Resonances in $D^0 \to \pi^+\pi^- l^+ l^-$ & sensitivity to New Physics

Rencontres de Moriond 2024

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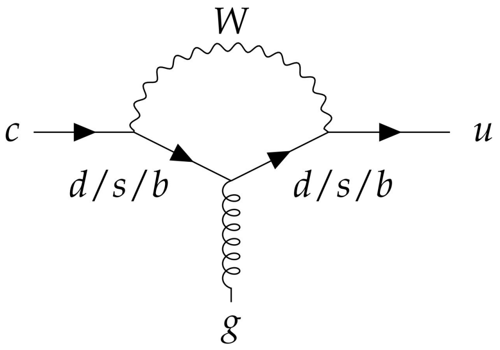
Why look into rare decays of quarks

Rare (in the SM) ⟶ more room for NP to show up!



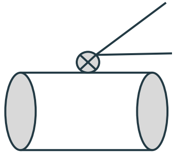
Part of indirect searches programme

Main suppression factor: **Flavor-changing neutral currents (FCNCs)**

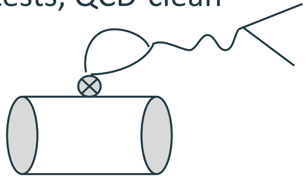
Long history:

- K⟶μμ and charm discovery
- b⟶s μμ and all associated processes and observables (lepton flavor universality tests, QCD-clean observables ...)

Main limiting factor for SM precision: **hadronic uncertainties**



B->K(*), φ,... form factors (lattice, LCSRs + z-expansion)

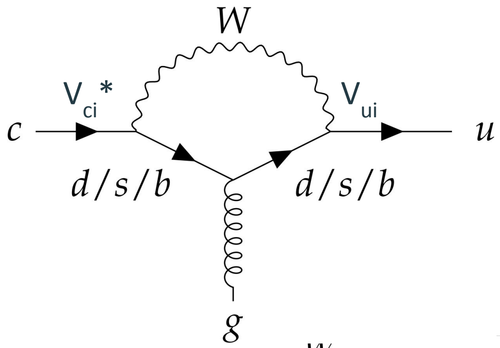


charm loop - long-distance form factors

Why look into rare *charm* decays

Charm is the only weakly decaying up-type quark bound in hadrons ⟶ offers unique ways to

- probe CKM mixing,
- test the SM and
- explore NP models with couplings to up-type quarks

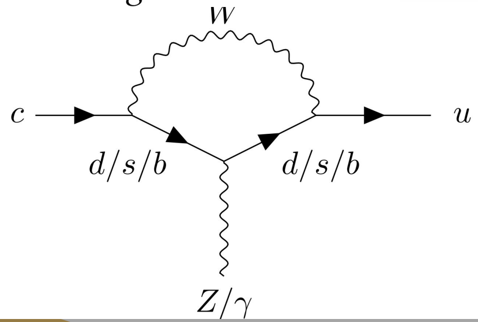


Up-type quark: FCNCs involve only **down-type quarks** i=d,s,b

All of them have $m_i/m_W \ll 1$

$\Sigma V_{ci}^* V_{ui} \otimes (\text{loop function}(m_i^2/m_W^2)) \sim 0$

because of CKM unitarity ⟶ **huge suppressions (GIM)**



Compare to bottom or strange FCNCs: $m_t/m_W > 1$

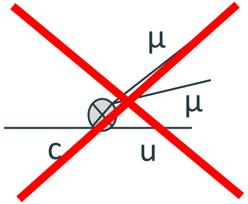
Rich experimental programme (LHCb, Belle II, BESIII, future facilities,...)

c⟶*ull* decays and intricacies

Effective weak Hamiltonian:

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}}\left[\sum_{i=1}^{2} C_i(\mu)\left(\lambda_d Q_i^d + \lambda_s Q_i^s\right) - \lambda_b\left(C_7(\mu)Q_7 + C_9(\mu)Q_9 + C_{10}(\mu)Q_{10}\right)\right] + \text{h.c.}$$

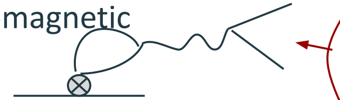
no semileptonic operators



above μ_b

<u>FCNC suppression translates into C7, C9, **C10** really small</u>

- only proceeds through quark-quark operators + electromagnetic interactions (non-local contributions)
- long-distance physics becomes important!
- some observables vanish in the SM! ***null tests of the SM***
- the observables that don't vanish are dominated by long-distance contributions



$$Q_1^d = (\bar{d}c)_{V-A}(\bar{u}d)_{V-A}\,,$$
$$Q_2^d = (\bar{u}c)_{V-A}(\bar{d}d)_{V-A}\,,$$
$$Q_1^s = (\bar{s}c)_{V-A}(\bar{u}s)_{V-A}\,,$$
$$Q_2^s = (\bar{u}c)_{V-A}(\bar{s}s)_{V-A}\,,$$
$$Q_7 = \frac{e}{8\pi^2} m_c \bar{u}\sigma_{\mu\nu}(\mathbf{1}+\gamma_5)F^{\mu\nu}c\,,$$
$$Q_9 = \frac{\alpha_{em}}{2\pi}(\bar{u}\gamma_\mu(\mathbf{1}-\gamma_5)c)(\bar{\ell}\gamma^\mu\ell)\,,$$
$$Q_{10} = \frac{\alpha_{em}}{2\pi}(\bar{u}\gamma_\mu(\mathbf{1}-\gamma_5)c)(\bar{\ell}\gamma^\mu\gamma_5\ell)$$

Goal: estimate the size of **NP**-induced Wilson coefficients (set bounds)

→ *important to control the hadronic parameters so as to probe NP as precisely as possible*

[Fajfer, Prelovsek '06; Cappiello, Cata, D'Ambrosio '13; Feldmann, Mueller, Seidel '17; De Boer, Hiller '18; Bharucha, Boito, Meaux '20...]

$D \rightarrow \pi\pi ll$: the resonance landscape

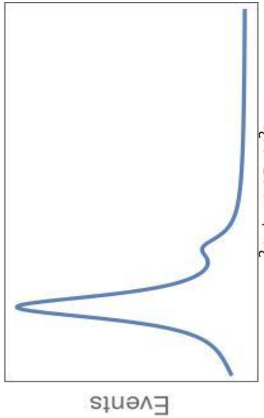
More limited kinematical region than B Physics - Resonance-driven

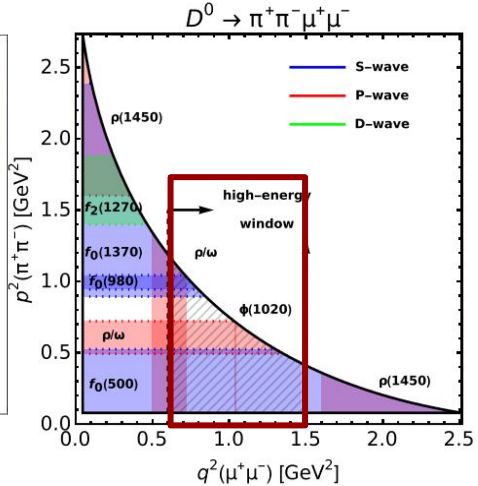
Not much room for discovery of NP in diff. branching ratios

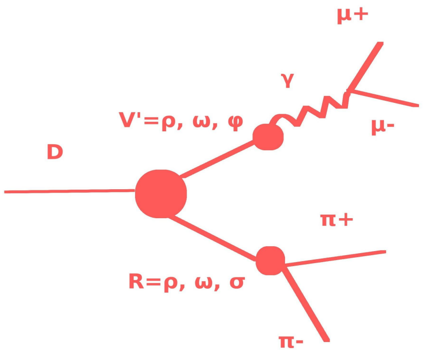
(Heavy-NP BR predictions below current sensitivity away from the resonances,

unfeasible to detect in the resonant region)

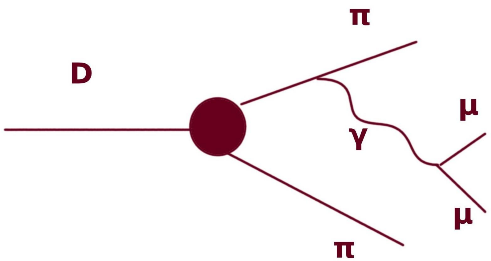
Driven mostly by quasi-two-body topologies

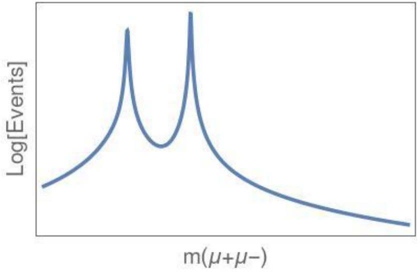






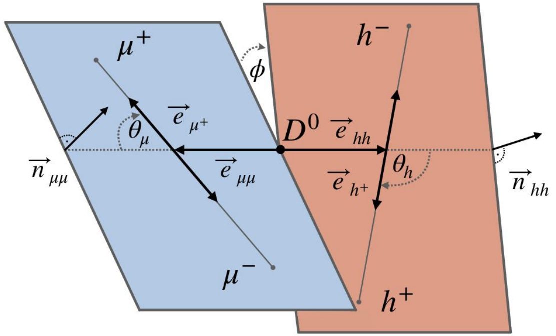
+Bremsstrahlung at low dimuon momenta, e.g.





$D\to \pi\pi ll$: plenty of observables

4-body decay: 5 kinematical variables



Angular observables I_i defined as

$$\sum_i c_i \int_{\theta_{\ell,i}}^{\theta_{\ell,i+1}} d\theta_\ell \sum_j c'_j \int_{\phi_i}^{\phi_{i+1}} d\phi\, d^5\Gamma(q^2, p^2, \theta_h, \theta_\ell, \phi)$$

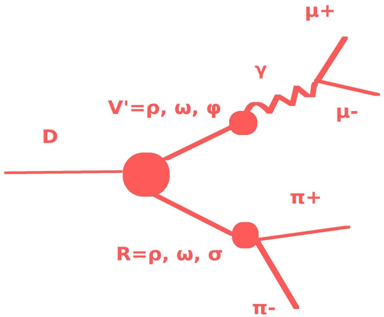
(for different combinations of c_i, c'_j)
further integrated as

$$\left(\int_0^1 d\cos\theta_h - \int_{-1}^0 d\cos\theta_h\right) I_i \quad \text{or} \quad \int_{-1}^1 d\cos\theta_h I_i$$

+ CP-*symmetrised* or *antisymmetrised*

- different observables ⟶ access to different interference patterns: S-wave, P-wave, NP
- vanishing WCs in the SM: a lot of observables are null tests

Framework



$$\langle \pi^+\pi^-\ell^+\ell^- | S | D^0 \rangle$$

$$= \langle \pi^+\pi^-\ell^+\ell^- | \int d^4x\, d^4w\, d^4y\, d^4z\, T\{\mathcal{H}^{\rm lept}_{em}(z)\, \mathcal{H}_{\mathcal{V}\gamma}(y)\, \mathcal{H}_{\mathcal{R}\pi\pi}(w)\, \mathcal{H}_{\mathcal{DRV}}(x)\} | D^0 \rangle$$

where $\mathcal{H}_{\mathcal{DRV}}$ corresponds to

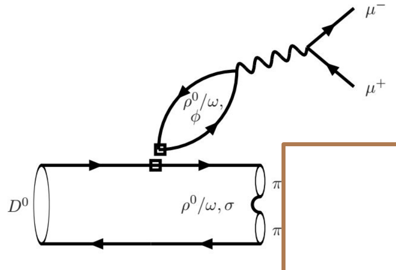
$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} \sum_{i=1}^{2} C_i(\mu) \left(\lambda_d Q_i^d + \lambda_s Q_i^s\right)$$

Start from factorization assumption: only $Q_2^d = (\overline{u}c)_{V-A}(\overline{d}d)_{V-A}\,,\quad Q_2^s = (\overline{u}c)_{V-A}(\overline{s}s)_{V-A}$, contribute

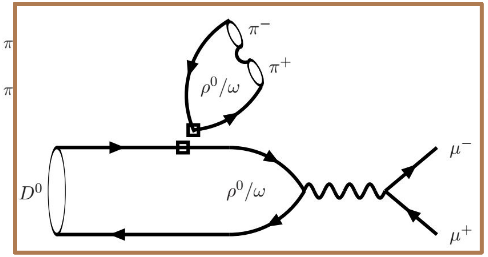
which can be

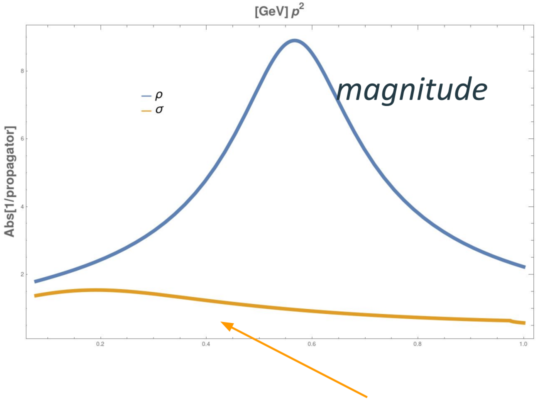
only in charm!

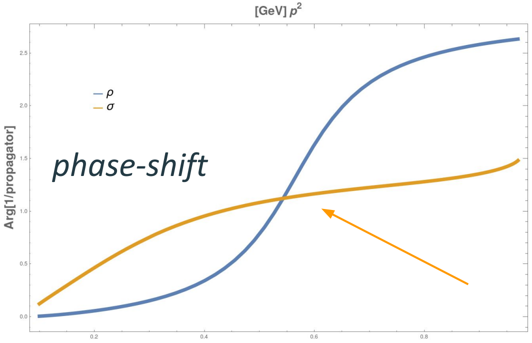
New: consider the effect of the scalar resonance $f_0(500) = \sigma$



or

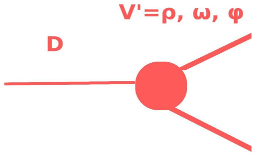






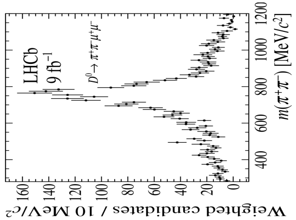
Corrections to factorization and free parameters

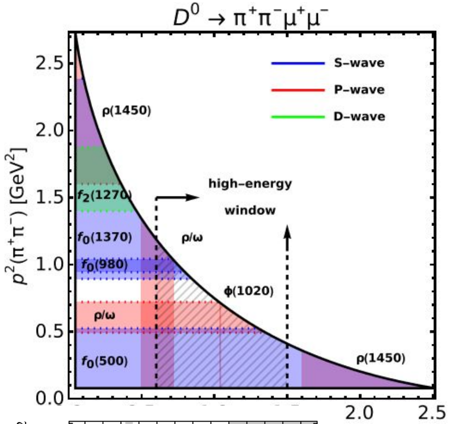
A free phase and a free normalization factor (=1 for exact factorization) at each D⟶(2 resonances) vertex: account for (some) strong rescattering effects

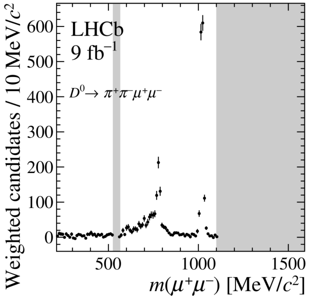


We fit to diff. branching ratios in dimuon and dipion masses

[LHCb, *Phys.Rev.Lett.* 128 (2022) 22, 221801]





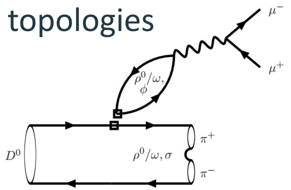


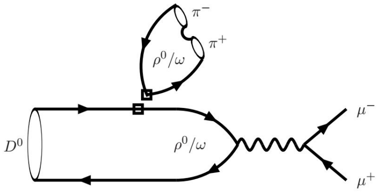
Technicalities:

- ω decaying to two pions is isospin-suppressed ⟶ grouped together with ρ as ρ/ω (similarly to e.g. e+e- ⟶π+π- analyses)
- D⟶ ρ(⟶ππ)ω(⟶μμ) suppressed because of the competition of the two factorization topologies

The fit is sensitive to:

- relative size & phase of ω⟶ ππ wrt ρ⟶ ππ
- relative size of σ⟶ ππ wrt ρ⟶ ππ
- relative phases between resonance pairs for pions in the same partial wave





Angular observables that demonstrate interference

$$\langle I_i\rangle_- \equiv \left[\int_0^{+1} d\cos\theta_\pi - \int_{-1}^{0} d\cos\theta_\pi\right] I_i\,, \quad \langle I_i\rangle_+ \equiv \int_{-1}^{+1} d\cos\theta_\pi I_i$$

We can assign an *approximate*, *effective*/*non-local* (dependent on the dimuon mass) C_9 Wilson coefficient for each partial wave S and P

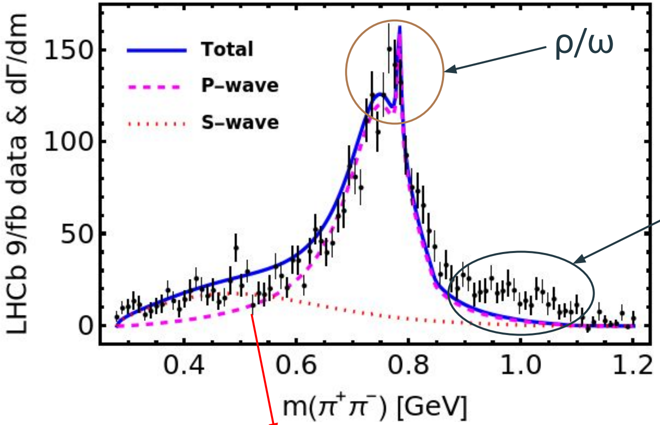
The resulting observables exhibit interesting interference patterns between C_9^S, C_9^P, C_{10} (the latter from NP only):

$\langle I_i\rangle_+$			
i	S-wave	Null test	WCs
$1^\dagger$	○		$\lvert C_9^{\mathrm{eff}:S}\rvert^2$, $\lvert C_9^{\mathrm{eff}:P}\rvert^2$
$2^\dagger$	○		$\lvert C_9^{\mathrm{eff}:S}\rvert^2$, $\lvert C_9^{\mathrm{eff}:P}\rvert^2$
$3^\dagger$	×		$\lvert C_9^{\mathrm{eff}:P}\rvert^2$
4	✓		$C_9^{\mathrm{eff}:S}\,(C_9^{\mathrm{eff}:P})^*$
5	✓	yes	$C_9^{\mathrm{eff}:S}\,C_{10}^* + C_{10}\,(C_9^{\mathrm{eff}:P})^*$
$6^\dagger$	×	yes	$\mathrm{Re}\left[C_9^{\mathrm{eff}:P}\,C_{10}^*\right]$
7	✓	yes	$C_9^{\mathrm{eff}:S}\,C_{10}^* + C_{10}\,(C_9^{\mathrm{eff}:P})^*$
8	✓		$C_9^{\mathrm{eff}:S}\,(C_9^{\mathrm{eff}:P})^*$
$9^\dagger$	×		$\lvert C_9^{\mathrm{eff}:P}\rvert^2$

$\langle I_i\rangle_-$			
i	S-wave	Null test	WCs
1	✓		$C_9^{\mathrm{eff}:S}\,(C_9^{\mathrm{eff}:P})^*$
2	✓		$C_9^{\mathrm{eff}:S}\,(C_9^{\mathrm{eff}:P})^*$
$4^\dagger$	×		$\lvert C_9^{\mathrm{eff}:P}\rvert^2$
$5^\dagger$	×	yes	$\mathrm{Re}\left[C_9^{\mathrm{eff}:P}\,C_{10}^*\right]$
$7^\dagger$	×	yes	$\mathrm{Re}\left[C_9^{\mathrm{eff}:P}\,C_{10}^*\right]$
$8^\dagger$	×		$\lvert C_9^{\mathrm{eff}:P}\rvert^2$

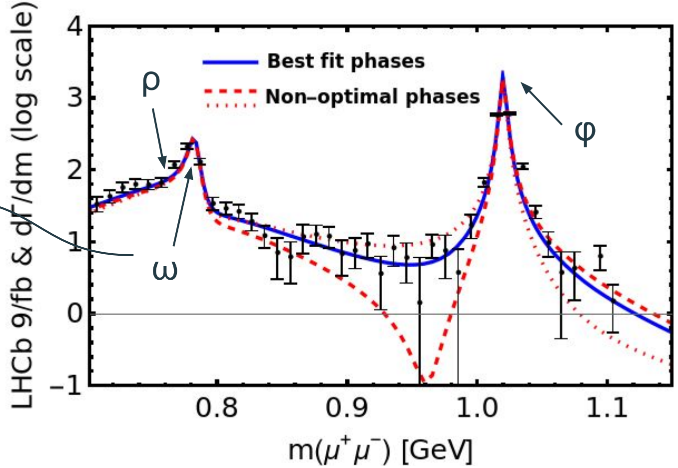
Some observables (8-, 9+) vanish for both SM and NP in the hadronic model employed

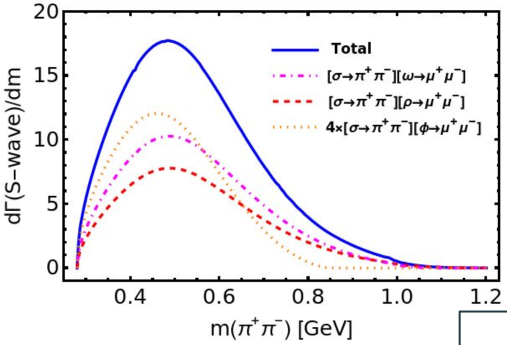
Results: branching ratios



KK threshold,
f0(980),...
(not taken into account)

from
D→σω





Fair agreement with m(ππ) data

Significant improvement with S-wave inclusion (~20% of total Br)

Fit prefers suppression of D→σφ

- Good agreement with m(μμ) data in the considered region, also for measured SM angular observables
- Relative phase between D→ρρ and D→ρφ (also D→σρ and D→σω) important for fitting in the inter-resonant region
- Probing this phase there results in different predictions for the high-energy region (important for NP searches)

using [LHCb *Phys.Rev.Lett.* 119 (2017) 18, 181805]

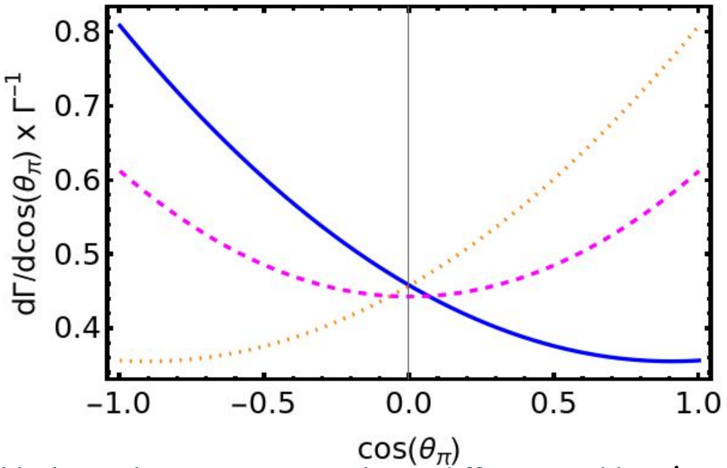
Overall normalisation factor (1 in perfect naive factorization) around 1.8

Results: observables probing the S-wave

Future experimental analyses can examine a series of so far unmeasured observables that depend on the S- and P-wave interference (& vanish in the absence of an S-wave)

Integration over whole kinematical region or in bins

$$\frac{d\Gamma}{d\cos\theta_\pi} = \langle I_1\rangle^r_{+,\rho^0/\omega} + \langle I_2\rangle^r_{+,\rho^0/\omega}\left(1 - 4\cos^2\theta_\pi\right) - \frac{4}{3}\langle I_2\rangle^r_{+,\sigma} - \frac{8}{3}\langle I_2\rangle^r_{-}\cos\theta_\pi$$

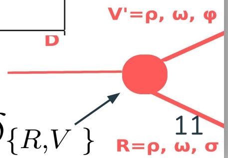


$\langle I_i\rangle_-$			
i	S-wave	Null test	WCs
1	✓		$C_9^{\text{eff}:S}\,(C_9^{\text{eff}:P})^*$
2	✓		$C_9^{\text{eff}:S}\,(C_9^{\text{eff}:P})^*$

$\langle I_i\rangle_+$			
i	S-wave	Null test	WCs
4	✓		$C_9^{\text{eff}:S}\,(C_9^{\text{eff}:P})^*$
5	✓	yes	$C_9^{\text{eff}:S}\,C_{10}^* + C_{10}\,(C_9^{\text{eff}:P})^*$
6†	×	yes	$\text{Re}\left[C_9^{\text{eff}:P}\,C_{10}^*\right]$
7	✓	yes	$C_9^{\text{eff}:S}\,C_{10}^* + C_{10}\,(C_9^{\text{eff}:P})^*$
8	✓		$C_9^{\text{eff}:S}\,(C_9^{\text{eff}:P})^*$

Will help probe new strong-phase differences like $\Delta_{SP} = \delta_{\{\sigma,\rho^0\}} - \delta_{\{\rho^0/\omega,\rho^0\}}$ and understand better the S-wave

S-wave angular observables <u>predicted to be sizeable</u>, same order as the measured ones (slightly smaller)



$\delta_{\{R,V\}}$

Results: null tests of the SM and sensitivity to NP

$\langle I_i \rangle_+$			
i	S-wave	Null test	WCs
5	✓	yes	$C_9^{\text{eff}:S}\, C_{10}^* + C_{10}\, (C_9^{\text{eff}:P})^*$
$6^\dagger$	×	yes	$\text{Re}\left[C_9^{\text{eff}:P}\, C_{10}^*\right]$
7	✓	yes	$C_9^{\text{eff}:S}\, C_{10}^* + C_{10}\, (C_9^{\text{eff}:P})^*$
$\langle I_i \rangle_-$			
$5^\dagger$	×	yes	$\text{Re}\left[C_9^{\text{eff}:P}\, C_{10}^*\right]$
$7^\dagger$	×	yes	$\text{Re}\left[C_9^{\text{eff}:P}\, C_{10}^*\right]$

O(few %)* (not included in last LHCb analysis)

O(few %) (included in last LHCb analysis)

~0 (hadronic model)

Exemplary case with non-vanishing C10, saturating present NP bounds [Fajfer and Kosnik, 1510.00965]

*S-wave-sensitive null tests depend on the unprobed phase

$$\Delta_{SP} = \delta_{\{\sigma,\rho^0\}} - \delta_{\{\rho^0/\omega,\rho^0\}}$$

Future analyses can determine this phase through the S-wave SM observables (last slide)

→ can implement the S-wave null test observables - **equally valuable as the P-wave null tests for setting bounds on NP**

With the current level of precision: extracting bounds on NP WCs not possible (exp. uncertainties are also O(few %))

Conclusions

Rare charm decays enjoy very advantageous features:

- ➢ Prominent GIM suppression ⟶ some WCs vanish ⟶ some observables completely devoid of SM contributions
- ➢ Those *null-test observables* are always of the form **SM⊗NP** ⟶ control over SM important for NP bounds
- ➢ Long-distance physics is dominant ⟶ need to address non-local hadronic effects

Naive factorization-based approach with intermediate resonances + allow room for further QCD effects

↓

Good description of the experimental data

Inclusion of S-wave improves the SM-exp agreement & gives access to new observables that can help probe NP

Feasible to set meaningful bounds on NP in the next analyses!

Further improvements of the hadronic model could be needed once experimental precision increases

BACKUP

$\int\langle I_i\rangle^r_+/\Gamma^r$		SM: $C_9^{NP} = C'_9 = C_{10} = C'_{10} = 0$		NP: $\tilde{C}_{10} = 0.43$, $C_9^{NP} = C'_9 = C'_{10} = 0$	
i	S-wave	WCs	value (%)	WCs	value (%)
$1^\dagger$	○	$\lvert C_9^{\text{eff}:S}\rvert^2$, $\lvert C_9^{\text{eff}:P}\rvert^2$	48	SM + $\lvert C_{10}\rvert^2$	48
$2^\dagger$	○	$\lvert C_9^{\text{eff}:S}\rvert^2$, $\lvert C_9^{\text{eff}:P}\rvert^2$	-7	SM + $\lvert C_{10}\rvert^2$	-7
$3^\dagger$	×	$\lvert C_9^{\text{eff}:P}\rvert^2$	-14	SM + $\lvert C_{10}\rvert^2$	-14
4	✓	$C_9^{\text{eff}:S}(C_9^{\text{eff}:P})^*$	± 2	SM + $\lvert C_{10}\rvert^2$	± 2
5	✓	–	0	$C_9^{\text{eff}:S}C_{10}^* + C_{10}(C_9^{\text{eff}:P})^*$	± 0.1
$6^\dagger$	×	–	0	Re $[C_9^{\text{eff}:P}C_{10}^*]$	± 0.3
7	✓	–	0	$C_9^{\text{eff}:S}C_{10}^* + C_{10}(C_9^{\text{eff}:P})^*$	± 0.4
8	✓	$C_9^{\text{eff}:S}(C_9^{\text{eff}:P})^*$	± 1	SM + $\lvert C_{10}\rvert^2$	± 1
$9^\dagger$	×	$\lvert C_9^{\text{eff}:P}\rvert^2$	~ 0	SM + $\lvert C_{10}\rvert^2$	~ 0

$\int\langle I_i\rangle^r_-/\Gamma^r$		SM: $C_9^{NP} = C'_9 = C_{10} = C'_{10} = 0$		NP: $\tilde{C}_{10} = 0.43$, $C_9^{NP} = C'_9 = C'_{10} = 0$	
i	S-wave	WCs	value (%)	WCs	value (%)
1	✓	$C_9^{\text{eff}:S}(C_9^{\text{eff}:P})^*$	∓ 2	SM + $\lvert C_{10}\rvert^2$	∓ 2
2	✓	$C_9^{\text{eff}:S}(C_9^{\text{eff}:P})^*$	± 2	SM + $\lvert C_{10}\rvert^2$	± 2
$4^\dagger$	×	$\lvert C_9^{\text{eff}:P}\rvert^2$	20	SM + $\lvert C_{10}\rvert^2$	20
$5^\dagger$	×	–	0	Re $[C_9^{\text{eff}:P}C_{10}^*]$	± 0.2
$7^\dagger$	×	–	0	Re $[C_9^{\text{eff}:P}C_{10}^*]$	~ 0
$8^\dagger$	×	$\lvert C_9^{\text{eff}:P}\rvert^2$	~ 0	SM + $\lvert C_{10}\rvert^2$	~ 0

q^2-bin r	Γ^r (SM)	$\frac{\Gamma^r_\sigma}{\Gamma^r}$ (%)	$\int\langle I_2\rangle^r_+ \times 100$	$\frac{\int\langle I_2\rangle^r_{+,\sigma}}{\int\langle I_2\rangle^r_+}$ (%)	$\int\langle I_3\rangle^r_+ \times 100$	$\int\langle I_4\rangle^r_- \times 100$
$r^{(\rho:\text{sup})}$	[0.64, 0.87]	[23, 43]	[−16, −8.5]	[59, 78]	[−7.2, −4.7]	[8.3, 13]
$r^{(\phi:\text{inf})}$	[1.6, 1.9]	[0.3, 8]	[−11, −6.2]	[3, 45]	[−30, −26]	[36, 41]
$r^{(\phi:\text{sup})}$	[1.2, 1.3]	[0.8, 10]	[−8.7, −4.3]	[8, 53]	[−22, −19]	[26, 29]

Angular observable predictions

q^2-bin r	$\int\langle I_2\rangle^r_- \times 100$
$r^{(\rho:\text{sup})}$	$[-6.6, -0.8]\, c_{SP} + [-2.3, -1.1]\, s_{SP}$
$r^{(\phi:\text{inf})}$	$[-7.7, 6.1]\, c_{SP} + [-5.3, 8.2]\, s_{SP}$
$r^{(\phi:\text{sup})}$	$[-7.1, 3.0]\, c_{SP} + [-5.0, 5.4]\, s_{SP}$

q^2-bin r	$\int\langle I_4\rangle^r_+ \times 100$
$r^{(\rho:\text{sup})}$	$[0.8, 5.9]\, c_{SP} + [0.4, 1.6]\, s_{SP}$
$r^{(\phi:\text{inf})}$	$[-6.7, 8.3]\, c_{SP} + [-8.6, 5.4]\, s_{SP}$
$r^{(\phi:\text{sup})}$	$[-3.1, 7.6]\, c_{SP} + [-5.9, 5.5]\, s_{SP}$

q^2-bin r	$\int\langle I_8\rangle^r_+ \times 100$
$r^{(\rho:\text{sup})}$	$[-3.0, -0.2]\, c_{SP} + [-0.4, 0.4]\, s_{SP}$
$r^{(\phi:\text{inf})}$	$[-4.6, 4.5]\, c_{SP} + [-3.4, 4.0]\, s_{SP}$
$r^{(\phi:\text{sup})}$	$[-2.6, 3.3]\, c_{SP} + [-1.7, 3.3]\, s_{SP}$

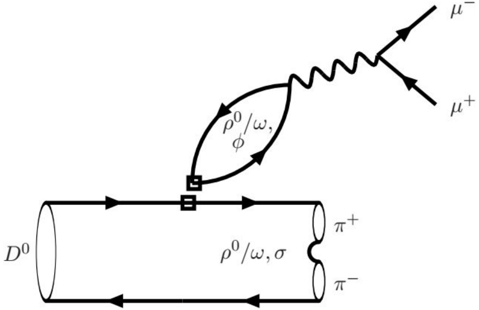
q^2-bin r	$\int\langle I_5\rangle^r_- \times 100$
$r^{(\rho:\text{sup})}$	$[0.49, 0.83]\, c_{\rho NP} + [-1.5, -1.3]\, s_{\rho NP}$
$r^{(\phi:\text{inf})}$	$[-0.36, 0.50]\, c_{\rho NP} + [-0.83, -0.60]\, s_{\rho NP}$
$r^{(\phi:\text{sup})}$	$[0.31, 0.66]\, c_{\rho NP} + [-0.09, 0.49]\, s_{\rho NP}$

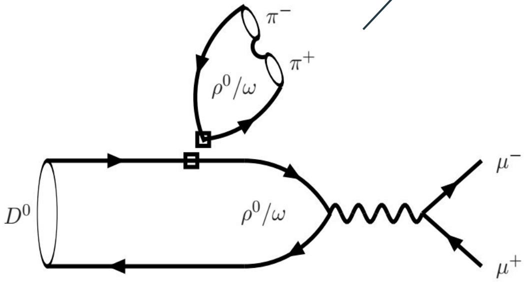
q^2-bin r	$\int\langle I_6\rangle^r_+ \times 100$
$r^{(\rho:\text{sup})}$	$[0.7, 1.2]\, c_{\rho NP} + [-2.1, -1.7]\, s_{\rho NP}$
$r^{(\phi:\text{inf})}$	$[-0.57, 0.78]\, c_{\rho NP} + [-1.3, -1.0]\, s_{\rho NP}$
$r^{(\phi:\text{sup})}$	$[0.5, 1.1]\, c_{\rho NP} + [-0.14, 0.78]\, s_{\rho NP}$

Effective C9

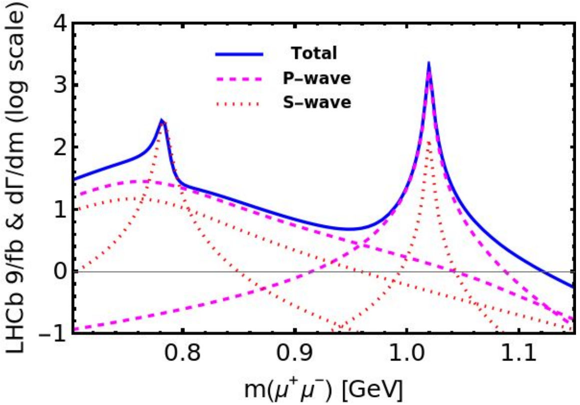
cannot be written as

$$\langle \pi^+\pi^- | (\bar{u}c)_{V-A} | D^0 \rangle \times \langle \ell^+\ell^- | (\bar{\ell}\ell)_V(x) | 0 \rangle$$





only for P-wave



For S-wave (D→σ V), we can write

$$C_9^{\text{eff}:S}(\mu; q^2) = 8\pi^2 C_2(\mu) \left(\frac{f_{\rho^0}^2}{2P_{\rho^0}(q^2)} B_{\rho^0}^{(S)} e^{i\delta_{\{\sigma,\rho^0\}}} - \frac{f_\omega^2}{6P_\omega(q^2)} B_\omega^{(S)} e^{i\delta_{\{\sigma,\omega\}}} - \frac{f_\phi^2}{3P_\phi(q^2)} B_\phi^{(S)} e^{i\delta_{\{\sigma,\phi\}}} \right)$$

For P-wave (D→ρ/ω V), we can *approximately* write

$$C_9^{\text{eff}:P}(\mu; q^2) \approx 8\pi^2 C_2(\mu) \left(\frac{f_{\rho^0}^2}{P_{\rho^0}(q^2)} B_{\rho^0}^{(P)} e^{i\delta_{\{\rho^0/\omega,\rho^0\}}} - \frac{f_\phi^2}{3P_\phi(q^2)} B_\phi^{(P)} e^{i\delta_{\{\rho^0/\omega,\phi\}}} \right)$$

No D→ ρ ω because of competing contributions

$$b_{\rho^0}\left(1 + a_\omega \, \text{RBW}_\omega(p^2)\right)\left(-\frac{1}{6}\right)\frac{f_\omega^2}{P_\omega(q^2)}$$

vs

$$b_{\rho^0}\left(1 - a_\omega \, \text{RBW}_\omega(p^2)\right)\frac{1}{6}\frac{f_\omega^2}{P_\omega(q^2)}$$

Observables vanishing because of hadronic model

$$\langle I_9 \rangle_+ = \frac{2}{3}\left[\mathrm{Re}(\mathcal{F}_\perp \mathcal{F}_\parallel^*)\,\mathrm{Im}\rho_2^+ + \mathrm{Im}(\mathcal{F}_\perp \mathcal{F}_\parallel^*)\,\mathrm{Re}\rho_2^-\right]$$ which for SM and no phases between C10, C10'

becomes $\langle I_9 \rangle_+ \propto \mathrm{Im}(\mathcal{F}_\perp \mathcal{F}_\parallel^*)\,\left(|C_9|^2 + |C_{10}|^2\right)$

(transversity form factors)

$$\mathcal{F}_\parallel = N \frac{b_{\rho^0/\omega}(p^2) F_{BW}(p^2)\sqrt{\beta_\ell(3-\beta_\ell^2)}\lambda_h^{3/4}\lambda_D^{1/4}}{P_{\rho^0}(p^2)} \frac{\sqrt{q^2}(m_D + m_{\rho^0})A_1(q^2)}{\sqrt{2}p^2},$$

$$\mathcal{F}_\perp = -N \frac{b_{\rho^0/\omega}(p^2) F_{BW}(p^2)\beta_\ell^{3/2}\lambda_h^{3/4}\lambda_D^{3/4}}{P_{\rho^0}(p^2)} \frac{\sqrt{q^2}V(q^2)}{(m_D + m_{\rho^0})p^2}$$

} same strong phase coming from ρ/ω (→ ππ) line shape (& no extra strong phase assigned individually)

Similarly, the null-test observable

$$\langle I_7 \rangle_- \propto \mathrm{Im}\left(\mathcal{F}_P \mathcal{F}_\parallel^*\right) \mathrm{Re}\left(C_9^P C_{10}^*\right)$$ vanishes→ cannot serve for NP detection

Have assumed D→ ρ form factors & a single pole parameterization for them -
Improve form factor description?
Maybe in the presence of more resonances? But need to be P-wave: only ρ(1450) present - affects small region of the phase space

A couple of tensions with I8-, I9+ (χ^2/dof=2.4) could indicate a need for improvement

$m(\mu^+\mu^-)$ [MeV/c^2]	$\langle S_8 \rangle$ [%]	$\langle S_9 \rangle$ [%]
< 525	$16 \pm 17 \pm 1$	$26 \pm 16 \pm 2$
525–565	–	–
565–780	$12.9 \pm 4.9 \pm 1.0$	$-0.1 \pm 4.9 \pm 0.9$
780–950	$1.4 \pm 6.9 \pm 0.7$	$-4.7 \pm 6.8 \pm 0.8$
950–1020	$2.6 \pm 4.3 \pm 0.9$	$16.9 \pm 4.3 \pm 1.0$
1020–1100	$0.7 \pm 4.1 \pm 0.9$	$7.8 \pm 4.0 \pm 1.7$
> 1100	–	–
Full range	$3.8 \pm 2.5 \pm 0.5$	$5.1 \pm 2.5 \pm 0.5$

Free parameters (the fit to dBr's is sensitive to), collected

$\alpha_\omega, \phi_\omega$ — ω→ ππ contribution relative to ρ→ ππ

$B_\rho^P \cdot A_1(0)$ — overall normalization

$\dfrac{\alpha_s(0) B_\rho^S}{B_\rho^P \cdot A_1(0)}$ — S-wave contribution relative to ρ/ω→ππ

$\dfrac{B_\phi^P}{B_\rho^P}$ — φ from D→ρφ ("fudge factor"/ correction to normalization) relative to D→ρρ

$\dfrac{B_\omega^S}{B_\rho^S}, \dfrac{B_\phi^S}{B_\rho^S}$ — ω, φ from D→σω, σφ ("fudge factor"/ correction to normalization) relative to D→σρ

$\delta_{\{\rho,\rho\}} - \delta_{\{\rho,\phi\}}$

$\delta_{\{\sigma,\rho\}} - \delta_{\{\sigma,\omega\}}$ — relative phases

Total of 9 parameters (+1 relative phase but which the fit is not sensitive to)