Hydrodynamic Fluctuations, Perturbations, and Sources

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Hydrodynamics and related observables in HIC
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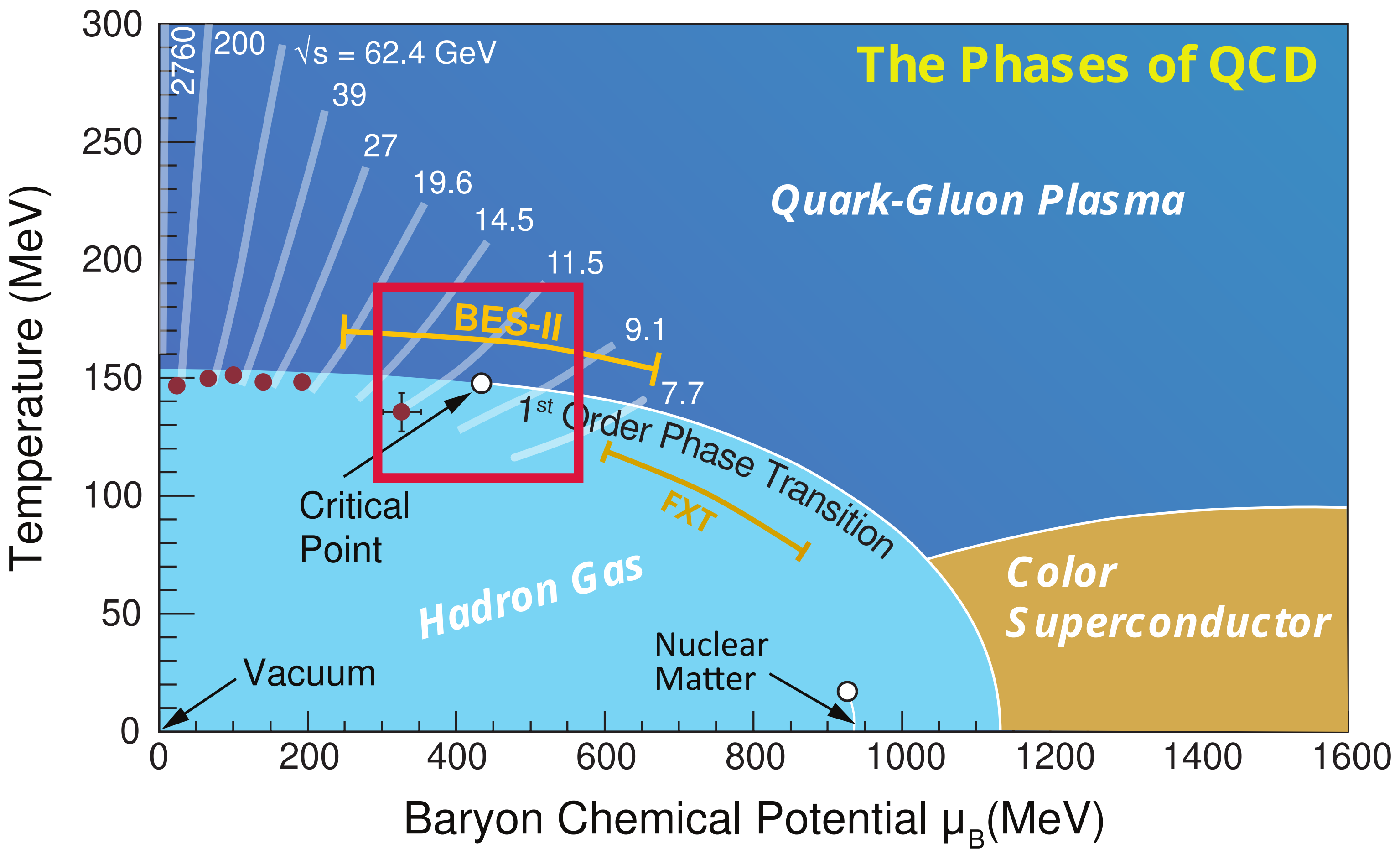
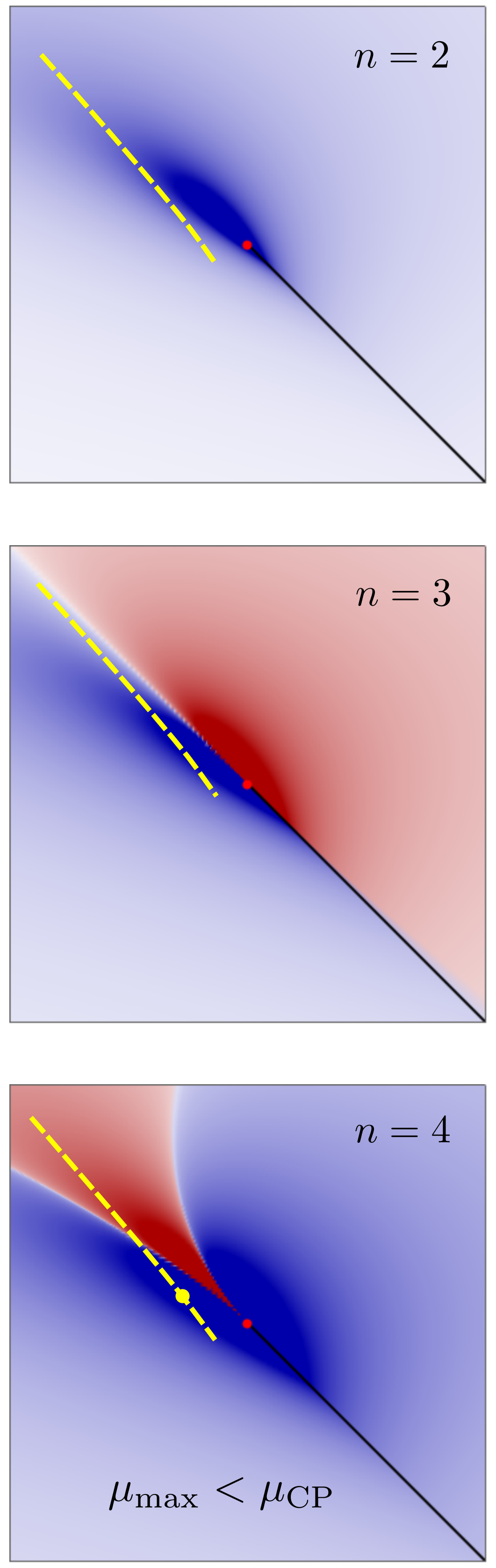
Hydrodynamic fluctuations

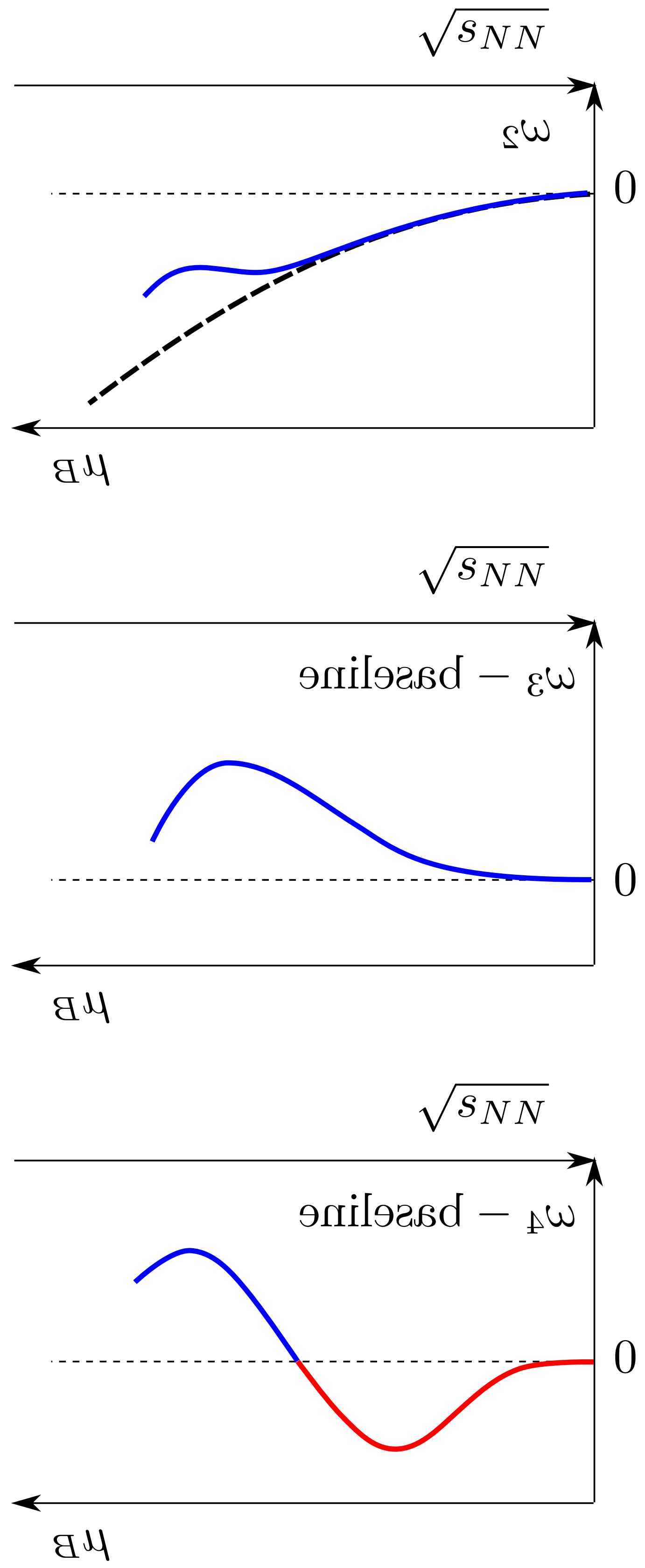
Theory vs Experiment

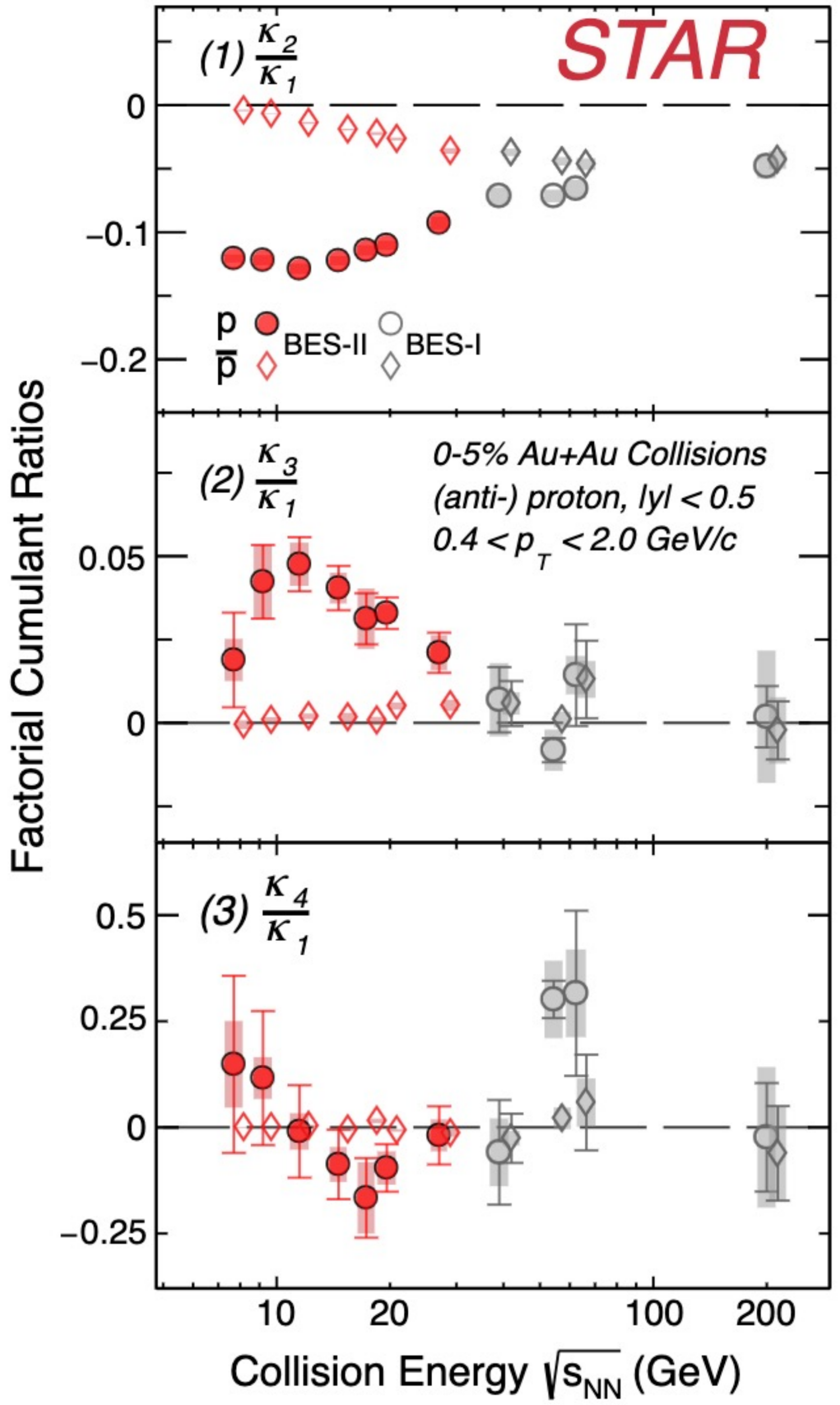
- BES-II data seems to advocate the intriguing hint of the QCD critical point from BES-I analysis, in a *qualitative* level based on *equilibrium* theory.

STAR, 2112.00240; Stephanov, 1104.1627, SQM24









What to do next

- Caveat:

Idealization

- Global equilibrium
- Static & homogeneous
- One common frame

Reality

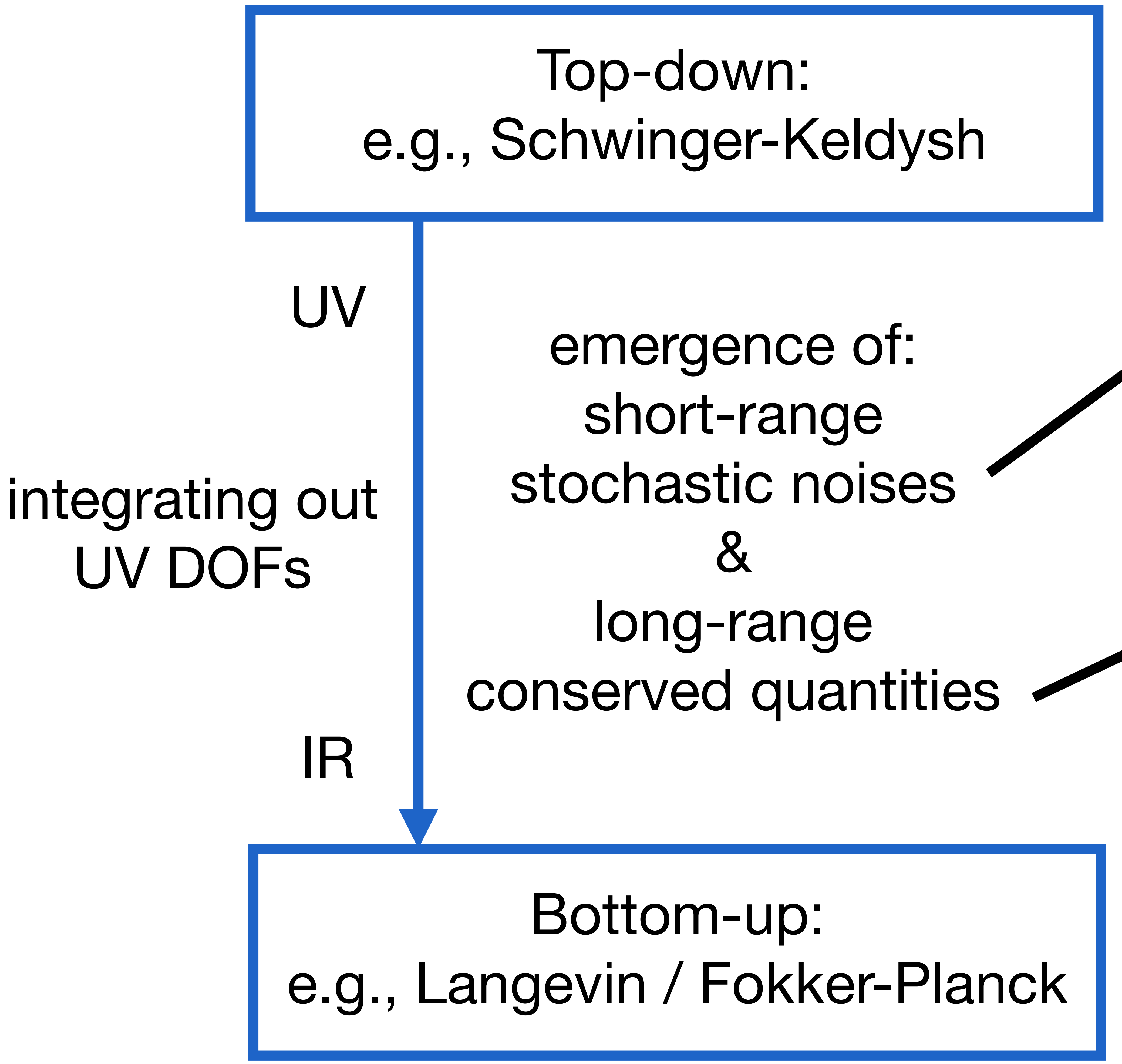
- Local equilibrium
- Dynamic & inhomogeneous
- Many local rest frames

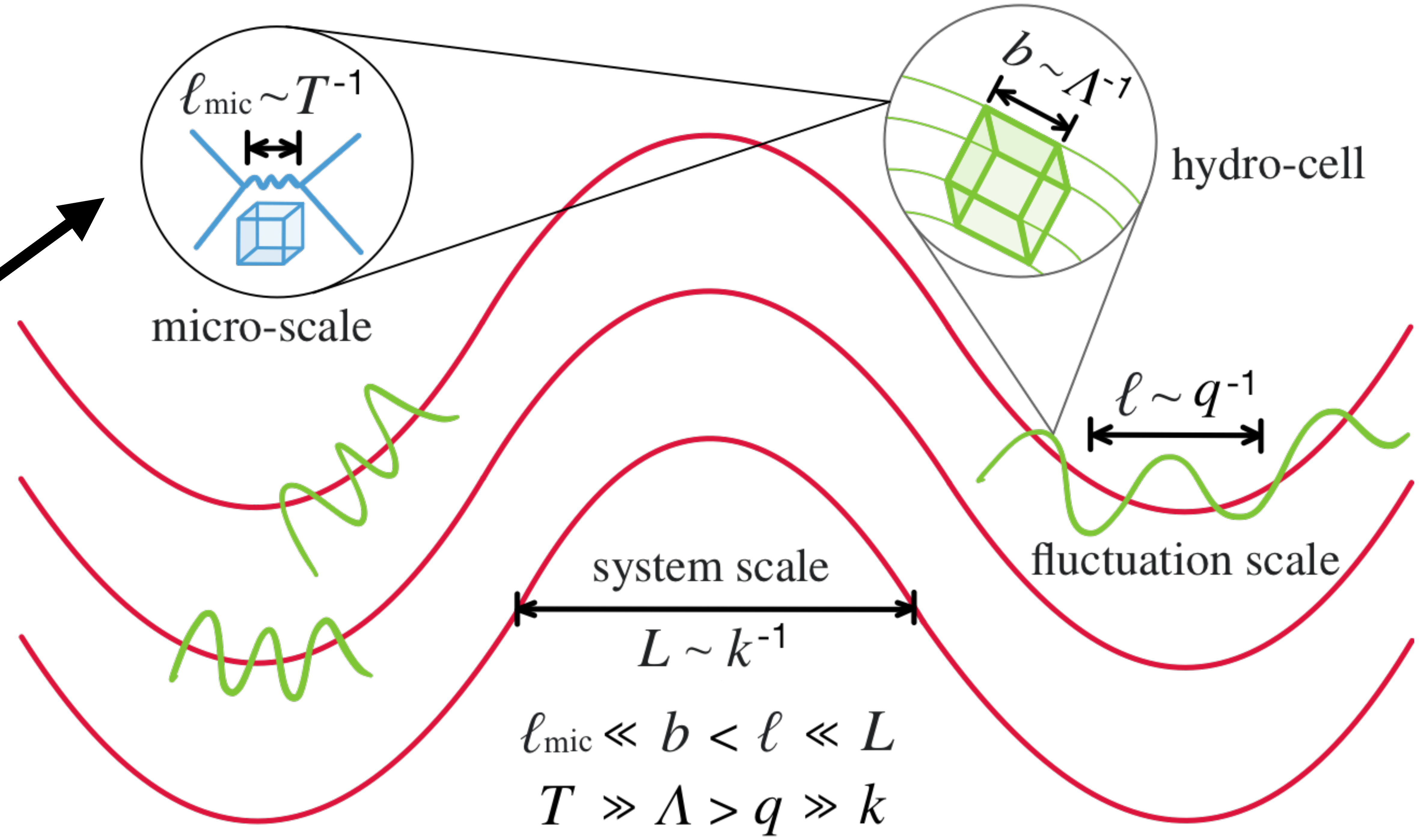
- Establish *quantitative* connection between the EOS and experiment, and use BES-II data in turn to constrain the EOS and transport coefficients.

    - Experiment results between 3-5 GeV.
    - Calibration of the non-critical baseline.
    - Dynamic modeling with hydro fluctuations and ME freezeout.

Stochastic hydrodynamics as an EFT

- Hydrodynamics + fluctuation & noise (source):





The scale hierarchy ensures:

1) local thermalization ($\ell_{\rm mic} \ll b$);
2) small Knudsen number ($\ell_{\rm mic} \ll L$);
3) separation of fluctuation and background ($\ell \ll L$).

Bottom-up approach

Stochastic

Langevin equation

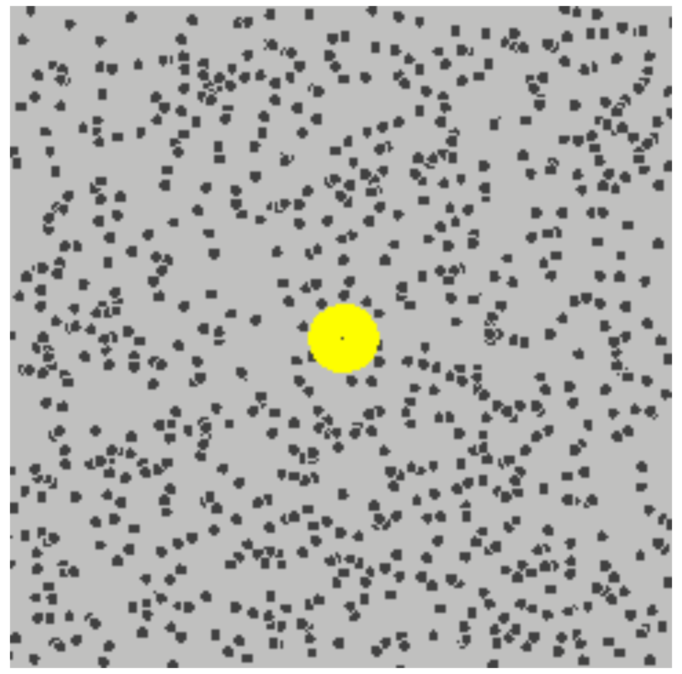
Newton's equation + noise

$$\partial_t \breve{\psi} = F[\breve{\psi}] + \eta[\breve{\psi}]$$

$$\langle \eta(x_1)\, \eta(x_2) \rangle = 2Q\, \delta^{(4)}(x_1 - x_2)$$

Langevin

Landau



Brownian motion

One equation, *Millions* of samples

cutoff-sensitive, multiplicative noise

Deterministic

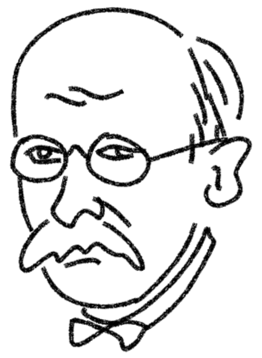
Fokker-Planck equation

probability evolution equation

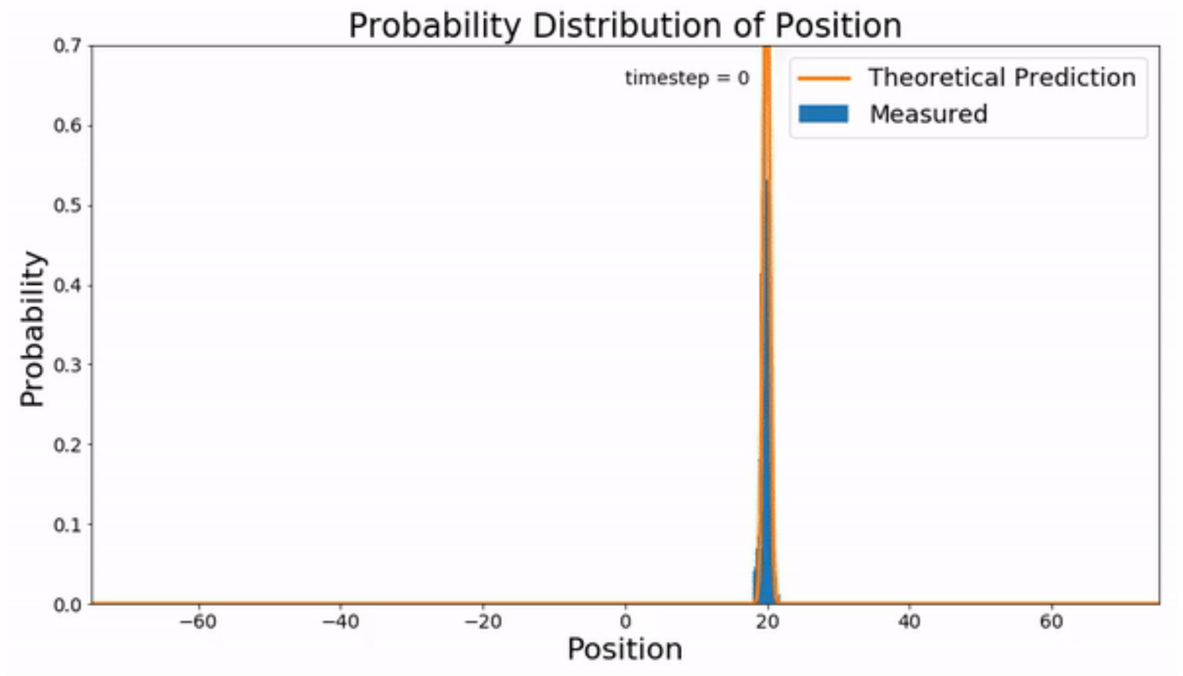
$$\partial_t P[\psi] = \partial_\psi \left(\mathrm{flux}[\psi] \right)$$

$$\mathrm{flux}[\psi] = -\,\mathrm{FP} + \partial_\psi(\mathrm{QP})$$

Fokker



Planck



(Wikipedia)

One sample, *Millions* of equations

cutoff-independent, analytically controllable

Pushing to non-Gaussian regime

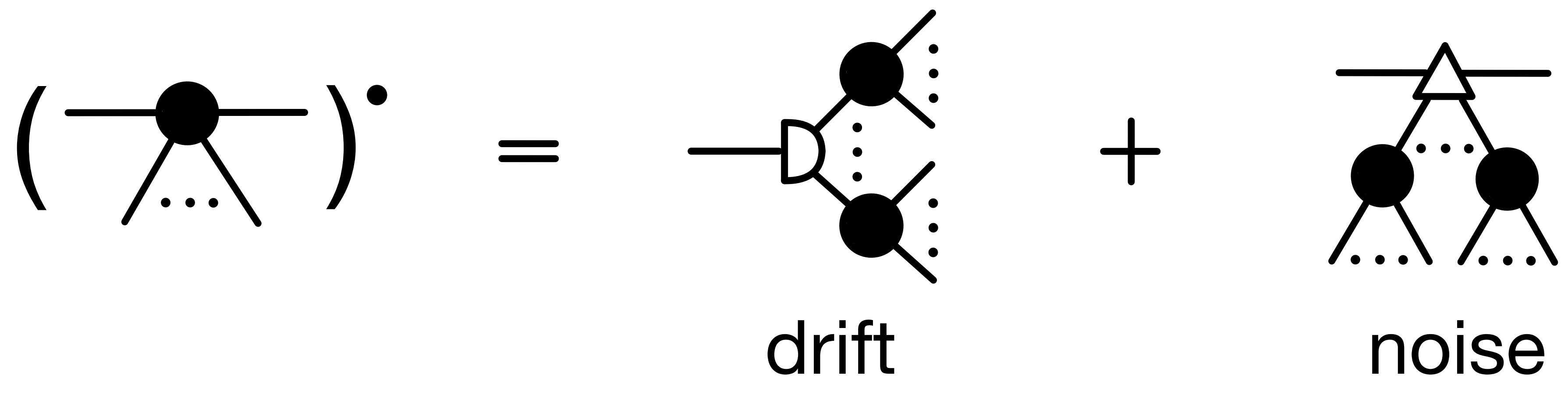
- The deterministic approach provides the truncated evolution equations for the fluctuation correlators $G_n = \langle \phi \ldots \phi \rangle$. XA et al, 2212.14029

NB: $\phi \equiv \psi - \langle \psi \rangle \sim 1/\sqrt{N}$ (CLT)

$$\partial_t G_n = \mathcal{F}\left[\langle \psi \rangle, G_2, G_3, \ldots, G_n\right] + \mathcal{O}(\varepsilon^n) \qquad \text{where} \qquad G_n \sim \varepsilon^{n-1}, \qquad F_i \sim 1, \qquad Q_{ij} \sim \varepsilon\,.$$

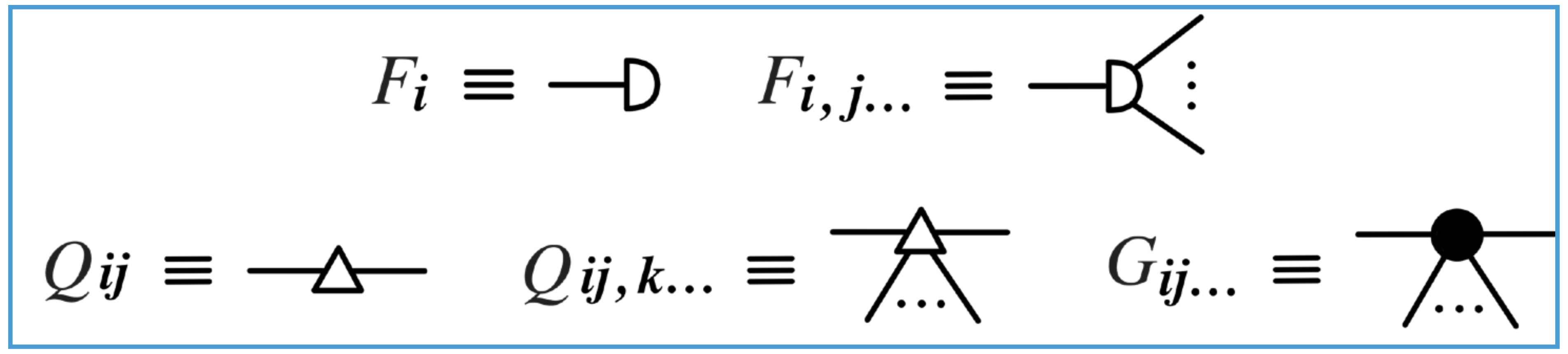
loop-expansion parameters: $\varepsilon \sim (\xi/\ell)^3 \sim 1/N$

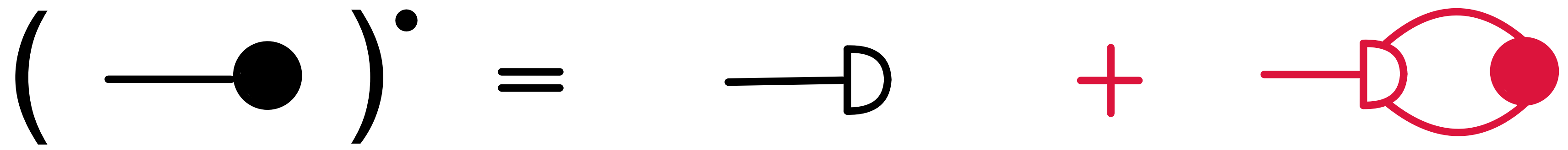
N: number of correlated volumes



all *combinatorial* configurations of trees

diagram ingredients:





1-pt equation including leading loop

conventional hydro equations

one loop (renormalization & long-time tails)

This is important for the 5th and 6th cumulants that are currently measured by STAR!

Pushing to relativistic formulation

- Goal: deal with relativity/covariance *as if one knows nothing about relativity* (i.e., like how one deals with non-relativistic theories in the lab).
- **1-pt**: covariantize Langevin equations

$$\partial_t \breve{\psi} = F[\breve{\psi}, \nabla \breve{\psi}] + \eta[\breve{\psi}, \nabla \breve{\psi}] \longrightarrow u \cdot \partial \breve{\psi} = F[\breve{\psi}, \Delta_{\mu\nu}\partial^{\nu}\breve{\psi}] + \eta[\breve{\psi}, \Delta_{\mu\nu}\partial^{\nu}\breve{\psi}]$$

e.g., $\breve{\Pi}^{\mu\nu} = -\frac{1}{\breve{\beta}}(2\breve{\eta}\Delta^{\mu\nu\lambda\kappa} + \breve{\zeta}\Delta^{\mu\nu}\Delta^{\lambda\kappa})\partial_{\lambda}(\breve{\beta}\breve{u}_{\kappa})$

spatial triad $e^a_\mu u^\mu = 0$

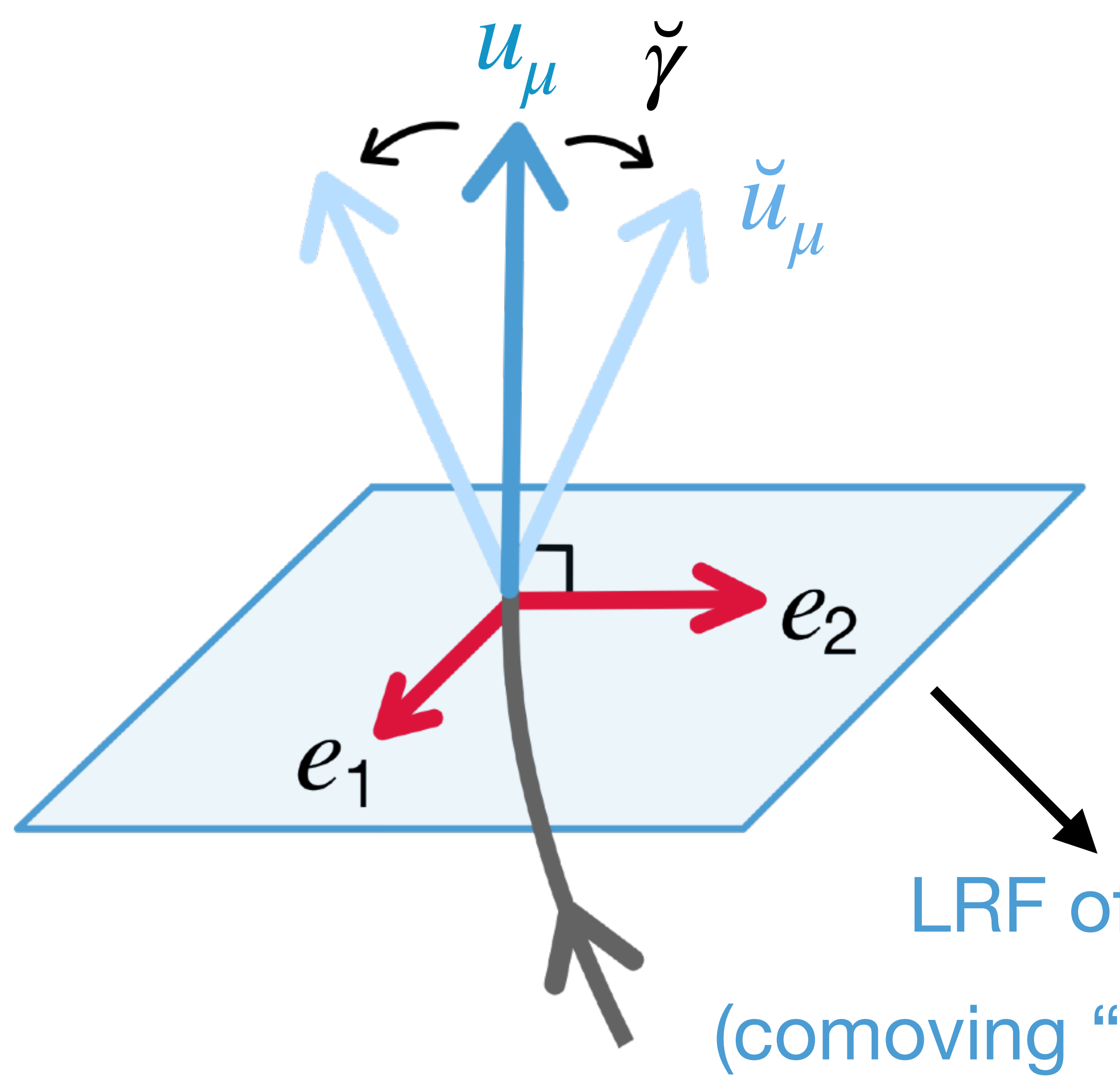
$$\breve{u}_\mu = \breve{u}_a e^a_\mu + \breve{\gamma}\, u_\mu$$

$\breve{\gamma} \equiv \gamma(\breve{u}_a) = (1 + \breve{u}_a^2)^{1/2}$

4-velocity fluctuation $\breve{u}_\mu$ is measured in terms of its independent 3-components $\breve{u}_a$ in the LRF of u_μ, (comoving "LF" of $\breve{u}_\mu$)

$\breve{T}^{\mu\nu}\breve{u}_\nu = -\breve{\epsilon}\breve{u}^\mu$

$T^{\mu\nu}u_\nu = -\epsilon u^\mu$



LRF of u_μ
(comoving "LF" of $\breve{u}_\mu$)

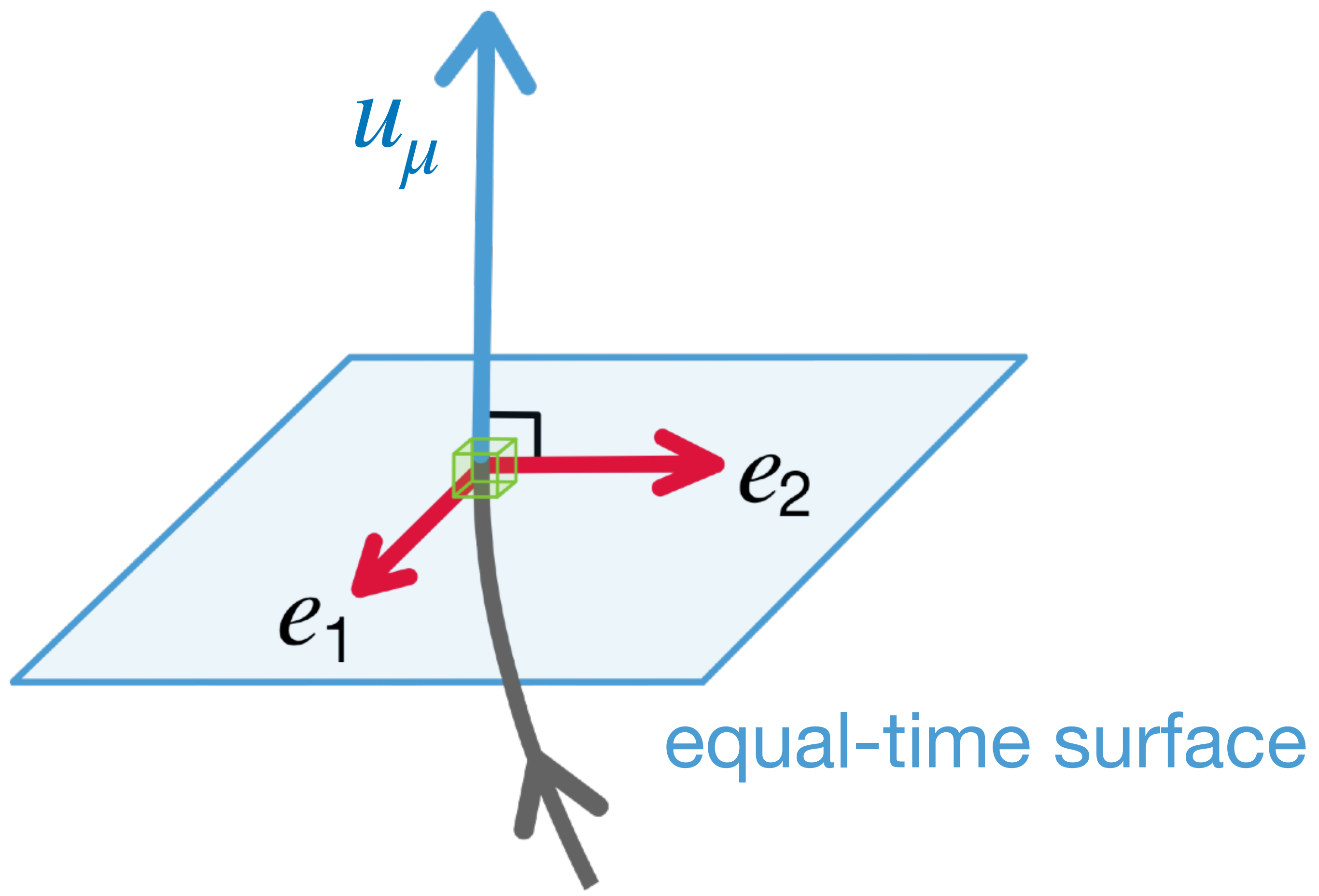
NB: $\langle \breve{u}_a \rangle = u_a$ while $\langle \breve{u}_\mu \rangle \neq u_\mu$.

Pushing to relativistic formulation

- Entropy is measured in the non-fluctuating LRF of u_μ in terms of fluctuating variables measured in the LRF of $\breve{u}_\mu$ (related by boost $\breve{\gamma}$).

$$S(\breve{\epsilon}, \breve{n}, \breve{u}_a) = \int_x \breve{\gamma}\breve{s} + \alpha\breve{\gamma}\breve{n} - \beta(\breve{\gamma}^2\breve{w} - \breve{p})$$

$\breve{w} = \breve{\epsilon} + \breve{p},\ \breve{p} = p(\breve{\epsilon}, \breve{n}),$
α and β: Lagrange multipliers

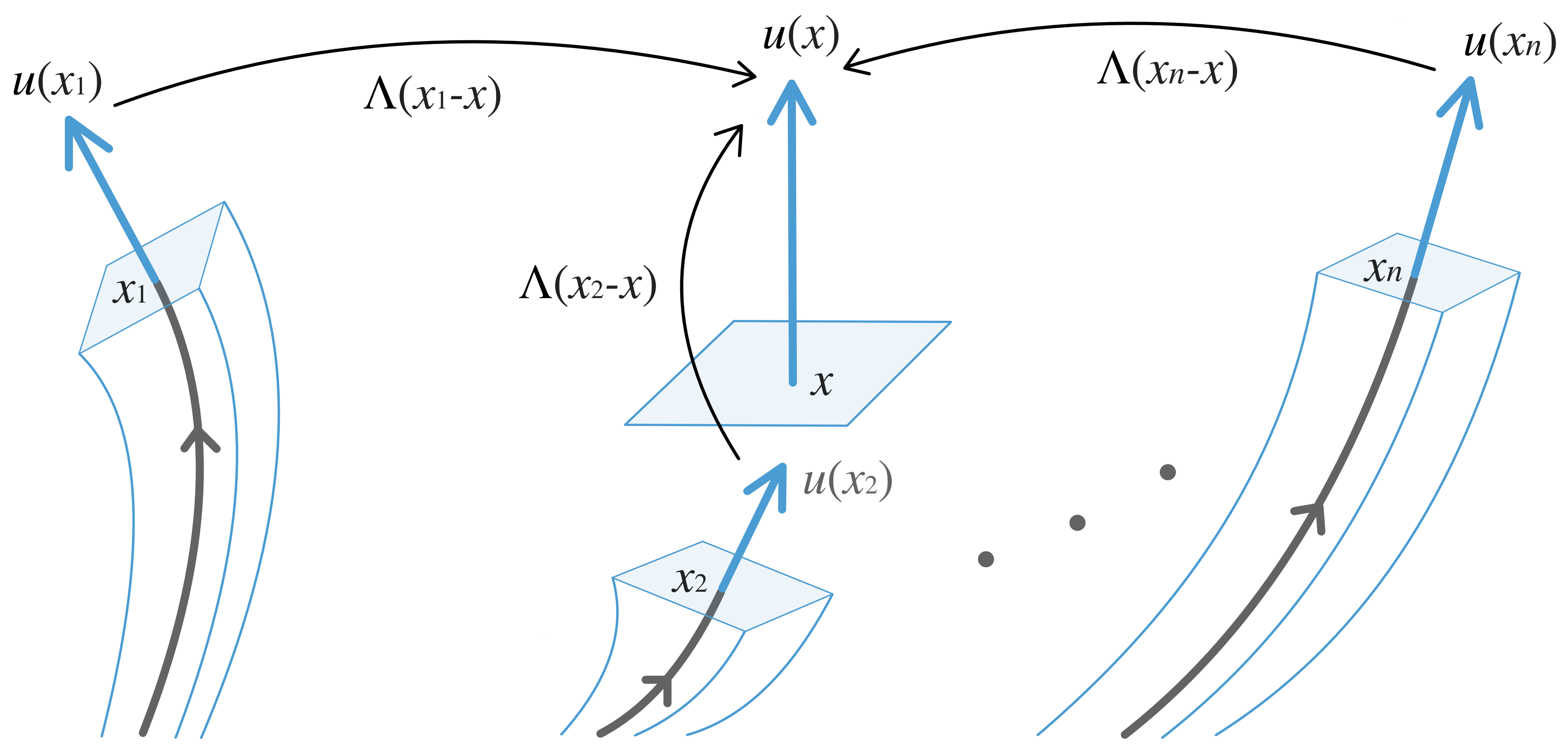


- **n-pt**: relative motion to the midpoint in the *equal-time* hypersurface needs to be described by 1-pt-like EOM along u_μ.

Confluent formulation: correlator and derivative

- Confluent formulation: covariant description for the comoving fluctuations (*as if you are in the lab frame*). See XA et al, 2212.14029 for more details

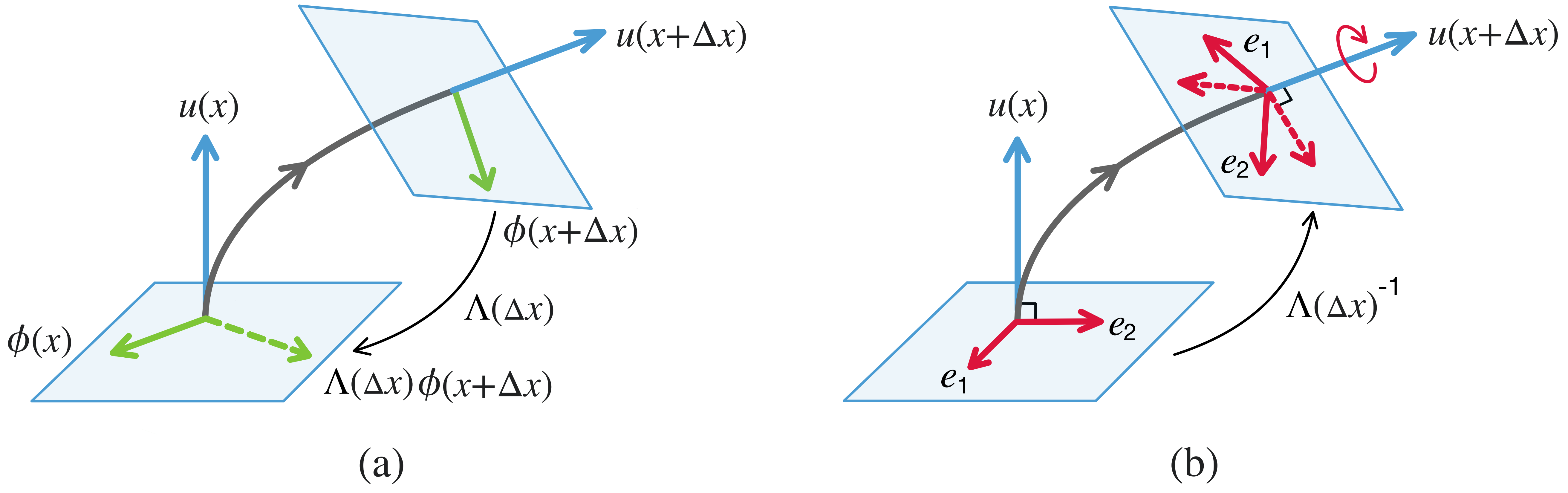
Confluent correlator $\bar{G}$



$$\bar{G}_{i_1 \ldots i_n} = \underbrace{\Lambda_{i_1}^{\ j_1}(x - x_1) \ldots \Lambda_{i_n}^{\ j_n}(x - x_n)}_{\text{boost}} G_{j_1 \ldots j_n}$$

boost all fields (measured at their own local rest frame) to **one same frame** (chosen at their midpoint)

Confluent derivative $\bar{\nabla}$



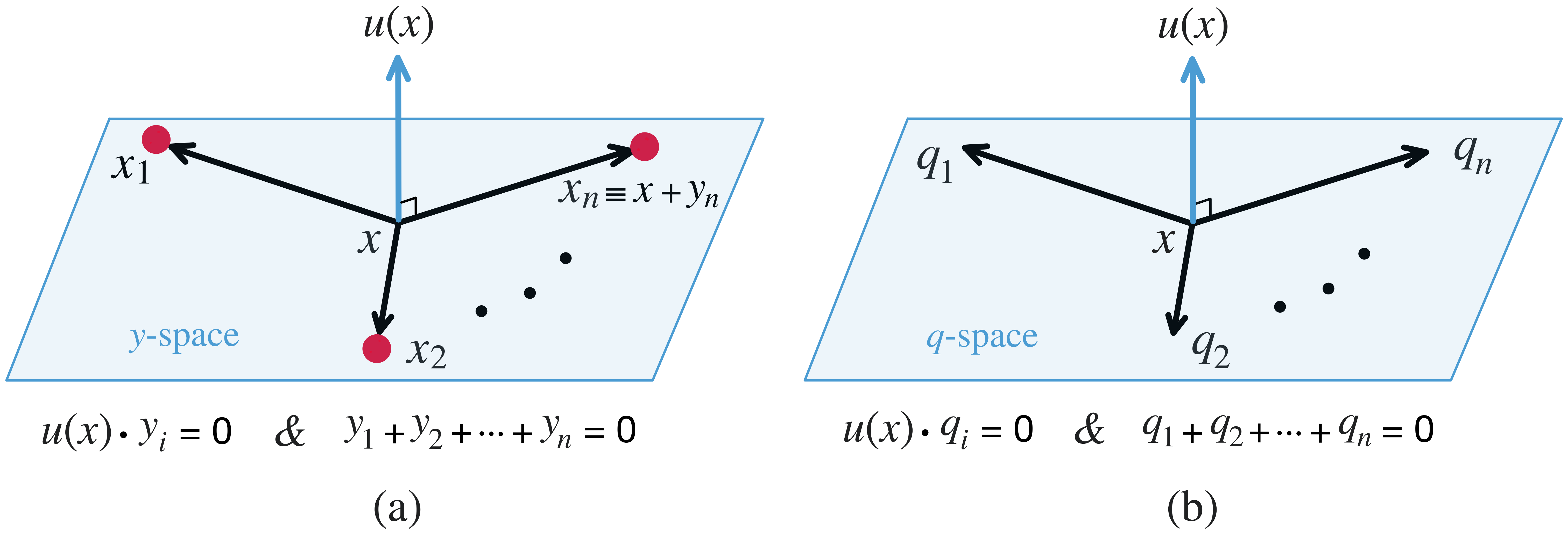
$$\bar{\nabla}_\mu \bar{G}_{i_1 \ldots i_n} = \partial_\mu \bar{G}_{i_1 \ldots i_n} - n \underbrace{\left(\bar{\omega}^{\ j_1}_{\mu i_1} \bar{G}_{j_1 \ldots i_n} + \mathring{\omega}^{\ a}_{\mu b}\, y_1^b \partial_a^{(y_1)} \bar{G}_{i_1 \ldots i_n} \right)_{\text{perm.}}}_{\text{connections}}$$

compare the difference of a given field along the time direction in **one same frame**, with the **equal-time** constraint preserved

Confluent formulation: Wigner function

- Confluent n-pt Wigner transform between 3-vectors y^a and q^a (*as if you are dealing with non-relativistic theories*). XA et al, 2212.14029

$$W_n(x; q_1^a, \ldots, q_n^a) = \underbrace{\int \prod_{i=1}^{n} \left(d^3 y_i^a \, e^{-i q_{ia} y_i^a}\right) \delta^{(3)}\left(\frac{1}{n}\sum_{i=1}^{n} y_i^a\right)}_{x \text{ independent integration kernal}} \bar{G}_n(\underbrace{x + e_a y_1^a}_{x_1}, \ldots, \underbrace{x + e_a y_n^a}_{x_n})$$



"While the bottom-up approach is useful in order to calculate two-point correlation functions, it is not immediately obvious how it should be generalized for the calculation of n-point correlation functions." Romatschke, 2019

Confluent fluctuation evolution equations

- Fluctuation evolution equations in the *impressionistic* form:

$$\mathscr{L}W_n = iqW_n - \gamma q^2(W_n - \ldots) - \partial\psi W_n + \ldots \quad \text{where} \quad \mathscr{L} = u\cdot\bar{\nabla}_x + f\cdot\nabla_q$$

sound/advection dissipation background

of which the solutions match results determined from entropy $S(\breve{m}, \breve{p}, \breve{u}_a)$.

m: entropy per baryon; p: pressure; u_a: three-velocity

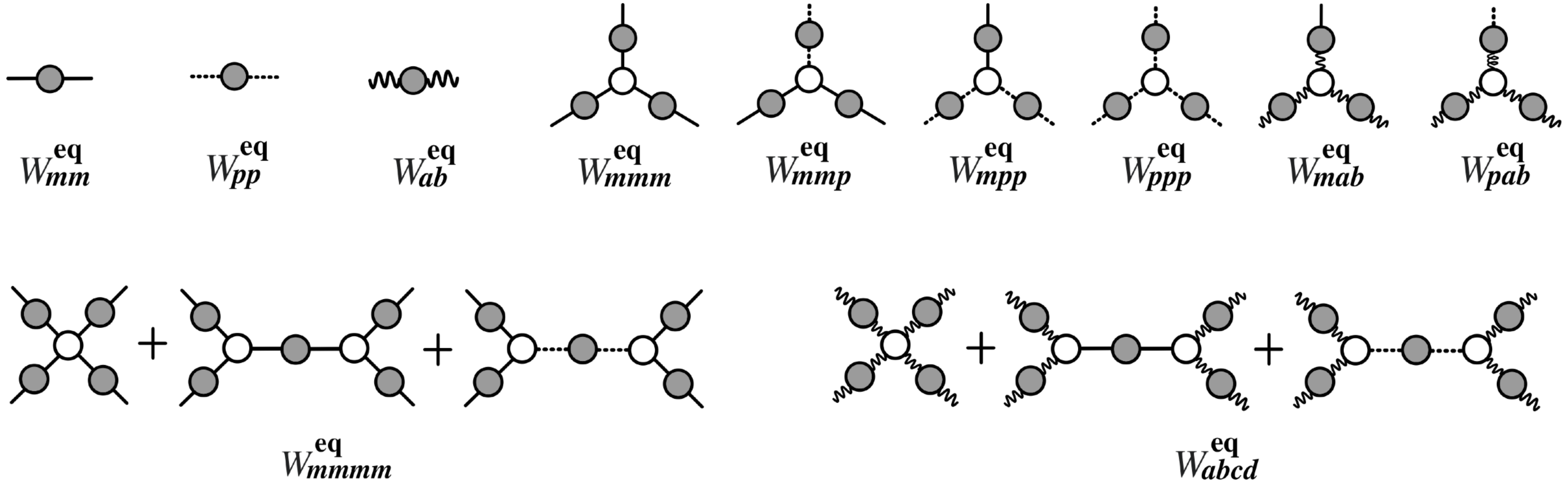
$$\mathscr{L}W_{ab}(\mathbf{q}_1, \mathbf{q}_2) = -\gamma_\eta(\mathbf{q}_1^2 + \mathbf{q}_2^2)(W_{ab} - W_{ab}^{\text{eq}}) + \ldots;$$

$$\mathscr{L}W_{abc}(\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3) = -\gamma_\eta(\mathbf{q}_1^2 + \mathbf{q}_2^2 + \mathbf{q}_3^2)W_{abc} + \ldots; \quad \ldots$$

$$W_{ab}^{\text{eq}} = -(\beta w)^{-1}\delta_{ab}$$

$$W_{abc}^{\text{eq}} = 0$$

$$W_{abcd}^{\text{eq}} \sim -3(\beta w)^{-3}\delta_{ab}\delta_{cd}$$



Rotating wave approximation

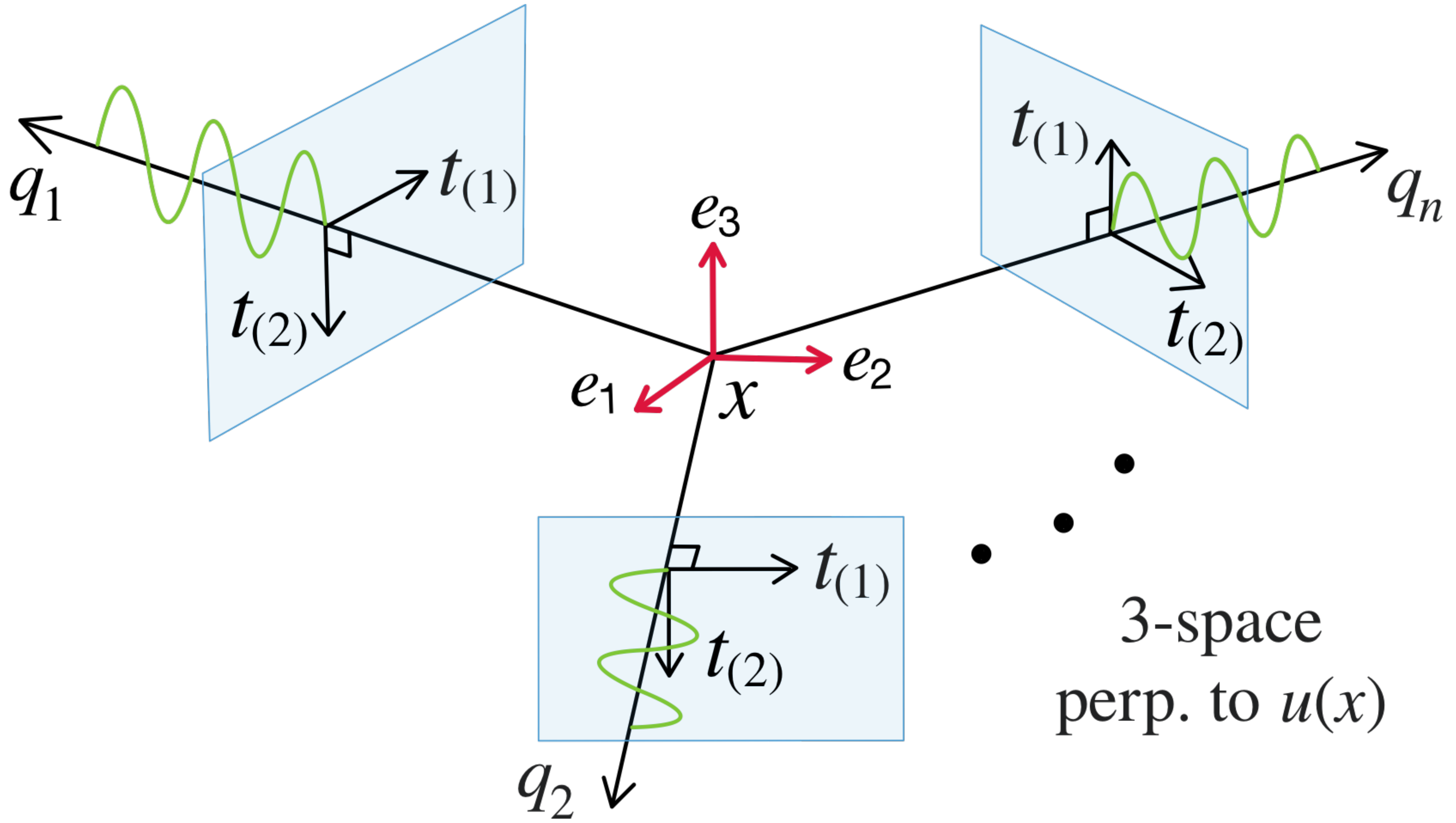
- We further introduce a local *spatial dyad* perpendicular to each $\mathbf{q}$, such that longitudinal velocity fluctuations decouple from their transverse partners.

$$\phi = \begin{pmatrix} \phi_m \\ \phi_p \\ \phi_a \end{pmatrix} = \begin{pmatrix} \delta m \\ \delta p \\ \delta u_a \end{pmatrix} \longrightarrow \Phi = \begin{pmatrix} \Phi_m \\ \Phi_\pm \\ \Phi_{(i)} \end{pmatrix} = \begin{pmatrix} \delta m \\ \delta p \pm c_s w \hat{\mathbf{q}}^a \delta u_a \\ t^a_{(i)} \delta u_a \end{pmatrix} \quad (i) = 1, 2$$

the "sound-front" basis with 5 eigenvalues

$$\lambda_\pm(\mathbf{q}) = \pm c_s |\mathbf{q}|, \; \lambda_m(\mathbf{q}) = \lambda_{(i)}(\mathbf{q}) = 0$$

$$\mathscr{L} W_{\Phi_1 \ldots \Phi_n} = \left(\sum_{i=1}^{n} \lambda_{\Phi_i}(\mathbf{q}_i) \right) W_{\Phi_1 \ldots \Phi_n} + \ldots$$



- In the "sound-front" basis RWA says

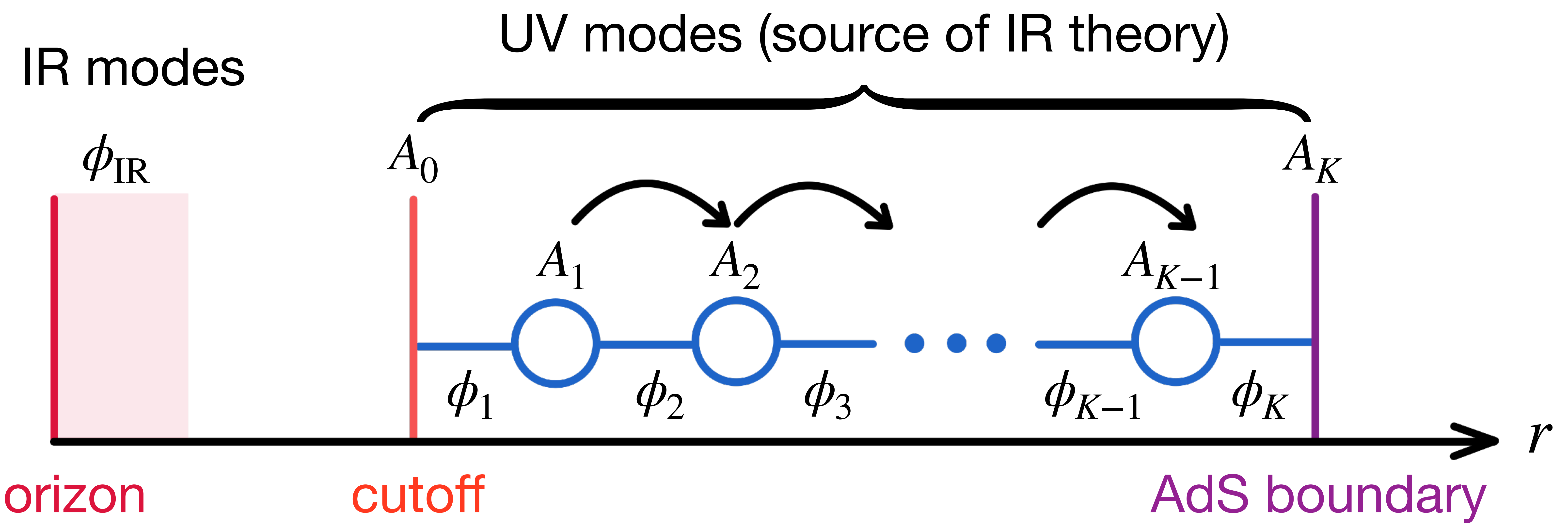
$$\text{if } \sum_{i=1}^{n} \lambda_{\Phi_i}(\mathbf{q}_i) \begin{cases} = 0 & \longrightarrow \text{ slow mode (kept)} \\ \neq 0 & \longrightarrow \text{ fast mode (averaged)} \end{cases}$$

a significant reduction of *independent* dynamic DOFs: $\mathcal{O}(10^2) \rightarrow \mathcal{O}(10)$!

Non-hydrodynamic perturbations

Non-hydro modes in holographic liquid

- Incorporate vector mesons as spontaneously broken gauge bosons of hidden local symmetry.



$$\mathscr{L}_{\rm UV} = \frac{1}{2}\int d^4x \sum_{n=0}^{K} \left(\Sigma_{t,n}^{-2} F_{n0}^2 - \Sigma_{s,n}^{-2} F_{ni}^2 + \kappa F_{i0}^2 - \kappa' F_{ij}^2 \right)$$

$$\Sigma_{t,0} = D/\sigma$$

$$\Sigma_{s,0} = \tau_R/\sigma$$

$$F_{n\mu} = \partial_\mu \phi_n - A_{n,\mu} + A_{n+1,\mu} \sim J_{n,\mu} \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Equation of motion

- EoM features a tower of conservation laws for $\mathbf{J} = (J_0^0, J_1^0, J_1^z, \ldots, J_K^0, J_K^z)$:

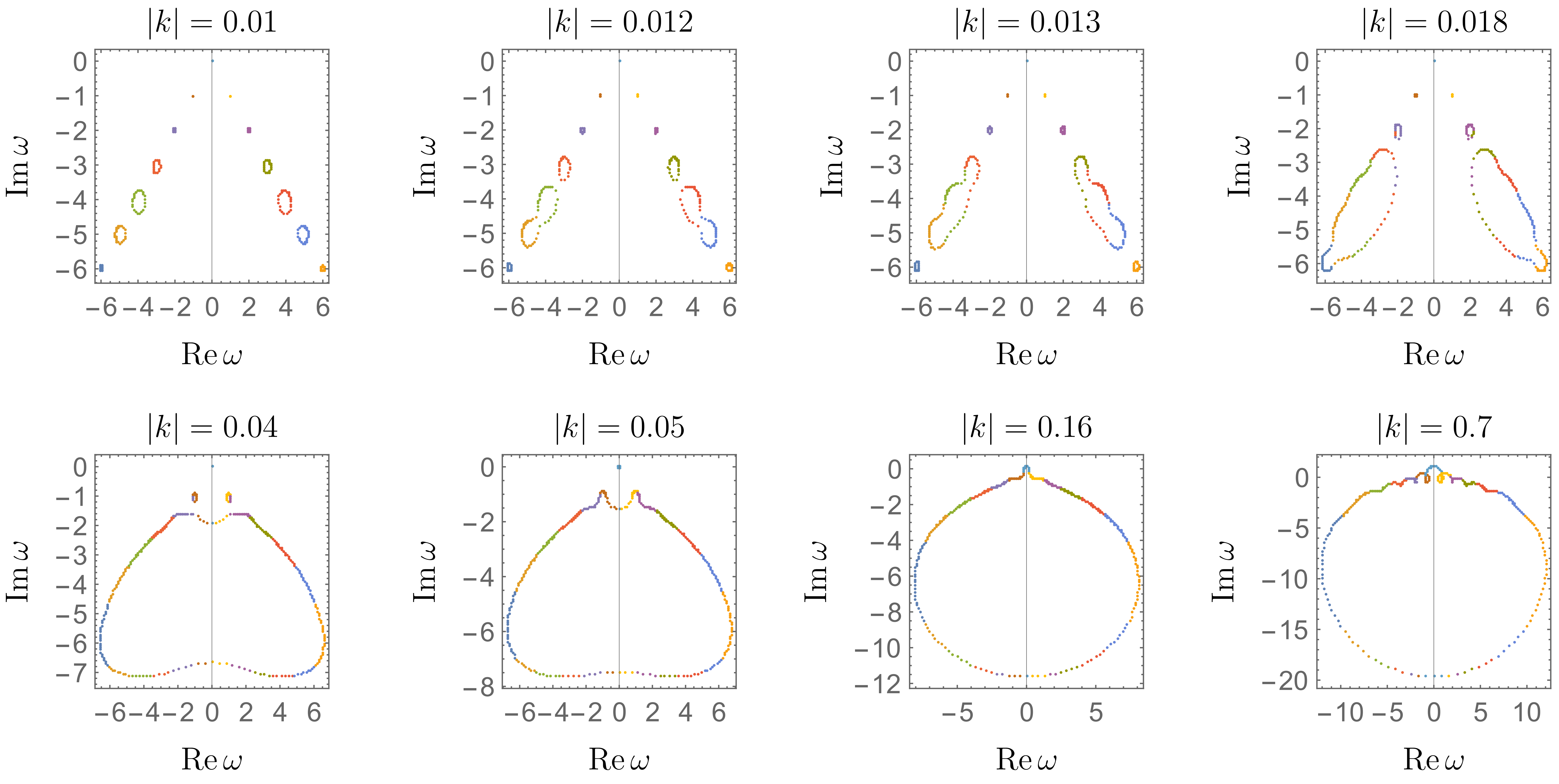
$$\partial_t \mathbf{J} = H\mathbf{J}$$

$$\partial_t \begin{pmatrix} J_0^0 \\ J_1^0 \\ J_1^z \\ J_2^0 \\ J_2^z \\ \vdots \\ J_K^0 \\ J_K^z \end{pmatrix} = \begin{pmatrix} \left(-\frac{1}{\kappa_1} + \Sigma_{t,0}k^2\right)\sigma & \frac{\sigma}{\kappa_1} & 0 & 0 & 0 & \ldots & 0 & 0 \\ 0 & 0 & -ik & 0 & 0 & \ldots & 0 & 0 \\ \frac{i}{\Sigma_{s,1}\kappa_1 k} & -\frac{i\left(\frac{1}{\kappa_1}+\frac{1}{\kappa_2}+\Sigma_{t,1}k^2\right)}{\Sigma_{s,1}k} & 0 & \frac{i}{\Sigma_{s,1}\kappa_2 k} & 0 & \ldots & 0 & 0 \\ 0 & 0 & 0 & 0 & -ik & \ldots & 0 & 0 \\ 0 & \frac{i}{\Sigma_{s,2}\kappa_2 k} & 0 & -\frac{i\left(\frac{1}{\kappa_2}+\frac{1}{\kappa_3}+\Sigma_{t,2}k^2\right)}{\Sigma_{s,2}k} & 0 & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ldots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \ldots & 0 & -ik \\ 0 & 0 & 0 & 0 & 0 & \ldots & -\frac{i\left(\frac{1}{\kappa_K}+\Sigma_{t,K}k^2\right)}{\Sigma_{s,K}k} & 0 \end{pmatrix} \begin{pmatrix} J_0^0 \\ J_1^0 \\ J_1^z \\ J_2^0 \\ J_2^z \\ \vdots \\ J_K^0 \\ J_K^z \end{pmatrix}.$$

$2K+1$ modes

Hydro and non-hydro modes

- Hydro modes do not appear alone, they interact non-hydro modes in the complex frequency plane above critical value of k_c. Hydrodynamics (attractor) works well as long as non-hydro modes can be neglected.



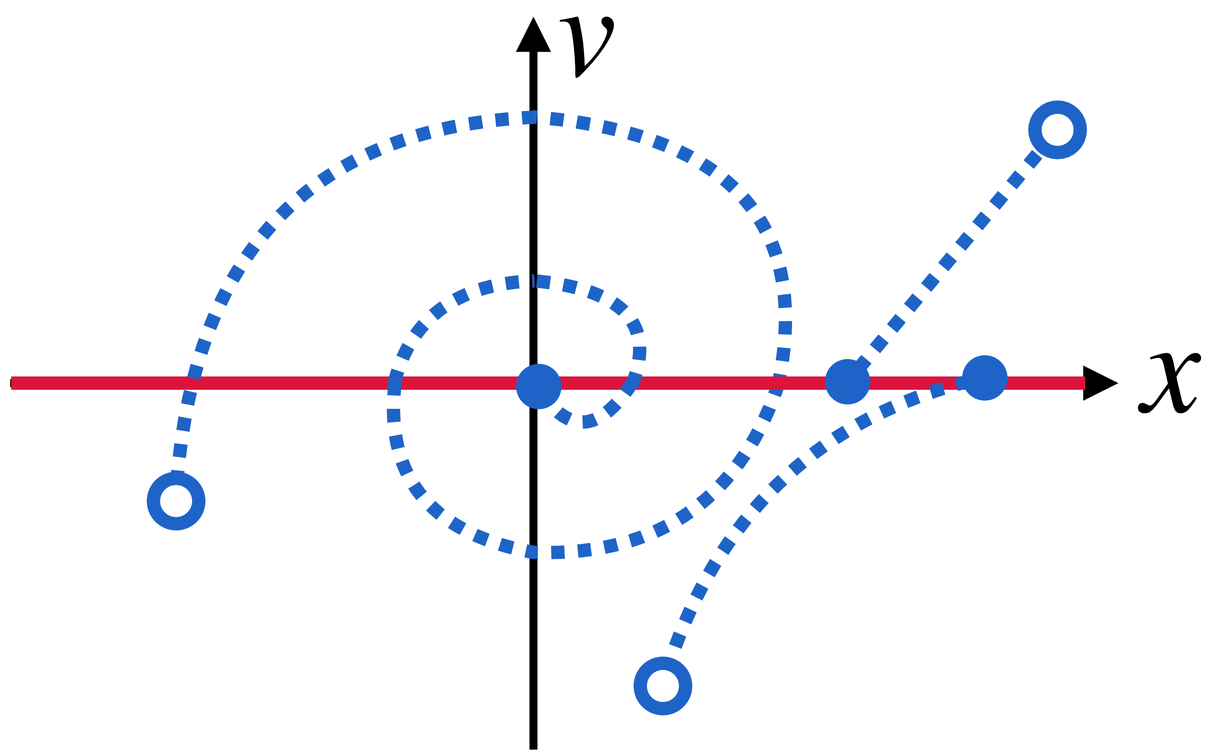
Level crossing of hydro/non-hydro modes at $K = 6$ (13 modes)

Attractor

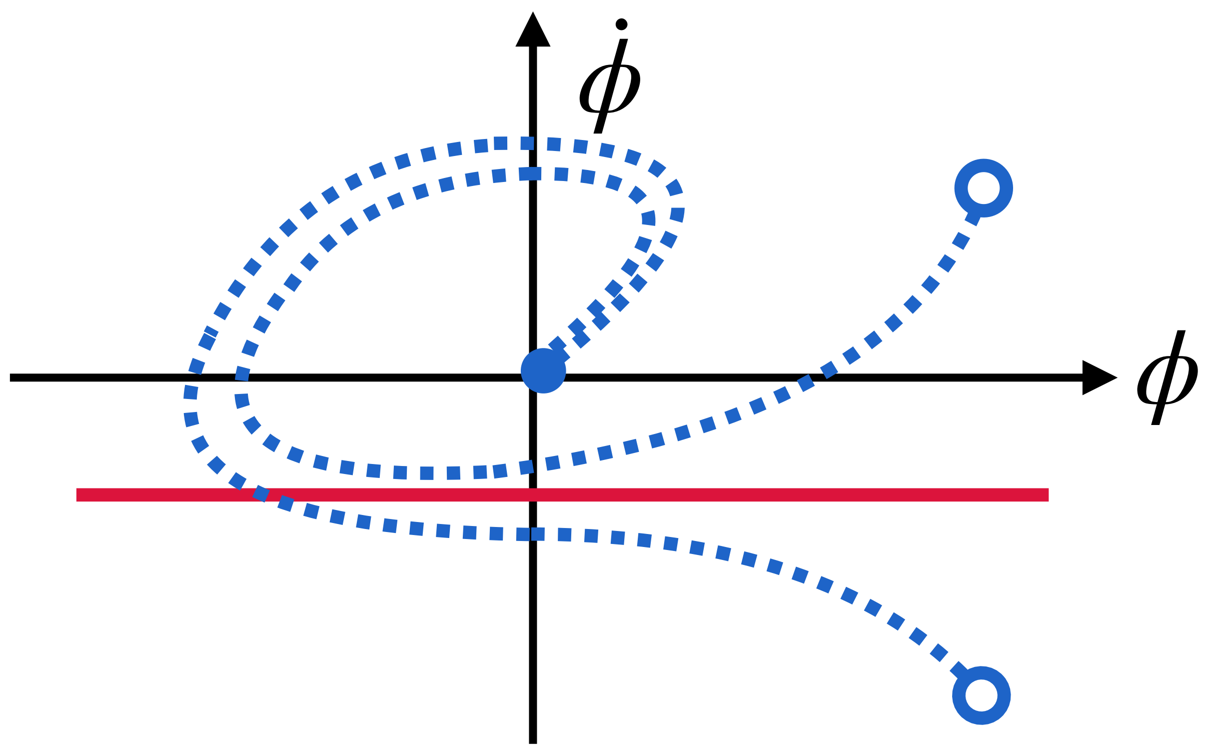
- In dynamical systems, an **attractor** is a set of states toward which a system tends to evolve, for a wide variety of initial conditions.

Examples:

1. Aristotle's law of motion, albeit wrong, implies a **dissipative attractor**.



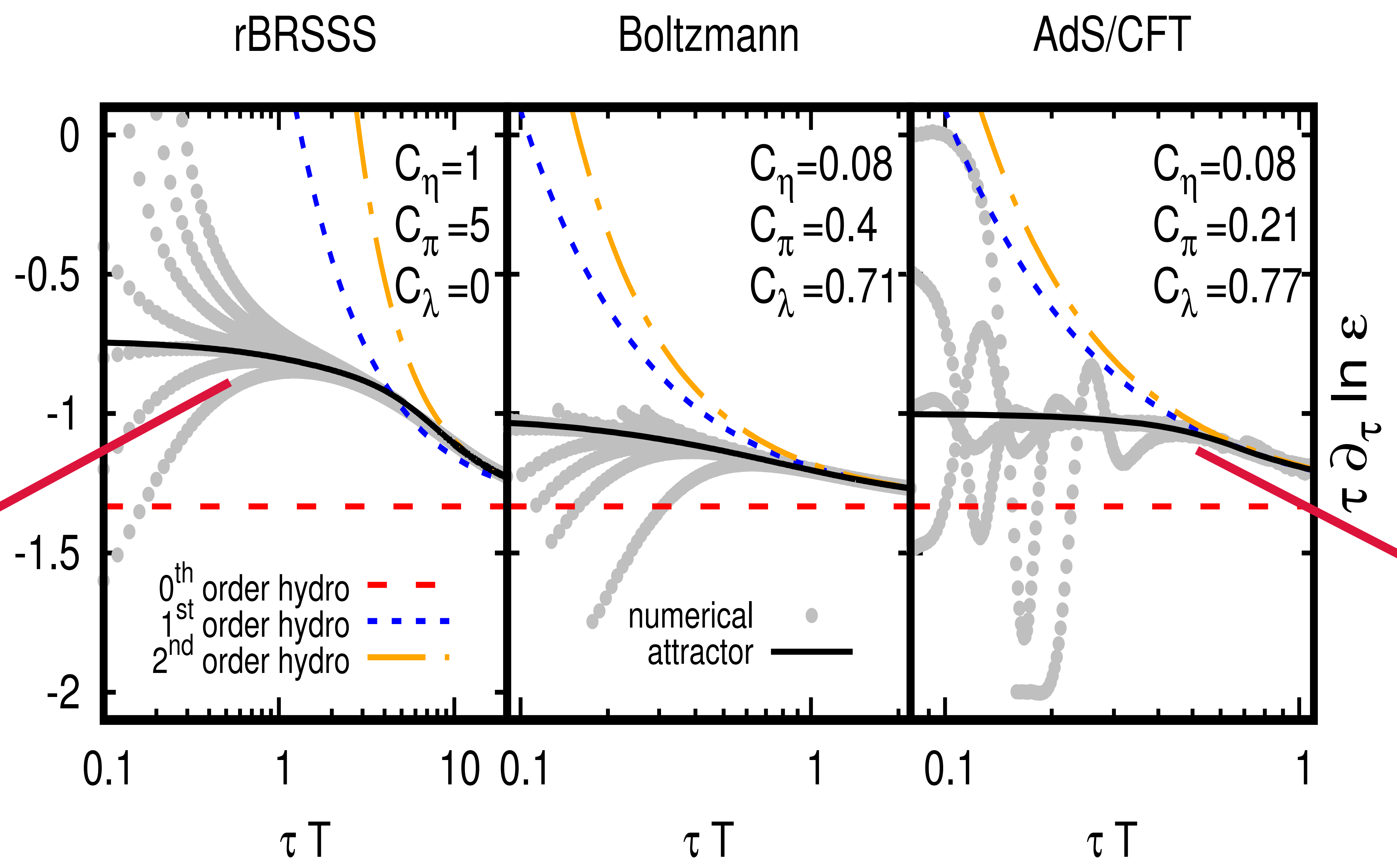
2. Inflation of the Universe at its early time implies a **slow-roll attractor**.



Hydrodynamic attractor

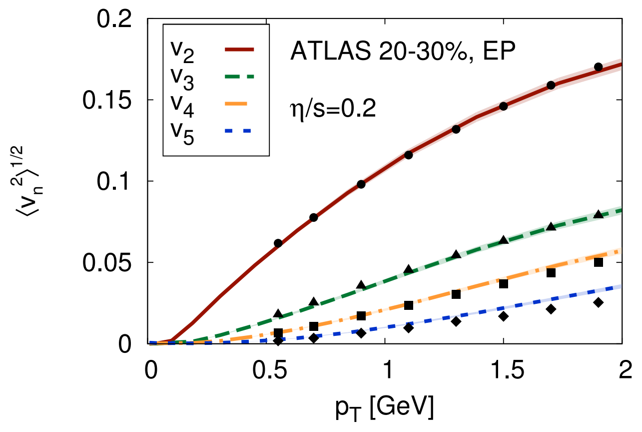
- Hydrodynamic attractor: a robust phenomenon in various models for fluids

Slow-roll attractor:
onset of
hydrodynamization at
early time

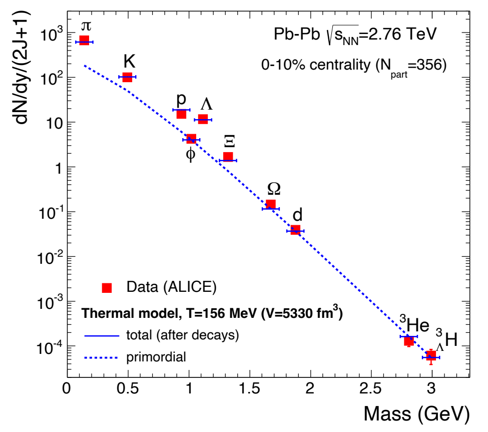


Florkowski et al, 1707.02282, Romatschke, 1712.05815

Dissipative attractor:
lost of initial information
at later time



Gale et al, 1301.5893



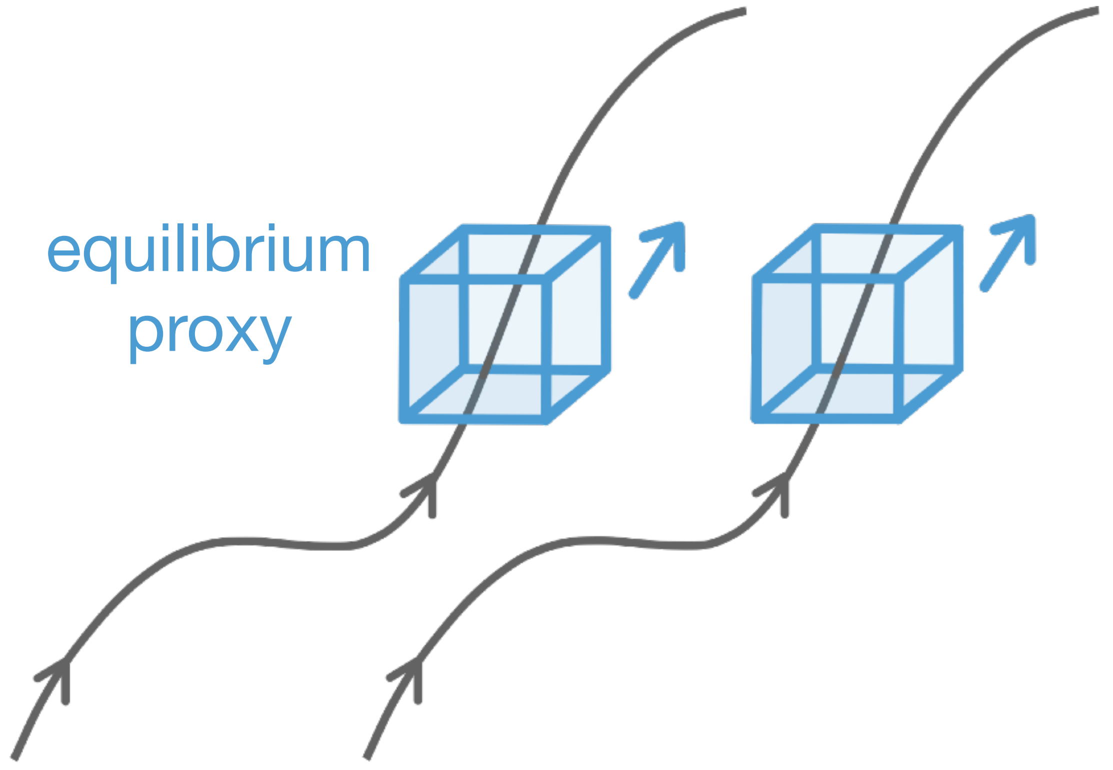
Andronic, 1407.5003

Fluid: from equilibrium to far-from-equilibrium

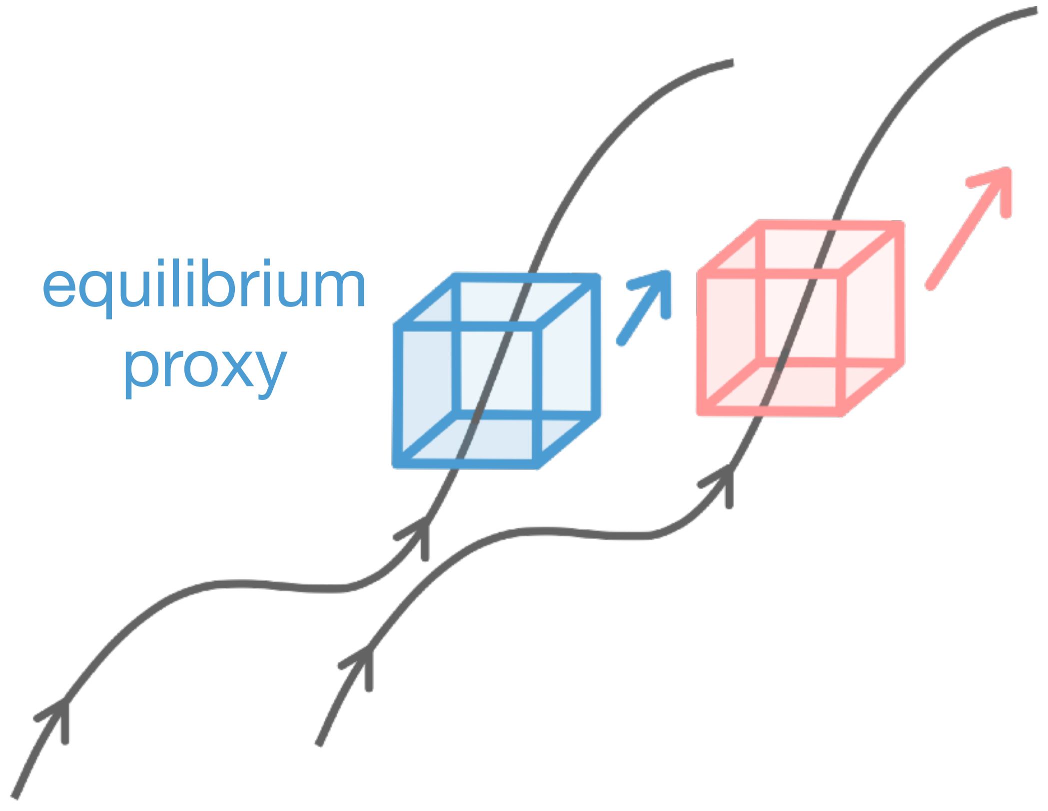
- Conservation equations

$$\partial_\nu T^{\mu\nu} = 0$$

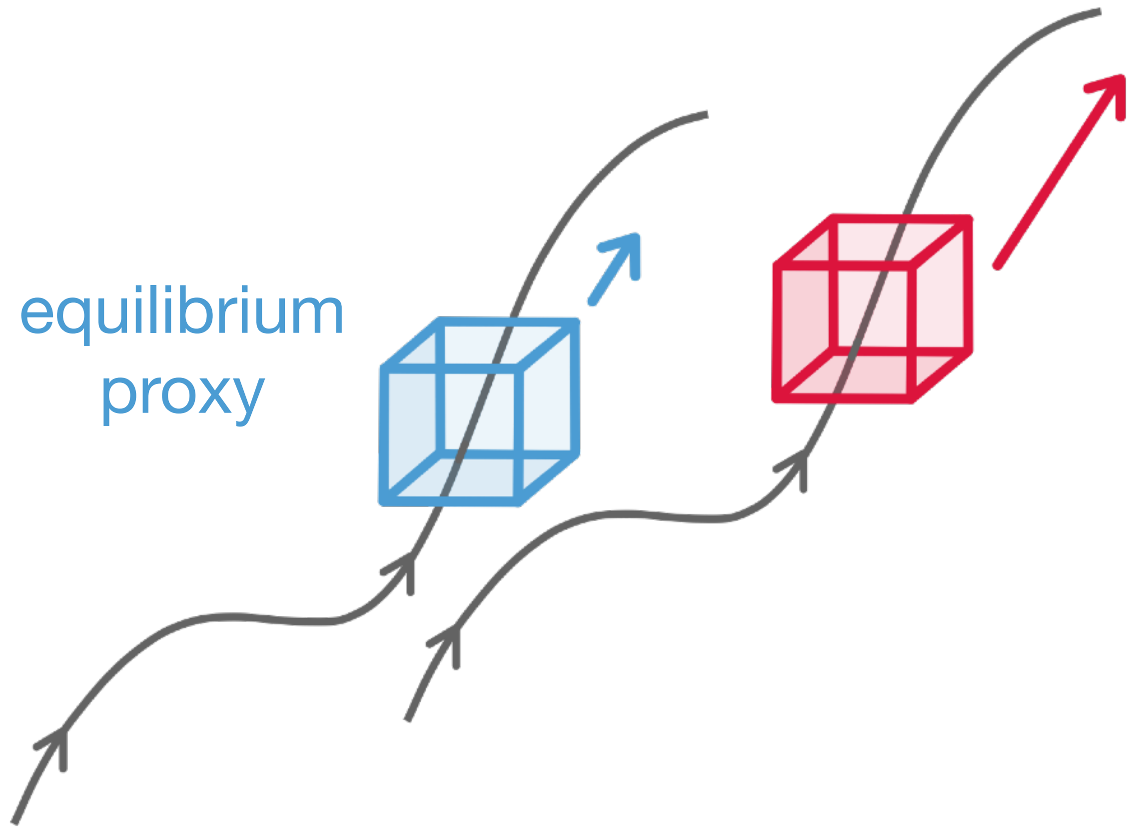
equilibrium	non-equilibrium	
$T^{\mu\nu}_{\text{eq}} = \Lambda^\mu(u)\Lambda^\nu(u)\text{diag}(\varepsilon, p, p, p)$	$T^{\mu\nu} = T^{\mu\nu}_{\text{eq}} + T^{\mu\nu}_{\text{neq}}$	
	$T^{\mu\nu}_{\text{neq}} = \partial T^{\mu\nu}_{\text{eq}} + \ldots$	$T^{\mu\nu}_{\text{neq}} = -\tau_{\text{UV}}\dot{T}^{\mu\nu}_{\text{neq}} + \partial T^{\mu\nu}_{\text{eq}} + \ldots$



in equilibrium



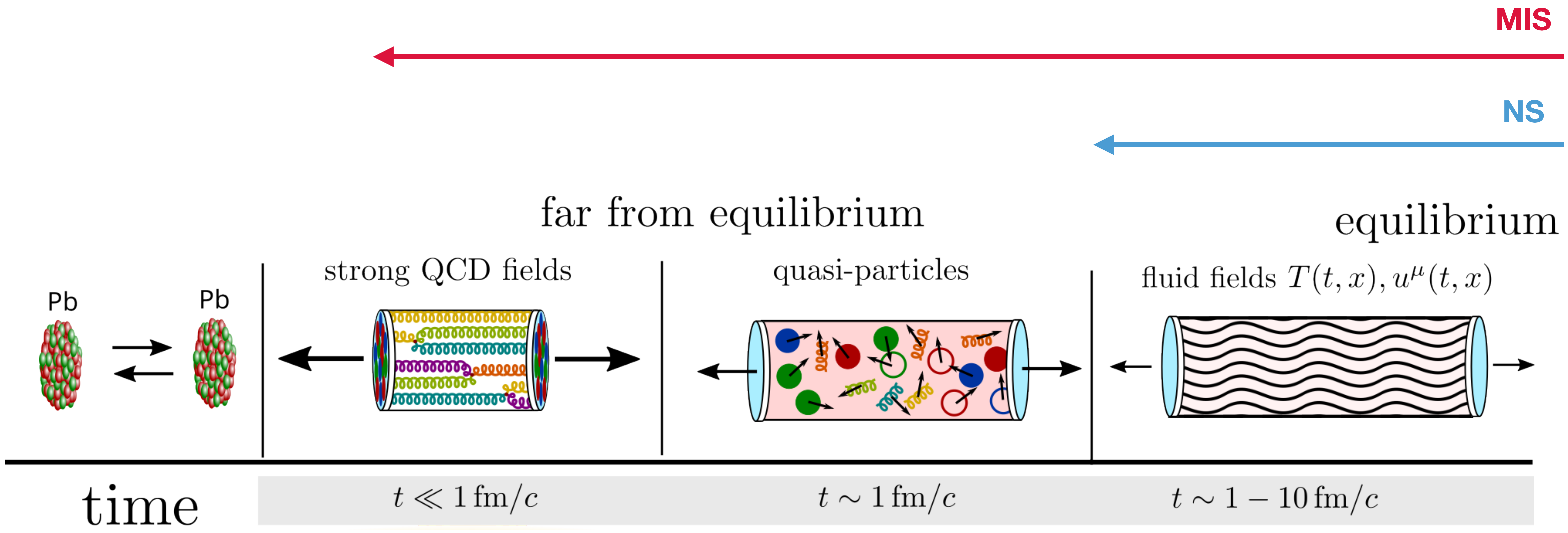
near equilibrium



far from equilibrium

From NS to MIS equations

- MIS-like theory *extend the applicability* of conventional hydrodynamics:



Courtesy of A. Mazeliauskas

The simplest MIS-like equations

- A simplest scenario: *0+1D conformal boost-invariant (Bjorken)* fluids.
  - τ (time) dependence only
  - T measures the effective energy scale: $\varepsilon = 3p = C_e T^4, \quad \eta = \frac{4}{3} C_e C_\eta T^3, \quad \tau_{\text{UV}} = C_\tau T^{-1}$
  - A measures the anisotropy (how far the system is from equilibrium):

$$T^{\mu\nu} = \begin{pmatrix} \varepsilon & & & \\ & p(1+\frac{A}{3}) & & \\ & & p(1+\frac{A}{3}) & \\ & & & p(1-\frac{2A}{3}) \end{pmatrix} \xrightarrow{\text{equilibration}} T^{\mu\nu}_{\text{eq}} = \begin{pmatrix} \varepsilon & & & \\ & p & & \\ & & p & \\ & & & p \end{pmatrix}$$

- The EOM for the dynamic system $\Psi = (T(\tau), A(\tau))$: Blaizot et al, 2106.10508

$$\tau \partial_\tau \Psi(\tau) = -M(\tau)\Psi(\tau) + V \qquad M(\tau) = \begin{pmatrix} 1/3 & -T(\tau)/18 \\ \tau A(\tau)/C_\tau & 2A(\tau)/9 \end{pmatrix} \qquad V = \begin{pmatrix} 0 \\ 8C_\eta/C_\tau \end{pmatrix}$$

solutions?

Asymptotic solutions

- Early-time *attractor* solutions:

XA et al, 2312.17237

$$T(\tau) \sim \mu(\mu\tau)^{-\frac{1-\alpha}{3}}(1+\ldots), \qquad A(\tau) \sim 6\alpha(1+\ldots) \qquad \alpha = \sqrt{C_\eta/C_\tau}$$

slow-roll attractor

μ: integration constant parametrizing attractor

- Later-time asymptotic solutions

$$T(\tau) \sim \Lambda(\Lambda\tau)^{-\frac{1}{3}}(1+\ldots) + C_\infty e^{-\frac{3}{2C_\tau}(\Lambda\tau)^{2/3}}(\Lambda\tau)^{-\frac{2}{3}(1-\alpha^2)}(1+\ldots)$$

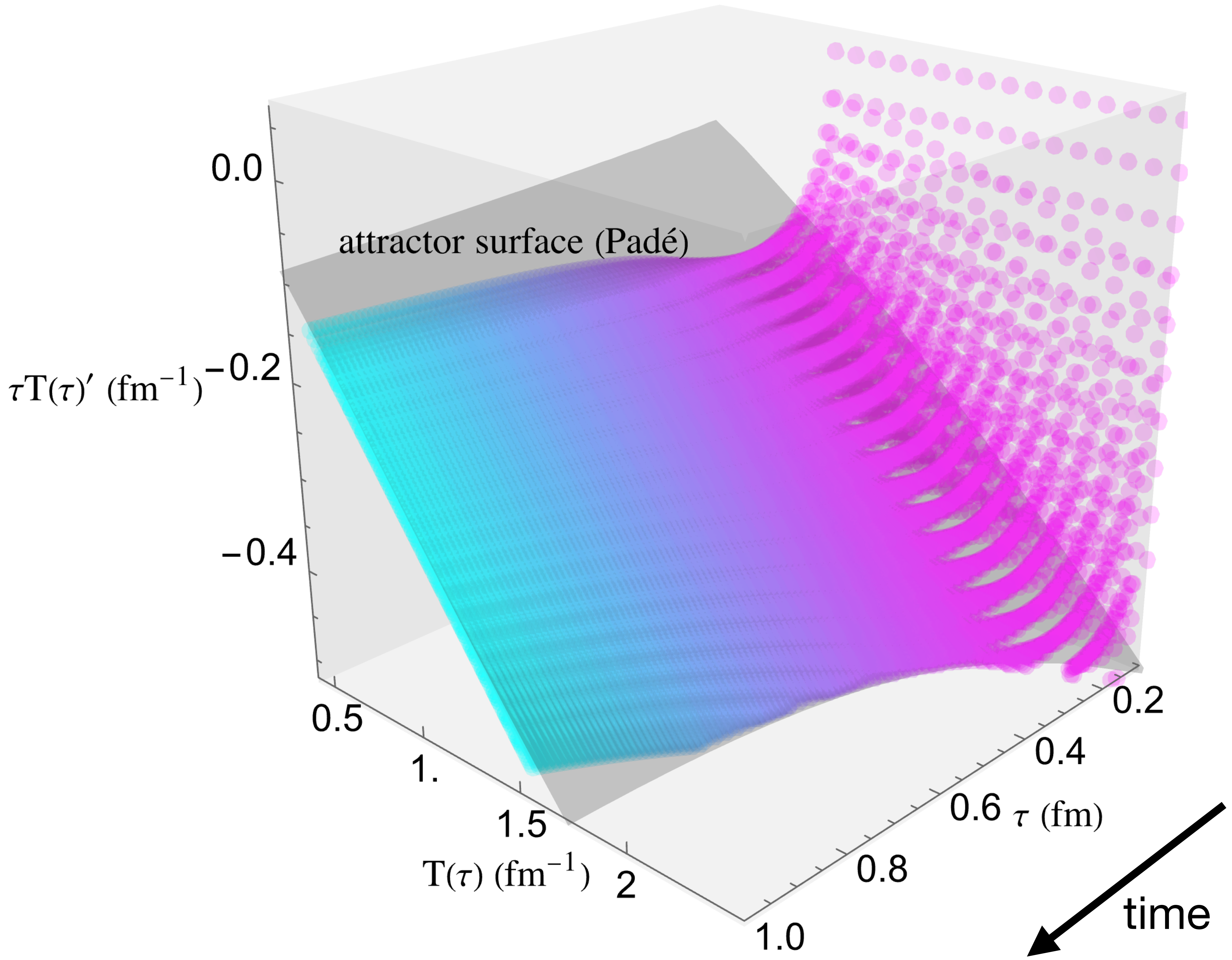
$$A(\tau) \sim 8C_\eta(\Lambda\tau)^{-\frac{2}{3}}(1+\ldots) + C'_\infty e^{-\frac{3}{2C_\tau}(\Lambda\tau)^{2/3}}(\Lambda\tau)^{-\frac{1}{3}+\alpha^2}(1+\ldots)$$

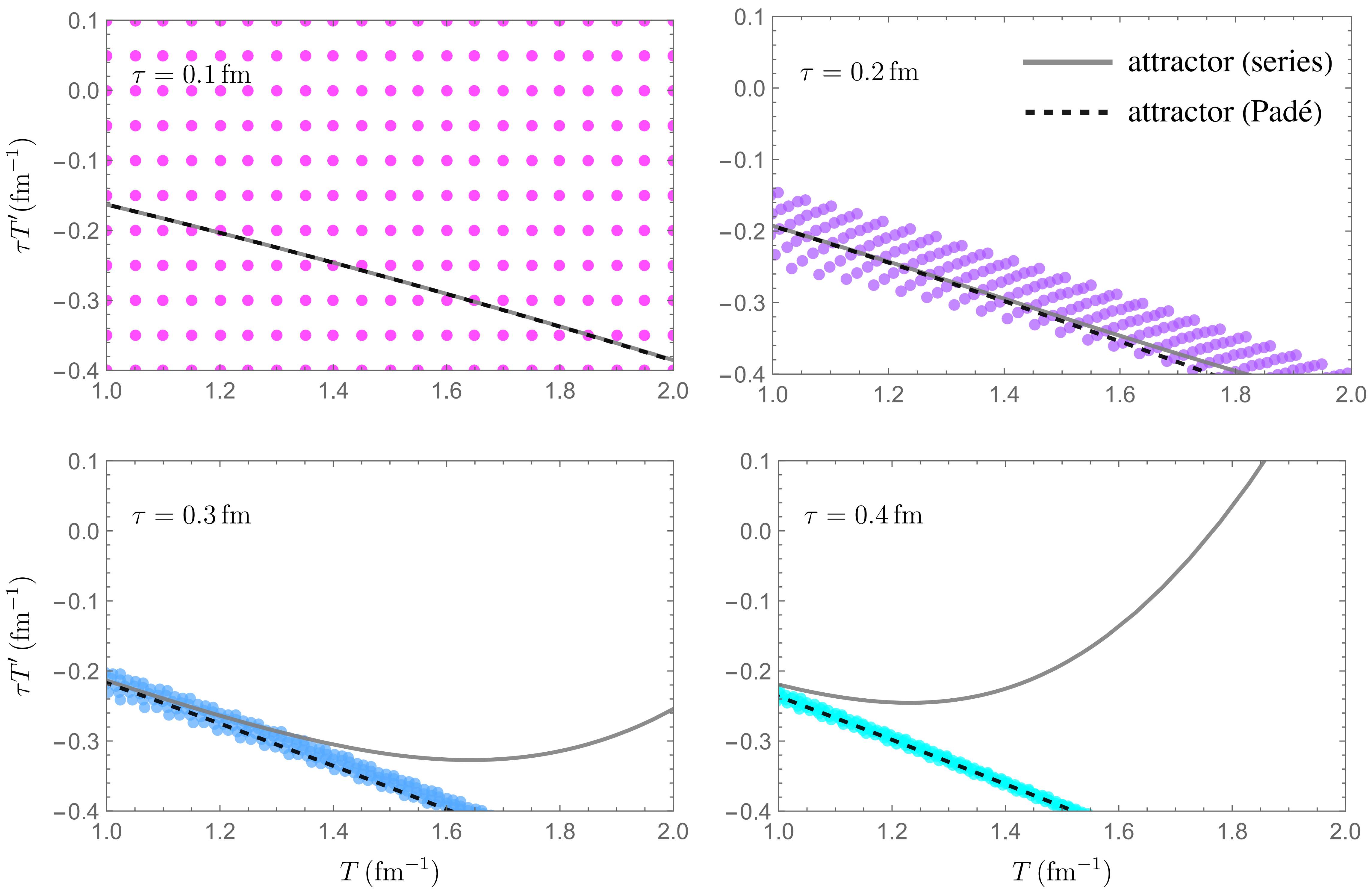
dissipative (hydrodynamic) attractor + transseries (non-hydrodynamic) modes

Λ, C_∞: integration constant

Early-time attractor in phase space

- Trajectories in phase space rapidly approach the early-time *attractor surface*.





snapshot of $(\tau T', T)$ plane at different τ

"Too simple to be true"

- 0+1D Bjorken model is highly idealized.
  - Does attractor exist in more complicated scenarios?
  - Will attractor wash out mostly everything?
  - What observables **can/cannot** be predicted from 0+1D Bjorken model?

✓

Multiplicity of hadrons

Thermal photon/dilepton spectrum

…

✗

Collective flow

Jet

…

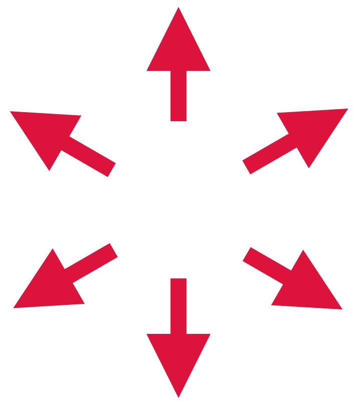
Linear perturbations

- The existence of attractor naturally allows one to linearization the full system around it:

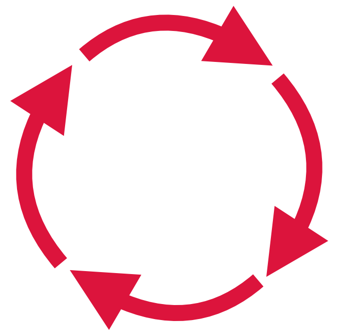
$$\partial_\nu T^{\mu\nu} = \partial_\nu (T^{\mu\nu}_{\text{attractor}} + \delta T^{\mu\nu}) = 0 \quad \longrightarrow \quad \begin{cases} \partial_\nu T^{\mu\nu}_{\text{attractor}} = 0, \\ \partial_\nu \delta T^{\mu\nu} = 0. \end{cases}$$

6 independent fields: $\phi = (\delta T, \delta\theta, \delta\omega, \delta\pi_{11}, \delta\pi_{22}, \delta\pi_{12})(\tau, \mathbf{x})$ $\quad i = 1{,}2$

fluid divergence $\delta\theta \equiv \partial_i \delta u_i$



vorticity $\delta\omega \equiv \epsilon_{ij}\partial_i \delta u_j$



shear stress tensor $\delta\pi_{ij}$

- The EOM for the dynamic system:

$$\partial_\tau \hat{\phi}_i(\tau, \mathbf{k}) = M_{ij}(\tau, \mathbf{k})\, \hat{\phi}_j(\tau, \mathbf{k})$$

solutions?

Asymptotic solutions at late time

- When $\tau \to \infty, k \neq 0$, solutions perturbed around attractor are transseries:

$$\delta\hat{T}(\mathbf{k}) \sim C_i\, e^{-S_i \tau^{b_i} + \dots}\, \tau^{a_i} (1 + \dots) \qquad i = 1,2,3,4$$

$$\delta\hat{\omega}(\mathbf{k}) \sim C_i\, e^{-S_i \tau^{b_i} + \dots}\, \tau^{a_i} (1 + \dots) \qquad i = 5,6$$

$C_1, \dots, C_6$: k-dependent integration constants

$\delta\hat{\theta}$ and $\delta\hat{\pi}_{ij}$ are determined independently

Attractor is asymptotically **stable** ($\mathrm{Re}\, \mathrm{S_i} > 0$) against transverse perturbations.

Non-hydrodynamic content is important at asymptotic later time.

Zero wavenumber modes

- When $\tau \to \infty$, $k = 0$ modes need to be considered separately:

$$\delta u_i \sim C_i \tau^{1/3}(1 + \ldots) \qquad i = 1,2$$

→ mild growth due to momentum conservation

$$\delta \hat{T} \sim C_3 \,(1 + \ldots) + C_4 \, e^{-\frac{3}{2C_\tau}\tau^{2/3}} \tau^{-\frac{2}{3}(1-\alpha^2)}(1 + \ldots)$$

$$\delta\hat{\pi}_{11} - \delta\hat{\pi}_{22} \sim C_5 \, e^{-\frac{3}{2C_\tau}\tau^{2/3}} \tau^{\frac{2}{3}\alpha^2}(1 + \ldots)$$

$$\delta\hat{\pi}_{12} \sim C_6 \, e^{-\frac{3}{2C_\tau}\tau^{2/3}} \tau^{\frac{2}{3}\alpha^2}(1 + \ldots)$$

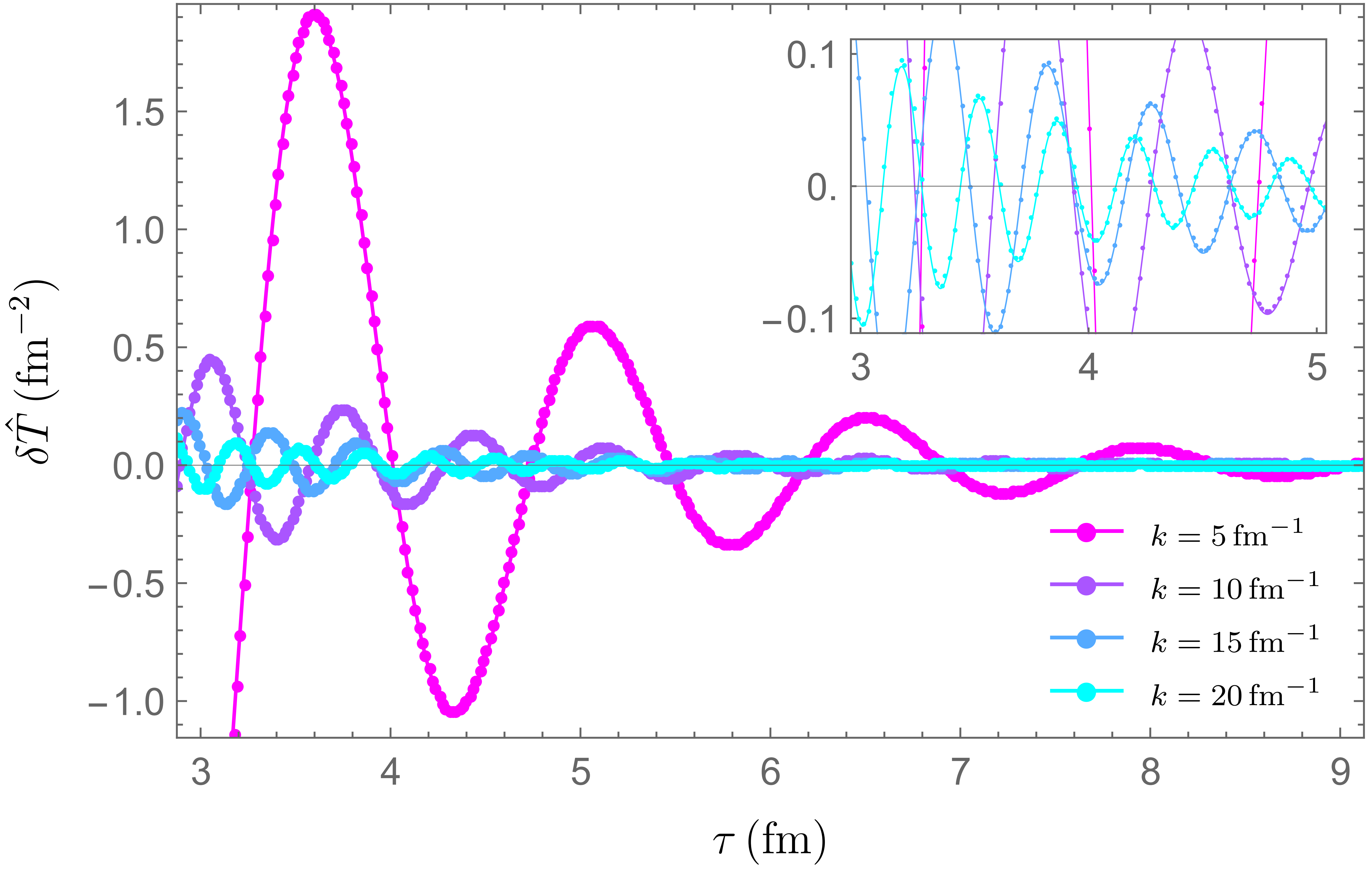
→ reproduces to background transseries solution

Observables are extracted from a **finite** set of asymptotic data $C_n(\mathbf{k})$.

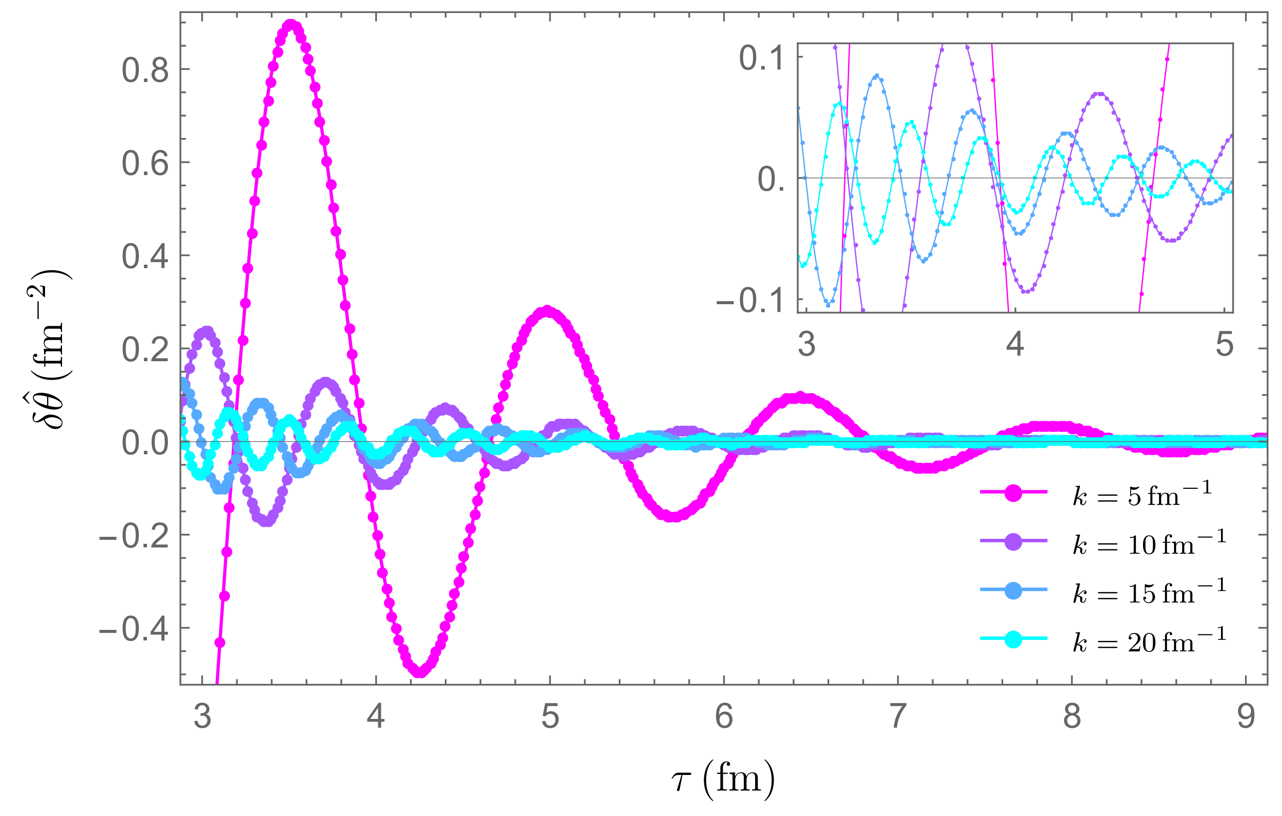
# of data: $6 \times N_k$

Matching to numerics

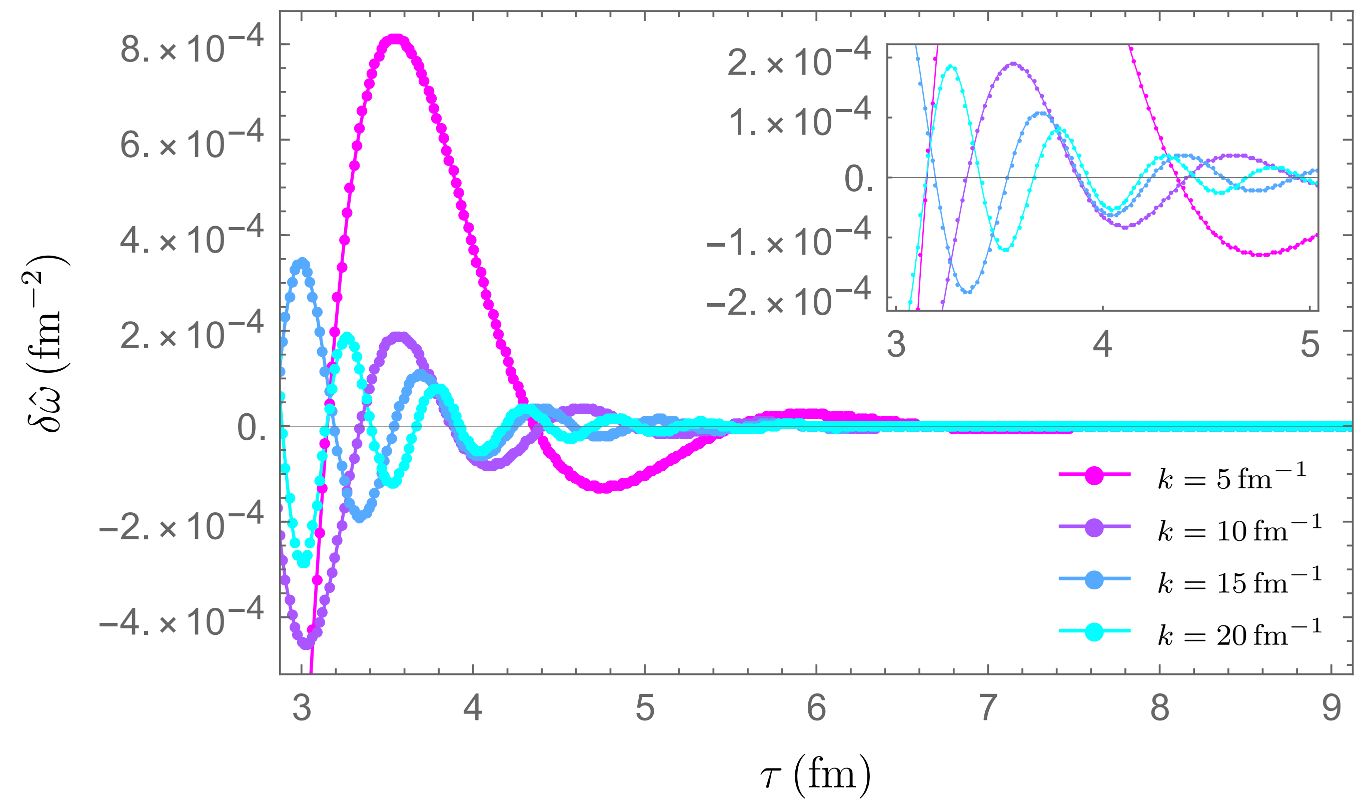
- The analytic solutions (solid curves) fit the numerics (discrete points) in a *wide range of time*.



temperature



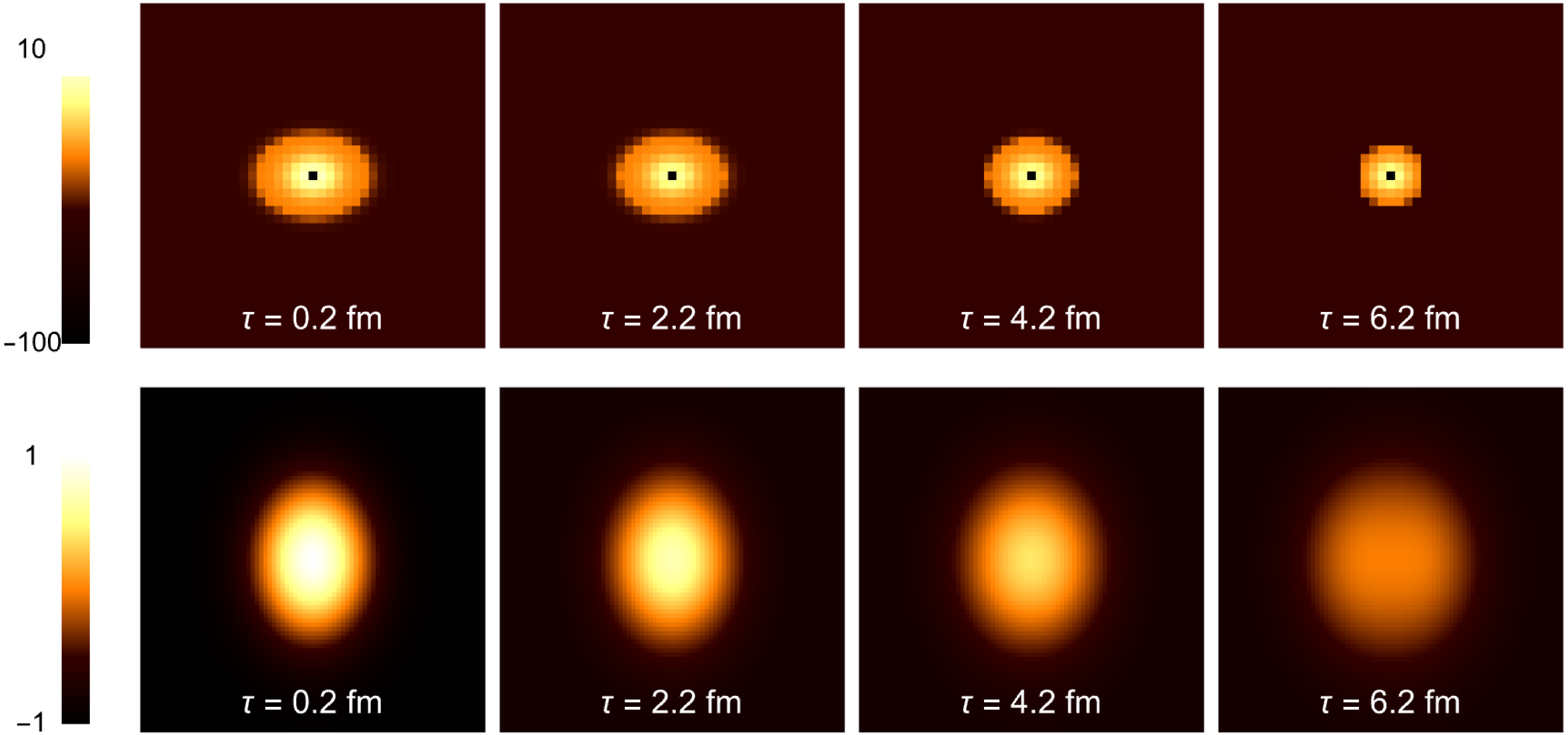
fluid divergence



vorticity

Transverse tomography

- Transverse information is encoded in a finite set of Fourier modes via FFT.



Evolution of temperature (energy density) in transverse spaces

Collectivity: analytic results

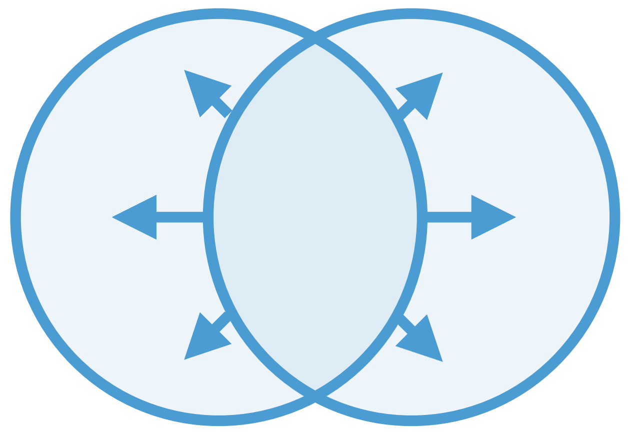
- Cooper-Frye freezeout

pressure anisotropy

$$\frac{dN(p_\perp,\phi)}{p_\perp dp_\perp d\phi dy} = \frac{m_\perp \tau \Sigma}{(2\pi)^3}\left\{\underbrace{2K_1(\hat{m}_\perp) + \frac{1}{12}\left[\hat{p}_\perp^2 K_1(\hat{m}_\perp) - 2\hat{m}_\perp K_2(\hat{m}_\perp)\right] A}_{F_0} + \text{perturbations}\right\}$$

- Collective expansion

$$\frac{dN(p_\perp,\phi)}{p_\perp dp_\perp d\phi dy} = v_0(p_\perp)\left(1 + \sum_{n=1}^{\infty} 2v_n(p_\perp)\cos(n\phi)\right)$$



$$v_0(\hat{p}_\perp) \sim \frac{m_\perp \tau_f \Sigma}{(2\pi)^3}\left(F_0 + \text{ perturbations}\right) \qquad v_1(\hat{p}_\perp), v_2(\hat{p}_\perp) \sim \frac{\text{perturbations}}{4F_0 + \text{ perturbations}}$$

Jet-medium interaction

- The total energy of jet and fluid system is conserved:

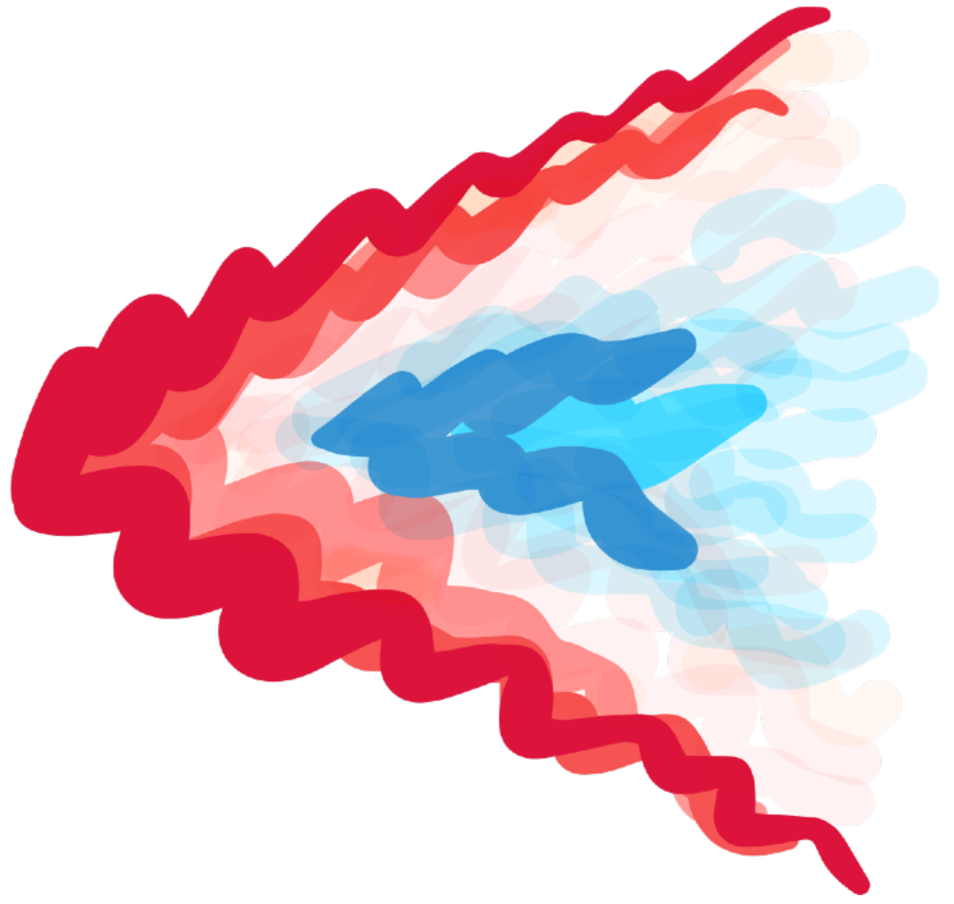
$$\partial_\nu T^{\mu\nu} = \partial_\nu \left(T^{\mu\nu}_{\text{attractor}} + \delta T^{\mu\nu} + T^{\mu\nu}_{\text{jet}} \right) = 0$$

- Attractor provides a background for the jet-medium interactions:

$$\begin{cases} \partial_\nu T^{\mu\nu}_{\text{attractor}} = 0, \\ \partial_\nu \delta T^{\mu\nu} = -\,\partial_\nu T^{\mu\nu}_{\text{jet}} = J^\mu\,. \end{cases}$$

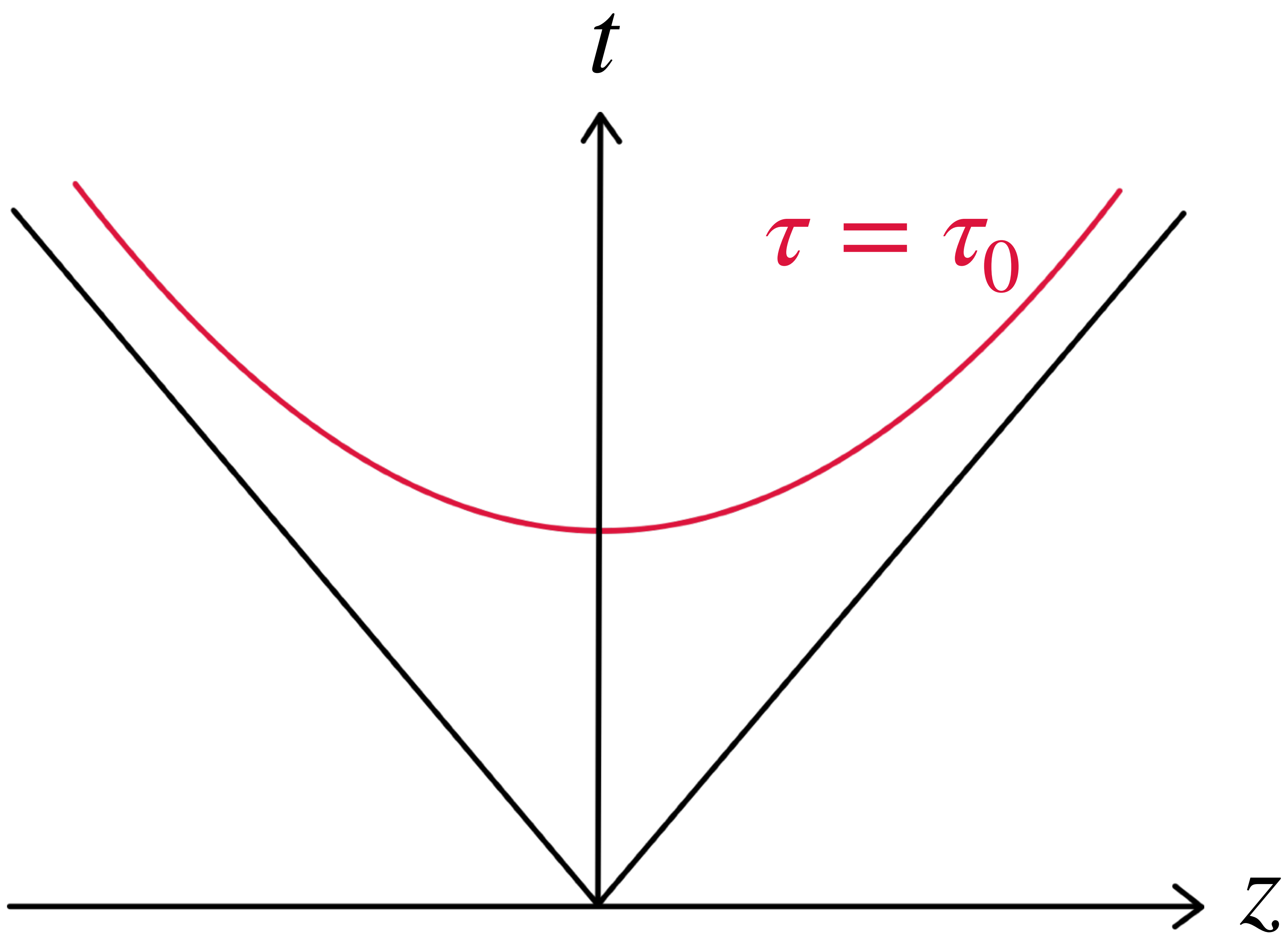
jet source

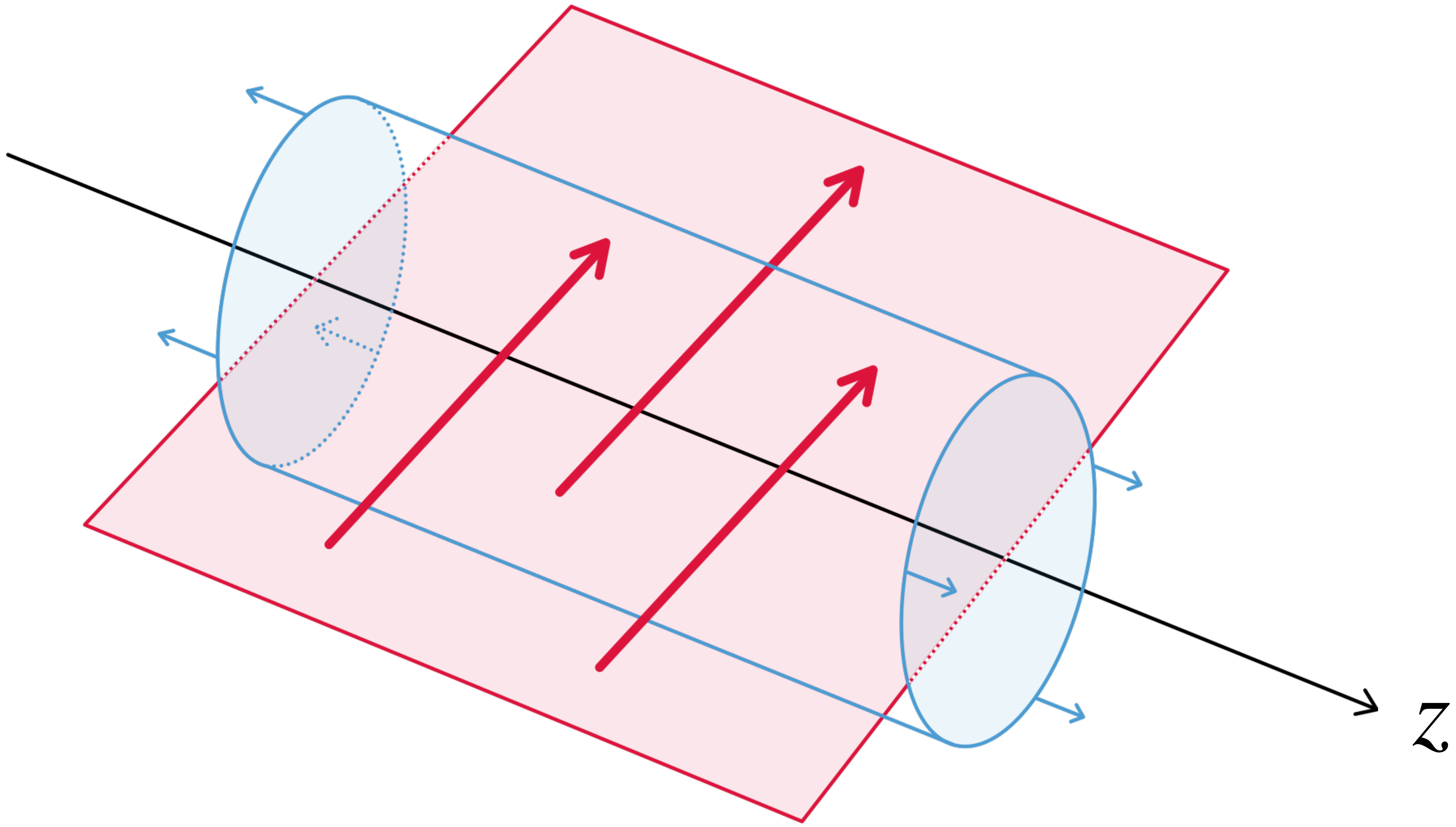
Chaudhuri et al, 0503028
Casalderrey-Solana et al, 0602183
Chesler et al, 0712.0050
Neufeld et al, 0802.2254
Qin et al, 0903.2255
Yan et al, 1707.09519
Casalderrey-Solana et al, 2010.01140
…



Boost-invariant jet

- A knife-shape jet resulted from boost-invariant assumption, which
  - captures main effects qualitatively;
  - corresponds to the longest wavelength modes along rapidity that are mostly relevant.





Jet source

- Jet source current:

$$J^\mu = f^\mu(t)\, n_{\rm jet}(t, \mathbf{x})$$

effective drag force $f^\mu(t) = \left(\frac{dE}{d\tau}, \frac{d\mathbf{P}}{d\tau}\right) = \frac{dE}{dt} u_s^\mu$

spatial distribution of source $n_{\rm jet}(t, \mathbf{x}) = (\tau\gamma_s)^{-1} n^\perp(t, \mathbf{x})$

The transverse distribution $n^\perp(t, \mathbf{x})$ and parton trajectory $\mathbf{x}_s(\tau)$ is arbitrary, e.g.,

Point-like distribution:

$$n^\perp(t, \mathbf{x}) \sim \delta^{(2)}(\mathbf{x} - \mathbf{x}_s(\tau))$$

Gaussian distribution:

$$n^\perp(t, \mathbf{x}) \sim e^{-(\mathbf{x}-\mathbf{x}_s(\tau))^2/2\sigma^2}$$

straight-line trajectory: $\mathbf{x}_s(\tau) = (\mathbf{x}_0 + \mathbf{v}_s(\tau - \tau_0))\Theta(\tau - \tau_0)$

Energy loss

- We assume the BBMG energy loss formalism

$$\frac{dE}{d\tau} = \kappa \left(\frac{E}{T}\right)^a (\tau T)^z T^2$$

κ: jet-medium coupling
a: jet-energy dependence
z: path-length dependence

model	(a, z)	**applicable regime**
Bethe-Heitler limit	(1,0)	additive single scattering
$N = 4$ SYM	(0,0)	pQCD elastic, non-relativistic heavy quark
LPM factorization limit	(0,1)	pQCD radiative, weakly coupled
AdS/CFT	(0,2)	light quark, strongly coupled

- Energy loss formula may fall into BBMG classification in certain limit, e.g.,

$$\frac{dE}{d\tau} = \frac{4E_{\rm in}\tau^2}{\pi \ell_{\rm stop}^2 \sqrt{\ell_{\rm stop}^2 - \tau^2}} \sim (\tau T)^2 T^2 \longrightarrow (0,2) \text{ class}$$

Chesler et al, 1402.6756

$\ell_{\rm stop} \gg \tau, R$
(energetic partons / small systems)

Asymptotic jet solutions at late time

- Inhomogeneous EOM

$$\partial_\tau \hat{\phi}_i(\tau, \mathbf{k}) = M_{ij}\hat{\phi}_j(\tau, \mathbf{k}) + J_i(\tau, \mathbf{k})$$

- The late-time asymptotic solutions can be found by Wronskian:

$$\delta\hat{\phi}(\tau, \mathbf{k}) = \sum_i C_i(k)\,\delta\hat{\phi}_i(\tau, \mathbf{k}) + \delta\hat{\phi}_p(\tau, \mathbf{k})$$

- The particular solutions have the universal *power-law* behavior, e.g.,

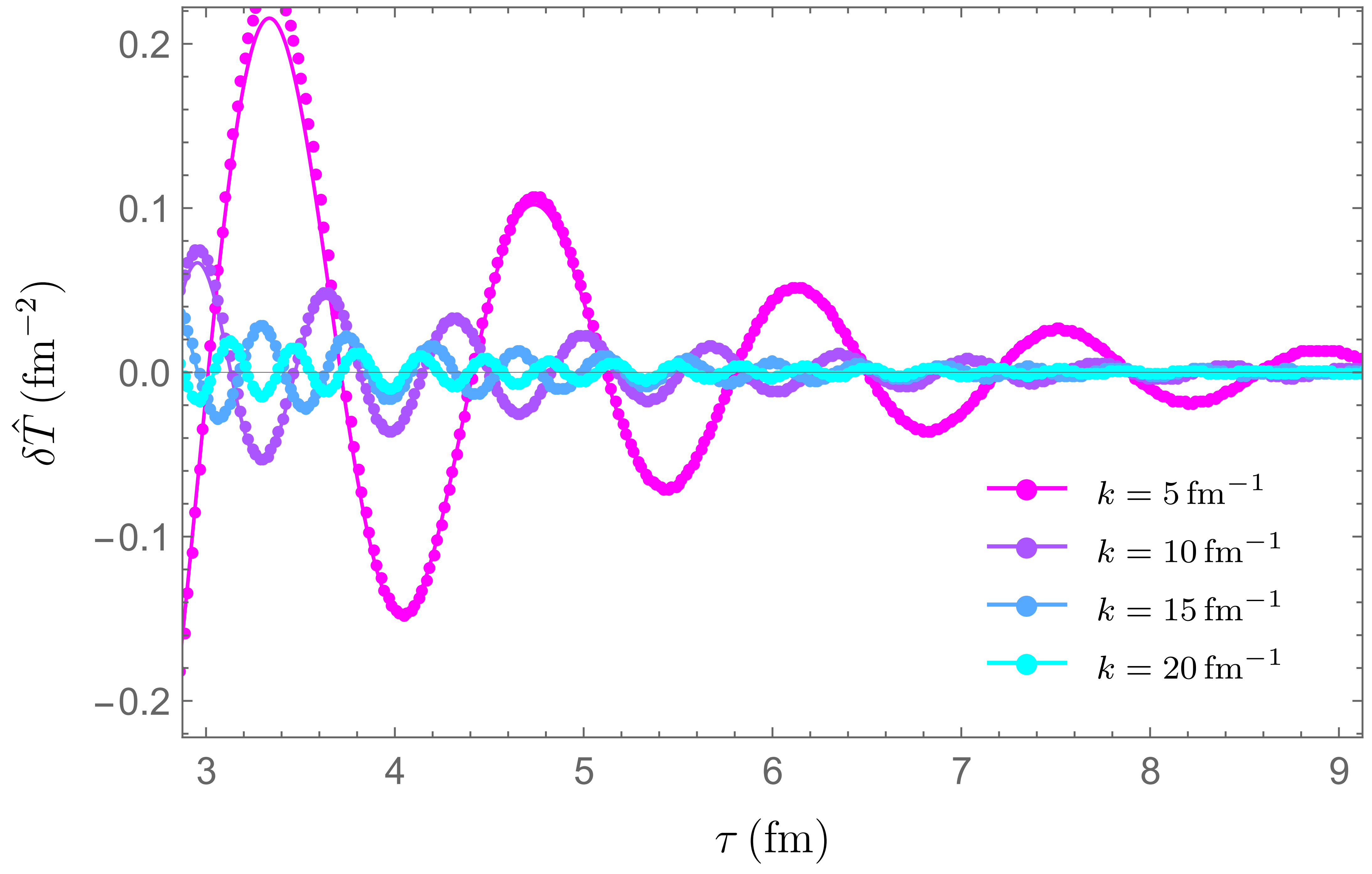
$$\delta\hat{T}_p(\tau, \mathbf{k}) \sim \frac{i\,n^{\perp}(\mathbf{k})\,e^{-i\mathbf{k}\cdot\mathbf{x}_s(\tau)}(2C_\eta - 3C_\tau(\hat{\mathbf{k}}\cdot\mathbf{v}_s)^2)}{2k\Lambda\hat{\mathbf{k}}\cdot\mathbf{v}_s\left(4C_\eta + C_\tau\left(1 - 3(\hat{\mathbf{k}}\cdot\mathbf{v}_s)^2\right)\right)}(\Lambda\tau)^{\frac{2z-1}{3}}\left(1 + \mathcal{O}(\tau^{-1/3})\right)$$

Poles at c_∞ (effective MIS speed of sound)

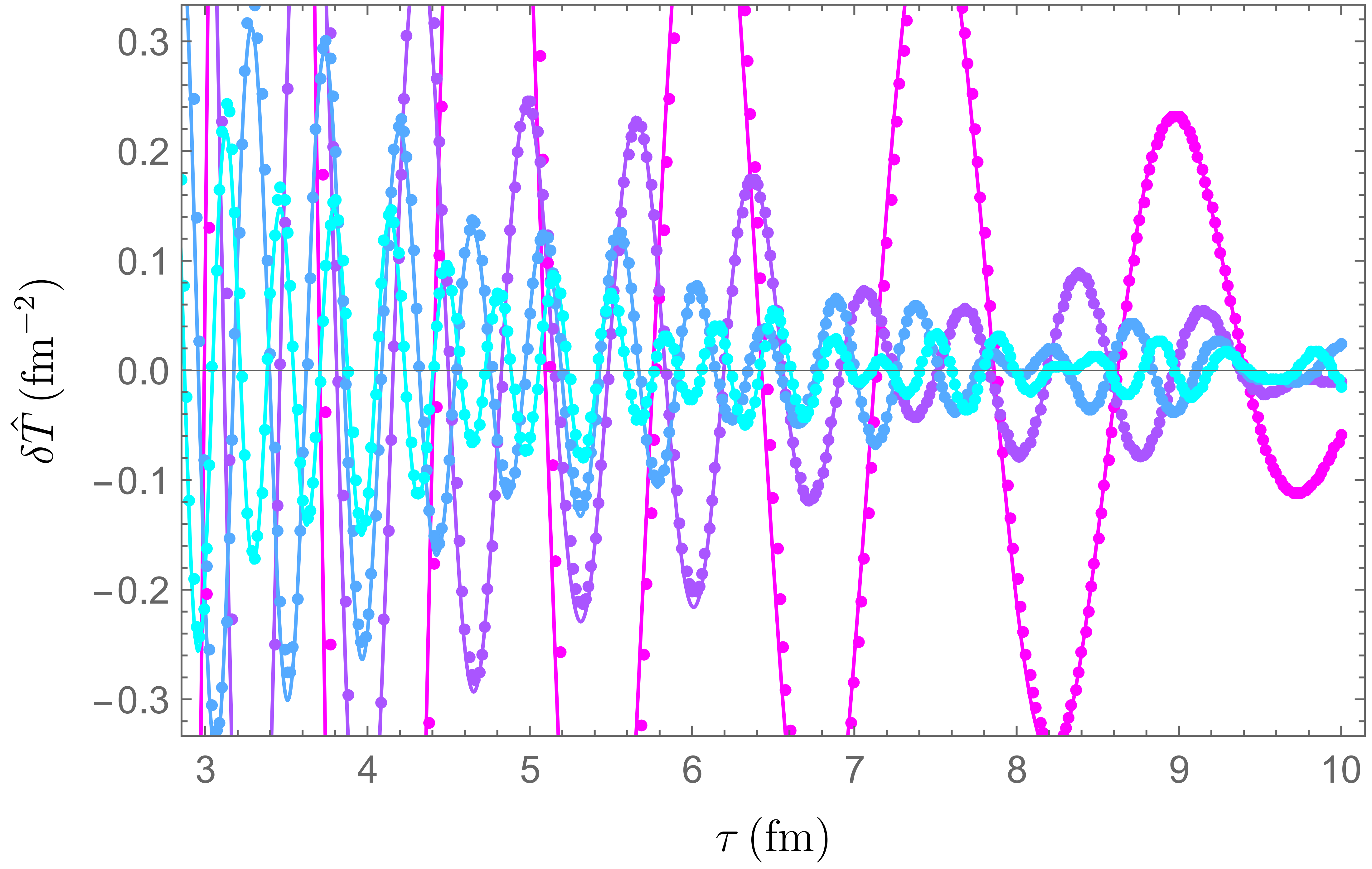
leading power-law decay is model dependent

Matching to numerics

- The analytic solutions (solid curves) fit the numerics (discrete points) in a *wide range of time* (with the same initial conditions).



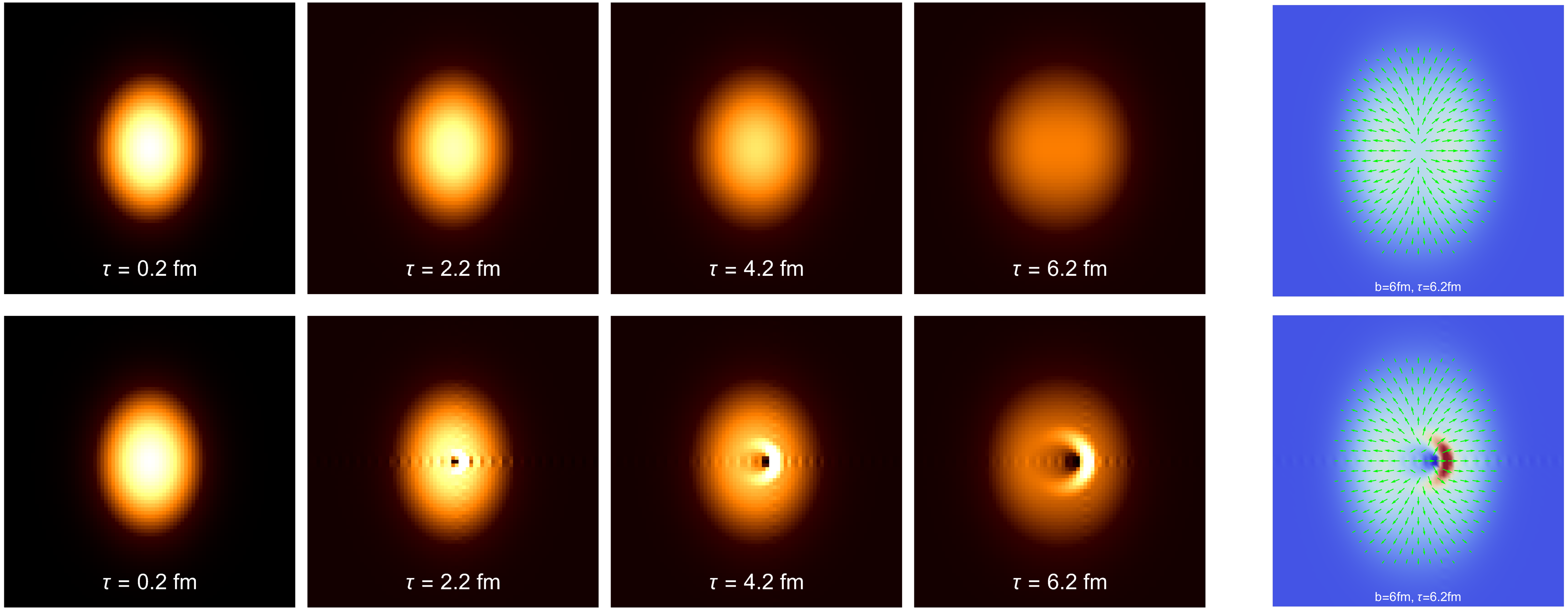
w/o jet



w/ jet

Jet wake

- The transverse tomography with jet wake

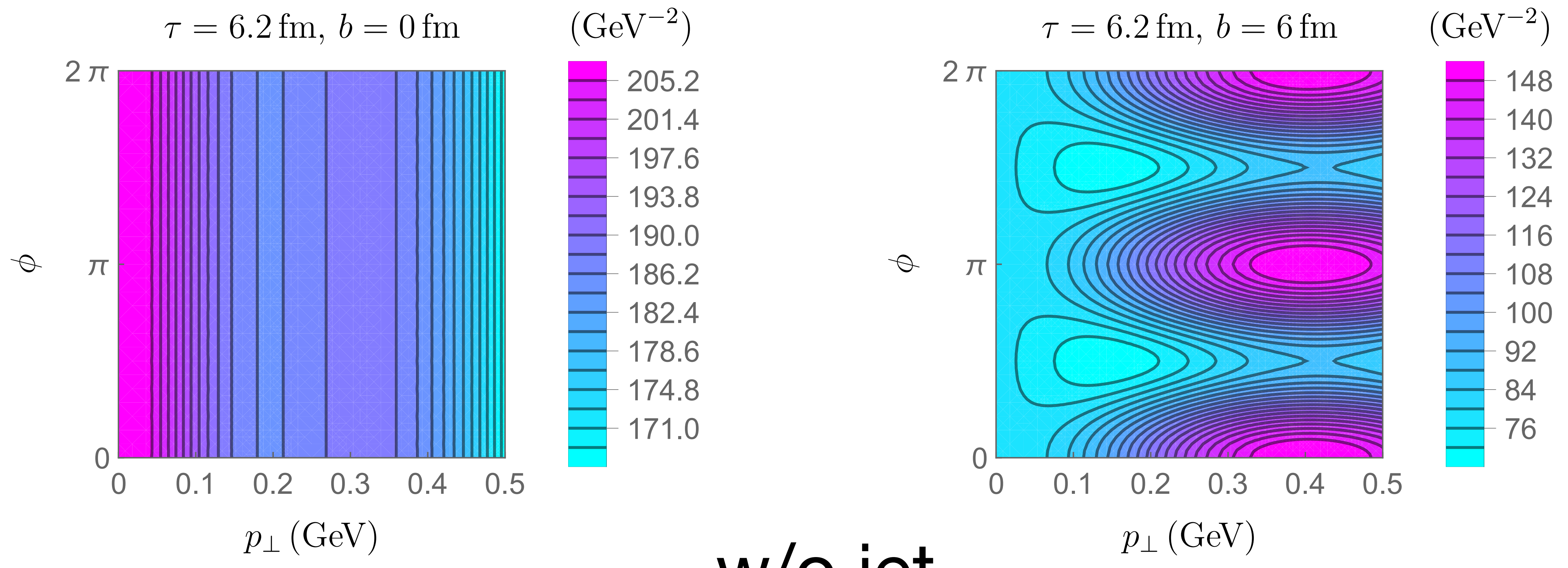


energy density

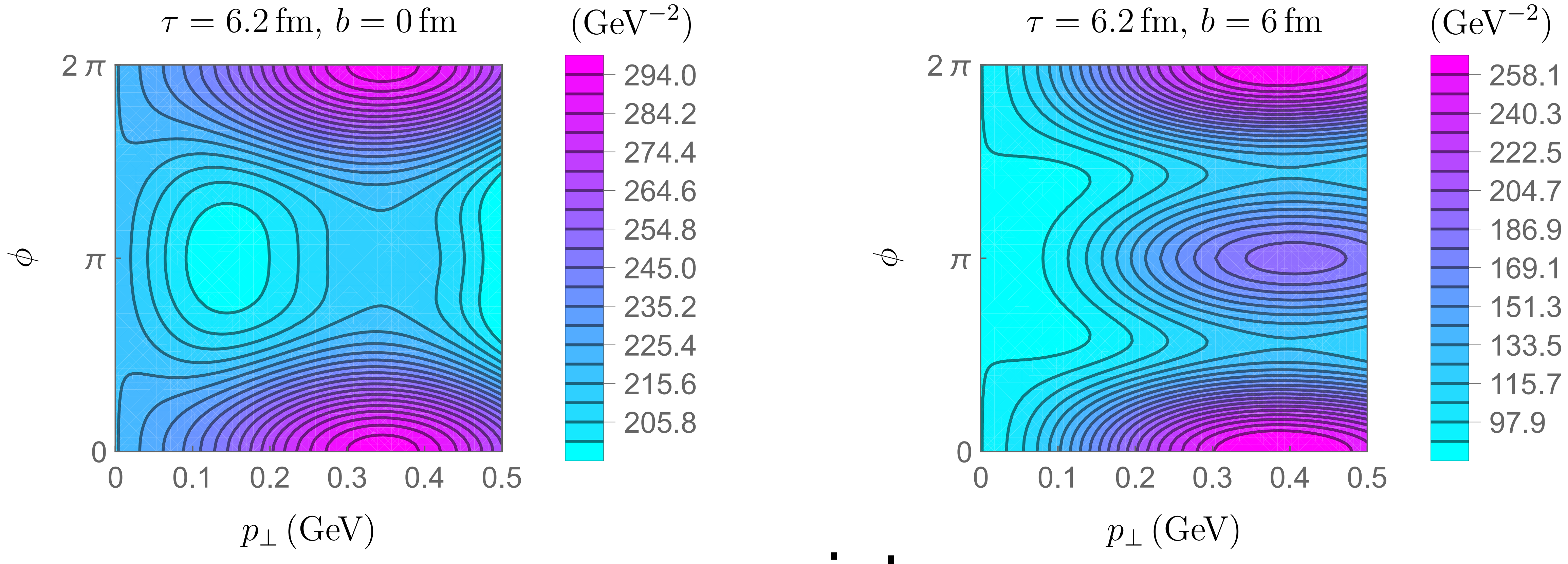
velocity

Collectivity with jet: multiplicities

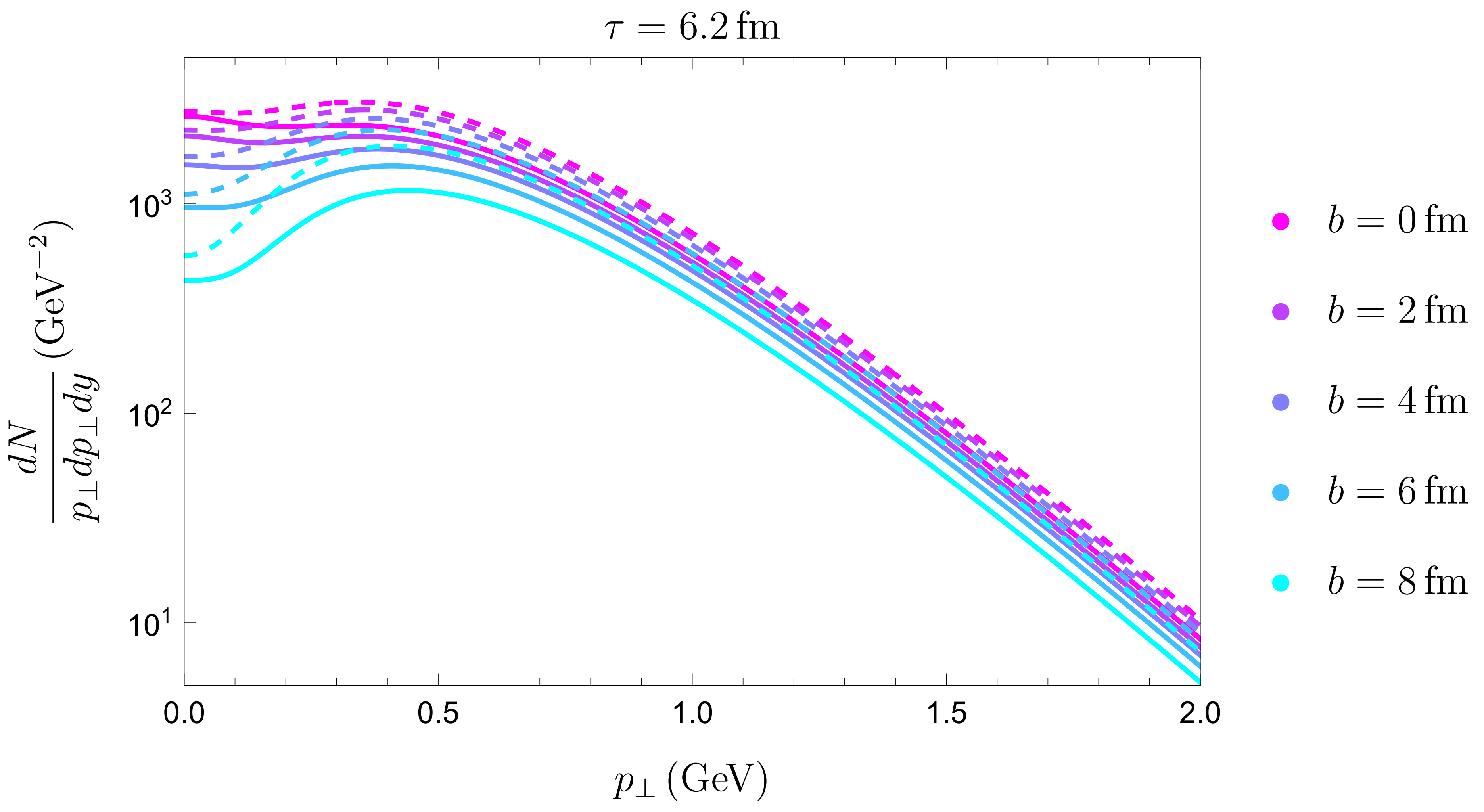
- Comparison of flow observables without jet (solid) and with jet (dashed)

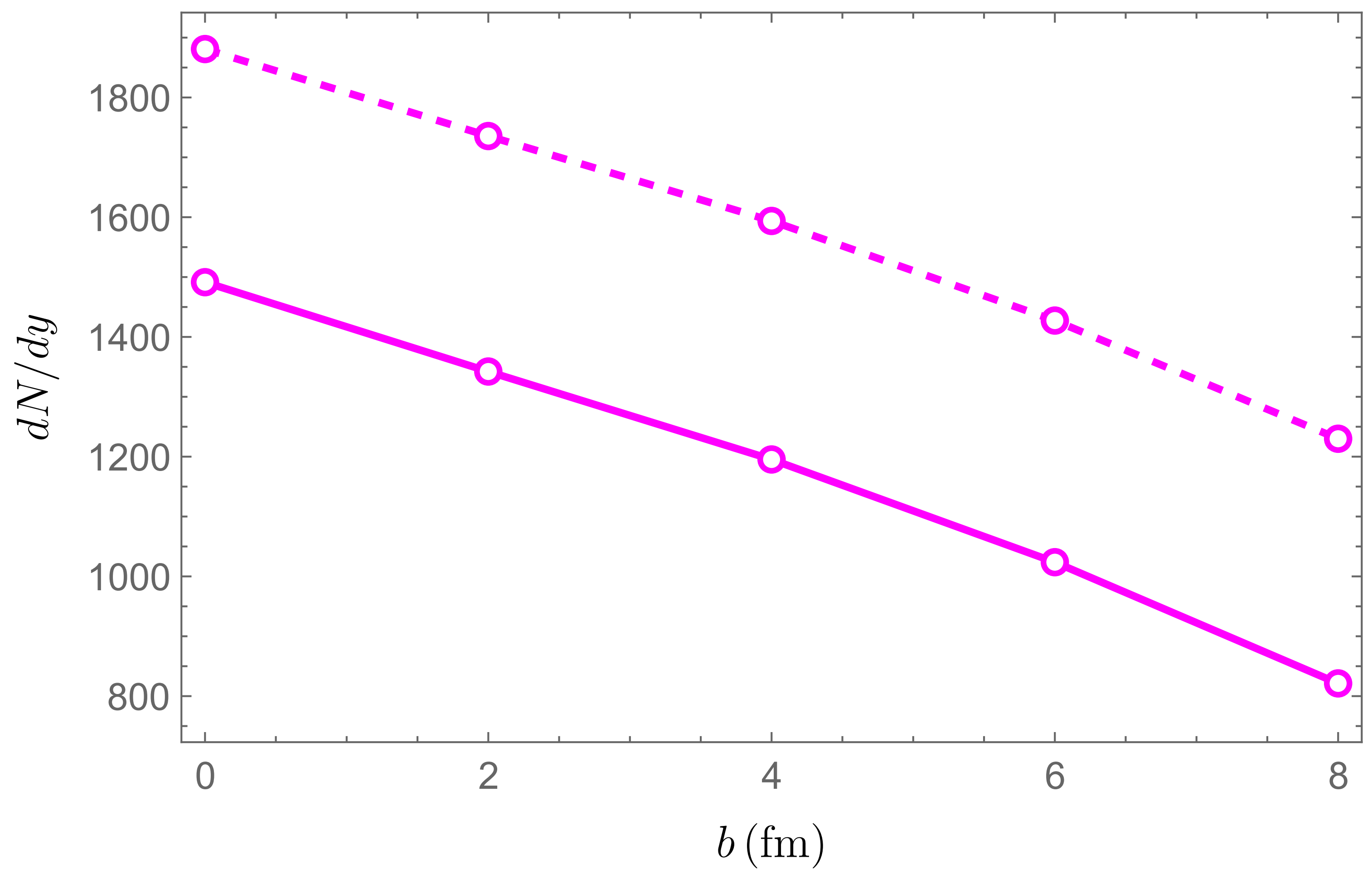


w/o jet



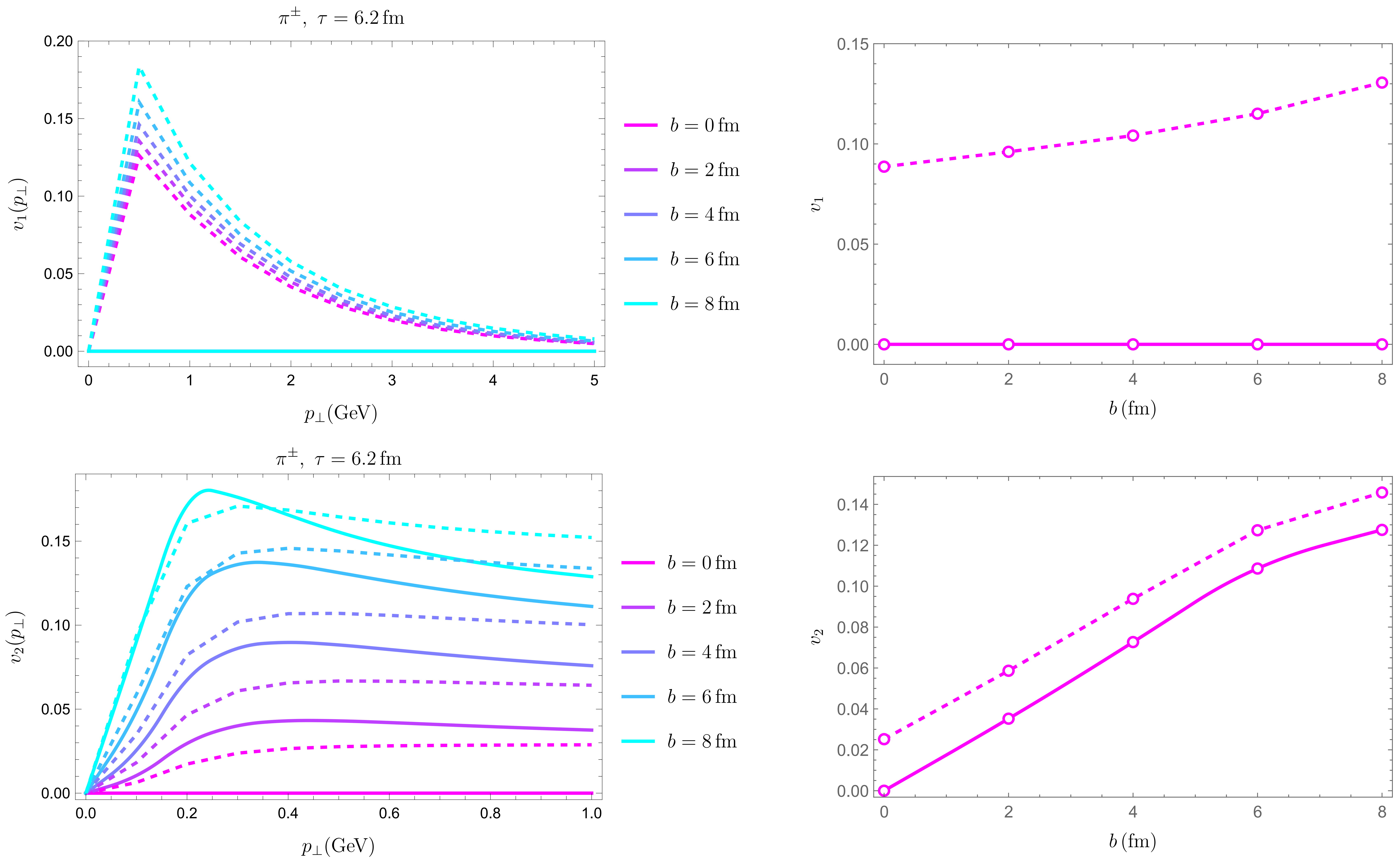
w jet





Collectivity with jet: collective coefficients

- Comparison of flow observables without jet (solid) and with jet (dashed)



Conclusion

Recap

- Formulating relativistic hydrodynamic fluctuations involving velocity in non-Gaussian regime is very nontrivial.

- The perturbation of attractors demonstrates the importance of non-hydrodynamic sector.

Outlook

- Fluctuations: establish quantitative connection between EOS and experiment, and use BES-II data in turn to constrain the EOS and transport coefficients.

- Stochastic fluctuations in the regime far from equilibrium, spin hydrodynamics.

Thank You!