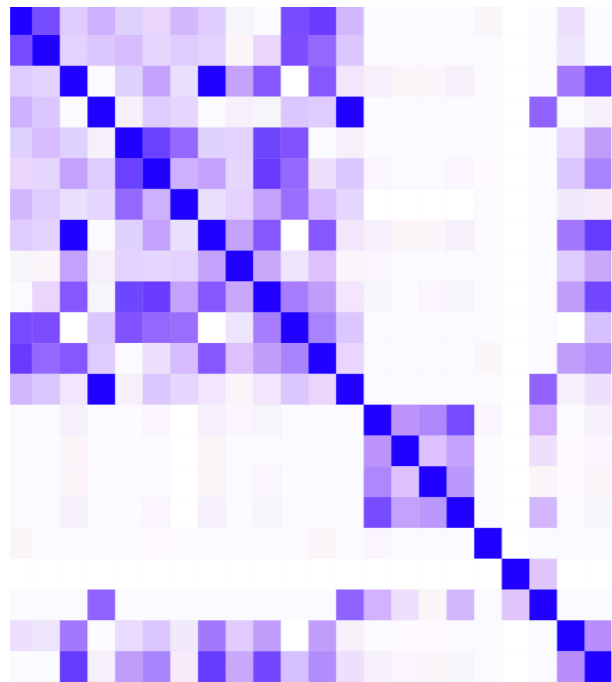


# Understanding the Multi-Dimensional SMEFT fit for Higgs factories



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At the LHC, SMEFT is used to quantify the uncertainties on precision tests of the Standard Model. The methodology is by now well established.

For each process, one picks out the dimension-6 SMEFT operators that make the most direct contributions, carries out a fit to the value of fiducial observables including the coefficients of these operators as free parameters, and quotes the results in terms of a  $\Lambda$  scale and uncertainties on the coefficients. Fits with more sophistication include more operators, but the full number of dimension-6 operators is large and gives an unwieldy basis set. So this set is restricted to the most relevant ones.

In principle, deviations from the Standard Model could be observed. In that case, the SMEFT fit can point to the dimension-6 operators most strongly induced, to give clues about the underlying BSM physics.

To discuss Higgs factories, SMEFT fitters should take a different attitude.

1. Because of higher precision, the set of operators should be **as complete as possible**.
2. Many operators contribute to other reactions measurable at a Higgs factory. **Determining their coefficients can assist the fit for the Higgs couplings**.
3. There is information **not accessible experimentally** — the Higgs total width. This should be determined in a manner as **model-independent** as possible.
4. Effects **outside of SMEFT** are possible. Don't be a purist !

Here I will describe fits carried out by the ILC group. See

Barklow et al, Phys.Rev.D 97 (2018) 5, 053003 [arXiv:1708.08912]

ILC Report to Snowmass, Chapter 12, arXiv:2203.07622 (2022).

The conclusions are extensively checked with the Snowmass 2021 SMEFT fitting group, arXiv:2206.08326, in which we participated. Special thanks to the members of that group, and especially to Junping Tian and Yong Du.

Build up the fit parameters. We want to include **all dimension-6 operators** that influence e+e- observables at the tree level. Here I will describe a more limited fit, but a larger fit still closes if the special assumptions are relaxed. The fit describes fits this expression to the results expected from the 250 GeV run of the ILC.

First, we must include the **4 basic SM parameters**:  $g, g', v, \lambda$ .

**operators with  $\gamma, Z, W, H$  only** (6 of these, not including  $C_6$  and CP-violating operators:

$$\begin{aligned} & \frac{c_H}{2v^2} \partial^\mu (\Phi^\dagger \Phi) \partial_\mu (\Phi^\dagger \Phi) + \frac{c_T}{2v^2} (\Phi^\dagger \overleftrightarrow{D} \Phi) (\Phi^\dagger \overleftrightarrow{D} \Phi) \\ & + \frac{g^2 c_{WW}}{v^2} \Phi^\dagger \Phi W_{\mu\nu}^a W^{a\mu\nu} + \frac{4gg' c_{WB}}{v^2} \Phi^\dagger t^a \Phi W_{\mu\nu}^a B^{\mu\nu} \\ & + \frac{g'^2 c_{BB}}{v^2} \Phi^\dagger \Phi B_{\mu\nu} B^{\mu\nu} + \frac{g^3 g' c_{3W}}{v^2} \epsilon_{abc} W_{\mu\nu}^a W^{b\nu}{}_\rho W^{c\rho\mu} \end{aligned}$$

$c_H$  modifies the Higgs field self energy and so appears as a rescaling of all Higgs couplings.

operators that couple leptons to the Higgs current. Here I assume lepton universality, so 3 of these:

$$+i\frac{c_{HL}}{v^2}(\Phi^\dagger \overleftrightarrow{D}^\mu \Phi)(\bar{L}\gamma_\mu L) + 4i\frac{c'_{HL}}{v^2}(\Phi^\dagger t^a \overleftrightarrow{D}^\mu \Phi)(\bar{L}\gamma_\mu t^a L) \\ +i\frac{c_{HE}}{v^2}(\Phi^\dagger \overleftrightarrow{D}^\mu \Phi)(\bar{e}\gamma_\mu e) .$$

2 additional linear combinations of operators modify the total widths of Z and W; these must also be included.

operators that modify additional Higgs couplings (5 of these, for b, c, g,  $\tau$ ,  $\mu$ )

$$\sum_f \frac{c_f}{v^2} |\Phi|^2 L_f^\dagger \cdot \Phi f_R + h.c. + \frac{c_{GG}}{v^2} \Phi^\dagger \Phi G_{\mu\nu}^a G^{a\mu\nu}$$

This is a total of 20 parameters. In a more extended but not conceptually different fit, adding 4-fermion operators, violating lepton universality, violating CP can be accounted by adding dimension-6 operators and additional observables. This fit closes in a similar way to this simplified one. See my talk at the recent **Snowmass** workshop for details.

The Higgs boson could also have non-SM decays, through the “Higgs portal”. The search for such decays is an important part of the precision Higgs study. Exotic decays can be in 1 of 3 classes:

1. **Completely invisible decays.** These can be searched for to  $BR \sim 0.3\%$  by searching for  $e^+e^- \rightarrow Z + (\text{missing})$ .
2. **Completely or partially visible decays.** These can be searched for directly, to  $BR \sim 10^{-3} - 10^{-4}$ .
3. Decays that belong to **categories not recognized** in searches.

To account for all three classes, **we add 2 parameters to the fit – the Higgs branching ratios to invisible decays and to other exotic decays.** This is consistent with the SMEFT description as long as the light final state particles do not couple directly to the Z boson in such a way as to significantly change the precision electroweak predictions. This is true if their coupling is through the Higgs portal.

**The resulting 22-parameter fit is strongly overdetermined.** In fact, the results are unchanged if  $G_F$  is not given as an input.

In the Barklow et al. paper, we suggested that this fit should replace the ad hoc  $\kappa$  parameter fit as the method for extracting Higgs couplings from collider data.

The overdetermined fit gives a derived value for the Higgs total width, with a precision of  $\sim 1.5\%$  for all Higgs factories. It is separately sensitive to changes in the Higgs width from short-distance BSM corrections and from exotic decays to new light species.

It is more powerful than the  $\kappa$  fit in that it includes input from additional processes, such as precision electroweak and  $e^+e^- \rightarrow W^+W^-$ .



Importantly, the structure of the BSM contributions to the Higgs couplings is different in the SMEFT fit from that in the  $\kappa$  fit.

Consider, for example, the  $\kappa$  fit results for the Z and W couplings, as quoted in the Snowmass Higgs report, Dawson et al., arXiv:2209.07510.

Higgs Coupling (%)	HL-LHC	ILC250 + HL-LHC	ILC500 +HL-LHC	ILC1000 + HL-LHC	FCC-ee + HL-LHC	CEPC240 + HL-LHC	CEPC360 +HL-LHC
$hZZ$	1.5	.22	.17	.16	.17	.074	.072
$hWW$	1.7	.98	.20	.13	.41	.73	.41
$hb\bar{b}$	3.7	1.06	.50	.41	.64	.73	.44

Why the asymmetry between Z and W ? Shouldn't these be related by custodial symmetry?

In dimension-6 SMEFT at the tree level, the Higgs boson coupling to WW and ZZ has the form

$$\begin{aligned} \delta\mathcal{L} = & (1 + \eta_W) \frac{2m_W^2}{v} H W^{+\mu} W_{\mu}^{-} + \zeta_W \frac{H}{v} W^{+\mu\nu} W_{\mu\nu} \\ & + (1 + \eta_Z) \frac{m_Z^2}{v} H Z^{\mu} Z_{\mu} + \zeta_Z \frac{H}{2v} Z^{+\mu\nu} Z_{\mu\nu} \end{aligned}$$

In SMEFT,

$$\eta_W = -\frac{1}{2}c_H + 2\delta m_W - \delta v$$

$$\eta_Z = 1 - \frac{1}{2}c_H + 2\delta m_Z - \delta v - c_T$$

also

$$\zeta_W = 2g^2 c_{WW} \quad \zeta_Z = 2g^2 \left( c_w^2 c_{WW} + s_w^2 c_{WB} + \left( \frac{s_w^4}{c_w^2} \right) c_{BB} \right)$$

so that  $(\eta_W, \eta_Z)$  and  $(\zeta_W, \zeta_Z)$  are **tied together** by constraints from precision electroweak,  $e^+e^- \rightarrow W^+W^-$ , and  $H \rightarrow \gamma\gamma$ .

On the other hand, the extraction of the HZZ coupling now becomes more involved.

In the  $\kappa$  method, total cross section for  $e^+e^- \rightarrow ZH$  and the partial width for  $H \rightarrow ZZ^*$  depend on  $\kappa_Z$  in a simple way:

$$\sigma(e^+e^- \rightarrow Zh) \sim \kappa_Z^2 \quad BR(h \rightarrow ZZ^*) \sim \kappa_Z^2 / \Gamma_{tot}$$

Using SMEFT, this is not so straightforward. The new tensor structure depends on whether the HZZ vertex has spacelike or timelike momentum transfer,

$$\sigma(e^+e^- \rightarrow ZH) = (SM) \cdot (1 + 2\eta_Z + 5.7 \zeta_Z)$$

$$\Gamma(H \rightarrow WW^*) = (SM) \cdot (1 + 2\eta_W - 0.78 \zeta_W)$$

$$\Gamma(H \rightarrow ZZ^*) = (SM) \cdot (1 + 2\eta_Z - 0.50 \zeta_Z)$$

More information from other variables is needed to extract the two parameters separately.

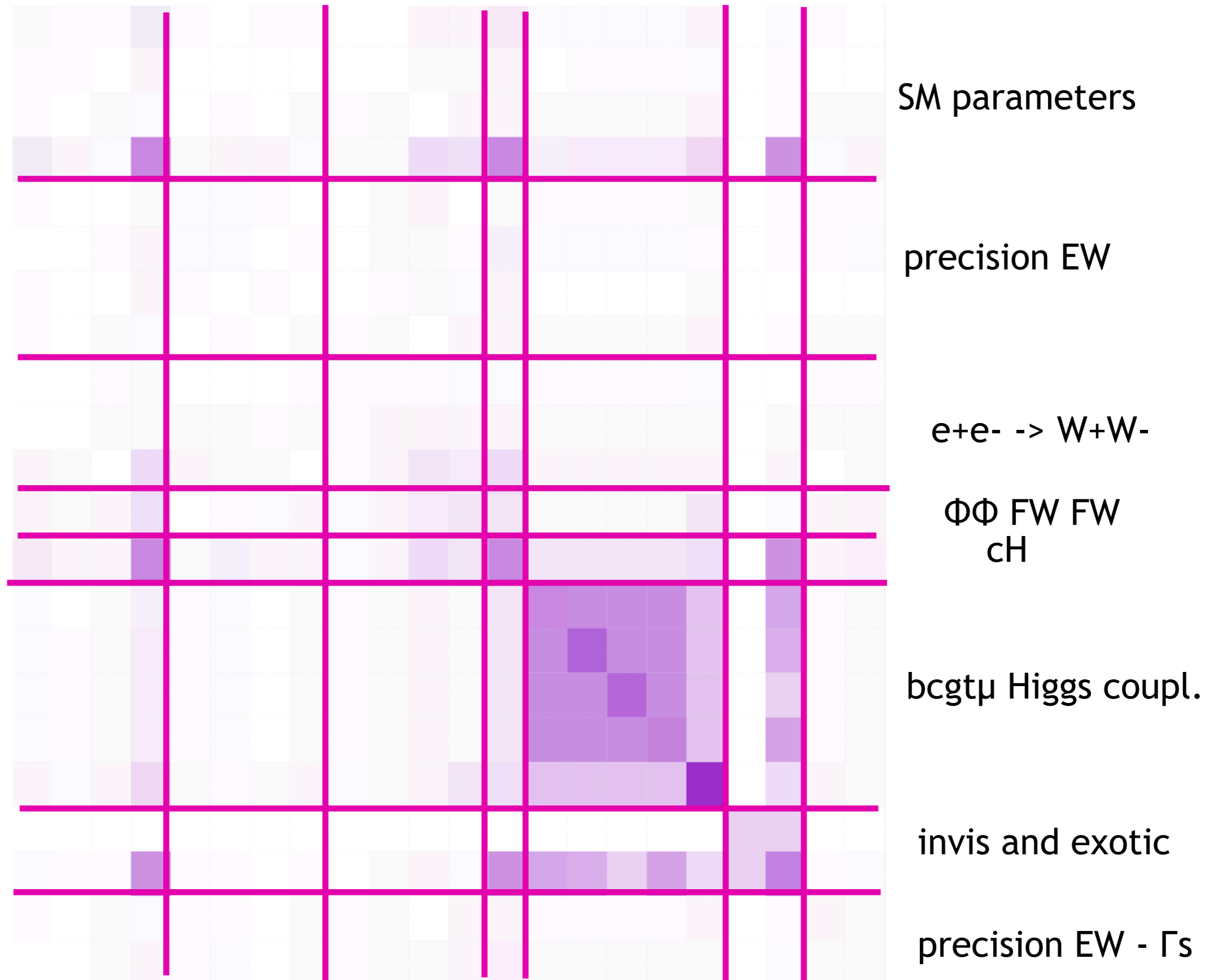
An observable that is very sensitive to  $\zeta_Z$  is the **beam polarization asymmetry in  $e^+e^- \rightarrow ZH$** .

In general, **SMEFT operators are chiral** and so corrections due to these operators are **sensitive to beam polarization**. Then this can potentially be used as a handle to improve the determination of SMEFT parameters. This is true in the determination of the Higgs couplings, and also in the determination of the VWW triple gauge couplings.

**In both analyses, high  $e^-$  beam polarization compensates about a factor 2.5 in luminosity.**

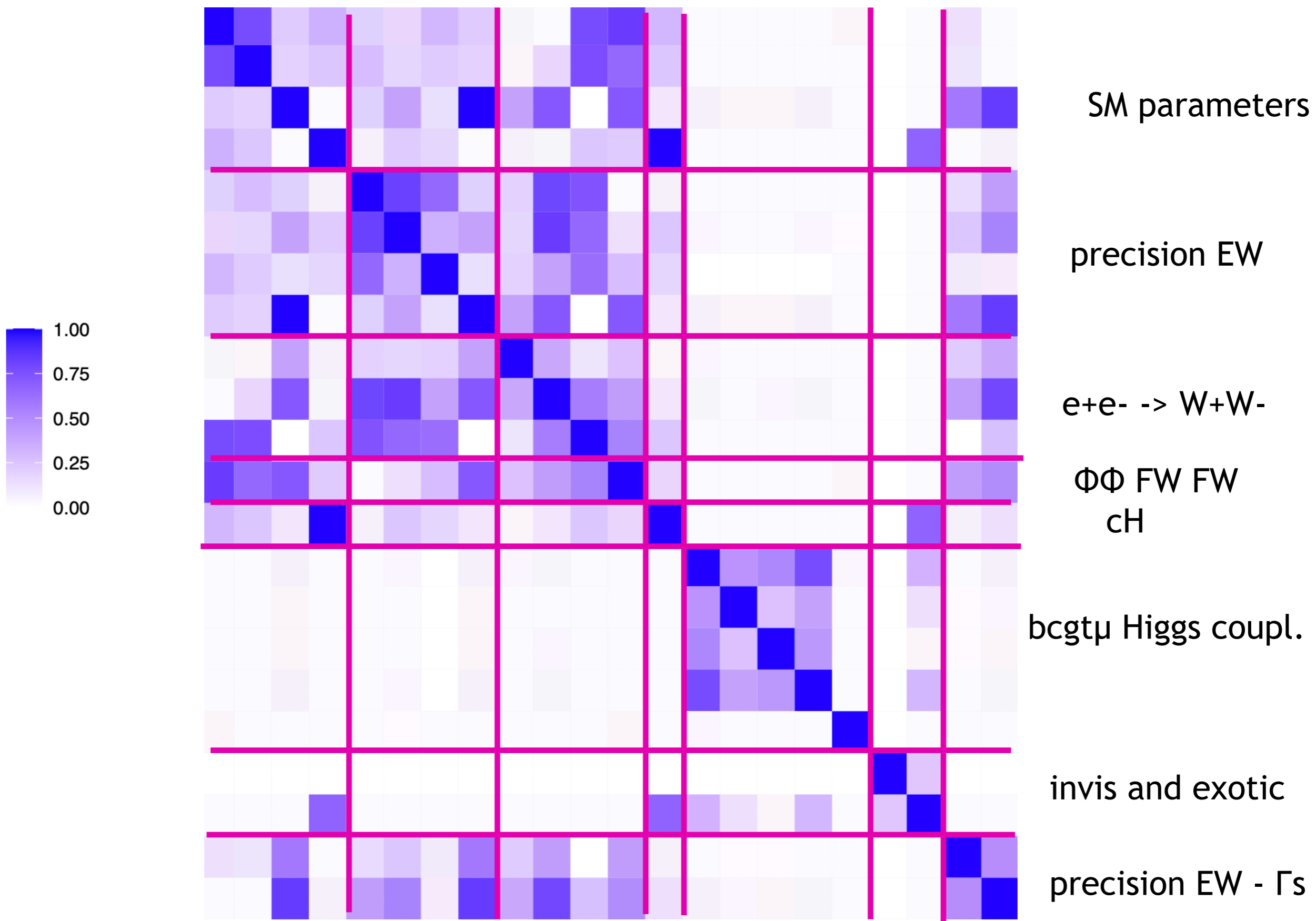
In the  $\kappa$  method, projections are independent of beam polarization. This already indicates that this method is not general or model-independent.

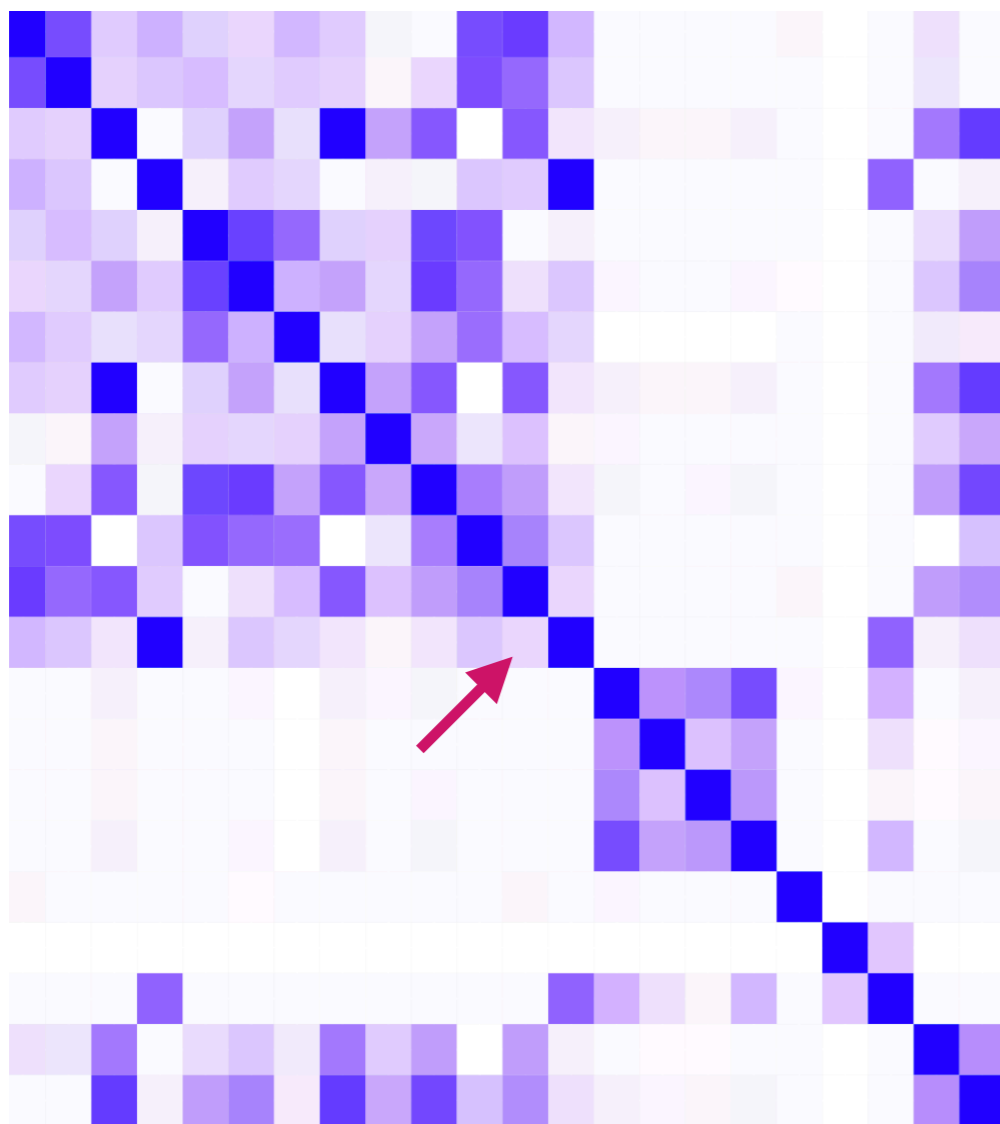
It is interesting to look into the correlation matrix of the SMEFT fit. Here is the  $C_{ij}$  matrix, with elements displayed on a log scale:



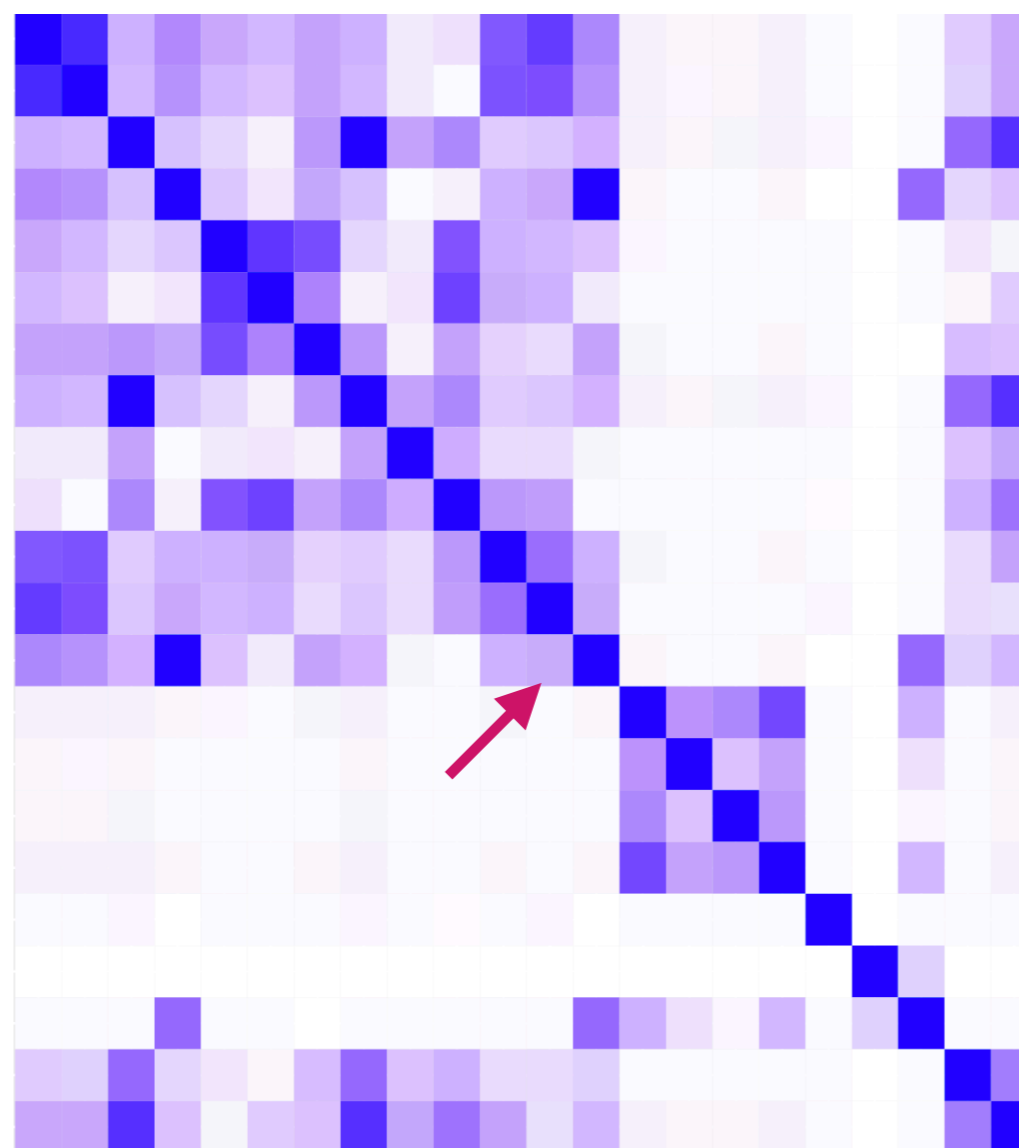
To make this easier to see, construct

$$C_{ij} = \frac{C_{ij}}{\sqrt{C_{ii}C_{jj}}}$$





80/30 e-/e+ polarization



zero polarization



Very recently, [Astieradis, Dawson, Giardino, and Szafron](#) have published the Next to Leading Order formulae for the  $e^+e^- \rightarrow ZH$  cross section in SMEFT, [arXiv:2406.03557](#), [arXiv:2409.11466](#). This is a very important piece of work. However, its interpretation is subtle.

[These corrections bring in additional operators not included in the fit that I have discussed.](#) CP-violating operators can be constrained at the tree level by additional measurements in  $e^+e^- \rightarrow ZH, WW$ . Operators involving the top quark can be constrained by explicit top quark measurements, but this might require  $e^+e^-$  at higher energies.

The extension of this analysis to Higgs decays brings in the quark anomalous magnetic moment operators and 4-fermion quark operators, which are more difficult to measure directly.

In principle, additional operators arising at NLO can ruin the closure of the fits. [We need a prescription to use the new information sensibly, so that it is an improvement rather than an confusion in this analysis.](#)

## Conclusions:

Determination of the Higgs boson couplings from data is **nontrivial** and requires a method to **recover missing information**, including the total Higgs width and the branching ratio to exotic final states.

The traditional  $\kappa$  parametrization is not up to the task. We suggest that **a method based on SMEFT** is more physical, complete, and model-independent.

In principle, SMEFT allows all processes measured at e+e- Higgs factories, including polarization observables, to **improve** the Higgs coupling determinations.

In principle, this method is improvable with higher-order calculations. However, this is subtle and requires **guidelines and interpretation** which have yet to be thought out.