The forgotten importance of impact parameter

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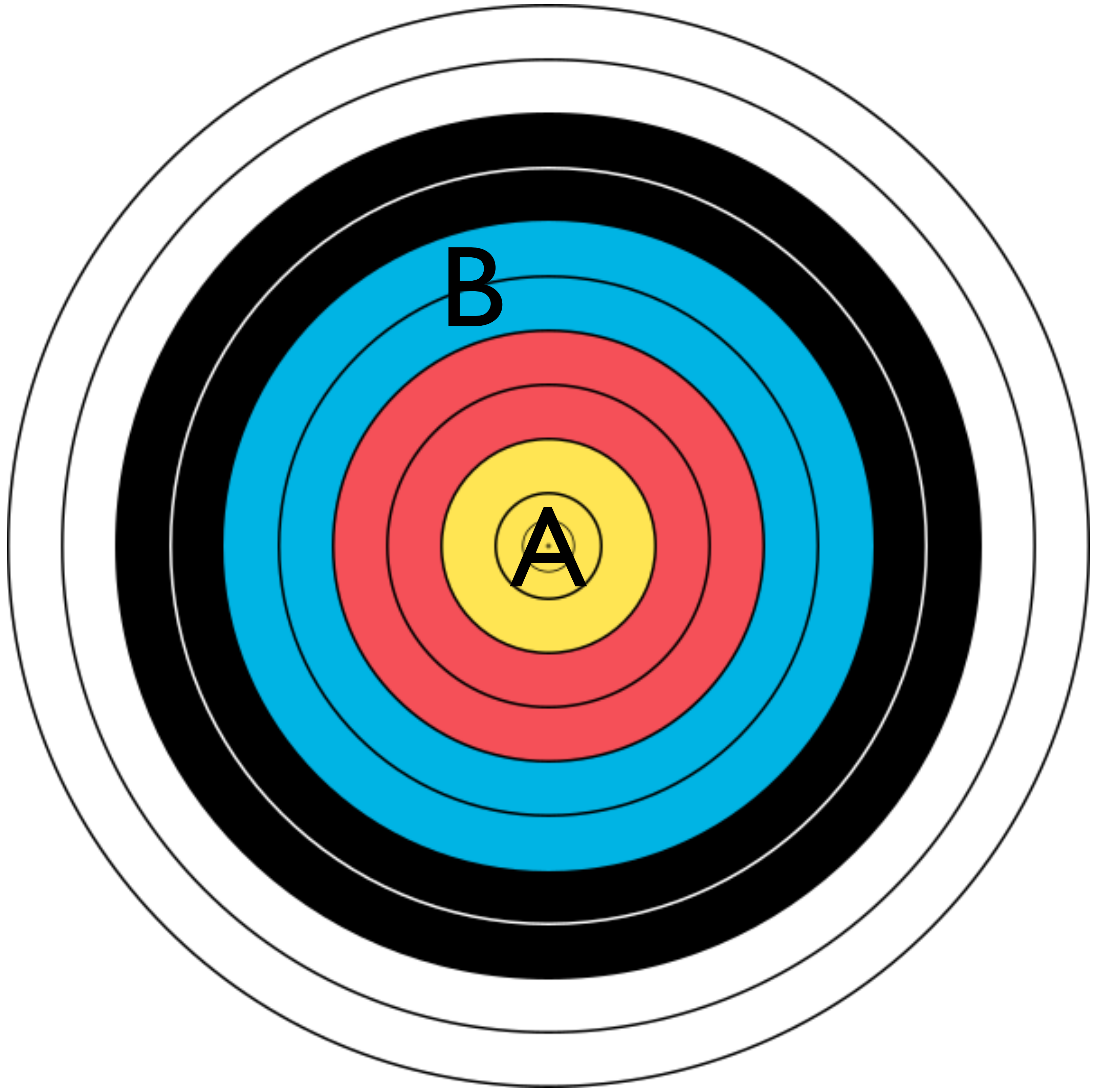
Workshop on
Hydrodynamics and related observables in heavy-ion collisions
Subatech, Nantes, Oct. 30, 2024

Centrality in experiment

- Analyses of experimental data at the LHC are always done in centrality classes.
- A specific observable is used as a centrality classifier: particle multiplicity N_{ch} in a detector (ATLAS), number of hits in a scintillator (ALICE), energy E_T deposited in a calorimeter (CMS, ATLAS).
- $0 - 5\,\%$ most central $\equiv 5\,\%$ of events with largest N_{ch} or E_T.

Centrality in theory

- Centrality originally refers to impact parameter $b \equiv$ distance between the centers of colliding nuclei A and B.
- The *true centrality* is $c \equiv \pi b^2/\sigma_{PbPb}$
- $c < 0.05$ corresponds to the $5\,\%$ of events with the smallest b.
- In this talk, "fixed centrality" means "fixed impact parameter".



Outline

1. Puzzling observations in ultracentral collisions
2. Reconstructing the probability distribution of the true centrality c
   Das Giacalone Monard JYO 1708.00081
3. Understanding anisotropic flow (v_n) fluctuations in ultracentral collisions
   Alqahtani Bhalerao Giacalone Kirchner JYO 2407.17308
4. Understanding mean transverse momentum ($[p_T]$) fluctuations in ultracentral collisions
   Samanta Bhatta Jia Luzum JYO 2303.15323
   Samanta Picchetti Luzum JYO 2306.09294
5. Summary and perspectives

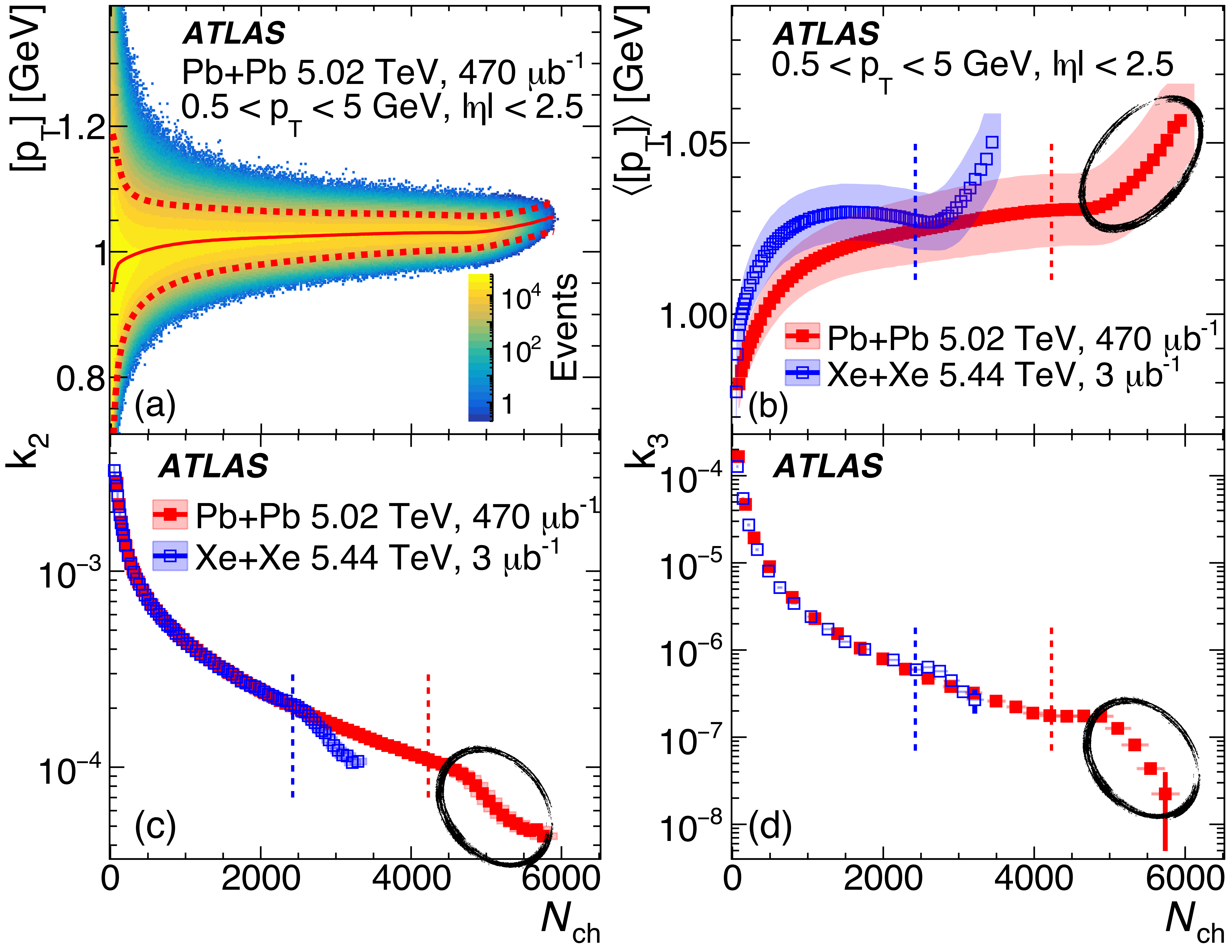
1. Puzzling observations in ultracentral collisions

Event-by-event fluctuations of $[p_T]$

$[p_T] \equiv$ transverse momentum per particle in an event

For very large N_{ch} (ultracentral collisions):

- The mean value increases
- The relative variance k_2 decreases
- The relative skewness k_3 decreases



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Correlation between $[p_T]$ and anisotropic flow v_n

$\rho_n \equiv$ Pearson correlation coefficient between $[p_T]$ and v_n^2

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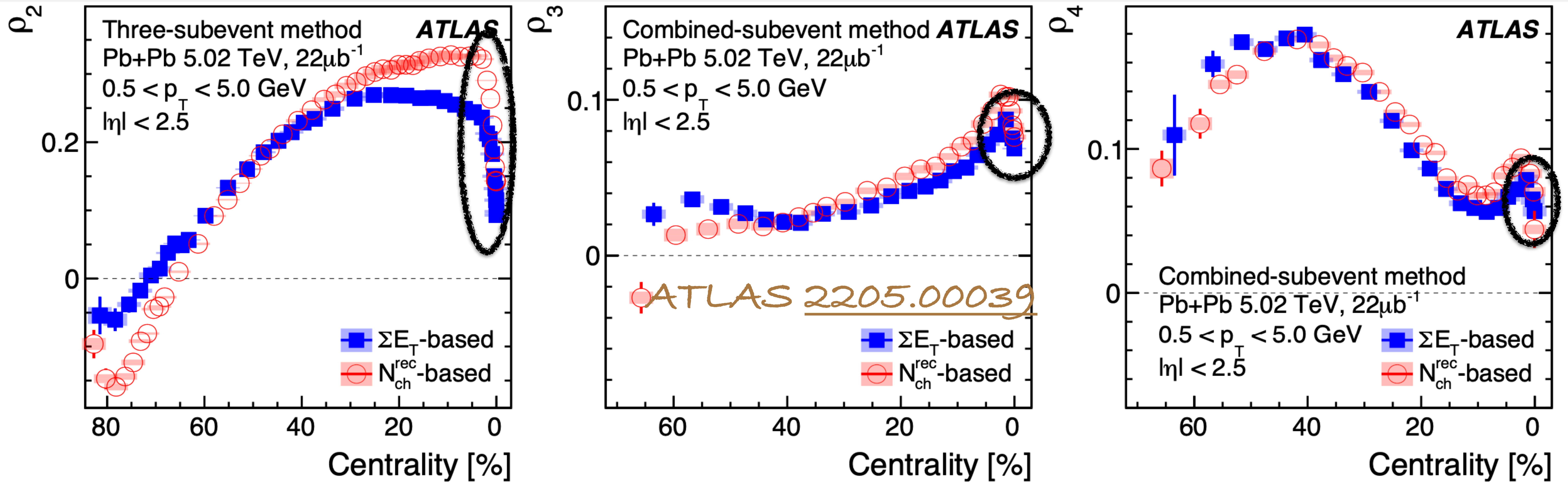

- Differs depending on whether centrality is defined using N_{ch} or E_T
- Decreases for ultracentral collisions

Anisotropic flow v_n fluctuations

Scaled cumulant (see talk by Koichi on Tuesday)

$$nc_n\{4\} \equiv \frac{\langle v_n^4 \rangle}{\langle v_n^2 \rangle^2} - 2$$

$= -1$ no fluctuations

$= 0$ Gaussian fluct.

ATLAS observes $nc_2\{4\} > 0$ in ultracentral collisions, and the value depends on the centrality classifier

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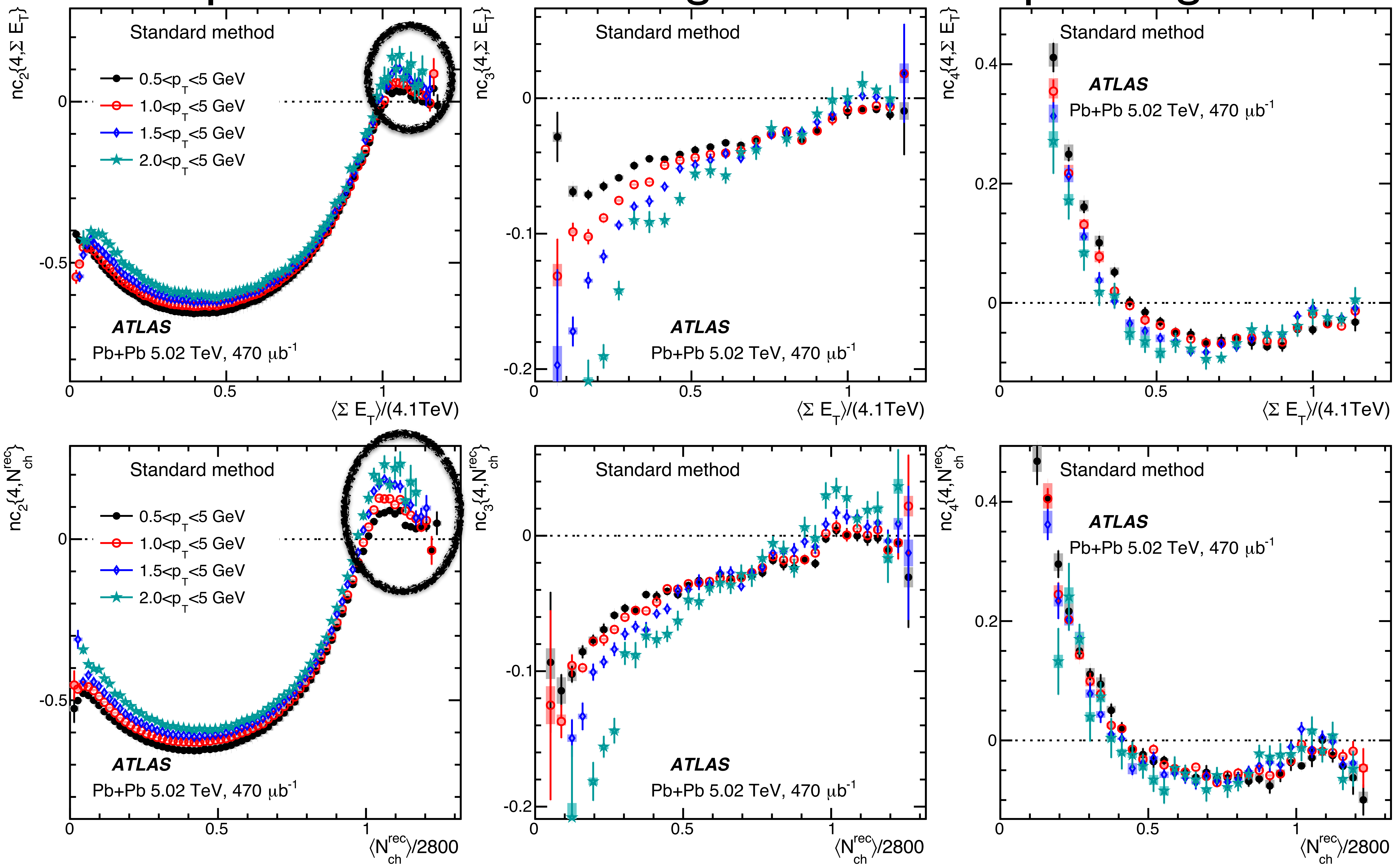
elliptic n=2 triangular n=3 quadrangular n=4



- In this talk, I show that these peculiarities are simple consequences of centrality fluctuations:
- A fixed value of the centrality classifier N_{ch} or E_T corresponds to a range of *true centralities*, which can be precisely determined from data.
- The general observation is that correlations and fluctuations decrease for ultracentral collisions. This is due to the gradual disappearance of centrality fluctuations: $c \approx 0$ for ultracentral (see next part).
- *No hydrodynamic modeling* here, just minimal theory input: central limit theorem, analyticity, symmetry (I'll use follow this colour code throughout this presentation)

2. Reconstructing the probability distribution of the true centrality c

- First, solve the inverse problem: what is the distribution of E_T (or N_{ch}) at fixed centrality?
- Then apply Bayes' theorem: $P(c \mid E_T) = \dfrac{P(E_T \mid c)P(c)}{P(E_T)} = \dfrac{P(E_T \mid c)}{P(E_T)}$

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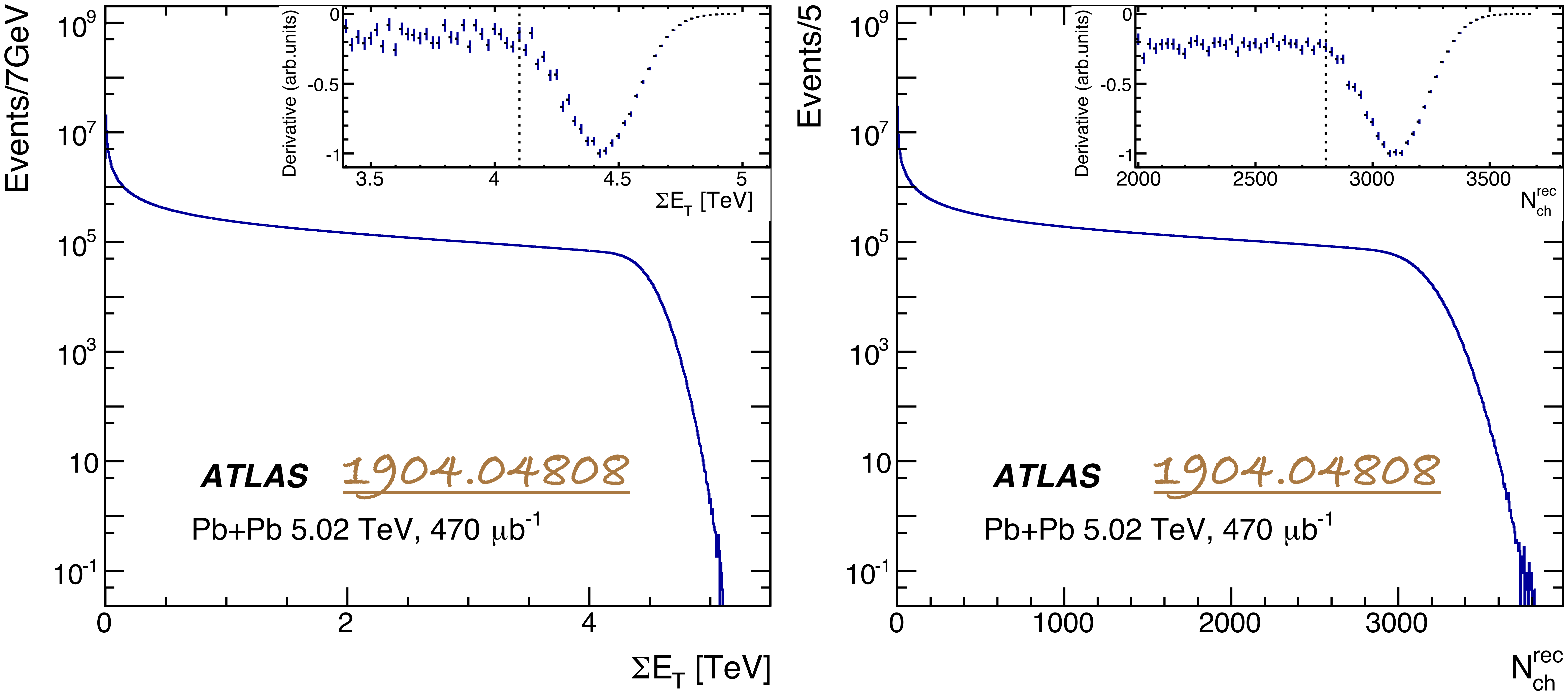
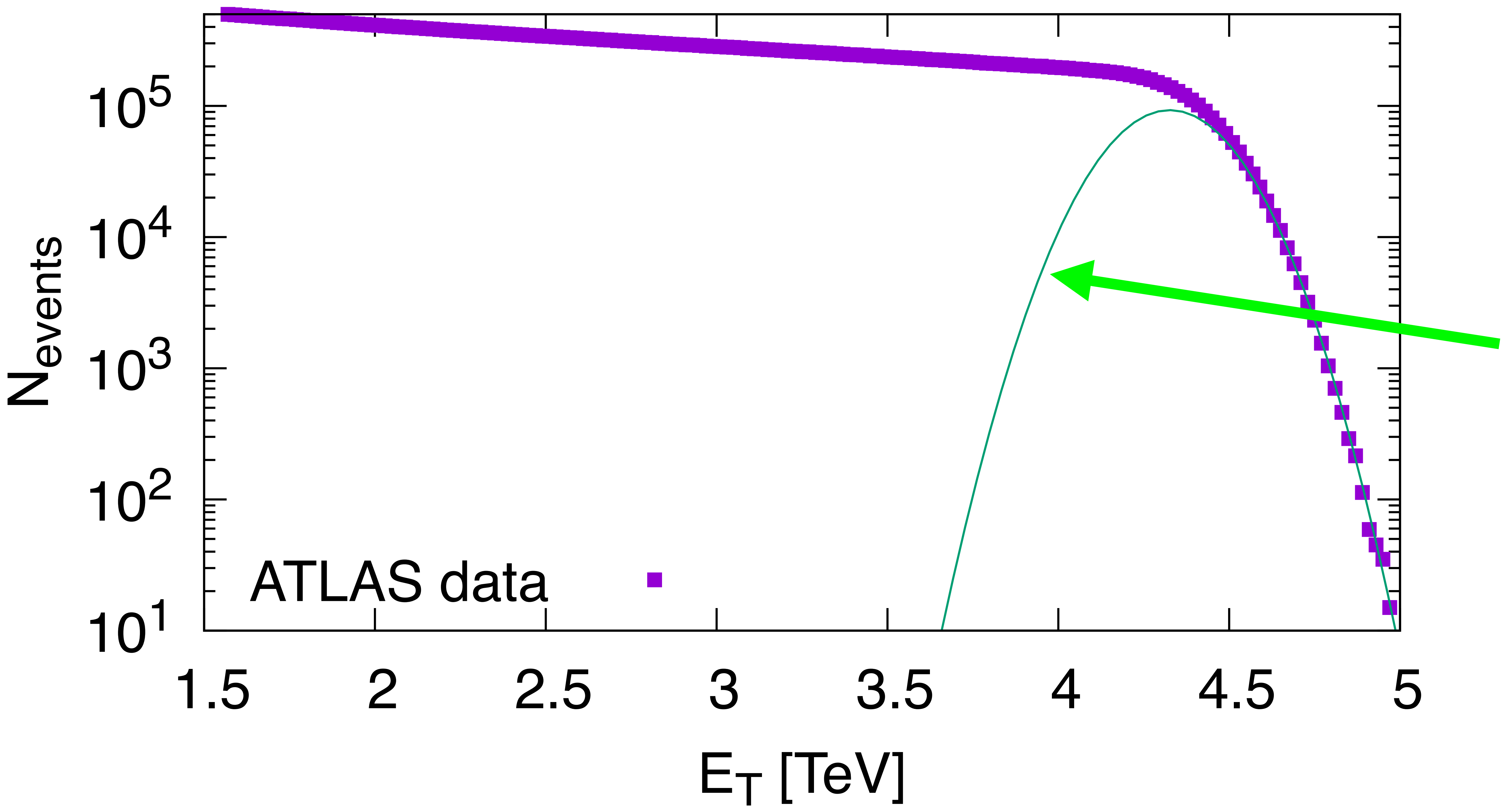
Input: distribution of the centrality classifier



We need experiments to provide the histogram of the centrality classifier.
Not all collaborations agree to share these data !

Basic assumption: Gaussian fluctuations



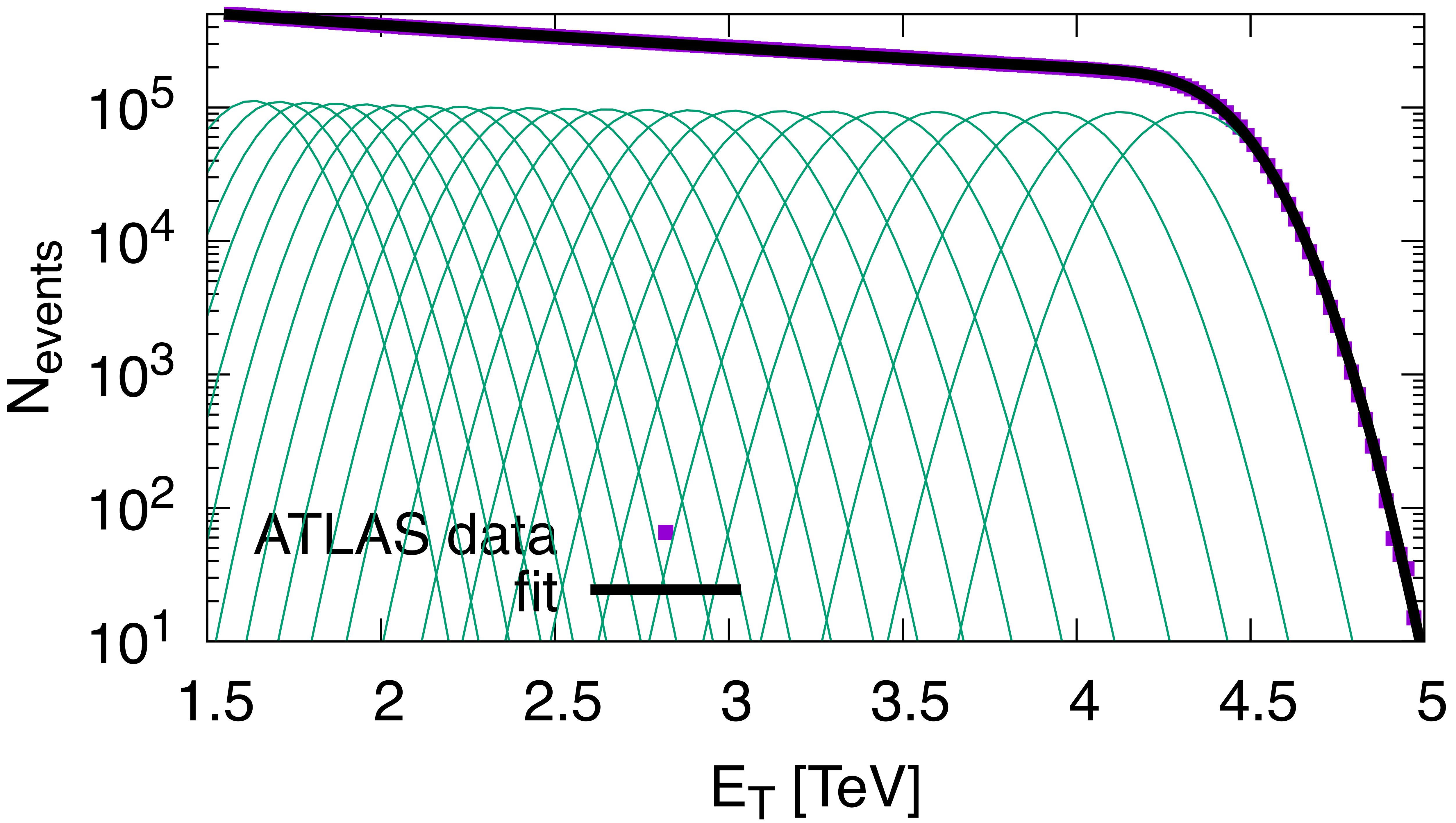
Distribution of E_T for $c = 0$ (head-on collisions)

We assume that the fluctuations of E_T at fixed c are Gaussian:

$P(E_T|c)$ is a Gaussian distribution, mean $\overline{E_T}$ and width σ_{E_T} are smooth functions of c.

The width $\sigma_{E_T}(c=0)$ can be read off from the tail of the distribution.

Fitting the distribution of E_T



different Gaussians ≡ successive increments of c by 1%

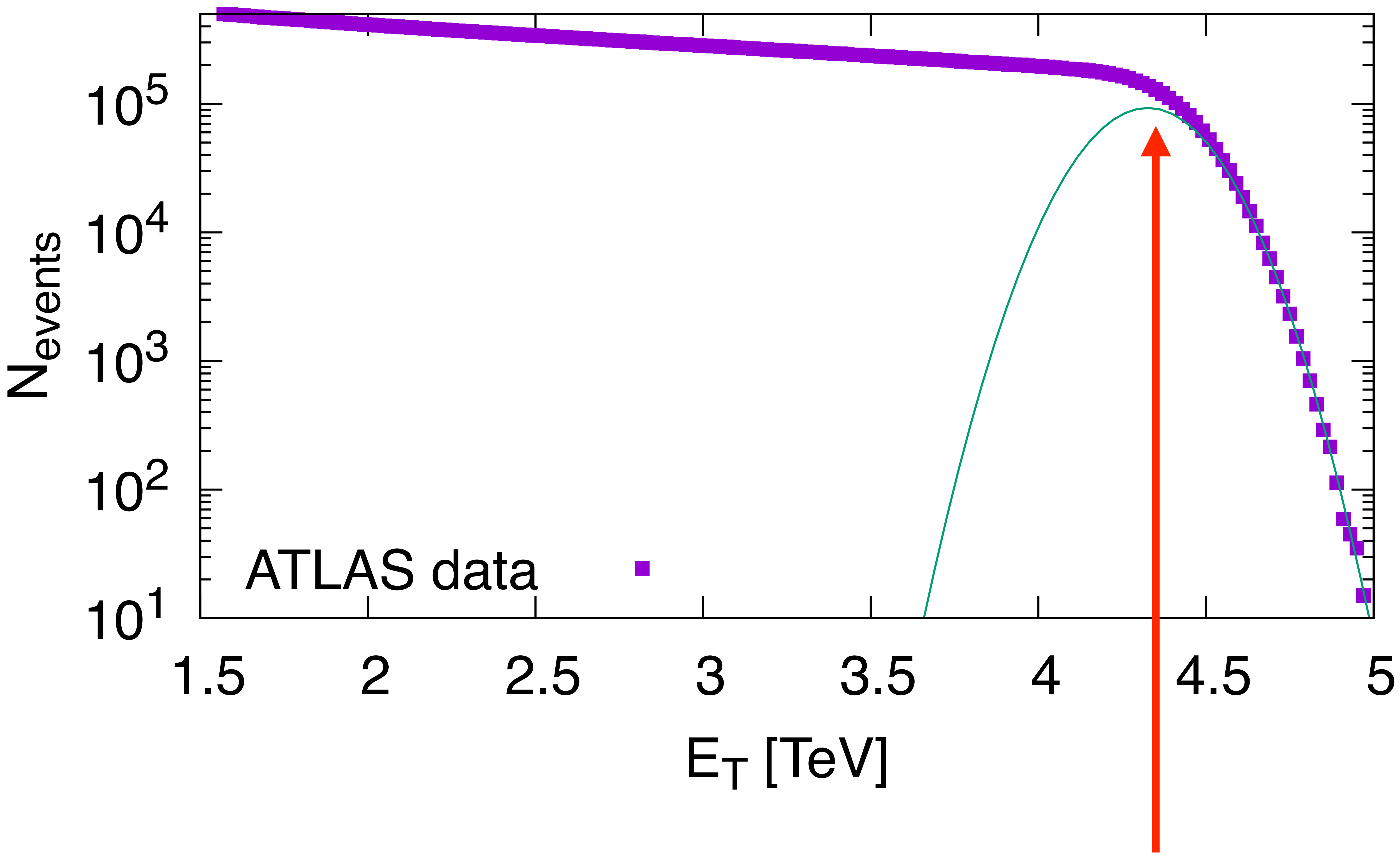
We fit the distribution of E_T as an integral of Gaussians over the centrality c.

The fit returns the mean value $\overline{E_T}(c)$ and the width $\sigma_{E_T}(c=0)$.

The variation of $\sigma_{E_T}(c)$ cannot be determined from data.

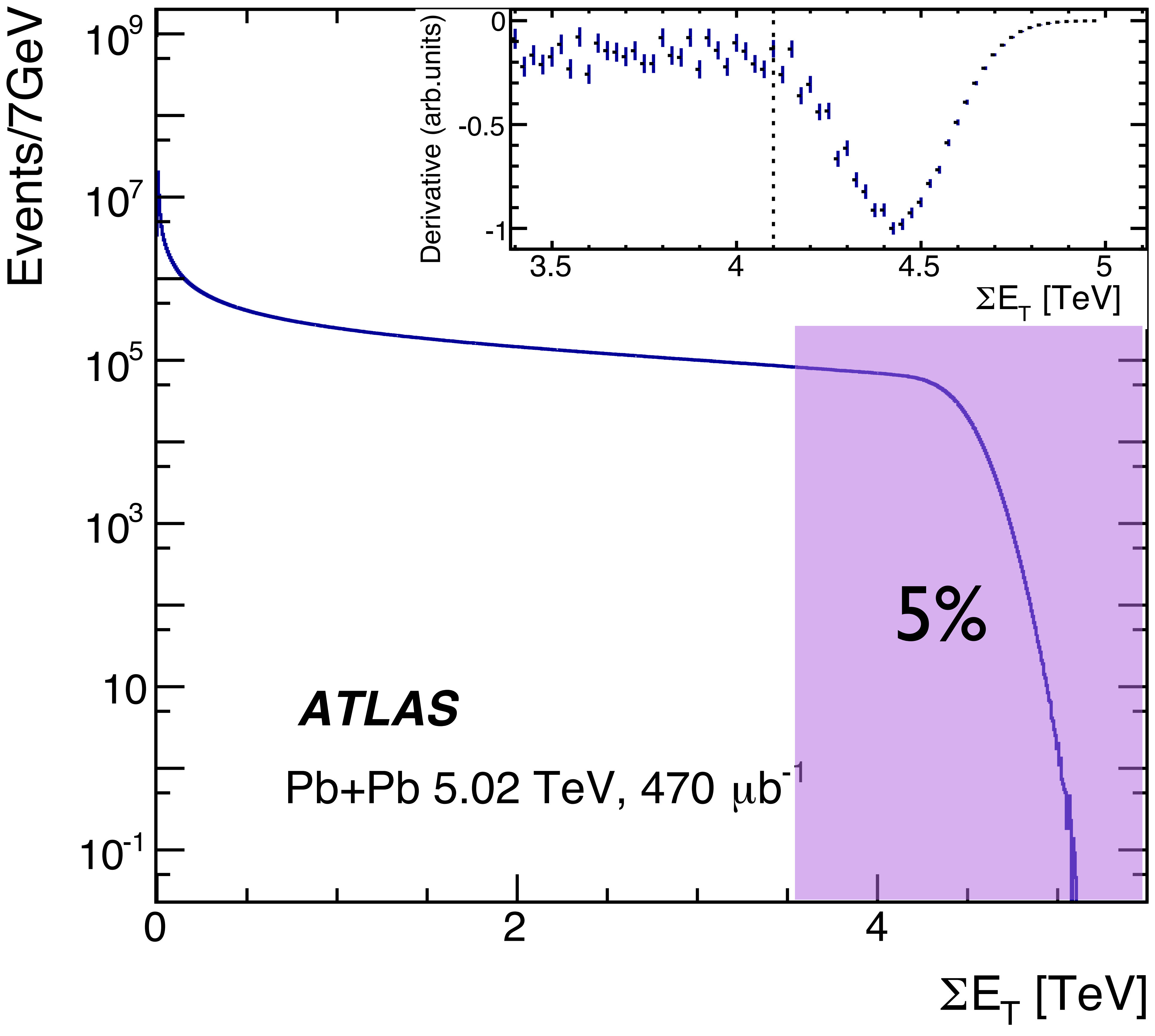
This is why we focus on 0-5% most central collisions, where $\sigma_{E_T}(c) \simeq \sigma_{E_T}(0)$

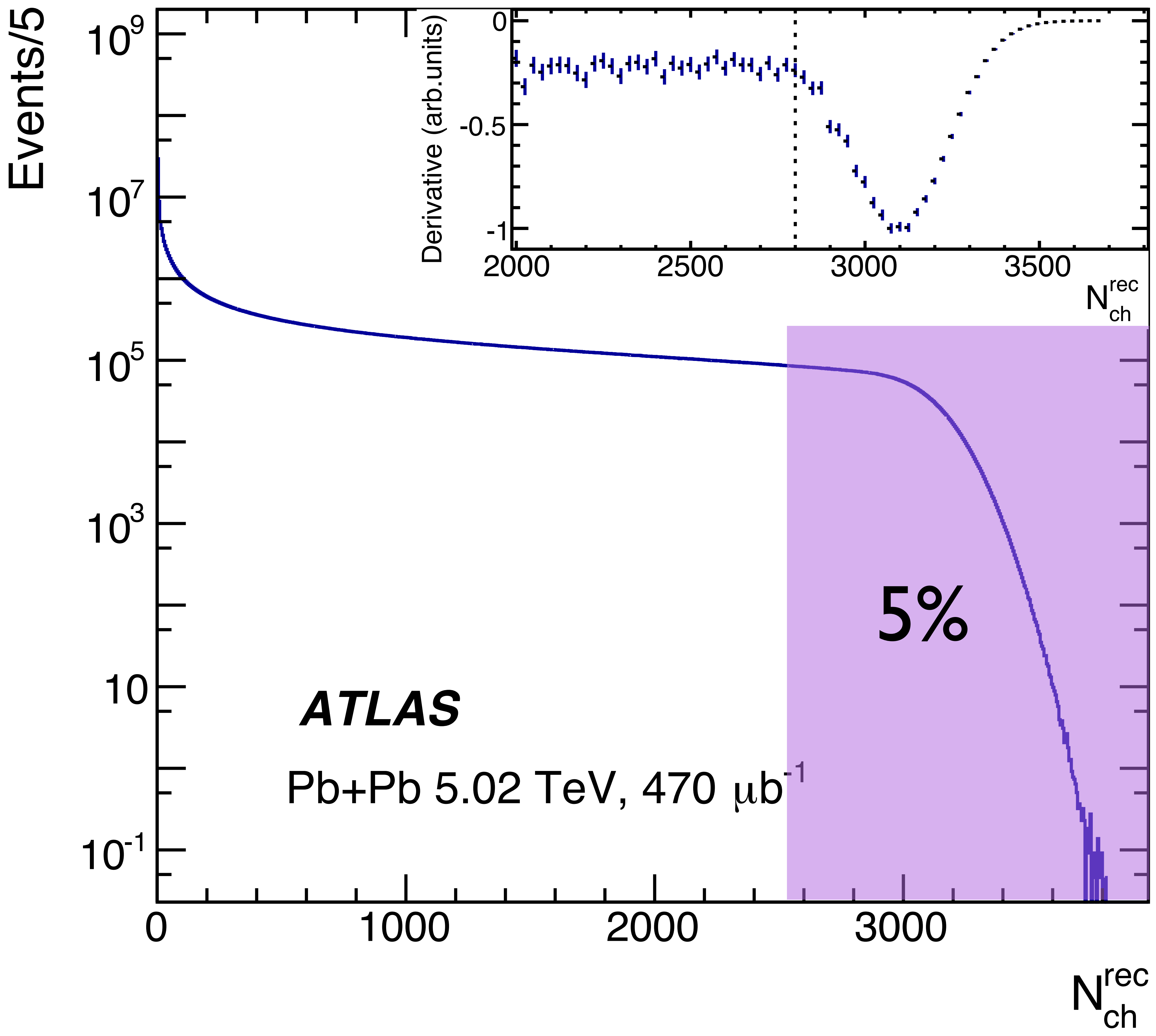
The knee



We define the knee of the distribution of E_T as the mean value of E_T for $c = 0$.
This is an output of the fit, and it is determined very precisely (typically 0.3% accuracy).
We propose to call *ultracentral* the events above the knee.

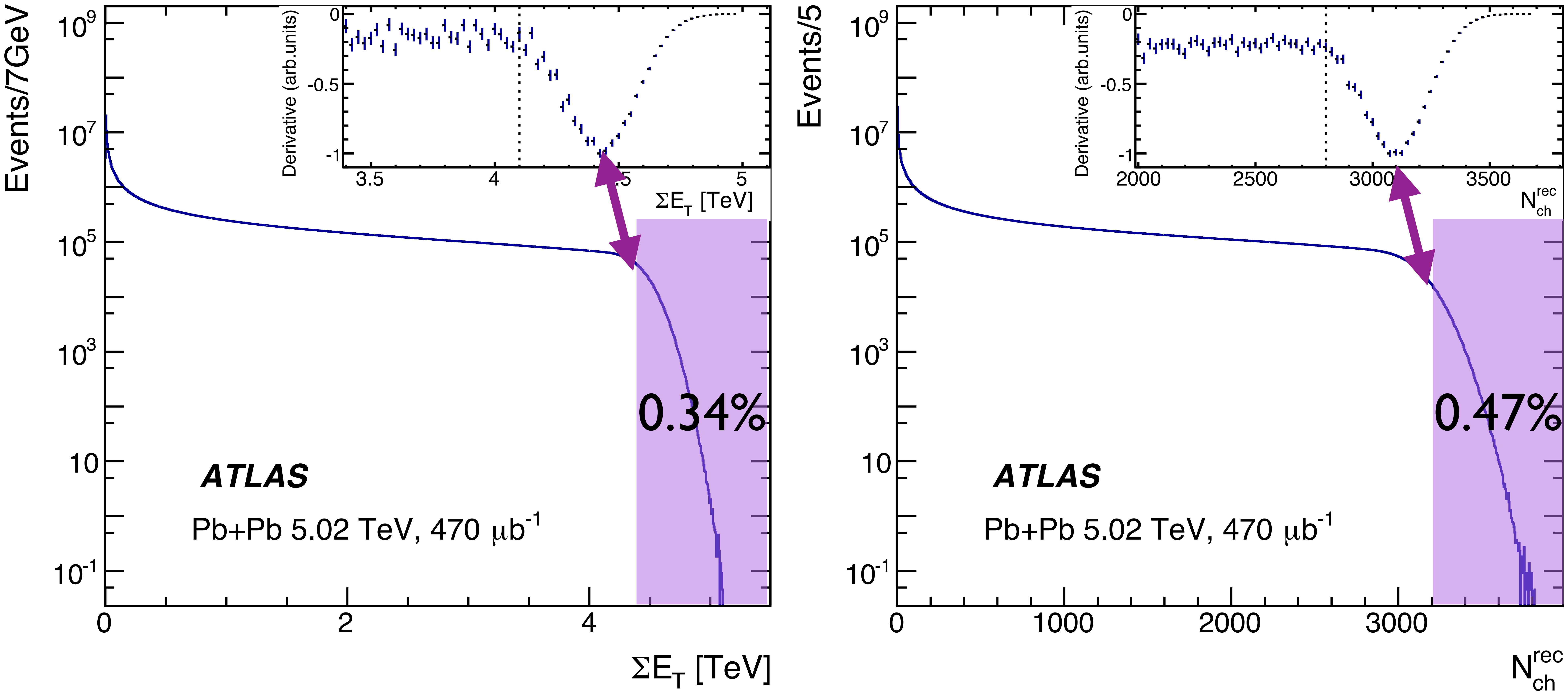
Central versus ultracentral collisions





Many analyses use 0-5% as the most central bin.

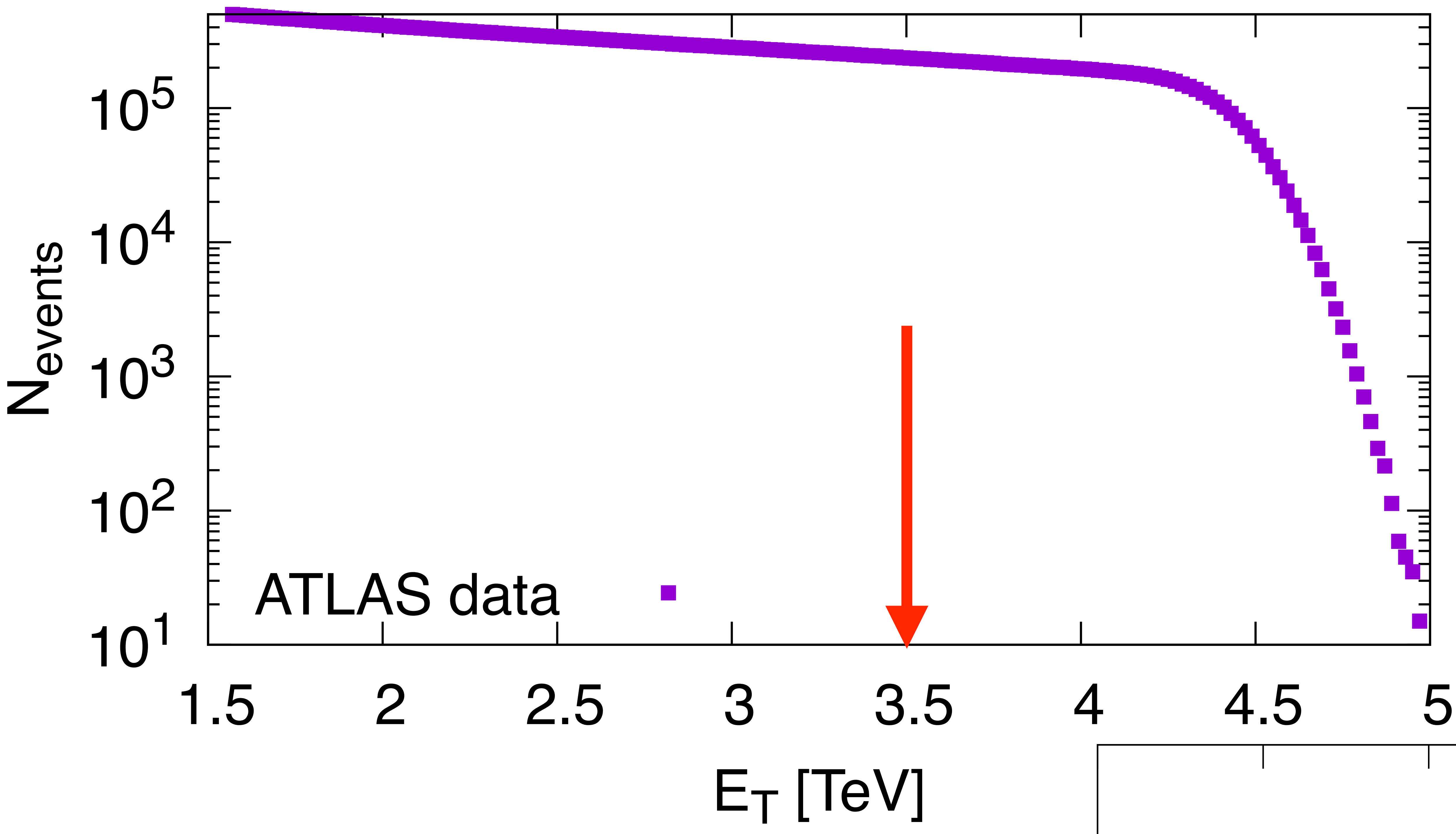
Central versus ultracentral collisions



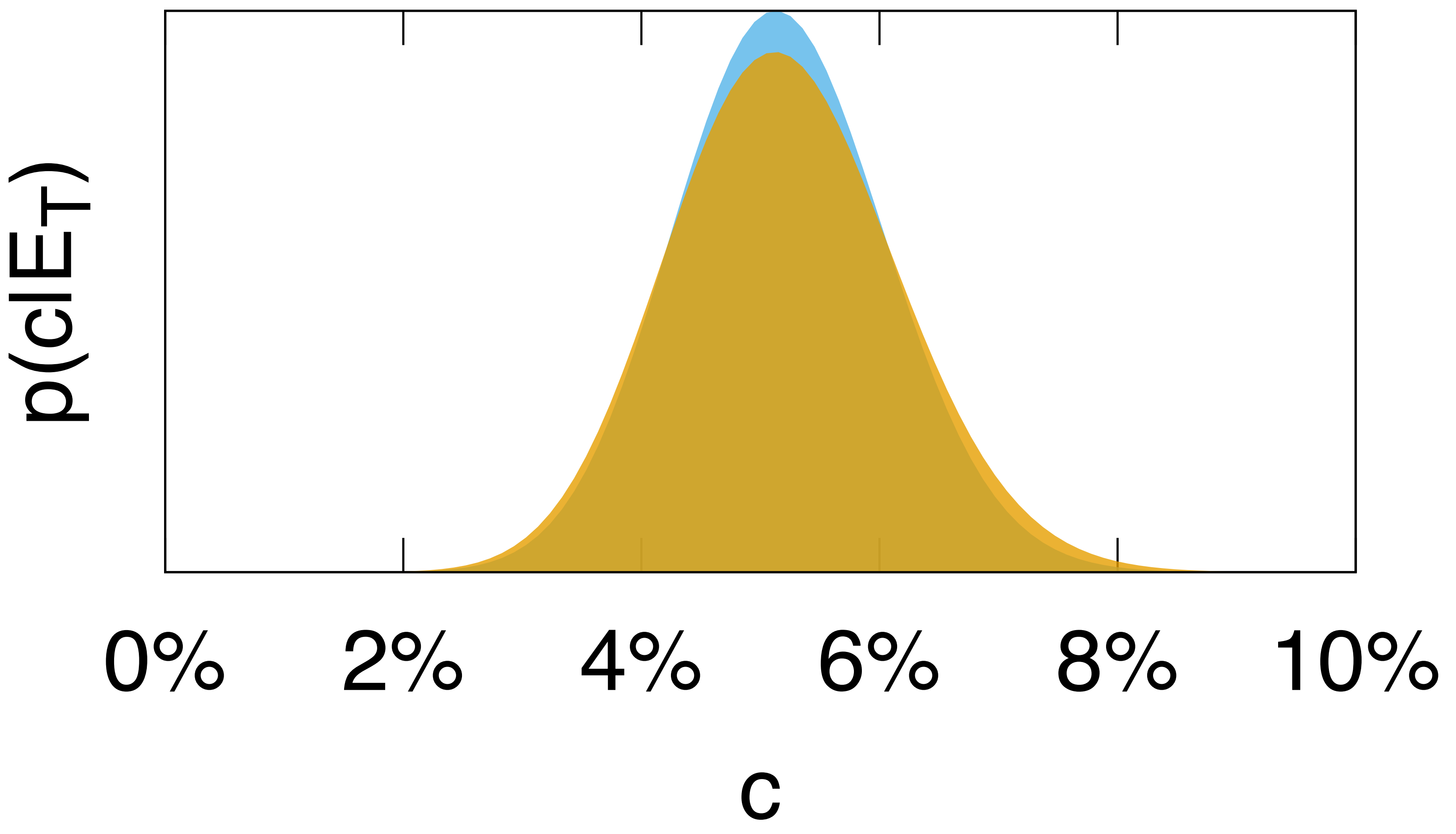
Ultracentral collisions are a much smaller fraction

(note: knee corresponds approximately to the max. slope of histogram on linear scale)

Distribution of centrality from Bayes' theorem



The distribution $p(c\,|\,E_T)$ is also Gaussian to a good approximation, with a std. dev. $\sigma_c \simeq 0.85\,\%$.
For N_{ch}, $\sigma_c \simeq 1.3\,\%$ (not shown).



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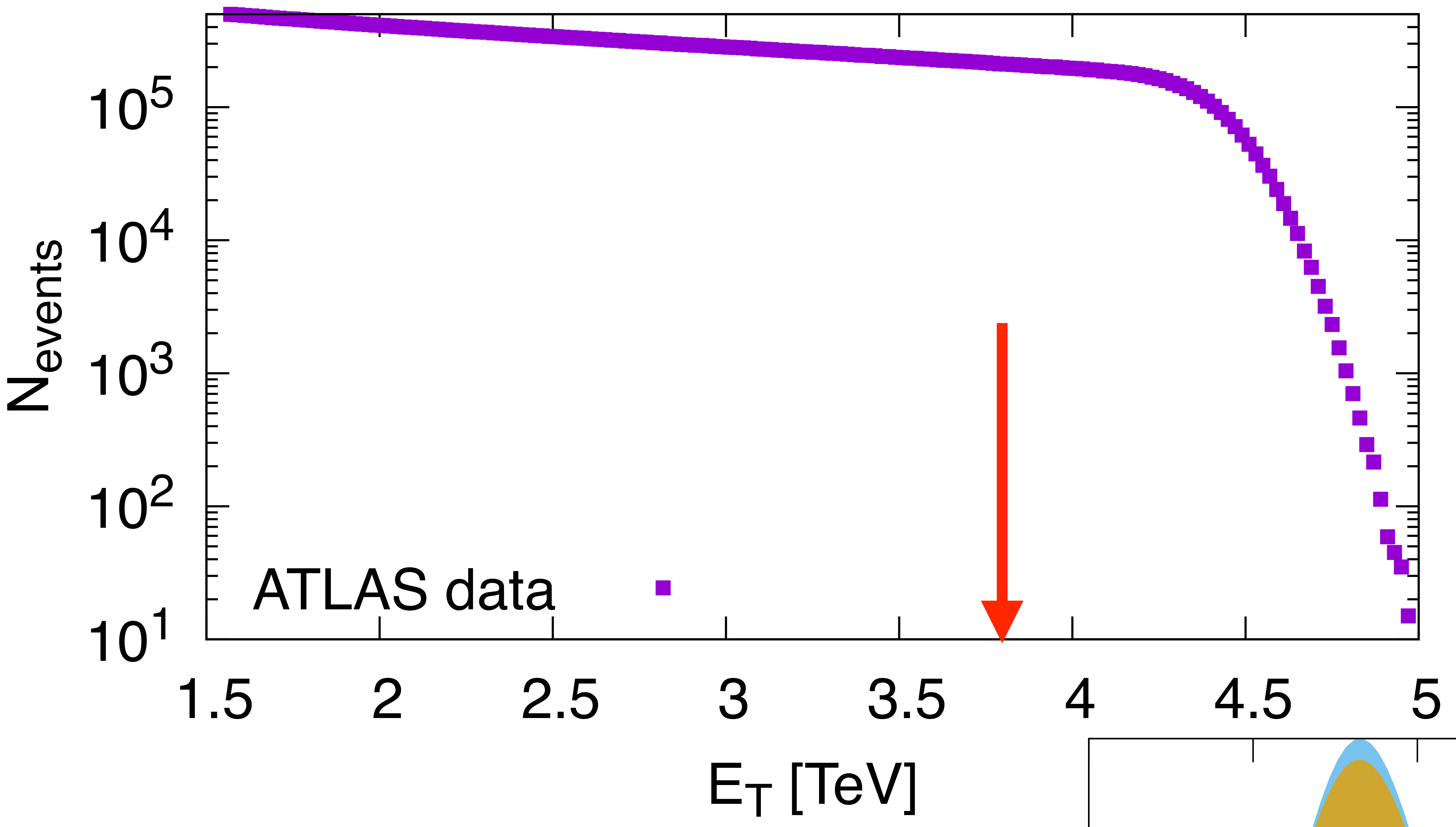
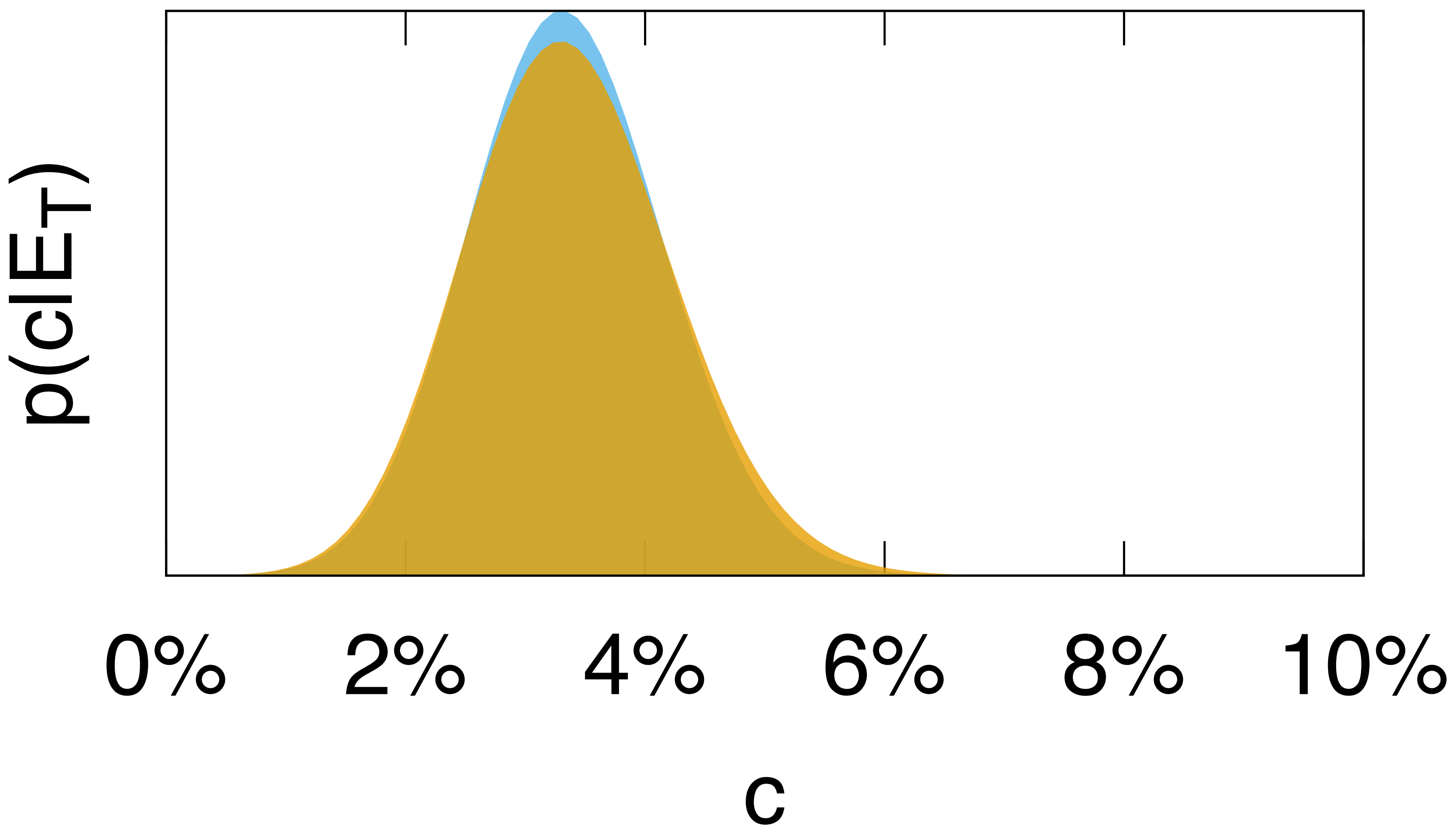


As E_T increases, the distribution of centrality gets shifted towards smaller values.



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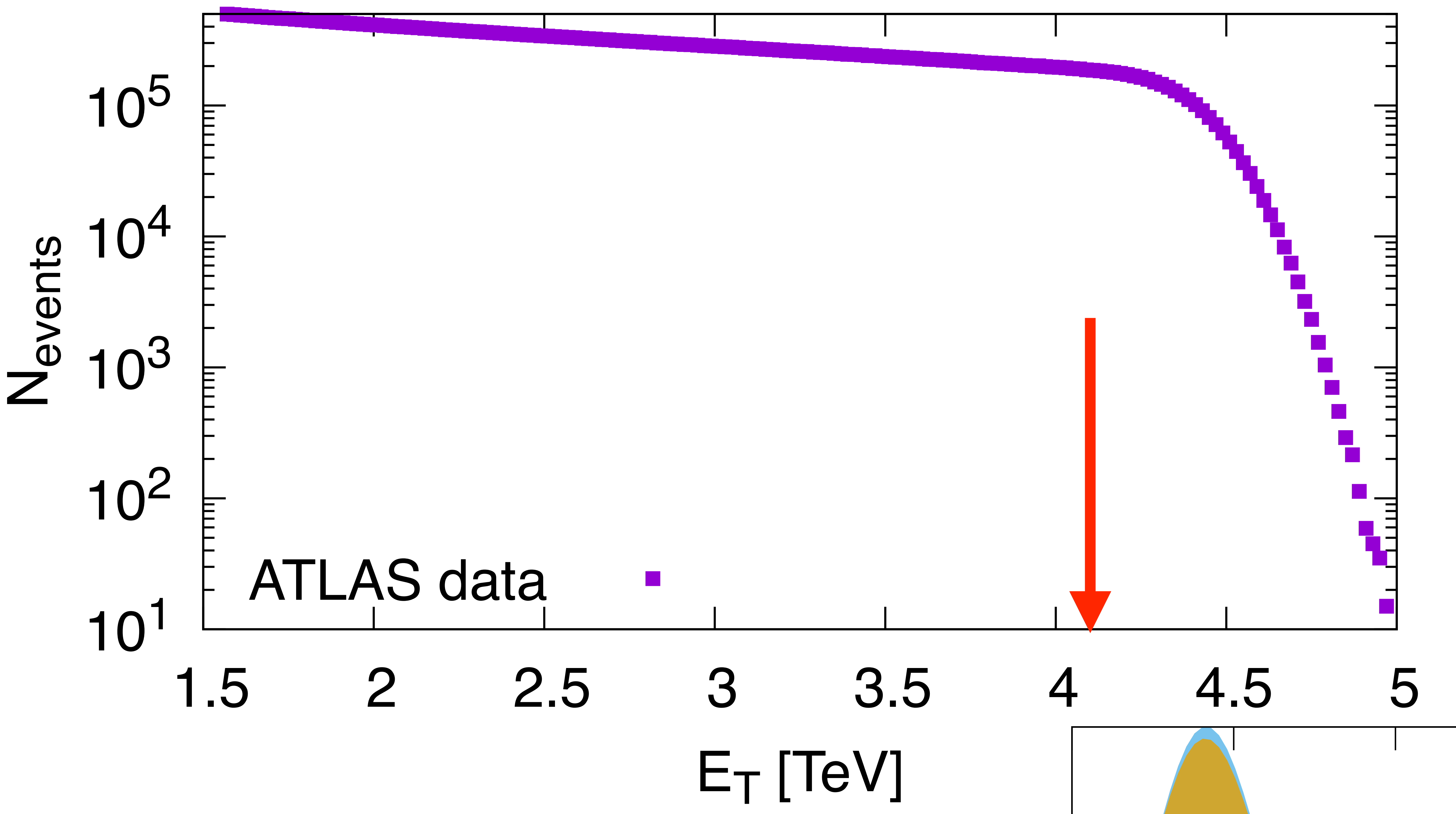

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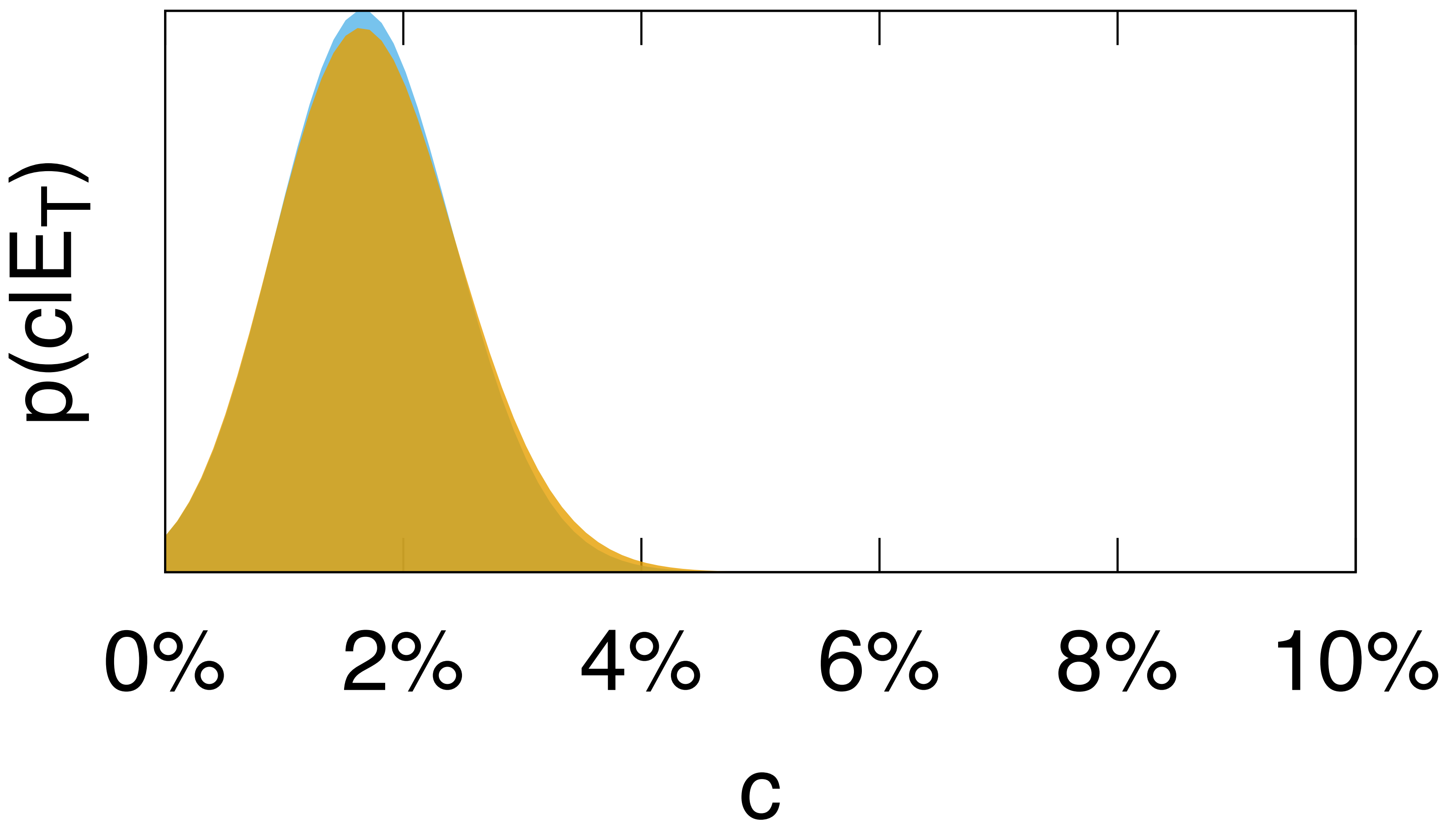

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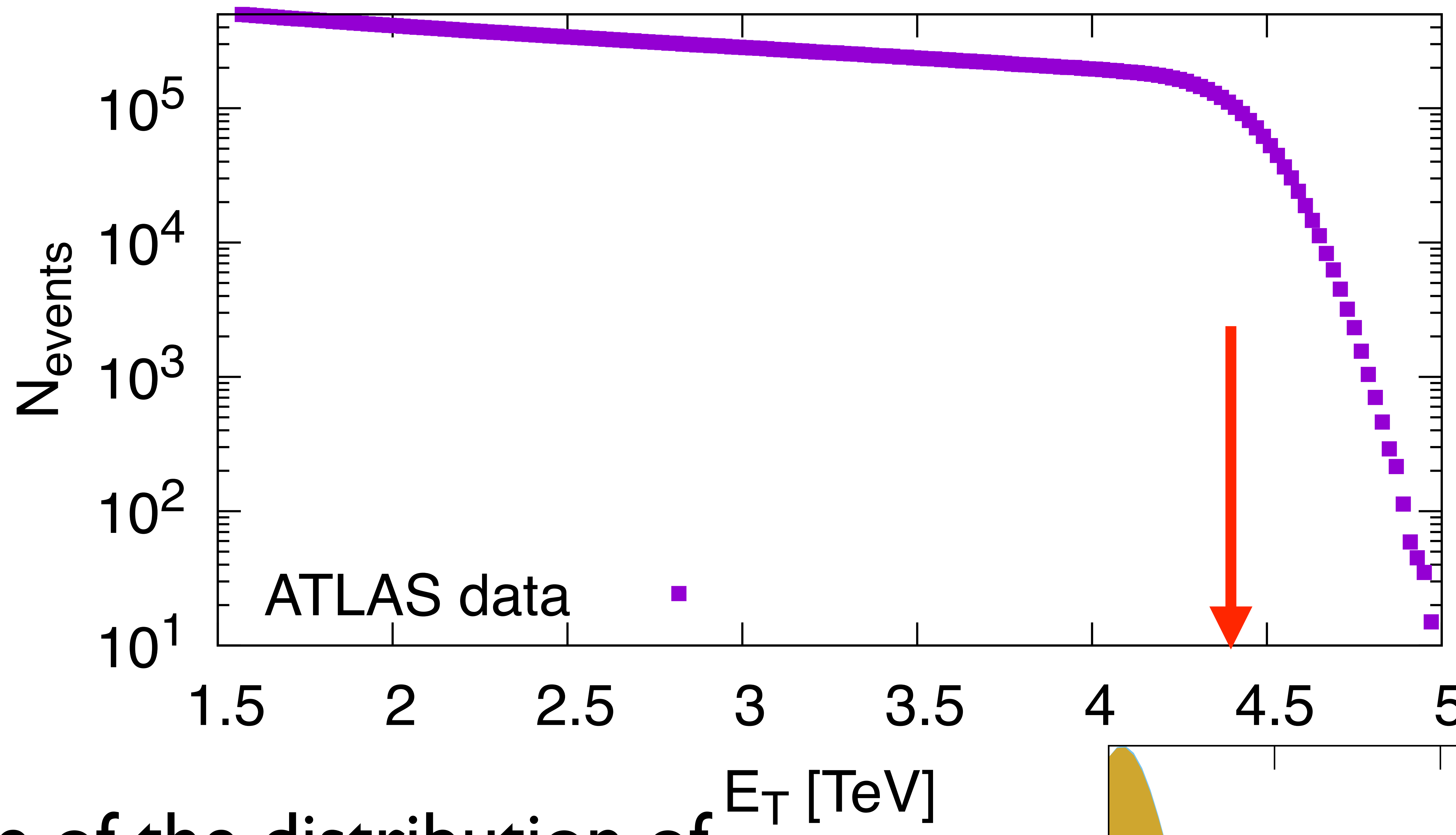
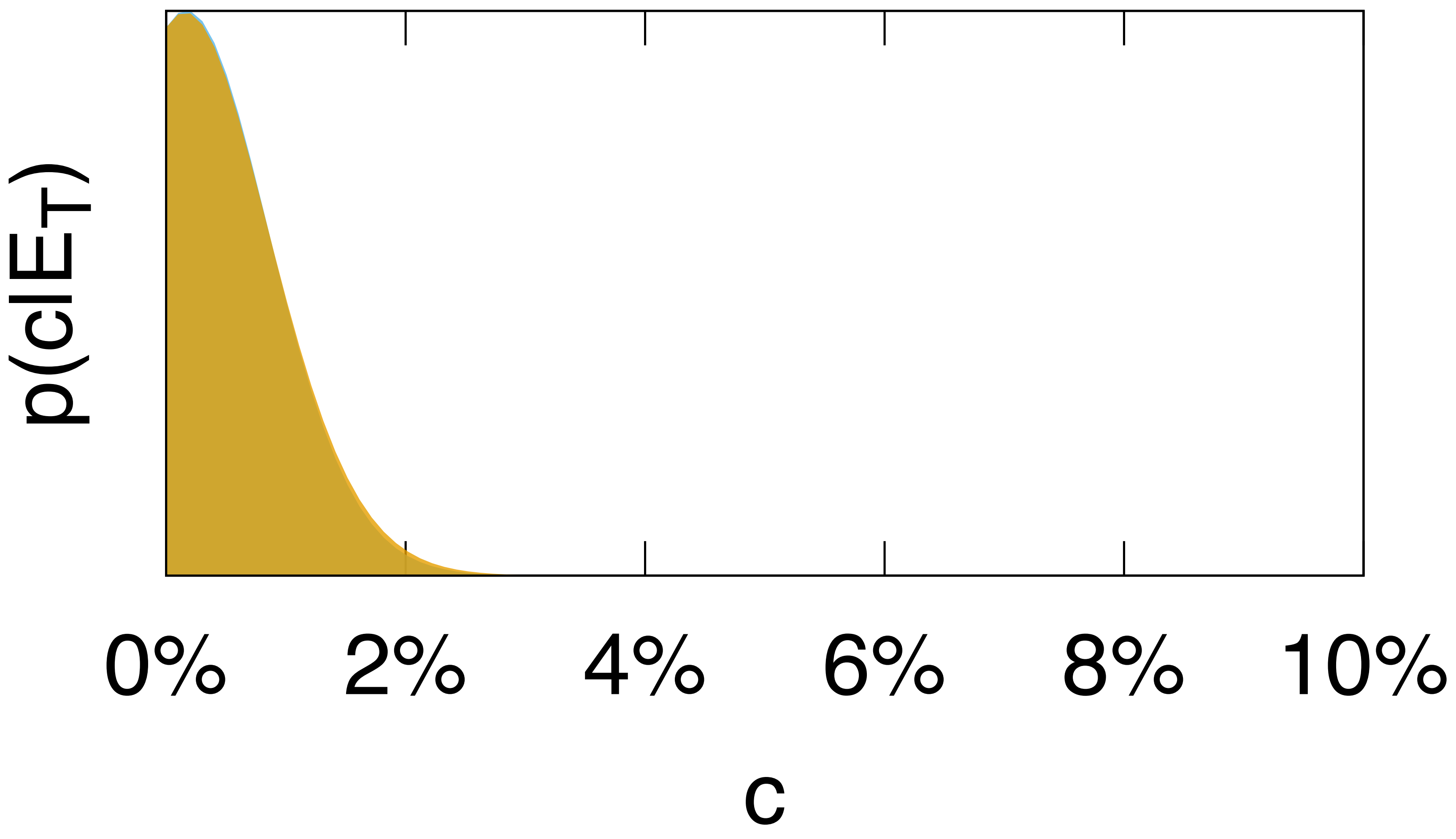


Above the knee of the distribution of E_T, the distribution hits the boundary at $c = 0$. No longer a Gaussian, but a *truncated* Gaussian.



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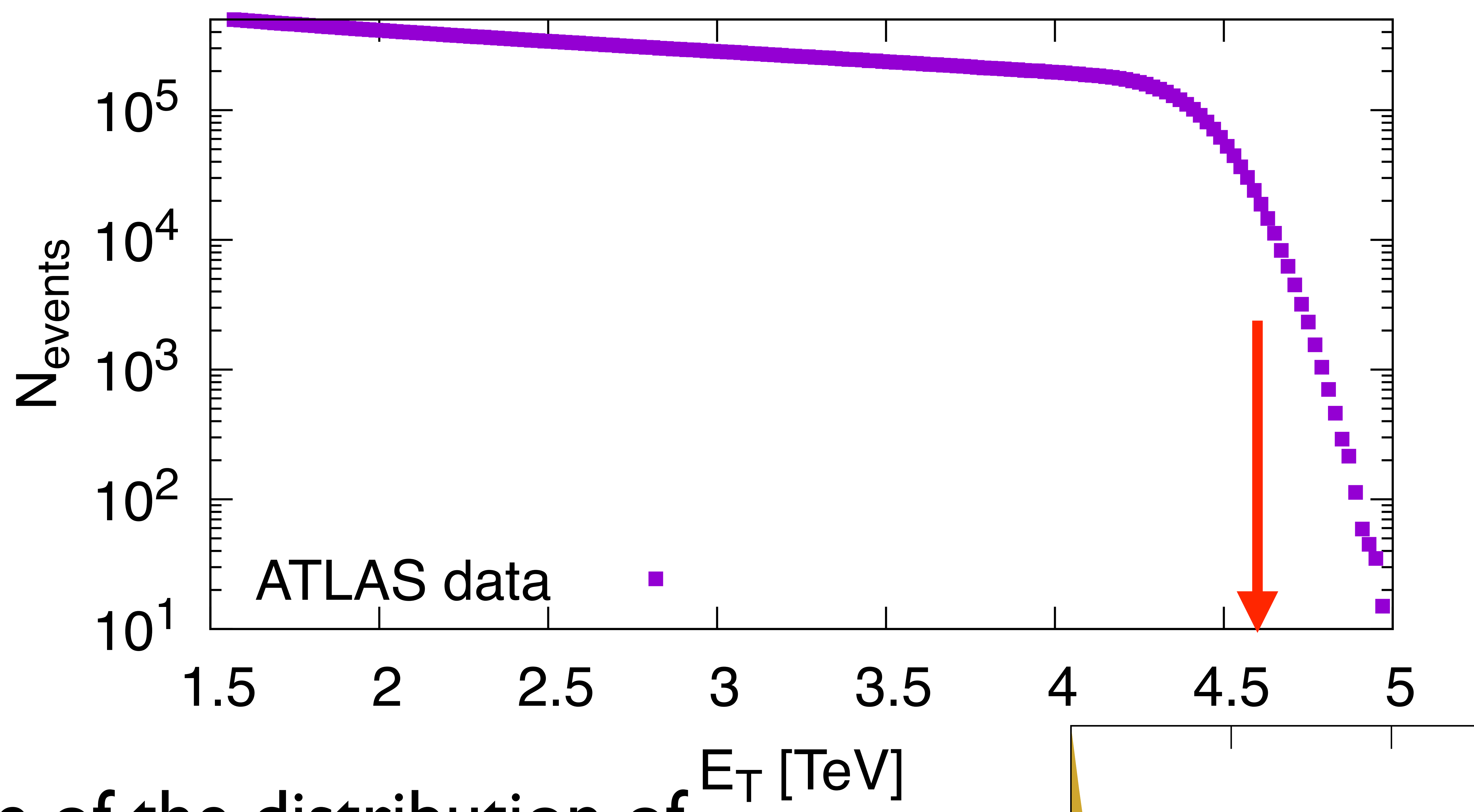

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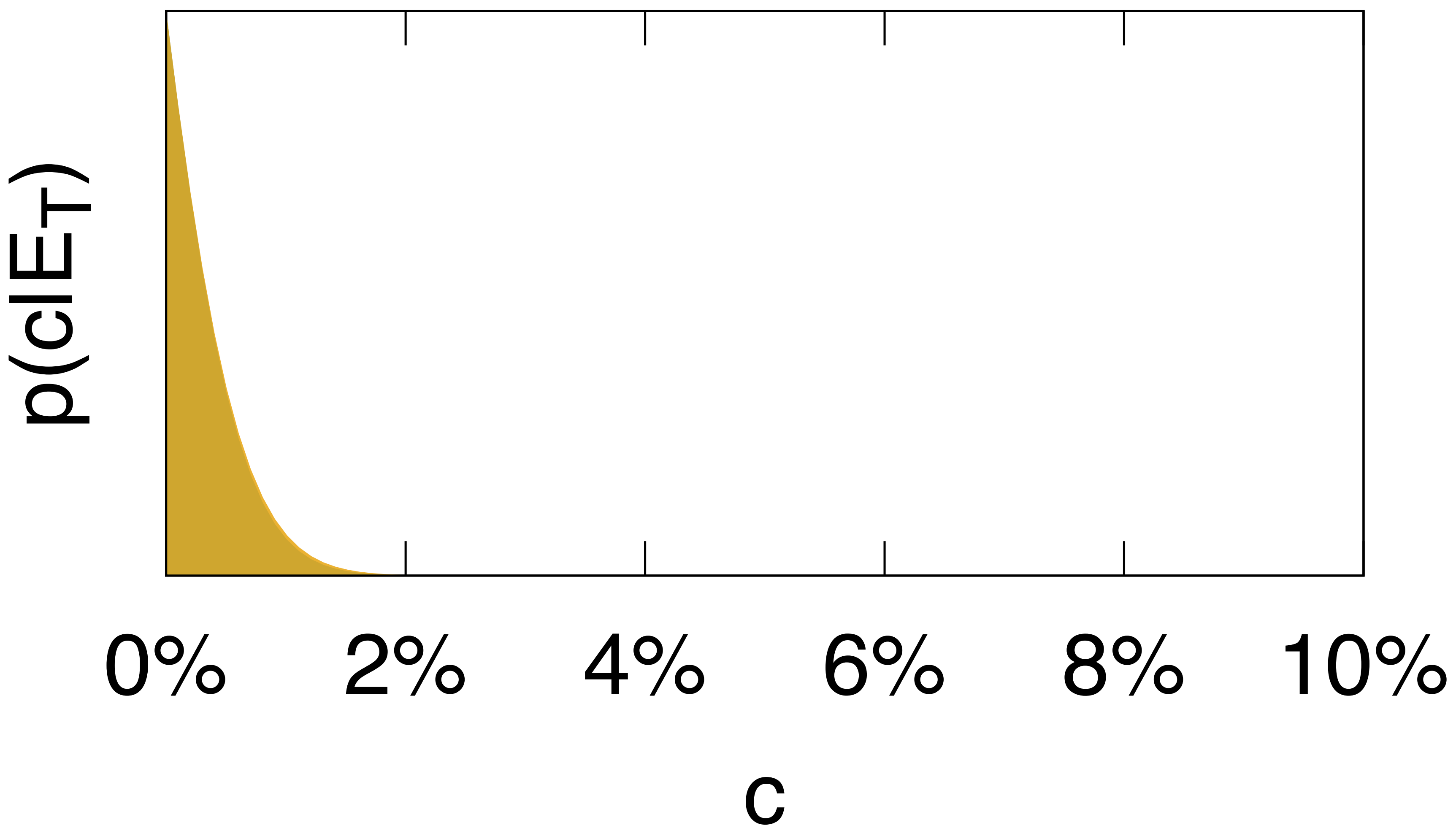
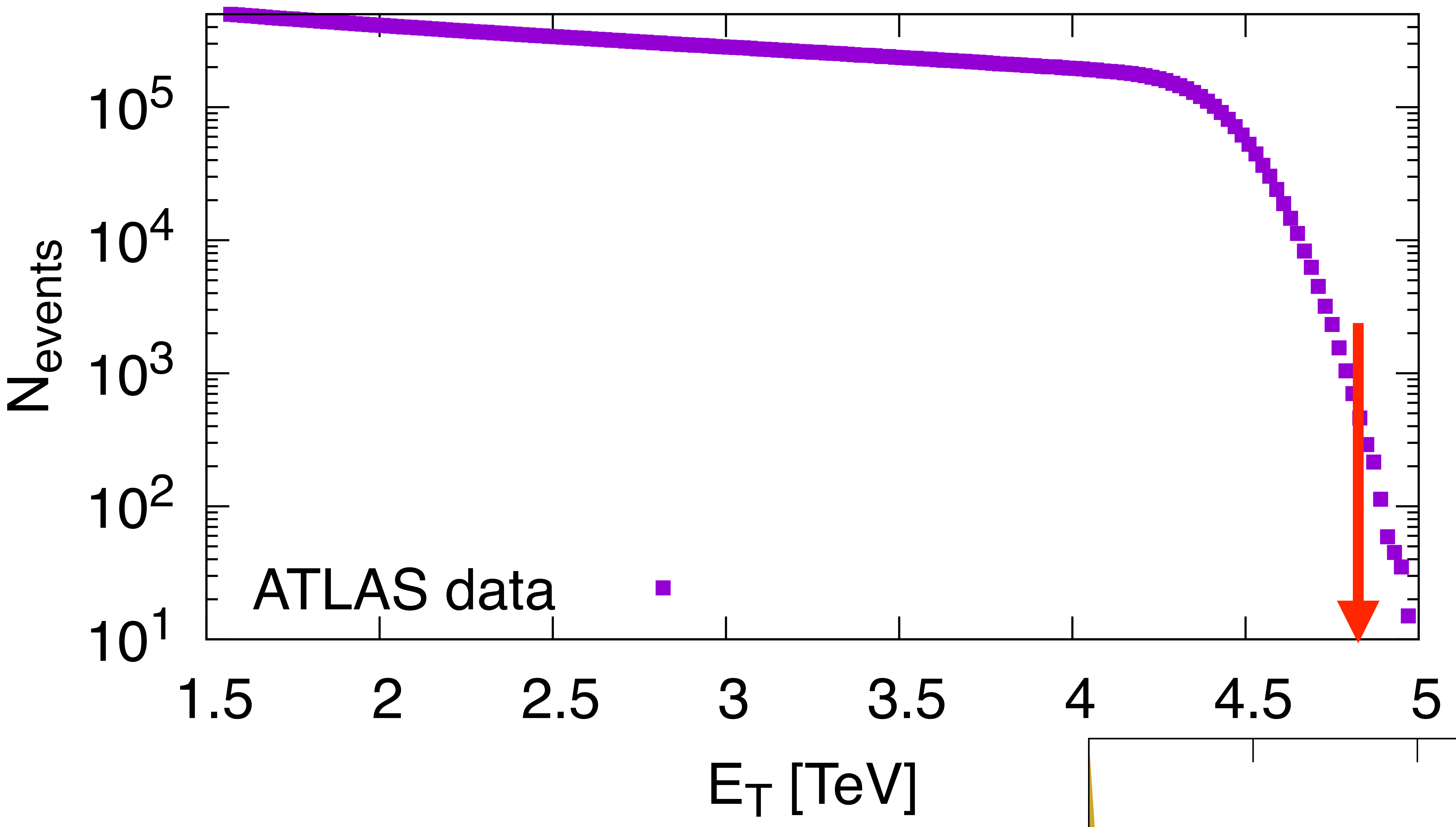
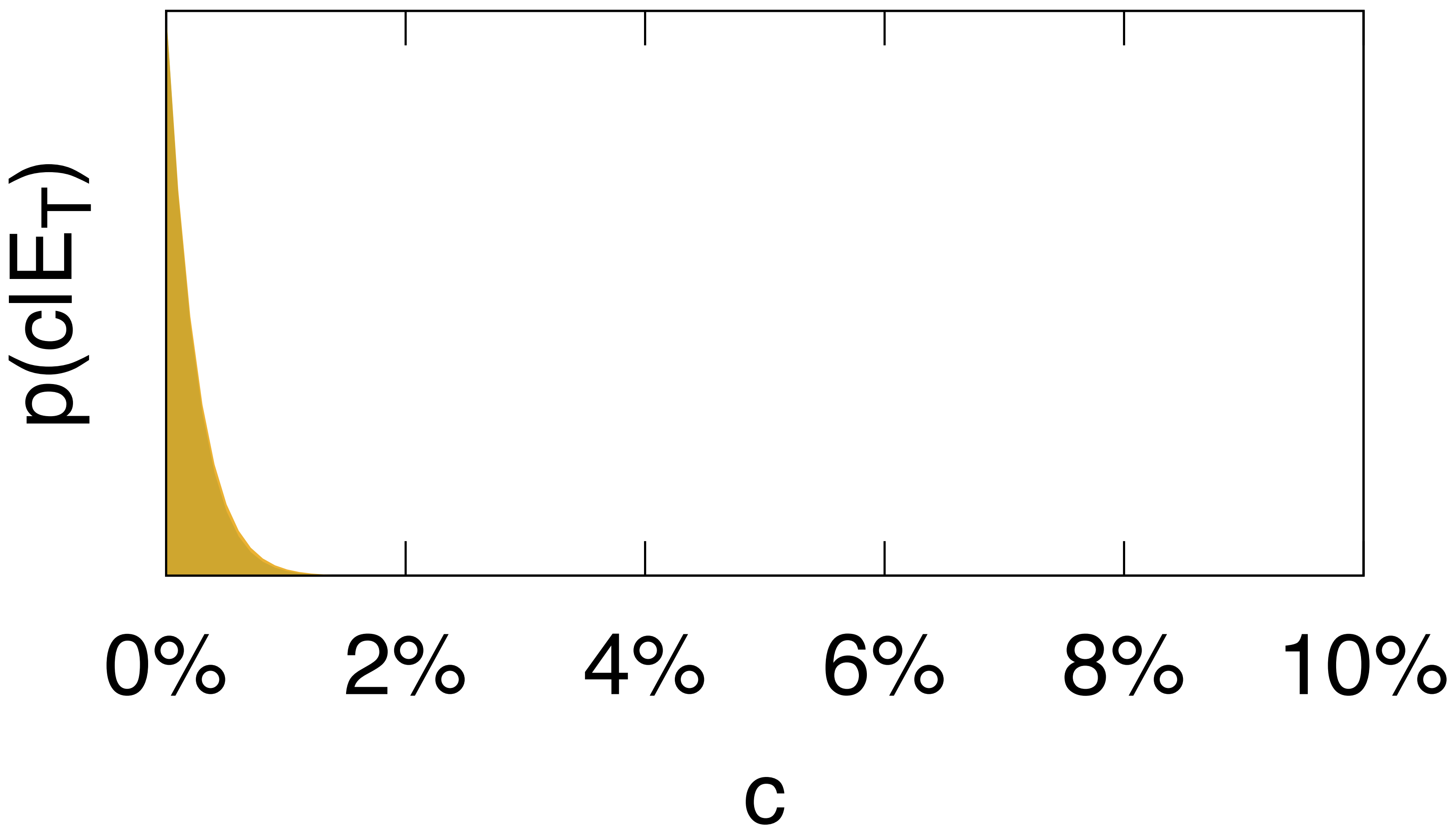


Distribution of centrality from Bayes' theorem



The larger E_T, the narrower the distribution: centrality fluctuations disappear.



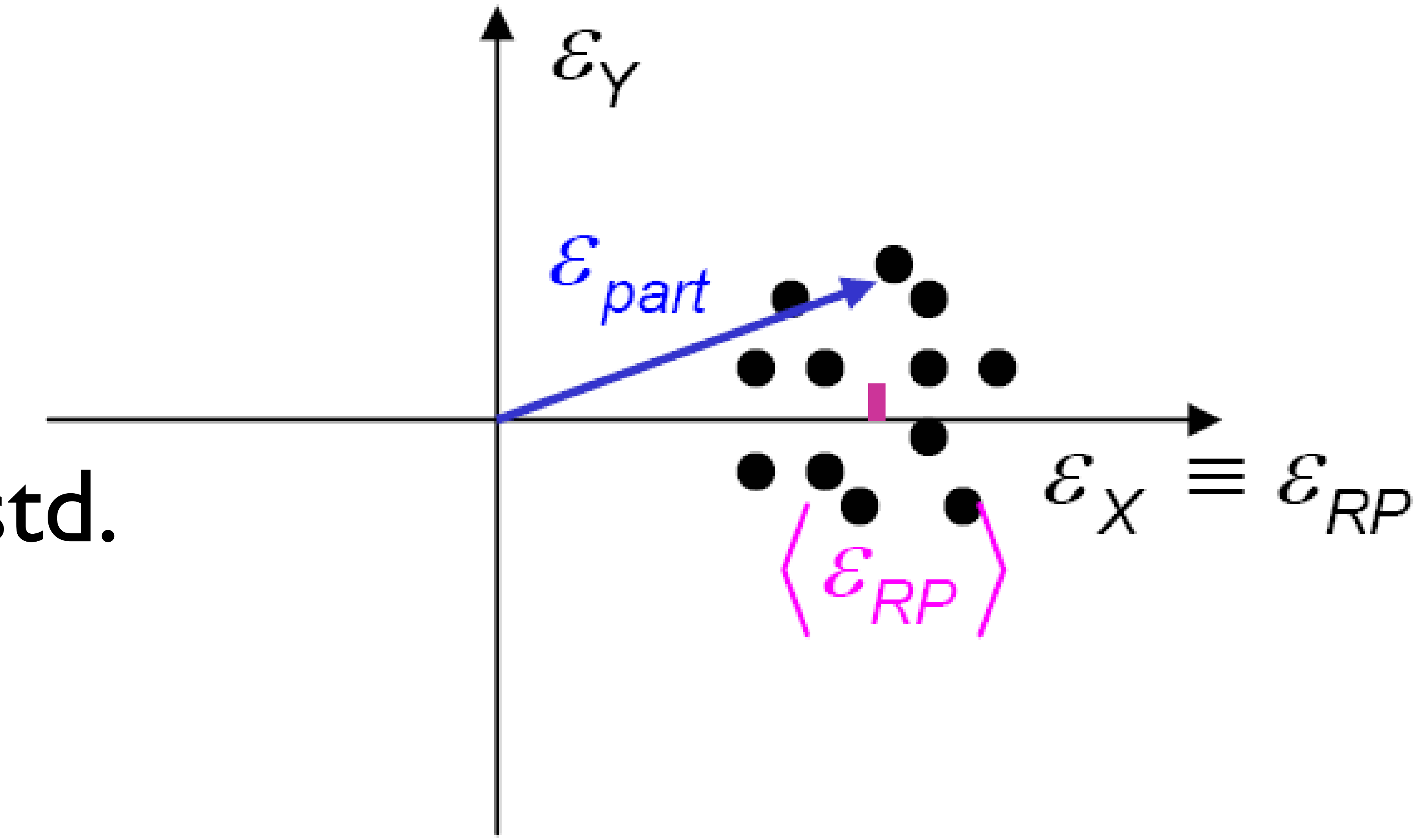
3. Understanding anisotropic flow (v_n) fluctuations in ultracentral collisions

I st step: Gaussian model

We assume that the fluctuations of elliptic flow
$v_{2x} \equiv \langle \cos(2\varphi) \rangle, v_{2y} \equiv \langle \sin(2\varphi) \rangle$ at fixed c are Gaussian
in the *intrinsic* frame where $x \parallel$ impact parameter:

$$p(v_{2x}, v_{2y}) = \frac{1}{\pi \sigma_v^2} \exp\left(-\frac{(v_{2x} - \overline{v})^2 + v_{2y}^2}{\sigma_v^2} \right).$$

the mean elliptic flow in the reaction plane, $\overline{v}$, and the std. dev. of flow fluctuations σ_v are smooth functions of c, which are our fit parameters.
Note that azimuthal symmetry requires $\overline{v}(c=0)=0$.



Voloshin Poskanzer Tang Wang
0708.0800

For v_3 and v_4, same, with $\overline{v}(c)=0$ (triangular flow is solely due to fluctuations)

Averaging over centrality

The moments of the distribution of v_2 at fixed c are those of the Gaussian:
$\langle v_2^2 \rangle = \bar{v}^2 + \sigma_v^2$
$\langle v_2^4 \rangle = \bar{v}^4 + 4\bar{v}^2\sigma_v^2 + 2\sigma_v^4$
If experiments were carried out at fixed centrality, the Gaussian model would give
$v_2\{2\} \equiv \left(\langle v_2^2 \rangle\right)^{1/2} = \sqrt{\bar{v}^2 + \sigma_v^2}$ (flow in reaction plane+fluctuations)
$v_2\{4\} \equiv \left(2\langle v_2^2 \rangle^2 - \langle v_2^4 \rangle\right)^{1/4} = \bar{v}$ (higher-order cumulants=flow in reaction plane)

In reality, centrality fluctuates: For a given value of E_T (or N_{ch}), we average moments over c using the probability distribution $P(c \mid E_T)$, before evaluating the cumulant $nc_2\{4\}$.

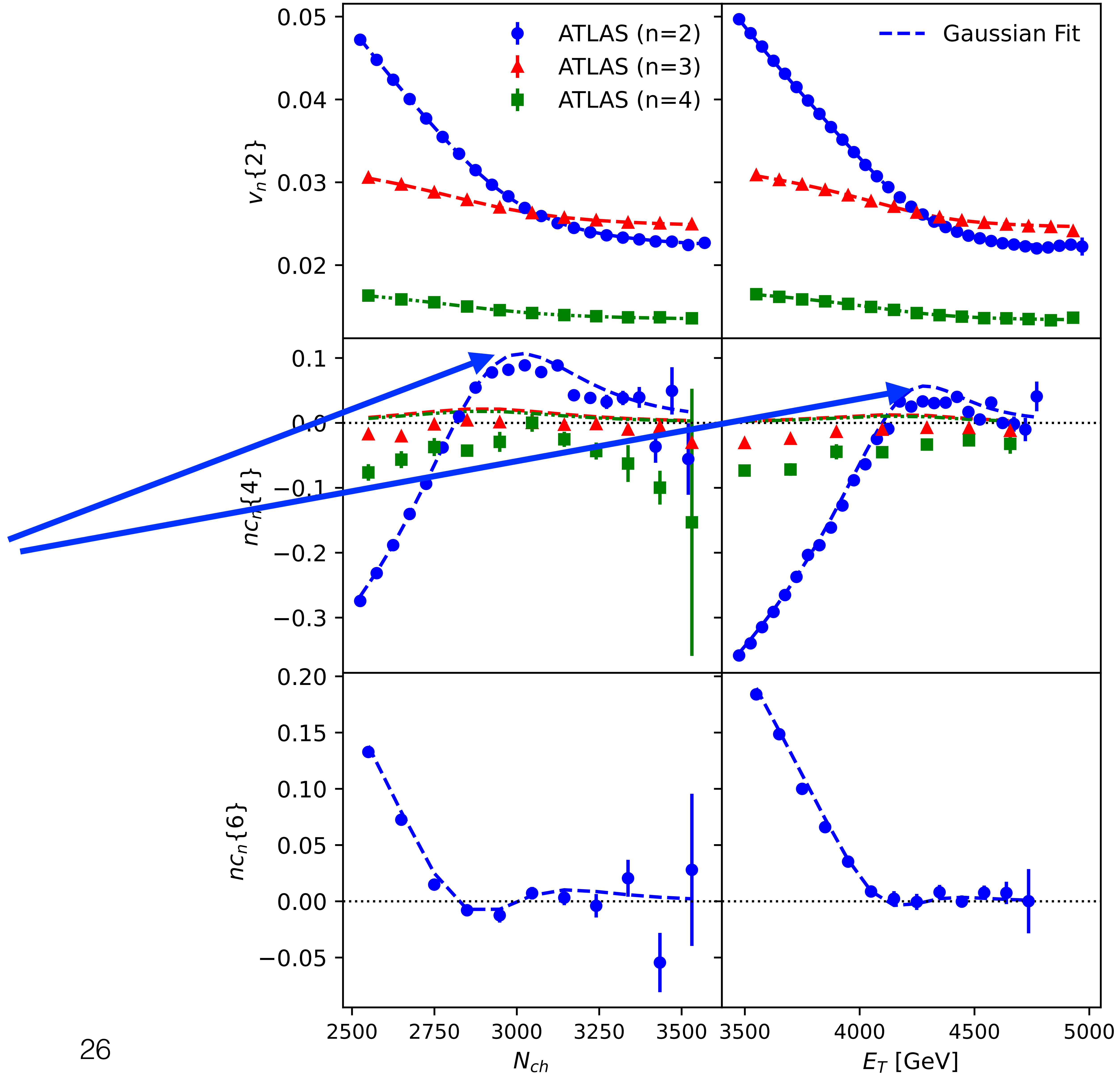
For v_3 and v_4, same, with $\bar{v} = 0$.

Fitting ATLAS data

The simple Gaussian model reproduces all elliptic flow (v_2) data: It fits simultaneously cumulants of order 2, 4, 6 are, with both centrality classifiers.

In particular, we get the change of sign of $nc_2\{4\}$ "for free".

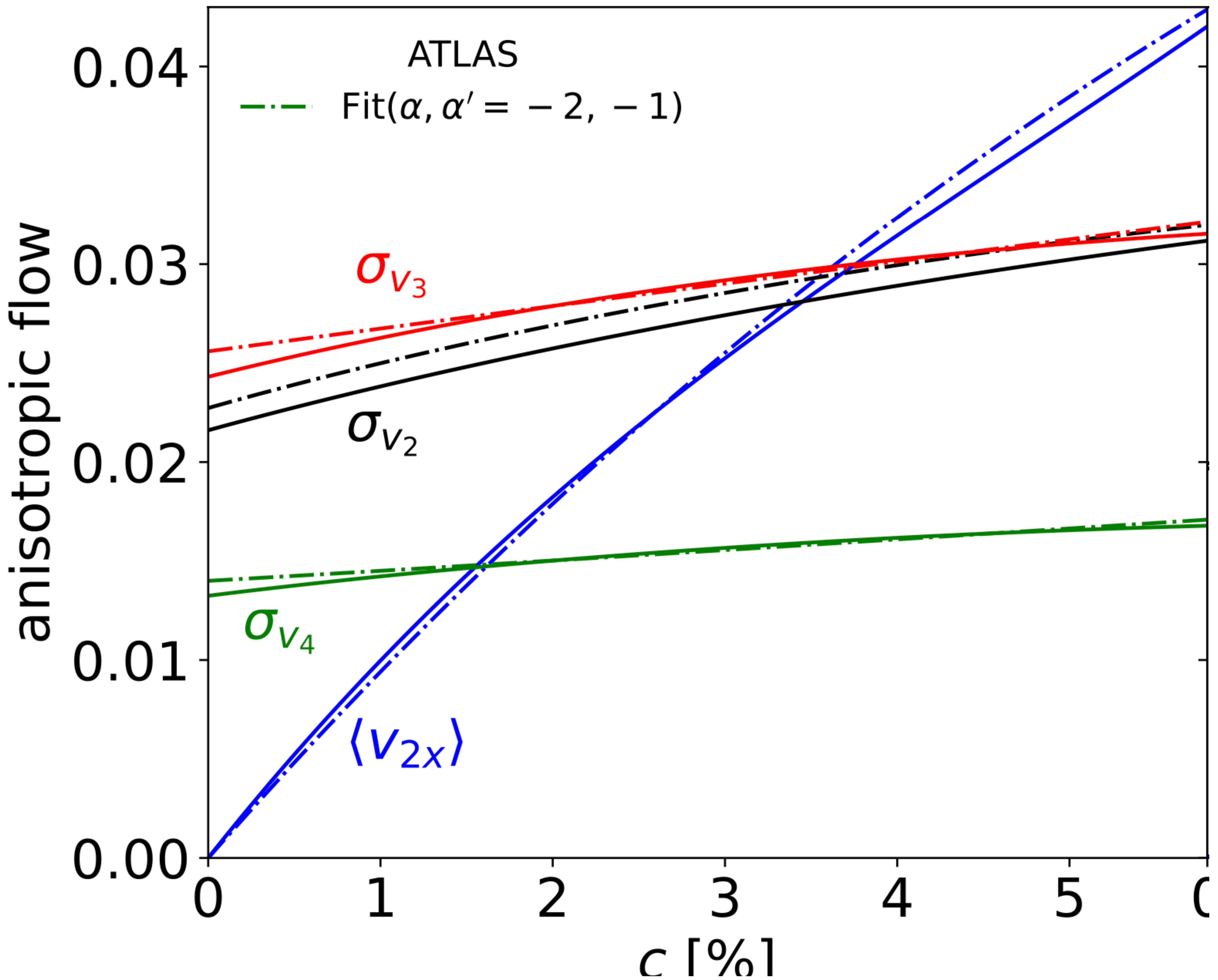
Implies that v_2 is driven by the true centrality, rather than N_{ch} or E_T.



Output of fit

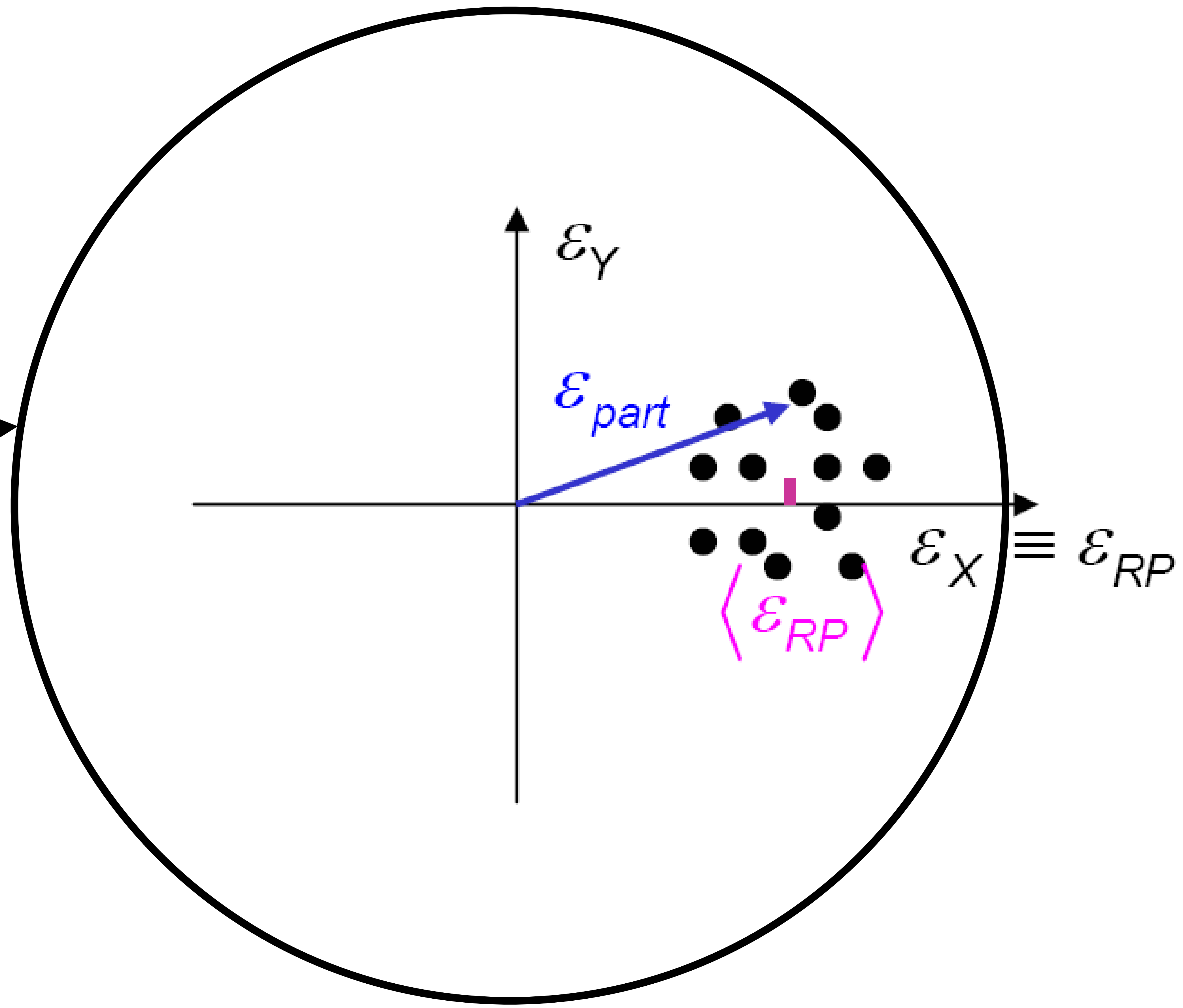
Our fit returns the variation of the mean elliptic flow in the reaction plane and of the width of flow fluctuations with the *true centrality*
(only look at solid lines...)

This facilitates comparison between theory and data. Running hydro calculations at fixed impact parameter is straightforward.



2nd step: Non-Gaussian corrections

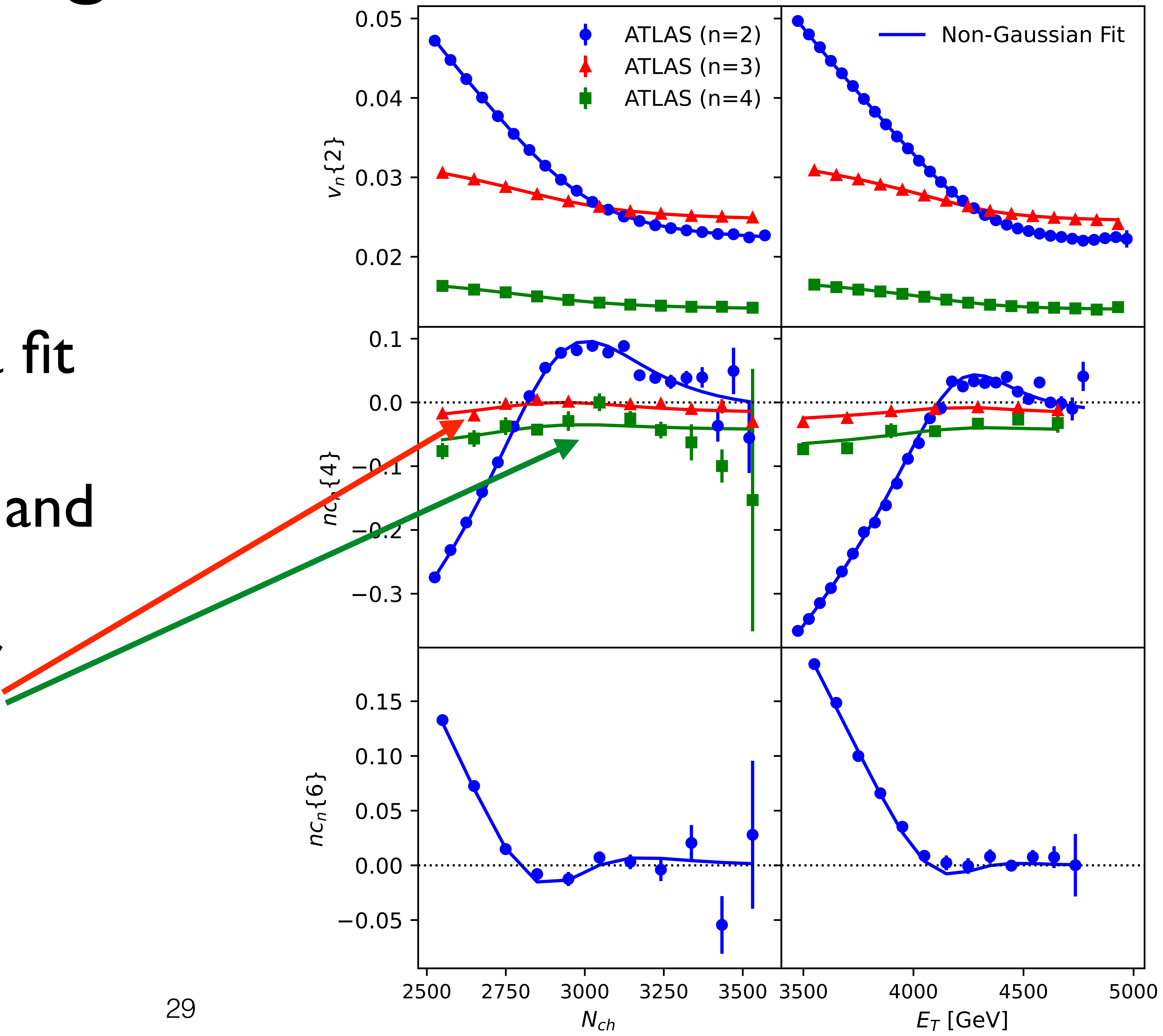
- Event-by-event hydrodynamic simulations have established that v_n is proportional to the initial anisotropy ε_n to a good approximation: $v_n = \kappa_n \varepsilon_n$
- ε_n is bounded by unity: $\varepsilon_x^2 + \varepsilon_y^2 < 1$
- This implies that the distribution of $(\varepsilon_x, \varepsilon_y)$ is narrower than a Gaussian. Generates a negative $nc_n\{4\}$.
- *This effect explains both $v_2\{4\} > 0$ in p+Pb, and $v_3\{4\} > 0$ in Pb+Pb.*



Yan JYO 1312.6555

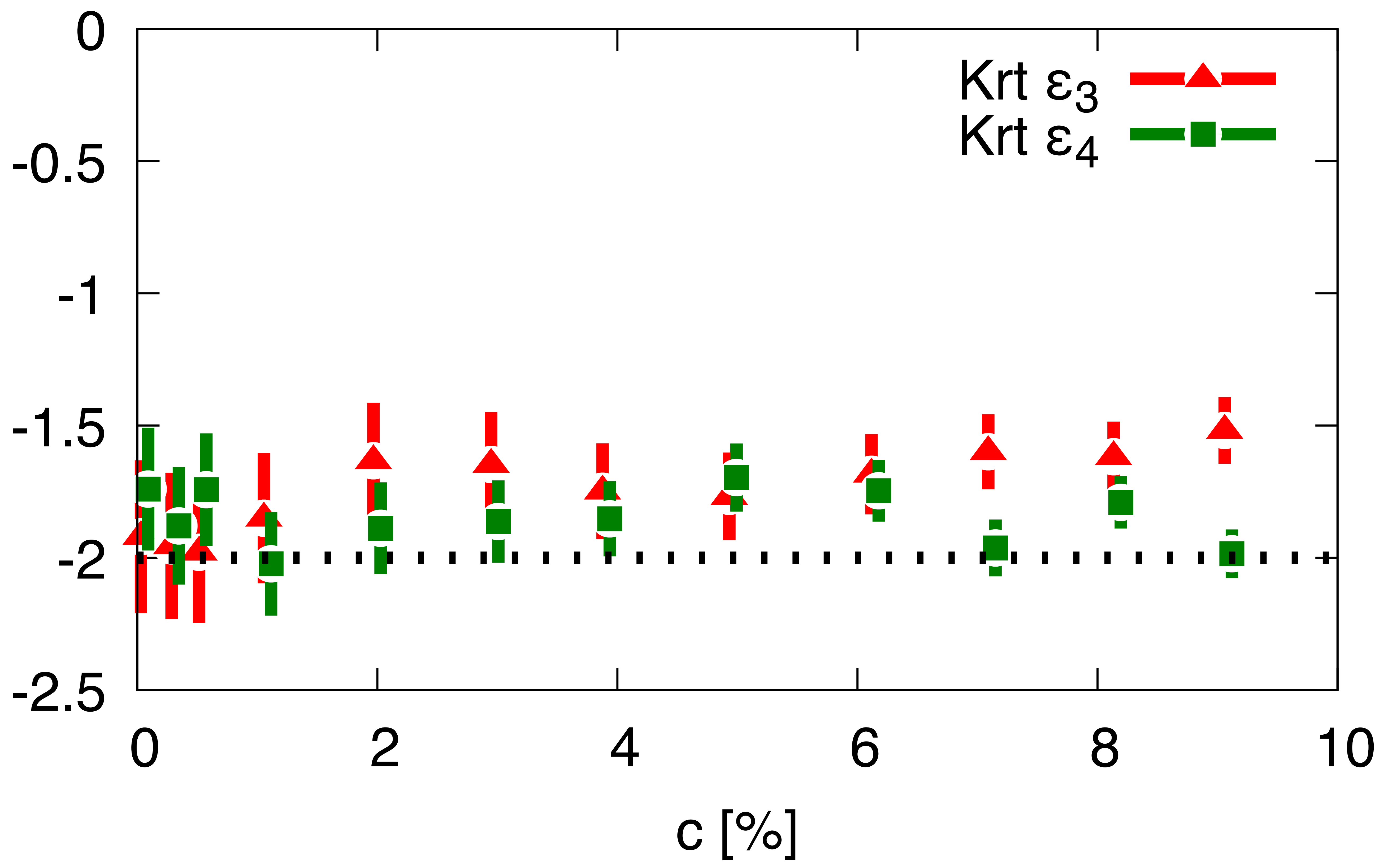
Refitting ATLAS data

- We add the leading non-Gaussian corrections to the distribution at fixed c: 1 extra fit parameter for v_3 and v_4 (kurtosis), 2 for v_2 (skewness and kurtosis).
- Fit quality much improved for $nc_3\{4\}$ and $nc_4\{4\}$



Are non-Gaussianities universal?

- Larger fluctuations are less Gaussian.
- Simple scaling arguments show thatthe ratio $nc_n\{4\}/\langle\varepsilon_n^2\rangle$ should depend weakly on system size (cf. ratio of kurtosis/variance, Nadine's talk on Tuesday).
- Initial state calculations at fixed c using the Trento model consistently return $nc_n\{4\}/\langle\varepsilon_n^2\rangle \simeq -2$



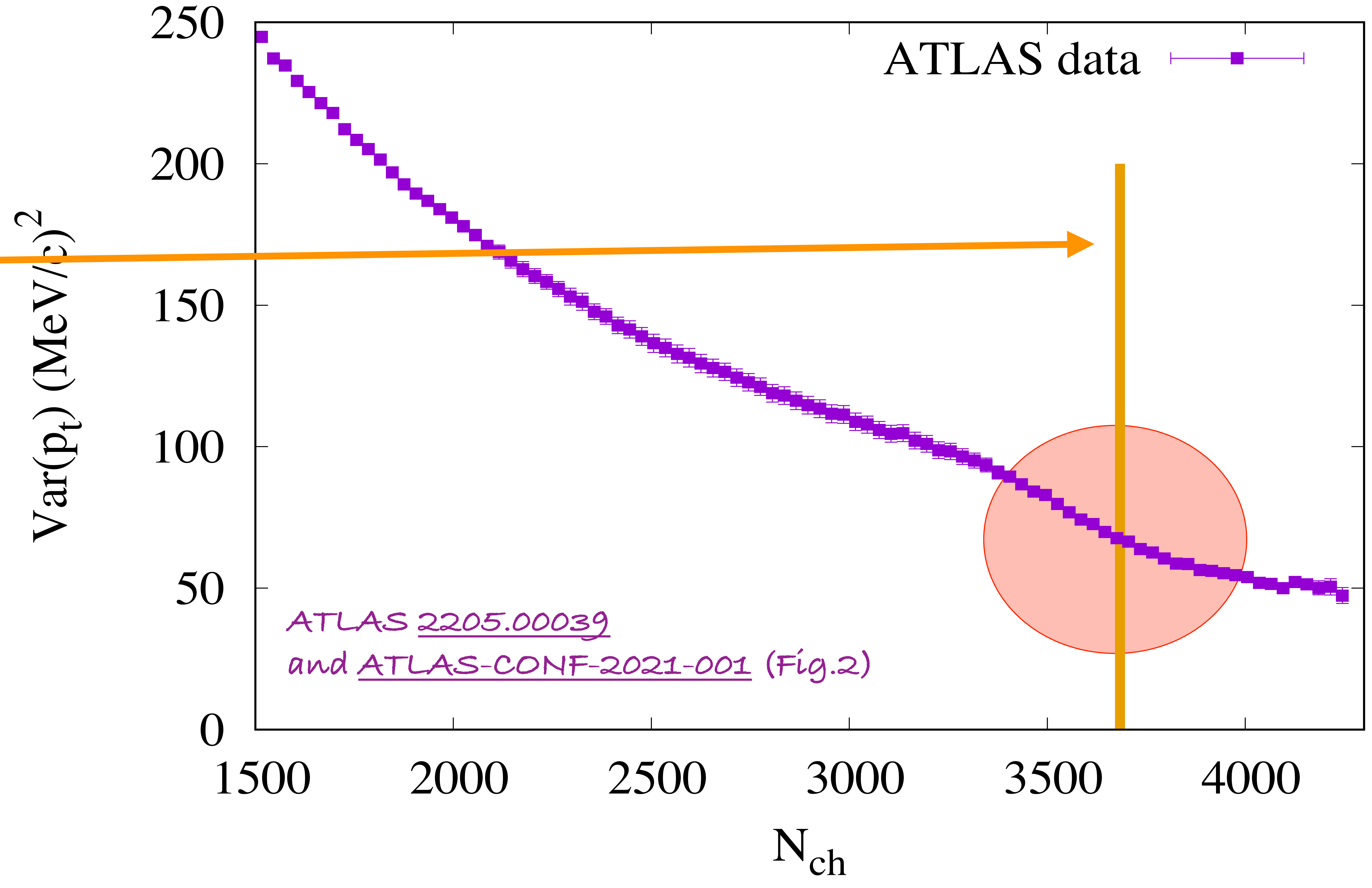
(speculative) data-driven estimate of hydro response

- If $v_n = \kappa_n \varepsilon_n$, where κ_n is the hydrodynamic response coefficient, then the boundary condition $\varepsilon_n < 1$ implies $v_n < \kappa_n$.
- Smaller κ_n implies less space for long tails, hence larger deviation from Gaussian.
- The larger non-Gaussianity for v_4 than for v_3 is in fact a natural consequence of the smaller hydrodynamic response in the higher harmonic.
- Assuming a universal non-Gaussianity at fixed c, $nc_n\{4\}/\langle \varepsilon_n^2 \rangle \simeq -2$, we obtain a data-driven estimate $0.09 < \kappa_4 < 0.11$.
- Bounds are tighter on κ_4 than on κ_2 and κ_3.
- Potentially interesting as κ_4 is more sensitive to viscosity than κ_2 or κ_3.

4. Understanding data on [p_t] fluctuations

ATLAS sees a fall of the variance of $[p_T]$ by a factor ~ 2 around the knee.

We model this in a way analogous to v_n fluctuations, by assuming that fluctuations of $[p_T]$ at fixed c are Gaussian.



Fluctuations at fixed centrality

What we have learned so far:

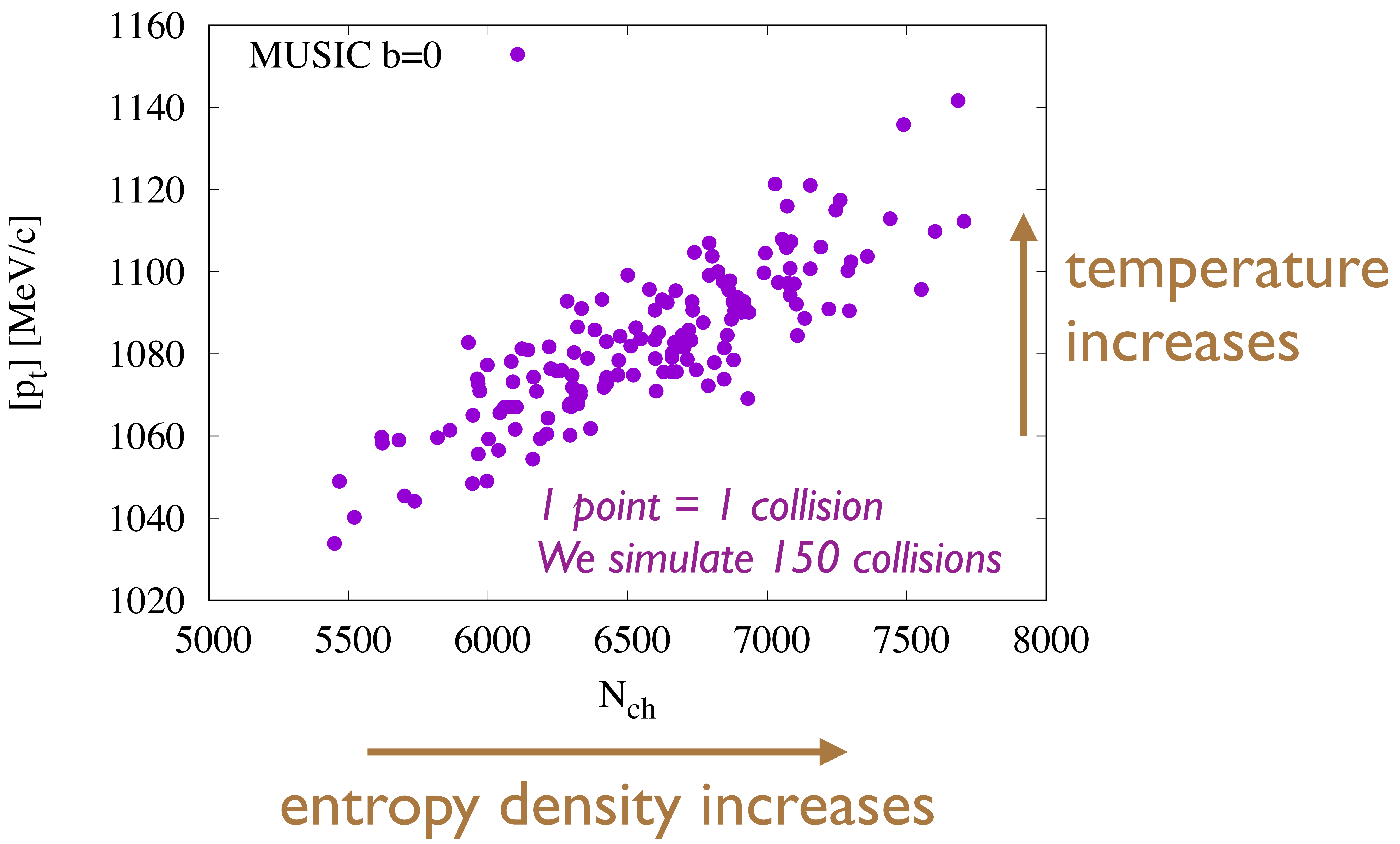
- Fluctuations of N_{ch} are Gaussian
- Fluctuations of the anisotropic flow vector $(v_{n,x}, v_{n,y})$ are (almost) Gaussian.
- One can neglect the correlation between $(v_{n,x}, v_{n,y})$ and N_{ch}.

Natural extension:

- Fluctuations of $[p_T]$ are Gaussian.
- Major difference is: the correlation between $[p_T]$ and N_{ch} is essential.

Event-by-event hydrodynamics at fixed c

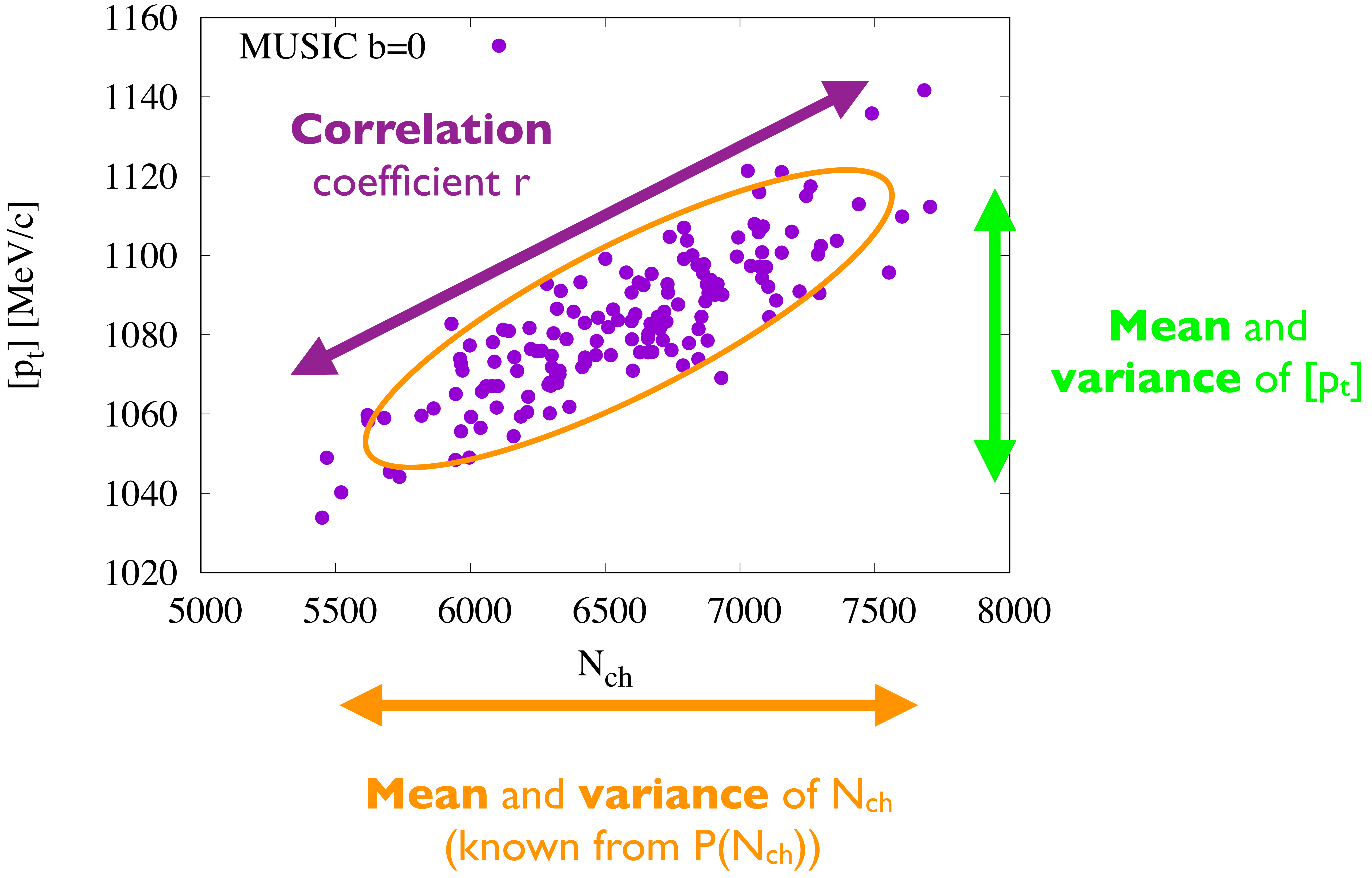
- Strong **correlation** between N_{ch} and $[p_T]$.
- In hydro, $[p_T]$ is proportional to the temperature of the QGP.
- The increase of $[p_T]$ with N_{ch} is driven by the speed of sound of the QGP.
- Consequence of **thermalization**.



Gardim Giacalone Luzum JYO
Nature Phys. 16 (2020) 6, 615

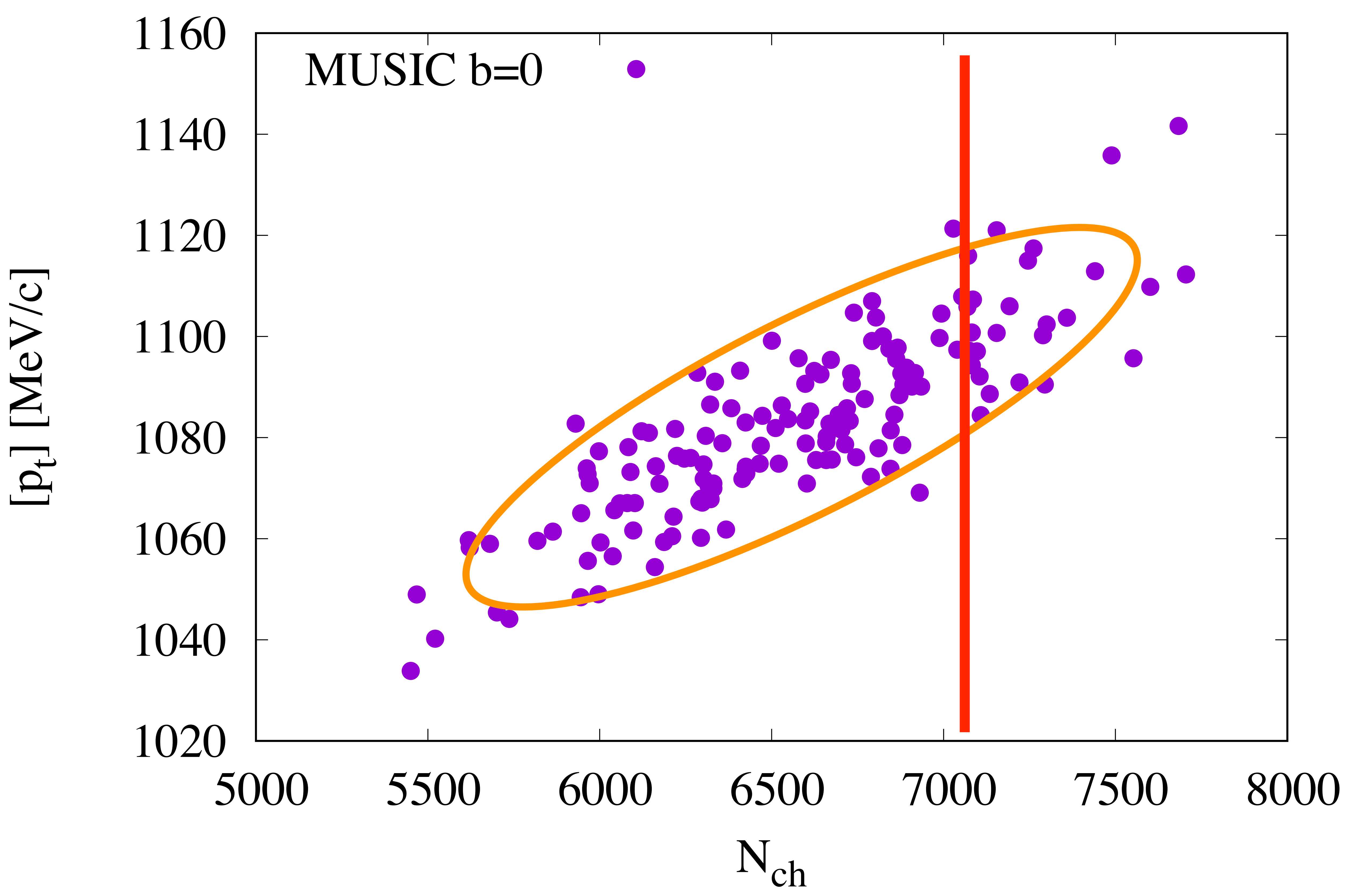
Gaussian parametrization

- We assume that the joint distribution of N_{ch} and $[p_T]$ is a correlated Gaussian, which has 5 parameters.
- 2 parameters are already known, 1 (mean p_t) is irrelevant.
- We assume that σ_{p_T} is a power law of multiplicity, and that r is constant:
- 3 fit parameters adjusted to ATLAS data.

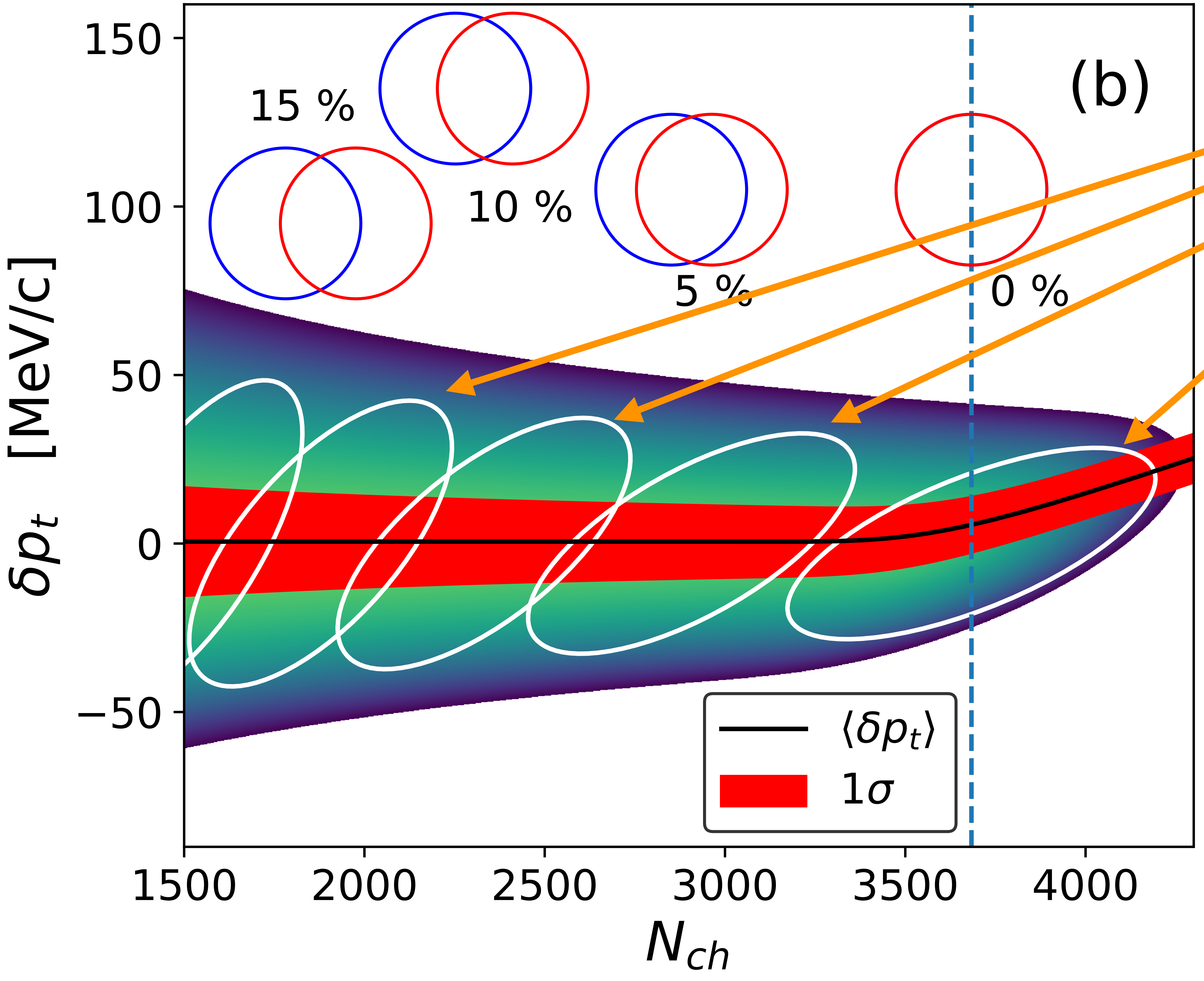


Averaging over centrality

- The distribution of $[p_T]$ at fixed N_{ch} and c is also a Gaussian (nice property of the multidimensional Gaussian distribution).
- Procedure = as for v_n:
  1. compute the moments $\langle [p_T]^n \rangle$ at fixed N_{ch} and c.
  2. average them over c.
  3. evaluate cumulants at fixed N_{ch} (variance, skewness), as measured in experiment.

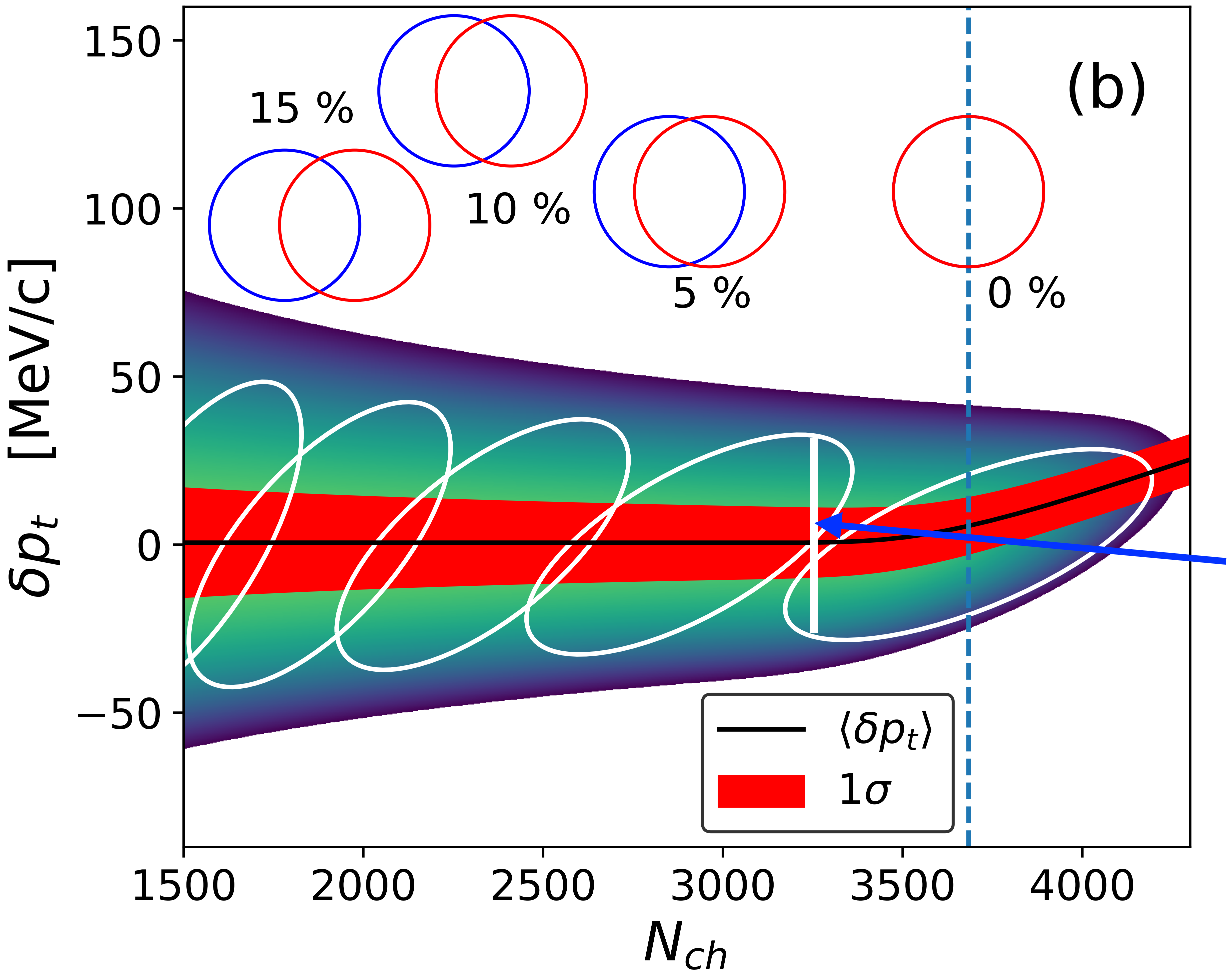


Fit results: P(N_{ch},δp_t)



Gaussian distributions at fixed c

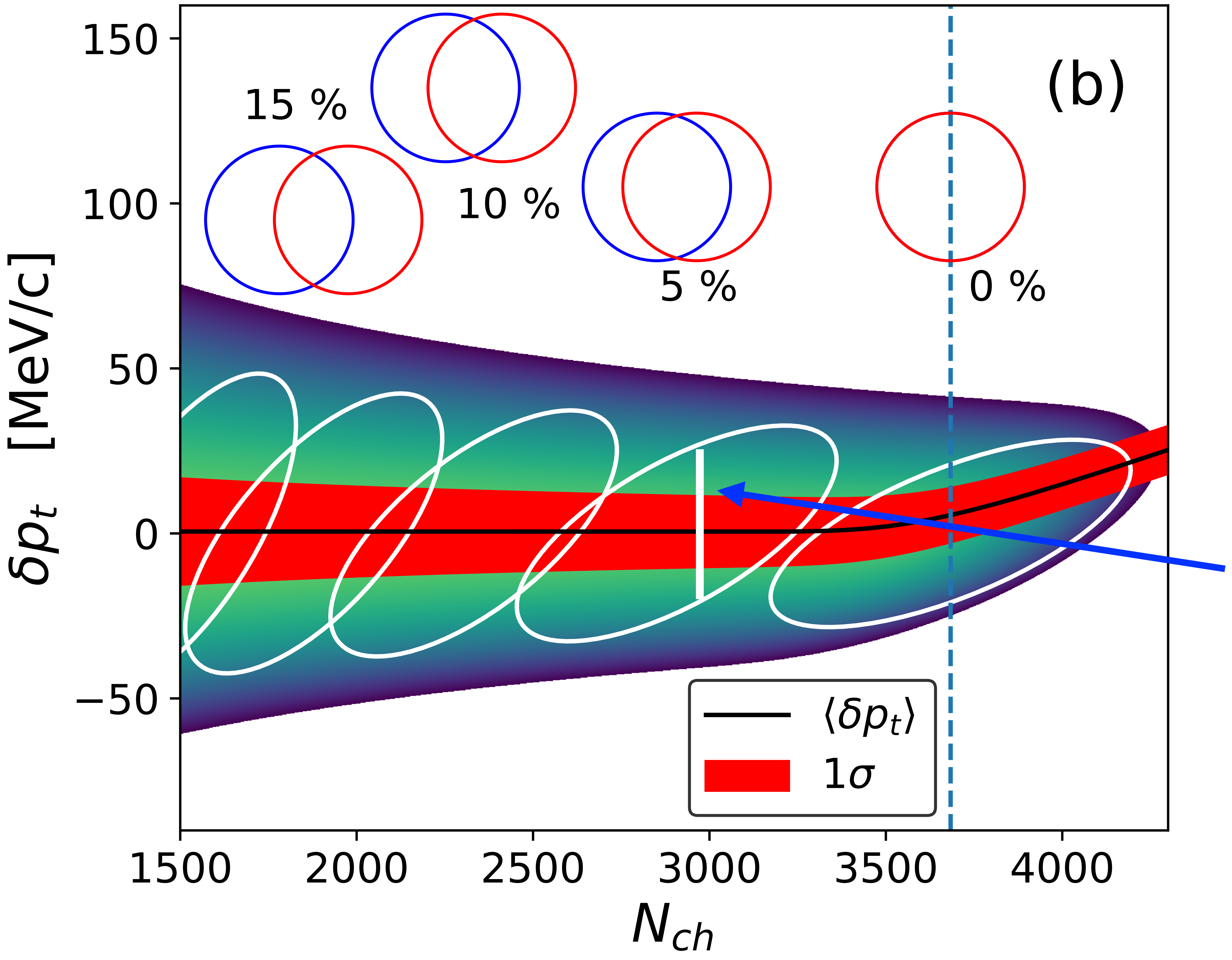
Fit results: P(N_{ch},δp_t)



At fixed N_{ch}, two contributions to the width in δp_t

1. fluctuations from the variation of b *(several ellipses contribute)*

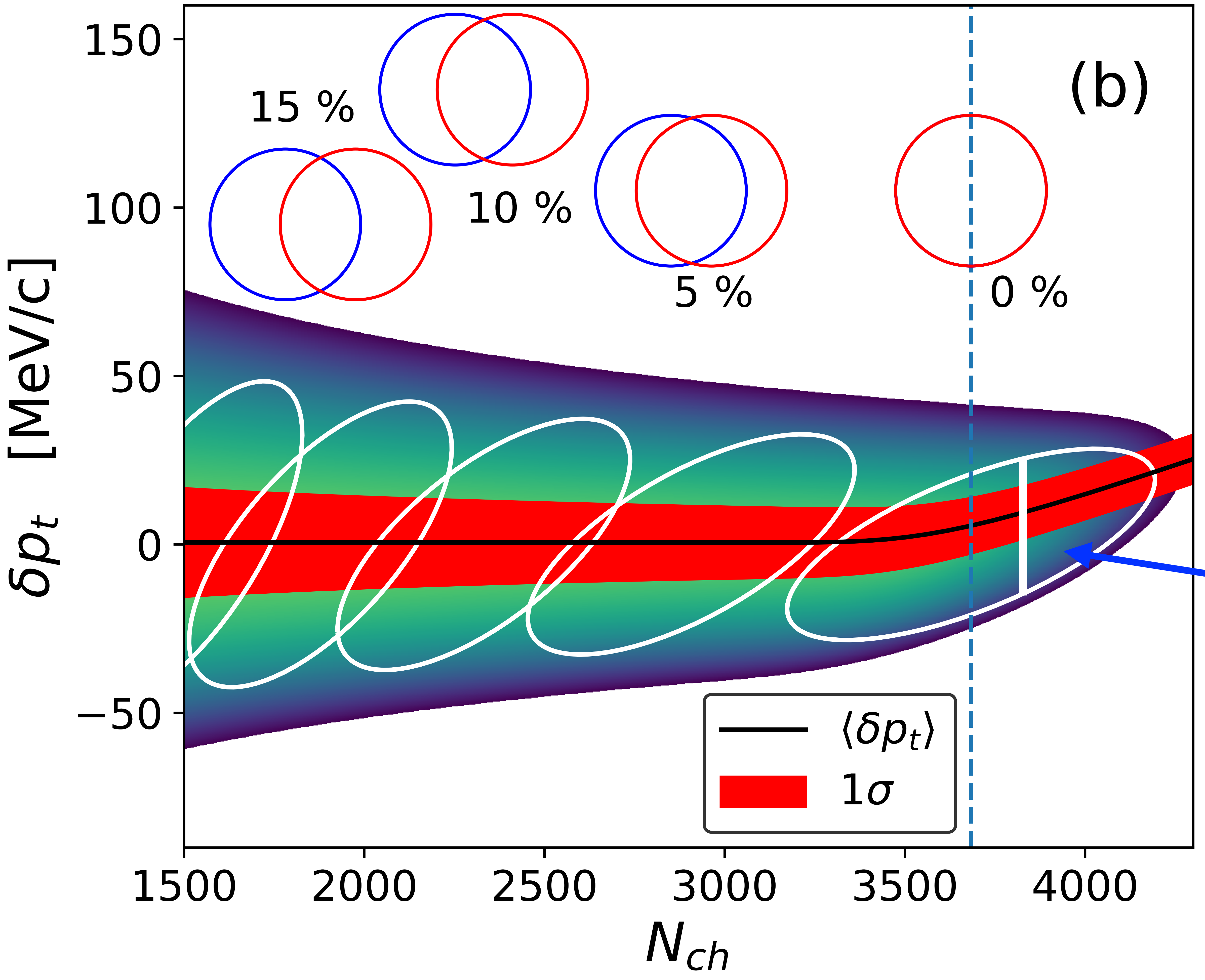
Fit results: P(N_{ch},δp_t)



At fixed N_{ch}, two contributions to the width in δp_t

2. fluctuations of δp_t at fixed b and N_{ch} *(height of a single ellipse)*

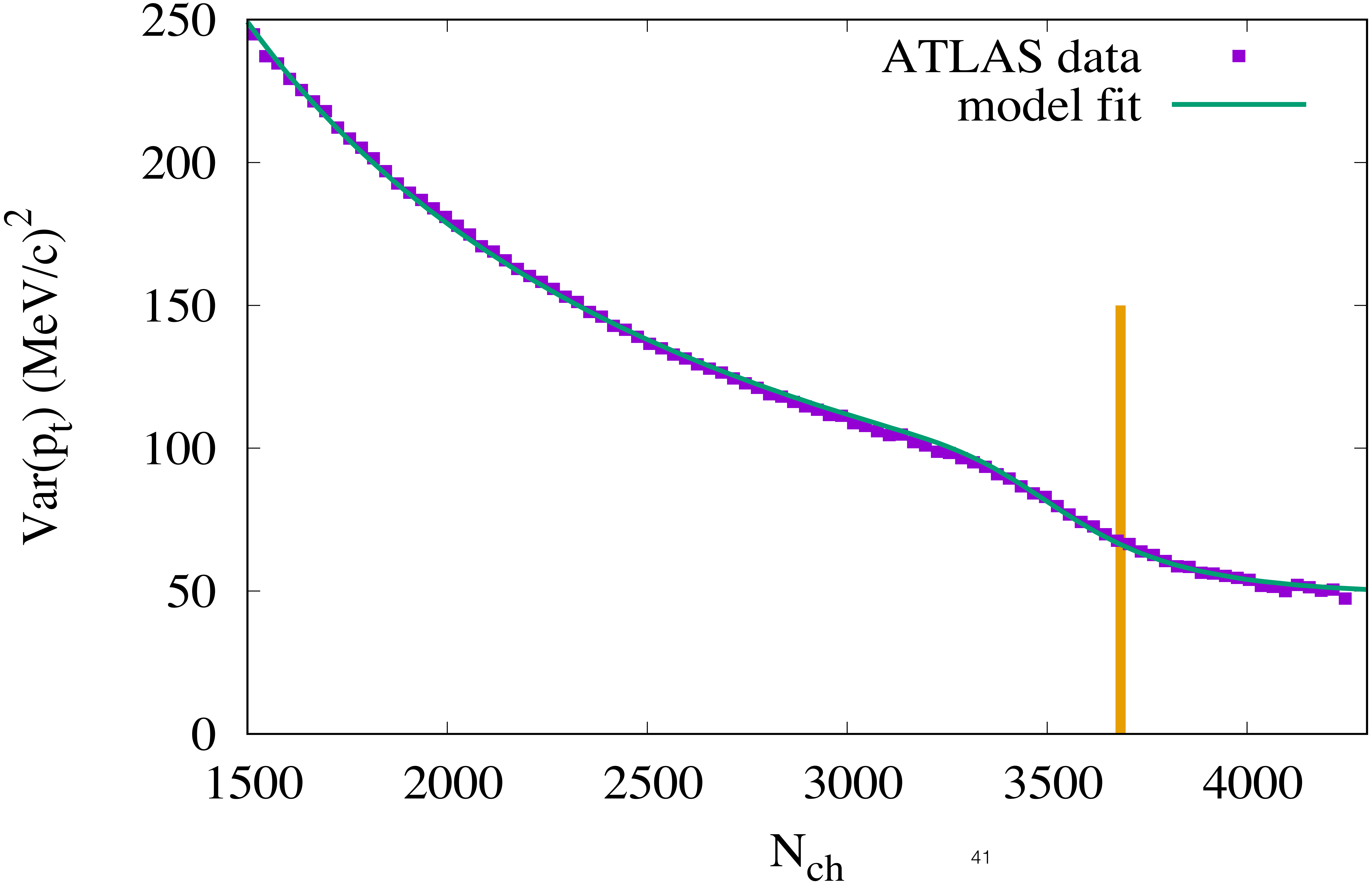
Fit results: P(N_{ch},δp_t)



At fixed N_{ch}, two contributions to the width in δp_t

2. Only this second term remains in ultracentral collisions

Fit results: Var($[p_t]$) versus N_{ch}



Our simple model naturally explains the observed fall in ultracentral collisions. It is the combination of two effects

1. Thermalization
2. Centrality fluctuations

Non-Gaussian fluctuations

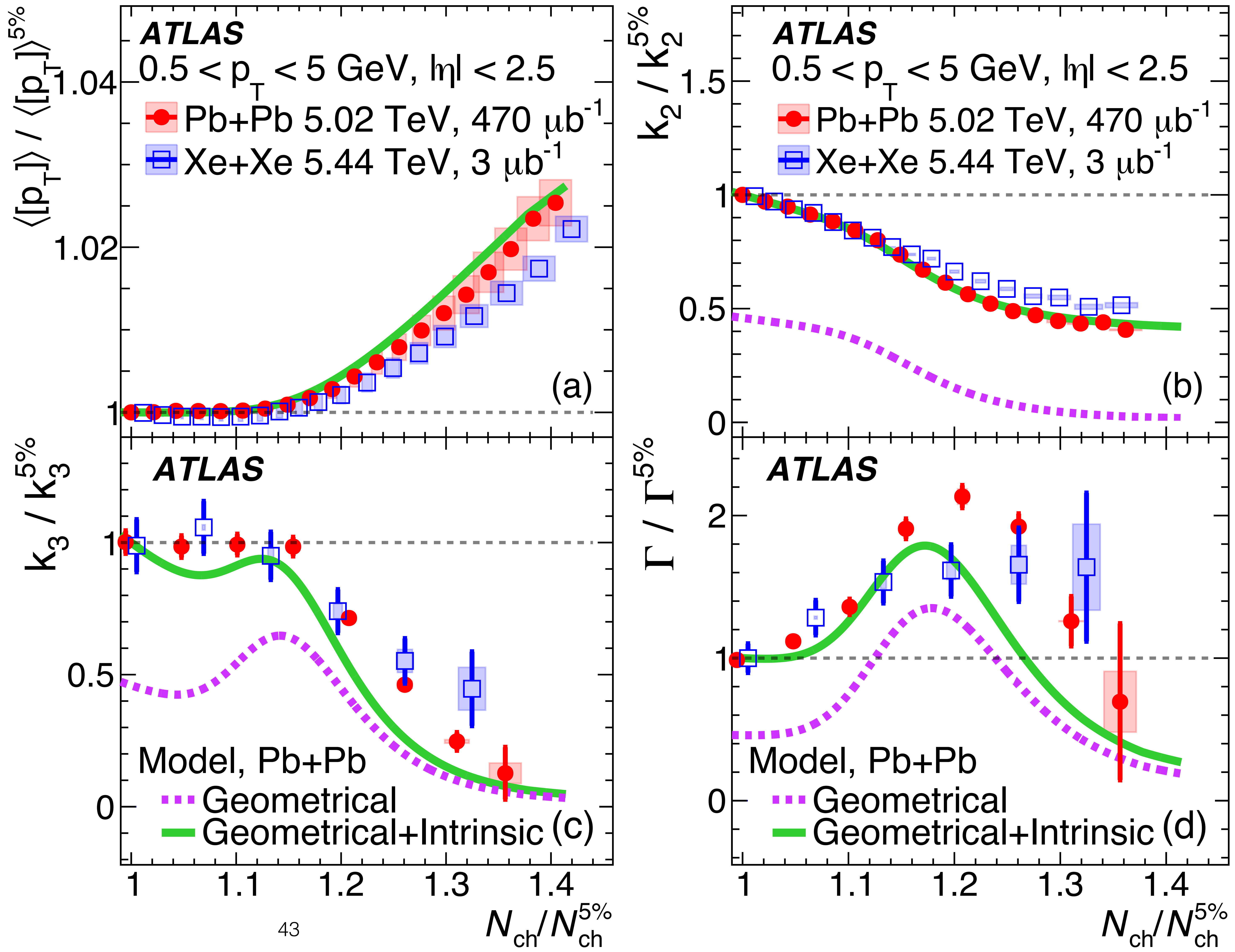
Samanta Picchetti Luzum JYO 2306.09294

- We predicted a significant skewness and kurtosis of $[p_T]$ fluctuations around the knee.
- Consider for simplicity that at fixed N_{ch}, $[p_T]$ increases linearly with centrality c.
- The distribution of centrality c is a truncated Gaussian around the knee.
- The skewness and kurtosis of $[p_T]$ fluctuations are those of the truncated Gaussian.

New ATLAS analysis

ATLAS 2407.06413

Our Gaussian model underestimates the skewness. Needs to be improved by skewing the Gaussian (intrinsic skewness).



Summary

- The quantum uncertainty on impact parameter is negligible in Pb+Pb collisions at the LHC: $\delta b = 4 \times 10^{-7}$fm
- Both the magnitude and orientation of impact parameter are classical quantities.
- Pb+Pb collisions with the same impact parameter differ only by quantum fluctuations.
- The impact parameter is an essential quantity for hydro modeling, as it determines the geometry.
- For technical reasons, this classical quantity cannot be measured, and it plays the role of a hidden variable, whose relevance is not always realized, both by theorists and experimentalists.

Perspectives

- I have shown only two types of observables, but the same approach can be generalized to many other correlations.
- Bożek's correlator between $[p_T]$ and v_n is also largely driven by centrality fluctuations: at fixed N_{ch}, collisions with larger c have both larger $[p_T]$ and larger v_n.

  Alqahtani Giacalone JYO, work in progress
- A similar reasoning applies to symmetric cumulants between, say, v_2 and v_3 (see talk by Magdalena on Monday).
- I want to extend this approach to semi-central collisions, but the magnitude of centrality fluctuations is only known in central collisions. I consider this a severe limitation for phenomenology.