Towards a fluid-dynamic description of an entire heavy-ion collision: from the colliding nuclei to the quark-gluon plasma phase

2410.08169

Andreas Kirchner, Federica Capellino, Eduardo Grossi, Stefan Floerchinger

Hydrodynamics and related observables in heavy-ion collisions, Oktober 2024

Overview

Introduction

Nuclei as static fluids

Interactionless limit

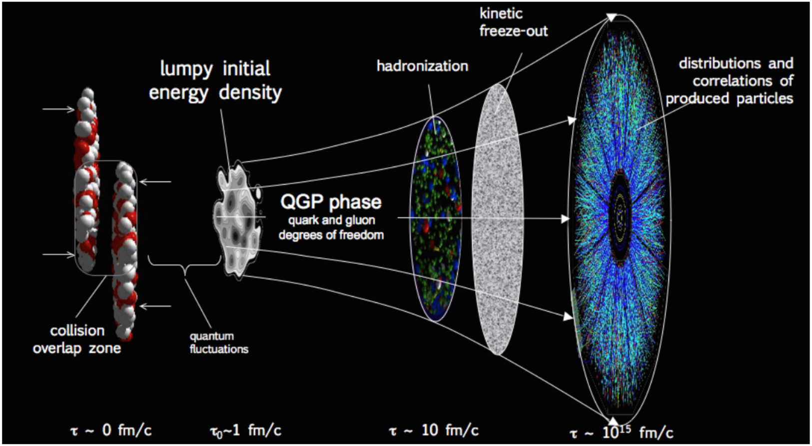
Fluid dynamics
& equation of state

Results

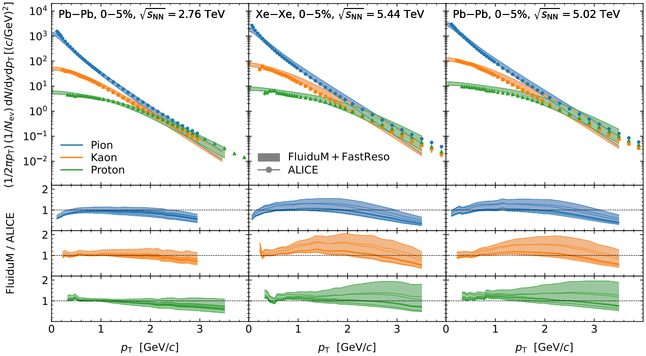
Conclusion & Outlook

Introduction

Standard model of HIC



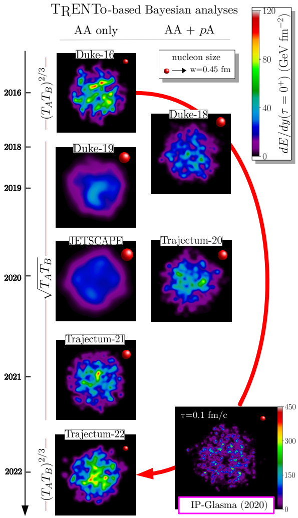
Observables



2308.16722

- Exp. data described well by fluid dynamics across different energies & systems
- Largest uncert. from fluid fields in initial state

Initial state models



2208.06839

- Many models for IC (TrenTo, IP-Glasma, CGC, free-streaming,...)
- Require add. parameters (e.g. Norm., τ_{hydro}, p, μ_{Q_s},$\tau_{\text{fs/EKT}}$)
- Fixed by fit/Bayesian analysis

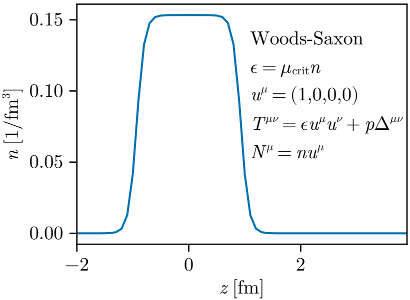
All fluid description?

- Fluid dynamics valid outside eq. (holographic models, eff. kin. theory)
  $\rightarrow$ Can fluid dynamics describe soft QCD?
  $\rightarrow$ Only inputs from QFT (EoS, transport coefficients)
- Idea not new: Weizäcker 1938 (liquid drop model), Landau 1953 (ideal fluid dynamics) and Stöcker et al., Katscher et al., Houvinen et al. & Karpenko et al. (multi fluid model)

Overview

Nuclei as static fluids

One nucleus



Fluid description:

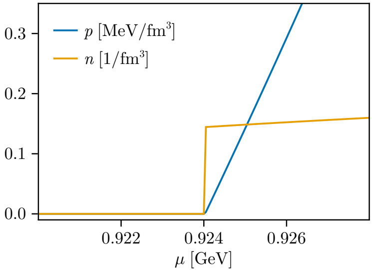
$$\nabla_\mu T^{\mu\nu} = 0$$

$$\nabla_\mu N^\mu = 0$$

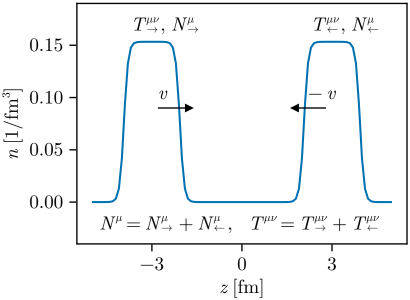
First order phase trans.

ensures stability!

$$(T = 0,\ \mu = \mu_{\mathsf{crit}})$$



Two approaching nuclei



Decomposition given by $N^\mu = n u^\mu + \nu^\mu$, $T^{\mu\nu} = \epsilon u^\mu u^\nu + (p + \pi_{\mathsf{bulk}})\Delta^{\mu\nu} + \pi^{\mu\nu}$

Fluid fields for full collision system via Landau frame matching $T^{\mu}{}_{\nu} u^\nu = -\epsilon u^\mu$

Stability conditions

- Gradients in T, μ, u^μ $\rightarrow$ dispersion
- Isolated nuclei at first order phase trans. $\rightarrow T$ & μ const.
- v changes sign around $z = 0$
  $\rightarrow$ Gradient in u^μ
  $\rightarrow$ Initially: $n = 0$, $\epsilon = 0 \Rightarrow$ Gradients no physical meaning
- Picture breaks down when nuclei start to overlap

Overview

Introduction

Nuclei as static fluids

Interactionless limit

Fluid dynamics
& equation of state

Results

Conclusion & Outlook

Interactionless limit

Interactionless limit

- QCD formally free at asymptotically high energies
  $\rightarrow$ Nuclei pass through each other
  $\rightarrow$ Use matching to gain insights into expectations of collision dynamics
- Fluid fields are obtained via matching procedure at different times

Matching procedure

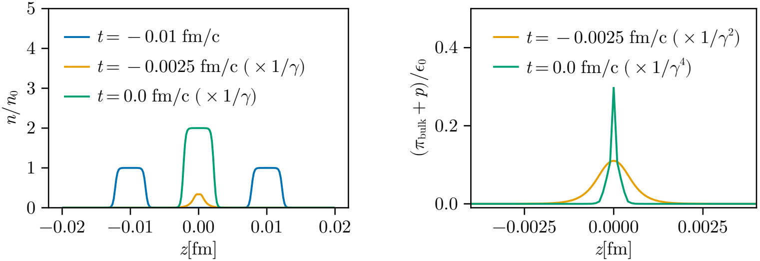
Composite system:

$N^\mu = nu^\mu + \nu^\mu,\ T^{\mu\nu} = \epsilon u^\mu u^\nu + (p + \pi_{\mathsf{bulk}})\Delta^{\mu\nu} + \pi^{\mu\nu}$

Extract fluid fields via

- $T^{\mu}{}_{\nu} u^\nu = -\epsilon u^\mu$
- $p + \pi_{\mathsf{bulk}} = \frac{1}{3}\Delta^{\mu\nu} T_{\mu\nu}$
- $\pi^{\mu\nu} = T^{\mu\nu} - \epsilon u^\mu u^\nu - [p + \pi_{\mathsf{bulk}}]\Delta^{\mu\nu}$
- Solve $nu^\mu + \nu^\mu = N^\mu_{\rightarrow} + N^\mu_{\leftarrow}$ for n, ν^i

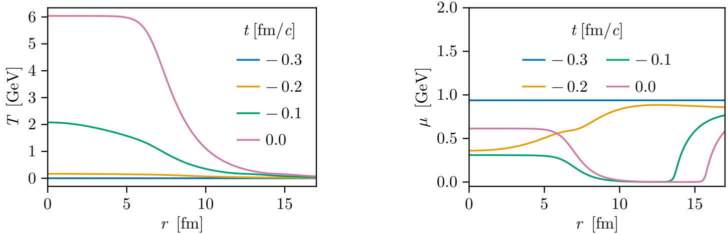
Fluid fields



$\gamma \approx 2960$

Large densities & viscous corrections require careful treatment of evolution equations

Temperature & chemical potential



Apply EoS to obtain T & μ:

- $T \to 0$ & $\mu \to 0.93\,$MeV for low densities
- Indications of expected trajectory, but T much to high
- Description including interactions needed

Overview

Introduction

Nuclei as static fluids

Interactionless limit

Fluid dynamics
& equation of state

Results

Conclusion & Outlook

Fluid dynamics & equation of state

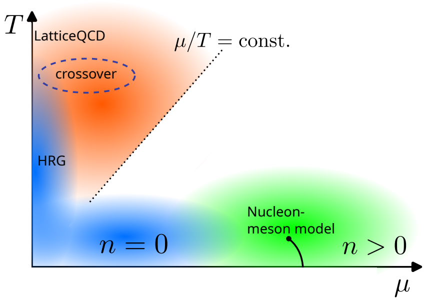
Second-order fluid dynamics

- Fluid dynamics based on conservation laws
  $\rightarrow$ Additional constituent relations needed for viscous fields
  $\rightarrow$ Israel-Stewart equations derived from $\nabla_\mu S^\mu \geq 0$
  $\rightarrow$ Entropy production $\nabla_\mu S^\mu = \frac{\pi_{\text{bulk}}^2}{\zeta T} + \frac{\pi^{\mu\nu}\pi_{\mu\nu}}{2\eta T} + \frac{(\epsilon+p)^2 \nu^\mu \nu_\mu}{\kappa (nT)^2}$
- Additional transport coefficients: viscosities & conductivities (η, ζ, κ) and relaxation times $(\tau_{\text{shear}}, \tau_{\text{bulk}}, \tau_{\text{heat}})$

Fluid dynamics outside equilibrium

- IS-theory includes relaxation times
  $\tau_{\text{bulk}} u^\mu \partial_\mu \pi_{\text{bulk}} + \zeta \nabla_\mu u^\mu + \pi_{\text{bulk}} + \ldots = 0$
  $\rightarrow$ Equations remain valid outside equilibrium
  $\rightarrow$ Glasma behaves like fluid
  $\rightarrow$ Kin. th. & fluid dynamics can give equivalent results

Equation of state

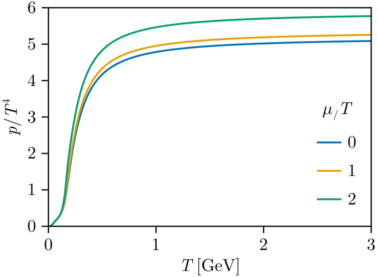


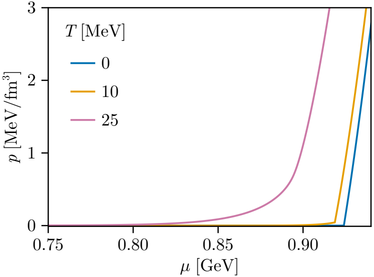
Composite equation of state from LQCD, HRG & nucleon-meson model interpolated by transfer functions

At phase transition: T & μ not sufficient for description

$\rightarrow$ Additional parameters needed: Volume ratio parameter r

Equation of State





- Smooth transition between LQCD & HRG
- Includes first order phase transition

Model system

- High compression & expansion of nuclear matter during initial moments of collision
- Model by homogen. universe filled with nucl. matter $\mathrm{d}s^2 = -\mathrm{d}t^2 + a(t)^2(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2)$
- Fluid fields reduce to $\Phi(t) = (T, \mu, \pi_{\mathsf{bulk}})$
- Expansion rate given by $H = \frac{\dot{a}(t)}{a(t)} = -\frac{\partial_t n}{3n}$

Overview

Introduction

Nuclei as static fluids

Interactionless limit

Fluid dynamics

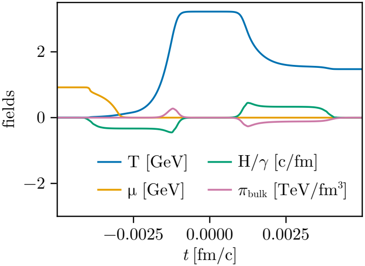
& equation of state

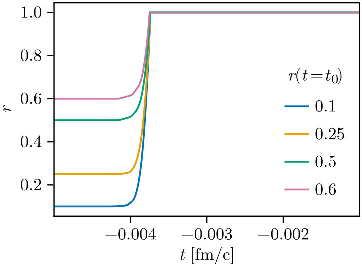
Results

Conclusion & Outlook

Results

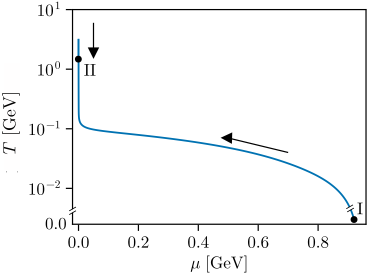
Fluid solutions

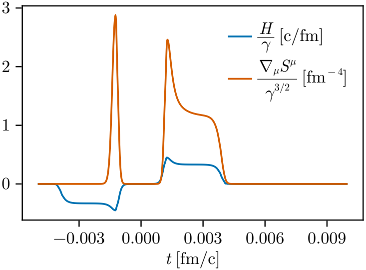




- Viscosity creates heat and bulk pressure
- Dynamics independent of initial r

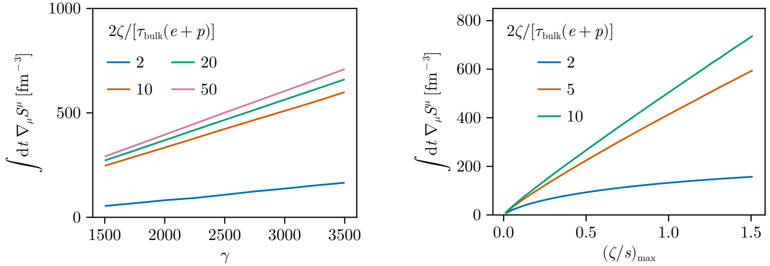
Phase diagram & entropy prod.





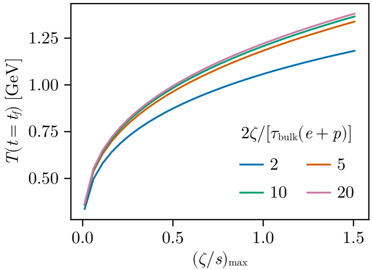
- Initial & final point of trajectory in phase diagram are not same due to viscosity
- Entropy production follows Hubble rate after delay

Produced entropy



- More entropy produced for higher γ and viscosity
- Dynamics almost independent of τ_{bulk} for τ_{bulk} small enough

Final temperature



- Final temp. scales with viscosity & inverse relaxation time
- $T(t = t_f)$ independent of relax. time for τ_{bulk} small enough
- Any value of $T(t = t_f)$ can be obtained

Overview

Introduction

Nuclei as static fluids

Interactionless limit

Fluid dynamics
& equation of state

Results

Conclusion & Outlook

Conclusion & Outlook

Conclusion

First steps toward fluid-dynamic description:

- Established set of equations & EoS to describe soft part of HIC
- Study entropy production during compression & expansion
- Bulk visc. & relaxation time can be chosen such that $T(t = t_f)$ is similar to temp. found in initial moments of fireball

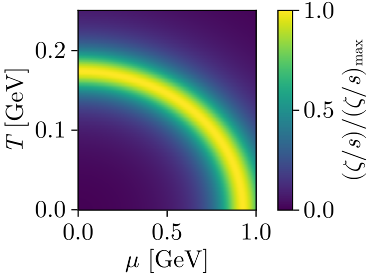
Outlook

- Apply QFT calculations to model
- More realistic $1+1D$ setup $\rightarrow$ Includes shear viscosity & baryon diffusion
- Include fluctuations & correlations

Thank you for your attention!

Backup

Bulk viscosity parametrization



Bulk viscosity

$$\zeta/s = \frac{(\zeta/s)_{\mathsf{max}}}{1+\left(\frac{\sqrt{T^2+k^2\mu^2}-24\,\mathsf{MeV}}{175\,\mathsf{MeV}}\right)}$$

Relaxation time

$$\frac{\zeta}{\tau_{\mathsf{bulk}}(\epsilon+p)} = \frac{\beta_0}{2}$$

Massieu potential

- Massieu potential $w = \beta p$
- Phase coexistence $w = (1 - r)w' + rw''$
- Evolution eq.for volume ratio parameter $u^\mu \partial_\mu r = -\frac{n_p(\gamma)G^{pq}(\gamma,r)f_q}{n_r(\gamma)G^{rs}(\gamma,r)[c''_s(\gamma)-c'_s(\gamma)]}$
- Modified conservation equation

$$u^\mu \partial_\mu \gamma^n + G^{nm}(\gamma, r)\left[f_m - [c''_m - c'_m]\frac{n_p(\gamma)G^{pq}(\gamma,r)f_q}{n_l(\gamma)G^{ls}(\gamma,r)[c''_s-c'_s]}\right] = 0.$$

Combining the EoS

- Composite EoS
  $p = \frac{1}{2}(1-f)p_{\mathsf{HRG}} + \frac{1}{2}(1+f)p_{\mathsf{LQCD}}$
- Transfer function
  $f(T,\mu) = \tanh\left(\frac{T - T_{\mathsf{trans}}(\mu)}{\Delta T_{\mathsf{trans}}}\right)$ with
  $T_{\mathsf{trans}}(\mu) = 0.1\,\mathsf{GeV} + 0.28\mu - 0.2\,\mathsf{GeV}^{-1}\mu^2$ &
  $\Delta T_{\mathsf{trans}} = 0.1 T_{\mathsf{trans}}(\mu = 0)$

LQCD & HRG

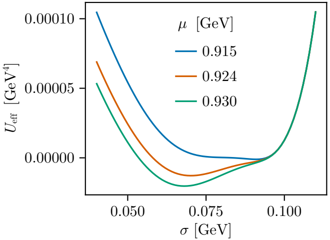
Lattice EoS: Expansion around eq. pressure

$p_{\mathsf{LQCD}}(T, \mu) = p(T) + \sum_{n=2,4,6} \frac{\chi_n(T)}{n!} \mu^n$

Hadron resonance gas: Total pressure is sum of partial pressures

$p_{\mathsf{HRG}}(T, \mu) = \sum_{\text{baryons}} d_i p_F(T, B_i \mu; m_i) + \sum_{\text{mesons}} d_i p_B(T, 0; m_i)$

Nucleon-meson model



- Effective model for cold, dense nuclear matter
- Interactions via scalar, pseudo-scalar and vector meson exchange

Apply mean-field approximation $\rightarrow$ Fields self-consistently determined by gap equations

$\Rightarrow$ First-order phase transition captured

Fluid-dynamic EoM

- EoM are set of coupled, first-order PDEs with discontinuous initial conditions
  $\rightarrow$ Weal solutions need to be expected
- Uniqueness of weak solutions given by Rankine-Hugoniot condition