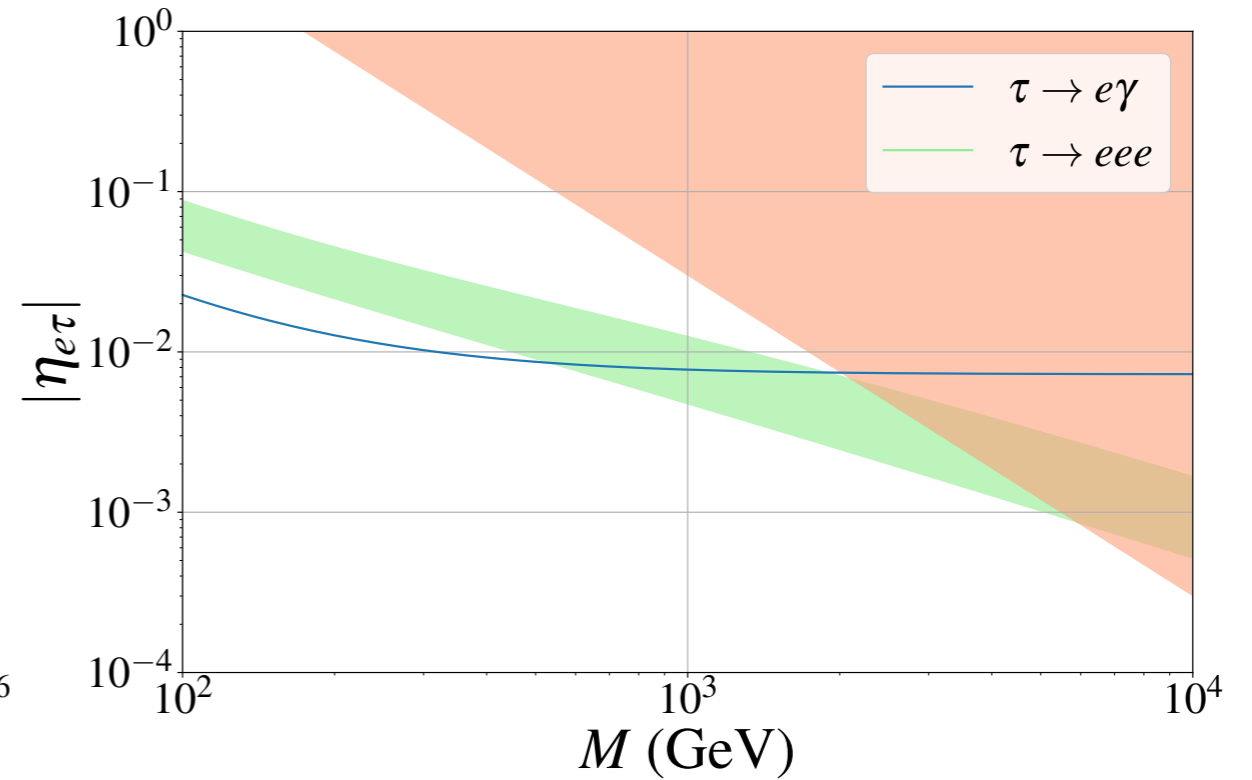
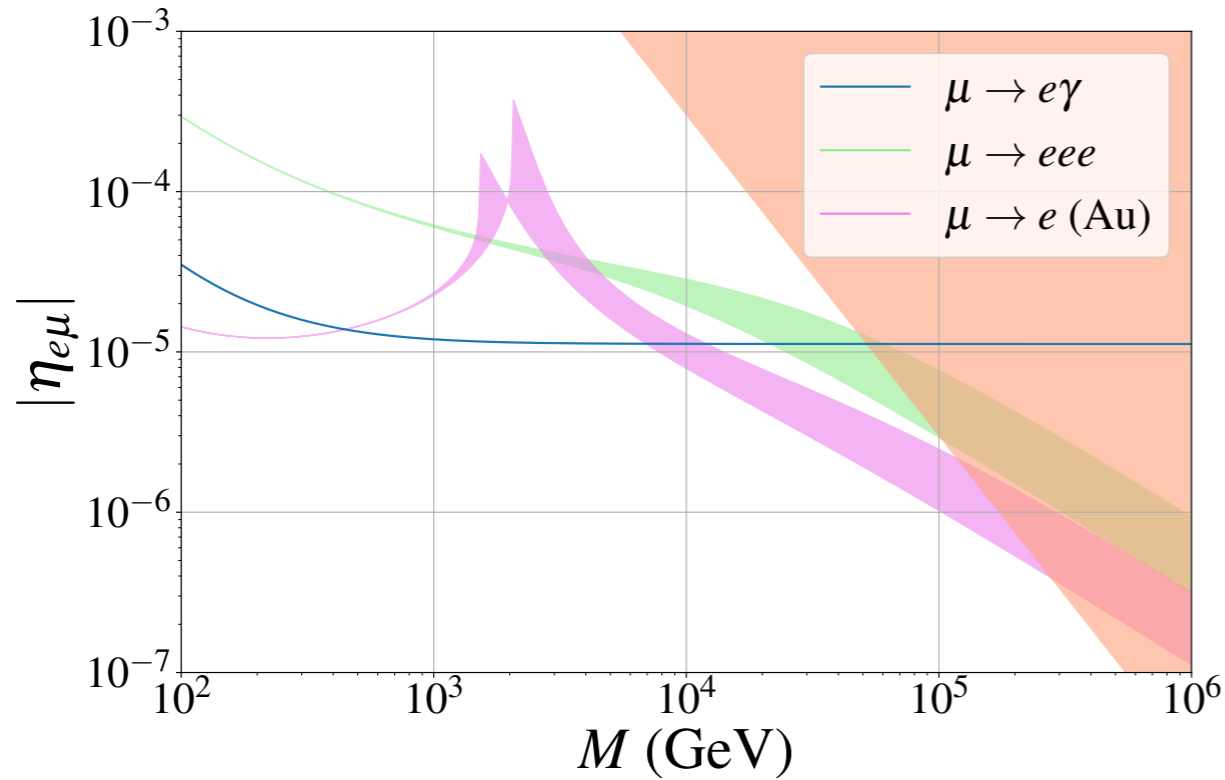


GUV	LFC Bound			LFV Bound	
	68%CL	95%CL		68%CL	95%CL
$\eta_{ee}$	$[0.56, 1.29] \cdot 10^{-3}$	$[0.20, 1.65] \cdot 10^{-3}$	$ \eta_{e\mu} $	$5.0 \cdot 10^{-6}$	$7.2 \cdot 10^{-6}$
$\eta_{\mu\mu}$	$[-8.2, -3.3] \cdot 10^{-4}$	$[-1.1, -0.088] \cdot 10^{-3}$	$ \eta_{e\tau} $	$3.4 \cdot 10^{-3}$	$4.9 \cdot 10^{-3}$
$\eta_{\tau\tau}$	$[-2.2, -0.38] \cdot 10^{-3}$	$[-3.1, 0.56] \cdot 10^{-3}$	$ \eta_{\mu\tau} $	$4.0 \cdot 10^{-3}$	$5.6 \cdot 10^{-3}$



# CHARGED LEPTON FLAVOR VIOLATION

— M. Blennow, E. Fernández Martínez, J. Hernández-García, J. López-Pavón, XM, D. Naredo-Tuero [2306.01040]—



■ Off-diag  $\eta$  entries induce **charged LFV**, e.g.

$$\text{BR}(\ell_\alpha \rightarrow \ell_\beta \gamma) \simeq \frac{3\alpha}{2\pi} |\eta_{\alpha\beta}|^2 \quad \text{— for } M \gg M_w \text{—}$$

■ Some cLFV obs have non-trivial mass dependence

— marginalize over  $M$  —

# RELATION TO OTHER PARAMETRIZATIONS

- Non-unitarity effects are also parametrized via the  $\alpha$ -parametrization

$$\alpha = \begin{pmatrix} \alpha_{11} & 0 & 0 \\ \alpha_{21} & \alpha_{22} & 0 \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix}$$

- Our results are **easily translated**, e.g. in the 2N-SS:

$$\text{NO: } \mathbb{I} - \alpha < \begin{pmatrix} 9.4 \cdot 10^{-6} & 0 & 0 \\ 2.4 \cdot 10^{-5} & 1.3 \cdot 10^{-4} & 0 \\ 4.4 \cdot 10^{-5} & 2.6 \cdot 10^{-4} & 2.1 \cdot 10^{-4} \end{pmatrix}$$

$$\text{IO: } \mathbb{I} - \alpha < \begin{pmatrix} 5.5 \cdot 10^{-4} & 0 & 0 \\ 2.6 \cdot 10^{-5} & 3.2 \cdot 10^{-5} & 0 \\ 2.8 \cdot 10^{-4} & 7.0 \cdot 10^{-5} & 4.5 \cdot 10^{-5} \end{pmatrix}$$

— See [2306.01040] for other scenarios —