Using the exclusive photoproduction of a photon-meson pair in collinear factorisation to extract generalised parton distributions

Nantes Seminar

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IJCLab

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Outline

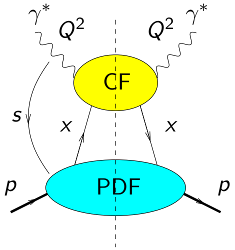
- Background
- Motivation for considering photon-meson pair photoproduction to probe GPDs
- Computation at LO and leading twist (quark GPD only)
- Results and prospects at experiments
- Collinear Factorisation breaking effects in gluon GPD contributions to $\pi^0\gamma$ photoproduction
- Conclusions and Outlook

- Deep Inelastic Scattering DIS: inclusive process

  $\Rightarrow$ 1-dimensional structure

  $\Rightarrow$ Collinear factorisation at the *cross section* level

Coefficient Function $\otimes$ Parton Distribution Function
(hard) (soft)



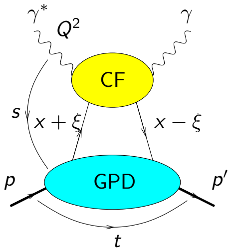
DVCS: exclusive process (non forward amplitude)

Fourier transf.: $t \leftrightarrow$ impact parameter

$\Rightarrow$ 3-dimensional structure

Collinear factorisation implies

Coefficient Function (hard) $\otimes$ Generalized Parton Distribution (soft)



x: *Average* mom. fraction of the nucleon carried by the parton
ξ: Mom. fraction of the nucleon *transferred* to hard part

[X. Ji: hep-ph/9609381]

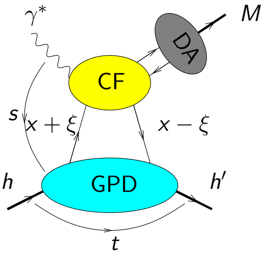
[A. Radyushkin: hep-ph/9604317, hep-ph/9704207]

[J. Collins, A. Freund: hep-ph/9801262]

[D. Müller, D. Robaschik, B. Geyer, F.-M. Dittes, J. Horejsi: hep-ph/9812448]

DVMP: γ replaced by $\rho,\ \pi, \cdots$

GPD	$\otimes$	CF	$\otimes$	Distribution Amplitude
(soft)		(hard)		(soft)



[J. Collins, L. Frankfurt, M. Strikman: hep-ph/9611433]

[A. Radyushkin: hep-ph/9704207]

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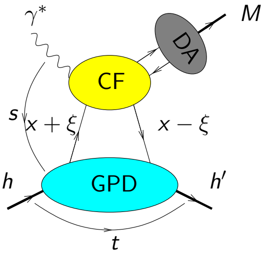
GPD (soft) $\otimes$ CF (hard) $\otimes$ Distribution Amplitude (soft)



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proofs valid only for some restricted cases

Very useful *Sudakov decomposition* of a generic 4-vector v in lightcone directions n_+ and n_-:

$$v^\mu = v^+ n_+^\mu + v^- n_-^\mu + v_\perp^\mu$$

with

$$\begin{aligned} n_+^2 = n_-^2 &= 0 \\ n_+ \cdot n_- &= 1 \\ v^\pm &= \frac{v^0 \pm v^3}{\sqrt{2}} \\ v^2 &= 2v^+ v^- + v_\perp^2 \end{aligned}$$

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In other words, n_+^μ (n_-^μ) defines a *lightlike* 4-vector with spatial components purely in the positive (negative) z-direction

Quark GPDs at twist 2 [M. Diehl: hep-ph/0307382]

without helicity flip (chiral-even Γ matrices): 4 chiral-even GPDs:
(Note: $\Delta = p' - p$)

$$F^q = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p'|\, \bar{q}(-\tfrac{1}{2}z)\, \gamma^+ q(\tfrac{1}{2}z)\, |p\rangle\Big|_{z^+=0,\, z_\perp=0}$$
$$= \frac{1}{2P^+}\left[H^q(x,\xi,t)\, \bar{u}(p')\gamma^+ u(p) + E^q(x,\xi,t)\, \bar{u}(p')\frac{i\,\sigma^{+\alpha}\Delta_\alpha}{2m} u(p)\right],$$

$$\tilde{F}^q = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p'|\, \bar{q}(-\tfrac{1}{2}z)\, \gamma^+\gamma_5\, q(\tfrac{1}{2}z)\, |p\rangle\Big|_{z^+=0,\, z_\perp=0}$$
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$H^q \xrightarrow{\xi=0,t=0}$ PDF q $\qquad$ $\tilde{H}^q \xrightarrow{\xi=0,t=0}$ polarised PDF Δq

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$$= \frac{1}{2P^+}\bar{u}(p')\left[H_T^q\, i\sigma^{+i} + \tilde{H}_T^q\, \frac{P^+\Delta^i - \Delta^+P^i}{m^2} + E_T^q\, \frac{\gamma^+\Delta^i - \Delta^+\gamma^i}{2m} + \tilde{E}_T^q\, \frac{\gamma^+P^i - P^+\gamma^i}{m}\right] u(p)\,,$$

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$H_T^q \xrightarrow{\xi=0,t=0}$ quark transversity PDFs δq

Note: $\tilde{E}_T^q(x,-\xi,t) = -\tilde{E}_T^q(x,\xi,t)$

- 3D parton tomography [M. Burkardt hep-ph/0207047; M. Diehl hep-ph/0205208]:
  In the $\xi \to 0$ limit, the Fourier transform of the GPD over $\Delta_\perp$ gives the *impact parameter dependent* PDF, $q(x, b_\perp)$:

$$q(x, b_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H^q(x, \xi = 0, \Delta_\perp^2) e^{ib_\perp \cdot \Delta_\perp}$$

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- Connection to nucleon spin:
  Ji sum rule [X. Ji hep-ph/9603249]

$$2J^q = \int_{-1}^{1} dx\, x \left(H^q(x, \xi, 0) + E^q(x, \xi, 0) \right)$$

Why consider a gamma-meson pair?
Understanding quark transversity

- Transverse spin content of the proton:

$$\begin{array}{ccc} |\uparrow\rangle_{(x)} & \sim & |\rightarrow\rangle + |\leftarrow\rangle \\ |\downarrow\rangle_{(x)} & \sim & |\rightarrow\rangle - |\leftarrow\rangle \\ \text{spin along } x & & \text{helicity states} \end{array}$$

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- Since (in the massless limit) QCD and QED are chiral-even $(\gamma^\mu, \gamma^\mu\gamma^5)$, the chiral-odd quantities $(1, \gamma^5, [\gamma^\mu, \gamma^\nu])$ which one wants to measure should appear in pairs.

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Can we probe quark transversity GPDs in DVMP?

- the leading DA (twist 2) of ρ_T is chiral-odd ($\sigma^{\mu\nu}$ coupling)

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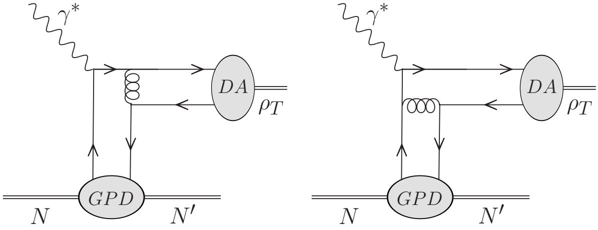
- lowest order diagrammatic argument:



$$\gamma^\alpha[\gamma^\mu, \gamma^\nu]\gamma_\alpha = 0$$

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- However processes involving twist 3 DAs may face problems with factorisation (end-point singularities)

  $\Rightarrow$ can be made safe in the high-energy k_T-factorisation approach

  [I. Anikin, D. Ivanov, B. Pire, L. Szymanowski, S. Wallon: 0909.4090]

Why consider a gamma-meson pair?

A convenient solution

Circumvent this using *3-body* final states:

- $\gamma N \to MMN'$:

  D. Ivanov, B. Pire, L. Szymanowski, O. Teryaev: [hep-ph/0209300]

  R. Enberg, B. Pire, L. Szymanowski: [hep-ph/0601138]

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- $\gamma N \to \gamma MN'$:

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  G. Duplančić, K. Passek-Kumerički, B. Pire, L. Szymanowski, S. Wallon: [1809.08104]

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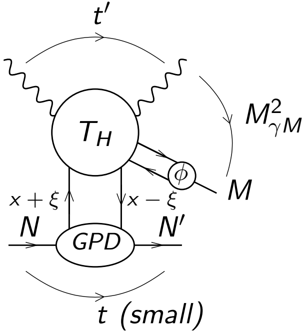
Moreover, the richer kinematics of the process allows the sensitivity of GPDs wrt x to be probed (beyond moment-type dependence, e.g. in DVCS): J. Qiu, Z. Yu: [2305.15397]

- Consider the process $\gamma N \to \gamma M N'$, M =meson. Collinear factorisation of the amplitude at large $M^2_{\gamma M}$, t', u', and small t.



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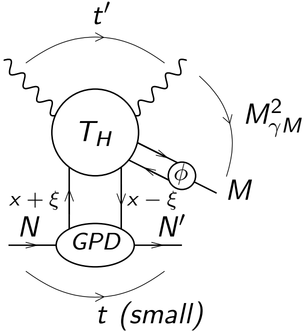

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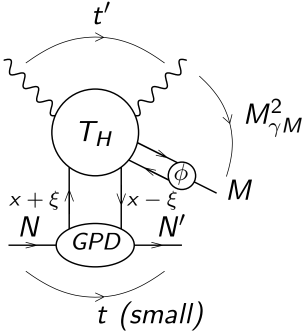
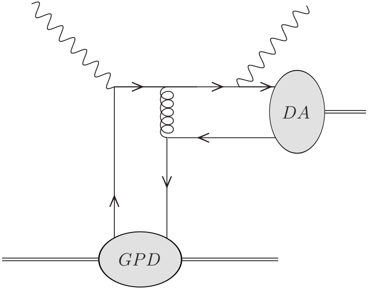


- Mesons considered in the final state: $\pi^{\pm}$, $\rho^{\pm,0}_{L,T}$.
- Leading order and leading twist.

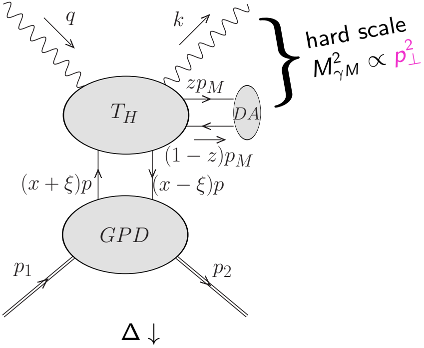
How does it work (at LO)?



Typical non-zero diagram for a transverse ρ meson

the σ matrices (from either the DA or the GPD) do not kill it anymore!

$$\gamma(q) + N(p_1) \rightarrow \gamma(k) + M(p_M, \varepsilon_M) + N'(p_2)$$

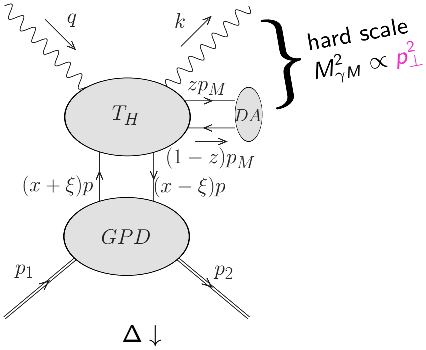


Useful Mandelstam variables:

$$t = (p_2 - p_1)^2 ,$$
$$u' = (p_M - q)^2 ,$$
$$t' = (k - q)^2 ,$$
$$S_{\gamma N} = (q + p_1)^2 .$$

- Factorisation requires:
  $-u' > 1 \text{ GeV}^2$, $-t' > 1 \text{ GeV}^2$ and $(-t)_{\min} \leqslant -t \leqslant .5 \text{ GeV}^2$
  $\implies$ sufficient to ensure large p_T.

$$\gamma(q) + N(p_1) \to \gamma(k) + M(p_M, \varepsilon_M) + N'(p_2)$$



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  $\Longrightarrow$ sufficient to ensure large p_T.
- Cross-section differential in $(-u')$ and $M^2_{\gamma M}$, and evaluated at $(-t) = (-t)_{\min}$, covering $S_{\gamma N}$ from $\sim 4\,\mathrm{GeV}^2$ to $20000\,\mathrm{GeV}^2$.

Computation
Method

$$\mathcal{A} = \int_{-1}^{1} dx \int_{0}^{1} dz \; T(x, \xi, z) \; H(x, \xi, t) \; \Phi_M(z)$$

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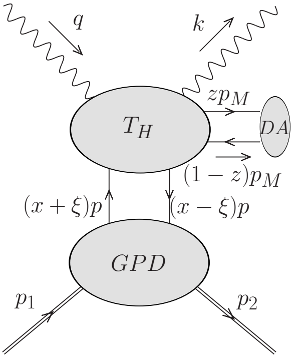
- Differential cross section:

$$\left.\frac{d\sigma}{dt\, du'\, dM^2_{\gamma M}}\right|_{-t=(-t)_{min}} = \frac{|\overline{\mathcal{A}}|^2}{32 S^2_{\gamma N} M^2_{\gamma M} (2\pi)^3} \, .$$



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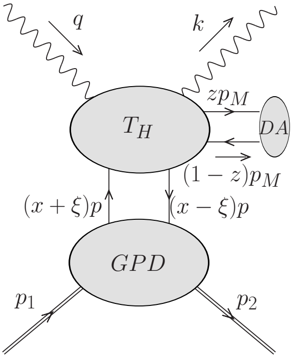

- Kinematic parameters: $S_{\gamma N}$, $M^2_{\gamma M}$, $-t$, $-u'$
- Useful dimensionless variables (hard part):

$$\alpha = \frac{-u'}{M^2_{\gamma M}} \,,$$

$$\xi = \frac{M^2_{\gamma M}}{2\,(S_{\gamma N} - m_N^2) - M^2_{\gamma M}} \,.$$

Quark GPDs are parametrised in terms of Double Distributions

[A. Radyushkin: hep-ph/9805342]

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For polarised PDFs Δq (and hence transversity PDFs δq), two scenarios are proposed for the parameterization:

- "standard" scenario, with flavor-symmetric light sea quark and antiquark distributions.
- "valence" scenario with a completely flavor-asymmetric light sea quark densities.

Computation
DAs used

- We take the simplistic asymptotic form of the DAs

$$\phi_{\mathrm{as}}(z) = 6z(1-z)\,.$$

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- We also investigate the effect of using a holographic DA:

$$\phi_{\rm hol}(z) = \frac{8}{\pi}\sqrt{z(1-z)}\,.$$

Suggested by

- AdS/QCD correspondence [S. Brodsky, G. de Teramond: hep-ph/0602252],
- dynamical chiral symmetry breaking on the light-front [C. Shi, C. Chen, L. Chang, C. Roberts, S. Schmidt, H, Zong: 1504.00689],
- recent lattice results. [X. Gao, A. Hanlon, N. Karthik, S. Mukherjee, P. Petreczky, P. Scior, S. Syritsyn, Y. Zhao: 2206.04084]

Is QCD collinear factorisation really justified?

- Recently, factorisation has been proved for the process $\pi N \to \gamma\gamma N'$ by J. Qiu, Z. Yu [2205.07846].
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- The proof relies on having large p_T, rather than large invariant mass (e.g. photon-meson pair).
- In fact, NLO computation has been performed for $\gamma N \to \gamma\gamma N'$ by O. Grocholski, B. Pire, P. Sznajder, L. Szymanowski, J. Wagner [2110.00048, 2204.00396]
- Also, NLO computation for $\gamma\gamma \to \pi^+\pi^-$ by crossing symmetry G. Duplancic, B. Nizic: [hep-ph/0607069].

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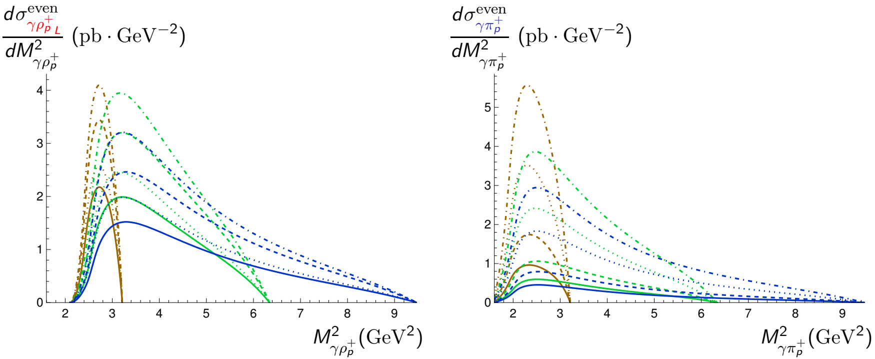
- This was extended to a wide range of $2 \to 3$ exclusive processes by J. Qiu, Z. Yu [2210.07995]
- The proof relies on having large p_T, rather than large invariant mass (e.g. photon-meson pair).
- In fact, NLO computation has been performed for $\gamma N \to \gamma\gamma N'$ by O. Grocholski, B. Pire, P. Sznajder, L. Szymanowski, J. Wagner [2110.00048, 2204.00396]
- Also, NLO computation for $\gamma\gamma \to \pi^+\pi^-$ by crossing symmetry G. Duplancic, B. Nizic: [hep-ph/0607069].

Issues with exclusive $\pi^0\gamma$ photoproduction...

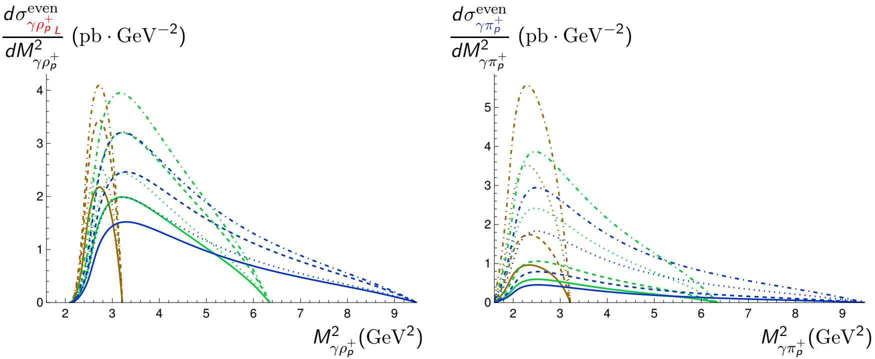


$S_{\gamma N} = 8,\ 14,\ 20\ \text{GeV}^2$

Dashed: Holographic DA non-dashed: Asymptotical DA

Dotted: standard scenario non-dotted: valence scenario

Single differential cross-section: $\gamma\rho_{p\,L}^{+}$ vs $\gamma\pi_{p}^{+}$



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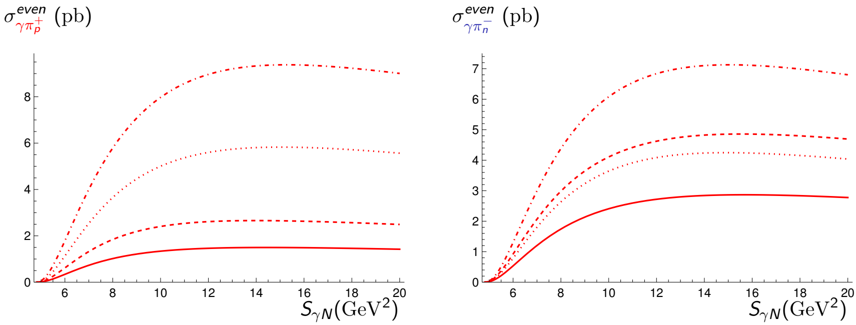
$\Longrightarrow$ Effect of GPD model more important on π_p^+ than on ρ_p^+

Results

Integrated cross-section: $\gamma\pi_p^+$ vs $\gamma\pi_n^-$



Dashed: Holographic DA non-dashed: Asymptotical DA

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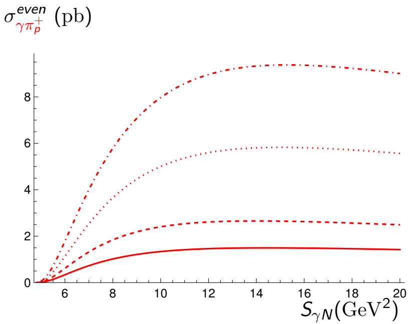
Results

Integrated cross-section: $\gamma\pi^+_p$ vs $\gamma\pi^-_n$



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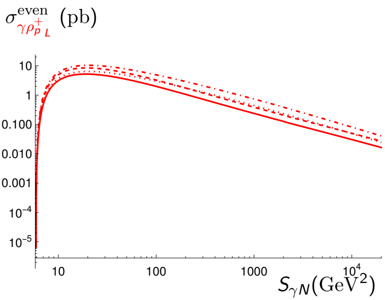
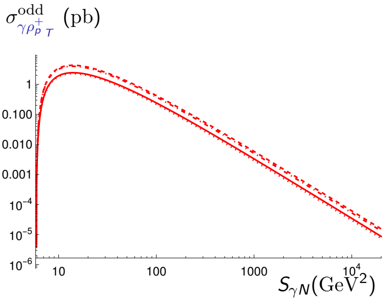
$\Longrightarrow$ Huge effect from GPD model in π^+_p case.

Results

Integrated cross-section: $\gamma\rho^+_{p\,L}$ vs $\gamma\rho^+_{p\,T}$





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Results
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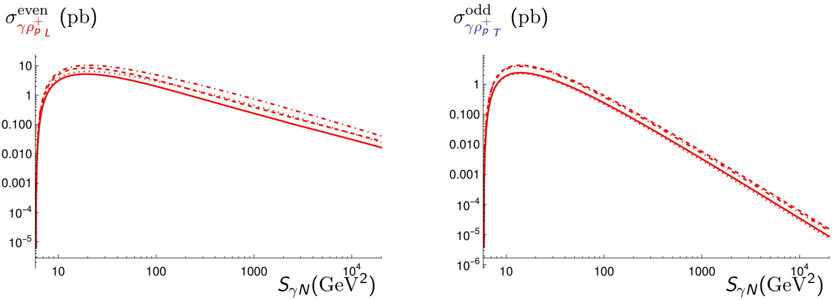

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$\Longrightarrow$ ξ^2 suppression in the chiral-odd case causes the cross-section to drop rapidly with $S_{\gamma N}$ ($\xi \approx \frac{M^2_{\gamma\rho}}{2S_{\gamma N}}$).

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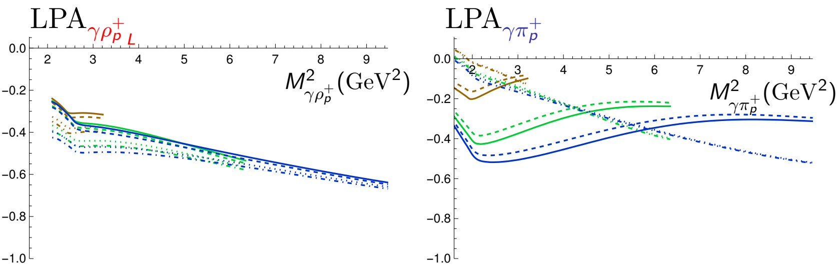
- In fact,

$$\mathrm{LPA}_{\mathrm{Lab}} = \mathrm{LPA}\cos(2\theta) \; ,$$

  where θ is the angle between the lab frame x-direction and $p_\perp$.
- Kleiss-Sterling spinor techniques used to obtain expressions.
- Both asymmetries zero in chiral-odd case!

Results

LPA wrt incoming photon: Single-differential level: $\gamma\rho^+_{p\,L}$ vs $\gamma\pi^+_p$



$$S_{\gamma N} = 8,\ 14,\ 20\ \mathrm{GeV}^2$$

Dashed: Holographic DA non-dashed: Asymptotical DA

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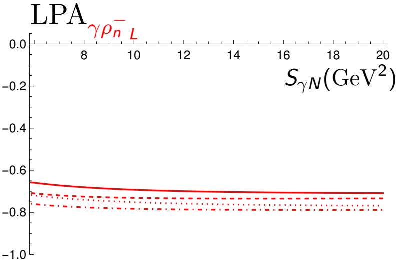
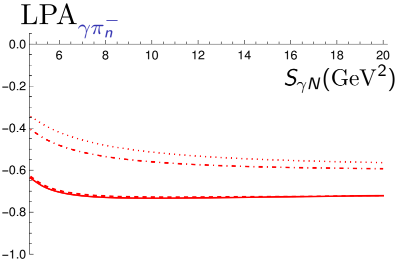
$\Longrightarrow$ GPD model changes the behaviour of the LPA completely in the π^+_p case!

LPA wrt incoming photon: Integrated level: $\gamma\rho^-_{n\ L}$ vs $\gamma\pi^-_n$





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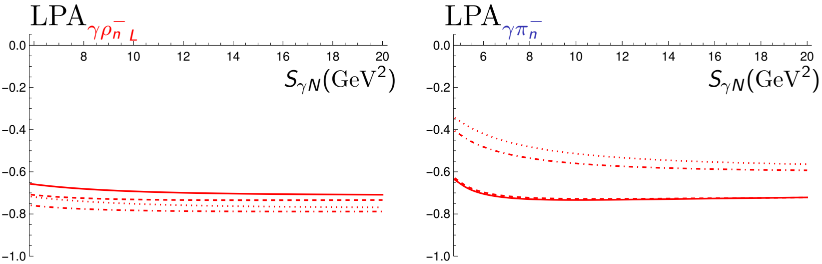

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$\Rightarrow$ LPAs are sizeable!

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  - ρ^0_L (on p) : $\approx 2.4 \times 10^5$
  - ρ^0_T (on p) : $\approx 4.2 \times 10^4$ (Chiral-odd)
  - ρ^+_L : $\approx 1.4 \times 10^5$
  - ρ^+_T : $\approx 6.7 \times 10^4$ (Chiral-odd)
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- No problem in detecting outgoing photon at JLab.

- At the future EIC, with an expected integrated luminosity of $10\,\mathrm{fb}^{-1}$ (about 100 times smaller than JLab):
  - ρ_L^0 (on p) : $\approx 2.4 \times 10^4$
  - ρ_T^0 (on p) : $\approx 2.4 \times 10^3$ (Chiral-odd)
  - ρ_L^+ : $\approx 1.5 \times 10^4$
  - ρ_T^+ : $\approx 4.2 \times 10^3$ (Chiral-odd)
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- Small ξ study:
  $300 < S_{\gamma N}/\,\mathrm{GeV}^2 < 20000$ ($5 \cdot 10^{-5} < \xi < 5 \cdot 10^{-3}$):
  - ρ_L^0 (on p) : $\approx 1.2 \times 10^3$
  - ρ_T^0 (on p) : ≈ 6.5 (Chiral-odd) (tiny)
  - ρ_L^+ : $\approx 9.3 \times 10^2$
  - π^+ : $\approx 5.0 \times 10^2$

For p-Pb UPCs at LHC (integrated luminosity of 1200 nb^{-1}):

- With future data from runs 3 and 4,
  - ρ^0_L : $\approx 1.6 \times 10^4$
  - ρ^0_T : $\approx 1.7 \times 10^3$ (Chiral-odd)
  - ρ^+_L : $\approx 1.1 \times 10^4$
  - ρ^+_T : $\approx 2.9 \times 10^3$ (Chiral-odd)
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  - $\pi^+ : \approx 9.3 \times 10^3$
- $300 < S_{\gamma N}/\,\text{GeV}^2 < 20000$ ($5 \cdot 10^{-5} < \xi < 5 \cdot 10^{-3}$):
  - $\rho_L^0 : \approx 8.1 \times 10^2$
  - $\rho_L^+ : \approx 6.4 \times 10^2$
  - $\pi^+ : \approx 3.4 \times 10^2$

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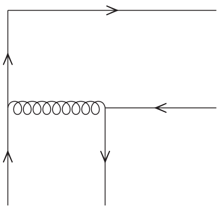
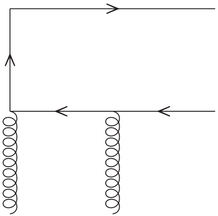
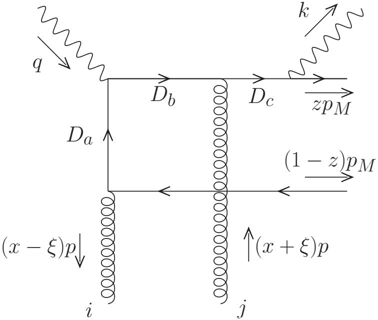
- Diagrams amount to connecting photons to the following two topologies.







$$CF \sim \frac{\mathrm{Tr}\left[\not{p}_M\gamma^5\not{\epsilon}_k\left(\not{k}+z\not{p}_M\right)\gamma^j\left(\not{q}-(x-\xi)\not{p}-\bar{z}\not{p}_M\right)\not{\epsilon}_q\left(-(x-\xi)\not{p}-\bar{z}\not{p}_M\right)\gamma^i\right]}{[2z\,kp_M]\left[-2\,(x-\xi)\,qp-2\bar{z}\,qp_M+2\bar{z}\,(x-\xi)\,pp_M+i\epsilon\right]\left[2\bar{z}\,(x-\xi)\,pp_M+i\epsilon\right]}$$

$$\xrightarrow{x\to\xi,\bar{z}\to 0}\propto \frac{x-\xi}{\left[(x-\xi)+A\bar{z}-i\epsilon\right]\left[\bar{z}\,(x-\xi)+i\epsilon\right]}\,, \qquad A\equiv\frac{qp_M}{qp}>0.$$

(Assuming p_M is along minus direction)

Need to dress coefficient function CF with gluon GPD $\left(\frac{H_g(x)}{(x-\xi+i\epsilon)(x+\xi-i\epsilon)}\right)$, and DA $(z\bar{z})$. This gives

$$\mathcal{A} \sim \frac{\bar{z}\,(x-\xi)\,H_g(x)}{(x-\xi+i\epsilon)\,[(x-\xi)+A\bar{z}-i\epsilon]\,[\bar{z}\,(x-\xi)+i\epsilon]}$$

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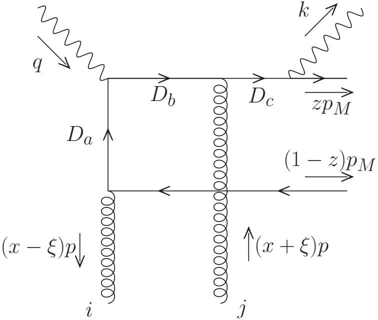
The integral over z and x diverges if the GPD $H_g(x)$ is non-vanishing at $x=\xi$:

$$\int_{-1}^{1} dx \int_{0}^{1} dz \frac{1}{[(x-\xi)+A\bar{z}-i\epsilon]\,[x-\xi+i\epsilon]}$$

$$\supset \int_{-1}^{1} dx \frac{\ln(x-\xi-i\epsilon)}{[x-\xi+i\epsilon]} \quad \Longrightarrow \text{ divergent imaginary part!}$$



$$\int_{-1}^{1} dx \int_{0}^{1} dz \frac{1}{[(x-\xi)+A\bar{z}-i\epsilon]\,[x-\xi+i\epsilon]}$$

$\Longrightarrow$ The "*pinching*" is caused by propagators D_a and D_b.

What about the sum of diagrams?

$$\sum \mathcal{A} \sim \frac{z\bar{z}\left(x^2-\xi^2\right)\left[-\alpha\left[\left(x^2-\xi^2\right)^2(1-2z\bar{z})+8x^2\xi^2 z\bar{z}\right]-\left(1+\alpha^2\right)z\bar{z}\left(x^4-\xi^4\right)\right]H_g(x)}{z\bar{z}\left[x-\xi+i\epsilon\right]^2\left[\bar{z}\left(x+\xi\right)-\alpha z\left(x-\xi\right)-i\epsilon\right]\left[z\left(x-\xi\right)+\alpha\bar{z}\left(x+\xi\right)-i\epsilon\right]}$$

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$$\xrightarrow{x\to\xi,\bar{z}\to 0} \propto \frac{\left[-\alpha\left[\left(x^2-\xi^2\right)^2(1-2z\bar{z})+\ 8x^2\xi^2 z\bar{z}\ \right]-\left(1+\alpha^2\right)z\bar{z}\left(x^4-\xi^4\right)\right]H_g(x)}{\left[x-\xi+i\epsilon\right]\left[2\xi\bar{z}-\alpha\left(x-\xi\right)-i\epsilon\right]\left[\left(x-\xi\right)+2\xi\alpha\bar{z}-i\epsilon\right]}$$

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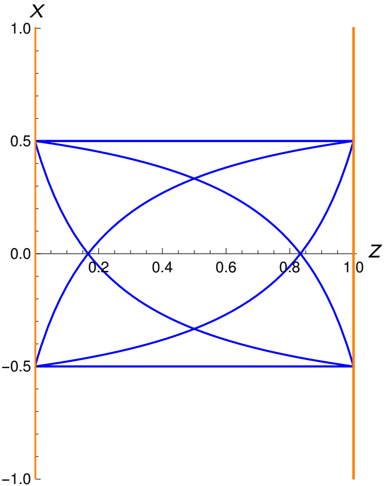
Full amplitude (anti)-symmetric in $x \to -x$ and $z \to \bar{z}$ for (anti)-symmetric GPD. (only symmetric result shown above)

$\Longrightarrow$ *divergence survives*, and actually adds up.

'Phase Space' for amplitude



$\xi = 0.5$

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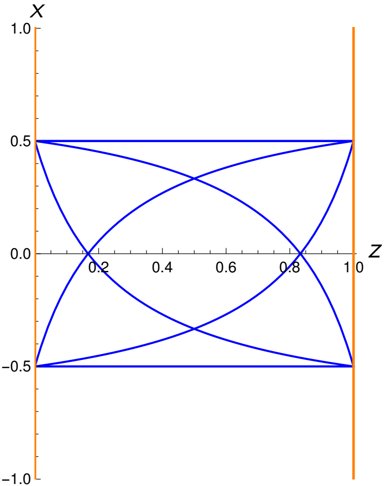

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- Unfortunately, no cancellations between the 4 corners.

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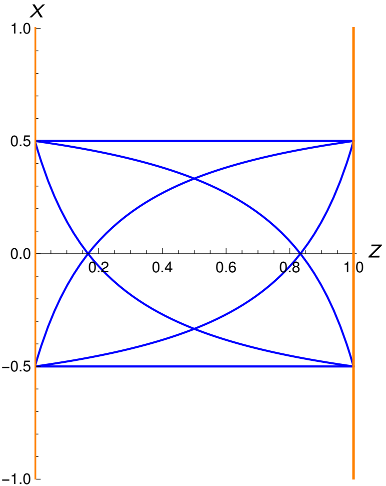

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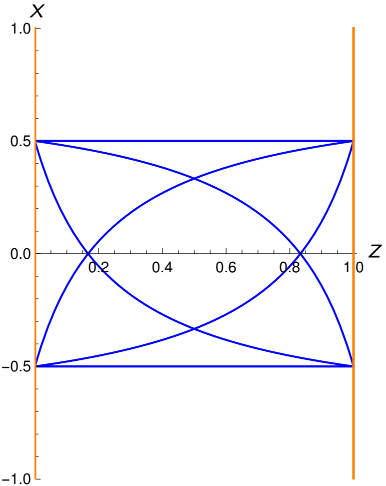

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  At twist-2??

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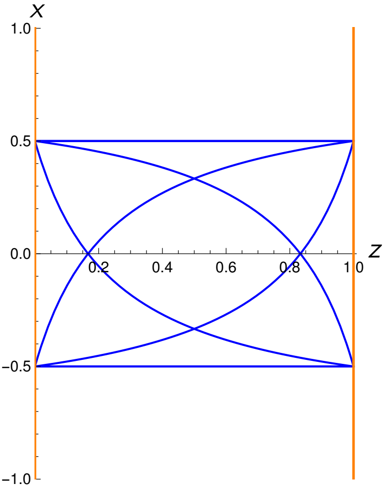

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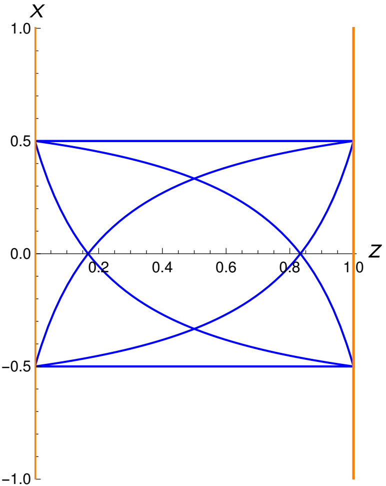
- *Can this divergence be understood from a theoretical point of view?*

Factorisation breaking effects in $\pi^0\gamma$ photoproduction

Gluon GPD contributions: Singularity structure of the full amplitude

'Phase Space' for amplitude



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  YES! $\Longrightarrow$ [**S. N.**, J. Schönleber, L. Szymanowski, S. Wallon: 2311.09146]

- How to obtain the dominant contribution of an amplitude (in QCD) in a certain specific kinematics (e. g. collinear)?
  $\Longrightarrow$ Libby-Sterman power counting rule [Phys.Rev.D 18 (1978) 3252; Phys.Rev.D 18 (1978) 4737]

Factorisation breaking effects in $\pi^0\gamma$ photoproduction
Reduced diagram analysis

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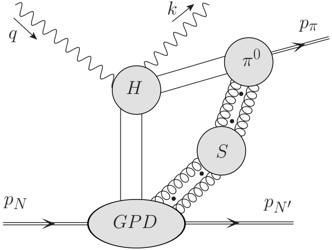
- Basic idea is to identify regions of loop momenta of partons (also number of partons), which gives the dominant contribution to the full amplitude.
- Collect all contributions to the *smallest* α:

$$\mathcal{A} = Q^\beta \sum_\alpha f_\alpha \lambda^\alpha \,, \qquad \lambda = \frac{\Lambda_{\rm QCD},\, m_\pi,\, m_N}{Q} \ll 1$$

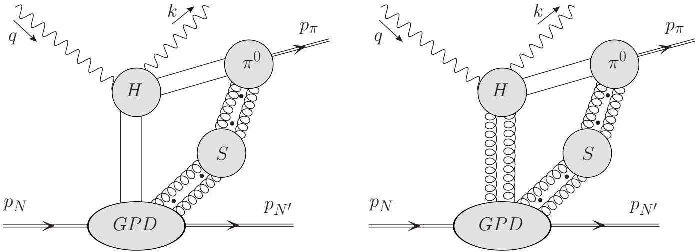


In both of the above cases, the power counting is [**S. N.**, J. Schönleber, L. Szymanowski, S. Wallon: 2311.09146]:

$$\mathcal{A} \sim Q^{-1}\lambda^{\alpha}\,, \qquad \lambda = \frac{\Lambda_{\rm QCD},\, m_\pi,\, m_N}{Q} \ll 1\,, \qquad \alpha = 1$$

Factorisation breaking effects in $\pi^0\gamma$ photoproduction

Reduced diagram analysis: Classic Collinear pinch

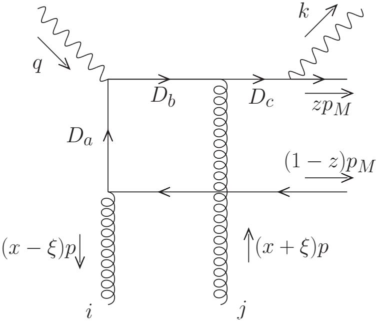


In both of the above cases, the power counting is [**S. N.**, J. Schönleber, L. Szymanowski, S. Wallon: 2311.09146]:

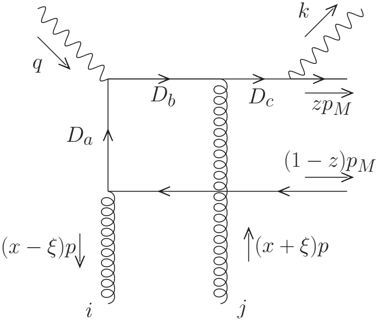
$$\mathcal{A} \sim Q^{-1}\lambda^{\alpha}\,, \qquad \lambda = \frac{\Lambda_{\mathrm{QCD}},\, m_\pi,\, m_N}{Q} \ll 1\,, \qquad \alpha = 1$$

Collinear factorisation at *all orders* and *leading power* provided:

- the above (classic) collinear pinch diagrams are the *only ones contributing to the leading power of $\alpha = 1$*
- the *soft factor S 'cancels'*



Divergence obtained when $(x-\xi)\,p$ and $(1-z)\,p_M$ lines become soft:
$\implies$ D_a becomes soft and D_b becomes collinear with respect to q.



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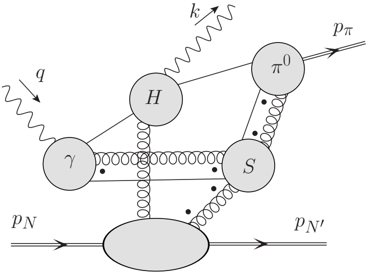
Is there a *leading* pinch diagram that corresponds to this region? *Yes!*

Factorisation breaking effects in $\pi^0\gamma$ photoproduction
Other pinch surfaces?



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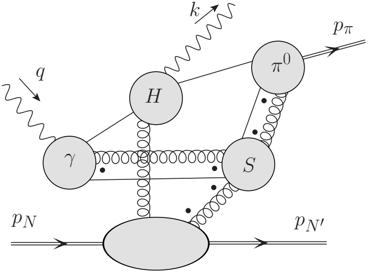

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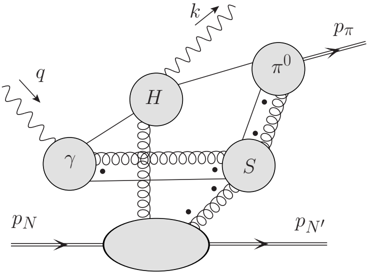

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Note: Corresponding reduced diagram for quark GPD case is power suppressed.

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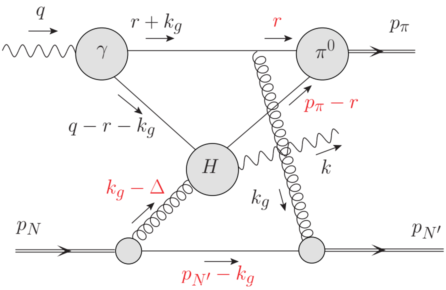
- Key Question: Is there a Glauber pinch that contributes at leading power?



(Notation: $(+, -, \perp)$)

$$p_N,\ p_{N'},\ \Delta \sim Q\left(1, \lambda^2, \lambda\right), \quad \Delta^+ < 0.$$
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$$q,\ k \sim Q\left(1, 1, 1\right), \quad q^2,\ k^2 \sim \lambda^2 Q^2$$
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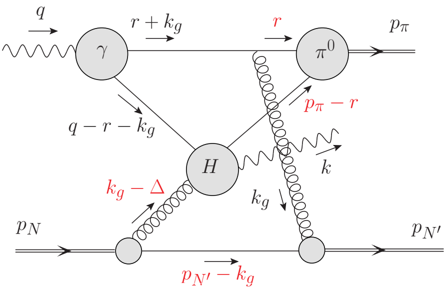

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Factorisation breaking effects in $\pi^0\gamma$ photoproduction

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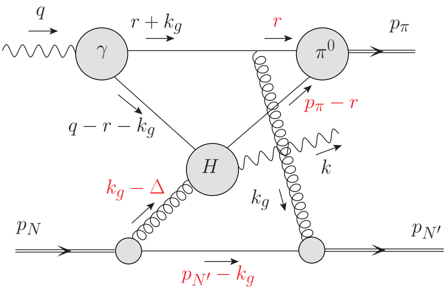

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$$\implies r^+ = \mathcal{O}(\lambda) - \text{sgn}(r^-)\, i0\,.$$

$$(p_\pi - r)^2 + i0 = -2p_\pi^- r^+ + \mathcal{O}(\lambda^2) + i0\,,$$
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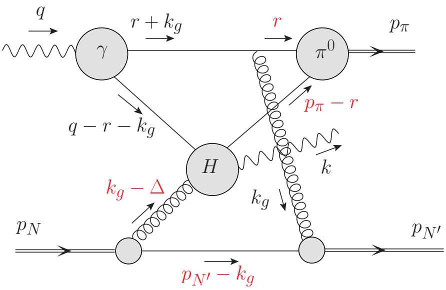

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k_g^- pinch:

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$$\implies k_g^- = \mathcal{O}(\lambda^2) - i0$$

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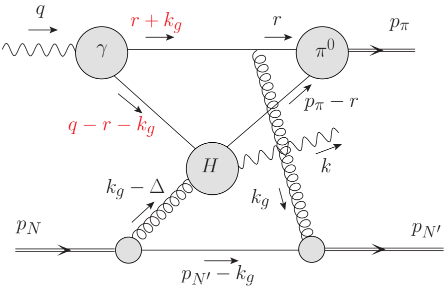

r^- pinch:

$$(q - r - k_g)^2 + i0$$
$$= -2q^+ r^- - 2q^- k_g^+ + \mathcal{O}(\lambda) + i0$$
$$\implies r^- = \mathcal{O}(\lambda) + i0$$
$$(r + k_g)^2 + i0 = 2k_g^+ r^- + \mathcal{O}(\lambda^2) + i0$$
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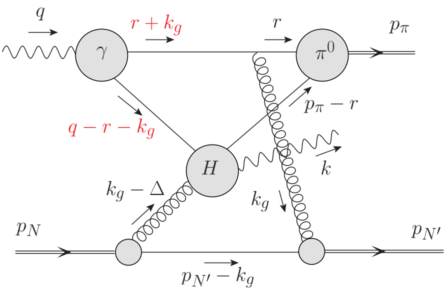

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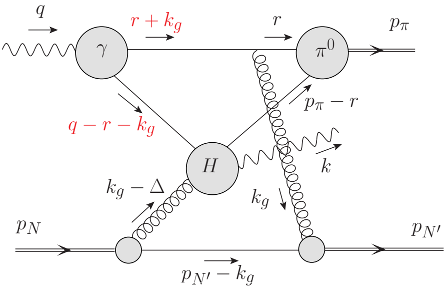
Pinch when $k_g^+ > 0 \implies$ DGLAP region



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Recall:

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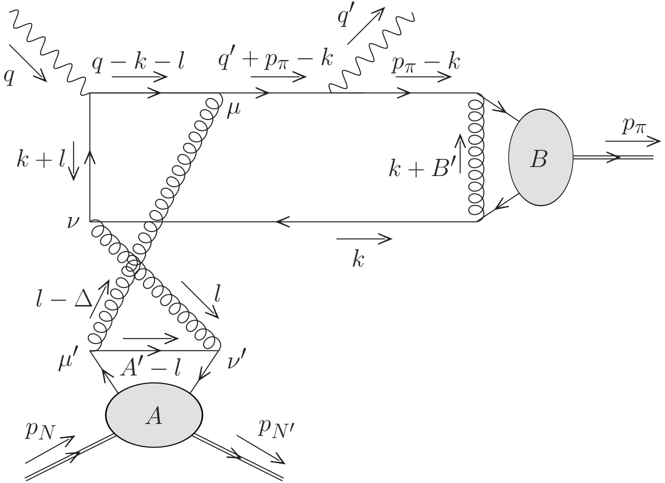
Factorisation breaking effects in $\pi^0\gamma$ photoproduction

Glauber pinch



Explicit 2-loop analysis shows that the Glauber pinch demonstrated previously is leading, i.e. it scales as λ^α, with $\alpha = 1$.

Very similar to the exclusive double diffractive process, where the Glauber gluon is pinched between the two pairs of incoming and outgoing collinear hadrons.

$$p(p_{N_1}) + p(p_{N_2}) \longrightarrow p(p_{N_1'}) + p(p_{N_2'}) + \gamma(k_1) + \gamma(k_2)$$

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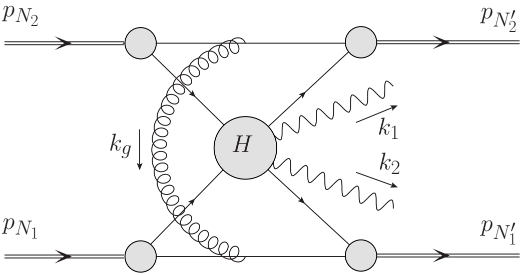

Here, the Glauber pinch corresponds to $k_g \sim (\lambda^2, \lambda^2, \lambda)$

Instead, here, the Glauber gluon (which corresponds to one of the active partons) is pinched between *a pair of collinear hadrons*, and *a soft line joining the outgoing pion and the incoming photon*.

So-called *generalised* Landau conditions [Collins] can also be used to prove the existence of the Glauber pinch here [ongoing]

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Conclusions:

- Collinear factorisation for the exclusive $\pi^0\gamma$ photoproduction *fails* due to the *gluon exchange channel*.
- The same thing happens for the exclusive process $\pi^0 N \to N\gamma\gamma$ discussed in J. Qiu, Z. Yu [2205.07846].
- Channels where *2-gluon exchanges are forbidden* ($\pi^\pm$ and $\rho^{0,\pm}$) are *safe from the effects discussed here*.

Conclusions

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- Small ξ limit of GPDs can be investigated by exploiting high energies available in collider mode such as EIC and UPCs at LHC.

Outlook

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  [ongoing]

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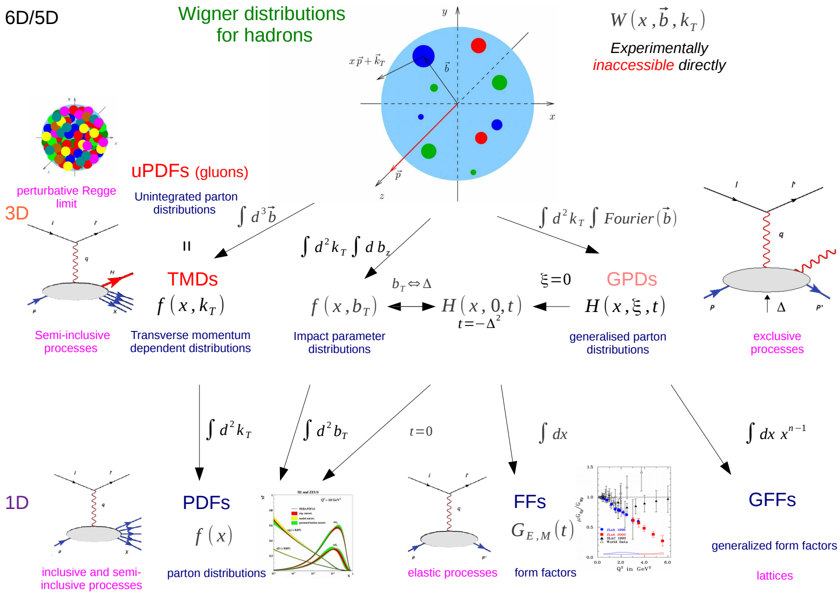
- Generalise to electroproduction ($Q^2 \neq 0$).
- Add Bethe-Heitler component (photon emitted from incoming lepton)
  - zero in chiral-odd case.
  - suppressed in chiral-even case.

BACKUP SLIDES

Introduction

From Wigner distributions to GPDs and PDFs



$$H^q(x,\xi,t=0) = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha\ \delta(\beta+\xi\alpha-x)\, f^q(\beta,\alpha)$$

- Ansatz for Double Distributions $f^q(\beta,\alpha)$:
  - chiral-even sector:

$$f^q(\beta,\alpha,t=0) = \Pi(\beta,\alpha)\, q(\beta)\Theta(\beta) - \Pi(-\beta,\alpha)\, \bar{q}(-\beta)\, \Theta(-\beta)\,,$$
$$\tilde{f}^q(\beta,\alpha,t=0) = \Pi(\beta,\alpha)\, \Delta q(\beta)\Theta(\beta) + \Pi(-\beta,\alpha)\, \Delta\bar{q}(-\beta)\, \Theta(-\beta)\,.$$

  - chiral-odd sector:

$$f_T^q(\beta,\alpha,t=0) = \Pi(\beta,\alpha)\, \delta q(\beta)\Theta(\beta) - \Pi(-\beta,\alpha)\, \delta\bar{q}(-\beta)\, \Theta(-\beta)\,.$$

  - $\Pi(\beta,\alpha) = \frac{3}{4}\frac{(1-\beta)^2-\alpha^2}{(1-\beta)^3}$: profile function

- simplistic factorised ansatz for the t-dependence:

$$H^q(x, \xi, t) = H^q(x, \xi, t = t_{\min}) \times F_H(t)$$

with $F_H(t) = \frac{(t_{\min} - C)^2}{(t - C)^2}$ a standard dipole form factor ($C = 0.71 \mathrm{GeV}^2$)

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p_2, \lambda_2 | \bar{\psi}_q \left(-\frac{1}{2}z^-\right) \gamma^+ \psi \left(\frac{1}{2}z^-\right) | p_1, \lambda_1 \rangle$$

$$= \frac{1}{2P^+} \bar{u}(p_2, \lambda_2) \left[H^q(x,\xi,t)\gamma^+ + E^q(x,\xi,t) \frac{i\sigma^{\alpha+}\Delta_\alpha}{2m} \right] u(p_1, \lambda_1)$$

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p_2, \lambda_2 | \bar{\psi}_q \left(-\frac{1}{2}z^-\right) \gamma^+ \gamma^5 \psi \left(\frac{1}{2}z^-\right) | p_1, \lambda_1 \rangle$$

$$= \frac{1}{2P^+} \bar{u}(p_2, \lambda_2) \left[\tilde{H}^q(x,\xi,t)\gamma^+\gamma^5 + \tilde{E}^q(x,\xi,t) \frac{\gamma^5\Delta^+}{2m} \right] u(p_1, \lambda_1)$$

- Take the limit $\Delta_\perp = 0$.
- In that case and for small ξ, the dominant contributions come from H^q and $\tilde{H}^q$.

$$
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+ \;& \left. E_T^q(x,\xi,t) \frac{\gamma^+\Delta^i - \Delta^+\gamma^i}{2m_N} + \tilde{E}_T^q(x,\xi,t) \frac{\gamma^+ P^i - P^+\gamma^i}{m_N} \right] u(p_1, \lambda_1)
\end{aligned}
$$

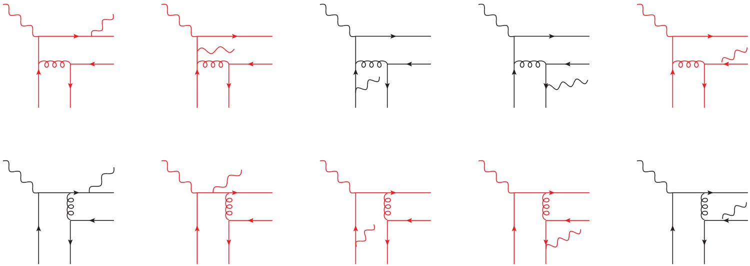
- Take the limit $\Delta_\perp = 0$.
- In that case and for small ξ, the dominant contributions come from H_T^q.

Sets of PDFs used to model GPDs

- $q(x)$: unpolarised PDF:
  - GRV-98 [M. Glück, E. Reya, A. Vogt: hep-ph/9806404]
  - MSTW2008lo [A. Martin, W. Stirling, R. Thorne, G. Watt: 0901.0002]
  - MSTW2008nnlo [A. Martin, W. Stirling, R. Thorne, G. Watt: 0901.0002]
  - ABM11nnlo [S. Alekhin, J. Blumlein, S. Moch: 1202.2281]
  - CT10nnlo [J. Gao, M. Guzzi, J. Huston, H. Lai, Z. Li, P. Nadolsky, J. Pumplin, D. Stump, C.P. Yuan: 1302.6246]
- $\Delta q(x)$ polarised PDF
  - GRSV-2000 [M. Glück, E. Reya, M. Stratmann, W. Vogelsang: hep-ph/0011215]
- $\delta q(x)$: transversity PDF:
  - Based on parameterisation for TMDs from which transversity PDFs obtained as limiting case [M. Anselmino, M. Boglione, U. D'Alesio, S. Melis, F. Murgia, A. Prokudin: 1303.3822]

Effects are not significant! But relevant for NLO corrections!

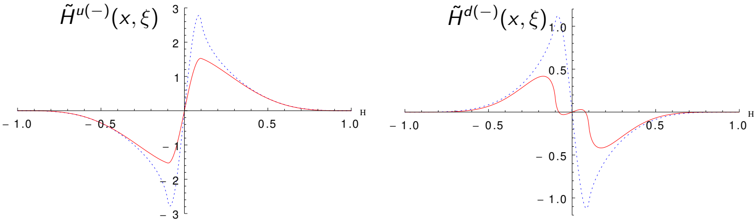
A total of 20 diagrams to compute



- We compute 10 diagrams: Other half related by $q \leftrightarrow \bar{q}$ (anti)symmetry.
- In fact, by choosing the right gauge, only 4 diagrams can be used to generate all the others by various symmetries (eg. photon exchange).
- Red diagrams cancel in the chiral-odd case

Typical kinematic point (for JLab kinematics):
$\xi = .1 \ \leftrightarrow\ S_{\gamma N} = 20\ \mathrm{GeV}^2$ and $M^2_{\gamma\rho} = 3.5\ \mathrm{GeV}^2$

$$\tilde{H}^{q(-)}(x,\xi,t) = \tilde{H}^{q}(x,\xi,t) - \tilde{H}^{q}(-x,\xi,t) \quad [C=-1]$$



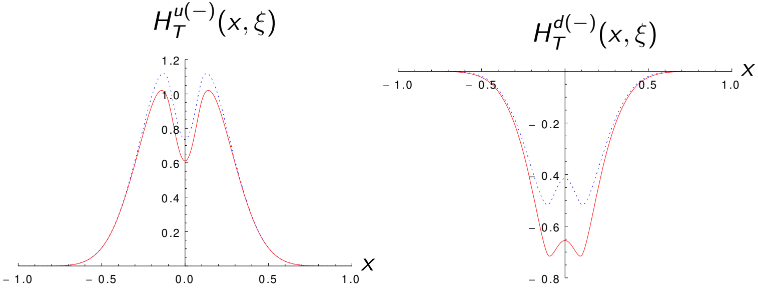
"valence" and "standard": two GRSV Ansätze for $\Delta q(x)$

Computation

Valence vs Standard scenarios in H_T (Chiral-odd)

Typical kinematic point (for JLab kinematics):
$\xi = .1 \;\leftrightarrow\; S_{\gamma N} = 20\ \mathrm{GeV}^2$ and $M^2_{\gamma\rho} = 3.5\ \mathrm{GeV}^2$

$$H_T^{q(-)}(x,\xi,t) = H_T^q(x,\xi,t) + H_T^q(-x,\xi,t) \quad [C = -1]$$



"valence" and "standard": two GRSV Ansätze for $\Delta q(x)$

$\Rightarrow$ two Ansätze for $\delta q(x)$

- Helicity conserving (vector) DA at twist 2: ρ_L

$$\langle 0|\bar{u}(0)\gamma^{\mu}u(x)|\rho^0_L(p)\rangle = \frac{p^{\mu}}{\sqrt{2}} f_{\rho} \int_0^1 du\ e^{-iup\cdot x}\phi_{\rho}(u)$$

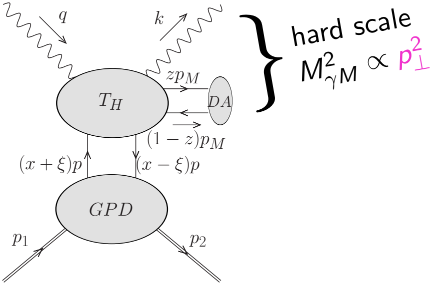
- Helicity flip (tensor) DA at twist 2: ρ_T

$$\langle 0|\bar{u}(0)\sigma^{\mu\nu}u(x)|\rho^0_T(p,s)\rangle = \frac{i}{\sqrt{2}}(\epsilon^{\mu}_{\rho}p^{\nu} - \epsilon^{\nu}_{\rho}p^{\mu}) f^{\perp}_{\rho} \int_0^1 du\ e^{-iup\cdot x}\ \phi_{\rho}(u)$$

- Helicity conserving (axial) DA at twist 2: $\pi^{\pm}$

$$\langle 0|\bar{u}(0)\gamma^{\mu}\gamma^{5}d(x)|\pi(p)\rangle = i p^{\mu} f_{\pi} \int_0^1 du\ e^{-iup\cdot x}\phi_{\pi}(u)$$

- Work in the limit of:
  - $\Delta_\perp \ll p_\perp$
  - $m_N^2,\ m_M^2 \ll M_{\gamma M}^2$



- initial state particle momenta:

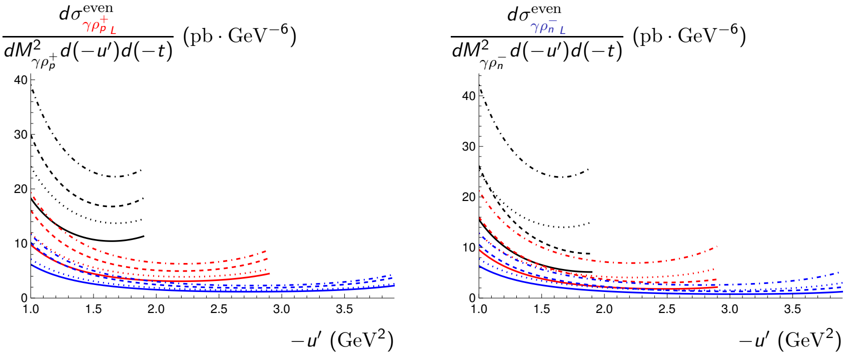
$$q^\mu = n^\mu,$$
$$p_1^\mu = (1+\xi)\,p^\mu + \frac{m_N^2}{s(1+\xi)}n^\mu$$

- final state particle momenta:

$$p_2^\mu = (1-\xi)\,p^\mu + \frac{m_N^2+\vec{p}_t^{\,2}}{s(1-\xi)}n^\mu + \Delta_\perp^\mu$$
$$k^\mu = \alpha\, n^\mu + \frac{(\vec{p}_t - \vec{\Delta}_t/2)^2}{\alpha s}\,p^\mu + p_\perp^\mu - \frac{\Delta_\perp^\mu}{2}\,, \qquad \Delta \downarrow$$
$$p_M^\mu = \alpha_M\, n^\mu + \frac{(\vec{p}_t + \vec{\Delta}_t/2)^2 + m_M^2}{\alpha_M s}\,p^\mu - p_\perp^\mu - \frac{\Delta_\perp^\mu}{2}\,,$$

Results

Fully-differential cross-sections: $\gamma\rho^+_{p\,L}$ vs $\gamma\rho^-_{n\,L}$



$$S_{\gamma N} = 20\ \text{GeV}^2,\ -t = (-t)_{\text{min}},\ M^2_{\gamma\rho} = 3, 4, 5\ \text{GeV}^2$$

Dashed: Holographic DA non-dashed: Asymptotical DA

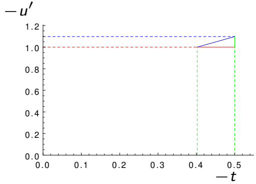
Dotted: standard scenario non-dotted: valence scenario

Results

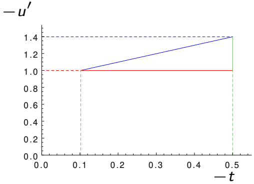
Phase space integration: Evolution in $(-t, -u')$ plane

large angle scattering: $M^2_{\gamma\rho} \sim -u' \sim -t' \quad (S_{\gamma N} = 20\ \mathrm{GeV}^2)$

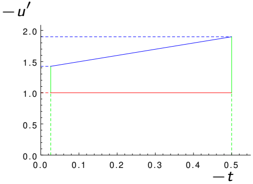
$\Longrightarrow\ -u' > 1\ \mathrm{GeV}^2$ and $-t' > 1\ \mathrm{GeV}^2$ and $(-t)_{\min} \leqslant -t \leqslant .5\ \mathrm{GeV}^2$



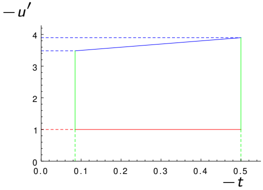
$M_{\gamma\rho} = 2.2\ \mathrm{GeV}^2$



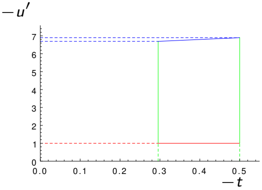
$M^2_{\gamma\rho} = 2.5\ \mathrm{GeV}^2$



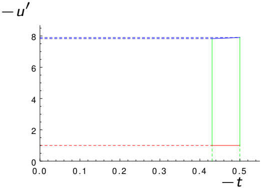
$M_{\gamma\rho} = 3\ \mathrm{GeV}^2$



$M_{\gamma\rho} = 5\ \mathrm{GeV}^2$



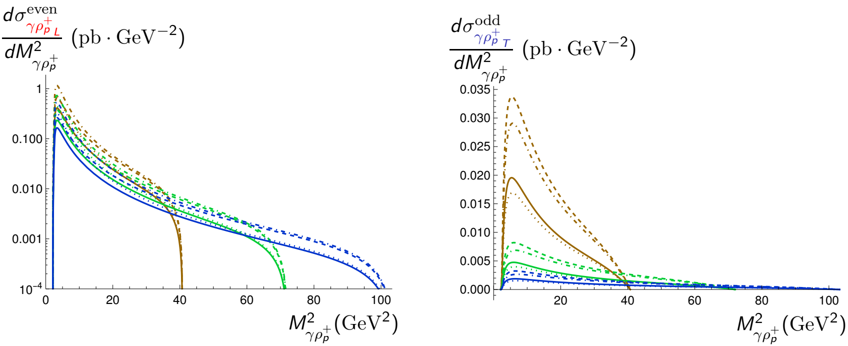
$M_{\gamma\rho} = 8\ \mathrm{GeV}^2$



$M_{\gamma\rho} = 9\ \mathrm{GeV}^2$

Results

Single differential cross-section: $\gamma\rho^+_{p\,L}$ vs $\gamma\rho^+_{p\,T}$



$S_{\gamma N} = 80, 140, 200$ GeV^2

Dashed: Holographic DA non-dashed: Asymptotical DA

Dotted: standard scenario non-dotted: valence scenario

$\Longrightarrow$ CO cross-section is suppressed by a factor of ξ^2 ($\xi \approx \frac{M^2_{\gamma\rho}}{2S_{\gamma N}}$): Measurable at small $S_{\gamma N}$, but drops rapidly with increasing $S_{\gamma N}$.

Results

Explaining the difference between chiral-even and chiral-odd plots

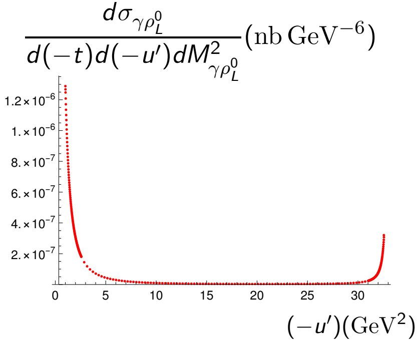
- $\xi = \frac{M^2_{\gamma M}}{2S_{\gamma N} - M_N^2} \approx \frac{M^2_{\gamma M}}{2S_{\gamma N}}$ for $M^2_{\gamma M} \ll S_{\gamma N}$
- Chiral-even (unpolarised) cross-section:

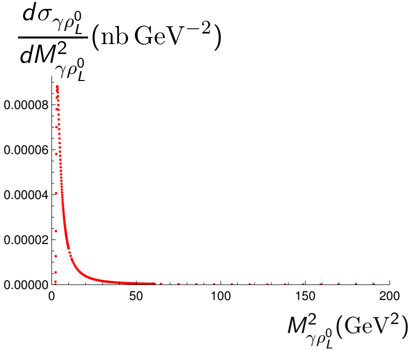
$$|\overline{\mathcal{M}}_{\mathrm{CE}}|^2 = \frac{2}{s^2}(1-\xi^2)C^2_{\mathrm{CE}}\left\{2\,|N_A|^2 + \frac{p_\perp^4}{s^2}\,|N_B|^2 + \frac{p_\perp^2}{s}\,(N_A N_B^* + c.c.) + \frac{p_\perp^4}{4s^2}\,|N_{A_5}|^2 + \frac{p_\perp^4}{4s^2}\,|N_{B_5}|^2\right\}.$$

- Chiral-odd (unpolarised) cross-section:

$$|\overline{\mathcal{M}}_{CO}|^2 = \frac{2048}{s^2}\xi^2(1-\xi^2)C^2_{\mathrm{CO}}\left\{\alpha^4\,|N_{T\,A}|^2 + |N_{T\,B}|^2\right\}.$$

- Note: $\alpha = \frac{-u'}{M^2_{\gamma M}}$.





- Need enough points at boundaries for distribution in $(-u')$
- Need enough points to resolve peak (at low $M^2_{\gamma\rho_L^0}$) for distribution in $M^2_{\gamma\rho_L^0}$

To obtain distribution in $S_{\gamma N}$, we exploit non-trivial mapping between 1 set of data at a fixed $S_{\gamma N}$ to other values $\tilde{S}_{\gamma N}$ *lower* than it.

$$\tilde{M}^2_{\gamma M} = M^2_{\gamma M} \frac{\tilde{S}_{\gamma N} - m_N^2}{S_{\gamma N} - m_N^2} \,,$$

$$-\tilde{u}' = \frac{\tilde{M}^2_{\gamma M}}{M^2_{\gamma M}} (-u') \,.$$

Implementing importance sampling $\Longrightarrow$ careful consideration of the various limits involved are needed.

Mapping possible since different sets of $(S_{\gamma N}, M^2_{\gamma M}, -u')$ correspond to the same (α, ξ).

$$\alpha = \frac{-u'}{M^2_{\gamma M}} \,, \qquad \xi = \frac{M^2_{\gamma M}}{2(S_{\gamma N} - m_N^2) - M^2_{\gamma M}} \,.$$

Consider

$$\gamma(q,\lambda_q) + N(p_1,\lambda_1) \rightarrow \gamma(k,\lambda_k) + \pi^{\pm}(p_\pi) + N'(p_2,\lambda_2) \ ,$$

where λ_i represent the helicities of the particles.

QED/QCD invariance under parity implies that [C. Bourrely, J. Soffer, E. Leader: Phys.Rept. 59 (1980) 95-297]

$$\mathcal{A}_{\lambda_2\lambda_k\,;\,\lambda_1\lambda_q} = \eta\,(-1)^{\lambda_1-\lambda_q-(\lambda_2-\lambda_k)}\mathcal{A}_{-\lambda_2-\lambda_k\,;\,-\lambda_1-\lambda_q} \ ,$$

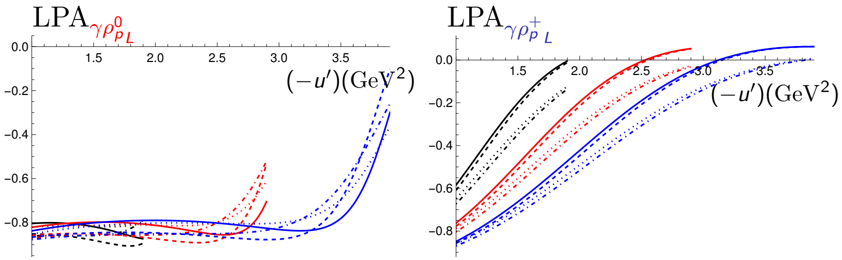
where η represents phase factors related to intrinsic spin.

Thus, at the cross-section level, it is clear that circular asymmetry will vanish, since

$$\sum_{\lambda_i,\,i\neq q} \left|\mathcal{A}_{\lambda_2\lambda_k\,;\,\lambda_1+}\right|^2 = \sum_{\lambda_i,\,i\neq q} \left|\mathcal{A}_{\lambda_2\lambda_k\,;\,\lambda_1-}\right|^2$$

Results

LPA wrt incoming photon: Fully-differential level: $\gamma\rho^0_{p\,L}$ vs $\gamma\rho^+_{p\,L}$



$$S_{\gamma N} = 20\ \mathrm{GeV}^2,\ -t = (-t)_{\min},\ M^2_{\gamma\rho} = 3, 4, 5\ \mathrm{GeV}^2$$

Dashed: Holographic DA   non-dashed: Asymptotical DA

Dotted: standard scenario   non-dotted: valence scenario

At COMPASS:

- Taking a luminosity of $\mathcal{L} = 0.1 \ \mathrm{nb}^{-1} s^{-1}$, and 300 days of run,
  - ρ_L^0 (on p) : $\approx 1.2 \times 10^3$
  - ρ_T^0 (on p) : $\approx 1.5 \times 10^2$ (Chiral-odd)
  - ρ_L^+ : $\approx 7.4 \times 10^2$
  - ρ_T^+ : $\approx 2.6 \times 10^2$ (Chiral-odd)
  - π^+ : $\approx 7.4 \times 10^2$
- Lower numbers due to low luminosity (factor of 10^3 less than JLab!)

- In ultraperipheral collisions (UPCs), hadronic interactions (QCD) are suppressed.

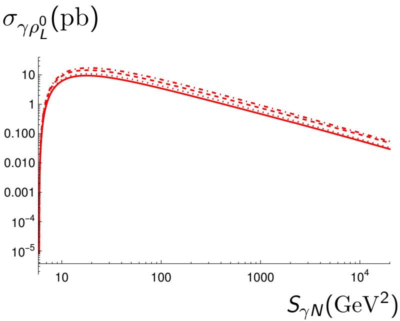
  $\Longrightarrow$ *Interactions between nuclei dominated by photon exchanges.*

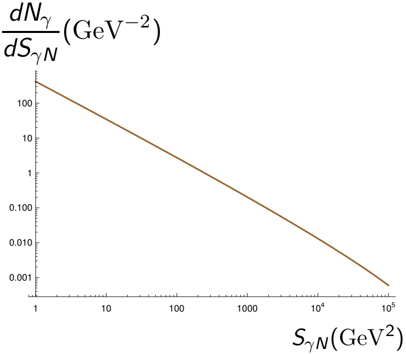
- Therefore, we can study p-Pb collisions, with the Pb nucleus acting as the photon source, since it has a much larger charge:

$$\frac{d^3N_\gamma}{dk d^2\vec{b}} = \frac{Z^2\alpha x^2}{\pi^2 k|\vec{b}|^2} K_1^2(x)\,, \qquad x = \frac{k|\vec{b}|}{\gamma\hbar c}$$

$$\frac{dN_\gamma(k)}{dk} = \int_{b_{\min}}^{b_{\max}} db\; 2\pi b\, \frac{d^3N_\gamma}{dk d^2\vec{b}}\, P_{\mathrm{NOHAD}}(b)\,,$$

- $P_{\mathrm{NOHAD}}(b)$ taken from STARlight [S. Klein, J. Nystrand, J. Seger, Y. Gorbunov, J. Butterworth: 1607.03838]

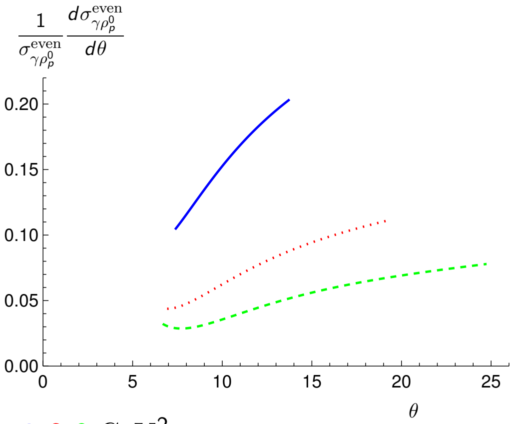




- Photon flux enhanced by a factor of Z^2, but drops rapidly with $S_{\gamma N}$.
- LHC great for high energy, but JLab far better in terms of luminosity.
- Still, LHC gives us access to the small ξ region of GPDs!

Angular cuts on outgoing photon at JLab

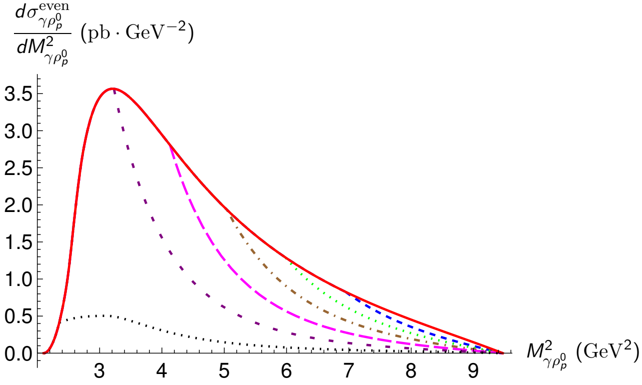
Angular distribution: $\rho_p^0\gamma$ photoproduction at $S_{\gamma N} = 20\,\mathrm{GeV}^2$



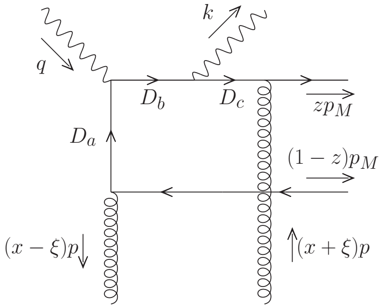
- $M^2_{\gamma\rho_p^0} = 4, 6, 8\ \mathrm{GeV}^2$.
- θ: Angle between outgoing photon and incoming photon in lab (proton rest) frame.

Angular cuts on outgoing photon at JLab

Single differential cross-section: $\rho_p^0\gamma$ photoproduction at $S_{\gamma N} = 20\,\mathrm{GeV}^2$



- no angular cut (solid red)
- $\theta \leq 35^\circ$ (dashed blue)
- $\theta \leq 30^\circ$ (dotted green)
- $\theta \leq 25^\circ$ (dashed-dotted brown)
- $\theta \leq 20^\circ$ (long-dashed magenta)
- $\theta \leq 15^\circ$ (short-dashed purple)
- $\theta \leq 10^\circ$ (dotted black)



$$D_a = ((x-\xi)p + \bar{z}p_M)^2 + i\epsilon$$
$$= s\bar{\alpha}\bar{z}\,[x-\xi+i\epsilon]\ ,$$

$$D_b = (k + zp_M - (x+\xi)p)^2 + i\epsilon$$
$$= -s\,[z\,(x-\xi-i\epsilon) + \alpha\bar{z}\,(x+\xi-i\epsilon)]\ ,$$

$$D_c = (zp_M - (x+\xi)p)^2 + i\epsilon$$
$$= -s\bar{\alpha}z\,[x+\xi-i\epsilon]$$

$\Longrightarrow$ *pinching of poles in the propagators (D_a and D_b) in the limit of $z \to 1$*

Extension of the calculation to NLO

Quark GPD case

At LO, there are 20 diagrams, but at NLO, this goes up to 422!

$\Longrightarrow$ Necessary to automate!

Our approach:

1. Generate diagrams using FeynArts
2. Reduce tensor loop integrals (which can go up to 6-point functions!) to a basis of *known scalar master integrals*.
   $\Longrightarrow$ Use ROLI (Reduction Of Loop Integrals), a private code based on Integration-By-Parts (IBP) reduction developed by Goran Duplancic, which is based on B. Nizic, G. Duplancic [hep-ph/0303184] .
3. Include GPD evolution and observe explicitly the cancellation of IR divergences.
4. Perform convolution over momentum fractions x (GPD) and z (DA).

Computation very similar to B. Nizic, G. Duplancic [hep-ph/0607069] for $\gamma\gamma \to \pi^+\pi^-$ ... except ...

- No $i\epsilon$ factors needed when calculating the convolution of coefficient function with 2 DAs in the $\gamma\gamma \to \pi^+\pi^-$ case.
- In $\gamma N \to \gamma MN$, since poles of propagators are crossed during the convolution, one requires $i\epsilon$ factors to be in place in arguments of logs and dilogs (easy), as well as in denominators (hard).
- Denominators can appear both through the IBP reduction procedure, or through the evalutation of master integrals themselves, where naive analytic continuation ($p_i^2 \to p_i^2 + i\epsilon$) does NOT lead to the correct prescription! [B. Nizic, G. Duplancic: hep-ph/0006249]

Extension of the calculation to NLO

Quark GPD case

- Finally, need to deal with numerical instabilities in convolution integral: These instabilities are present even in the $\gamma\gamma \to \pi^+\pi^-$ calculation, due to the introduction of spurious singularities that should cancel in the end...
- With $i\epsilon$ in denominators, the situation becomes much more complicated.
- This was actually a significant bottleneck in the NLO computation of $\gamma N \to \gamma\gamma N$, performed by O. Grocholski, B. Pire, P. Sznajder, L. Szymanowski, J. Wagner [2110.00048, 2204.00396], where a finite $i\epsilon$ was kept for the numerics.
  $\implies$ However, calculation significantly simpler than our case, since only one convolution integration to perform, and also have up to 5-point functions to reduce.