Renormalized critical dynamics and fluctuations in model A

arXiv:2408.06438

Nadine Attieh, Nathan Touroux, Marcus Bluhm, Masakiyo Kitazawa, Taklit Sami, Marlene Nahrgang

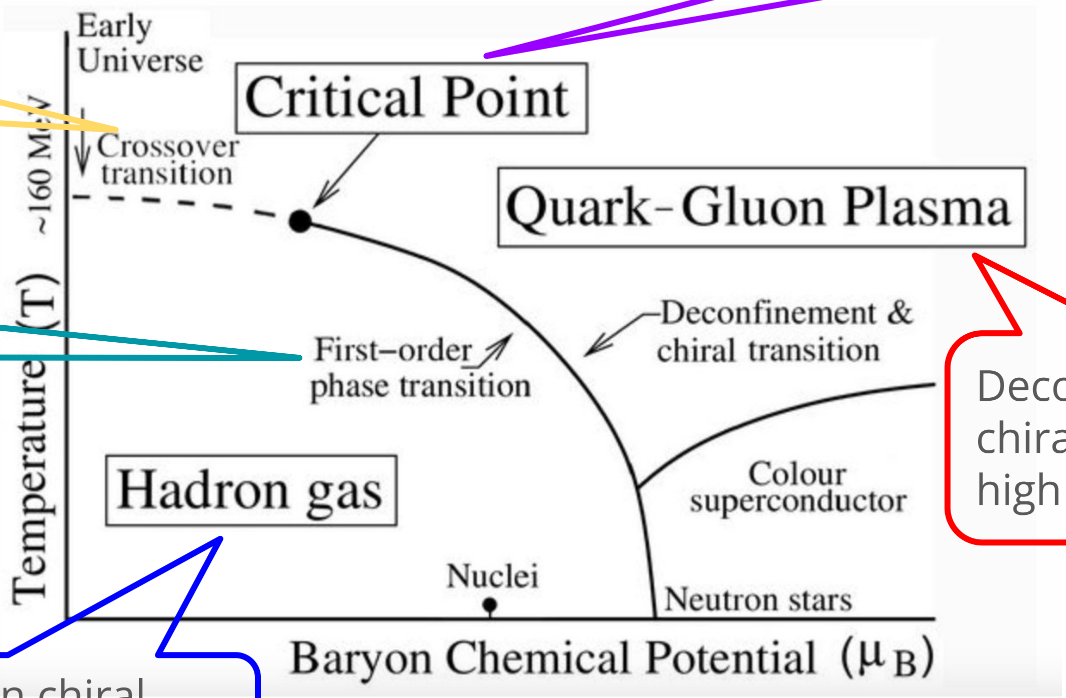
Hydrodynamics and related observables in heavy-ion collisions Workshop Oct 29, 2024
Subatech / IMT-Atlantique, Nantes

QCD phase diagram

End of 1st order transition

From lattice QCD

From effective models

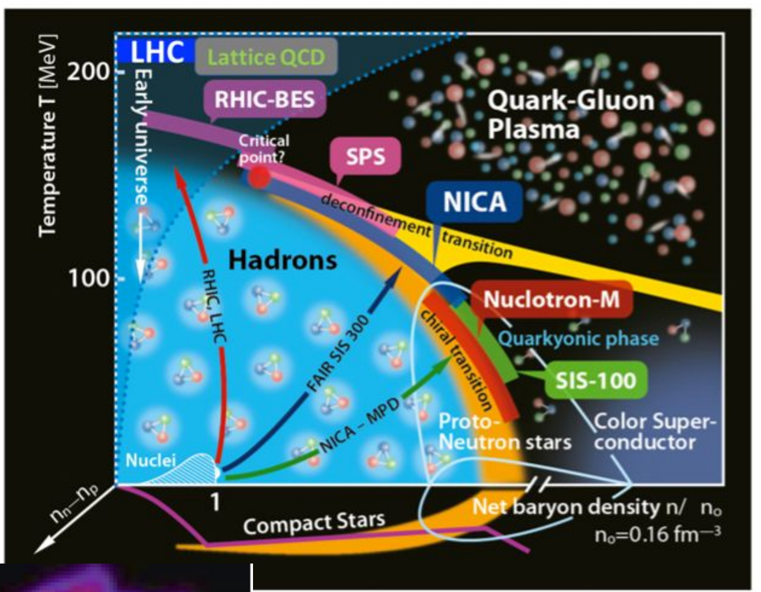


Bhalerao, 1404.3294v1

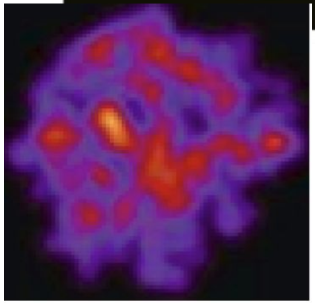
Deconfined, chirally symmetric, high T and high μ_B

Confined, broken chiral symmetry, low T and low μ_B

Heavy Ion Collisions



Sahoo and Nayak, arXiv:2201.00202v1



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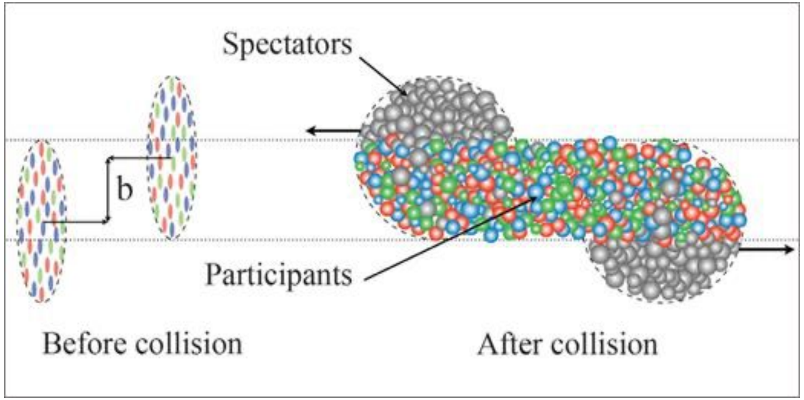
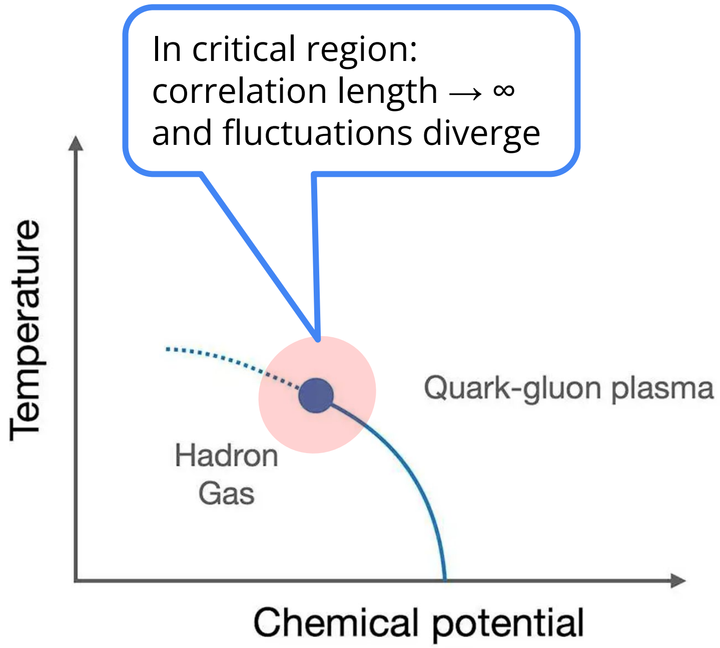


LHCb collaboration, arXiv:2111.01607v1

Heavy Ion Collision is highly dynamical:

- Short lived
- Small size
- Inhomogeneous
- Out of equilibrium evolution

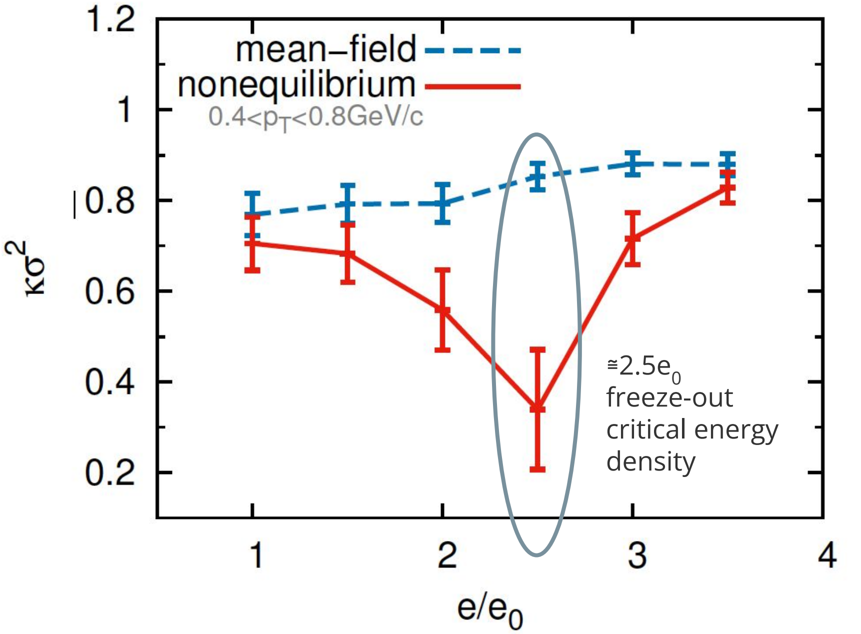
Fluctuations Near Critical Point



Higher moments and their derived quantities are promising fluctuation observables close to critical point:

- Sensitive to powers of the correlation length
- Non-monotonic behavior
- Non-Gaussian behavior

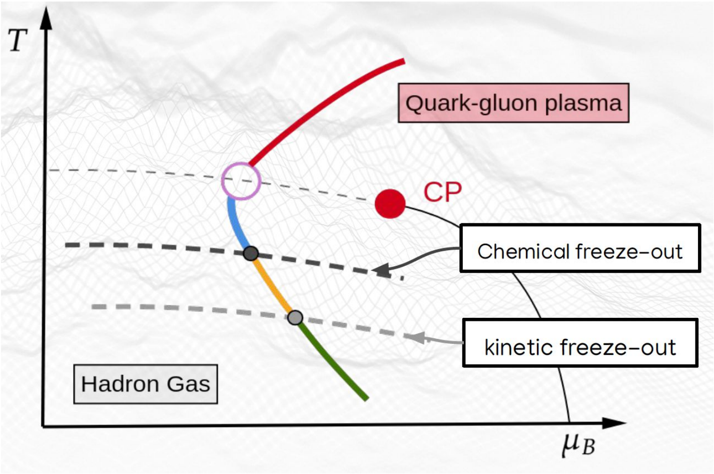
Fluctuations Near Critical Point



Large dip in kurtosis of net-proton number on expected crossover side of critical point

Herold, Nahrgang, Yan and Kobdaj 1601.04839v1
Nahrgang, Leupold, Herold, Bleicher 1105.0622
Bluhm, Jiang, Nahrgang, Pawlowski, Rennecke, Wink 1808.01377v1

Signals of phase transition and critical point detected after freeze-out



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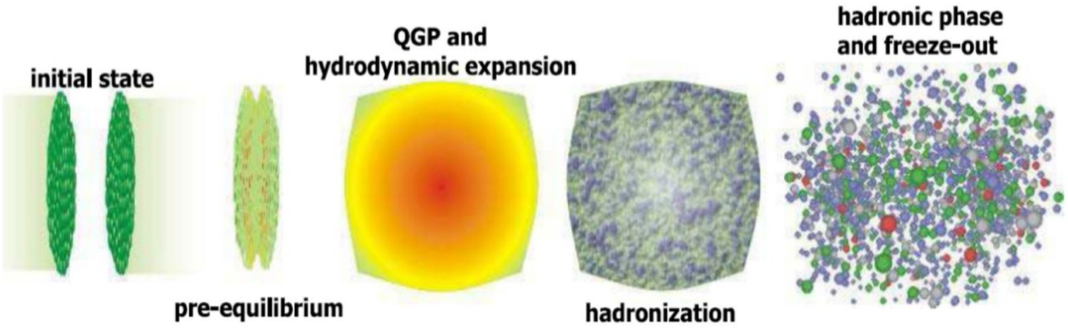
Coupled fluid dynamics

A dynamical model coupling a fluctuating initial state to a final hadronic phase where fluctuations survive is necessary to study out-of-equilibrium effects at the QCD phase transitions and critical point

Couple the dynamics of an order parameter to coarse-grained hydrodynamic evolution of the hot medium to study evolution of fireball created in HIC

Fluid dynamical description of HIC

- Successful and “simple” description of systems created in HIC, even when they are small and rapidly evolving
  Romatschke and Romatschke, arXiv:1712.05815v3



©Berndt Mueller, arXiv:1309.7616v2

- The Quark-Gluon Plasma is considered the most ideal fluid ever created RHIC, 2005
- Describes QGP-HG phase transition by including an adapted equation of state

Fluid dynamical description of HIC

For time-dependent phase transitions, the dynamics of an order parameter needs to be explicitly included

Current models limited to event-averaged quantities.

2 approaches to couple the dynamics of an order parameter, to the hydrodynamics of the hot medium

Deterministic (Hydro+)

Stephanov, Yin 1712.10305
Rajagopal, Ridgway, Weller, Yin 1908.08539
An, Basar, Stephanov, Yee 1902.09517, 1912.13456
Pradeep, Rajagopal, Stephanov, Yin 2204.00639

Stochastic Fluid Dynamics

Challenge: lattice spacing dependence introduced by noise

Stochastic Chiral Fluid Dynamics

Explicitly propagate the fluctuating chiral order parameter in dynamically expanding fluid

$$\frac{\partial^2 \varphi(\vec{x},t)}{\partial t^2} - \nabla^2 \varphi(\vec{x},t) + \eta \frac{\partial \varphi(\vec{x},t)}{\partial t} + \frac{\partial V_{\text{eff}}[\varphi]}{\partial \varphi(\vec{x},t)} = \xi(\vec{x},t)$$

Damping coefficient (η)

noise ($\xi(\vec{x},t)$)

$$\partial_\mu T^{\mu\nu} = -\partial_\mu T^{\mu\nu}_\varphi \equiv S^\mu$$

Source term gives rise to the coupling

Unphysical lattice spacing dependence poses a significant challenge to this model

Relaxation Model

Focus on non-conserved order parameter in model A: Stochastic Relaxation Equation

$$\frac{\partial^2\varphi(\vec{x},t)}{\partial t^2} - \nabla^2\varphi(\vec{x},t) + \eta\frac{\partial\varphi(\vec{x},t)}{\partial t} + \frac{\partial V_{\text{eff}}[\varphi]}{\partial\varphi(\vec{x},t)} = \xi(\vec{x},t)$$

Effective potential

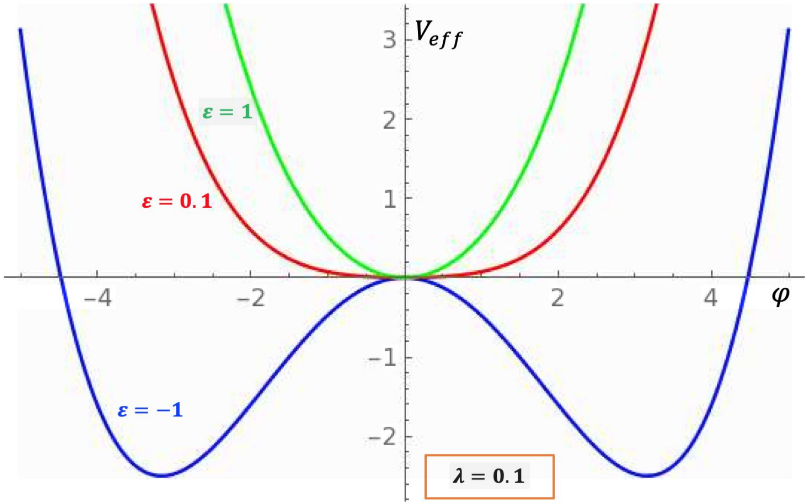
$$V_{\text{eff}}(\varphi) = \frac{1}{2}\epsilon\varphi^2 + \frac{1}{4}\lambda\varphi^4$$

The noise ξ is defined by

$$\langle\xi(\vec{x},t)\rangle = 0$$

$$\langle\xi(\vec{x},t)\xi(\vec{x}',t')\rangle = 2\eta T\ \delta(\vec{x}-\vec{x}')\delta(t-t')$$

Ensures that φ relaxes to correct equilibrium value, guarantees proper equilibrium distribution and satisfies fluctuation dissipation theorem



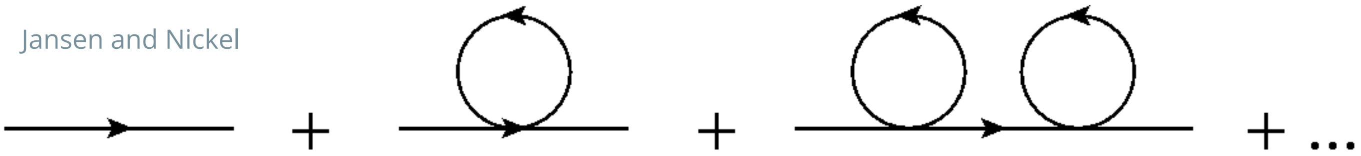
Lattice Spacing Dependence

➔ UV divergences caused by the noise translate as non-physical lattice spacing dependence in numerical simulations $\delta(\vec{x}' - \vec{x}) \rightarrow \frac{1}{dx^3}$

➔ Finite lattice requires a UV cutoff which contributes to the lattice spacing dependence

➔ Loop corrections in the φ^4 theory also introduce UV divergences

Jansen and Nickel



The tadpole diagram in the expansion of 2-point function gives rise to a correction term

Lattice Spacing Dependence

Currently, going around the noise term includes:

- Coarse-graining over grid with larger spacing
  Nahrgang et al. arXiv:1704.03553, Bluhm et a. arXiv:1804.03493
- Filtering large momentum modes Singh arXiv:1807.05451
- Smearing by a Gauss distribution
  Murase and Hirano arXiv:1601.02260, Hirano, Kurita, and. Murase,arXiv:1809.04773

Effects unknown especially on fluctuation observables

Lattice theory may no longer correspond to continuum theory

Improve solution: lattice regularisation

Numerical simulations

- 3D system at fixed temperature: cubic lattice of sides *L=20 fm*, volume L^3
- N cells in each direction → Lattice spacing (use *dx* for simplicity)

$$dx = dy = dz = \frac{L}{N}$$

- Discretize time: repeat simulations for a number of time steps until equilibrium is reached
- Periodic boundary conditions
- Code on GPU: input equations and parameters ➞ evaluate the dynamical variable ➞ derive relevant observables (correlation function, different moments, etc.)

Linear Approximation of V_{eff} ϵ=1, λ=0

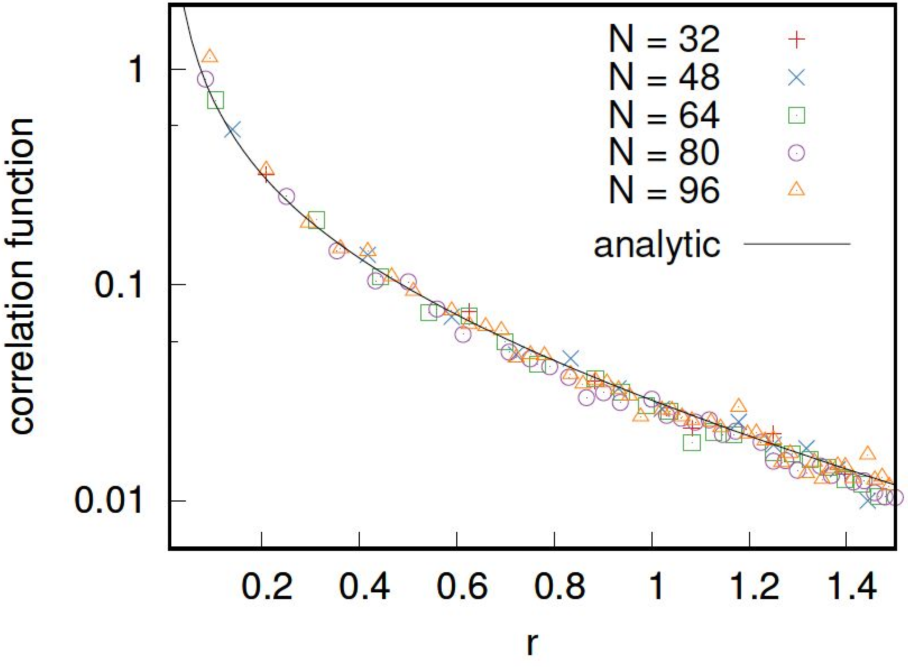
The 2-point function is

$$C(r) = \frac{T}{4\pi r} e^{-\frac{r}{r_c}}$$

Correlation length

$$r_c = \sqrt{1/\epsilon} \quad r = |\vec{x} - \vec{x}'|$$

- Reproduced analytic result
- Benchmarked correlation function
- No dx dependence for finite distances: introduced close to 0



$T = 1, \epsilon=1, r_c=1, \lambda=0$

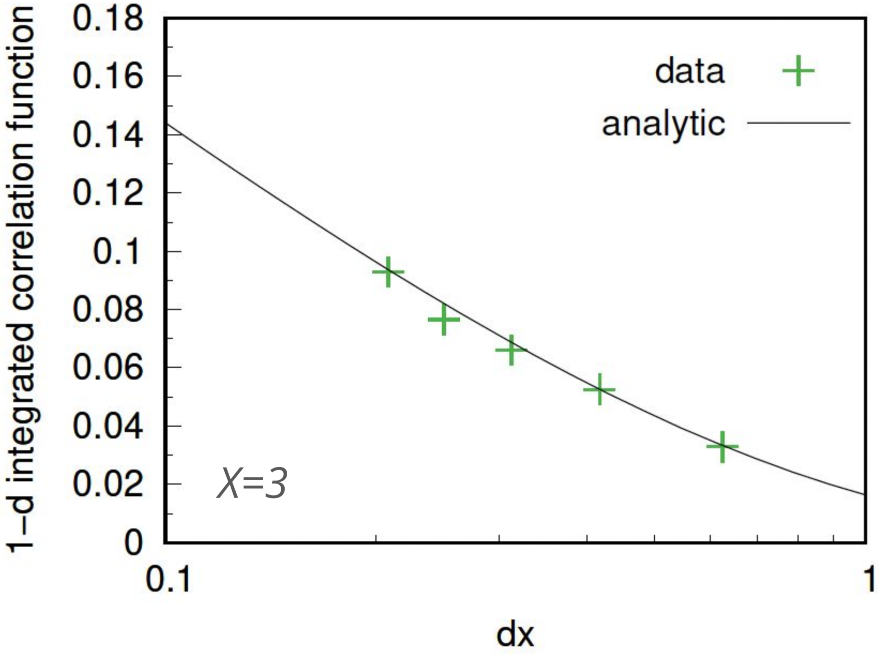
Linear Approximation of V_{eff} ϵ=1, λ=0

Integrate correlation function in 1d to benchmark logarithmic dx dependence

$$\int_{dx}^{X} \frac{T}{4\pi r} e^{-\frac{r}{rc}} \, dr \propto \ln(X) - \ln(dx)$$

- Reproduced analytic result
- Benchmarked dx dependence

➥ include nonlinear terms



Observables: Mean, Variance and Kurtosis

Volume average of the order parameter:

$\varphi_V(t) = \frac{1}{V}\int_V d^3x\, \varphi(\vec{x},t)$

Mean ➜ $\Phi = \langle \varphi_V(t) \rangle_{\text{conf}}$

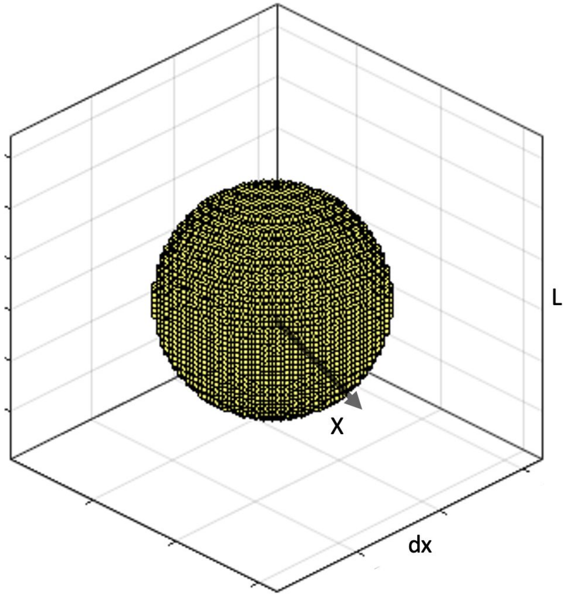
We are interested in fluctuation variables:

Variance ➜ $\sigma^2 = \langle (\varphi_V(t) - \Phi)^2 \rangle_{\text{conf}}$

➜ $\kappa\sigma^2$

Where the volume-independent kurtosis is

$\kappa = \frac{\langle (\varphi_V(t) - \Phi)^4 \rangle_{\text{conf}}}{\sigma^4}$



$X \leq \frac{L}{2} \Rightarrow V \propto V_{sphere}$

Lattice Regularisation

Equilibrium counterterm to correct V_{eff} from mass renormalization procedure

$$\mathcal{V}_{ct} = \{-\frac{3\lambda\Sigma}{4\pi}\frac{T}{dx} + \frac{3\lambda^2 T^2}{8\pi^2}[ln(\frac{6}{dxM}) + \zeta]\}\frac{\varphi^2}{2}$$

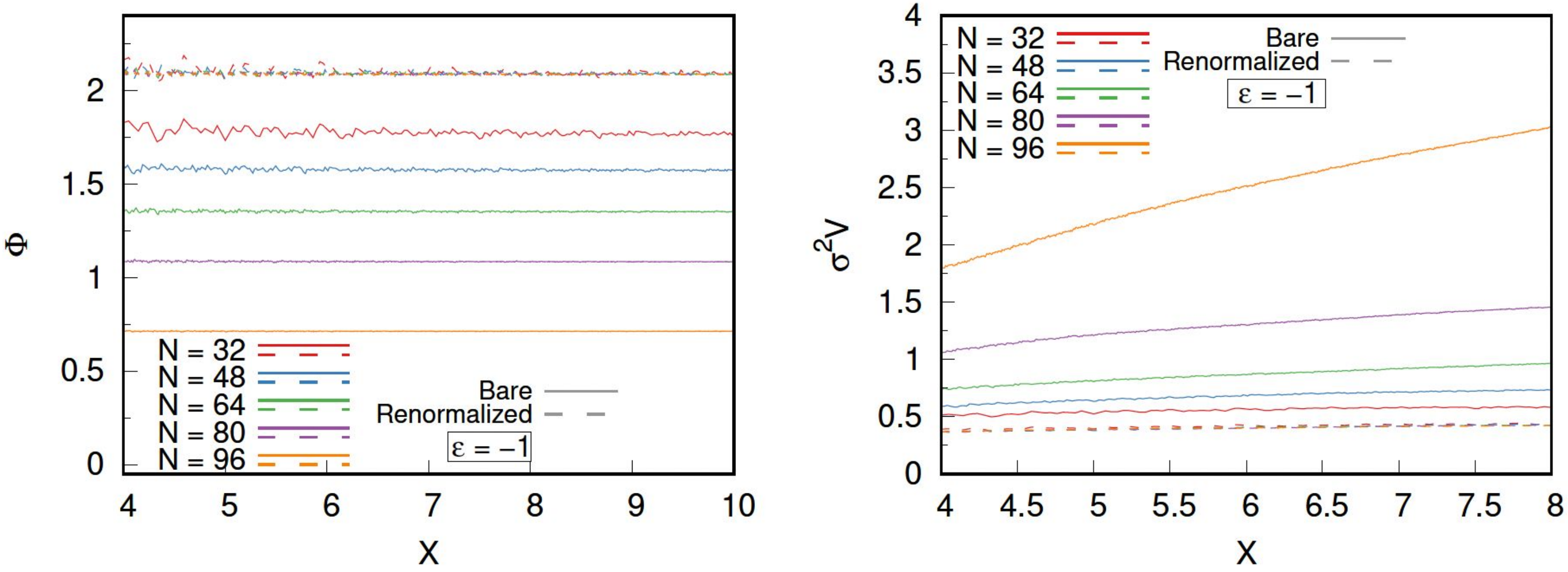
Cassol-Seewald et al. 0711.1866
Farakos et al. 9412091, 9404201v1
Gleiser, Ramos 9311278v1

- *Σ≈3.1759, ζ≈0.09*
- M renormalization scale
- Leading *1/dx* dependence

Equilibrium and dynamical evolution, with and without counterterm, for

- $\epsilon<0$ (broken symmetry)
- ϵ small and positive (close to critical point)
- $\lambda=0.25$
- $T=M=\eta=1$
- All quantities are dimensionless

At equilibrium ϵ=-1 Broken Symmetry

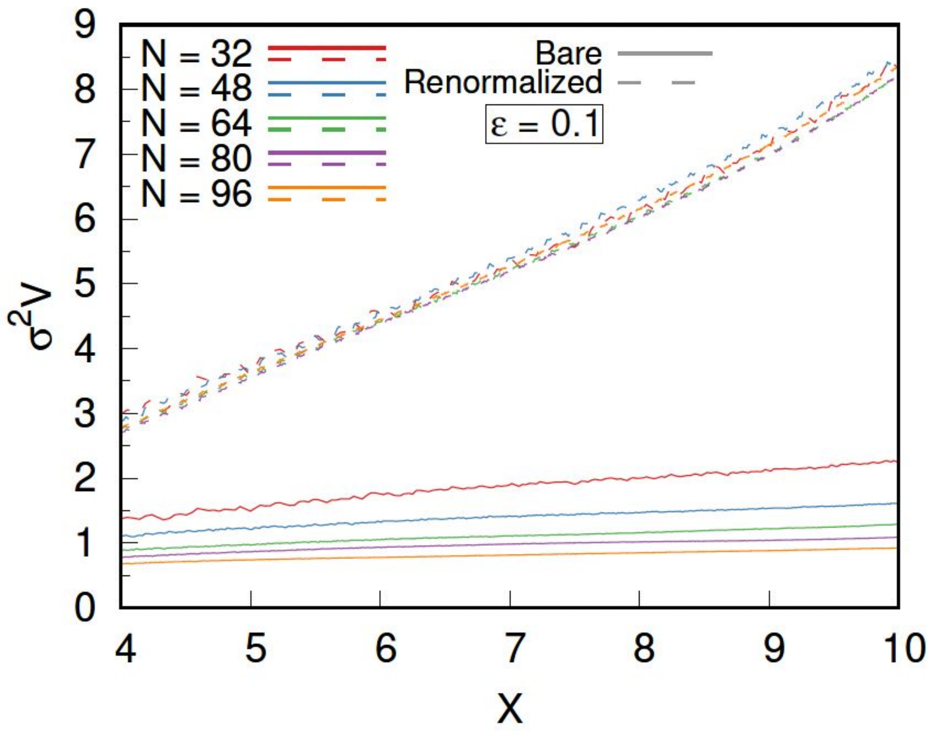


Lattice spacing dependence corrected by the same counterterm, correct value of mean recovered

Consistent with previous equilibrium results for the mean

Cassol-Seewald et al. 0711.1866 and references therein

At equilibrium ϵ=*0.1* close to Critical Point



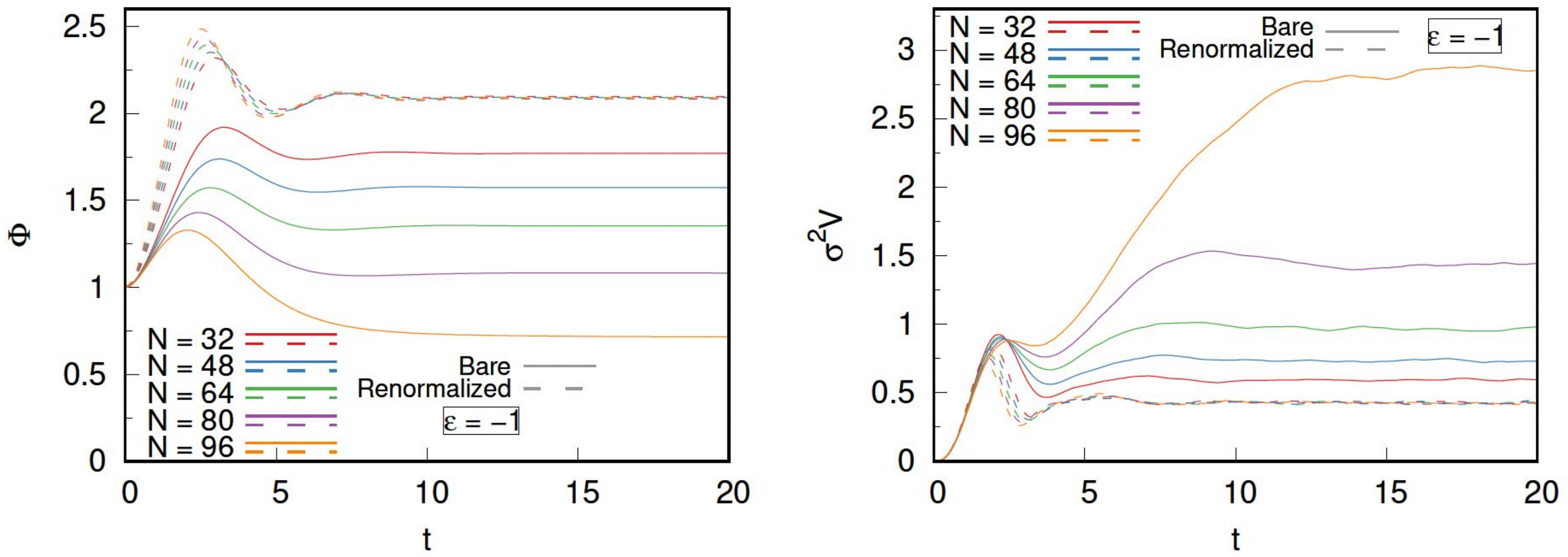
Close to the critical point, as ϵ decreases, the correlation length diverges

$$r_c = \sqrt{1/\epsilon}$$

Long-range fluctuations add up and the variance increases with the volume

Same counterterm corrects lattice spacing dependence close to critical point

Dynamical Evolution ϵ=-1 Broken Symmetry



At early times and despite being evaluated in equilibrium conditions, the counterterm significantly alleviates the dx dependence

Dynamical Evolution ϵ=*0.1* close to Critical Point

The counterterm causes positive ϵ to become negative and dx-dependent

V_{eff} initially becomes a double well potential

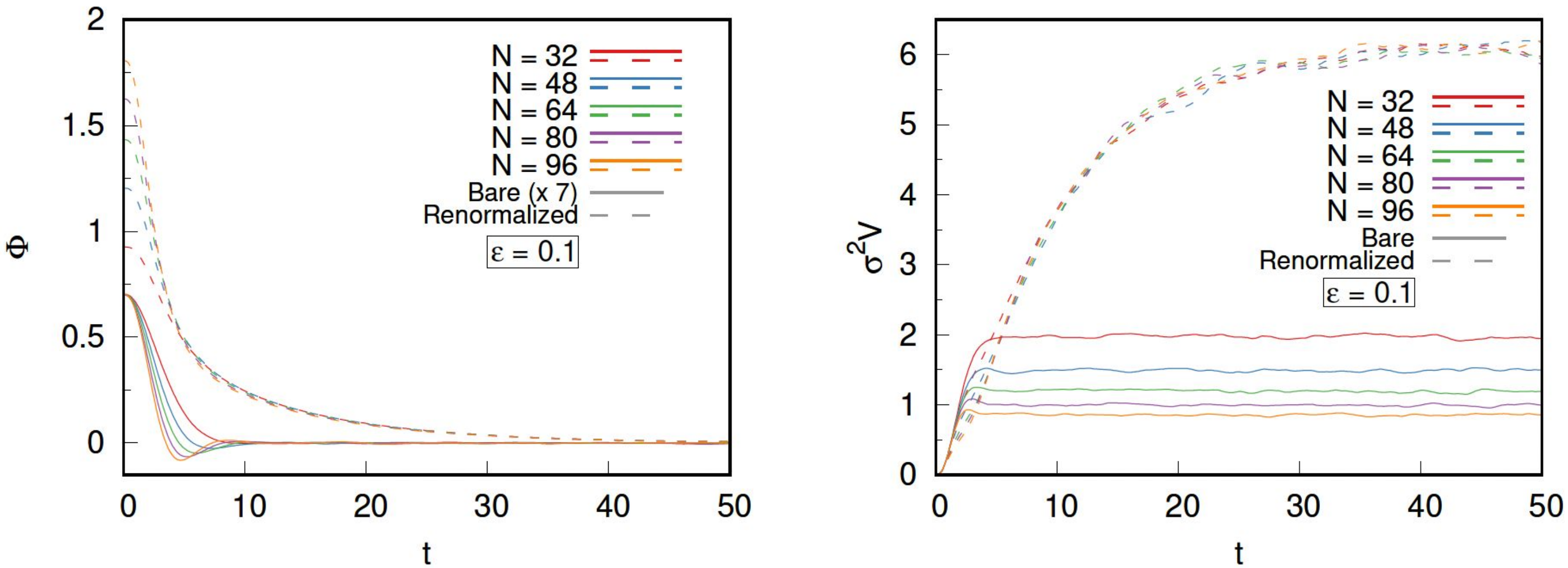
For small fixed φ_0 the restoring force is different for different dx: initially the field moves towards dx-dependent positive minima instead of correct $\phi = 0$

We adapt φ_0 such that

$$\frac{\partial (V_{\text{eff}})^{\text{ren}}}{\partial \varphi}\Big|_{\varphi_0=\varphi_0(dx)} = \frac{\partial (V_{\text{eff}})^{\text{bare}}}{\partial \varphi}\Big|_{\varphi_0=0.1}$$

The fluctuations can then kick in and restore the flattened single well effective potential.

Dynamical Evolution ϵ=*0.1* close to Critical Point



Mean: the dx-dependent counterterm artificially introduces a dx sensitivity at early times ➜ adapt φ_0*(dx)*.
The dx-dependence is cured in the renormalized cases.

Dynamical Evolution of Kurtosis $\kappa\sigma^2$

Higher order cumulants scale with higher powers of the correlation length, they are therefore considered more indicative of criticality

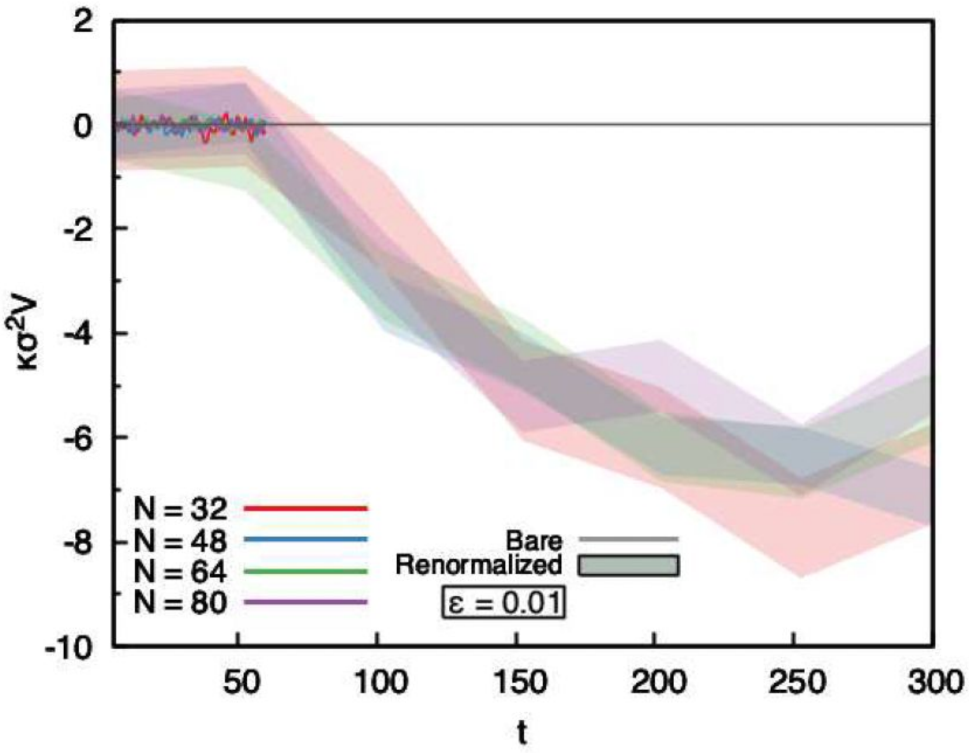
$\kappa\sigma^2$ corresponds to the ratio of the 4^{th} order cumulant to the variance

The kurtosis should take a non-zero negative value

For ϵ=0.1, $\kappa\sigma^2$ shows only a small shift towards negative values: no solid conclusion can be reached with available statistics.

We then look at the kurtosis closer to the critical point and take ϵ=0.01.

Dynamical Evolution of Kurtosis $\varkappa\sigma^2$ for ϵ=0.01



Higher order moments require more time to equilibrate

t_{fin}=60 for bare system and t_{fin}=300 for renormalized system;

Limited statistics ⇒ results in shaded areas for 4 values of *dx*

Recover expected non-Gaussian behavior in renormalized system: kurtosis takes on a negative finite value

Carbon footprint

- Numerical calculations were carried out on GPUs at the in2p3 computing center
- Number of GPU/CPU usage hours and TDP (thermal design power) of GPU/CPU to evaluate energy consumption → average CO_2/kWh in France for 2023

Cautious estimate ***1.8 tCO_2eq***

For results shown here, as well as tests, trials, errors and repeats

1 tCO_2eq emitted for one passenger on a round trip flight Paris-New York or for an average car over 5000 km
TDP is used for simplicity, but is considered a poor proxy for actual energy consumption

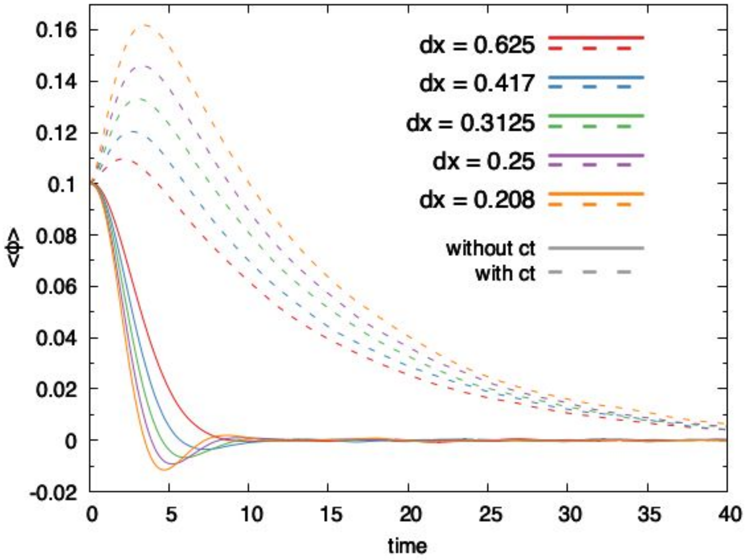
Summary

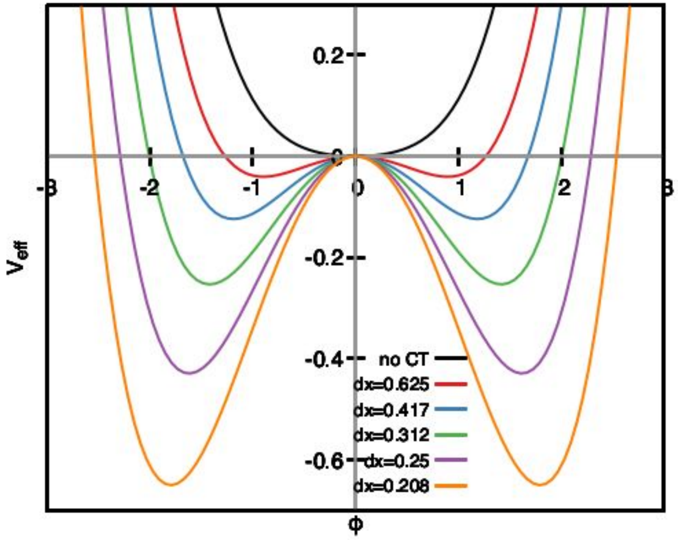
- Properly benchmarked our code
- Observed lattice spacing dependence of mean and variance in bare system
- Same counterterm cures dx dependence of mean and variance both in the vicinity of the critical point and in the broken symmetry phase
- Recover correct behavior of mean and variance
    - Correct mean in chrally broken phase
    - Volume dependence of variance close to critical point
- Recover expected non-Gaussian behavior of kurtosis (finite non-zero value) in renormalized system

Outlook

- ➔ Apply approach to the coupled chiral fluid dynamics: derive proper counterterm(s)
- ➔ Useful for studying diffusive dynamics of conserved order parameter
- ➔ Renormalize the full stochastic hydrodynamics to study heavy-ion collisions near a critical point, using realistic initial conditions for HIC and equation of state of QCD

Fixed φ_0 for ϵ=0.1





$\varkappa\sigma^2$ for ϵ=0.1