

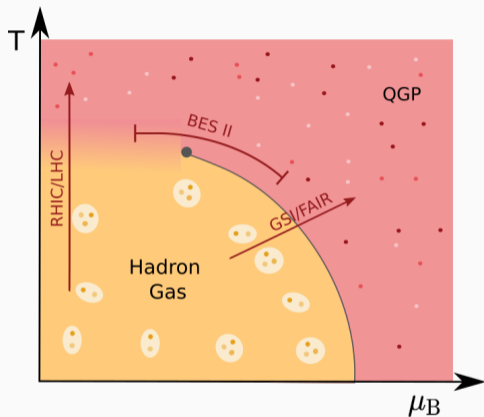
Bayesian analysis of a (3+1)D hybrid approach with initial conditions from hadronic transport

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Exploring the QCD phase diagram



- Want to study the properties of nuclear matter at different densities and temperatures
- Hybrid approaches, combining hadronic transport and viscous hydrodynamic evolution, describe dynamics of heavy-ion collisions well
- Require as an input an equation of state, transport coefficients as well initial condition
- Transport coefficients are hard to determine from first principles \rightarrow want to extract them from comparing model to data

Karpenko et al.: PRC 91 (2015)

Akamatsu et al.: PRC 98 (2018)

Du et al.: Comp.Phys.Com. 251 (2020)

Nandi et al.: PRC 102 (2020)

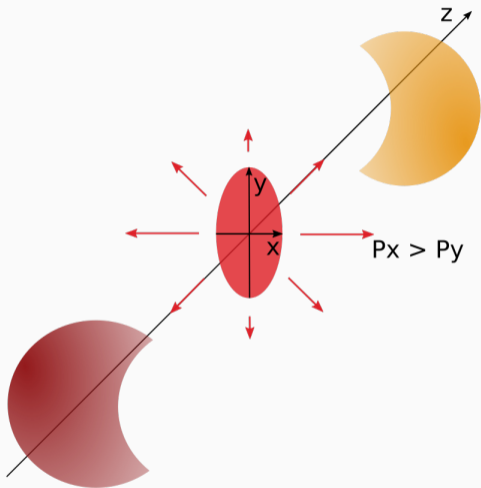
Schäfer et al.: 2112.08724 Shen: 2001.11858

Everett et al.: Phys. Rev. Lett. 126

Kovtun et al.: JHEP 0310 (2003)

Ghiglieri et al.: JHEP 1803 (2018)

Flow in heavy ion collisions



- Off-central collisions result in an eccentricity of the fireball, which results depending on the value of the shear viscosity into anisotropic flow in the final state
- Both the initial state eccentricities as well as the viscosity affect the flow measured in the final state
- In order to determine the transport coefficients, the initial state has to be under control
- Due to the lack of easy-accessible observables, there is significant theoretical uncertainty

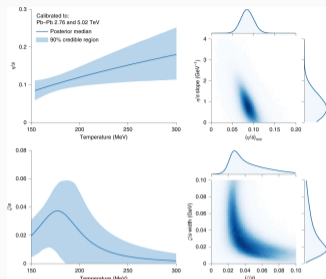
Shen: 2001.11858

Everett et al.: Phys. Rev. Lett. 126

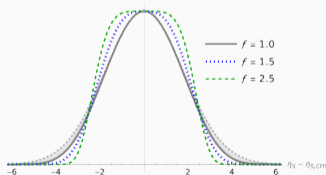
Schenke et al.: PRC 86 (2012)

Moreland et al.: PRC 92 (2015)

Bayesian inference



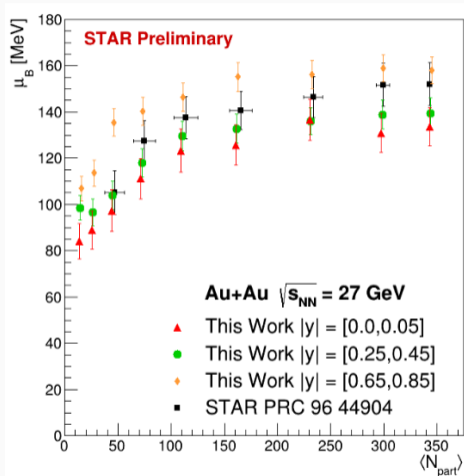
Bernhard et al. : Nature Phys 15 (2021)



- Bayesian inference gives probability distribution of model parameters by taking into account theoretical knowledge (prior) and experimental data
- Applied to heavy-ion collisions to constrain shear and bulk viscosities
- Often used with parametric initial conditions → initial conditions generated by hadronic transport reduce parameter space

Nijs et al.: PRL 126 (2020) Everett et al.: PRC 103 (2021)
Parkila et al. : PRB 835 (2022) Heffernan et al. : PRL 132 (2024)

Towards 3D Bayesian inference

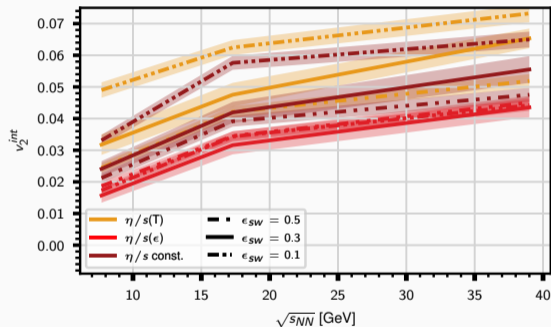


STAR EPJ 296 (2024)

- Earlier Bayesian analysis of HIC simulations focused (2+1)D simulations due to the high computational cost of 3D simulations
- Advances in simulation allow to investigate the full rapidity range, giving additional constraints
- 3D observables give access to a wider area in the $T-\mu_B$ -space than midrapidity data alone
- As a result, 3D Bayesian analysis can give stronger constraints

Auvinen et al.: PRC 97 (2018) Shun et al.: PRL 132 (2024)

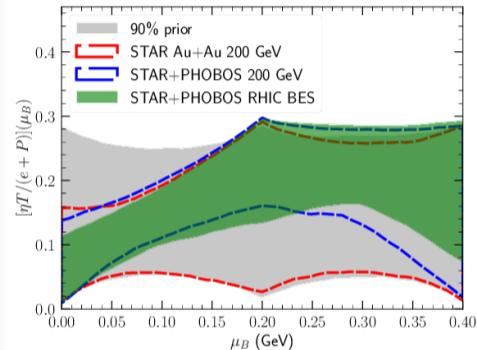
T and μ_B dependent viscosities



Götz et al.: PRC 106 (2022)

- Earlier studies in hybrid models show substantial effect of including T and μ_B dependence in the shear viscosity
- Flow predictions show reduced dependence on technical parameters
- However, this did not include effect of bulk viscosity and tuning to experimental data
- Ideally, the full range of reasonable dependencies is studied

T and μ_B dependent viscosities



Shun et al.: PRL 132 (2024)

- Initial studies focused on constraining η/s as a constant value or function of temperature only
- Additionally, not all studies included ζ/s , the presence of which can also influence signals relevant for the shear viscosity
- Reaching the point of being able to give constraints for η/s and ζ/s in the **full T - μ_B -plane**
- Due to high model uncertainties, predictions from different models are important to correctly assess our understanding!

- Extract (T, μ_B) -dependent parameterizations of viscosities from RHIC BES data
- Study the effect of including different observables on the constraints
- Study the sensitivity of observables on parameters both of the hydrodynamics and of the hadronic transport
- Constrain both the hydrodynamization of initial conditions from hadronic transport as well as late stage rescattering

SMASH-vHLE-hybrid



- Modular hybrid approach for the description of intermediate and high energy heavy-ion collisions
- Open-source and public
- [Available on Github](#)
- Conserves all charges (B , Q , S)

Schäfer et al.: EPJA 58 (2022)

Weil et al.: PRC 94 (2016)

DOI: 10.5281/zenodo.3484711

Karpenko et al.: PRC 91, 064901 (2015)

Karpenko et al.: CPC 185 (2014)

Most et al.: PRD 107 (2023)

SMASH

- Hadronic transport approach
- Initial conditions

+

vHLE

- 3+1D viscous hydrodynamics
- CORNELIUS routine to determine freezeout surface
- Chiral mean field equation of state

+

smash-hadron-sampler

- Cooper-Frye sampler
- Particlization of fluid elements

+

SMASH

- Hadronic transport approach
- Evolution of the late hadronic rescattering stage

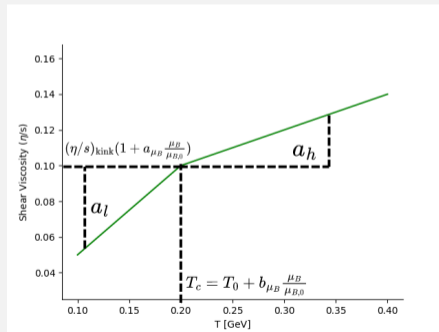
Parameterization of transport coefficients

Shear viscosity

$$\eta T / (\epsilon + P) = \max \left(0, \left((\eta/s)_{\text{kink}} + \begin{cases} a_l (T - T_c), & T < T_c \\ a_h (T - T_c) & T > T_c \end{cases} \right) \times \left(1 + a_{\mu_B} \frac{\mu_B}{\mu_{B,0}} \right) \right)$$

$$T_c = T_0 + b_{\mu_B} \frac{\mu_B}{\mu_{B,0}}$$

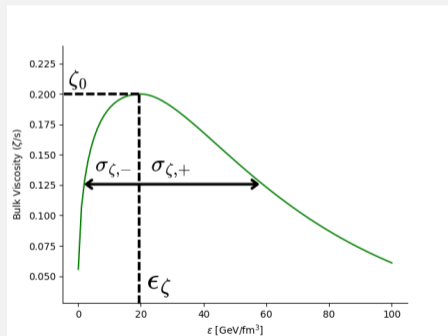
- Linear dependence on temperature
- Scaling with baryochemical potential
- Minimum can change with baryochemical potential
- Allows for constant shear viscosity



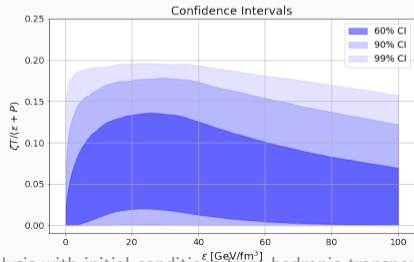
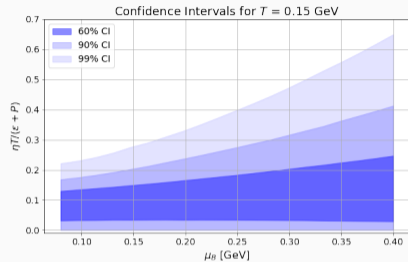
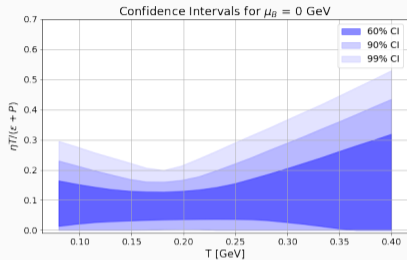
Bulk viscosity

$$\zeta T/(\epsilon + P) = \zeta_0 \times \begin{cases} \exp\left(-\beta \left(\frac{(\epsilon^{1/4} - \epsilon_\zeta^{1/4})^2}{2\sigma_{\zeta,-}^2}\right)\right), & \epsilon < \epsilon_\zeta \\ \exp\left(-\beta \left(\frac{(\epsilon^{1/4} - \epsilon_\zeta^{1/4})^2}{2\sigma_{\zeta,+}^2}\right)\right) & \epsilon > \epsilon_\zeta \end{cases}$$

- Asymmetric Gaussian peak along constant energy density line (implicit baryochemical potential dependence)
- Energy density scaled by power of 1/4 to emulate form of Gaussians with temperature dependency

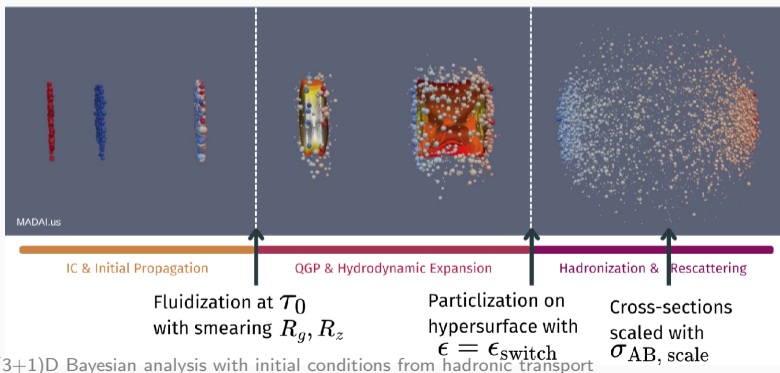


Priors of viscosities



Other Parameters

- τ_0 scaling: transition time scaling between initial transport and hydro; $\in [0.8, 1.5]$ fm
- R_g : transversal smearing width during fluidization; $\in [0.2, 2.2]$ fm
- R_z : longitudinal smearing width during fluidization $\in [0.2, 2.2]$ fm
- ϵ_{switch} : energy density of the hadronization hypersurface; $\in [0.25, 0.75]$ GeV/fm³
- $\sigma_{\text{AB, scale}}$: afterburner cross section scaling; $\in [0.8, 1.2]$



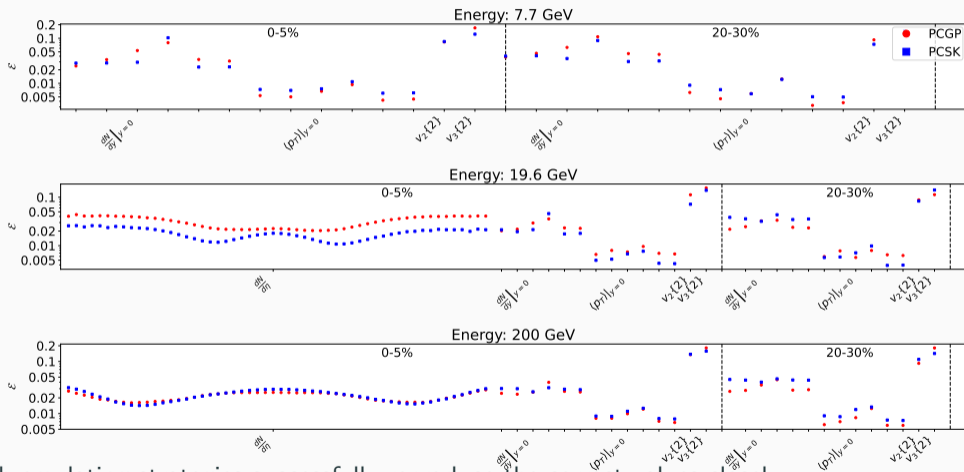
Data for inference

Energy [GeV]	0-5% Centrality	15-25% Centrality	20-30% Centrality
7.7	$dN/dy _{y=0}(\pi^+, \pi^-, K^+, K^-, p)$ $\langle p_T \rangle _{y=0}(\pi^+, \pi^-, K^+, K^-, p)$ $v_{2/3}\{2\}$ of charged particles	-	$dN/dy _{y=0}(\pi^+, \pi^-, K^+, K^-, p)$ $\langle p_T \rangle _{y=0}(\pi^+, \pi^-, K^+, K^-, p)$ $v_{2/3}\{2\}$ of charged particles
19.6	$dN/dy _{y=0}(\pi^+, \pi^-, K^+, K^-, p)$ $\langle p_T \rangle _{y=0}(\pi^+, \pi^-, K^+, K^-, p)$ $v_{2/3}\{2\}$ of charged particles $dN/d\eta$ of charged particles	$dN/d\eta$ of charged particles	$dN/dy _{y=0}(\pi^+, \pi^-, K^+, K^-, p)$ $\langle p_T \rangle _{y=0}(\pi^+, \pi^-, K^+, K^-, p)$ $v_{2/3}\{2\}$ of charged particles
200	$dN/dy _{y=0}(\pi^+, \pi^-, K^+, K^-)$ $\langle p_T \rangle _{y=0}(\pi^+, \pi^-, K^+, K^-)$ $v_{2/3}\{2\}$ of charged particles $dN/d\eta$ of charged particles	$dN/d\eta$ of charged particles $v_2(\eta)$ of charged particles	$dN/dy _{y=0}(\pi^+, \pi^-, K^+, K^-)$ $\langle p_T \rangle _{y=0}(\pi^+, \pi^-, K^+, K^-)$ $v_{2/3}\{2\}$ of charged particles

Further datasets to be included: STAR $v_n(p_T)$, $dN/d\eta$ and identified particle p_T -spectra, BRAHMS identified particles dN/dy , PHENIX $v_n(p_T)$

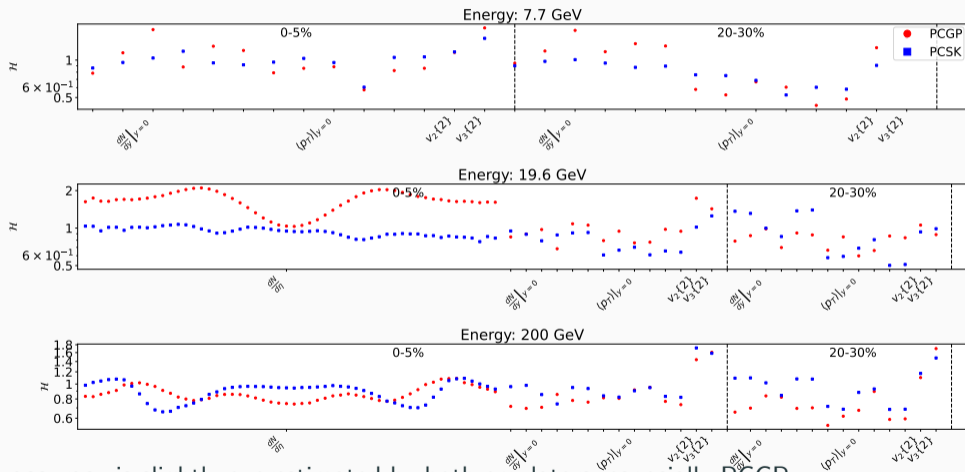
STAR PRC 79 (2009) STAR PRC 96 (2017) STAR PRC 98 (2018) PHOBOS PRC 72 (2005) PHOBOS PRC 74 (2006)

Emulator validation



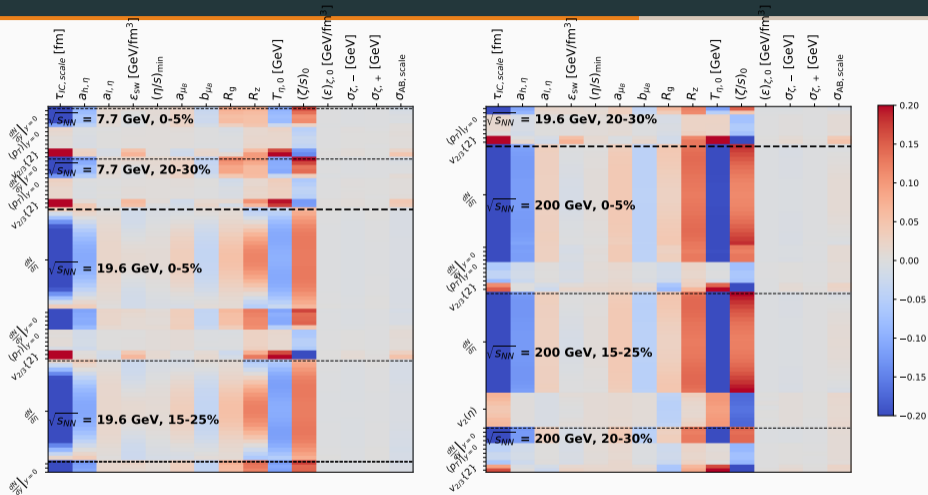
Both emulating strategies successfully reproduce the correct values closely

Emulator validation



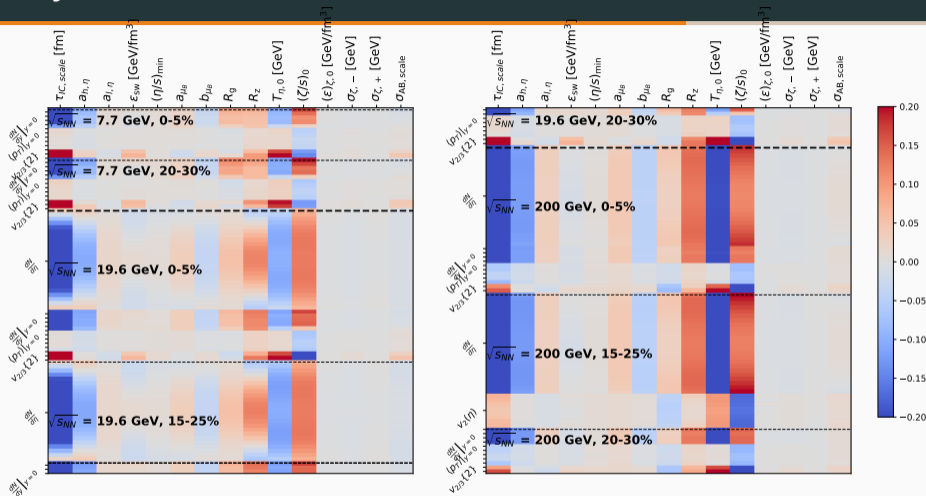
The accuracy is slightly overestimated by both emulators, especially PCGP

Sensitivity



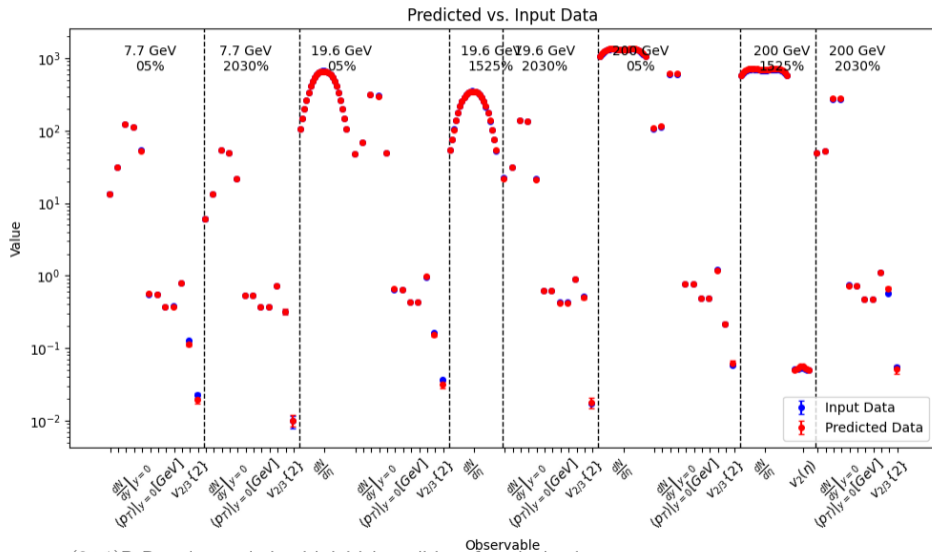
- Bulk viscosity (except for scale) and afterburner cross section scaling only weakly constrained

Sensitivity

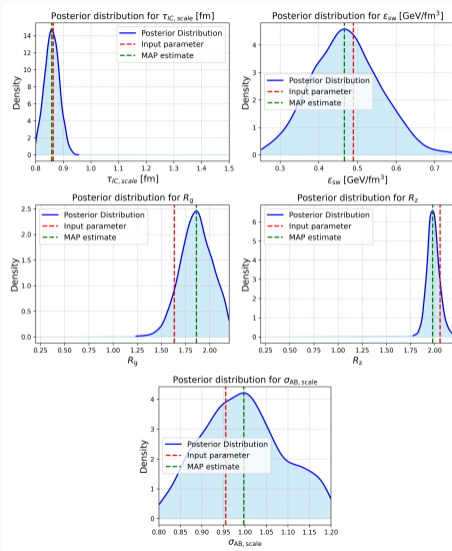


- Flow sensitive to fluidization time and shear kink temperature
- Pseudorapidity spectra sensitive to a wide range of parameters, effect depends on collision energy

Closure test: Reproduction of posterior

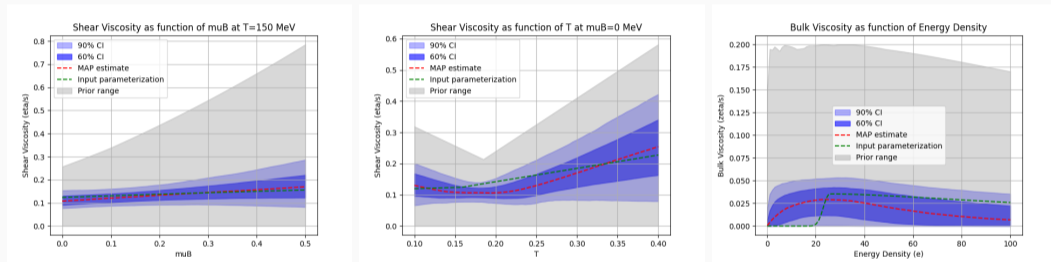


Closure test: Non-viscous parameters



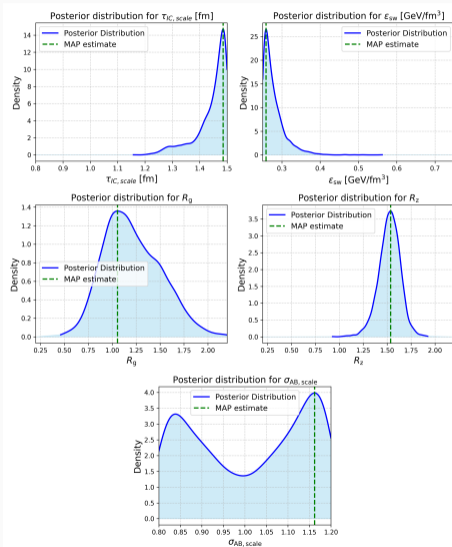
Most base simulation parameters can be reproduced well. R_g and afterburner scaling are weaker constrained.

Closure test: Viscosities



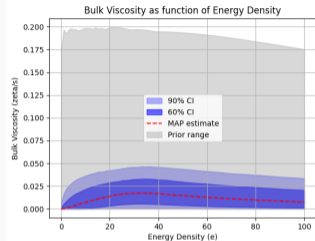
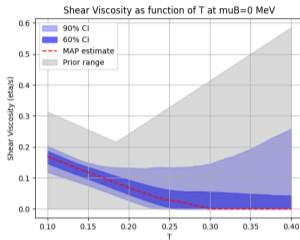
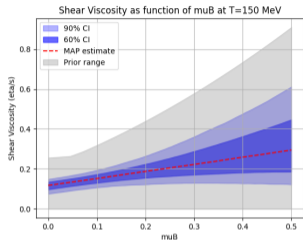
Parameterization of viscosities is mostly successfully reproduced. Some parameters/regions of phase diagram are more challenging.

Preliminary results: Non-viscous parameters



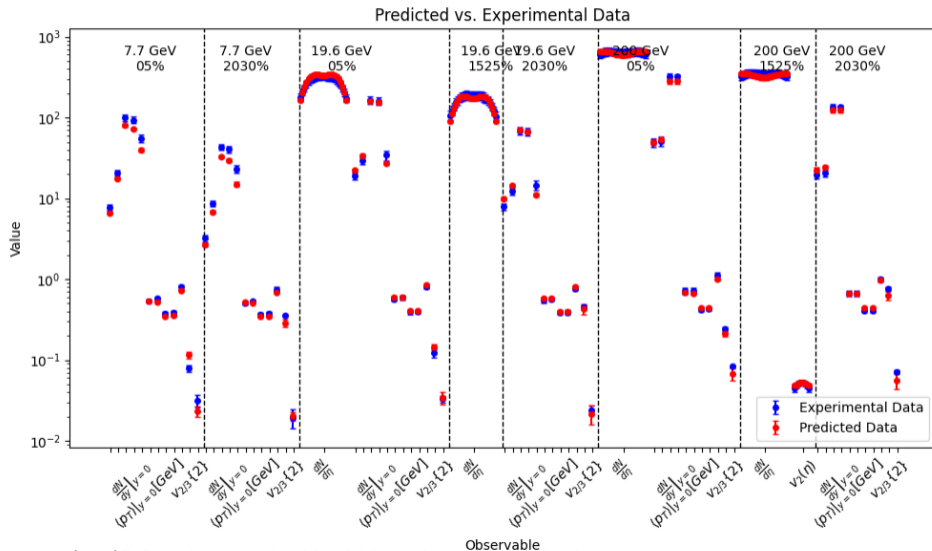
- Prefers late start of hydrodynamics, but running hydro down to very low energy density
- Considerable difference to [Auvinen et al.: PRC 97 \(2018\)](#), which prefers early start of hydro and early end
- Prefers enhanced interactions in afterburner

Preliminary results: Viscosities

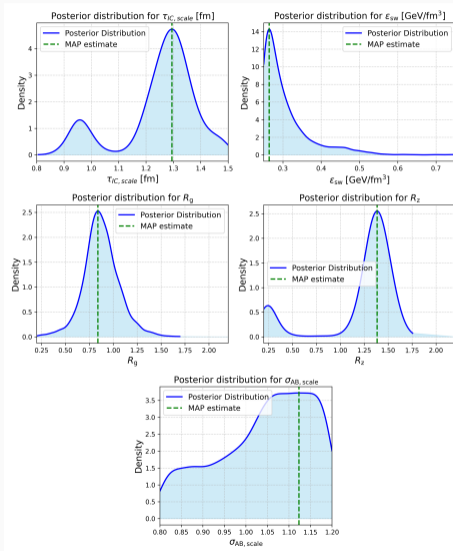


- Increasing $\eta/s(\mu_B)$ preferred, good constraints till 250 MeV
→ Nice agreement with [Shun et al.: PRL 132 \(2024\)](#)
- Constant $\eta/s(T)$ ruled out, solid constraints till 200 MeV, for higher energies small to vanishing η/s
- Very small bulk viscosity till vanishing preferred
→ difference to [Shun et al.: PRL 132 \(2024\)](#)

Preliminary results: Reproduction of posterior

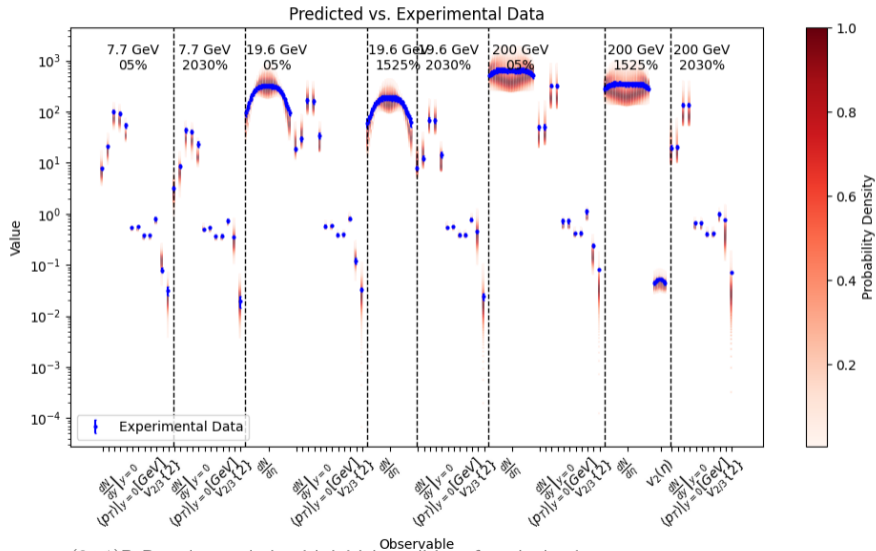


Open problem: Stability of posterior



- Removing forward/backward information changes preferred parameter range
- Origin of this effect under current investigation

Prior vs experimental data



- First (3+1)D Bayesian analysis of heavy ion collisions with T and μ_B dependent parameterizations of η/s and ζ/s and a hadronic initial state
- Strong constraints for decreasing shear viscosity for low temperatures, as well increasing shear viscosity with μ_B
- Bulk viscosity preferred small
- Further points of study: include additional data sets, study the impact of high- η data and role of hydro lifetime

SMASH and SMASH-vHLLC-hybrid are available at <https://github.com/smash-transport>.

More information can be also found at <https://smash-transport.github.io/>.

