# Three fluid model for BES energies

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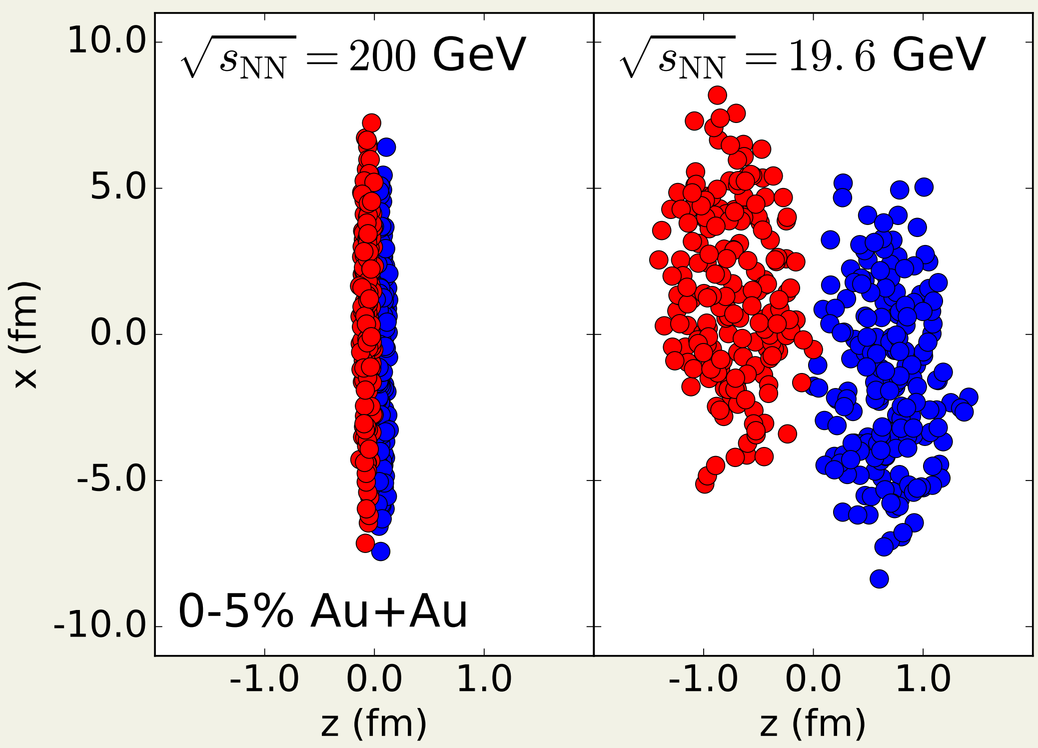
**Hydrodynamics and related observables in heavy-ion collisions**

**October 29, 2024, Subatech/IMT-Atlantique, Nantes**

**with Jakub Cimerman, Iurii Karpenko, Boris Tomasik**
**and Clemens Werthmann**
**PRC107, 044902 (2023)**

# Challenge

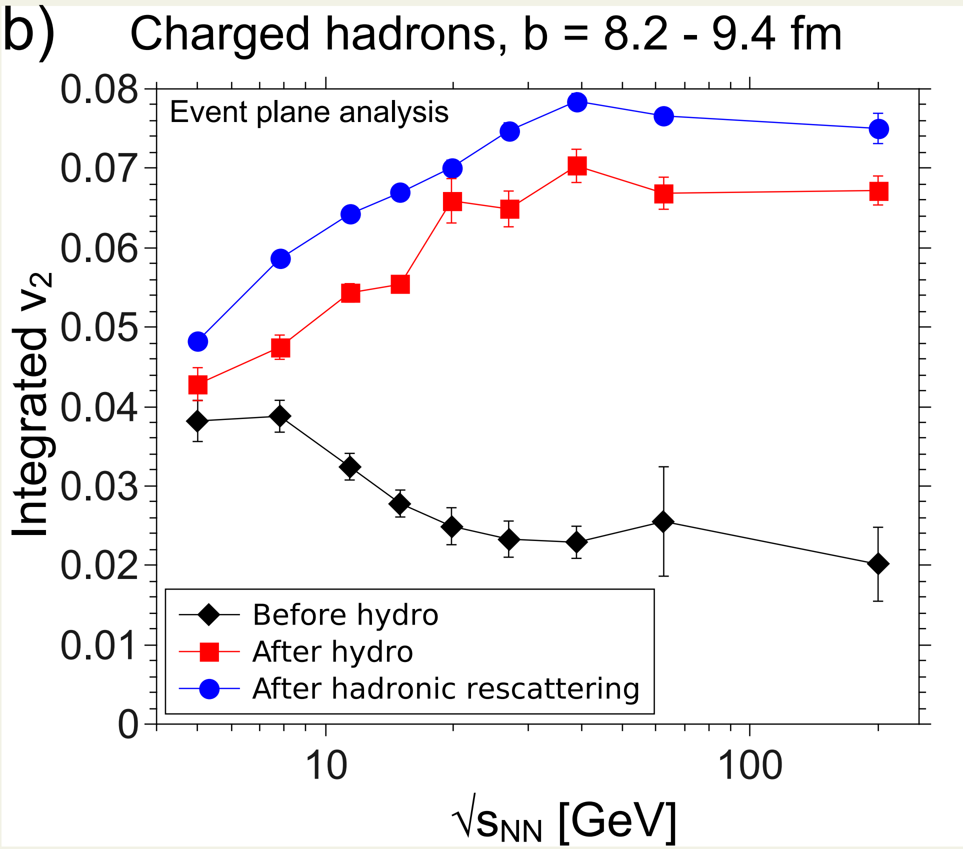
- **primary collisions overlap with secondary collisions**




**Shen & Schenke, PRC97, 024907 (2018)**

# Solutions

- "Sandwich hybrid"
  - cascade until the nuclei have passed each other
  - fluid until hadronisation
  - cascade until freeze out



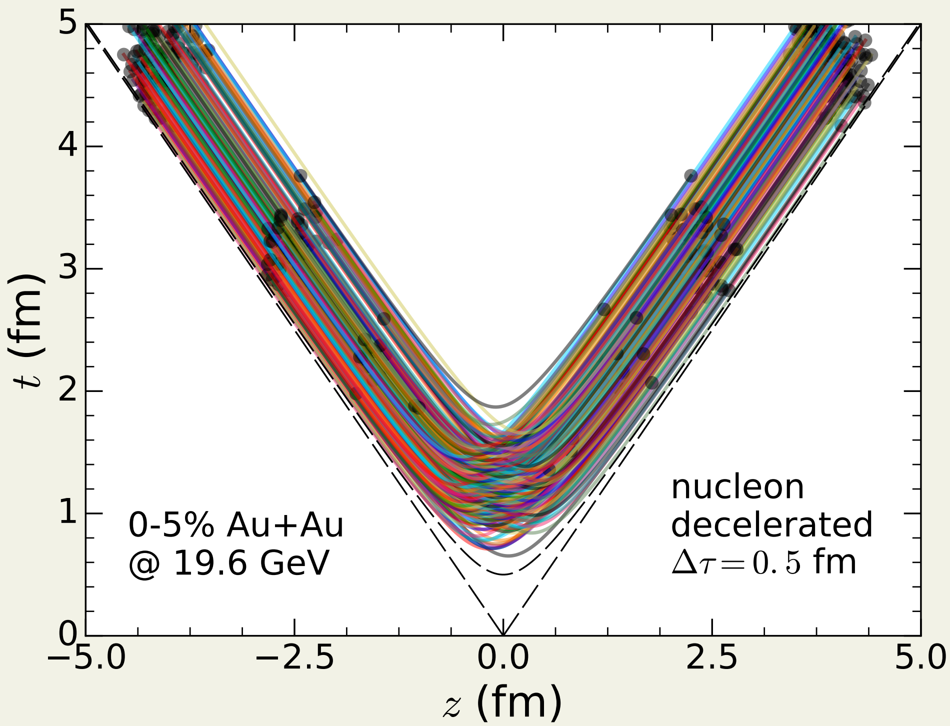

**Auvinen & Petersen, PRC88, 064908 (2013)**

- at $\sqrt{s_{NN}} < 10$ GeV not much happens during the hydro stage

- sensitivity to EoS?

# Solutions

- **Dynamical initialisation**
  - **each primary collision a source term for fluid**
  - $\partial_\mu T^{\mu\nu} = J^\nu$
  - $\partial_\mu N_B^\mu = \rho_B$




**Shen & Schenke, PRC97, 024907 (2018)**

- **no interaction between incoming nucleons and produced particles**

# 3-fluid dynamics

$$0 = \partial_\mu T^{\mu\nu}$$

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$$0 = \partial_\mu T^{\mu\nu}$$
$$= \partial_\mu T_{\mathrm{t}}^{\mu\nu}$$

$T_{\mathrm{t}}^{\mu\nu}$ = **target fluid**

# 3-fluid dynamics

$$
\begin{aligned}
0 &= \partial_\mu T^{\mu\nu} \\
&= \partial_\mu T^{\mu\nu}_{\mathrm{t}} + \partial_\mu T^{\mu\nu}_{\mathrm{p}}
\end{aligned}
$$

$T^{\mu\nu}_{\mathrm{t}}$ = **target fluid**

$T^{\mu\nu}_{\mathrm{p}}$ = **projectile fluid**

# 3-fluid dynamics

$$
\begin{aligned}
0 &= \partial_\mu T^{\mu\nu} \\
&= \partial_\mu T^{\mu\nu}_{\mathrm{t}} + \partial_\mu T^{\mu\nu}_{\mathrm{p}} + \partial_\mu T^{\mu\nu}_{\mathrm{fb}}
\end{aligned}
$$

$T^{\mu\nu}_{\mathrm{t}}$ = **target fluid**

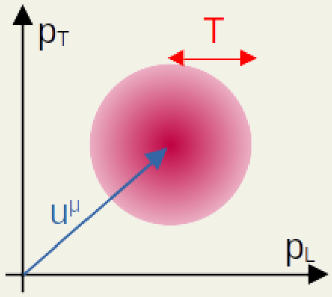
$T^{\mu\nu}_{\mathrm{p}}$ = **projectile fluid**

$T^{\mu\nu}_{\mathrm{fb}}$ = **fireball fluid**

- **target and projectile represent colliding nucleons**
- **fireball (loosely) represents produced particles**
- **three fluids, each with temperature and flow velocity of its own**
- **all fluids coexist in the same point in coordinate space**

# 3-fluid dynamics

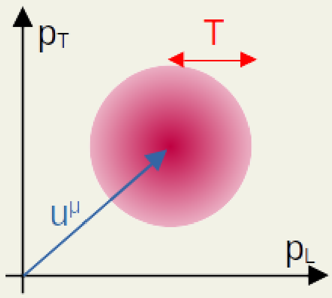
- **distributions in momentum space**



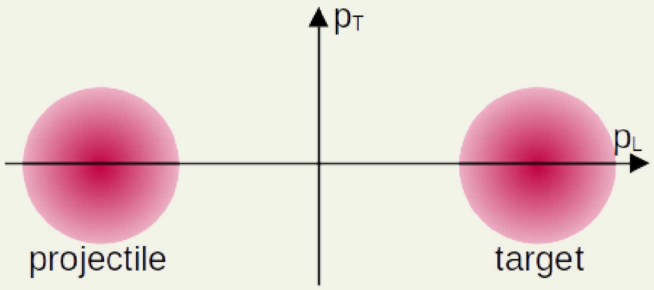

**one fluid**

# 3-fluid dynamics

- **distributions in momentum space**



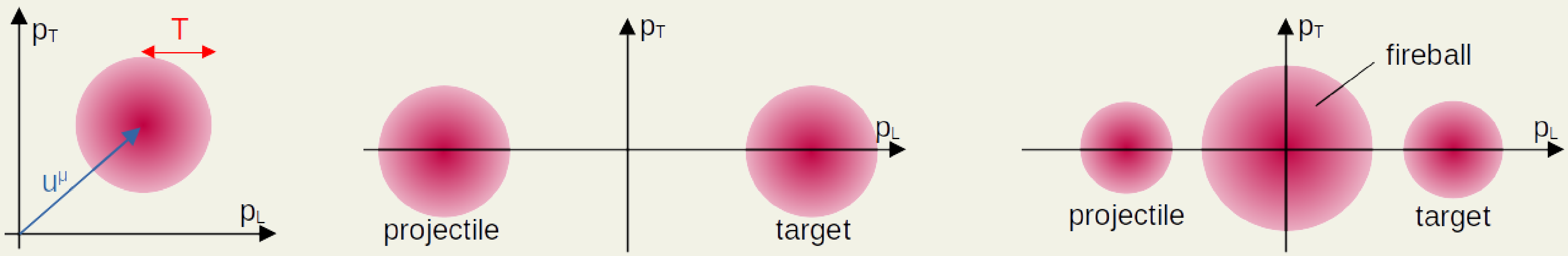

**one fluid**




**2 fluids**

# 3-fluid dynamics

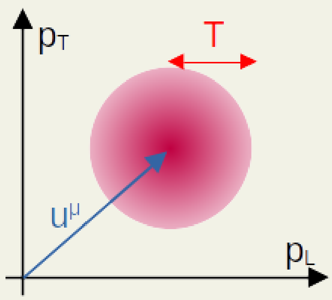
- **distributions in momentum space**



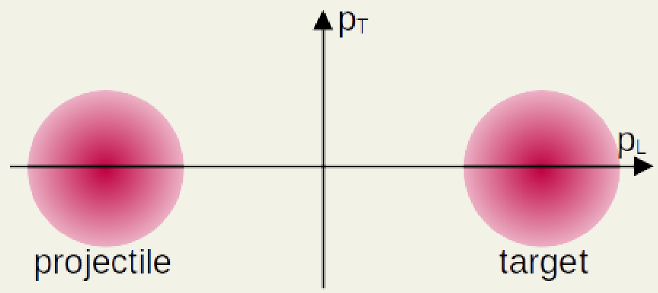

**one fluid** **2 fluids** **3 fluids**

# 3-fluid dynamics

- **distributions in momentum space**




**one fluid**

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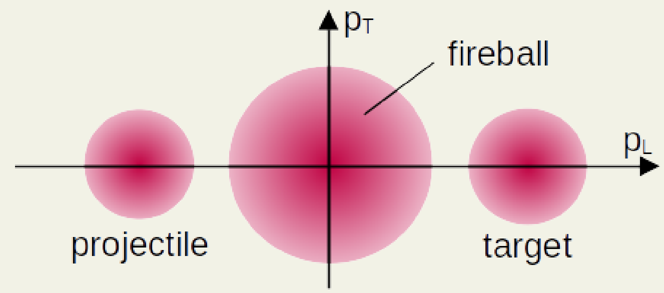
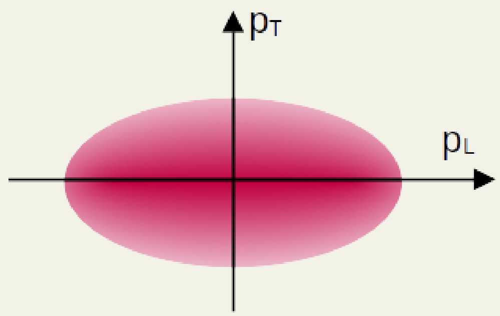


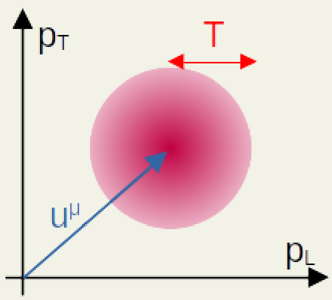

**3 fluids**




**anisotropic hydro**

# 3-fluid dynamics

- **distributions in momentum space**

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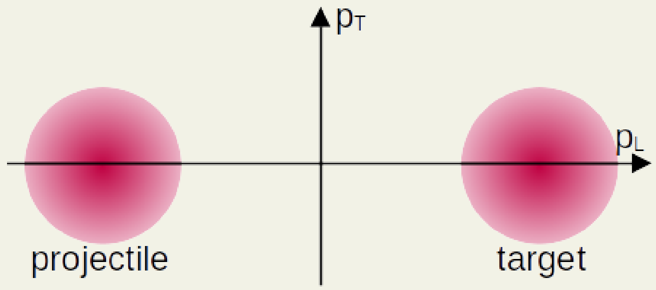


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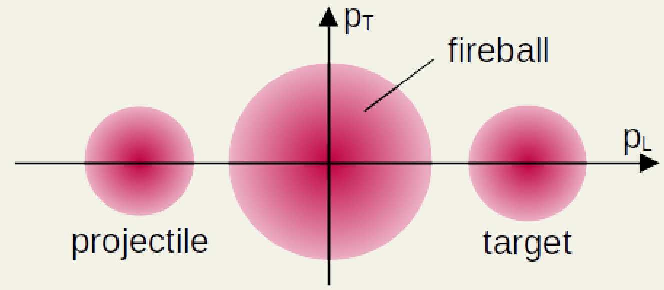


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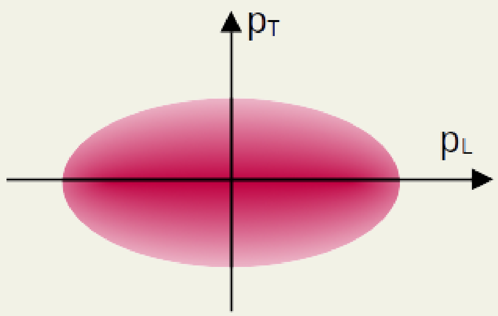
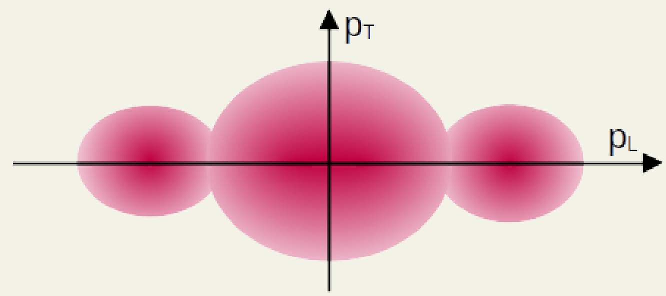


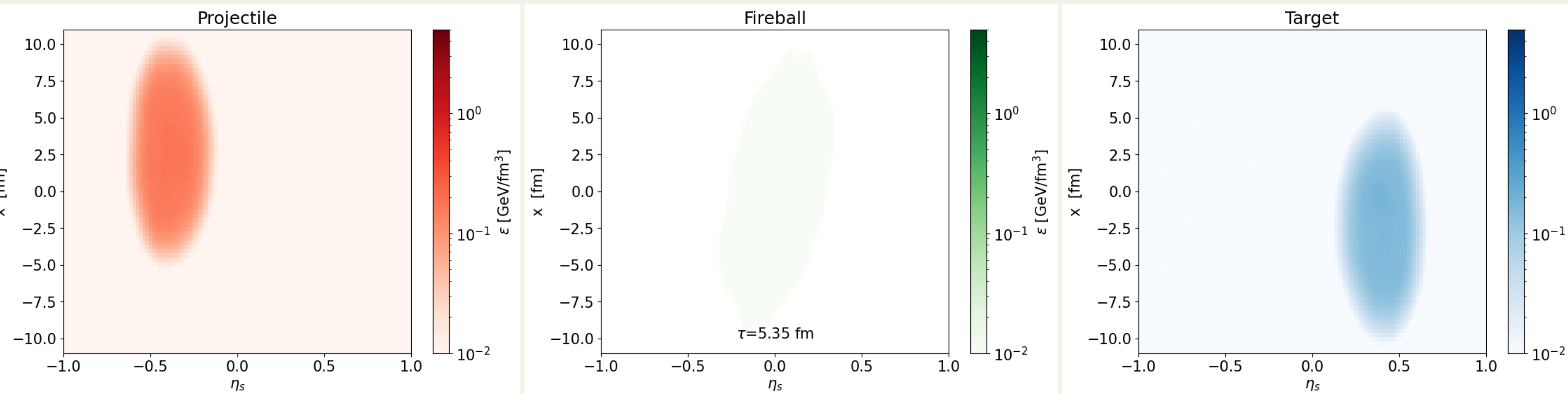

**anisotropic hydro**




**somewhat realistic distribution**

# Evolution of energy density

- **Au+Au collision at $\sqrt{s_{\mathrm{NN}}} = 7.7$ GeV**



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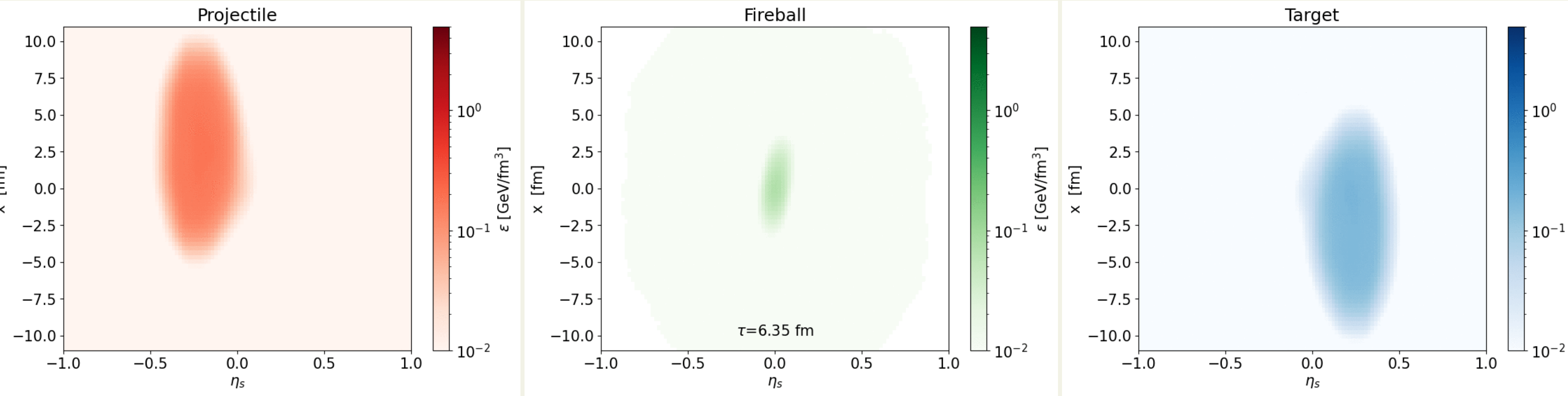

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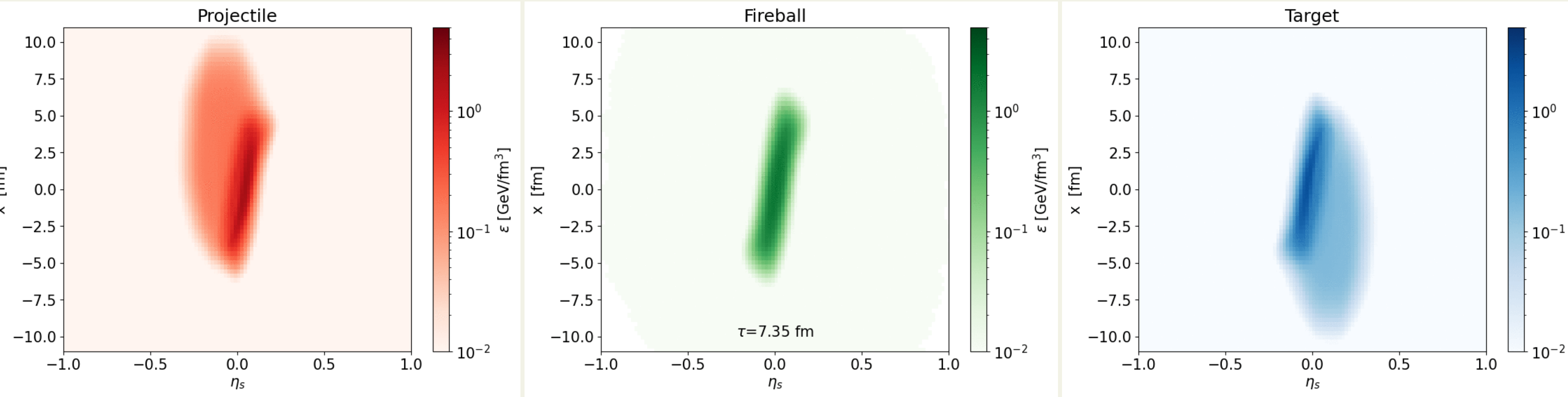

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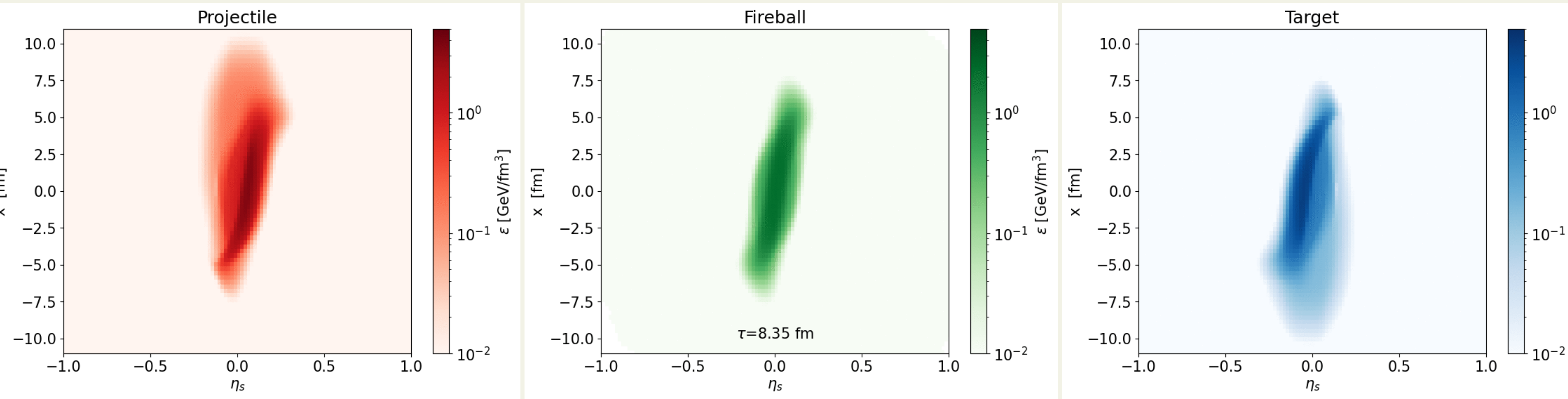

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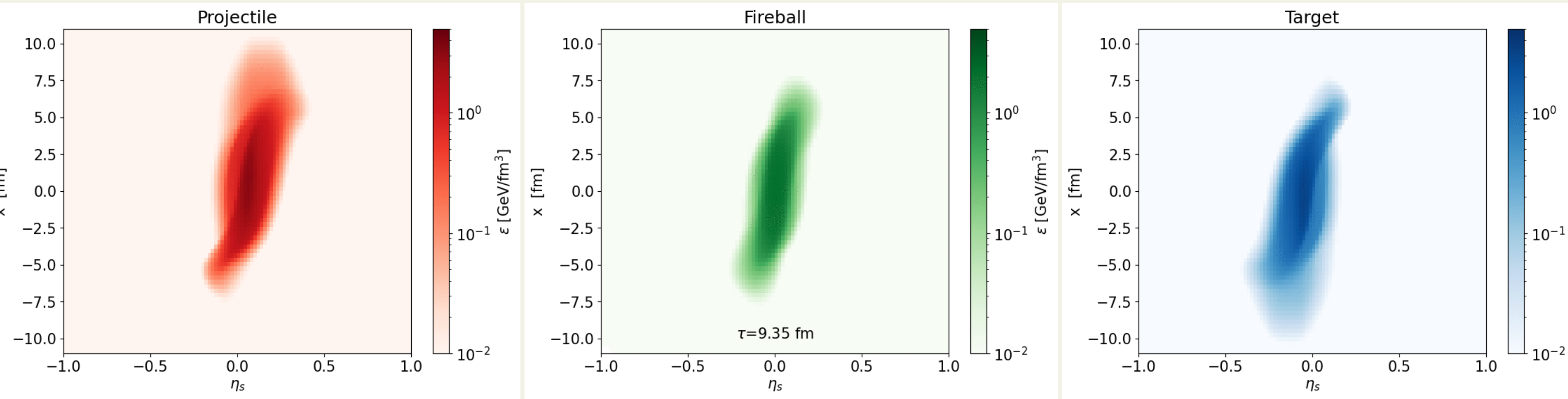

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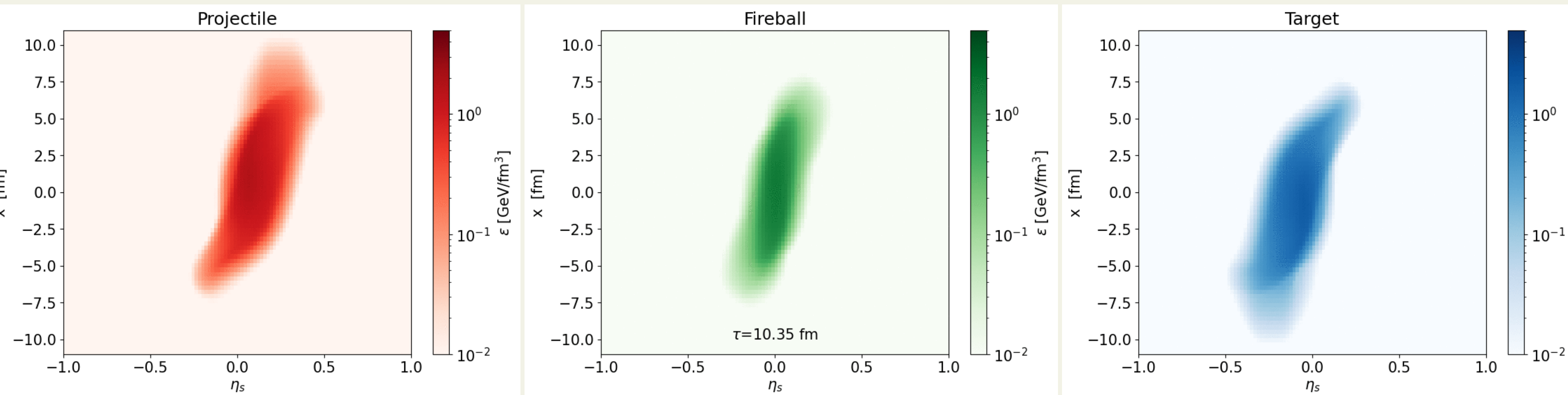

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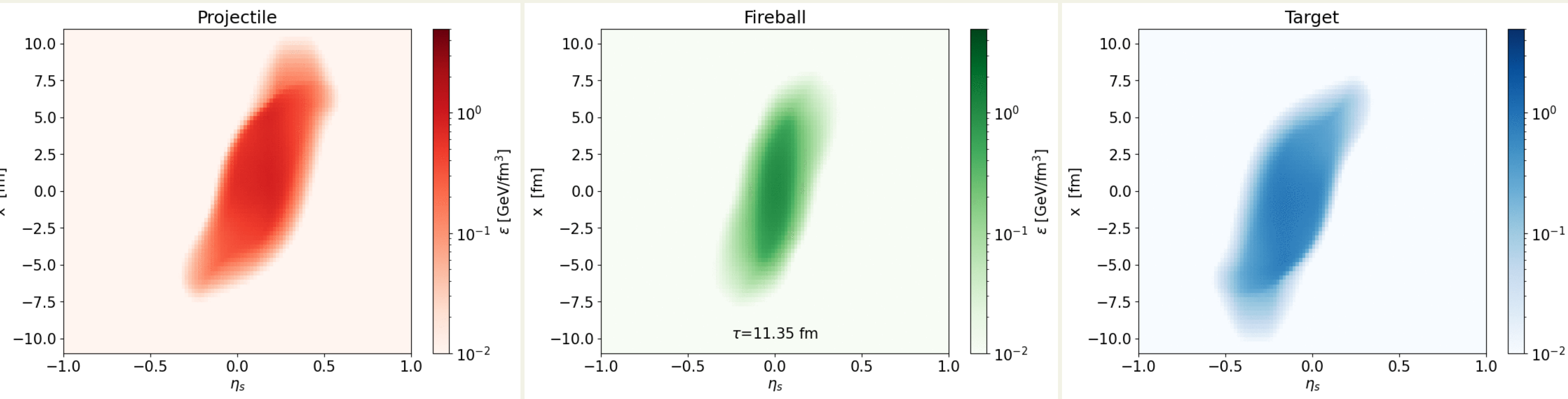

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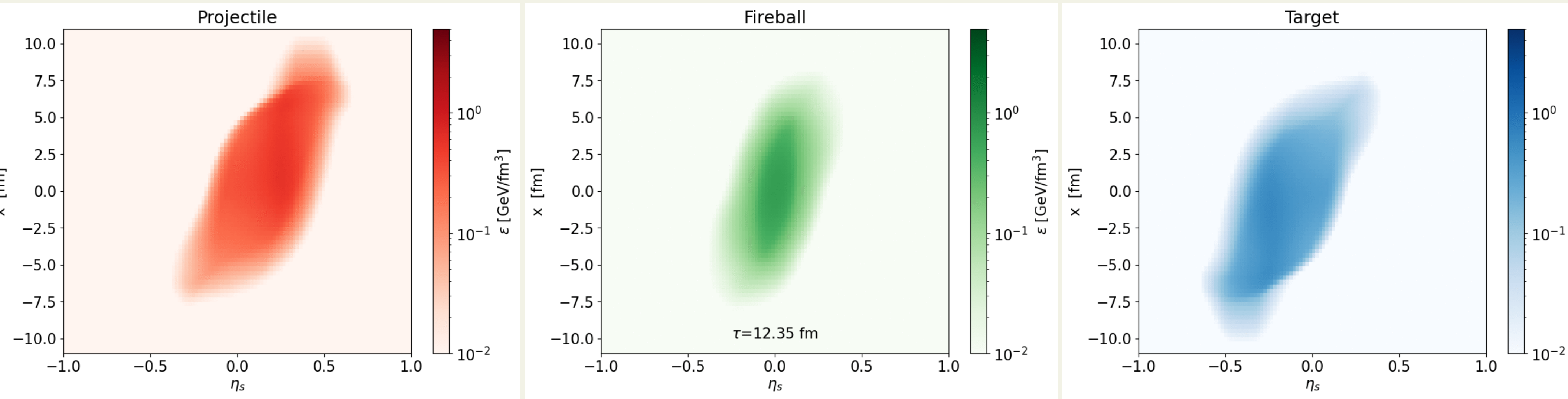

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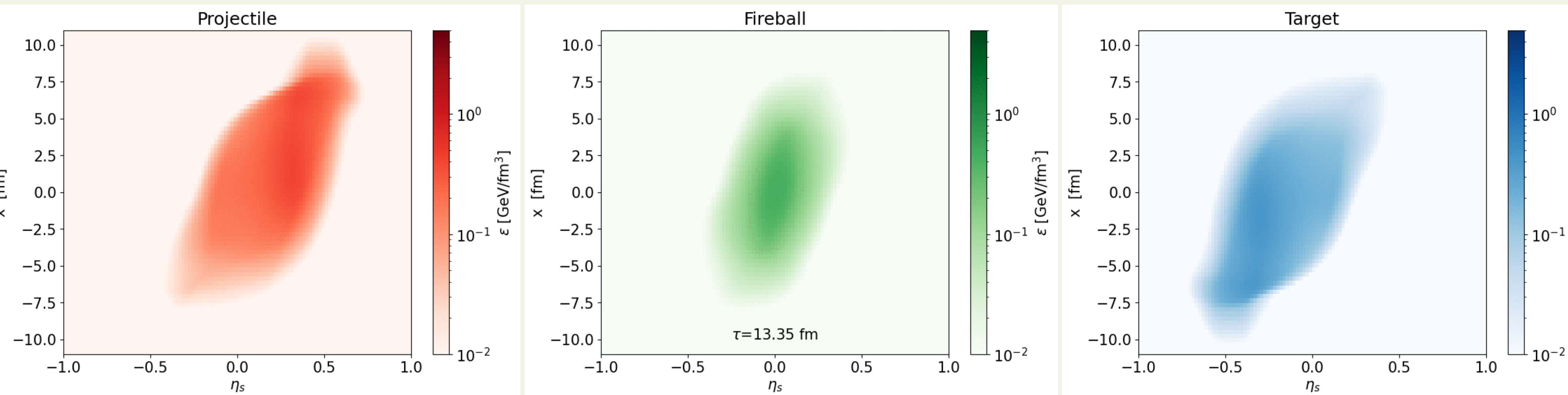

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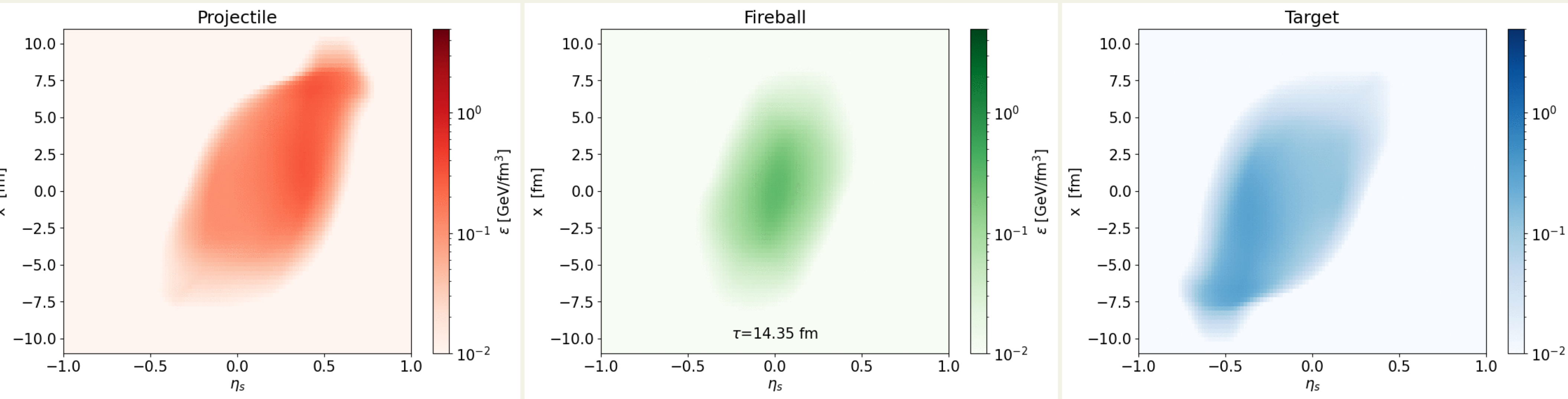


# 3-fluid dynamics

$$\partial_\mu T^{\mu\nu}_{\rm t}(x) = -F^\nu_{\rm t}(x) + F^\nu_{\rm ft}(x)$$
$$\partial_\mu T^{\mu\nu}_{\rm p}(x) = -F^\nu_{\rm p}(x) + F^\nu_{\rm fp}(x)$$
$$\partial_\mu T^{\mu\nu}_{\rm fb}(x) = F^\nu_{\rm p}(x) + F^\nu_{\rm t}(x) - F^\nu_{\rm fp}(x) - F^\nu_{\rm ft}(x)$$

- **interaction between target and projectile: friction terms** $-F^\nu_{\rm t}(x)$ **and** $-F^\nu_{\rm p}(x)$
- **interaction between fireball and target/projectile: friction terms** $F^\nu_{\rm fp}(x)$ **and** $F^\nu_{\rm ft}(x)$

# Friction from kinetic theory

**Boltzmann equation for three fluids**

$$p^\mu \partial_\mu f_i = C_i[f_p, f_t, f_f] = \sum_{j,k} C_i^{jk}[f_j, f_k], \qquad i, j, k \in \{p, t, f\}$$

$C_i^{jk}$**: change in distribution/fluid** $i$ **due to interactions of particles in** $j$ **and** $k$

**for given** $C_i^{jk}$**, friction obtained as**

$$\partial_\mu T_i^{\mu\nu} = \int \frac{\mathrm{d}^3 p}{p^0} p^\nu C_i = F_i^\nu\,, \quad \partial_\mu J_{B,i}^\mu = B_i \int \frac{\mathrm{d}^3 p}{p^0} C_i = R_{B,i}$$

# Friction from kinetic theory

**collision integrals in terms of scattering cross sections**

$$C_i^{ij}[f_i, f_j](p_i) \;=\; \int \mathrm{d}^3 p_j\, p_i^0 \Big[ \underbrace{-f_i(p_i) f_j(p_j) v_{\mathrm{rel}} \sigma_{ij\to X}}_{\text{loss}} + \underbrace{\int \mathrm{d}^3 q_i f_i(q_i) f_j(p_j) v_{\mathrm{rel}} \frac{\mathrm{d}\sigma_{ij\to iX}}{\mathrm{d}^3 p_i}}_{\text{gain}} \Big]$$

**from these, approximative friction formulae are derived**

<u>**problems:**</u>

- **cross sections may not be fully measured in experiment**
- **what stays in a fluid, what's moved to another?**
- **d.o.f. change in deconfinement transition**

# Csernai approach

**Csernai, Lovas, Maruhn, Rosenhauer, Zimányi, Greiner PRC 26, 149 (1982)**

- **all that scatters goes to the fireball**

- **projectile and target stay cold**

$$F^{\mu}_{p/t} = u^{\mu}_{p/t}\, m_N\, R^B_{p,t}(\sigma(\sqrt{s}))$$

# Csernai approach

Csernai, Lovas, Maruhn, Rosenhauer, Zimányi, Greiner PRC 26, 149 (1982)

- all that scatters goes to the fireball
- projectile and target stay cold

But:

- no baryon transparency!

# Csernai approach

Csernai, Lovas, Maruhn, Rosenhauer, Zimányi, Greiner PRC 26, 149 (1982)

- all that scatters goes to the fireball
- projectile and target stay cold

But:

- no baryon transparency!
- dynamical initialization is analogous to this approach!
- finite formation time & spatial distribution $\Rightarrow$ baryon transparency

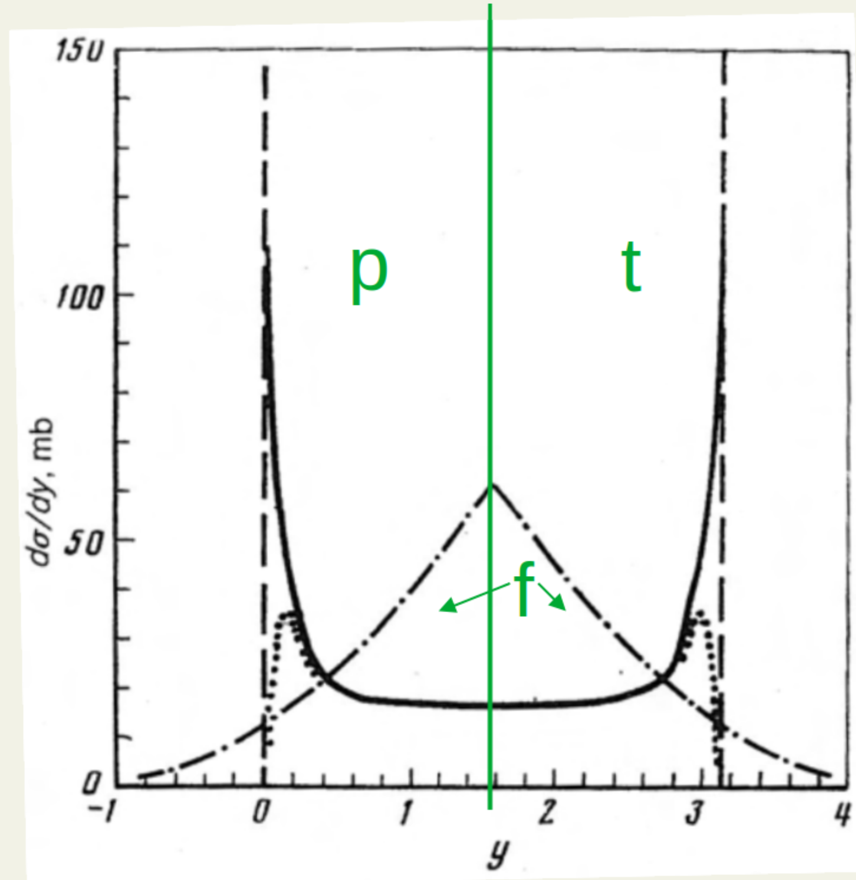
# Satarov/Ivanov approach

**Ivanov, Russkikh, Toneev PRC 73, 044904 (2006)**

- **N+N scattering: N strongly peaked at ingoing rapidities, $\pi$ at midrapidity $\Rightarrow$ in p-t friction: N stay in p/t, $\pi$ go to f**

- **$\pi + N$ mostly resonance formation $\Rightarrow$ all outgoing particles from p-f friction go to p**

- **uncertainty in deconfined phase: densities multiplied with $\sqrt{s}$-dependent prefactor**



# Satarov/Ivanov friction terms

**Projectile-target friction:**

$$F^{\nu}_{\alpha} = \rho^b_p \xi_h(s_{pt}) \rho^b_t \xi_h(s_{pt}) m_N V^{pt}_{\text{rel}} [(u^{\nu}_{\alpha} - u^{\nu}_{\bar{\alpha}}) \sigma_P(s_{pt}) + (u^{\nu}_p + u^{\nu}_t) \sigma_E(s_{pt})]$$

**Projectile/target-fireball friction:**

$$F^{\nu}_{f\alpha} = \rho^b_{\alpha} \xi_{f\alpha}(s_{f\alpha}) V^{f\alpha}_{\text{rel}} \frac{T^{0\nu}_{f(eq)}}{u^0_f} \sigma^{N\pi\to R}_{tot}(s_{f\alpha})$$

**Fitting factors:**

$$\xi_h = 1.8 \sqrt{\frac{2m_N}{\sqrt{s_{pt}}}}, \qquad \xi_{f\alpha} = 0.15 \frac{m^2_N}{s_{f\alpha}},$$

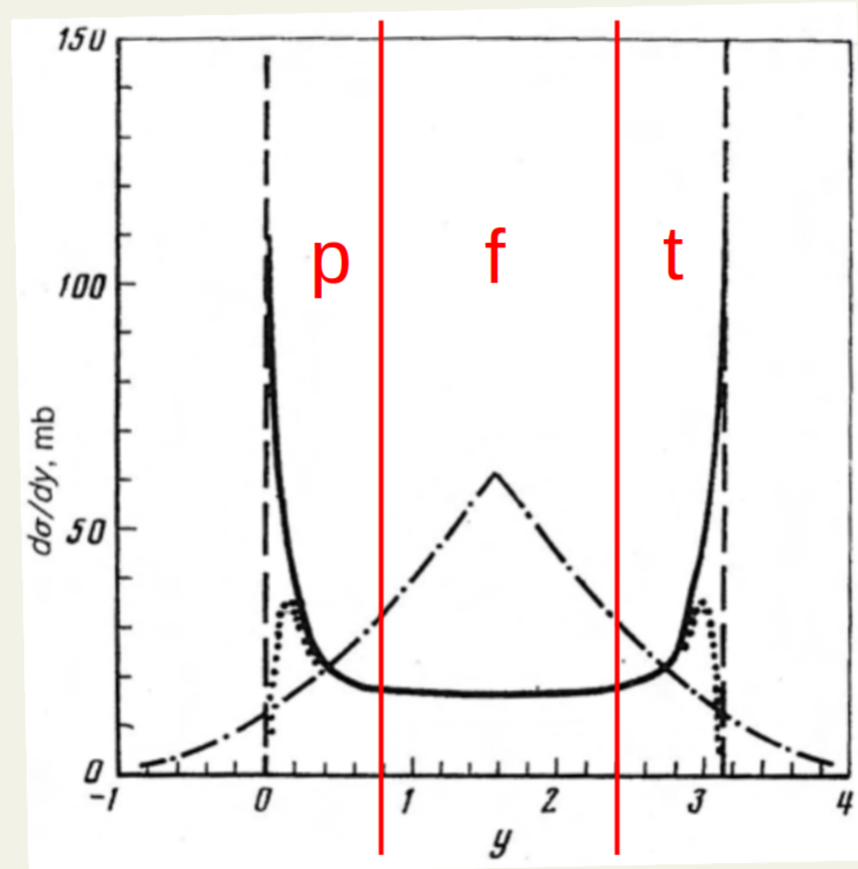
**pros:** only need total cross sections
can describe the double peak in baryon distributions!
**cons:** $\mu_B = 0$ in fireball

# modified Satarov/Ivanov approach

- for our purposes:
  need high $\mu_B$ also in fireball!

- idea: divide outgoing N from N+N
  into 3 regions
  $\Rightarrow$ modified p+t friction moves B
  to fireball

  but: need doubly differential cross sections! $(y, E)$



# modified friction terms

**Projectile-target friction:**

$$F^{\nu}_{\alpha} = \rho^{b}_{p}\rho^{b}_{t} m_N V^{pt}_{\text{rel}}[(u^{\nu}_{\alpha} - u^{\nu}_{\bar{\alpha}})\bar{\sigma}_P(s_{pt}) + (u^{\nu}_{p} + u^{\nu}_{t})\bar{\sigma}_E(s_{pt}) - u^{\nu}_{\alpha}\bar{\sigma}_B(s_{pt})]$$

- **effective cross sections modified**
- **term due to baryon transfer added**
- **no $\sqrt{s}$ dependence in fitting factors**
- **projectile/targer-fireball friction unchanged**

# MUlti-Fluid simulation for Fast IoN collisions (MUFFIN)

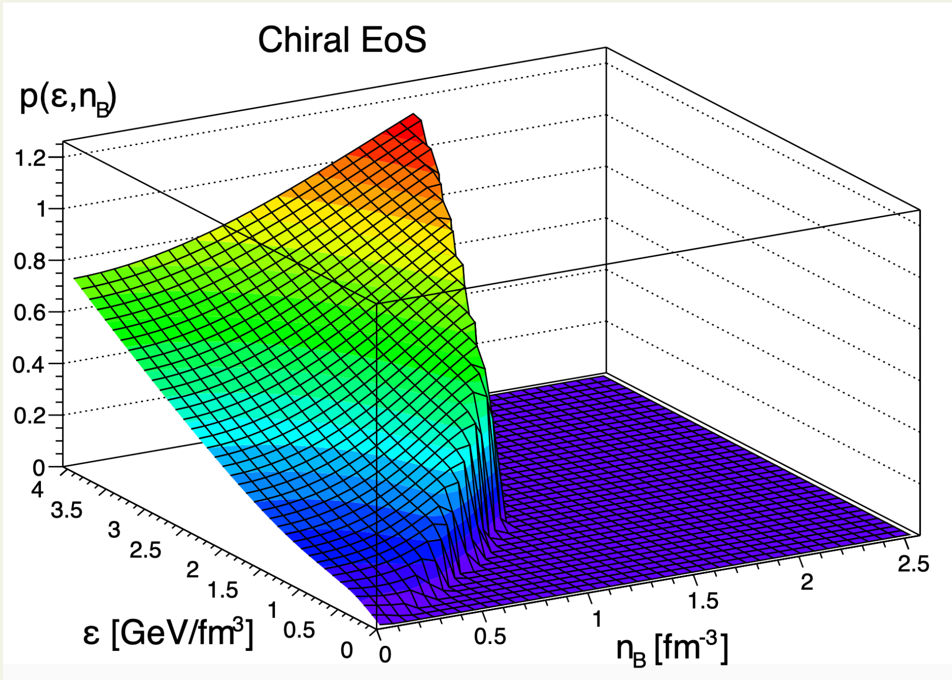
- **implementation of 3-fluid dynamics based on vHLLE**
- **coupled to SMASH hadron cascade (MUFFIN-SMASH)**
- **ideal only so far**

# Equation of State

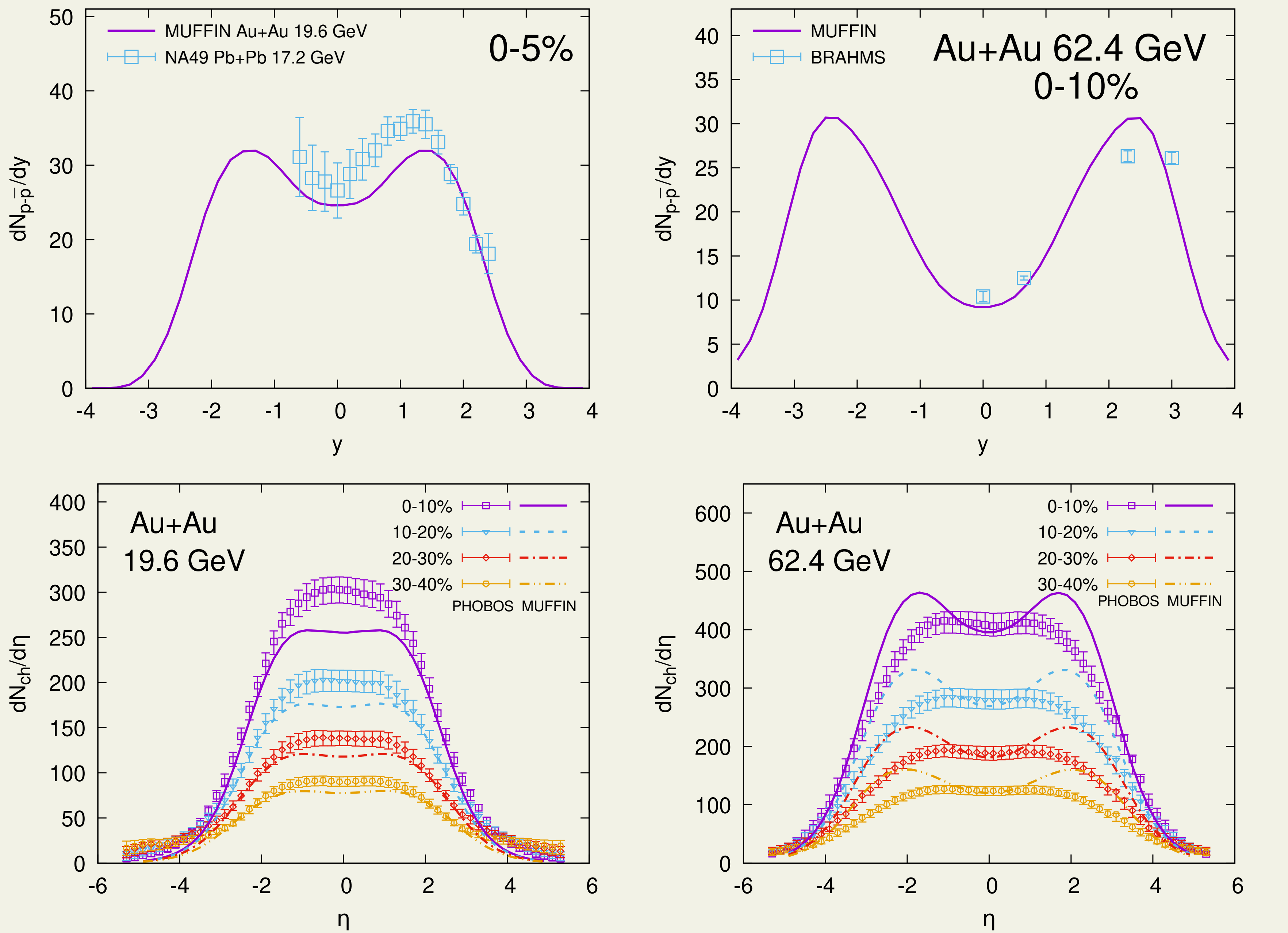
**So far we've used chiral model Equation of State**

**[J. Steinheimer *et al.*, J. Phys. G 38 (2011) 035001]**

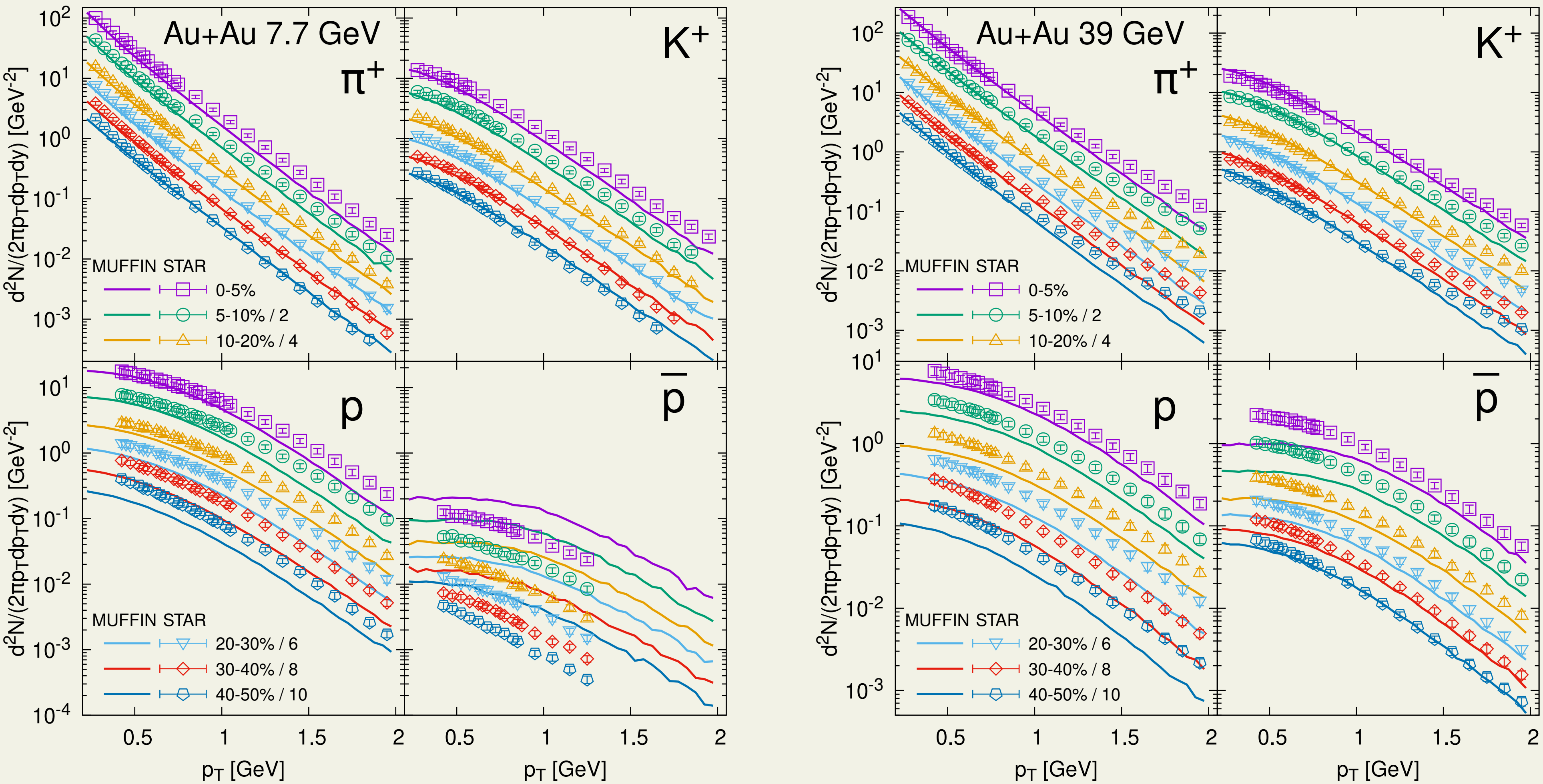
- **good agreement with lattice QCD at $\mu_B = 0$**
- **crossover type phase transition do deconfined phase at all $\mu_B$**



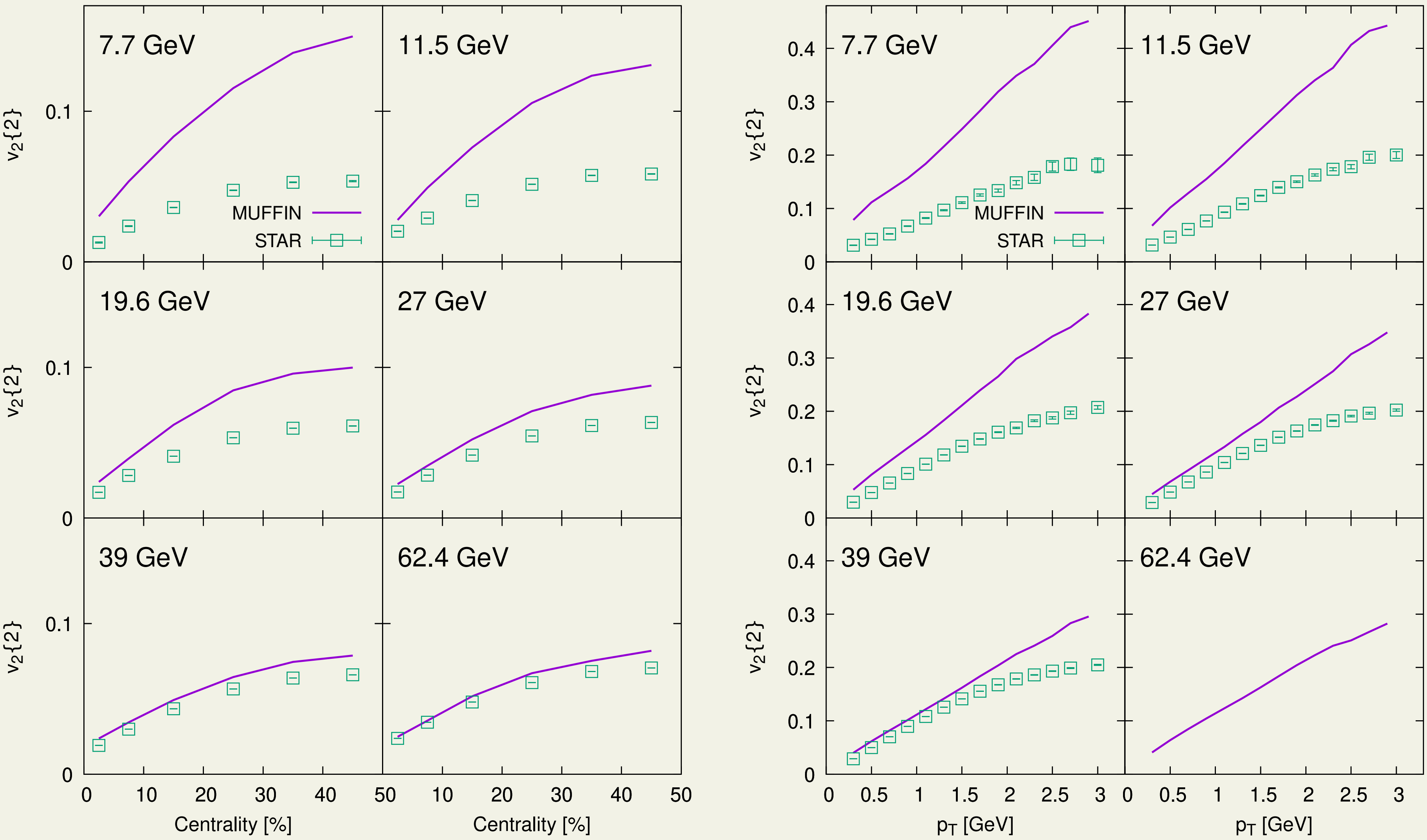
# Results: (pseudo)rapidity distributions



# Results: transverse momentum distributions



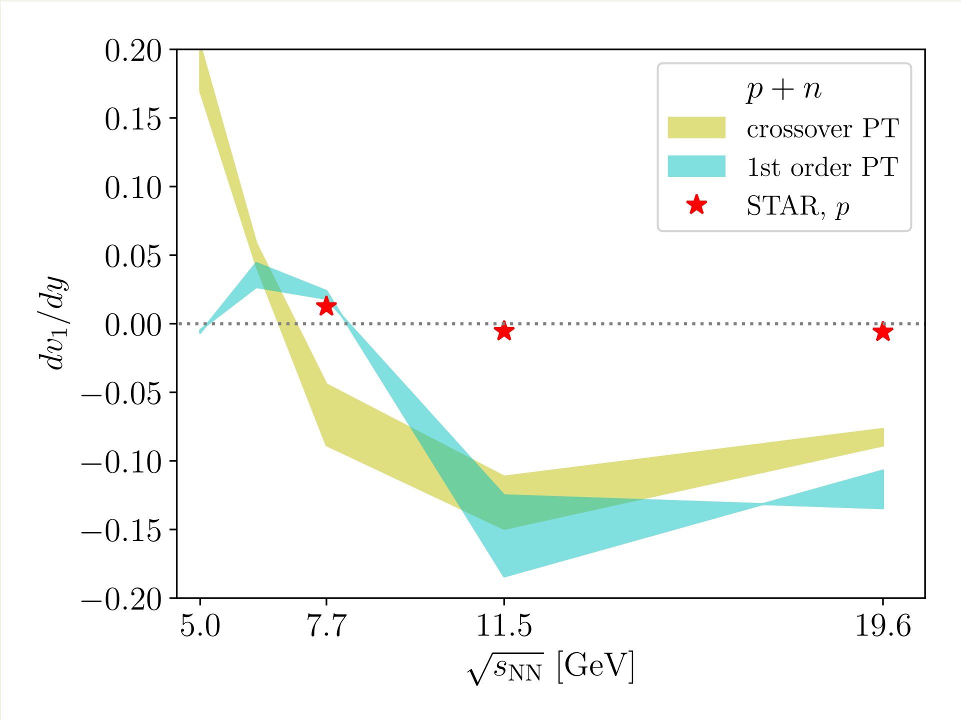
# Results: elliptic flow

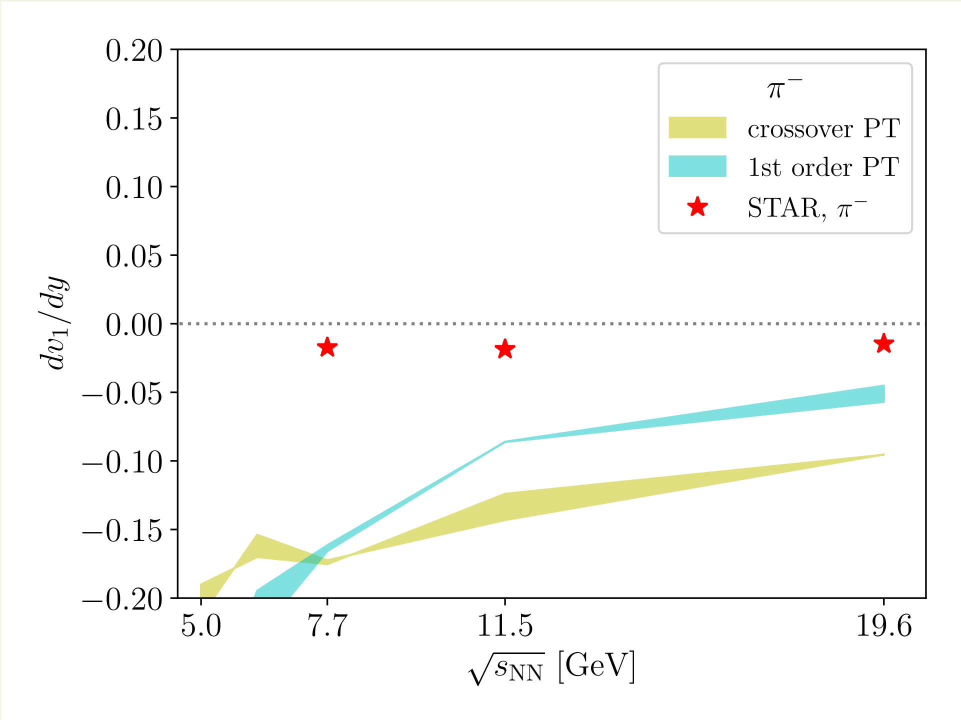



**Viscosity not yet included!**

# Results: directed flow

- old results: $v_1$ should be sensitive to EoS




- only weak sensitivity to EoS
- much stronger $v_1$ than measured

# Dissipation

$$T_i^{\mu\nu} = \epsilon_i u_i^\mu u_i^\nu + P_i \Delta_i^{\mu\nu} + \pi_i^{\mu\nu}, \qquad i \in \{t, p, f\}$$

$$\partial_\mu T_i^{\mu\nu} = \partial_\mu(\epsilon_i u_i^\mu u_i^\nu) + \partial_\mu(P_i \Delta_i^{\mu\nu}) + \partial_\mu \pi_i^{\mu\nu} = F_i^\nu$$

**where** $\pi_i^{\mu\nu}$ **obeys**

$$u^\alpha \partial_\alpha \pi_i^{\mu\nu} = -\frac{1}{\tau_\pi}\left(\pi_i^{\mu\nu} - 2\eta \nabla^{\langle\mu} u_i^{\nu\rangle}\right) + \cdots$$

**independent of** $F_i^\mu$**?**

$\Longrightarrow$ **corrections to the evolution equations needed**

# Dissipation

- **shear stress ($\pi^{\mu\nu}$) evolution equation (similarly $\Pi$ and $V^{\mu}$):**

$$D\pi^{\langle\mu\nu\rangle} = \cdots + \int \frac{\mathrm{d}^3p}{(2\pi)^3E(p)^2} p^{\langle\mu}p^{\nu\rangle} C_{\alpha}^{\beta,\gamma}[f_\beta, f_\gamma]$$

- **dissipative quantities modify $f_\alpha$:**

$$f_\alpha(p) = f_{\alpha,\mathrm{eq}}(p)\left[1 - \Pi\frac{3}{m^2}\mathcal{H}_0^{(0)}(p) + V^\mu p_\mu \mathcal{H}_0^{(1)}(p) + \pi^{\mu\nu}p_\mu p_\nu \mathcal{H}_0^{(2)}(p)\right]$$

$\Rightarrow$ **affects all terms involving $C_{\alpha}^{\beta\gamma}[f_\beta, f_\gamma]$, even energy & momentum transfer!**

# MUFFIN-SMASH

- **MUlti-Fluid simulation for Fast IoN collisions (MUFFIN)**
  - **three-fluid hydro to model collisions at RHIC BES energies**
  - **projectile, target, produced particles described as separate fluids**
- **coupled to SMASH hadron cascade**
- **rough reproduction of rapidity and $p_T$ distributions**
- **overshoots anisotropies—no viscosity**

# MUFFIN-SMASH

- **MUlti-Fluid simulation for Fast IoN collisions (MUFFIN)**

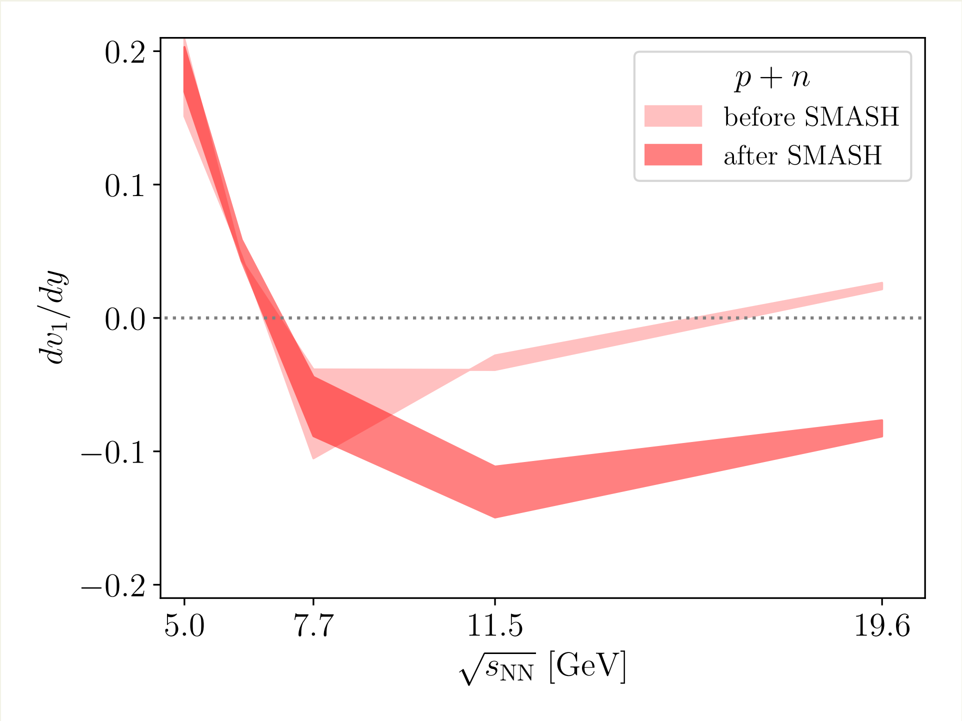
- **work in progress—stay tuned!**

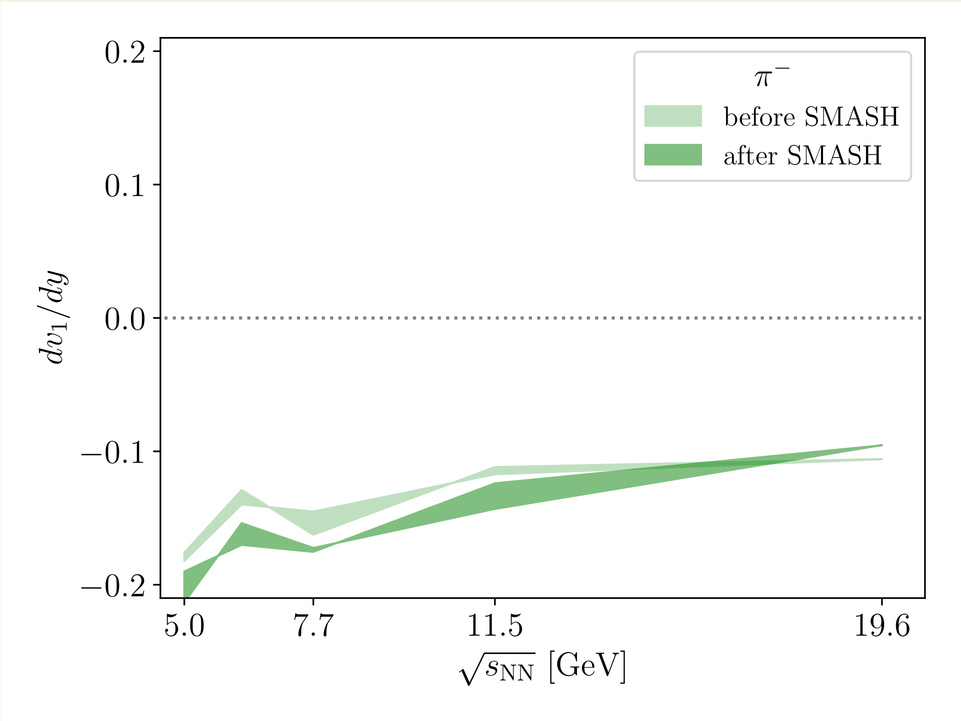
# Extreme QCD 2025
# University of Wroclaw, Poland


July 2-4, 2025

# Results: directed flow

- **old results: $v_1$ should be sensitive to EoS**




- **hadron cascade has a strong effect on $v_1$**

# Switching from fluid to cascade (particlization)

- **determine "effective energy" $\varepsilon_{sw}$ from diagonalised $T^p_{\mu\nu} + T^f_{\mu\nu} + T^t_{\mu\nu}$**

- **one particlisation hypersurface where $\varepsilon_{sw} = 0.5$ GeV/fm$^3$**

- **exclude parts of hypersurface where matter flows in**