Fluid dynamic and transport modelling of HIC at FAIR energies

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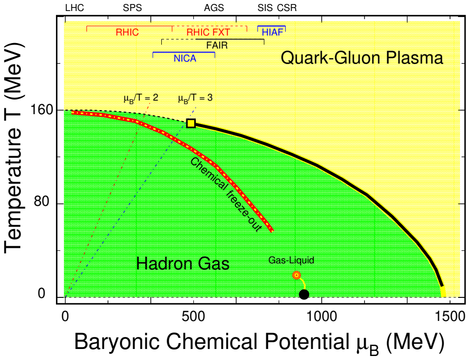
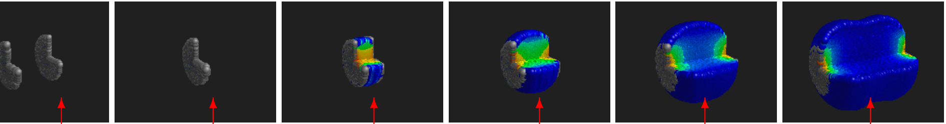
A generic motivation slide



taken from: Bedangadas Mohanty, Nu Xu, arXiv:2101.09210

Heavy ion collision in pictures

https://www.jyu.fi/fysiikka/tutkimus/suurenergia/urhic/anim1.gif/image_view_fullscreen



incoming nuclei

initial stage of the reaction

dense medium (QGP)

expansion

phase transition

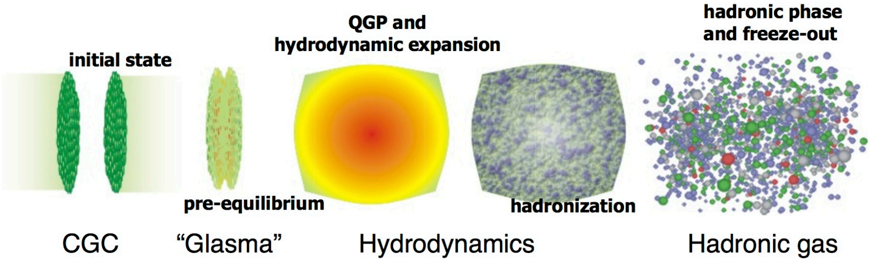
hadrons

Hybrid models

fluid dynamics + hadronic transport

Fluid dynamics $\Leftarrow$ (arbitrary) equation of state $\Leftarrow$ phase transition to QGP.

Most of the existing hybrids come from "high energies": LHC or top RHIC:

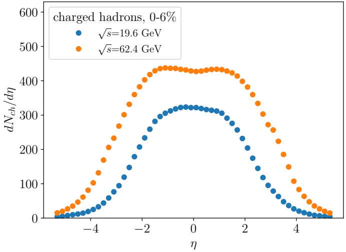


- There is a relatively clear time separation between the initial state and the fluid stage
- Symmetries: longitudinal boost invariance, near-zero baryon density.

Adapting hybrids to lower energies - RHIC BES

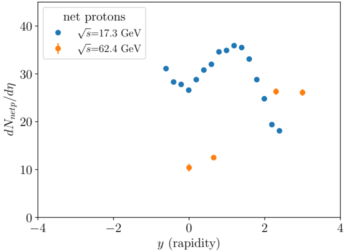
There is no boost invariance

- 3D initial state
  (just a different input)
- 3D hydro evolution
  (most of the codes are 3D)



Baryon and electric charge densities are significant

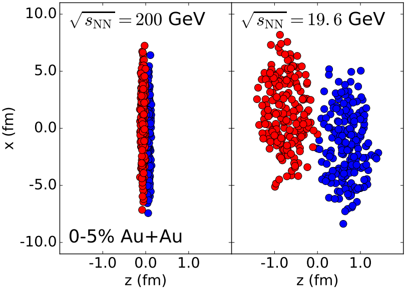
- EoS at finite n_B for hydro evolution
  (just a different input)



Adapting for *even* lower energies - RHIC FXT, FAIR

The paradigm of "thin pancakes" gradually loses its applicability.

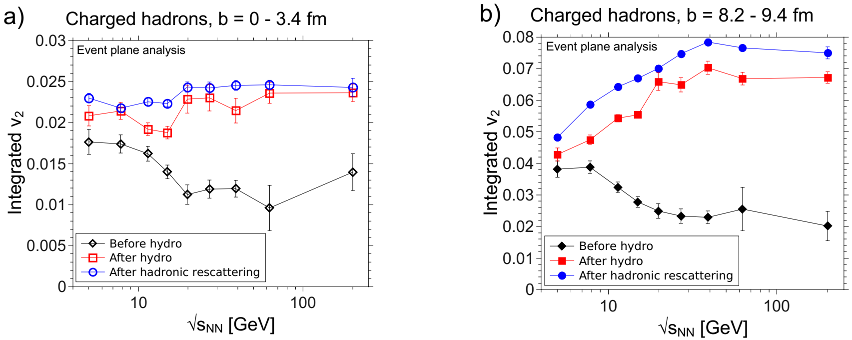
- **Nuclei pass through each other slowly**
  (the passage can last as long as subsequent fluid stage)
- There is no clear separation of the initial state and the fluid stage.
- Need to model the early-time dynamics differently!



picture credit: C. Shen, B. Schenke, Phys. Rev. C 97, 024907 (2018)

A lot of evolution is happening before the nuclei have completely passed through each other.

Simulation: UrQMD IS + ideal hydro + UrQMD afterburner
J. Auvinen, H. Petersen, Phys.Rev.C 88 (2013), 064908



Not only the initial state model is important, but the way of switching from initial state to fluid-dynamical picture.

Taxonomy of existing hybrids for RHIC BES and FAIR energies

In terms of initial state

There is abundance of models for $\sqrt{s_{\mathrm{NN}}}$ =7...62 GeV!

There are different strategies to initialise the fluid stage:

- Parametrise the initial state from the fit to experimental data.
- Run transport, wait until all primary scatterings are over, and melt their products $\Rightarrow$ miss the sensitivity to the early, dense stage of evolution (**sandwich approach**)
- Run transport and fluid dynamics in a tandem and fluidise dense regions of fireball dynamically (**dynamical fluidisation**)
- Assume everything is fluid from the beginning (**multi-fluid dynamics**)

All the models feature 3D initial state, 3D hydro and EoS at finite n_B.

- Final conditions: particlization at fixed energy density + hadronic cascade (except for multi-fluid models)

Existing hybrids do not contain criticality, and models with criticality aren't hybrids.

Hybrids and their initial states

Type 1: Parametrized initial state

Chun Shen, Sahr Alzhrani, Phys.Rev.C 102 (2020) 1, 014909

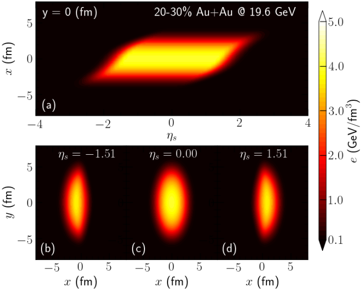
superMC (initial state) + MUSIC (hydro) + iSS (hadron sampling) + UrQMD

$$e(x,y,\eta_s;y_{CM}) = \mathcal{N}_e(x,y) \times \exp\left[-\frac{(|\eta_s - y_{CM}| - \eta_0)^2}{2\sigma_\eta^2}\theta(|\eta_s - y_{CM}| - \eta_0)\right].$$

$$\mathcal{N}_e(x,y) = \frac{M(x,y)}{2\sinh(\eta_0) + \sqrt{\frac{\pi}{2}}\sigma_\eta e^{\sigma_\eta^2/2} C_\eta} C_\eta = e^{\eta_0}\mathrm{erfc}\left(-\sqrt{\frac{1}{2}}\sigma_\eta\right) + e^{-\eta_0}\mathrm{erfc}\left(\sqrt{\frac{1}{2}}\sigma_\eta\right).$$

+scaling initial flow, $v_z = z/t$

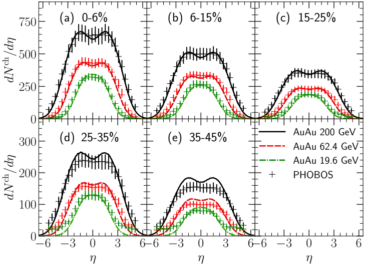
$\sqrt{s_{NN}}$ (GeV)	τ_0 (fm/c)	η_0	σ_η	$\eta_{B,0}$	$\sigma_{B,\mathrm{in}}$	$\sigma_{B,\mathrm{ou}}$
AuAu & dAu @ 200	1.0	2.5	0.6	3.5	2.0	0.1
AuAu & dAu @ 62.4	1.0	2.25	0.3	2.7	1.9	0.2
AuAu & dAu @ 39	1.3	1.9	0.3	2.2	1.6	0.2
AuAu@27	1.4	1.6	0.3	1.8	1.5	0.2
AuAu & dAu @ 19.6	1.8	1.3	0.3	1.5	1.2	0.2
AuAu@14.5	2.2	1.15	0.3	1.4	1.15	0.2
AuAu@7.7	3.6	0.9	0.2	1.05	1.0	0.1
PbPb@17.3	1.8	1.25	0.3	1.6	1.2	0.2
PbPb@8.77	3.5	0.95	0.2	1.2	1.0	0.1

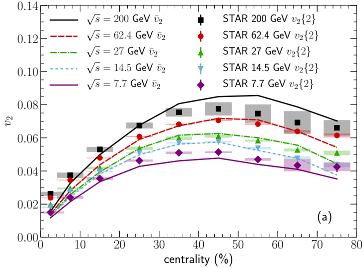


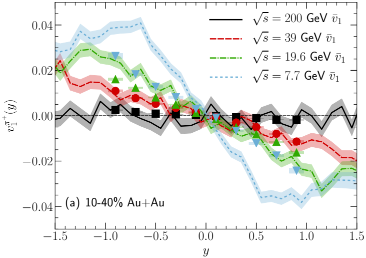
EoS: NEOS-BQS, 1902.05095.

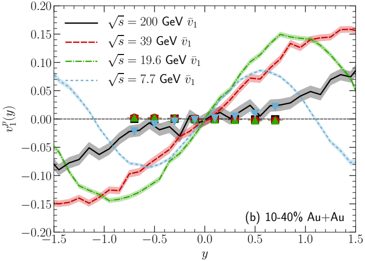
Chun Shen, Sahr Alzhrani, Phys.Rev.C 102 (2020) 1, 014909

Carve the initial state to fit the data.









Type 2: Transport model for initial state (sandwich approach)

`UrQMD` (initial state) + fluidisation at fixed τ_0 + `vHLLE` (hydro) + `UrQMD`
IK, Huovinen, Petersen, Bleicher, Phys.Rev. C91 (2015) no.6, 064901

- Initial state from $t=0$ till $\tau=\tau_0$: `UrQMD`

- At $\tau=\tau_0$:
  Gaussian smearing of energy, momentum and charges at fluidization:

$$\Delta P^{\alpha}_{ijk} = P^{\alpha}\cdot C\cdot\exp\left(-(\Delta x_i^2+\Delta y_j^2)/R_\perp^2 - \Delta\eta_k^2\gamma_\eta^2\tau_0^2/R_\eta^2\right)$$
$$\Delta N^{0}_{ijk} = N^{0}\cdot C\cdot\exp\left(-(\Delta x_i^2+\Delta y_j^2)/R_\perp^2 - \Delta\eta_k^2\gamma_\eta^2\tau_0^2/R_\eta^2\right)$$

  from each hadron that crosses the $\tau=\tau_0$ surface

- Assume that the resulting energy and momentum corresponds to a flowing fluid:

$$E/\Delta V = (\varepsilon+p)(u^0)^2 - p, \quad P^i/\Delta V = (\varepsilon+p)u^0u^i$$

- Run fluid dynamics.

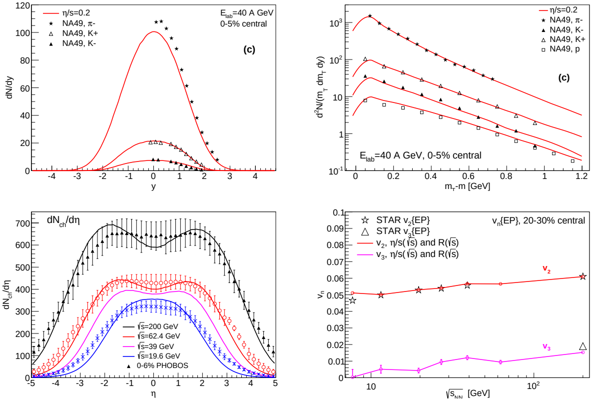
- ...

- compute the observables

Type 2: Transport model for initial state (sandwich approach)

UrQMD (initial state) + fluidisation at fixed τ_0 + vHLLE (hydro) + UrQMD
IK, Huovinen, Petersen, Bleicher, Phys.Rev. C91 (2015) no.6, 064901

Fix parameters of fluidisation procedure ($R_\perp$, R_η) and shear viscosity η/s to fit the data.



- ⇒ decent agreement with a mix of RHIC BES + NA49 + PHOBOS data

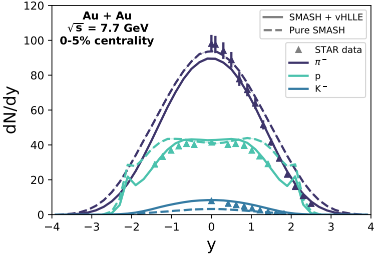
EoS: chiral model EoS.

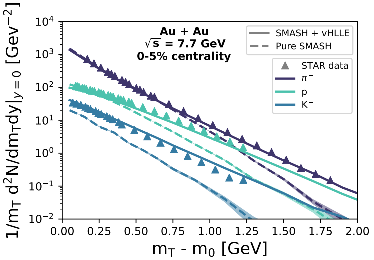
Type 2: Transport model for initial state

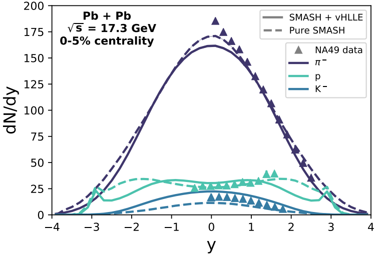
SMASH (initial state) + fluidisation at fixed τ_0 + vHLLE (hydro) + SMASH

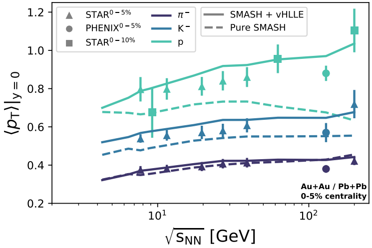
A. Schäfer, IK, Xiang-Yu Wu, J. Hammelmann, H. Elfner, Eur.Phys.J.A 58 (2022) 11, 230

Same set of free parameters.





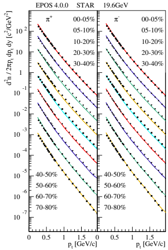


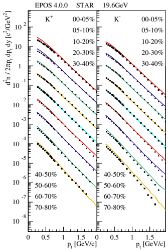


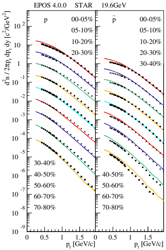
EPOS 4.0.0

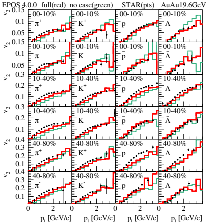
Werner, Jahan, IK, Pierog, Stefaniak, Vintache, 2401.11275

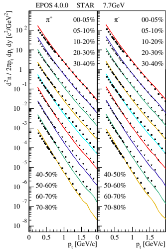
EPOS IS + fluidisation at τ_0 + vHLLE + UrQMD. core-corona included. BEST EoS

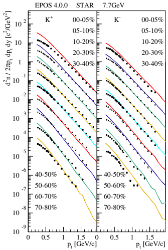


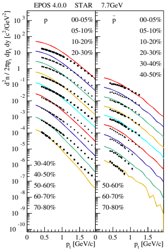


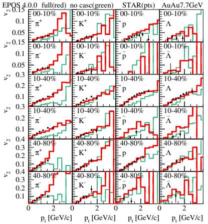












Type 3: Dynamical fluidisation (strings)

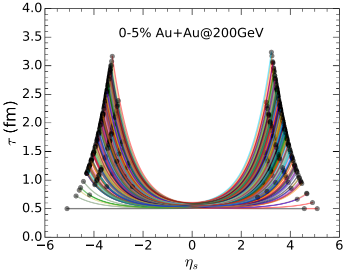
Chun Shen, Bjoern Schenke, Phys. Rev. C 97, 024907 (2018)

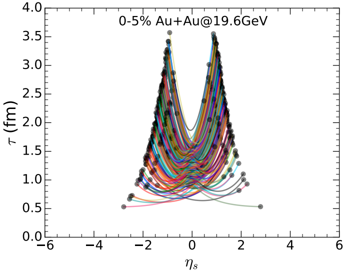
Decelerated strings melt into fluid, at different proper times. EoS: unclear.

- 1 string per colliding nucleon pair
- The string ends decelerate according to:

$$\frac{dE}{dz} = -\sigma, \quad \frac{dP_z}{dz} = -\sigma$$

- Once string ends come to a halt, its evolution stops.
- The products fluidize.
- Fluid dynamics starts from the beginning of the reaction, with energy-momentum being fed in gradually.





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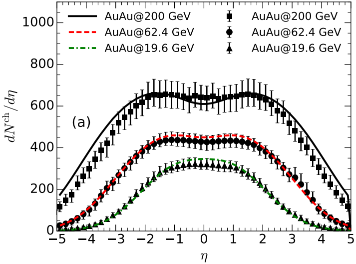
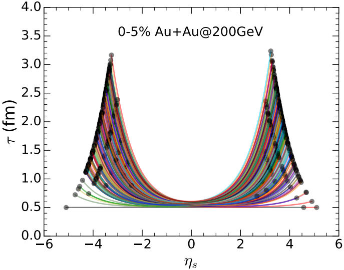
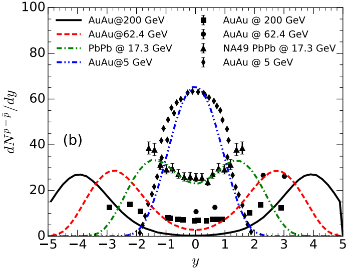
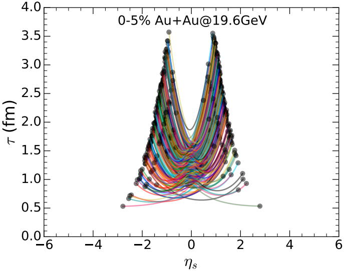








Type 3: Dynamical fluidisation (JAM)

JAM + dynamical fluidisation + hydro + JAM

Y. Akamatsu, M. Asakawa, T. Hirano, M. Kitazawa, K. Morita, K. Murase, Y. Nara, C. Nonaka, A. Ohnishi, Phys. Rev. C 98, 024909 (2018)

JAM IS: HIJING string excitation + PYTHIA6 fragmentation + rescatterings of produced hadrons.

Hadrons are converted to fluid if the local energy density $e > e_f = 0.5$ GeV/fm^3.

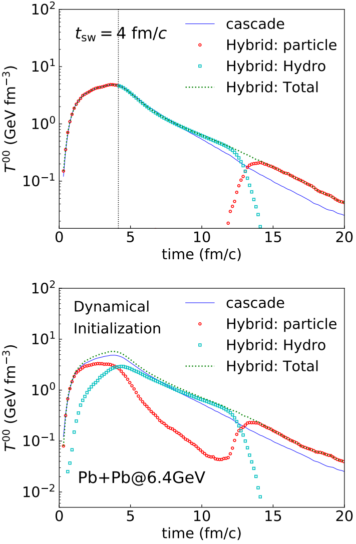
$$\partial_\mu T_f^{\mu\nu} = J^\nu, \quad \partial_\mu N_f^\mu = \rho$$

$$J^\mu(r) = \frac{1}{\Delta t}\sum_i p_i^\mu G(r - r_i(t))$$

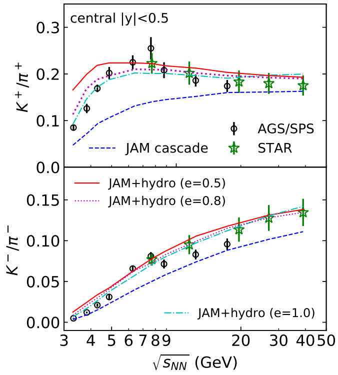
$$\rho(r) = \frac{1}{\Delta t}\sum_i B_i G(r - r_i(t))$$

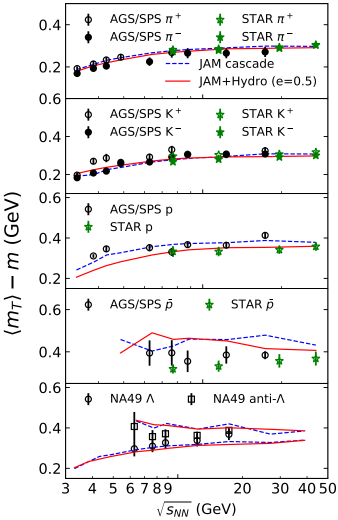
$G(r)$ is a Gaussian smearing profile.

EoS: EoS Q, a.k.a. BM+HRG EoS



JAM + dynamical fluidisation + hydro + JAM



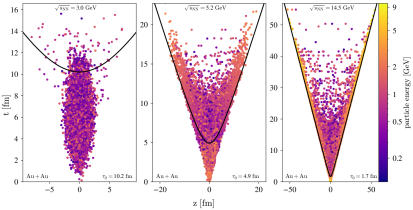


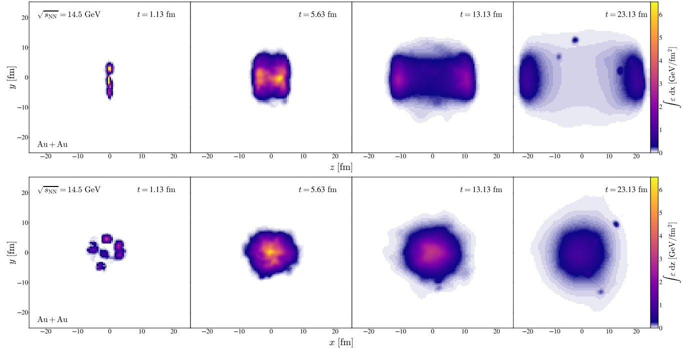
Type 3: Dynamical fluidisation (SMASH cascade)

SMASH + dynamical fluidisation + hydro + SMASH

WIP: Renan Hirayama, Zuzana Paulínyová, IK, Hannah Elfner

Same idea but with SMASH cascade.





Does transport prove the fluid dynamics is applicable?

Not really.

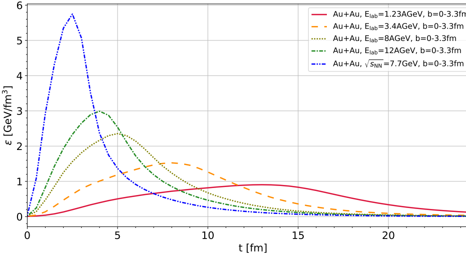
Gabriele Inghirami, Hannah Elfner, Eur. Phys. J. C 82, 796 (2022)

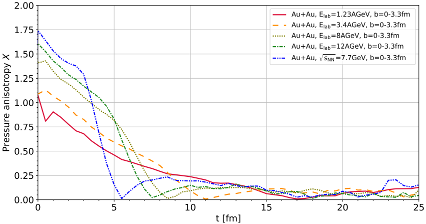
From $E_{\mathrm{lab}} = 1.23$ GeV up to $\sqrt{s} = 7.7$ GeV

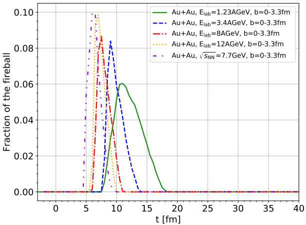
$$T^{\mu\nu} = \sum_i \frac{p_i^\mu p_i^\nu}{p_i^0} K(\mathbf{r} - \mathbf{r}_i, \mathbf{p_i})$$

$$X \equiv \frac{|\langle T_L^{11}\rangle - \langle T_L^{22}\rangle| + |\langle T_L^{22}\rangle - \langle T_L^{33}\rangle| + |\langle T_L^{33}\rangle - \langle T_L^{11}\rangle|}{\langle T_L^{11}\rangle + \langle T_L^{22}\rangle + \langle T_L^{33}\rangle},$$

$$Y \equiv \frac{3(|\langle T_L^{12}\rangle| + |\langle T_L^{23}\rangle| + |\langle T_L^{13}\rangle|)}{\langle T_L^{11}\rangle + \langle T_L^{22}\rangle + \langle T_L^{33}\rangle}$$



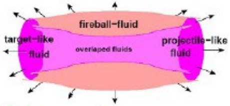




For most of the system, when simulated with transport, $T^{\mu\nu}$ does not look like a fluid.

Type 4: Everything is Fluid

3-Fluid Dynamics



Produced particles populate mid-rapidity
$\Rightarrow$ fireball fluid

Target-like fluid: $\partial_\mu J^\mu_t = 0$ $\quad$ $\partial_\mu T^{\mu\nu}_t = -F^\nu_{tp} + F^\nu_{ft}$

Leading particles carry bar. charge $\quad$ exchange/emission

Projectile-like fluid: $\partial_\mu J^\mu_p = 0,$ $\quad$ $\partial_\mu T^{\mu\nu}_p = -F^\nu_{pt} + F^\nu_{fp}$

Fireball fluid: $J^\mu_f = 0,$ $\quad$ $\partial_\mu T^{\mu\nu}_f = F^\nu_{pt} + F^\nu_{tp} - F^\nu_{fp} - F^\nu_{ft}$

Baryon-free fluid $\quad$ Source term $\quad$ Exchange

The source term is delayed due to a formation time $\tau \sim 1$ fm/c

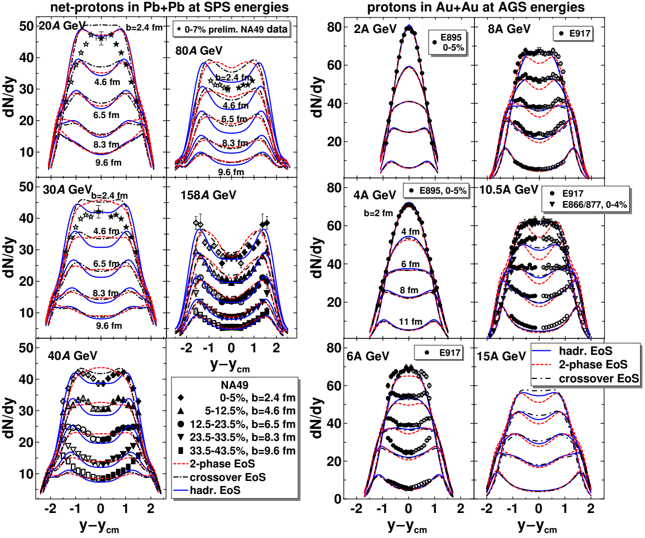
Total energy-momentum conservation:

$$\partial_\mu (T^{\mu\nu}_p + T^{\mu\nu}_t + T^{\mu\nu}_f) = 0$$

http://theory.gsi.de/~ivanov/mfd/

Yu.B. Ivanov, V.N. Russkikh and V.D. Toneev, Phys. Rev. C73, 044904 (2006)

Yu.B. Ivanov, Phys. Rev. C 87, 064904



What also works:

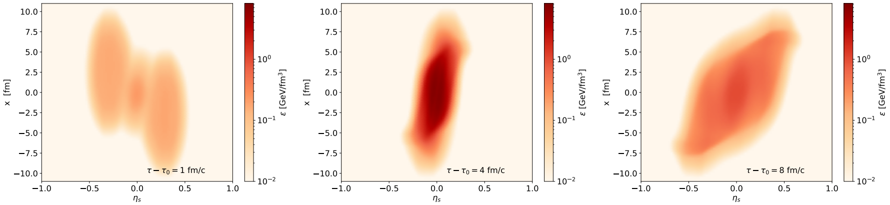
- pion dN/dy
- kaon dN/dp_T
  Yu.B. Ivanov, Phys. Rev. C 87, 064905
- proton, pion dN/dp_T
  Yu.B. Ivanov, Phys. Rev. C 89, 024903 (2014)
- elliptic flow
  Ivanov, Soldatov, Phys. Rev. C 91, 024914 (2015)

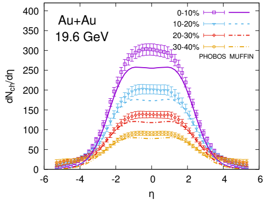
Relatively low sensitivity to the PT type in the EoS; rather worse agreement for purely hadronic EoS.

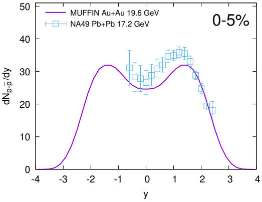
Type 4: Improved 3-fluid dynamics: MUFFIN

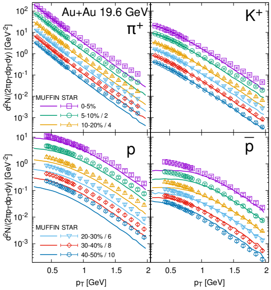
3-fluid version of vHLLE + hadron sampling + SMASH

WIP but published version corresponds to Ivanov's friction: Cimerman, IK, Tomasik, Huovinen, Phys.Rev.C 107 (2023) 4, 044902









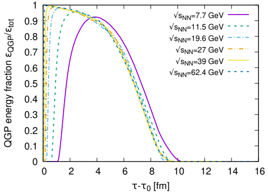
Acceptable data reproduction with chiral model EoS (crossover)

Some conclusions from the modelling

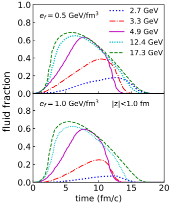
1. High densities are reached at low energies

- high-density medium is formed down to 7.7 GeV. According to EoS it has to be in the QGP phase.

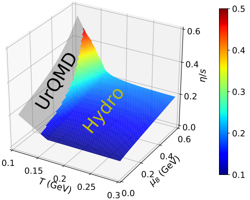
MUFFIN (MFH)

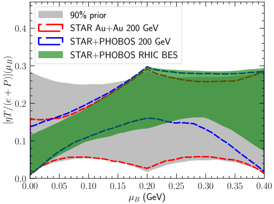


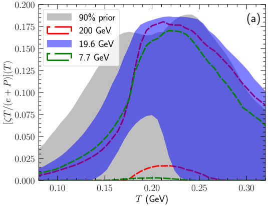
JAM+hydro



- η/s decreases with T (Geom.IS 2003.05852; String decceleration 2310.10787).

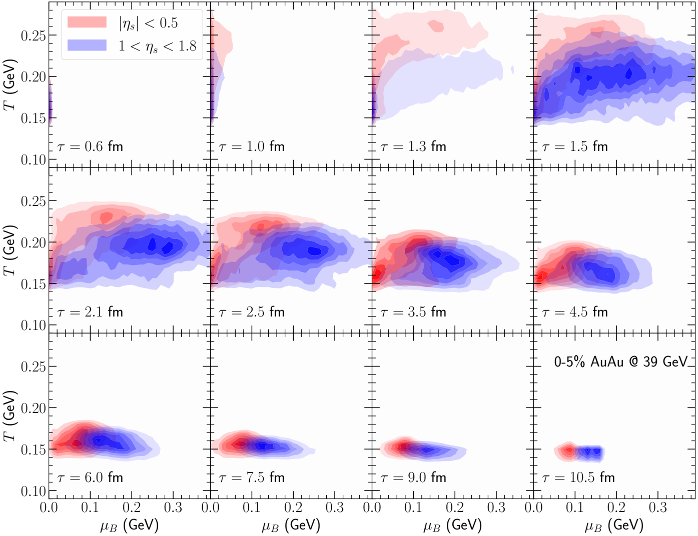






Some models (MFH-based) do without viscosity but overestimate v_2.

2. The system is very inhomogeneous

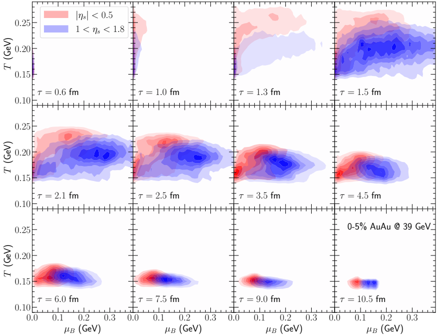


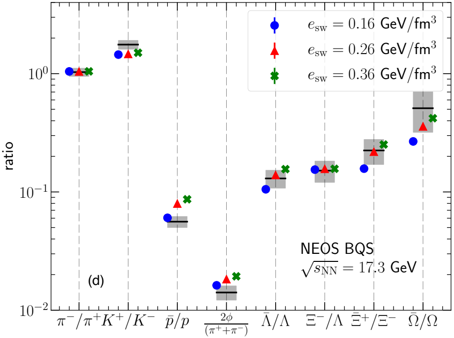
Simulation:

3D MC-Glauber + string deceleration + MUSIC

taken from: Chun Shen, 2108.04987, other models show a similar picture

2. The system is very inhomogeneous (2)

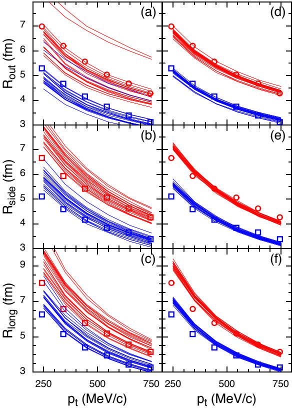


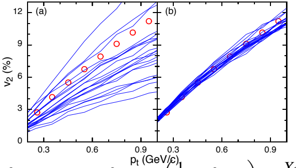


- Hybrid models generally fit dN/dy, dN/dp_T and v_2 **together**

Constraining the EoS of QGP matter - at high energies

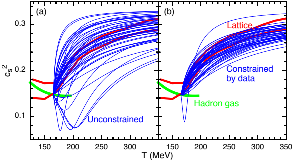
S. Pratt, E. Sangaline, P. Sorensen, H. Wang, Phys. Rev. Lett. 114, 202301 (2015)





$$c_s^2(\varepsilon) = c_s^2(\varepsilon_h) + \left(\frac{1}{3} - c_s^2(\varepsilon_h)\right) \frac{X_0 x + x^2}{X_0 x + x^2 + X'^2} \quad (1)$$

$$X_0 = X' R c_s(\varepsilon)\sqrt{12}, \quad x \equiv \ln \varepsilon/\varepsilon_h,$$



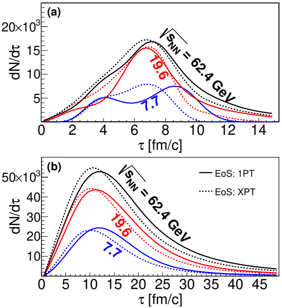
A similar analysis should be possible at lower energies, e.g. hypothetically in the FAIR range.

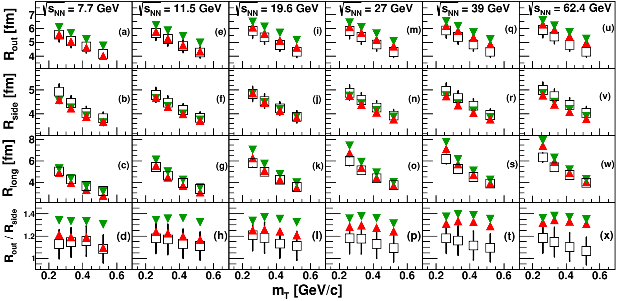
EoS/PT sensitivity via HBT/femtoscopy

Initial idea: enhancement of $R^2_{\text{out}} - R^2_{\text{side}}$ would signal a passage through the mixed phase.

"Reality check":
`UrQMD` (initial state) + fluidisation at fixed τ_0 + `vHLLE` (hydro) + `UrQMD`

Batyuk, IK, Lednicky, Malinina, Mikhaylov, Rogachevsky, Wielanek, Phys. Rev. C 96, 024911 (2017)





1PT = 1st order PT, XPT = crossover; crossover EoS is red, 1PT EoS is green

There is weak EoS sensitivity, crossover EoS is preferred.

EoS/PT sensitivity: directed flow?

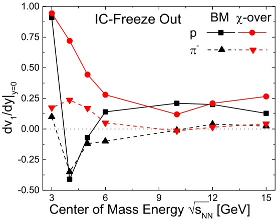
Initial idea: non-monotonic behaviour of v_1 would signal a passage through the mixed phase.

"Reality check":

J. Steinheimer, J. Auvinen, H. Petersen, M. Bleicher, H. Stöcker, Phys. Rev. C 89 (2014) 054913, arXiv:1402.7236

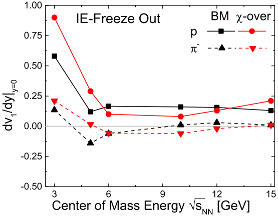
- 1-fluid model with iso-time freezeoout:

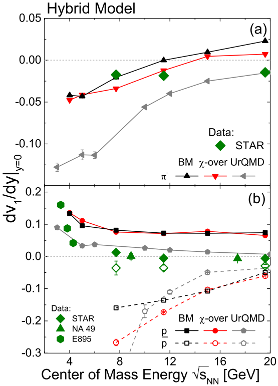
  sign change of dv_1/dy with 1st-order PT EoS



- 1-fluid model with iso-T freezeoout:

  NO sign change of dv_1/dy with 1st-order PT EoS



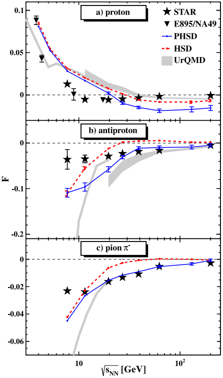


- Full hybrid model: no sign change of dv_1/dy, weak EoS dependence and no agreement with the data

Full-fledged models generally struggle to reproduce the v_1

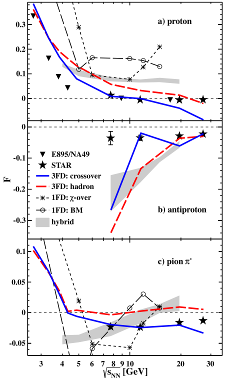
PHSD/HSD/UrQMD:

Konchakovski, Cassing, Ivanov, Toneev,



3-fluid/1-fluid models:

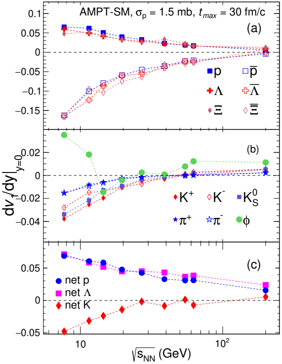
Phys. Rev. C 90, 014903 (2014)



AMPT:

K. Nayak, S. Shi, Nu Xu, Zi-Wei Lin,

Phys. Rev. C 100, 054903 (2019)

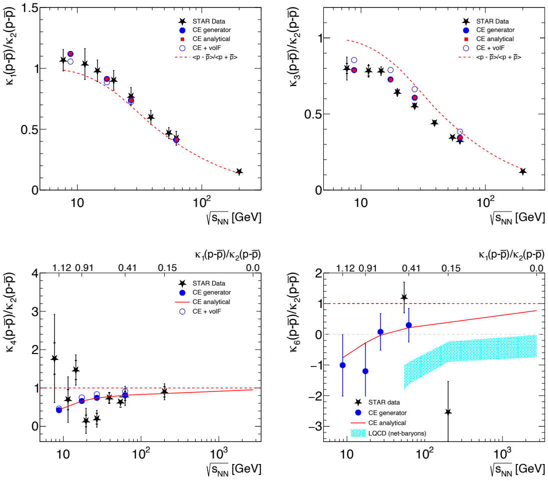


Net proton cumulants

Non-critical baseline for proton cumulants is non-trivial

Braun-Munzinger, Friman, Redlich, Rustamov, Stachel, Nucl. Phys. A 1008, 122141 (2021)

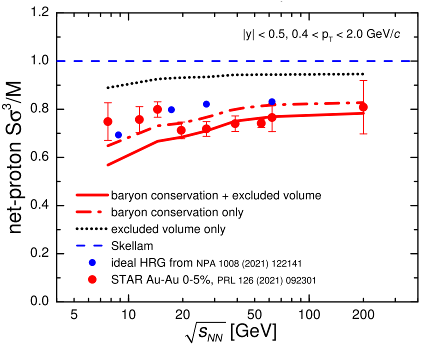
Effect of global baryon number conservation + volume fluctuations

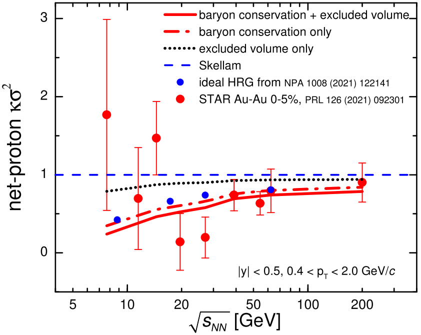


Non-critical baseline from 3D hydro

Vovchenko, Koch, Shen, Phys. Rev. C 105, 014904 (2022)

Parametrized initial state (Type 1) + MUSIC
Global baryon conservation using SAM-2.0 method



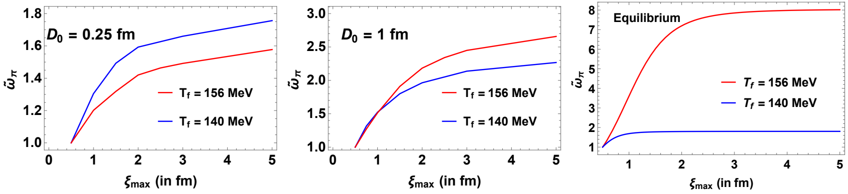


Progress on critical fluctuations in hydro

Maneesha Pradeep, Jamie Karthein, Krishna Rajagopal, Misha Stephanov, Yi Yin,
Phys.Rev.D 106 (2022) 3, 036017

Hydro+ : deterministic hydrodynamics + non-hydrodynamic modes ϕ
No concrete result just yet.

$\tilde{\omega}_\pi = \frac{\omega_\pi}{\omega_\pi^{\mathrm{nc}}}$, where $\omega_A = \frac{\langle \delta N_A^2 \rangle}{\langle N_A \rangle}$



where D is a diffusion coefficient which inversely scales with the correlation length ξ.

Conclusions

- There is abundance of hybrid models for the RHIC BES energy range, with different assumptions about initial state dynamics (or absence thereof).
- Most of the models desribe most of the basic observables.
  A future Bayesian analysis would tell us whether some EoS are excluded by the coherent data reproduction
- EoS sensitivity is generally not strong; it is not clear yet what observables would be very sensitive to the type of PT in the EoS.
- Baryon/proton cumulants: we know a non-critical baseline.
- There are no hybrids with critical behaviour included yet.
- We need more modelling and **more consistent experimental data**.

[backup]

Equation of state

- Chiral model EoS
- BM + HRG EoS
- NEOS-B, NEOS-BQS
- BEST EoS

Final conditions

- Particlization at fixed energy density + hadronic cascade (except for multi-fluid models)

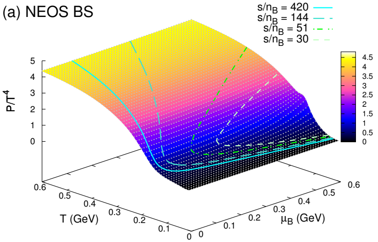
EoS in use (1)

All-crossover cases:

NEOS-B, NEOS-BQ, NEOS-BQS:

- lattice QCD at $\mu_B = 0$
- Tylor expansion in μ_i/T using susceptibilities
- free hadron-resonance gas at low T
- crossover transition at all μ_B

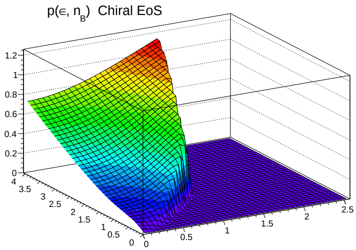
Monnai, Schenke, Shen,
Phys. Rev. C 100, 024907 (2019)



Chiral model EoS

- a single model for hadronic and quark phases
- hadronic SU(3) model
- extension to quark DOF in analogy to PNJL
- crossover transition at all densities

Steinheimer, Schramm, Stocker,
J. Phys. G 38 035001

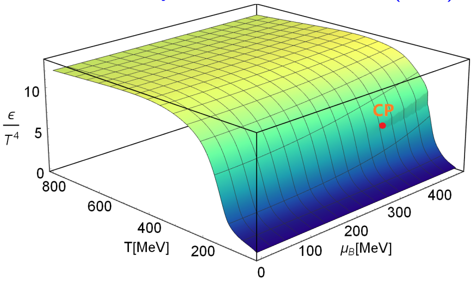


EoS in use (2)

BEST EoS

- lattice QCD at $\mu_B = 0$
- Tylor expansion in μ_i/T using susceptibilities
- critical contribution inspired by a 3D ising model

Parotto et al, Phys. Rev. C 101, 034901 (2020)



Bag model + HRG EoS a.k.a. EoS Q

- Bag model for QGP phase
- HRG model with repulsion
- Maxwell construction ⇒ 1st order PT.

P.F. Kolb, et al, Phys.Rev. C 62, 054909 (2000)