Tamas Gombor

Based on: arXív:2110.07960 arXív:2311.04870 and recent unpublished work REN **GIGICI**

Exact overlaps for boundary states





• Motivation

Overlaps of two site states for gl(2)
Overlaps of two site states for gl(N)
Overlaps of matrix product states for

	spín		naín.	c			
	Shew			3			
)	spir	nc	hain	Ś			
-0				n chi			

Motivation

why boundary state overlaps?

boundary state g

 $\langle \Psi | \overline{u} \rangle$

Motivation

Why boundary state overlaps?

In statistical physics

boundary state h

Time evolution from initial state $|\psi\rangle$ The overlaps are input for Quench Action

 $\langle \Psi | \overline{u} \rangle$

Motivation

why boundary state overlaps?

In statistical physics

boundary state h

Time evolution from initial state $|\psi\rangle$ The overlaps are input for Quench Action

 $\langle \Psi | \overline{u} \rangle$

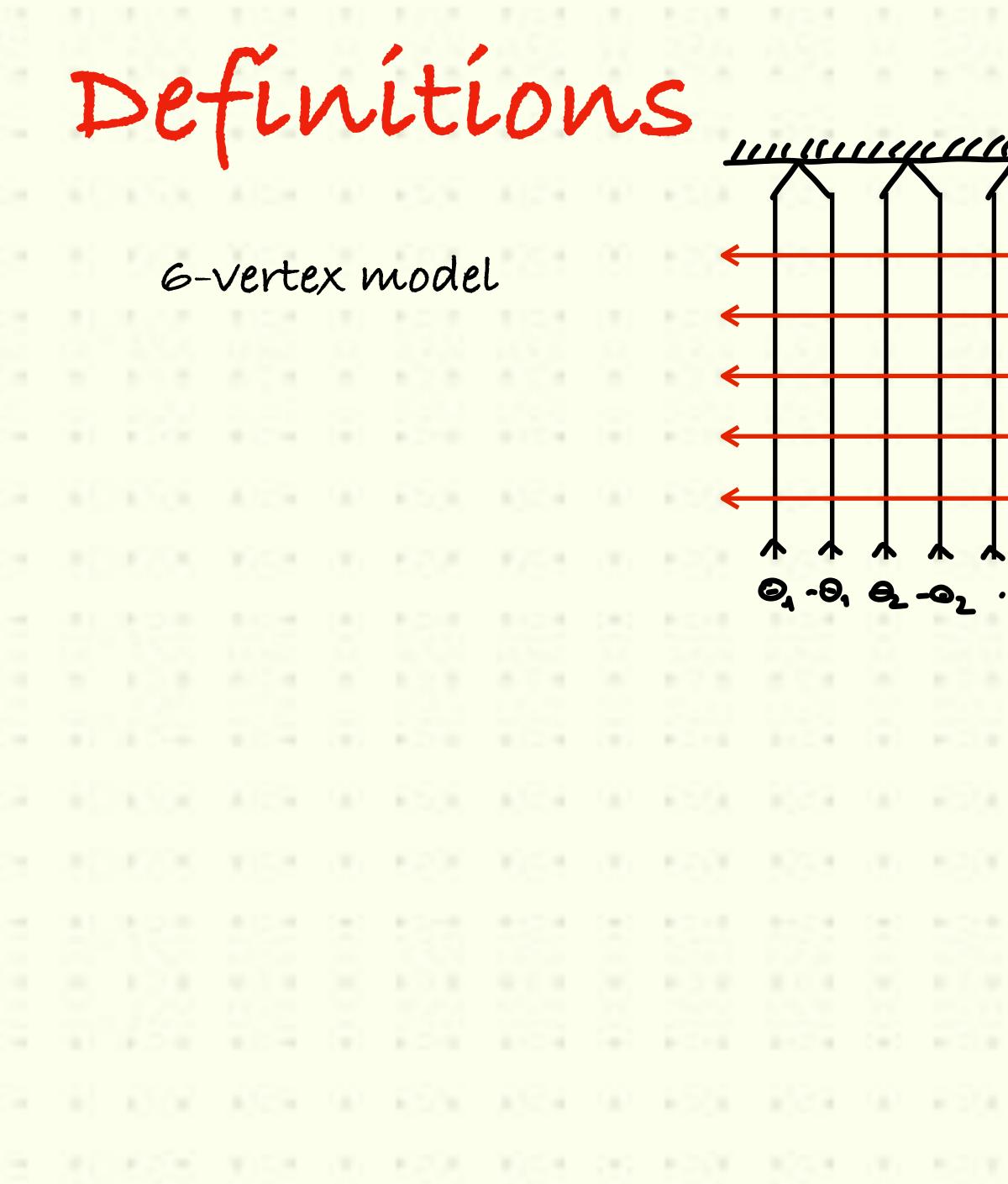
GBethe state

The overlaps correspond to 1-pt functions of defect theories

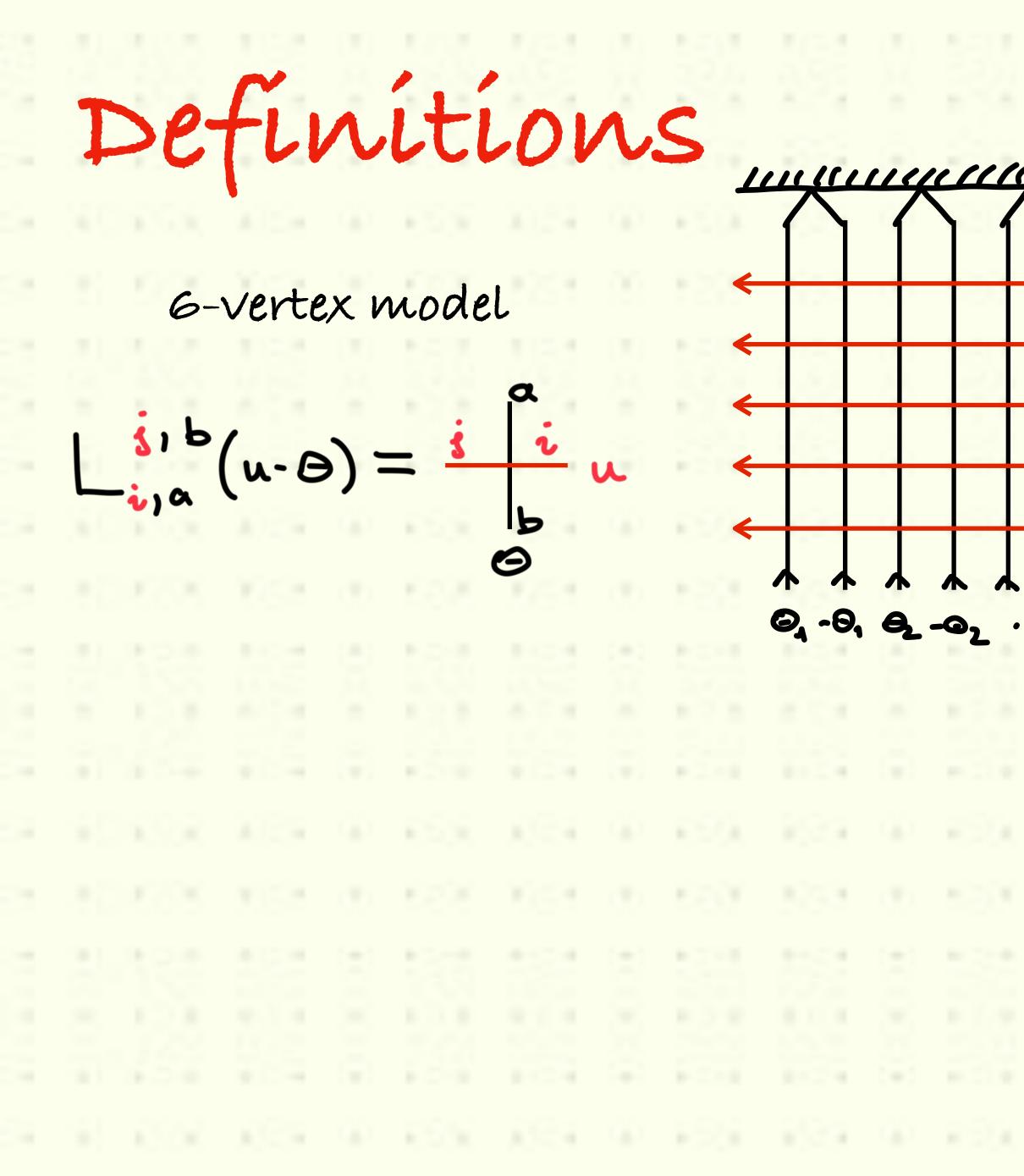


1.0							
				turo	site	c+0	tac
					SLLE	SLU	ics i

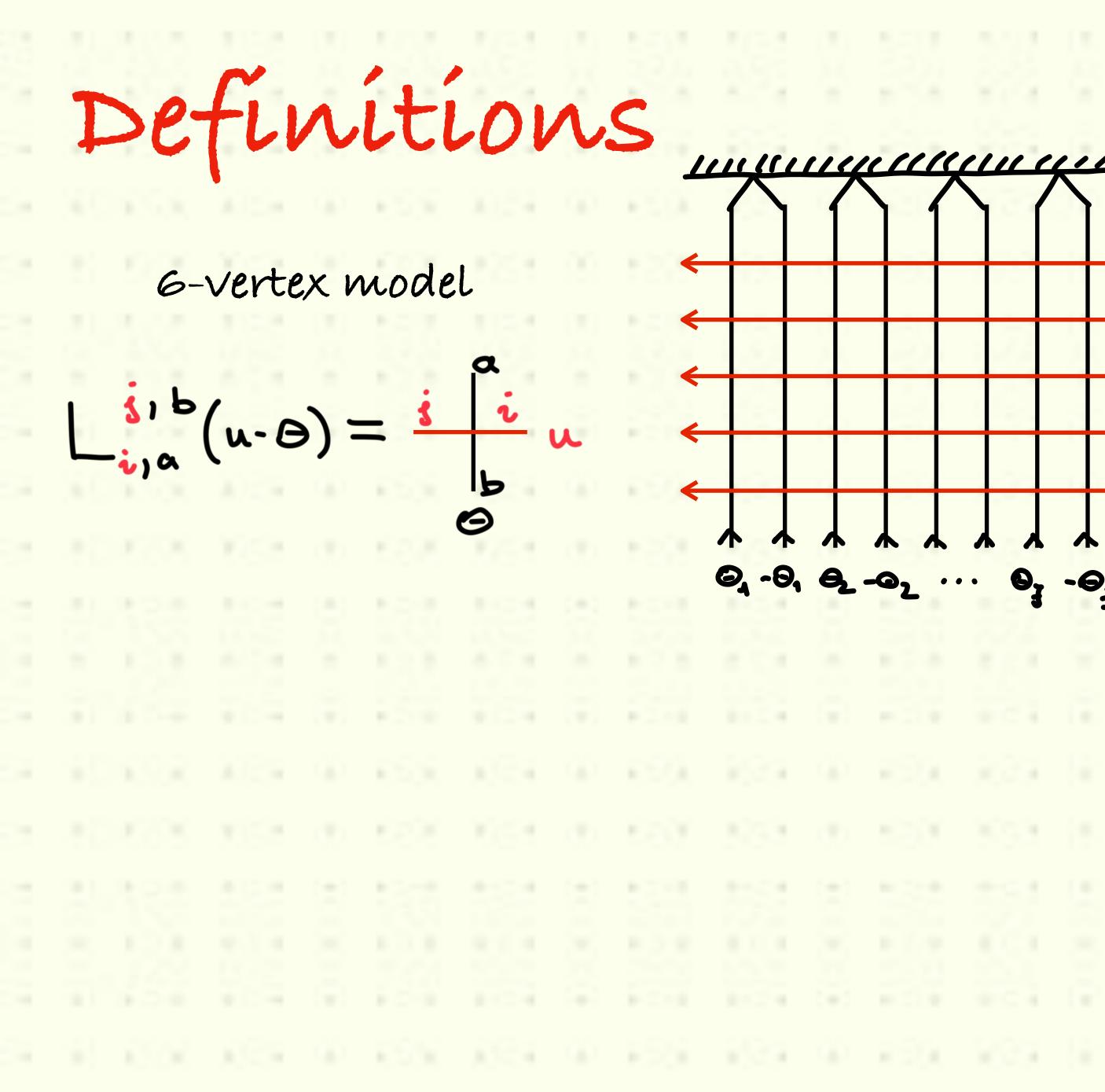
of gl(2) spin chains 4



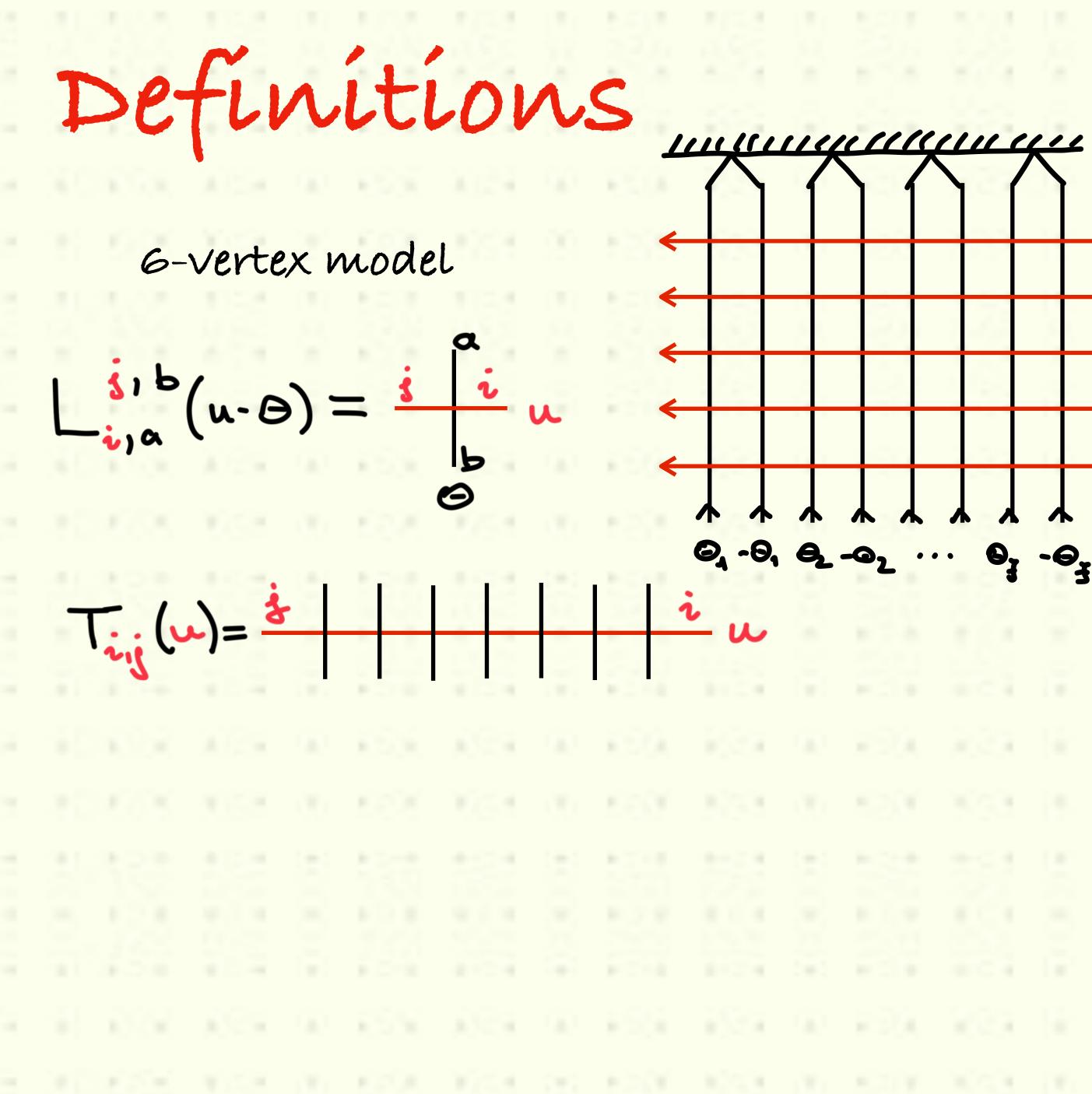
									3
4	une	<u> </u>							-
ĺ) (140
t			→ U4						12
Ĺ			→ U₂ → ·						2
		+							
┢		+	→ur						1.0
									1
									14
									1
									1
1	10.00	191	2.11	A 11, 40 - 14	2112	10.00	21-2	1000	



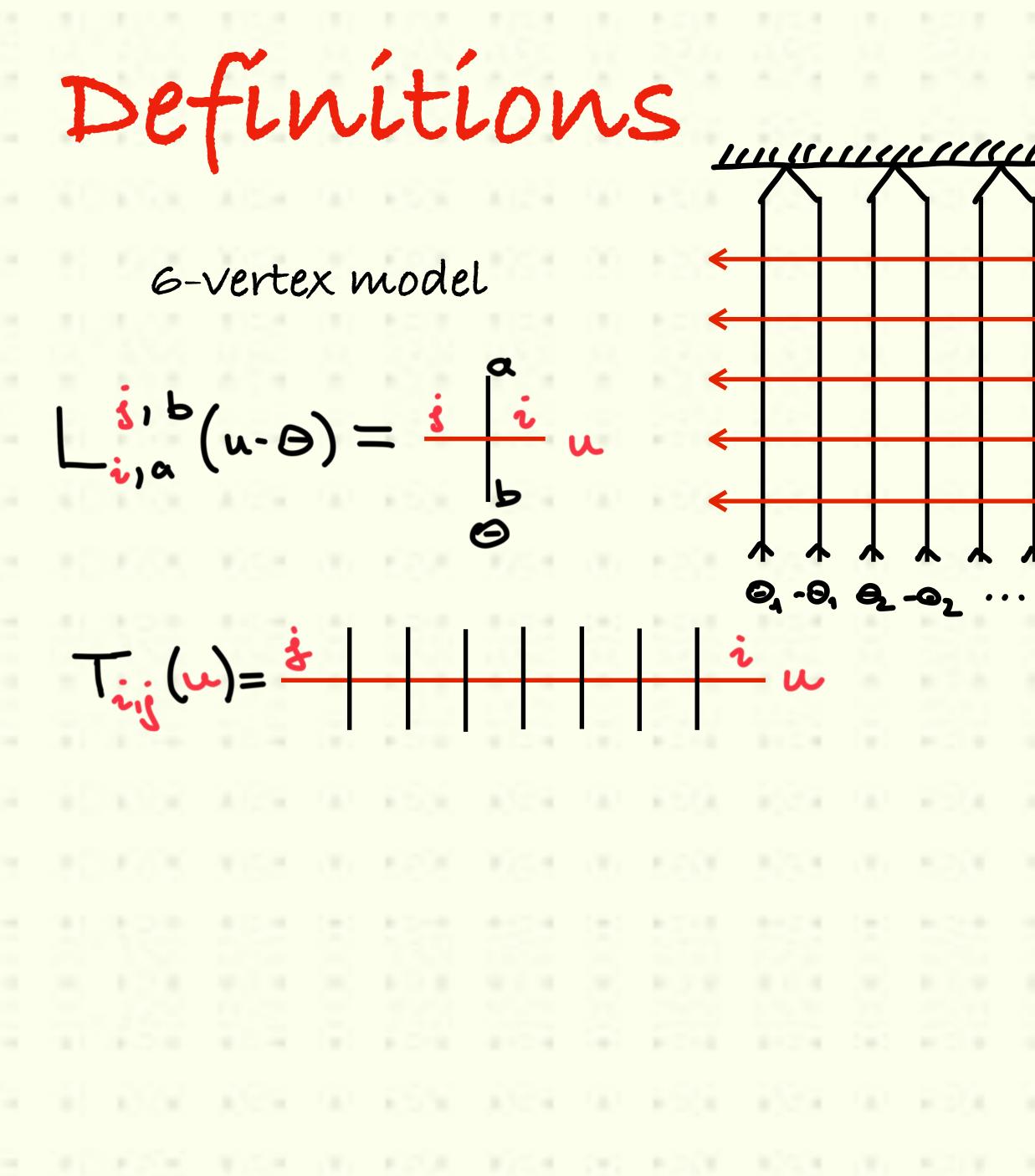
									3
4	une	<u> </u>							-
ĺ) (140
t			→ U4						12
Ĺ			→ U₂ → ·						2
		+							
┢		+	→ur						1.0
									1
									14
									1
									1
1	10.00	191	2.11	A 11, 40 - 14	2112	10.00	21-2	1000	



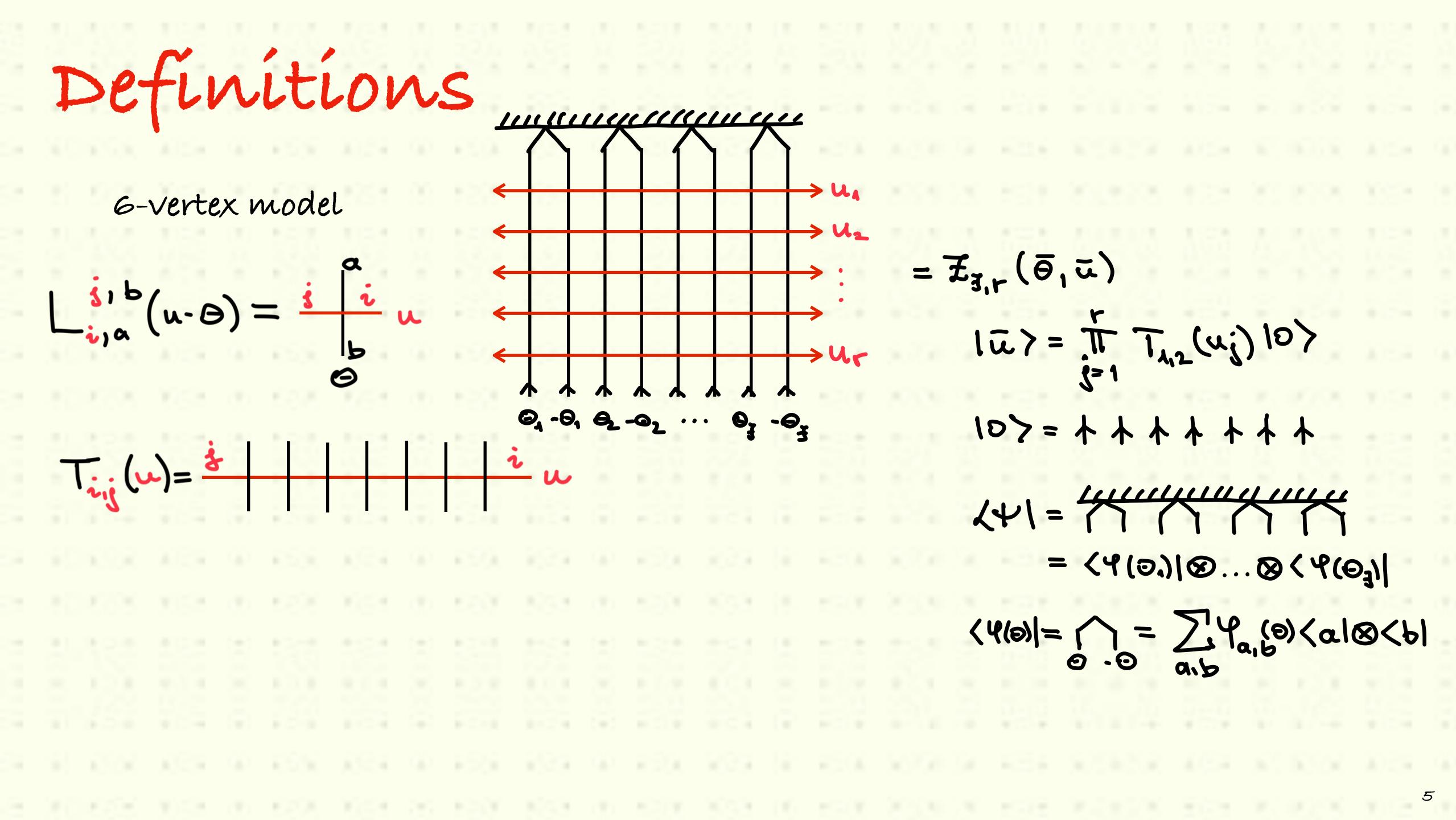
 $= \mathcal{I}_{3,r}(\bar{\Theta},\bar{u})$ $\Theta_1 - \Theta_1 \Theta_2 - \Theta_2 \cdots \Theta_r - \Theta_r$ -5



 $= \mathcal{I}_{3,r}(\bar{\Theta},\bar{u})$ -5

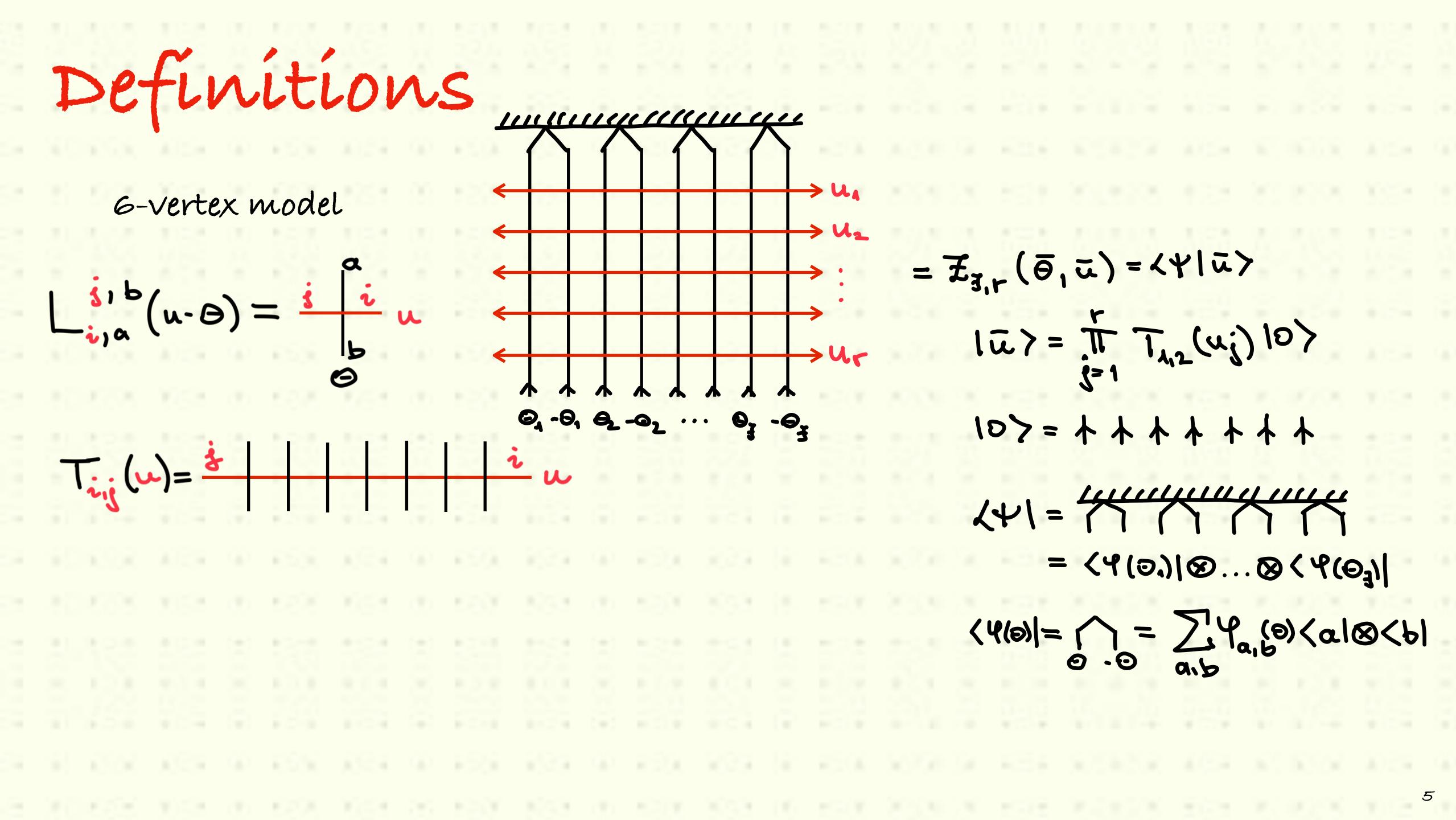


 $= \mathcal{I}_{\mathfrak{Z},r}(\bar{\mathfrak{O}},\bar{\mathfrak{u}})$ $|\bar{u}\rangle = \pi T_{1/2}(u_j)|0\rangle$ 10>= ト ト ト ト ト ト -5



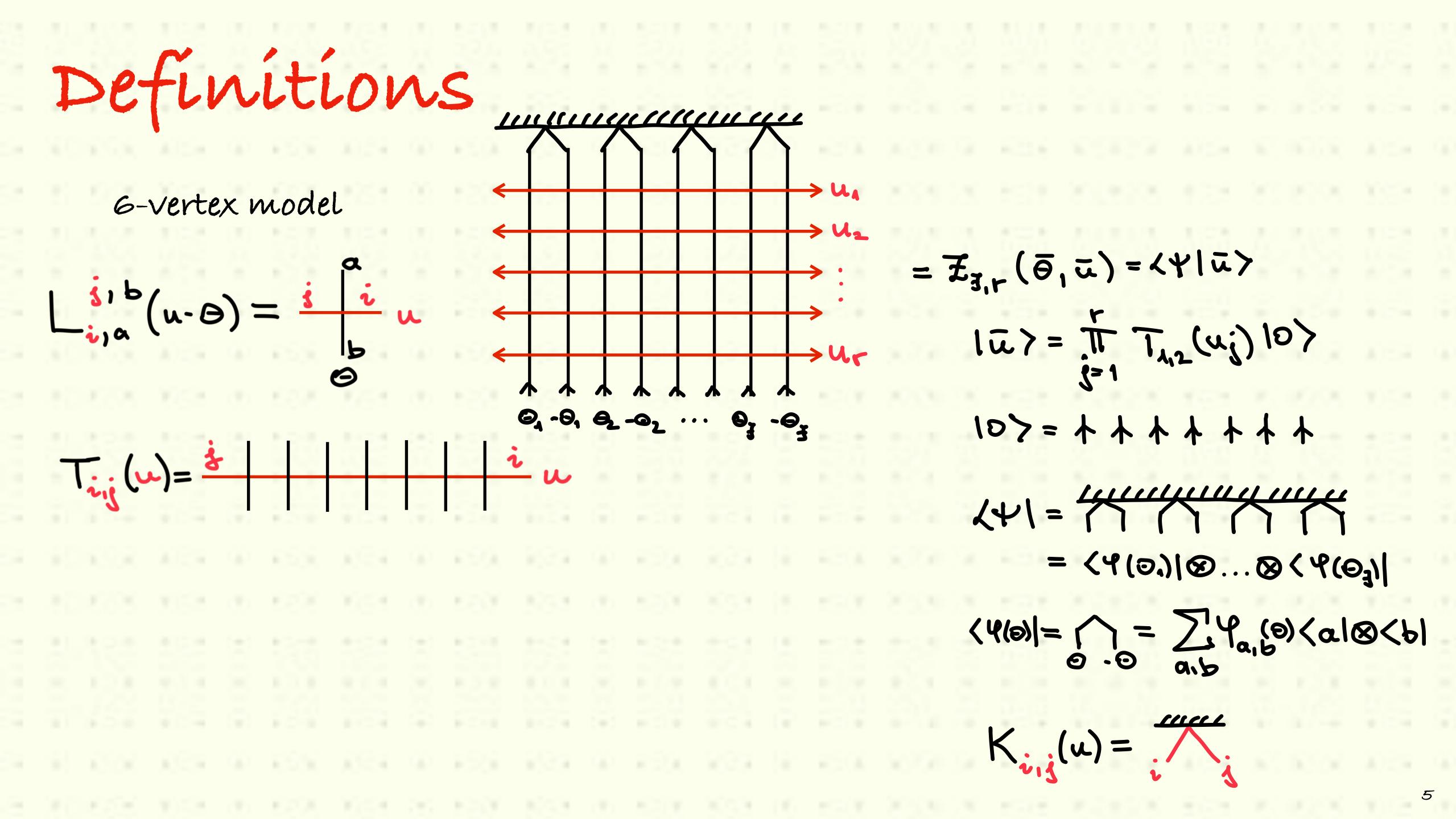
 $= \mathcal{I}_{\mathfrak{Z},r}(\bar{\mathfrak{O}},\bar{\mathfrak{u}})$ $|\bar{u}\rangle = \frac{r}{\pi} T_{1/2}(u_j)|0\rangle$ 10>= イイイイイイ Hund me 141= $= \langle \Psi(\Theta_1) | \otimes \ldots \otimes \langle \Psi(\Theta_1) |$ $= \sum Y_{a,b}(0) \langle a | \otimes \langle b |$ < 4(6) =





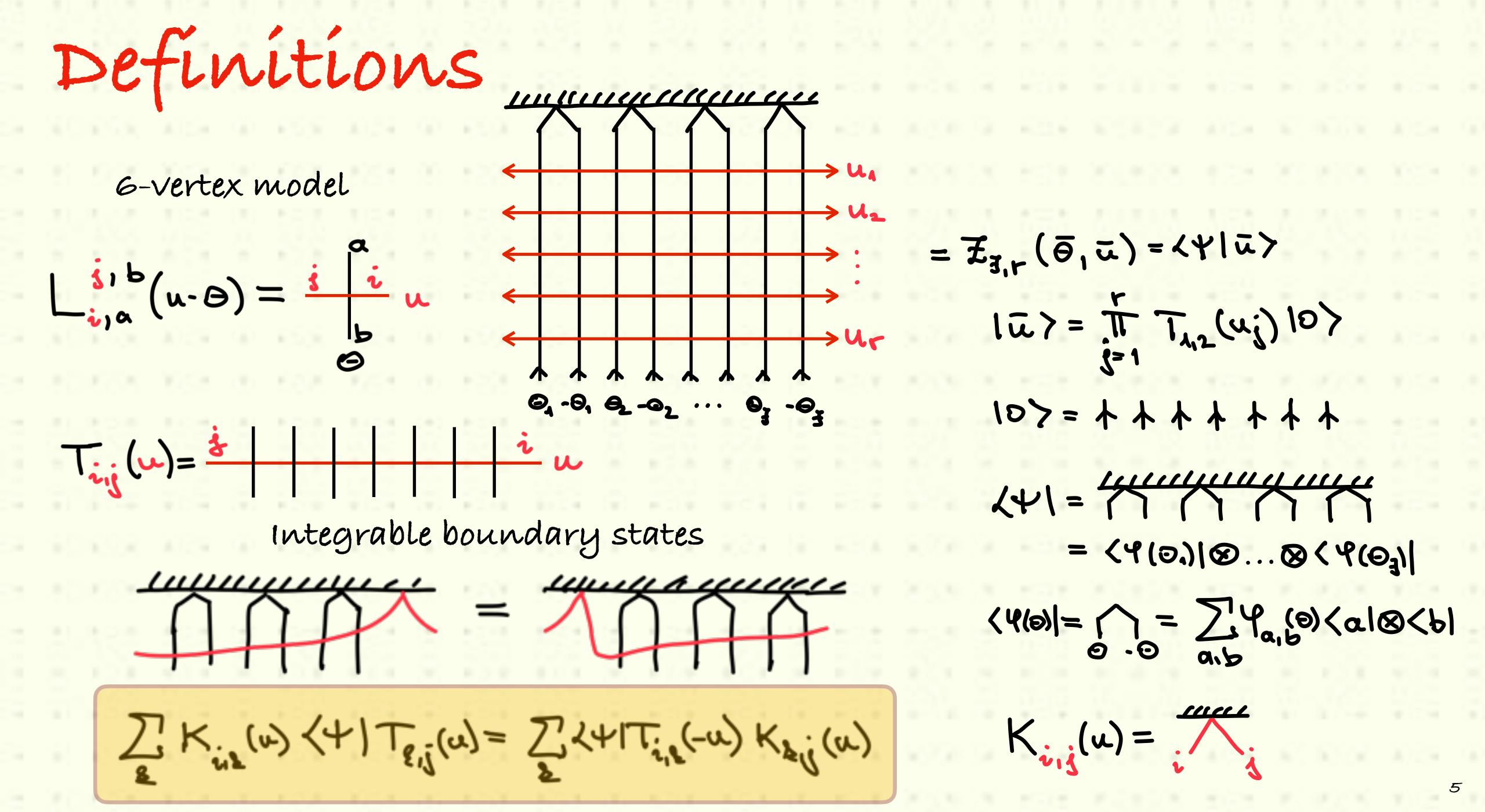
 $= \mathcal{I}_{3,r}(\bar{\Theta},\bar{u}) = \langle Y | \bar{u} \rangle$ $|\bar{u}\rangle = \frac{r}{\pi} T_{1/2}(u_j)|0\rangle$ 10>= イ イ イ ト イ イ Hunny my イヤ = $= \langle \Psi(\Theta_1) | \otimes \ldots \otimes \langle \Psi(\Theta_1) |$ $= \sum Y_{a,b}(\theta) \langle a | \otimes \langle b |$ (4(0)=

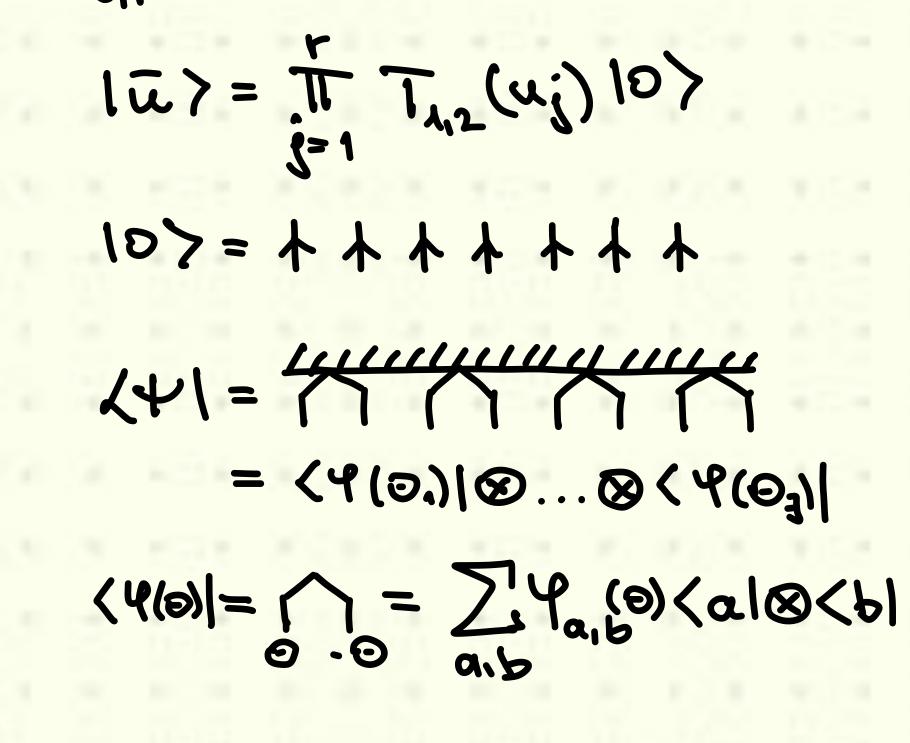




 $= \mathcal{I}_{3,r}(\bar{\Theta},\bar{u}) = \langle \Psi | \bar{u} \rangle$ $|u_{r}\rangle = \prod_{i=1}^{r} T_{u_{i}2}(u_{i})|0\rangle$ 10>= トトトトトトト Hun hun イヤ = $= \langle \Psi(\Theta_1) | \otimes \ldots \otimes \langle \Psi(\Theta_1) |$ $= \sum Y_{a,b}(\theta) \langle a | \otimes \langle b |$ く 4(の) =







 $= \mathcal{I}_{3,r}(\bar{\Theta},\bar{u}) = \langle \Psi | \bar{u} \rangle$



$K_{o}(z) < 4 | T_{o}(z) = < 4 | T_{o}(-z) K_{o}(z)$



<u>Umili k IIIIII</u>



Properties of the KT-relation <u>ummunner</u> $K_{o}(z)(4|T_{o}(z)) = (4|T_{o}(-z)K_{o}(z))$

1																			
2																			
24																			
2																			
24																			
	1	1.5	$W := \mathbb{R}^{n}$	191	6295	$V \leq 0$	121	1201	1223	 520 V	10.00	11	$g \leq 1/\ell$	20200-22	0.803	1000	100	(\mathbf{r}_{i})	0.08

Compatibility with the RTT-relation $R_{12}(u-v) T_{1}(w) T_{2}(v) = T_{2}(v) T_{1}(w) R_{12}(u-v)$



 $K_{0}(z)(4|T_{0}(z)) = (4|T_{0}(-z)K_{0}(z))$

 $\langle \Psi | T_{1}(z_{1}) T_{2}(z_{2}) = R \langle \Psi | T_{2}(z_{2}) T_{1}(z_{1}) R = ... = (...) \langle \Psi | T_{1}(z_{1}) T_{2}(z_{2}) (...)$

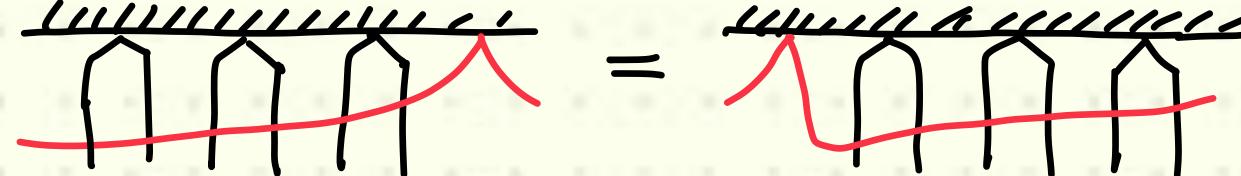
Compatibility with the RTT-relation $R_{12}(u-v) T_{1}(w) T_{2}(v) = T_{2}(v) T_{1}(w) R_{12}(u-v)$

10.00	15.01	1000	193	1000	101	10.00

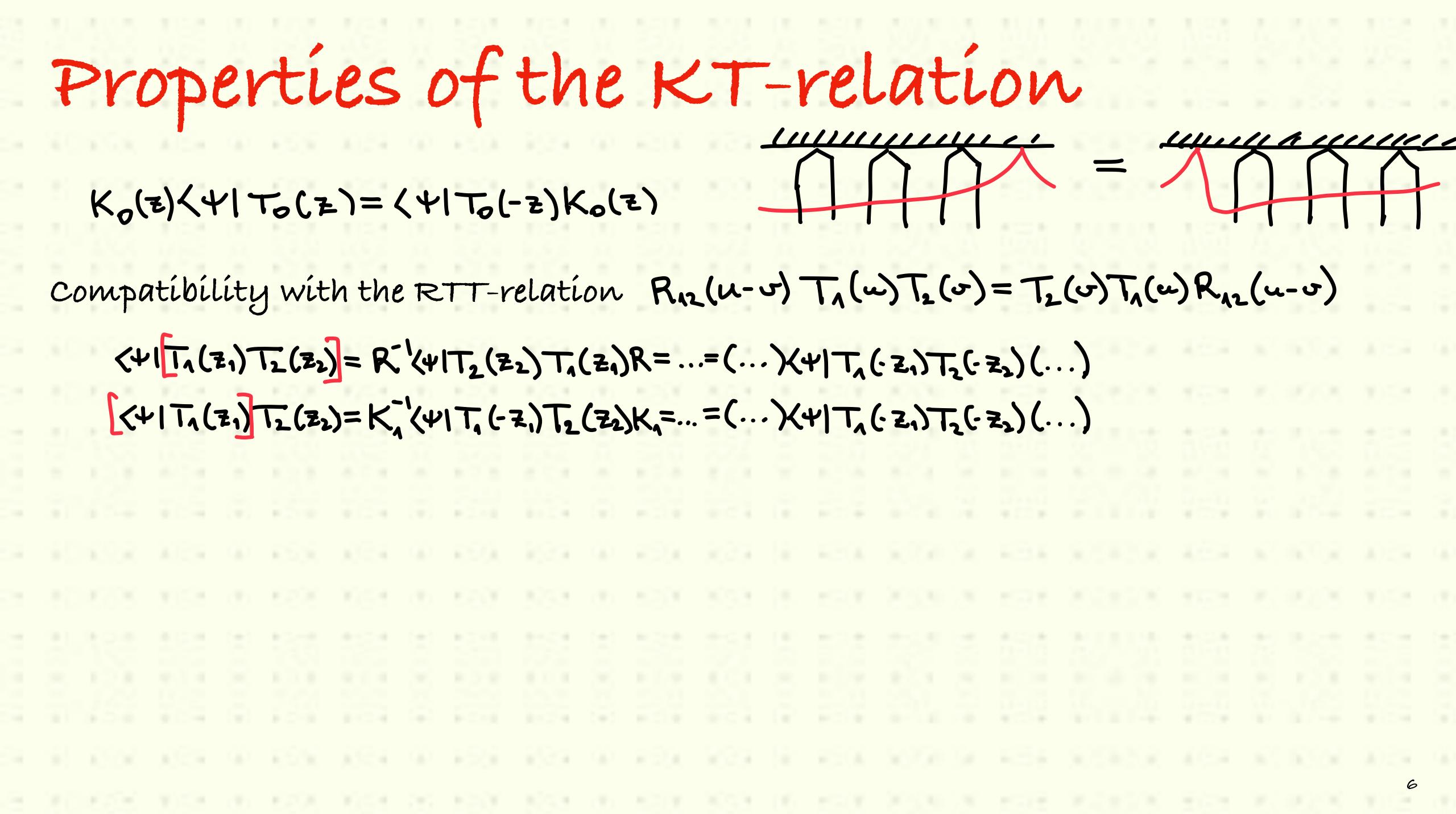


 $K_{0}(z)(4|T_{0}(z)) = (4|T_{0}(-z)K_{0}(z))$

 $\langle \Psi | T_{1}(z_{1}) T_{2}(z_{2}) = R \langle \Psi | T_{2}(z_{2}) T_{1}(z_{1}) R = ... = (...) \langle \Psi | T_{1}(\cdot z_{1}) T_{2}(\cdot z_{2}) (...)$ $(+|T_{1}(z_{1})T_{2}(z_{2})=K_{1}'(+|T_{1}(-z_{1})T_{2}(-z_{2})K_{1}=...=(...)K_{1}|T_{1}(-z_{1})T_{2}(-z_{2})(...)$



Compatibility with the RTT-relation $R_{12}(u-s) T_{1}(w) T_{2}(s) = T_{2}(s) T_{1}(w) R_{12}(u-s)$



 $K_{o}(z)(4|T_{o}(z)) = (4|T_{o}(-z)K_{o}(z))$

 $\langle \Psi | T_{4}(z_{1}) T_{2}(z_{2}) = R' \langle \Psi | T_{2}(z_{2}) T_{4}(z_{4}) R = ... = (...) \langle \Psi | T_{4}(\cdot z_{1}) T_{2}(\cdot z_{2}) (...)$ $(+|T_1(z_1)T_2(z_2)=K_1(+|T_1(-z_1)T_2(z_2)K_1=...=(...)X_1|T_1(-z_1)T_2(-z_2)(...)$

 \Rightarrow reflection equation $R_n(u \cdot \sigma)K_1(-u)R_n(u + \sigma)K_2(-\sigma)=K_2(-\sigma)R_n(u + \sigma)K_1(-u)R_n(u - \sigma)$

<u> Mullallelle</u>

- Compatibility with the RTT-relation $R_{12}(u-v) T_{1}(w) T_{2}(v) = T_{2}(v) T_{1}(w) R_{12}(u-v)$





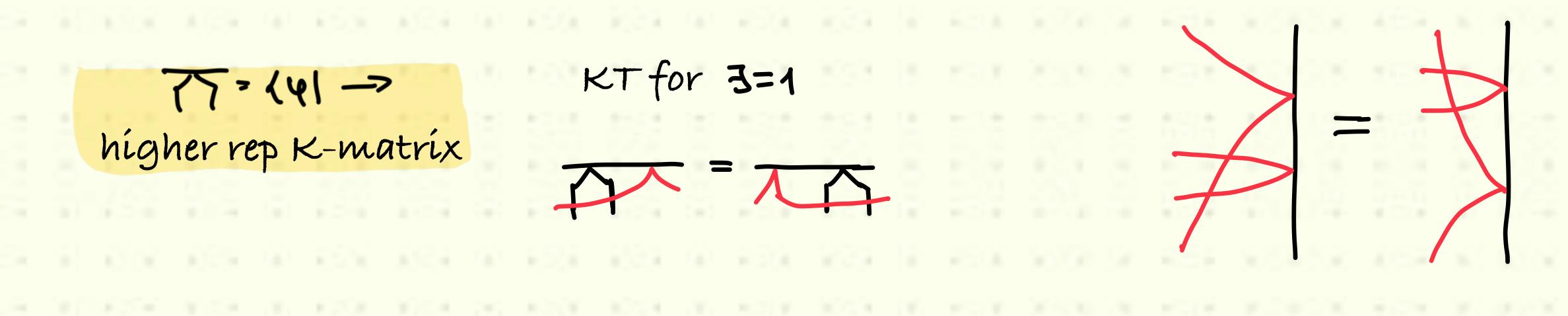
 $K_{0}(z)(4|T_{0}(z)) = (4|T_{0}(-z)K_{0}(z))$

 $\langle \Psi | T_{1}(z_{1}) T_{2}(z_{2}) = R' \langle \Psi | T_{2}(z_{2}) T_{1}(z_{1}) R = ... = (...) \langle \Psi | T_{1}(z_{1}) T_{2}(z_{2}) (...)$ $(+|T_{1}(z_{1})T_{2}(z_{2})=K_{1}'(+|T_{1}(-z_{1})T_{2}(z_{2})K_{1}=...=(...)X+|T_{1}(-z_{1})T_{2}(-z_{2})(...)$

KT for 3=1 フラーイターー

higher rep K-matrix

- Compatibility with the RTT-relation $R_{12}(u-v) T_{1}(w) T_{2}(v) = T_{2}(v) T_{1}(w) R_{12}(u-v)$
 - \Rightarrow reflection equation $R_n(u \cdot \sigma)K_1(-u)R_n(u + \sigma)K_2(-\sigma)=K_2(-\sigma)R_n(u + \sigma)K_1(-u)R_n(u \sigma)$





calculation of the Off-shell overlap (2,2) component of the KT-relation $K_{2,1}(z)(\psi|T_{1,2}(z) + K_{2,2}(z)(\psi|T_{2,2}(z) = \chi + |T_{2,1}(-z)K_{4,2}(z) + (\psi|T_{2,1}(-z)K_{2,2}(z))$ \mathcal{F}

calculation of the Off-shell overlap (2,2) component of the KT-relation $K_{2,1}(z)(\psi|T_{1,2}(z) + K_{2,2}(z)(\psi|T_{2,2}(z) = \chi + |T_{2,1}(-z)K_{1,2}(z) + \langle \psi|T_{2,1}(-z)K_{2,2}(z)$ Assuming Kinto we can express 441 The with 2+1722 or 2+1721 \mathcal{F}

calculation of the Off-shell overlap (2,2) component of the KT-relation $K_{2,1}(z)(\psi|T_{1,2}(z) + K_{2,2}(z)(\psi|T_{2,2}(z) = \chi + |T_{2,1}(-z)K_{4,2}(z) + (\psi|T_{2,1}(-z)K_{2,2}(z))$ Assuming K2, 70 we can express 41 The with 2+1722 or 2+1721

 \mathcal{F}

Creation diagonal annihilation

calculation of the Off-shell overlap

(2,2) component of the KT-relation

 $K_{2,1}(z)(\psi|T_{1,2}(z) + K_{2,2}(z)(\psi|T_{2,2}(z) = \chi + |T_{2,1}(-z)K_{4,2}(z) + (\psi|T_{2,1}(-z)K_{2,2}(z))$

Assuming Kinto we can express 41 The with 2+1722 or 2+1721 Creation diagonal annihilation

off-shell overlap

 $S(\overline{u}) = \lambda + |\overline{u}\rangle$

1										
-										
	10.000	$V \geq 0$	(\mathbf{v})	6.95	$Y \leq t$	20	120	1243	11	100

 \mathcal{F}

calculation of the Off-shell overlap

(2,2) component of the KT-relation

 $K_{2,1}(z)(\psi|T_{1,2}(z) + K_{2,2}(z)(\psi|T_{2,2}(z) = \chi + |T_{2,1}(-z)K_{4,2}(z) + \langle \psi|T_{2,1}(-z)K_{2,2}(z)$

Assuming $K_{2,1} \neq 0$ we can express $4 + 1 T_{12}$ with $2 + 1 T_{22}$ or $4 + 1 T_{21}$ Creation diagonal annihilation

off-shell overlap

 $S(u) = \lambda + |u]$

1										
-										
	10.000	$V \geq 0$	(\mathbf{v})	6.95	$Y \leq t$	20	120	1243	11	100

 $|\overline{u}\rangle = \frac{m}{|\overline{u}|} T_{12}(u_j)|0\rangle$

calculation of the Off-shell overlap

(2,2) component of the KT-relation

 $K_{2_{1}}(z)(\psi|T_{12}(z) + K_{22}(z)(\psi|T_{2_{12}}(z) = \chi+|T_{2_{1}}(-z)K_{42}(z) + (\psi|T_{2_{12}}(-z)K_{22}(z))$

Assuming $K_{2,1} \neq 0$ we can express $441T_{1,2}$ with $2+1T_{2,2}$ or $4+1T_{2,1}$ Creation diagonal annihilation

off-shell overlap

ら(む)=ん+1む7

1										
-										
	10.000	$V \geq 0$	(\mathbf{v})	6.95	$Y \leq t$	20	120	1243	11	100

 $|\overline{u}\rangle = \frac{n}{|\overline{u}|} T_{12}(u_j)|0\rangle$ $T_{i,i}(u) |0\rangle = \lambda_i(u) |0\rangle$ \mathcal{F}

Calculation of the Off-shell overlap

(2,2) component of the KT-relation

 $K_{2_{1}}(z)(\psi|T_{12}(z) + K_{22}(z)(\psi|T_{2_{12}}(z) = \chi + |T_{2_{1}}(-z)K_{42}(z) + (\psi|T_{2_{12}}(-z)K_{22}(z))$

Assuming Kinto we can express 41712 with 2+1722 or 2+1721 Creation diagonal annihilation

off-shell overlap

ら(む)=ん+しむう

 $S_{A}(iz, u) = \langle + | T_{u2}(z) | u \rangle =$

 $|u\rangle = \frac{m}{\sqrt{2}} T_{42}(u_j)|0\rangle \qquad T_{i_ji}(u_j)|0\rangle = \lambda_i(u_j)|0\rangle$ \mathcal{F}

Calculation of the Off-shell overlap

(2,2) component of the KT-relation

 $K_{2_{1}}(z)(\psi|T_{1,2}(z) + K_{2_{2}}(z)(\psi|T_{2_{1}}(z) = \chi + |T_{2_{1}}(-z)K_{4_{2}}(z) + (\psi|T_{2_{1}}(-z)K_{2_{2}}(z))$

Assuming $K_{2,1} \neq 0$ we can express $441T_{12}$ with $2+1T_{2,2}$ or $4+1T_{2,1}$ Creation diagonal annihilation

 \mathcal{F}

off-shell overlap $S(\overline{u}) = \lambda + |\overline{u}\rangle$

 $|\overline{u}\rangle = \frac{n}{|\overline{u}|} T_{12}(u_j)|_0\rangle \qquad T_{i,i}(u_j)|_0\rangle = \lambda_i(u_j)|_0\rangle$

 $S_{g}(i_{z_{1}}\bar{u}_{f}) = \langle \psi|T_{u_{2}}(z)|\bar{u}\rangle = \frac{K_{2,1}(z)}{K_{2,1}(z)} [\psi|T_{2,2}(-z)|\bar{u}\rangle - \langle \psi|T_{2,2}(z)|\bar{u}\rangle] + \frac{K_{4,2}(z)}{K_{2,1}(z)} \langle \psi|T_{2,2}(-z)|\bar{u}\rangle$

Calculation of the Off-shell overlap

(2,2) component of the KT-relation

 $K_{2_{1}}(z)(\psi|T_{1,2}(z) + K_{2_{2}}(z)(\psi|T_{2_{1}}(z) = \chi + |T_{2_{1}}(-z)K_{4_{2}}(z) + (\psi|T_{2_{1}}(-z)K_{2_{2}}(z))$

Assuming $K_{2,1} \neq 0$ we can express $441T_{12}$ with $2+1T_{2,2}$ or $4+1T_{2,1}$ Creation diagonal annihilation

off-shell overlap $S(\overline{u}) = \lambda + |\overline{u}\rangle$

 $S_{\lambda}(\{z,\bar{u}\}) = \sum(...)S_{\lambda}(\bar{w})$ 死こ {え-ころひん ¥ビッキ⊗

 $|\overline{u}\rangle = \frac{n}{|\overline{u}|} T_{12}(u_j)|_0\rangle \qquad T_{i,i}(u_j)|_0\rangle = \lambda_i(u_j)|_0\rangle$

 $S_{g}(i_{z_{1}}\bar{u}_{f}) = \langle \psi|T_{u_{2}}(z)|\bar{u}\rangle = \frac{K_{2,1}(z)}{K_{2,1}(z)} [\psi|T_{2,2}(-z)|\bar{u}\rangle - \langle \psi|T_{2,2}(z)|\bar{u}\rangle] + \frac{K_{4,2}(z)}{K_{2,1}(z)} \langle \psi|T_{2,2}(-z)|\bar{u}\rangle$

1) sum formula $S_{\bar{\lambda}}(\bar{u}) = \sum_{\bar{u}=\bar{u}_{\bar{u}}\bar{v}\bar{u}_{\bar{u}}} W(\bar{u}_{\bar{u}}(\bar{u}_{\bar{u}})\lambda_{\bar{u}}(\bar{u}_{\bar{u}})\lambda_{\bar{u}}(\bar{u}_{\bar{u}}))$

 $\lambda_{k}(\bar{u}) = T \lambda_{k}(u_{j})$



1) sum formula $S_{\bar{\lambda}}(\bar{u}) = \sum_{\bar{u}=\bar{u}_{I}\bar{v}\bar{u}_{I}} W(\bar{u}_{I}\bar{u}_{I}) \lambda_{I}(\bar{u}_{I}) \lambda_{2}(\bar{u}_{I})$

2) W($\bar{u}_{r}|\bar{u}_{r}$) = f(\bar{u}_{r},\bar{u}_{r}) Z(\bar{u}_{r}) Z(\bar{u}_{r})

 $\lambda_{k}(\bar{u}) = T \lambda_{k}(u_{j})$

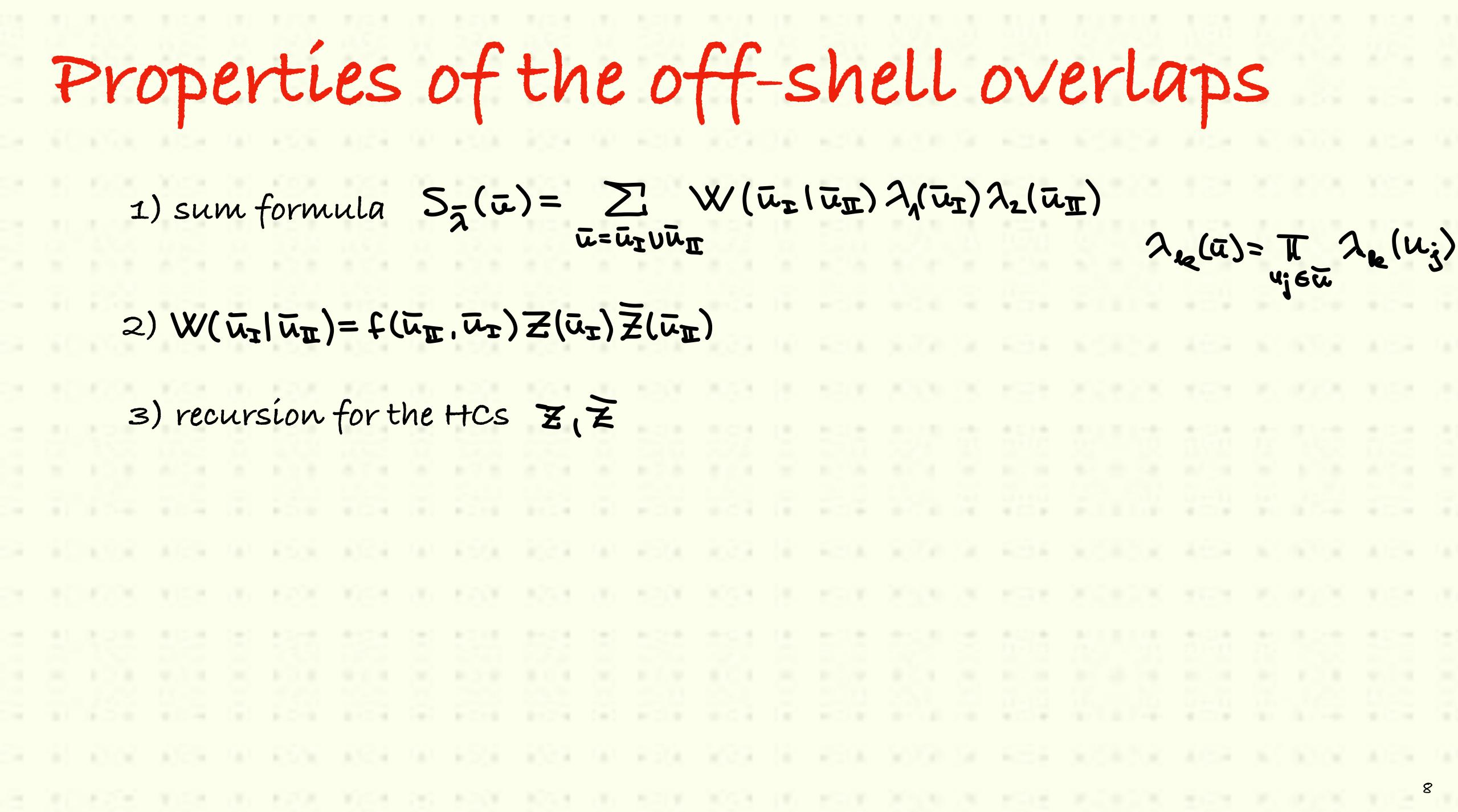


1) sum formula $S_{\bar{\lambda}}(\bar{u}) = \sum_{\bar{u}=\bar{u}_{I}\bar{v}\bar{u}_{I}} W(\bar{u}_{I}\bar{u}_{I}) \lambda_{I}(\bar{u}_{I}) \lambda_{2}(\bar{u}_{I})$

2) $W(\bar{u}_{I}|\bar{u}_{I}) = f(\bar{u}_{I},\bar{u}_{I})Z(\bar{u}_{I})\overline{Z}(\bar{u}_{I})$

3) recursion for the HCs Z,Z

 $\lambda_{k}(\bar{u}) = T \lambda_{k}(u_{j})$



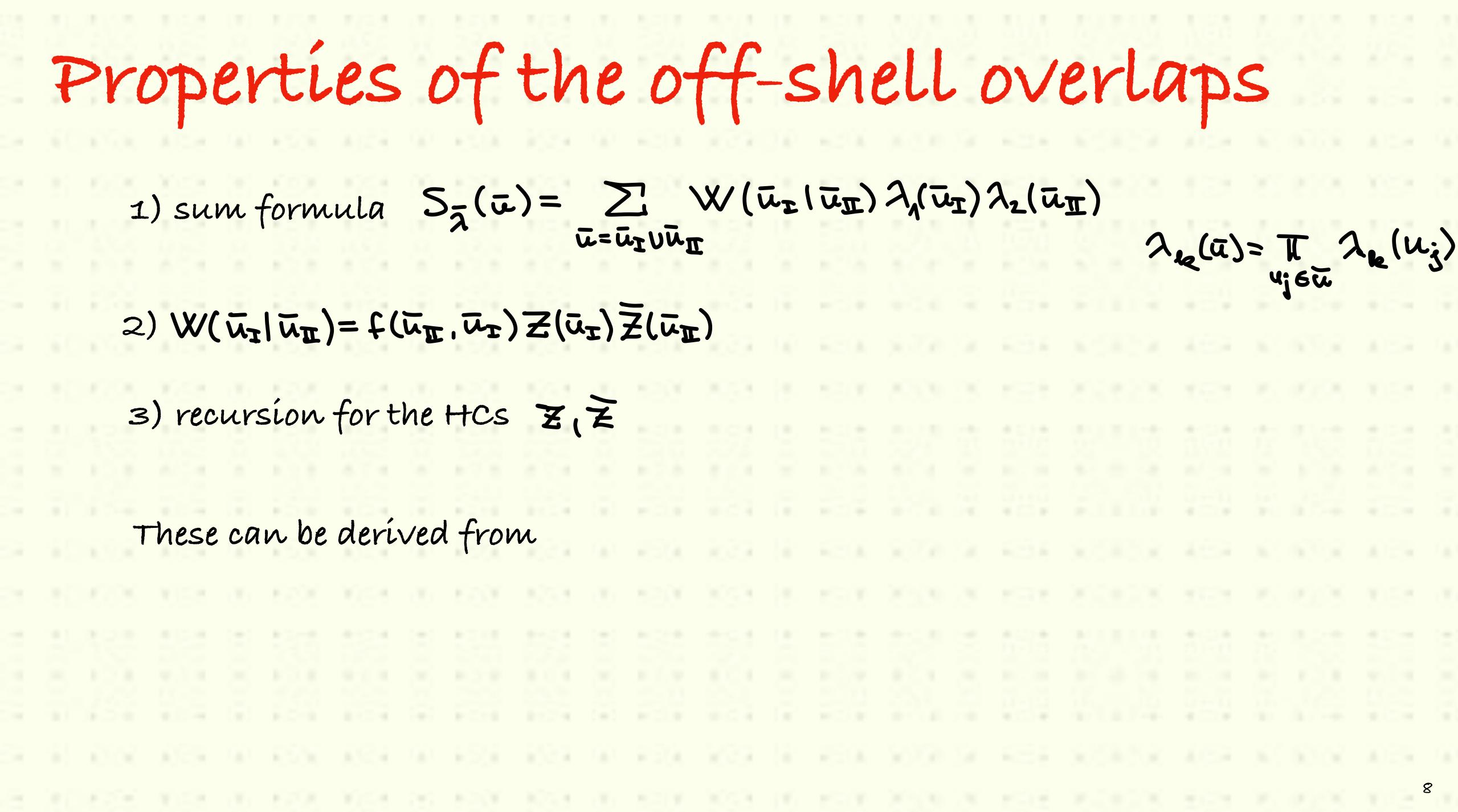
1) sum formula $S_{\bar{\lambda}}(\bar{u}) = \sum_{\bar{u}=\bar{u}_{I}\bar{v}\bar{u}_{I}} W(\bar{u}_{I}\bar{u}_{I}) \lambda_{I}(\bar{u}_{I}) \lambda_{2}(\bar{u}_{I})$

2) $W(\bar{u}_{I}|\bar{u}_{I}) = f(\bar{u}_{I},\bar{u}_{I})Z(\bar{u}_{I})\overline{Z}(\bar{u}_{I})$

3) recursion for the HCs Z,Z

These can be derived from

 $\lambda_{k}(\bar{u}) = T \lambda_{k}(u_{j})$



1) sum formula $S_{\bar{\lambda}}(\bar{u}) = \sum_{\bar{u}=\bar{u}_{I}} W(\bar{u}_{I} | \bar{u}_{I}) \lambda_{I}(\bar{u}_{I}) \lambda_{2}(\bar{u}_{I})$

2) $W(\bar{u}_{I}|\bar{u}_{I}) = f(\bar{u}_{I},\bar{u}_{I})Z(\bar{u}_{I})\overline{Z}(\bar{u}_{I})$

3) recursion for the HCs Z,Z

These can be derived from

 $\lambda_{k}(\bar{u}) = T \lambda_{k}(u_{j})$

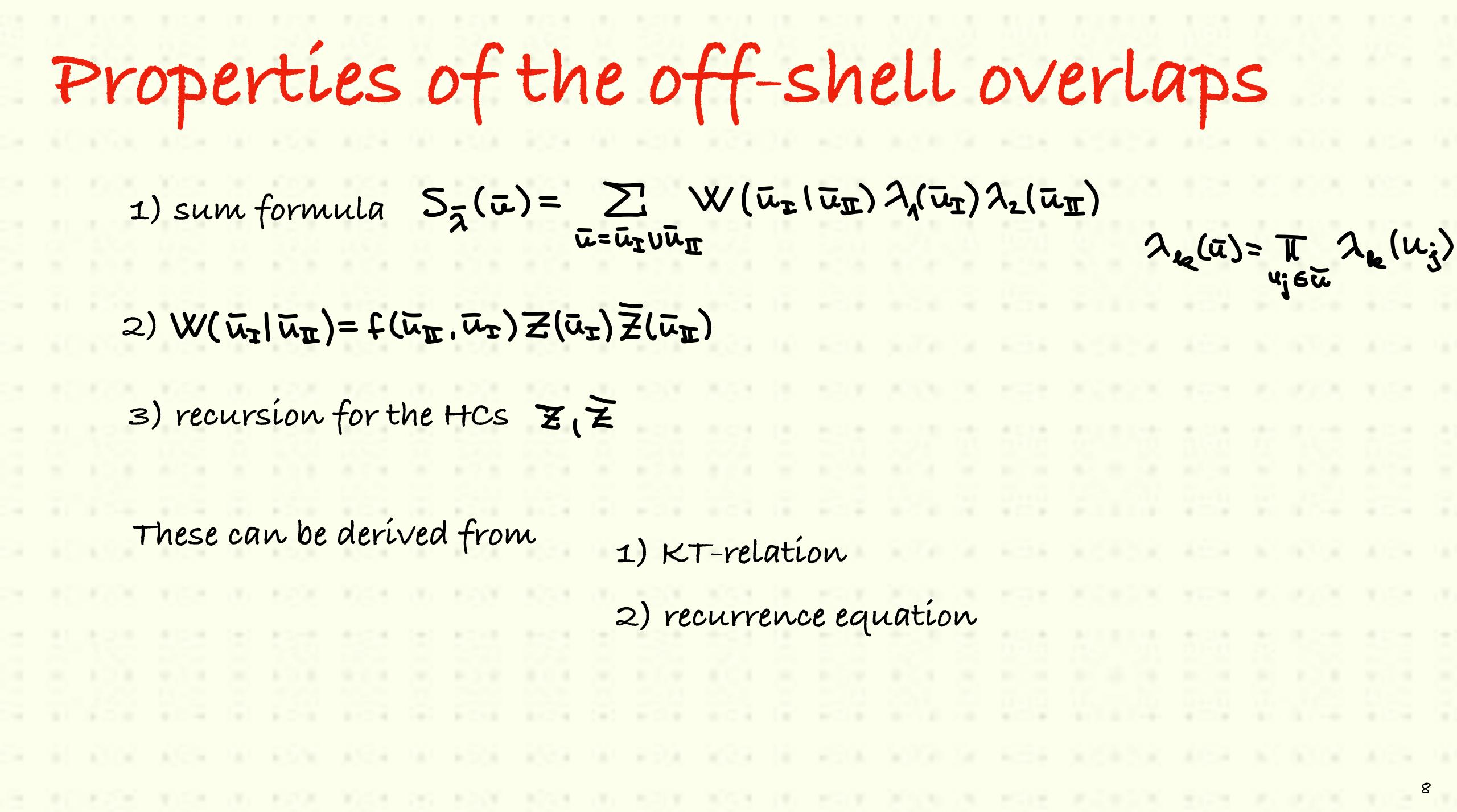
1) KT-relation



1) sum formula $S_{\bar{\lambda}}(\bar{u}) = \sum_{\bar{u}=\bar{u}_{I}} W(\bar{u}_{I}(\bar{u}_{I})\lambda_{I}(\bar{u}_{I})\lambda_{I}(\bar{u}_{I}))$

2) $W(\bar{u}_{I}|\bar{u}_{I}) = f(\bar{u}_{I},\bar{u}_{I})Z(\bar{u}_{I})Z(\bar{u}_{I})$ 3) recursion for the HCs Z,Z These can be derived from 1) KT-relation 2) recurrence equation

 $\lambda_{k}(\bar{u}) = T \lambda_{k}(u_{j})$

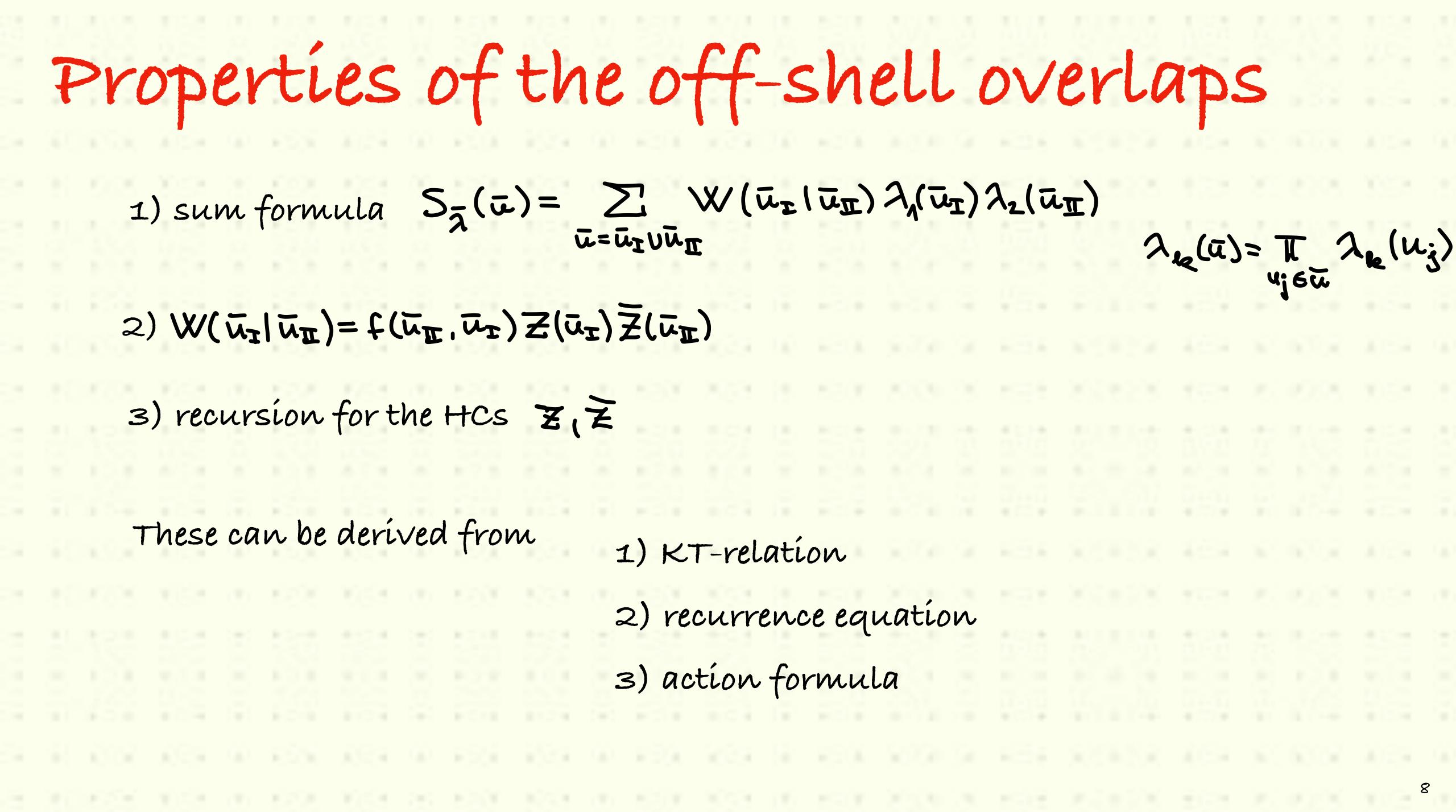


Properties of the off-shell overlaps

1) sum formula $S_{\bar{\lambda}}(\bar{u}) = \sum_{\bar{u}=\bar{u}_{\bar{u}}\bar{v}\bar{u}_{\bar{u}}} W(\bar{u}_{\bar{u}}(\bar{u}_{\bar{u}})\lambda_{\bar{u}}(\bar{u}_{\bar{u}})\lambda_{\bar{u}}(\bar{u}_{\bar{u}}))$

2) $W(\bar{u}_{I}|\bar{u}_{I}) = f(\bar{u}_{I},\bar{u}_{I})Z(\bar{u}_{I})Z(\bar{u}_{I})$ 3) recursion for the HCs Z,Z These can be derived from 1) KT-relation 2) recurrence equation 3) action formula

 $\lambda_{k}(\bar{u}) = T \lambda_{k}(u_{j})$

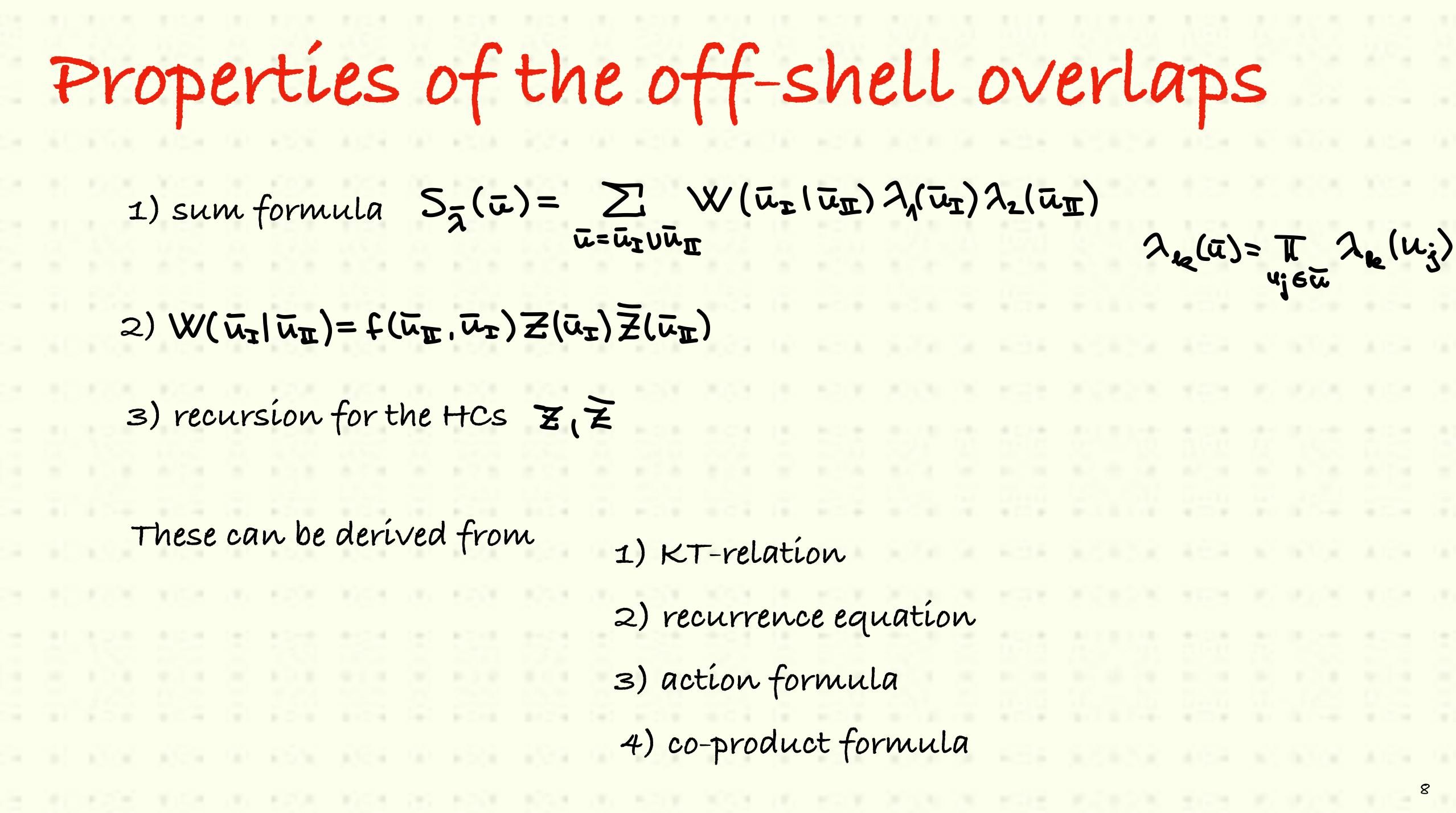


Properties of the off-shell overlaps

1) sum formula $S_{\bar{\lambda}}(\bar{u}) = \sum_{\bar{u}=\bar{u}_{\bar{u}}\bar{v}\bar{u}_{\bar{u}}} W(\bar{u}_{\bar{u}}(\bar{u}_{\bar{u}})\lambda_{\bar{u}}(\bar{u}_{\bar{u}})\lambda_{\bar{u}}(\bar{u}_{\bar{u}}))$

2) $W(\bar{u}_{I}|\bar{u}_{I}) = f(\bar{u}_{I},\bar{u}_{I})Z(\bar{u}_{I})Z(\bar{u}_{I})$ 3) recursion for the HCs Z,Z These can be derived from 1) KT-relation 2) recurrence equation 3) action formula 4) co-product formula

 $\lambda_{k}(\bar{u}) = T \lambda_{k}(u_{j})$



Properties of the off-shell overlaps

1) sum formula $S_{\bar{\lambda}}(\bar{u}) = \sum_{\bar{u}=\bar{u}_{\bar{u}}\bar{v}\bar{u}_{\bar{u}}} W(\bar{u}_{\bar{u}} | \bar{u}_{\bar{u}}) \lambda_{\bar{\lambda}}(\bar{u}_{\bar{u}}) \lambda_{\bar{\lambda}}(\bar{u}_{\bar{u}})$

2) $W(\bar{u}_{r}|\bar{u}_{r}) = f(\bar{u}_{r},\bar{u}_{r})Z(\bar{u}_{r})\overline{Z}(\bar{u}_{r})$

3) recursion for the HCs $Z_{1}Z$

These can be derived from

 $\lambda_{k}(\bar{u}) = T \lambda_{k}(u_{j})$

1) KT-relation $\langle \Psi | T_{A,2} \rightarrow \langle \Psi | T_{2,2} \& \langle \Psi | T_{2,n} \rangle$ 2) recurrence equation $|\{z,\overline{w}\}\rangle = T_{A,2}(z)|\overline{w}\rangle$ Ting(2) (2) = 2(...) (...) 3) action formula $|\overline{u}\rangle = \sum (...) |\overline{u}_{\mathrm{I}}\rangle \otimes |\overline{u}_{\mathrm{I}}\rangle^{(2)}$ 4) co-product formula



transfer matrix $\Upsilon(u) = \sum_{i=1}^{2} T_{i}$ g

On-shell limit transfer matrix $\langle \Psi | \Upsilon(u) = \langle \Psi | \Upsilon(-u)$ $\Upsilon(u) = \sum_{i=1}^{2} T_{i}$ g

transfer matrix $\Upsilon(u) = \sum_{i=1}^{2} T_{i}$

on-shell Bethe states

 $T(z)|\overline{u}\rangle = \mathcal{L}(z|\overline{u})|\overline{u}\rangle$

 $\langle \psi | \Upsilon(u) = \langle \psi | \Upsilon(-u)$ g

transfer matrix $\Upsilon(u) = \sum_{i=1}^{2} T_{i,i}$

> non-vaní on-shell o

on-shell Bethe states

くそしてくま

 $T(z)|u\rangle = f(z|u)|u\rangle$

11	=	(4)	176	-2)				
•			-	-				
is	hing	2						
		5						
<u>^</u>	'erla-	nc						
		23						
+	0							
+								
	2.2		1.11	C (3.5)	1.112	1000	110.1	2 C 12 C 13

On-shell limit

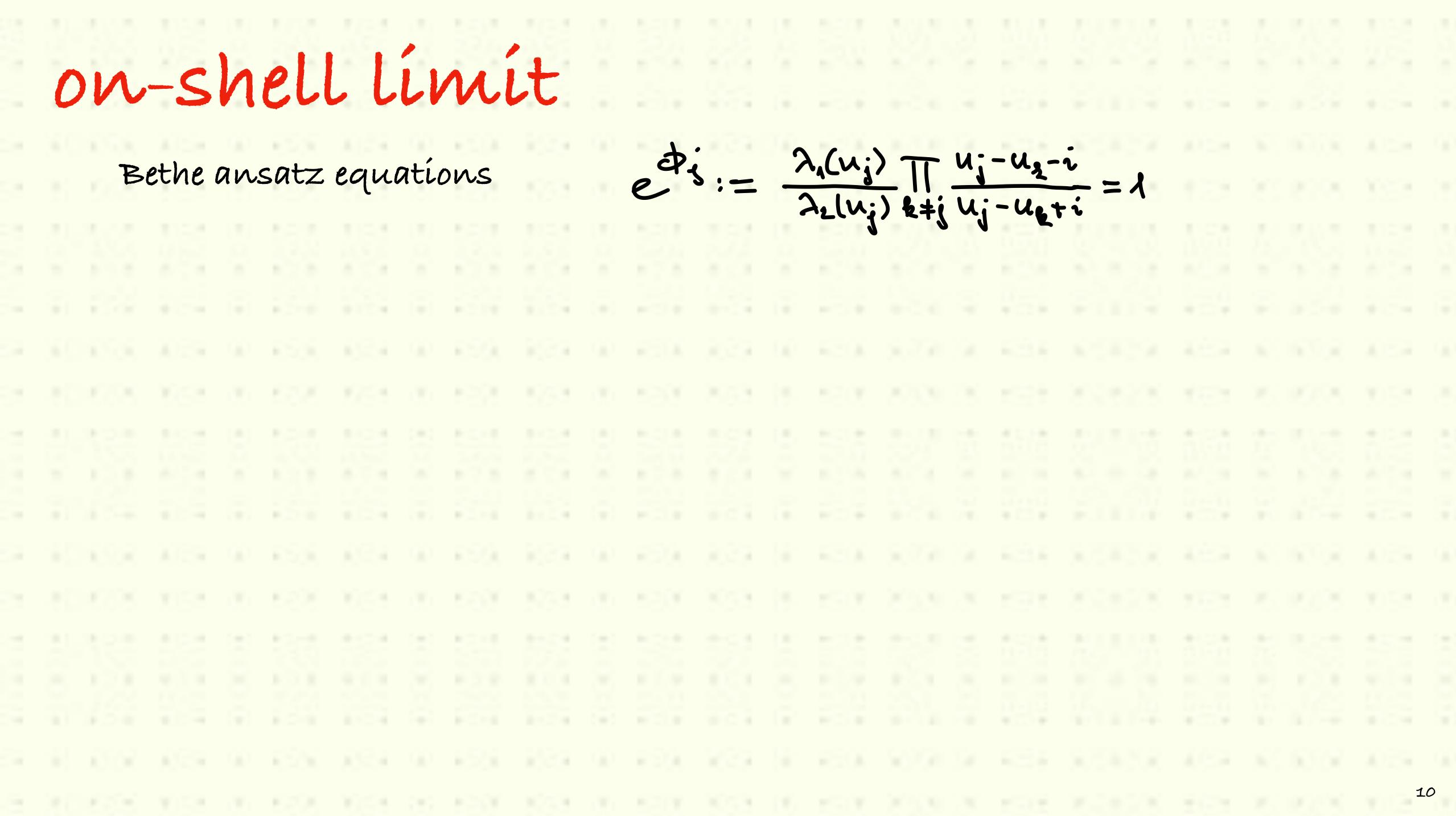
transfer matrix $\Upsilon(u) = \sum_{i,i}^{2} T_{i,i}$

> non-vanishing on-shell overlaps

on-shell Bethe states

$\langle \Psi | \Upsilon(u) = \langle \Psi | \Upsilon(-u)$ \rightarrow $\mathcal{L}(z \mid \overline{u}) = \mathcal{L}(-z \mid \overline{u})$

	T (z))しょう	= /	£(z1	む)10	i>		くそうすう				
								paír structure				
							600					
								$ \rightarrow \left\{ u_{j} \right\}_{j=1}^{r} \left\{ - u_{j} \right\}_{j=1}^{r} $				
								Q(z) = (-i) Q(-z)				
24									604			
1												



 $e^{\frac{1}{2}i} = \frac{\lambda_i(u_j)}{\lambda_i(u_j)} \prod_{k\neq j} \frac{u_j - u_{k-1}}{u_j - u_{k+1}} = 1$

on-shell limit $e^{\frac{1}{2}i} = \frac{\lambda_i(u_j)}{\lambda_i(u_j)} \prod_{\substack{\substack{i \neq j \\ i \neq j}}} \frac{u_j - u_{j-i}}{u_j - u_{j+i}} = 1$ Bethe ansatz equations Gaudín matrix Gjiz = Quz log \$ 10

Bethe ansatz equations

Gaudín matrix

Gjiz = Quzlog \$

on-shell norm

 $e^{\frac{1}{2}i} := \frac{\lambda_i(u_j)}{\lambda_i(u_j)} \prod_{\substack{k \neq j \\ i \neq j}} \frac{u_j - u_{k-1}}{u_j - u_{k+1}} = 1$ 10

Bethe ansatz equations

Gaudín matrix

Gjiz = Quz log Ø;

(uiu)~ detG

on-s	hell	norm
------	------	------

				pro	vec	x by
3						0
2						
24						
1.1						

 $e^{\varphi_{j}} := \frac{\lambda_{i}(u_{j})}{\lambda_{2}(u_{j})} \prod_{k \neq j} \frac{u_{j} - u_{k} - i}{u_{j} - u_{k} + i} = \lambda$

j Korepín's critería

Bethe ansatz equations

Gaudín matrix

Gjz= Quzlog Øj

on-shell norm

(ain)~ detG

ひ= む いむ-

pair structure

 $\frac{\lambda_i(u_j)}{\lambda_i(u_j)} \prod_{k\neq j} \frac{u_j - u_{k-1}}{u_j - u_{k+1}} = 1$

e :=

proved by Korepín'S critería

Bethe ansatz equations

Gaudín matrix

Gjiz = Quz log \$

on-shell norm

(uiu)~ detG

proved by Korepín's critería

10

pair structure

 $e^{\varphi_{j}} := \frac{\lambda_{i}(u_{j})}{\lambda_{i}(u_{j})} \prod_{k\neq j} \frac{u_{j} - u_{k} - i}{u_{j} - u_{k} + i} = 1$

 $\overline{u} = \overline{u}^{\dagger} \vee \overline{u}^{-} \qquad G_{j,k}^{\dagger} = [\partial_{u_{k}} \pm \partial_{u_{k}}] \log \phi_{j}^{\dagger}$

Bethe ansatz equations

Gaudín matrix

Gjiz = Quz log \$

on-shell norm

(aiu)~ detG

pair structure

 $det G = det G^{+} det G$

factorisation

 $e^{\varphi_{j}} := \frac{\lambda_{i}(u_{j})}{\lambda_{1}(u_{j})} \prod_{\substack{u_{j} - u_{2} - i \\ \lambda_{2}(u_{j})}} \prod_{\substack{u_{j} - u_{2} + i \\ u_{j} - u_{2} + i}} = 1$

proved by Korepín's critería

$\overline{u} = \overline{u}^{\dagger} \vee \overline{u}^{-} \qquad G_{\underline{j},\underline{v}}^{\pm} = [\partial_{u\underline{v}} \pm \partial_{u\underline{v}}] \log \varphi_{\underline{j}}^{\dagger}$

Bethe ansatz equations

Gaudín matrix

Gjiz = Quz log \$

on-shell norm

(aiu)~ detG

pair structure

factorisation

 $det G = det G^{+} det G$

Korepín's critería

 $e^{\varphi_{j}} := \frac{\lambda_{i}(u_{j})}{\lambda_{1}(u_{j})} \prod_{\substack{u_{j} - u_{2} - i \\ \lambda_{2}(u_{j})}} \prod_{\substack{u_{j} - u_{2} + i \\ u_{j} - u_{2} + i}} = 1$

proved by Korepín'S critería

$\overline{u} = \overline{u}^{\dagger} \vee \overline{u}^{-} \qquad G_{\underline{j},\underline{v}}^{\pm} = \left[\partial_{u\underline{v}} \pm \partial_{u\underline{v}} \right] \log \varphi_{\underline{j}}^{\dagger}$

Bethe ansatz equations

Gaudín matrix

Gjiz = Quz log \$

on-shell norm

<ūıū)∼ detG

proved by Korepín's critería

pair structure

factorisation

 $det G = det G^{+} det \overline{G}$

Korepín's crítería \longrightarrow $(4)u^7 \sim det G^4$

 $e^{\varphi_{j}} := \frac{\lambda_{i}(u_{j})}{\lambda_{1}(u_{j})} \prod_{\substack{u_{j} - u_{2} - i \\ \lambda_{2}(u_{j})}} \prod_{\substack{u_{j} - u_{2} + i \\ u_{j} - u_{2} + i}} = 1$

$\overline{u} = \overline{u}^{\dagger} \vee \overline{u}^{-} \qquad G_{\underline{j},\underline{v}}^{\pm} = [\partial_{u\underline{v}} \pm \partial_{u\underline{v}}] \log \varphi_{\underline{j}}^{\dagger}$

Normalized on-s $\frac{\langle \Psi | \overline{u} \rangle}{\langle \overline{u} | \overline{u} \rangle} = \frac{\mathcal{F}(\overline{u})}{\int \frac{d d d G}{d \sigma}}$

이는 것이 많이 좀 가지 않는 것이 같이 같이 많이 많이 많이 많이 많이 많이 많이 많이 했어?

	rel		01	lerl	.01	DS		
					124			
r.	10.0	$\{ \psi \}_{i \in \mathbb{N}}$	$g \sim 10^{-1}$	1000	r > 2	1000	100	10.00



Normalized on-s

 $\frac{\langle \Psi | \overline{u} \rangle}{\langle \overline{u} | \overline{u} \rangle} = \mathcal{F}(\overline{u}) \int_{\overline{u}} \frac{d d G}{d d G}$

universal part det det depends or

			01			S		
n	the	Bet	the st	tate				
1	10.0	141	$\mathcal{T} = \mathcal{T}$	1000	1.12	1202	11.11	1000





								lerl			
		へる	<u>1 u7</u> u.u7	=	∓(元	Jet C					
	uníve							tate			
								b.s.			
2											



Normalized on-s

 $\frac{\langle \Psi | \overline{u} \rangle}{\langle \overline{u} | \overline{u} \rangle} = \frac{\mathcal{F}(\overline{u})}{\int \frac{d}{d} \overline{c}}$



boundary part $\mp(\overline{u}) = \underbrace{\pi}_{f} \mp(u_{f}) de$

100	115.8	1.91	622	1.5.5	121	1200	10.00	1.91	1.11

sl	nel		01	le	rl	.a1		
on	, the	Bet	the st	tate				
deț	sends	s oi	n the	e b.s.	3			
OVE	eauín	Jal	ently	y Kl	'u)			
			100 million (100 million)					



Normalized on-s

 $\frac{\langle \Psi | \overline{u} \rangle}{\langle \overline{u} | \overline{u} \rangle} = \mathcal{T}(\overline{u}) \int_{\overline{u}}^{\overline{u}} \frac{d d c^{\dagger}}{\sqrt{u}}$







general solution of the reflection equ

$$K(u) = \frac{a}{b} + A = \begin{pmatrix} \frac{a}{b} + b_{1} \\ \frac{b}{b} \\ \frac{b}{b} \end{pmatrix}$$

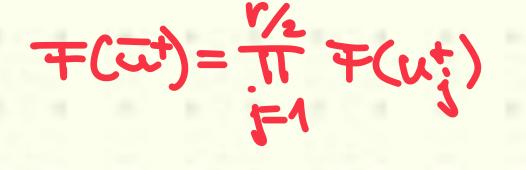
A2 = 81

chell mierlanc	
on the Bethe state	
depends on the b.S.	
anation	
biz	
æ -641	
8=-detA	







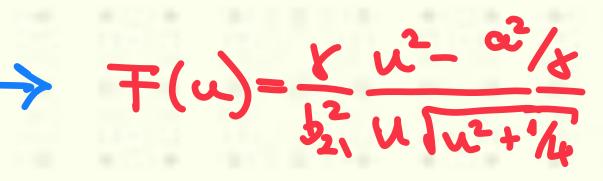


Normalized on-shell overlaps $\frac{\langle \Psi | \overline{u} \rangle}{\langle \overline{u} | \overline{u} \rangle} = \mp (\overline{u}) \int_{\frac{d}{d}\overline{u}} \frac{d}{\overline{u}}$ universal part det det depends on the Bethe state boundary part $\mp(\overline{u}) = \frac{\pi}{\pi} \mp(u_{1})$ depends on the b.S. or equivalently K(w) general solution of the reflection equation

a - 6 11

$$K(u) = \frac{a}{b} + A = \begin{pmatrix} \frac{a}{b} + b_{1} \\ \frac{b}{b} \\ \frac{b}{b} \end{pmatrix}$$

A2 = 81



8=-detA



			24								
		にい		ū	0	gl(:					
						2)	- 1				
						100					
						202					
						-					
		ミンコ		ū ⁴ ū ²		gl(
		-la',i		- ū 3	-0-0	N)					
		x²,		•••	-0						
		., īc ^{N-} 'S	636	ūn-1	-0						

Generalisation to gl(N) spin chains 이 승규에는 비교적 이번 승규에는 비교적 이번 승규에 가지 않고 한 것이에 가지 않고 한 바라에 비교적 것이 없다. two types of KT-relations 13

24										
	10.00	115.0	191	2.272	1.5.5	125	100	10.00	191	2011

Generalisation to gl(N) spin chains

two types of KT-relations

non-crossed K(u) (4) = (4) (-u) K(u)Crossed $K(u)(t)(\tau(u) = (t)(\tau(u)))$ 13

Generalisation to gl(N) spin chains

non-crossed $K(u) \downarrow \downarrow T(u) = \downarrow \downarrow \downarrow (-u) K(u)$

two types of KT-relations

Crossed $K(w) \langle \psi | T(w) = \chi \psi | \hat{T}(-w) K(w)$

inverse monodromy matrix

	 1128	w.	69.	1064	20	120	12.2	 507	10.0	11	2017	200	:03	1.1	28	3.03	7	1	1	13

 $\widehat{\tau}^{t}(\omega) T(\omega) = 4$

Generalisation to gl(N) spin chains

two types of KT-relations

Crossed

ínverse monodromy matrix

compatibility conditions

- non-crossed K(u) (4) = (4) (-u) K(u)
 - $K(u) \langle \psi | T(u) = \langle \psi | \hat{T}(-u) K(u)$
 - $\widehat{T}^{t}(\omega) T(\omega) = 4$
- $R_{12}(u-\sigma)K_{(-u)}R_{n}(u+\sigma)K_{2}(\sigma) = K_{2}(\sigma)R_{n}(u+\sigma)K_{1}(-u)R_{n}(u-\sigma)$ $R_{12}(u-v)K_{(-u)}\overline{R_{n}(u+v)}K_{2}(v) = K_{2}(v)\overline{R_{n}(u+v)}K_{(-u)}R_{n2}(u-v)$
- 13



Symmetries and pair structures

Non-crossed K-matrices



Symmetries and pair structures Non-crossed K-matrices K(m)= = 1 + A A²=1



Symmetries and pair structures K(n)= = + A $A^2 = 1$ Non-crossed K-matrices A~ diag($+,+,\dots,+,-,-,\dots,-$)



Symmetries and pair structures

Non-crossed K-matrices

 $K(w) = \frac{2}{2}I + A$ $A^2 = I$

residual symmetry

 $A \sim diag(+,+,\dots,+,-,-,\dots,)$ $g(M) \oplus g(N-M)$

1000	11	$g \leq 1/\ell$	2010/01/01	t > 2	1000	3.03	10000



Non-crossed K-matrices

			Yest in t														
			Crossed	l K-matri	ces												
2																	
2																	
	10	100	$V := \{v_1, \dots, v_n\} \in \mathbb{R}$	20. XXX 23	1000	1.253	(m. 823)	v - 10000	11	100	1000	$(-1)^{-1}$	1000	100	(\mathbf{r}_{i})	0.00	

 $A^2 = 4$ K(w)= = + A

residual symmetry

A~ diag(+,+,...,+,-,-,...) $g(M) \oplus g(N-M)$



Non-crossed K-matrices

Crossed K-matrices

1									
	1000	10.52	0.000	1053	121	120	12.53	11	500

residual symmetry K(い)= 佘1 + A $A^2 = 4$

 $A \sim diag(+,+,\dots,+,-,-,\dots,-)$ $g(M) \oplus g(N-M)$

 $\vee^{t} = \pm \vee$ K(w) = VNAMES AND ADDRESS AND ADDRES



Non-crossed K-matrices

Crossed K-matrices

1									
	1000	10.52	0.000	1053	121	120	12.53	11	500

residual symmetry $A^2 = 4$ K(w)= = + A

 $A \sim diag(+,+,\dots,+,-,-,\dots,-)$ $g(M) \oplus g(N-M)$

O(N) $\vee^{t} = \pm \vee$ K(w) = VSP(N)

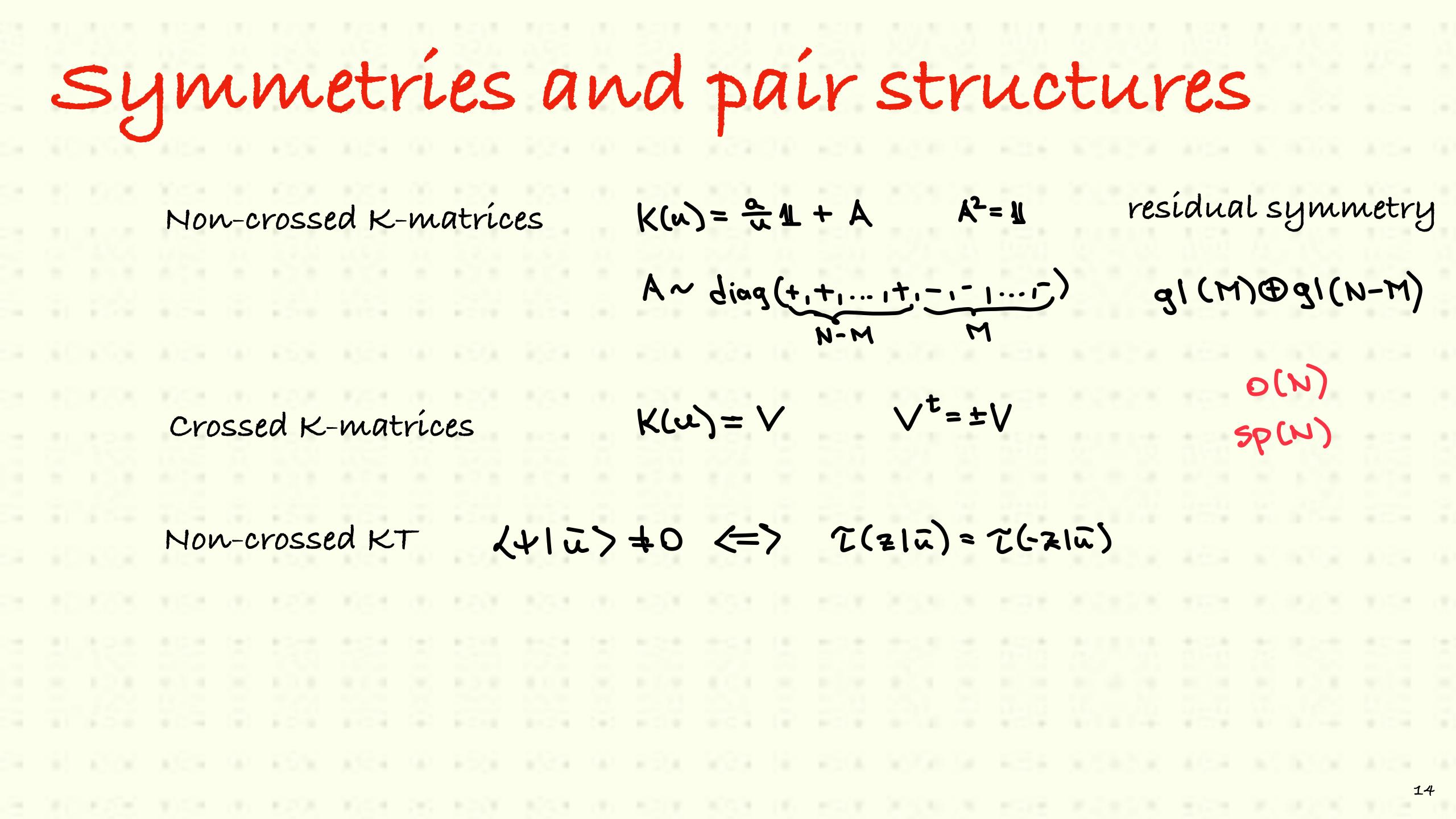


Non-crossed K-matrices

Crossed K-matrices

Non-crossed KT 人112>キ0 (=>

- resídual symmetry K(w)= 유1+ A $A^2 = 4$
- $A \sim diag(+,+,\dots,+,-,-,\dots,-)$
- K(u)=V $\vee^{t} = \pm V$
- て(えし)= て(えし)



0(N)

SP(N)

Symmetries and pair structures

Non-crossed K-matrices

Crossed K-matrices

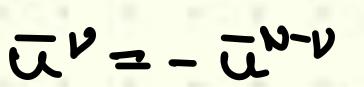
Non-crossed KT 人1112 +0

achiral pair structure

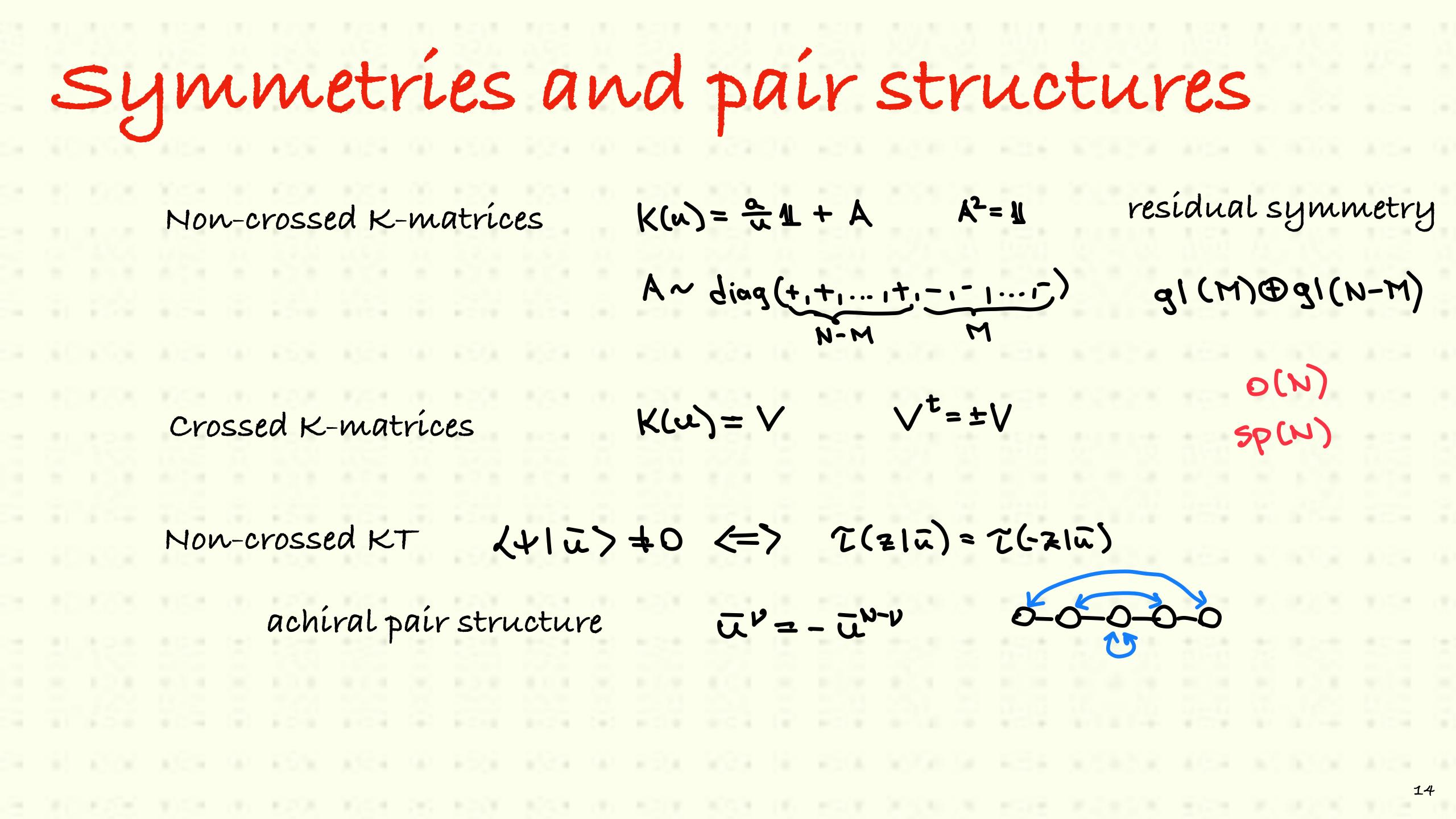
- residual symmetry K(い)= 유1+ A $A^2 = 4$
- A~ diag(+,+,...,+,-,-,....) N-M M
- K(w)=V $\vee^{t} = \pm \vee$

0(N) Sp(N)

 $\mathcal{I}(z|\overline{u}) = \mathcal{I}(-z|\overline{u})$







Symmetries and pair structures

Non-crossed K-matrices

Crossed K-matrices

Non-crossed KT 人1112 +0

achiral pair structure

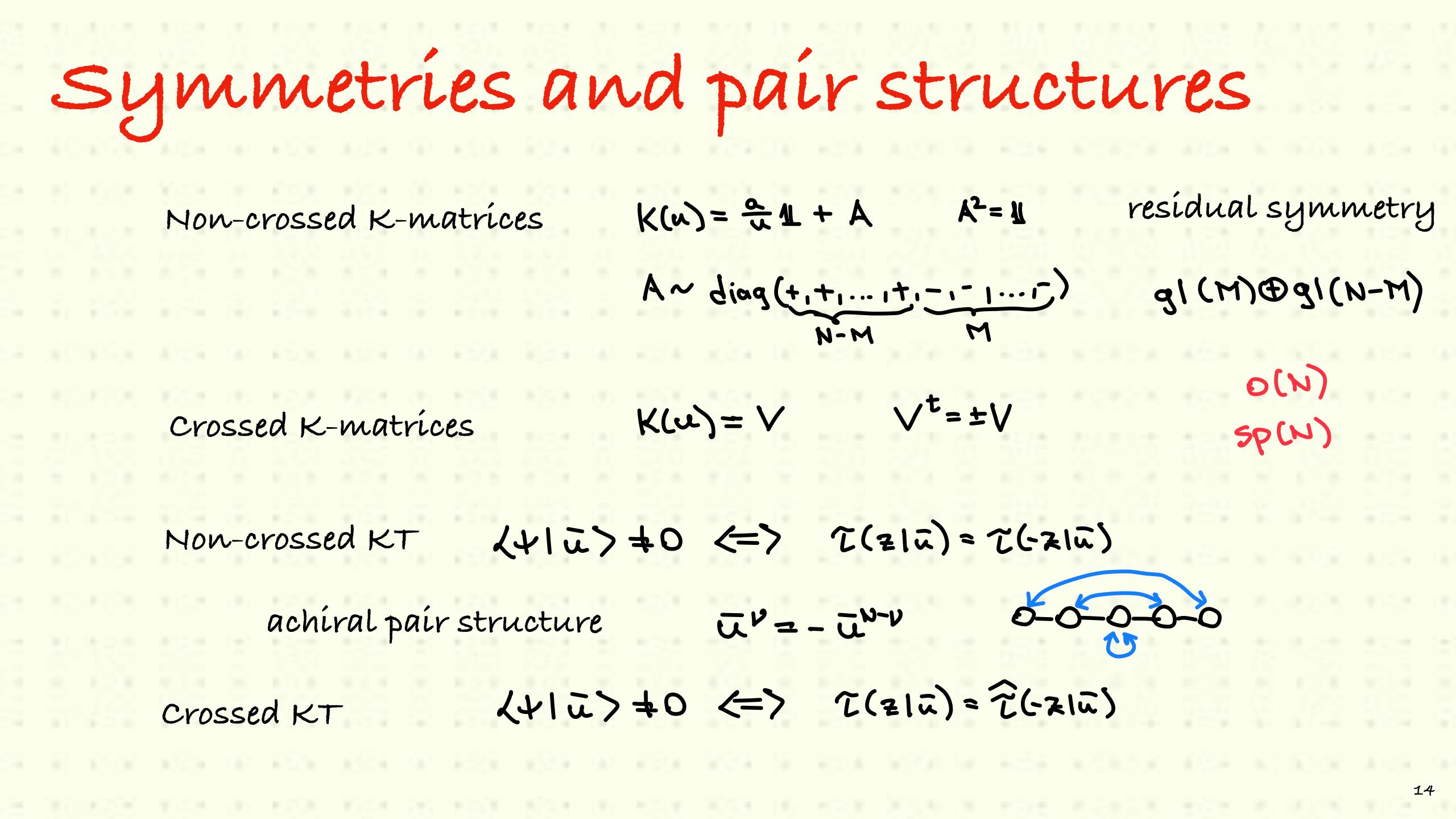
- residual symmetry K(い)= 유1+ A $A^2 = 4$
- A~ diag(+,+,...,+,-,-,....) N-M M
- K(w)=V $\vee^{t} = \pm \vee$

0(N) Sp(N)

- $\mathcal{I}(z|\overline{u}) = \mathcal{I}(-z|\overline{u})$



Crossed KT $(41\pi) \neq 0 \iff \mathcal{I}(21\pi) = \widehat{\mathcal{I}}(-21\pi)$



Symmetries and pair structures

Non-crossed K-matrices

Crossed K-matrices

Non-crossed KT 人1112 +0

achiral pair structure

chiral pair structure $\overline{u}^{\prime} = -\overline{u}^{\prime}$

- residual symmetry K(い)= 유1+ A $A^2 = 4$
- A~ diag(+,+,...,+,-,-,....) N-M M
- K(w)=V $\vee^{t} = \pm \vee$

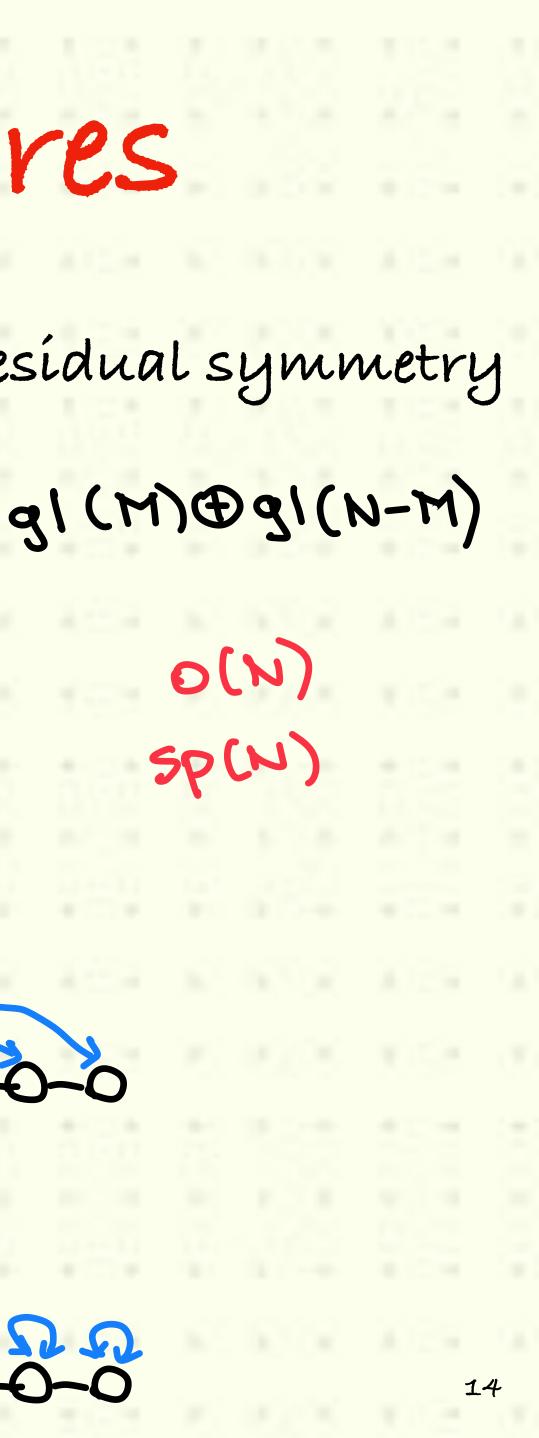
0(N) Sp(N)

- $\mathcal{I}(z|\overline{u}) = \mathcal{I}(-z|\overline{u})$



Crossed KT $(41\pi) \neq 0 \iff \mathcal{I}(21\pi) = \widehat{\mathcal{I}}(-21\pi)$

19 20 20 20



off-shell overlaps

List of criteria for off-shell overlaps

15

$S_{\chi}(\bar{u}) = \sum W(\bar{u}_{r} | \bar{u}_{T}) \prod_{\mu=1}^{N-1} \lambda_{\nu}(\bar{u}_{r}) \lambda_{\mu_{\mu}}(\bar{u}_{T})$



Off-shell overlaps

List of criteria for off-shell overlaps

1) KT-relation: creation to annihilation

 $S_{\chi}(\bar{u}) = \sum W(\bar{u}_{r}|\bar{u}_{r}) \prod_{\mu=1}^{N-1} \lambda_{\nu}(\bar{u}_{r}') \lambda_{\mu_{\mu}}(\bar{u}_{r}')$

ation (41 This > 241 The R>1

1	$\{ i, j \in I \}$	11	$g \leq 1/\ell$	20000	$t \in \{0, 0\}$	1000	$\widetilde{\mathcal{T}} \subset \mathcal{T}$	1000



Off-shell overlaps

List of criteria for off-shell overlaps

1)KT-relation: creation to annihilation <+1 This > 2+1 There 2>1

2) recurrence formula $|\{z, \bar{u}\}, \bar{u}^2, ... \rangle = Z(..., T_{ij}(z)|\bar{u}', \bar{v}^2, ... \rangle$

$S_{\chi}(\bar{u}) = \sum W(\bar{u}_{I} | \bar{u}_{I}) \prod_{j=1}^{N-1} \lambda_{j}(\bar{u}_{I}) \lambda_{m_{j}}(\bar{u}_{I})$



off-shell overlaps

List of criteria for off-shell overlaps 1)KT-relation: creation to annihilation <+1 This >> <+1 The 2>1 2) recurrence formula $\{z,\bar{u}\},\bar{u}^2,...\}=\mathbb{Z}(...)T_{4,j}(z)|\bar{u}',\bar{v}^2,...\}$ $T_{i,j}(z)|u\rangle = Z(...)|w\rangle$ 3) action formula

 $S_{\chi}(\bar{u}) = \sum W(\bar{u}_{I} | \bar{u}_{I}) \prod_{\nu=1}^{N-1} \lambda_{\nu}(\bar{u}_{I}^{\nu}) \lambda_{\nu_{1}}(\bar{u}_{I}^{\nu})$

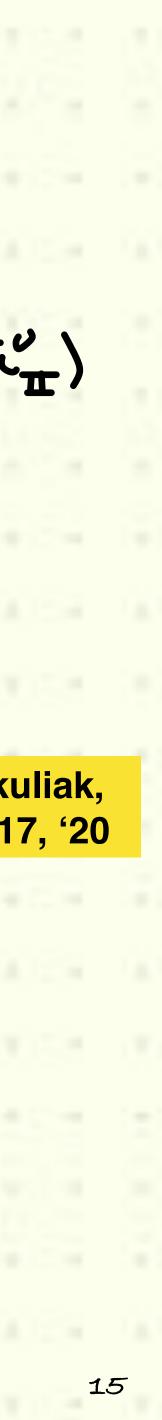


off-shell overlaps List of criteria for off-shell overlaps 1)KT-relation: creation to annihilation 2) recurrence formula з)action formula 4) co-product formula 15

 $S_{\chi}(\bar{u}) = \sum W(\bar{u}_{\pi} | \bar{u}_{\pi}) \prod_{\nu=1}^{N-1} \lambda_{\nu}(\bar{u}_{\pi}^{\nu}) \lambda_{\nu_{\eta}}(\bar{u}_{\pi}^{\nu})$ <+1 This >> <+1 The 2>1 $\{z,\bar{u}\},\bar{u}^2,...\}=Z(...)T_{ij}(z)|\bar{u}',\bar{v}^2,...\}$ $T_{i,j}(z)(i,z) = \sum_{i=1}^{\infty} (...)(i,z)$ $|\bar{u}\rangle = \sum (...) |\bar{u}_{r}\rangle^{(n)} \otimes |\bar{u}_{r}\rangle^{(2)}$



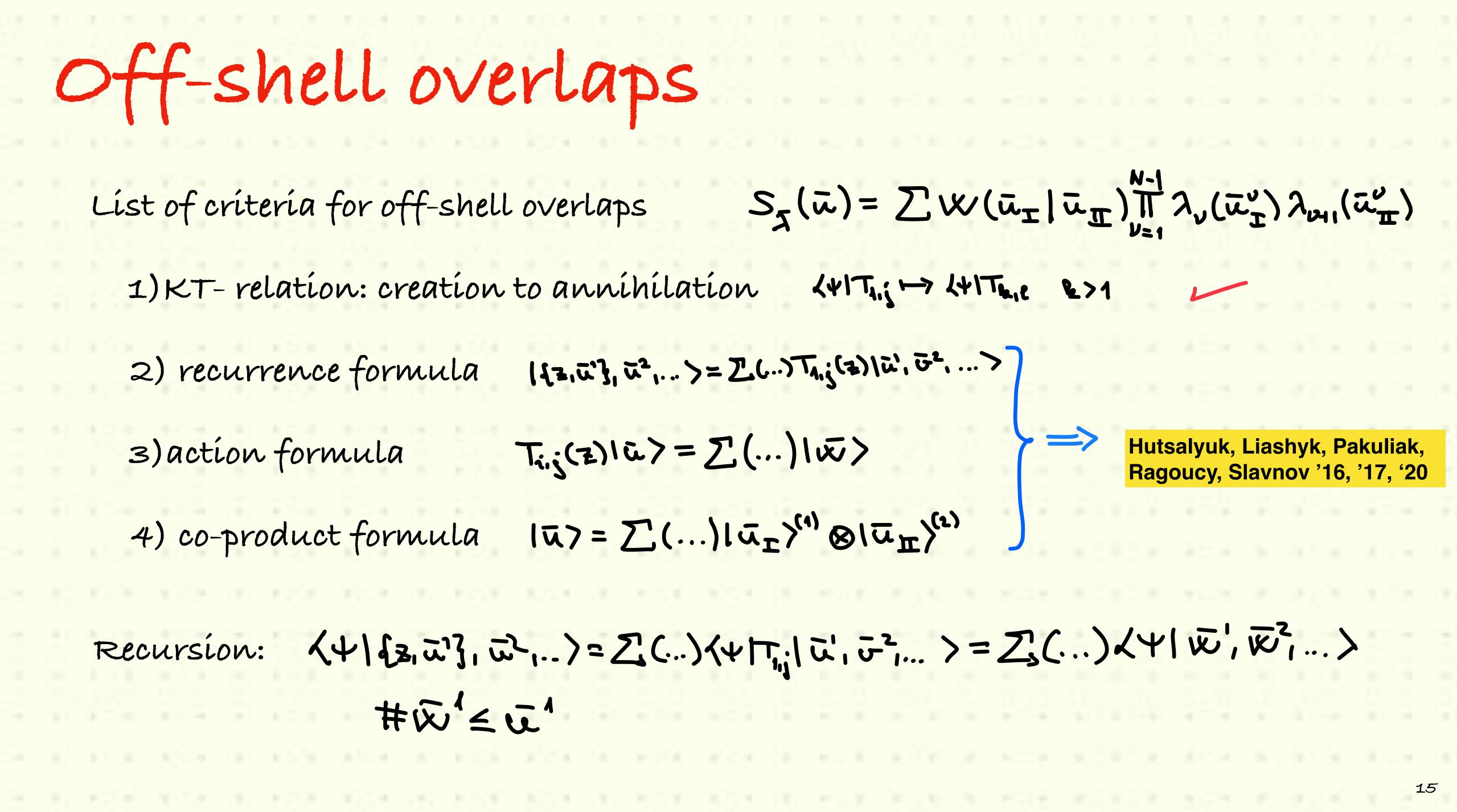
off-shell overlaps $S_{\chi}(\bar{u}) = \sum W(\bar{u}_{I} | \bar{u}_{I}) \prod_{\nu=1}^{N-1} \lambda_{\nu}(\bar{u}_{I}^{\nu}) \lambda_{\nu_{1}}(\bar{u}_{I}^{\nu})$ List of criteria for off-shell overlaps 1)KT-relation: creation to annihilation <+ ITA > + ITA , E 2>1 $\{z,\bar{u}^{2}\},\bar{u}^{2},...\}=Z(...)T_{4,j}(z)|\bar{u}^{1},\bar{v}^{2},...\}$ 2) recurrence formula $T_{i,j}(z)(i,z) = Z(...)(i,z)$ з)action formula Hutsalyuk, Liashyk, Pakuliak, Ragoucy, Slavnov '16, '17, '20 $|\overline{u}\rangle = \sum (...) |\overline{u}_{r}\rangle^{(n)} \otimes |\overline{u}_{m}\rangle^{(2)}$ 4) co-product formula 15



off-shell overlaps List of criteria for off-shell overlaps 1)KT-relation: creation to annihilation 2) recurrence formula $T_{i,j}(z)|u\rangle = \sum_{i=1}^{\infty} (...)|w\rangle$ з)action formula 4) co-product formula Recursion: $\#\omega' \leq \omega'$

- $S_{\chi}(\bar{u}) = \sum W(\bar{u}_{r}|\bar{u}_{r}) \prod_{\nu=1}^{N-1} \lambda_{\nu}(\bar{u}_{r}^{\nu}) \lambda_{\nu_{n}}(\bar{u}_{r}^{\nu})$
- <+ IT + + + The e 2>1
- $\{z,\bar{u}\},\bar{u}^{2},...\}=Z(...)T_{4,j}(z)|\bar{u}',\bar{v}^{2},...\}$
- $|\overline{u}\rangle = \sum (...) |\overline{u}_{r}\rangle^{(n)} \otimes |\overline{u}_{m}\rangle^{(n)}$

Hutsalyuk, Liashyk, Pakuliak, Ragoucy, Slavnov '16, '17, '20

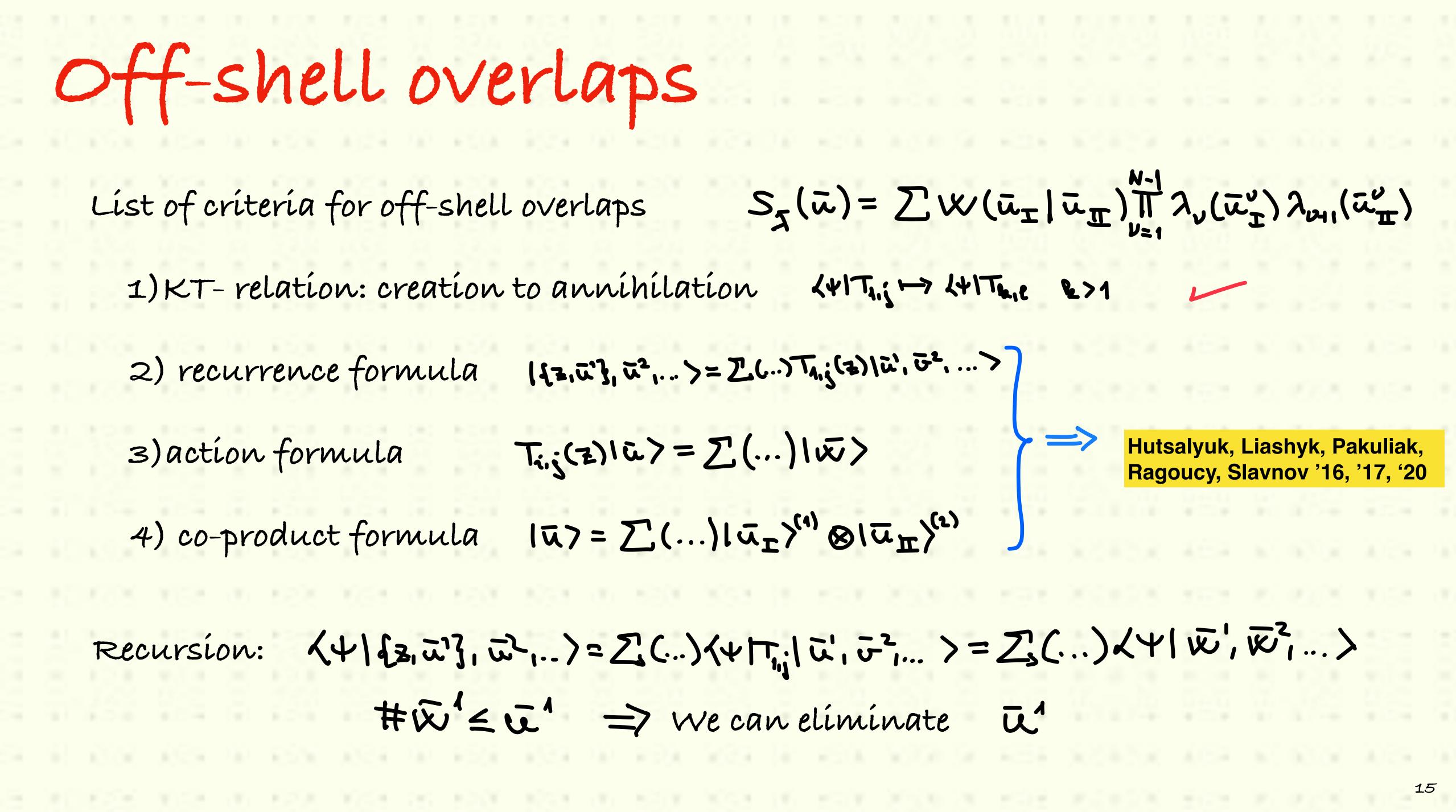


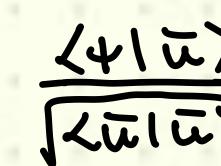
off-shell overlaps List of criteria for off-shell overlaps 1)KT-relation: creation to annihilation 2) recurrence formula $T_{i,j}(z)|u\rangle = \sum_{i=1}^{\infty} (...)|w\rangle$ з)action formula 4) co-product formula Recursion: $\#\overline{w}' \leq \overline{u}' \Rightarrow we can eliminate \overline{u}'$

- $S_{\chi}(\bar{u}) = \sum W(\bar{u}_{r}|\bar{u}_{r}) \prod_{\nu=1}^{N-1} \lambda_{\nu}(\bar{u}_{r}^{\nu}) \lambda_{\nu_{n}}(\bar{u}_{r}^{\nu})$
- <+ IT + + + The e 2>1
- $\{z,\bar{u}\},\bar{u}^{2},...\}=Z(...)T_{4,j}(z)|\bar{u}',\bar{v}^{2},...\}$
- $|\overline{u}\rangle = \sum (...) |\overline{u}_{r}\rangle^{(n)} \otimes |\overline{u}_{m}\rangle^{(n)}$

Hutsalyuk, Liashyk, Pakuliak, Ragoucy, Slavnov '16, '17, '20

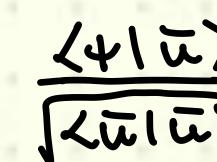
- $\langle \Psi | \{z, \overline{\omega}\}, \overline{\omega}, \ldots \rangle = \Sigma(\ldots) \langle \Psi | \overline{\omega}, \overline{\omega}, \ldots \rangle = Z(\ldots) \langle \Psi | \overline{\omega}, \overline{\omega}, \ldots \rangle$







Korepin's criteria $\longrightarrow \qquad \langle 4 | \overline{u} \rangle = T T T (\overline{u}) \int \frac{\det G}{\det G}$





G[±] depends on the pair structure

Korepín's crítería $\longrightarrow \qquad \frac{\langle \psi | \overline{u} \rangle}{\langle \overline{u} | \overline{u} \rangle} = \overline{T} \overline{T} \overline{T} (\overline{u}) \sqrt{\frac{\det G}{\det G}}$

1000	(\mathbf{v})	$g \leq 0$	2000	100	1000	$\overline{\mathcal{C}}$	10.00





Korepín's crítería $\longrightarrow \qquad \langle 4 | \overline{u} \rangle = T T T (\overline{u}) \int \frac{\det G}{\det G}$

G[±] depends on the pair structure

F, (w) given by the K-matrix

10.0	110	$g \leq 0$	2010.00	1018	1000	101	10.000







Fy (w)

 $Q_{M}(a)$ det G^{\dagger} $N = \frac{N}{2}$ Qn(D)Qn(=) det G 16

· g(M)@g((w·M)

2									
	12	19	6.95	1063	20	120	1223	19	100



Korepin's criteria $\longrightarrow \qquad \frac{\langle \psi | \overline{u} \rangle}{\langle \overline{u} | \overline{u} \rangle} = \prod_{v} \overline{F_{v}(\overline{u}^{v})} \frac{\det G}{\det G}$

 G^{\pm} depends on the pair structure

given by the K-matrix







F, (w)

Qm(a) det G⁺ $N = \frac{N}{2}$ Qn(D)Qn(z) det G 16

· gl(M)@gl(w·M)

2									
	12	19	6.95	1063	20	120	1223	19	100

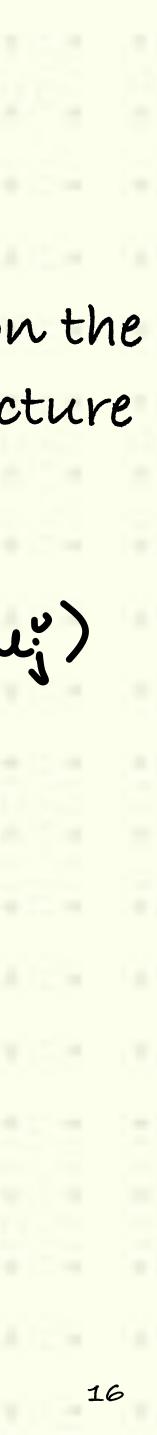


Korepín's crítería $\longrightarrow \qquad \frac{\langle \psi | \overline{u} \rangle}{\langle \overline{u} | \overline{u} \rangle} = \prod_{v} \overline{F_{v}(\overline{u}^{v})} \frac{\det G^{t}}{\det G}$

G[±] depends on the pair structure

given by the K-matrix

 $Q_{y}(z) = \prod_{j=1}^{r_{y}} (z - u_{j}^{y})$







F, (w)

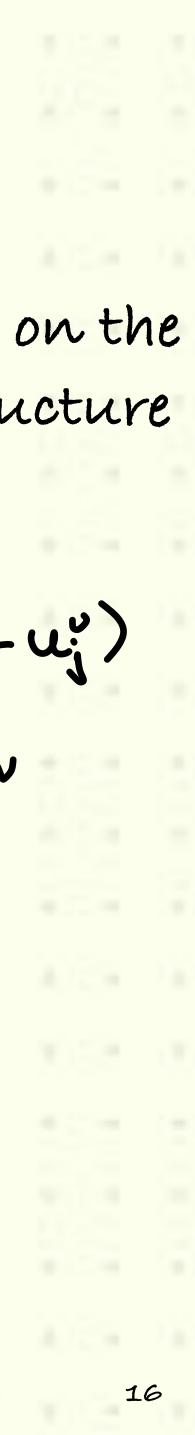
							$T_{y}(w)$ given by the K-matrix Q_{y}							2 _v (_₹)	$(z) = \prod_{j=1}^{r_{p}} (z - u)$				
	• 6)⊕	g1(v)-M)		6	$\frac{2}{n(0)Q}$) い(注	de	tG ⁺	٦		とえ		ichiva)	ī	(= - 1	
10	1.5	10.0	19	6.0	10<4	10	120	1.22.3									101	$[0,1] \in \mathcal{O}$	

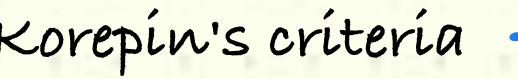


Korepín's crítería $\longrightarrow \qquad \langle \psi | \overline{u} \rangle = T T(\overline{u}) \int \frac{\det G}{\det G}$

given by the K-matrix

depends on the pair structure



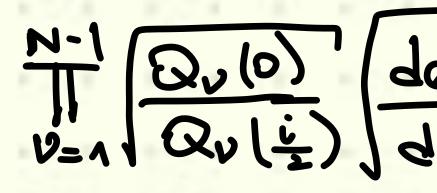




F, (u)

Qm(a) de 12、(の)の、(法)」

· gl(M)@gl(w·M)



 $\frac{Q_{2\nu}(3)Q_{2\nu}(\frac{i}{2})}{Q_{2\nu-1}(3)Q_{2\nu-1}(\frac{i}{2})}$

. 0(N)



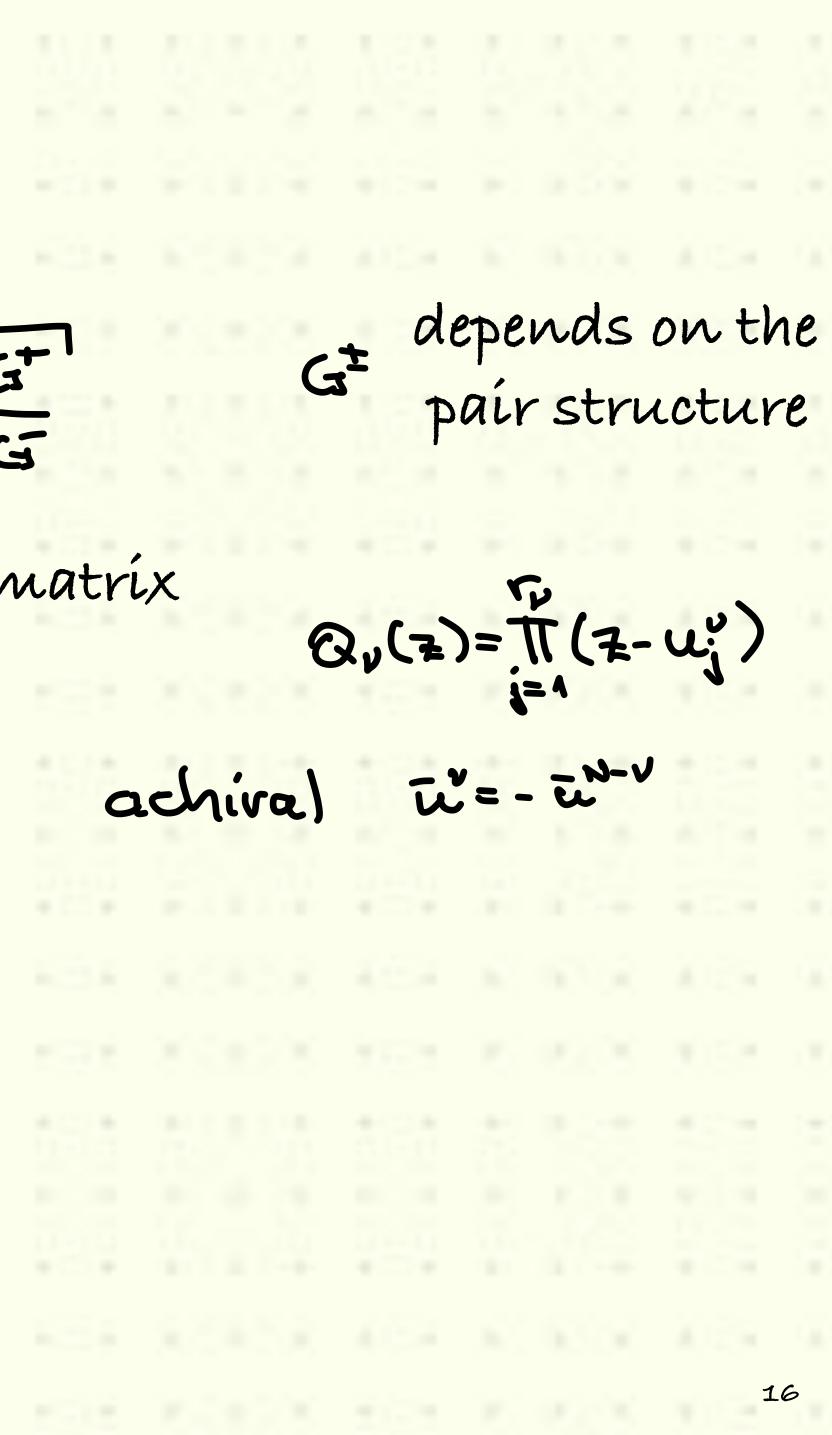
Korepin's criteria $\longrightarrow \qquad \frac{\langle \psi | \overline{u} \rangle}{\langle \overline{u} | \overline{u} \rangle} = T_{\overline{v}} T_{\overline{v}} (\overline{u}) \frac{\det G^{T}}{\det G}$

given by the K-matrix

G[±] depends on the pair structure

			ビスノ(モ)= ハ(モー)						
etG ⁺ etG ⁻	N=	N 2	C	ichiva)	ĩť	$= - \frac{N-V}{C}$			
0.04									
letG ⁺									
det G									
$1 + c^+$									

det G^T det G^T

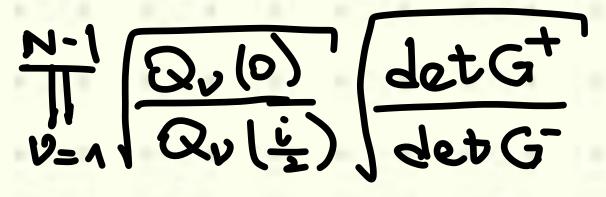






 $\frac{Q_{m}(\alpha)}{Q_{n}(0)Q_{n}(\frac{1}{2})} \det G$

 $\cdot g(M) \oplus g(N \cdot M)$



• SP(N) $\prod_{v} \frac{Q_{2v}(s)Q_{2v}(\frac{1}{2})}{Q_{2v-1}(s)Q_{2v-1}(\frac{1}{2})} \frac{\det G^{\dagger}}{\det G^{\dagger}}$

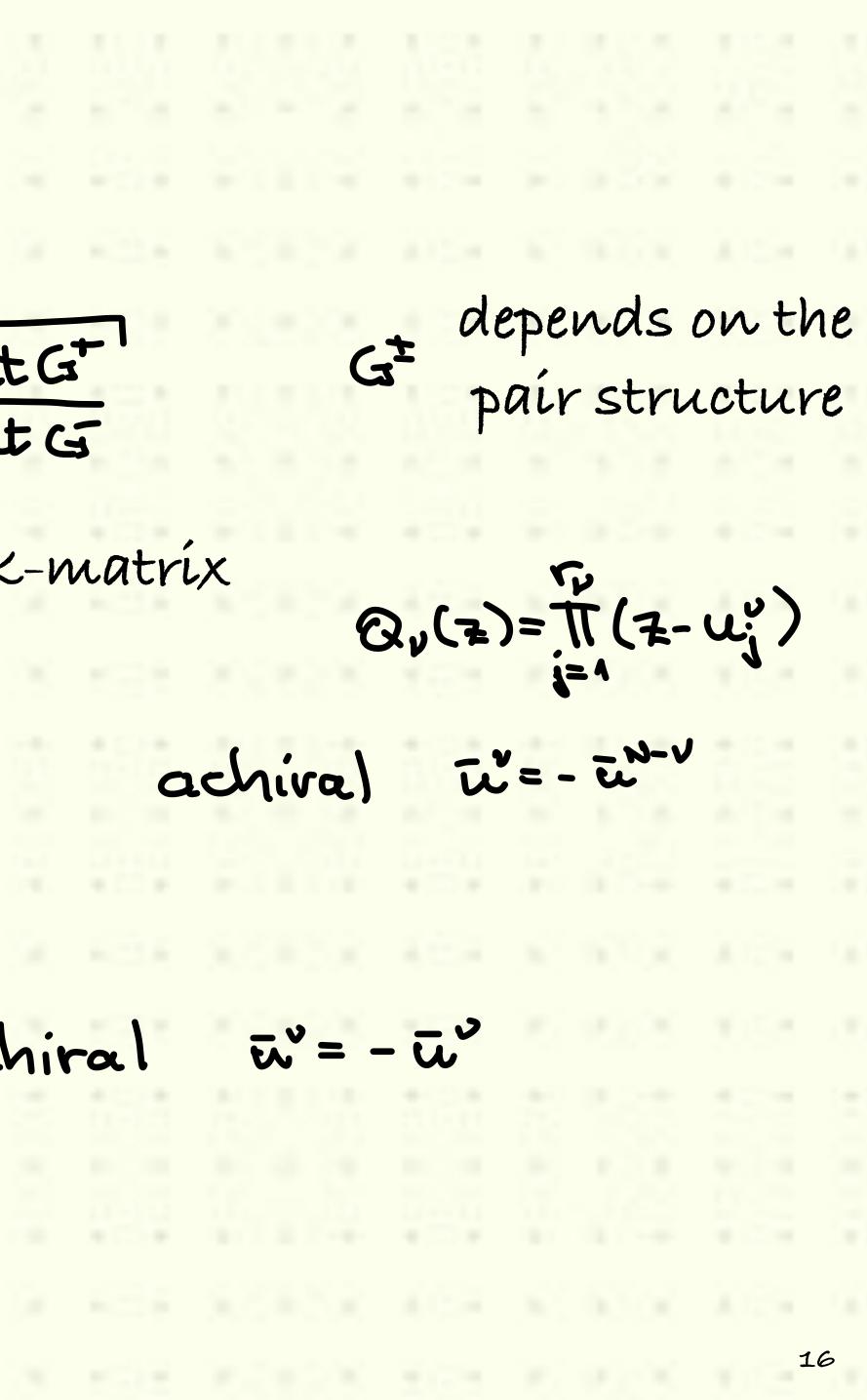
• 0(N)



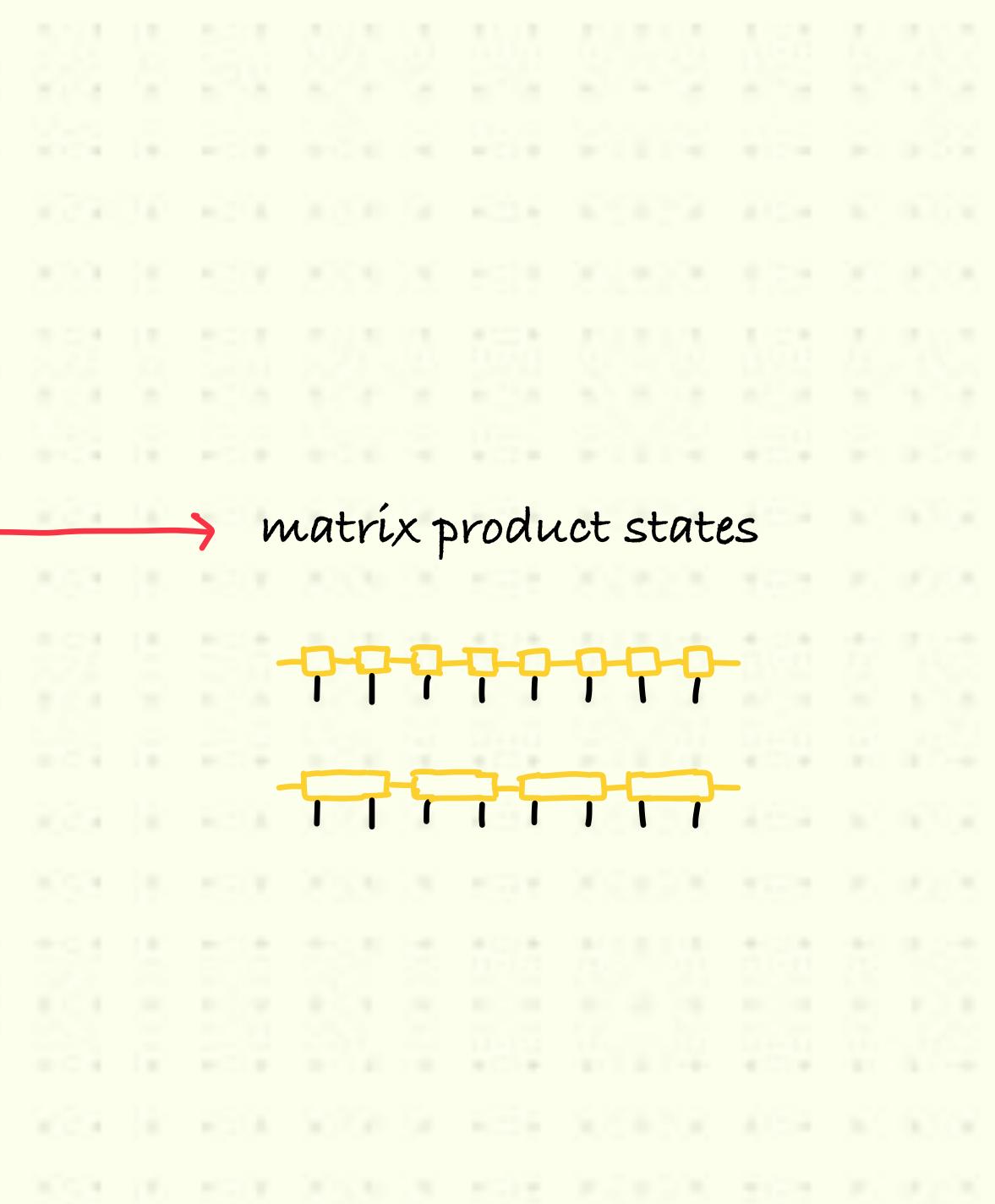
 $\frac{\langle \psi | \overline{u} \rangle}{\langle \overline{u} | \overline{u} \rangle} = \prod_{v} \overline{T}_{v} (\overline{u}^{v}) \frac{\det G}{\det G}$

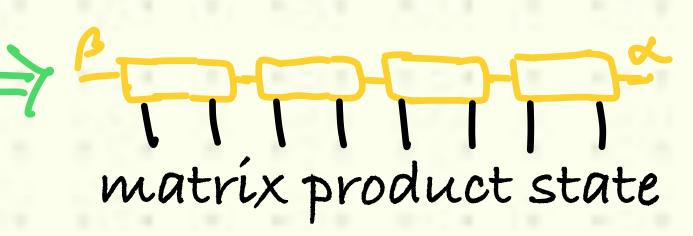
G[±] depends on the pair structure

F, (w) given by the K-matrix $Q_{\nu}(z) = \prod_{j=1}^{r_{\nu}} (z - u_{j}^{\nu})$ achival $\overline{u}^{*} = -\overline{u}^{N-v}$ $N = \frac{N}{2}$ $\overline{u}' = -\overline{u}'$ > chiral



two-site states — matrix product states $1\mathcal{F}$





1										
2										
	1000	$\forall 1 \leq 2$	(V)	6.05	$V \leq t$	121	$t \geq t^{\prime}$	12.53	(V)	100

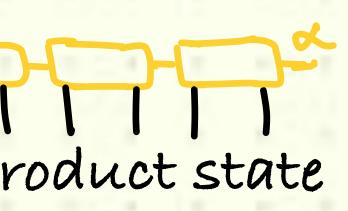


 $L + _{Lp} = \sum_{i=1, j_{23}} (M_{i}(\Theta_{i}) \dots M_{i}(\Theta_{i})) L_{i_{1}, j_{2}, \dots, j_{23}, j_{3}}$



 $\chi_{\mu} = \sum_{i=1}^{n} (M_{i}(\Theta_{i}) \dots M_{i}(\Theta_{i})) \chi_{\mu} \chi_$ i=1,...,d fl=[Cd]

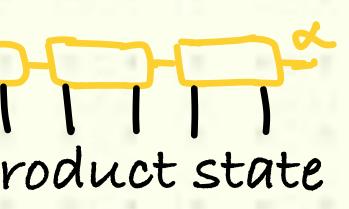




 $L + L = \sum_{i=1}^{n} (M_{i}(\Theta_{i}), M_{i}(\Theta_{i})) + L = \sum_{i=1}^{n} (M_$

<YI= ZKt apl⊗eupeft ⊗ End(H)







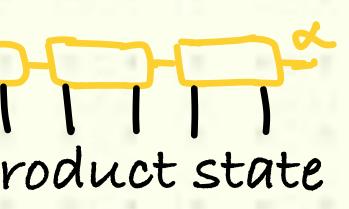
10.0	11	100	1000	1913	1000	101	10.00

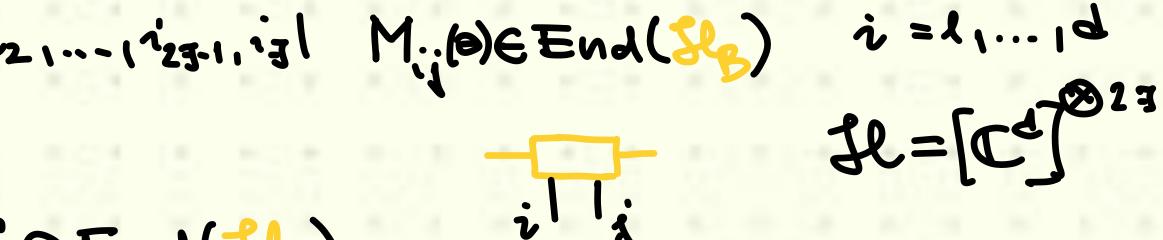
 $LY_{2} = \sum_{i_{2} \in I_{2}} (M_{i}(\Theta_{i}), M_{i}(\Theta_{i})) L_{i_{1}} L_{i_{1}} L_{i_{2}} L_{i_{2}} L_{i_{2}} L_{i_{2}} (M_{i_{2}}) = \sum_{i_{1} \in I_{2}} (M_{i_{2}}(\Theta_{i_{2}}), M_{i_{2}}(\Theta_{i_{2}})) L_{i_{1}} L_{i_{2}} L$

<YI= Z<+x, 1⊗ exp ∈ fet ⊗ End(H)

K(u) E End (CNOK)



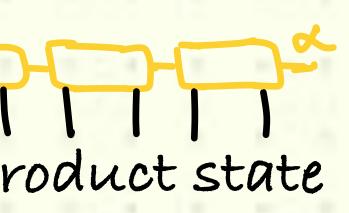


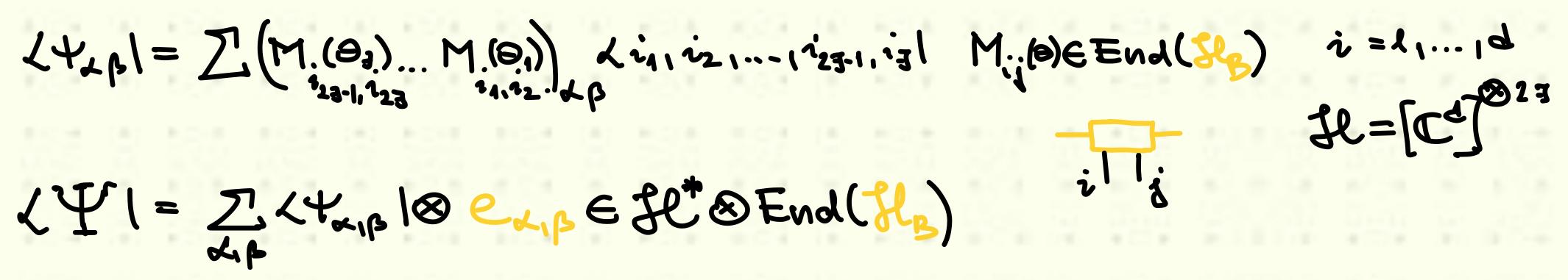


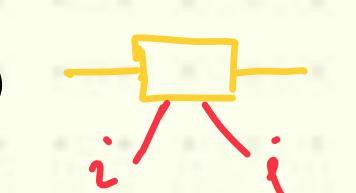
10.0	11	100	1000	1913	1000	101	10.00

K(u) E End (CNOLE) Kij (u) E End (Le)







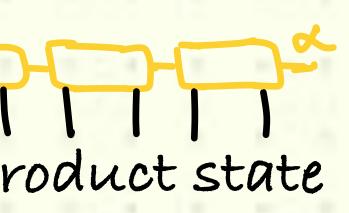


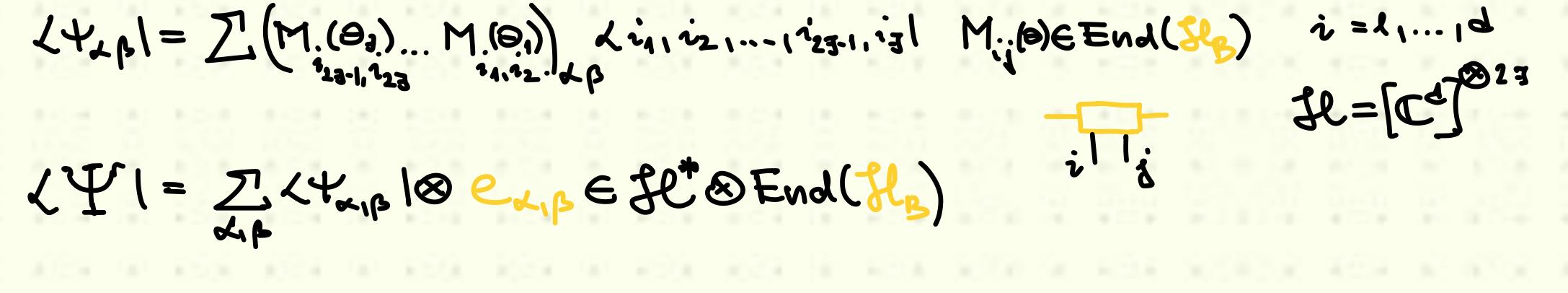
 $L + \mu = \sum_{i=1}^{n} (M_{i}(\Theta_{i}) M_{i}(\Theta_{i})) L_{i_{1}} L_{i_{1}} L_{i_{2}} L_{i_{$

K(u) E End (CNORB) Kij (u) E End (RB)

KT-relation







 $\sum_{j=1}^{N} K_{ij}(z) \langle \Psi|T_{j}(z) = \sum_{j=1}^{N} \langle \Psi|T_{ij}(-z) K_{j}(z) = \sum_{j=1}^{N} \langle \Psi|T_{ij}(-z) K_{j}(-z) K_{j}(z) = \sum_{j=1}^{N} \langle \Psi|T_{ij}(-z) K_{j}(-z) K_{j}(z) = \sum_{j=1}^{N} \langle \Psi|T_{ij}(-z) K_{j}(-z) K_{j}(z) = \sum_{j=1}^{N} \langle \Psi|T_{ij}(-z) K_{j}(-z) K_{j}($



Integrability con the second s

1	di	ti	.01					
	10.0	(\mathbf{v})	$g \leq 1^{-1}$	2010/01/01	1993	1000	102	10.00



Integrability con

$\langle MPS | = \frac{1}{1} - \frac{1}{1} = \sum_{n=1}^{n} \langle \Psi_{nn} \rangle$

1	di	ti	.01					
	10.0	(\mathbf{v})	$g \leq 1^{-1}$	2000	:03	1000	303	10.00



Integrability con a law alter lat attend alter lat attend with a lat at

$\langle MPS | = \frac{1}{11} - \frac{1}{11} = \sum_{n=1}^{n} \langle \Psi_{nn} |$

 $\Rightarrow \langle MPS|T(u) = \langle MPS|T(-u)$ or

1	di	ti	.01						
(}	1PSIT	(n)	= < M	psite	- u)				
٢.,	1.5.5	121	2.12	20.5	1.1	2.12	1.11.1	21-21	1000



Integrability con

$\langle MPS | = \frac{1}{1} - \frac{1}{1} = \sum_{n=1}^{n} \langle \Psi_{nn} |$

$\Rightarrow \langle MPS|T(u) = \langle MPS|T(-u)$ or

homogeneous lím

1	di	ti	.01	1					
(}	1 P S)7	r(n)	- <m< th=""><th>PSITC</th><th>-u)</th><th></th><th></th><th></th><th></th></m<>	PSITC	-u)				
ú	-								
	C								
	1000	141	$\mathcal{C} \subseteq \mathcal{C}$	2010		$t \in \{1, 2\}$	10.000	$\overline{\mathcal{T}} \subset \mathcal{T}$	10.00



Integrability con

$\langle MPS | = \frac{1}{1} - \frac{1}{1} = \sum_{n=1}^{n} \langle \Psi_{n,n} \rangle$

 $\Rightarrow \langle MPS|T(u) = \langle MPS|T(-u)$ or

Den tellikone eller isi kote etter isi kote isi kote

homogeneous lím

di	ti	.ov				
MPSIT	(u)	= < MF	psite	· u)		
3.53						
it $\overline{0}=0$						



Integrability con

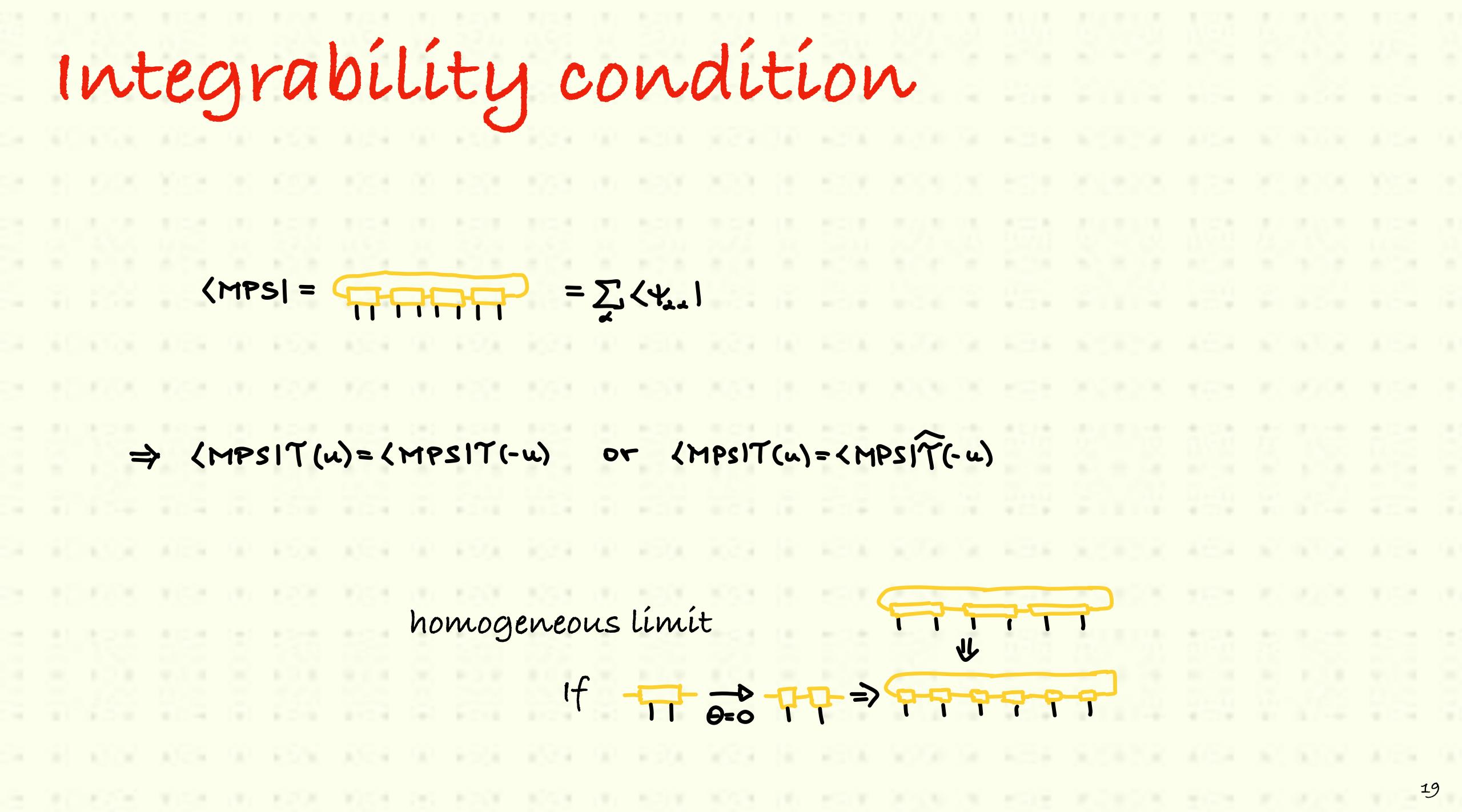
$\langle MPS| = - - - - - - - - = \sum_{x} \langle \Psi_{xx} \rangle$

tal attra ditta tal attra attra ditta attra

 $\Rightarrow \langle MPS|T(u) = \langle MPS|T(-u)$ or

homogeneous lim

,(di	ti	.01				
ر ۲							
/~	T/201			PSITC			
	16211	(~)					
c í t							
		-0-1	┍╴╼>		14		



classification of

compatibility of the KT with RTT -

	8					Ne	
tn	e	K	-M	at	rici	es	
7							
$\{ i, j \in I \}$	$\{ \psi \}_{i}$	$g \leq 1^{-1}$	2010/01/01	$g(x) \in \mathbb{R}$	1000	10.1	10.000

Classification of the K-matrices compatibility of the KT with RTT -> Reflection equation (byBe) 20

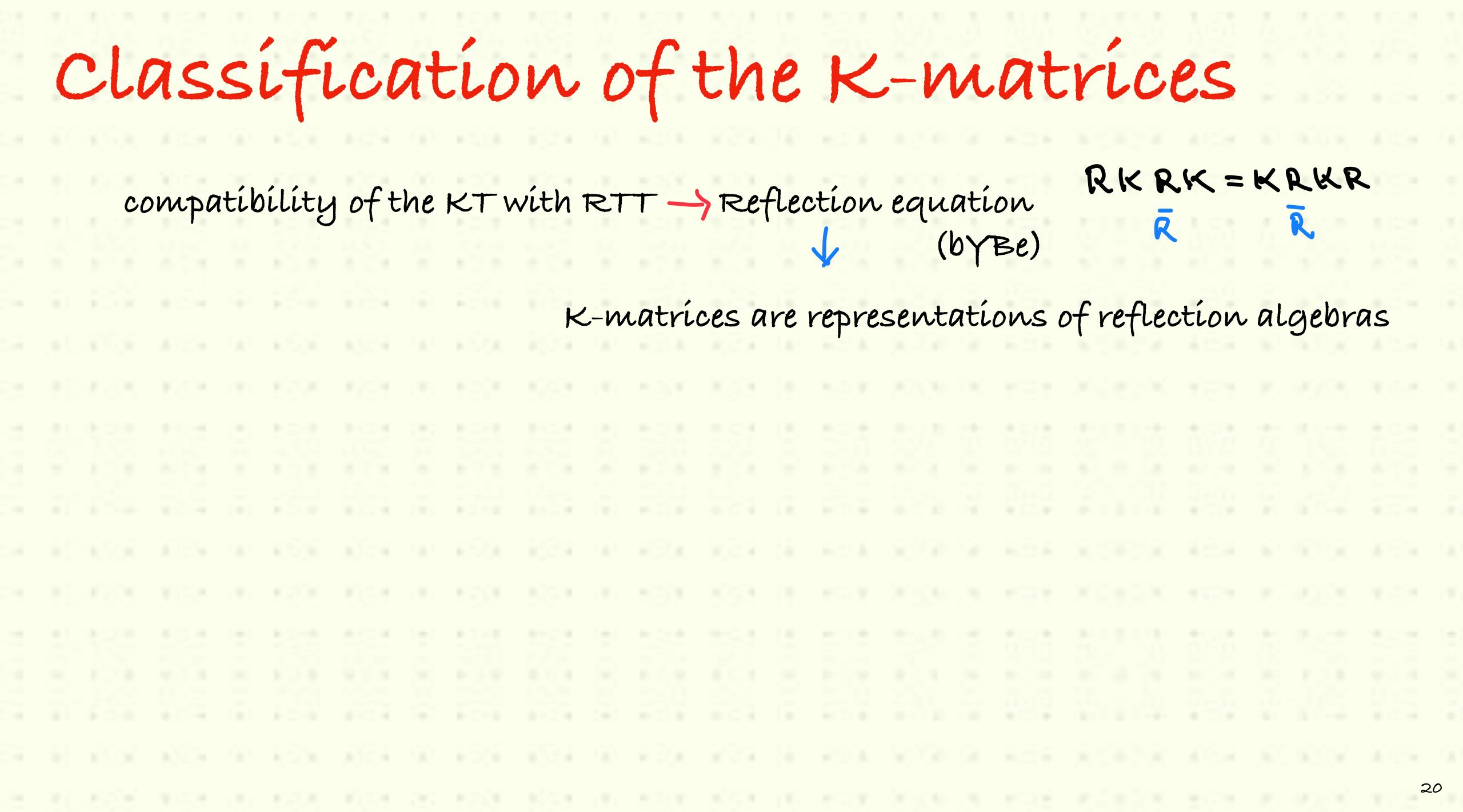
Classification of the K-matrices

compatibility of the KT with RTT \rightarrow Reflection equation (bYBe) RKRK = KRKR

compatibility of the KT with RTT -> Reflection equation RKRK = KRKR (byBe)



K-matrices are representations of reflection algebras



compatibility of the KT with RTT \rightarrow Reflection equation (by Be) RKRK = KRKR

K-matrices are representations of reflection algebras

non-crossed by Be $\longrightarrow B(N,M)$ algebra

algebra 20



compatibility of the KT with RTT -> Reflection equation

K-matrices are representations of reflection algebras

non-crossed by Be $\longrightarrow B(N,M)$

 \rightarrow Reflection equation (by Be) RKRK = KRKR

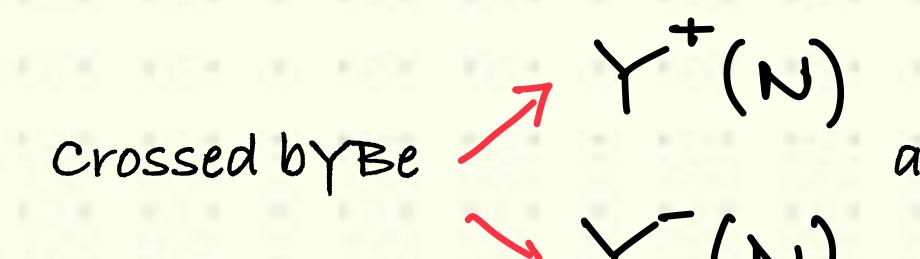
algebra → resídual symmetry: gl(M)⊕gl(N-M) 20



compatibility of the KT with RTT -> Reflection equation

K-matrices are representations of reflection algebras

B(N,M) algebra non-crossed by Be



residual symmetry: $g(M) \oplus g(N-M)$ $\rightarrow Y(N)$ 20

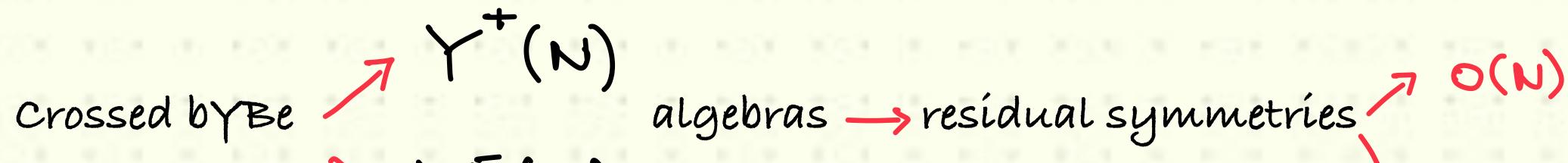
RKRK = KRKR (byBe)



compatibility of the KT with RTT -> Reflection equation

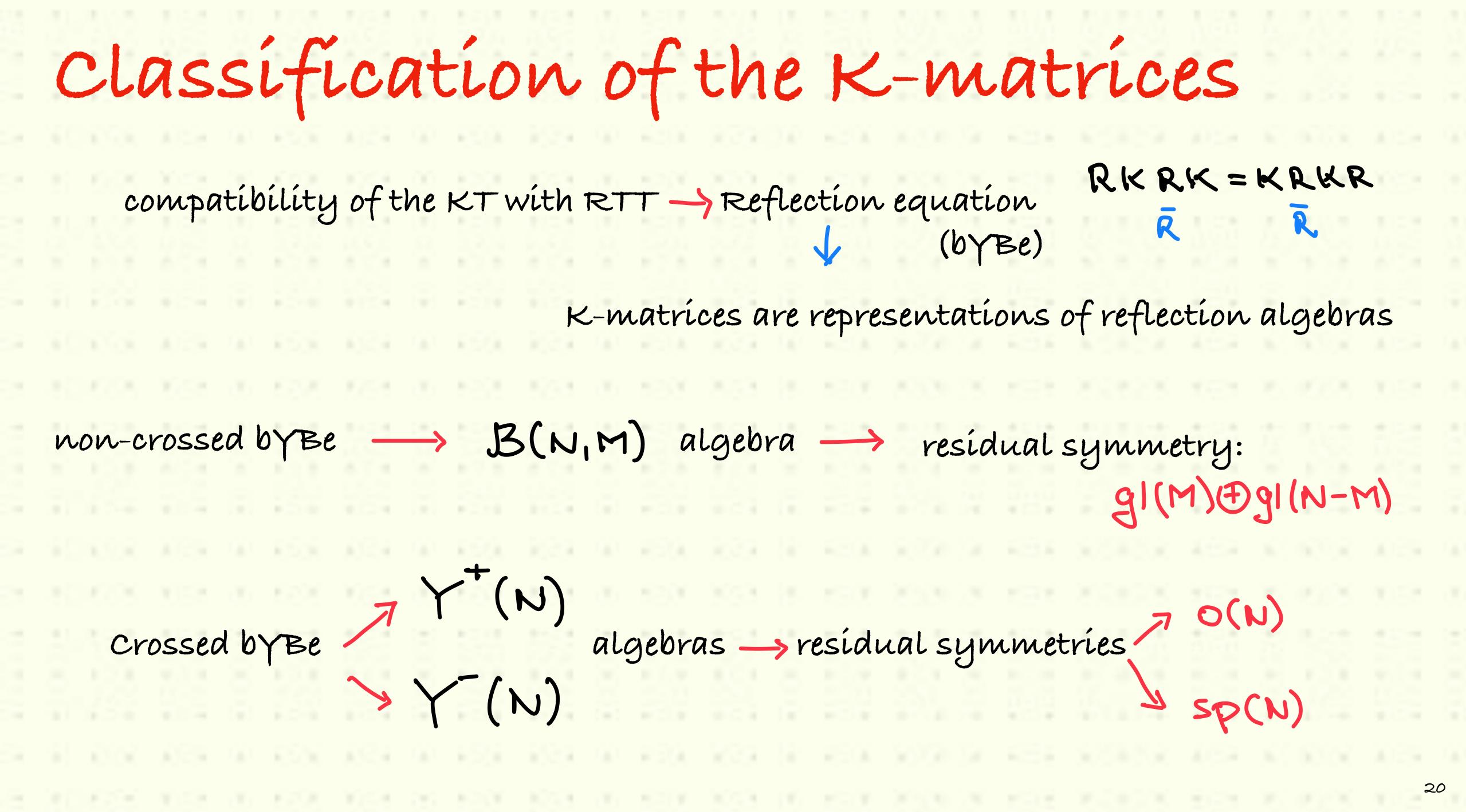
K-matrices are representations of reflection algebras

non-crossed by Be B(N,M) algebra



RKRK = KRKR (bybe)

residual symmetry: $g(M) \oplus g(N-M)$



Recursion for off-shell overlaps for crossed KT 21



Recursion for off-shell overlaps for crossed KT -> 21



•				ing	k	6.11

21

is invertible



•				ing	k	6.11

is invertible -recursion for u roots 21



for crossed KT -> assuming Kin is invertible -> recursion for u roots $\langle \psi | \overline{\omega}^{*}, \overline{\omega}^{*}, \dots \rangle = \sum (\dots) \langle \psi | \varphi, \overline{\omega}^{*}, \dots \rangle$ 21



 $\langle \Psi | \tilde{u}, \tilde{u}, ... \rangle = \Sigma(...) \langle \Psi | \phi, \tilde{w}, ... \rangle \longrightarrow g(N-i)$ overlaps

21

for crossed KT -> assuming Kin is invertible -> recursion for u roots



 $\langle \Psi | \overline{u}, \overline{u}, ... \rangle = \Sigma(...) \langle \Psi | \phi, \overline{w}, ... \rangle \longrightarrow g(N-i) overlaps$

gl(N-1) KT-relation?

for crossed KT -> assuming Kin is invertible -> recursion for in roots



 $\langle \Psi | \tilde{u}, \tilde{u}, ... \rangle = \Sigma(...) \langle \Psi | \phi, \tilde{w}, ... \rangle \longrightarrow g(N-i)$ overlaps

gl(N-1) KT-relation?

for crossed KT -> assuming Kin is invertible -> recursion for u roots

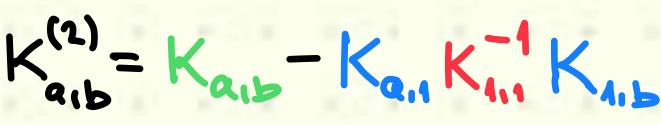


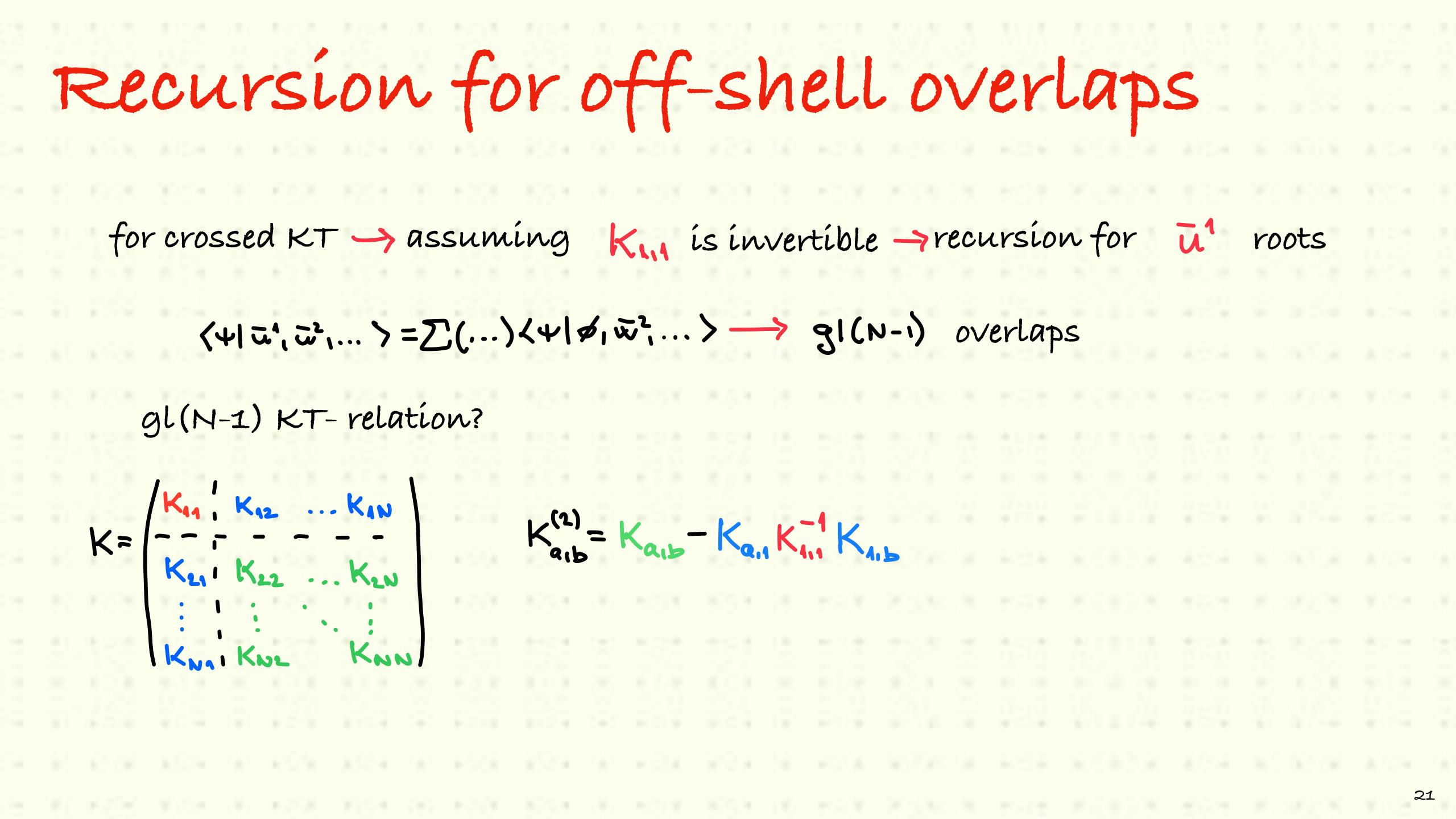
Recursion for off-shell overlaps

 $\langle \Psi | \tilde{u}, \tilde{u}, ... \rangle = \Sigma(...) \langle \Psi | \phi, \tilde{w}, ... \rangle \longrightarrow g(N-i)$ overlaps

gl(N-1) KT-relation?

for crossed KT -> assuming Kin is invertible -> recursion for I1 roots





Recursion for off-shell overlaps

 $\langle \Psi | \bar{\omega}^{*}, \bar{\omega}^{*}, ... \rangle = \Sigma(...) \langle \Psi | \phi_{1} \bar{\omega}^{*}, ... \rangle \longrightarrow g(N-1)$ overlaps

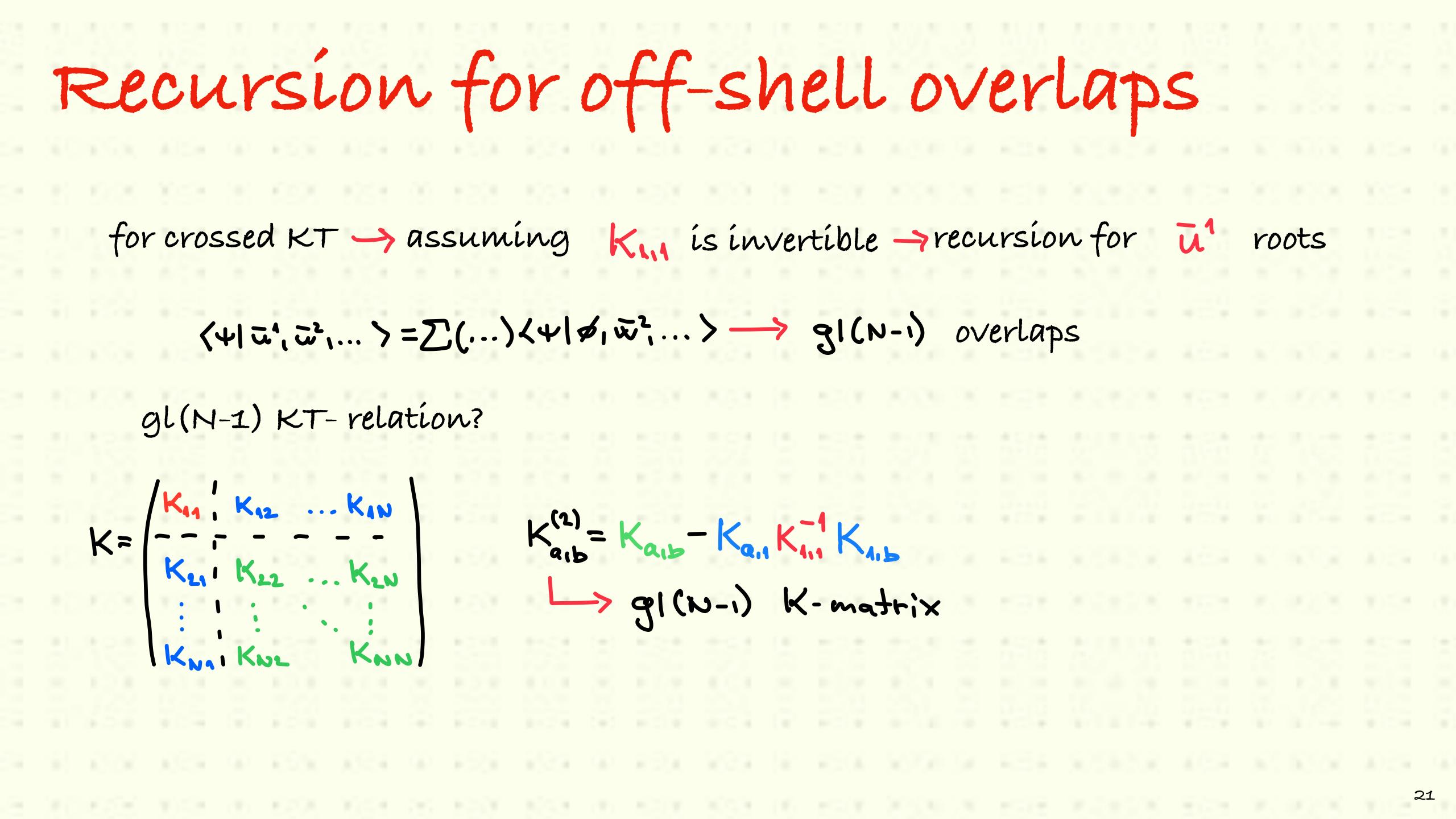
gl(N-1) KT-relation?

 $K = \begin{bmatrix} K_{11} & K_{12} & \dots & K_{1N} \\ - & - & - & - & - & - \\ K_{21} & K_{22} & \dots & K_{2N} \end{bmatrix}$

for crossed $KT \rightarrow assuming K_{M}$ is invertible $\rightarrow recursion$ for \overline{u}^{1} roots

 $K_{a,b}^{(2)} = K_{a,b} - K_{a,1} K_{a,1}^{-1} K_{a,b}$

-> g((N-i) K-matrix



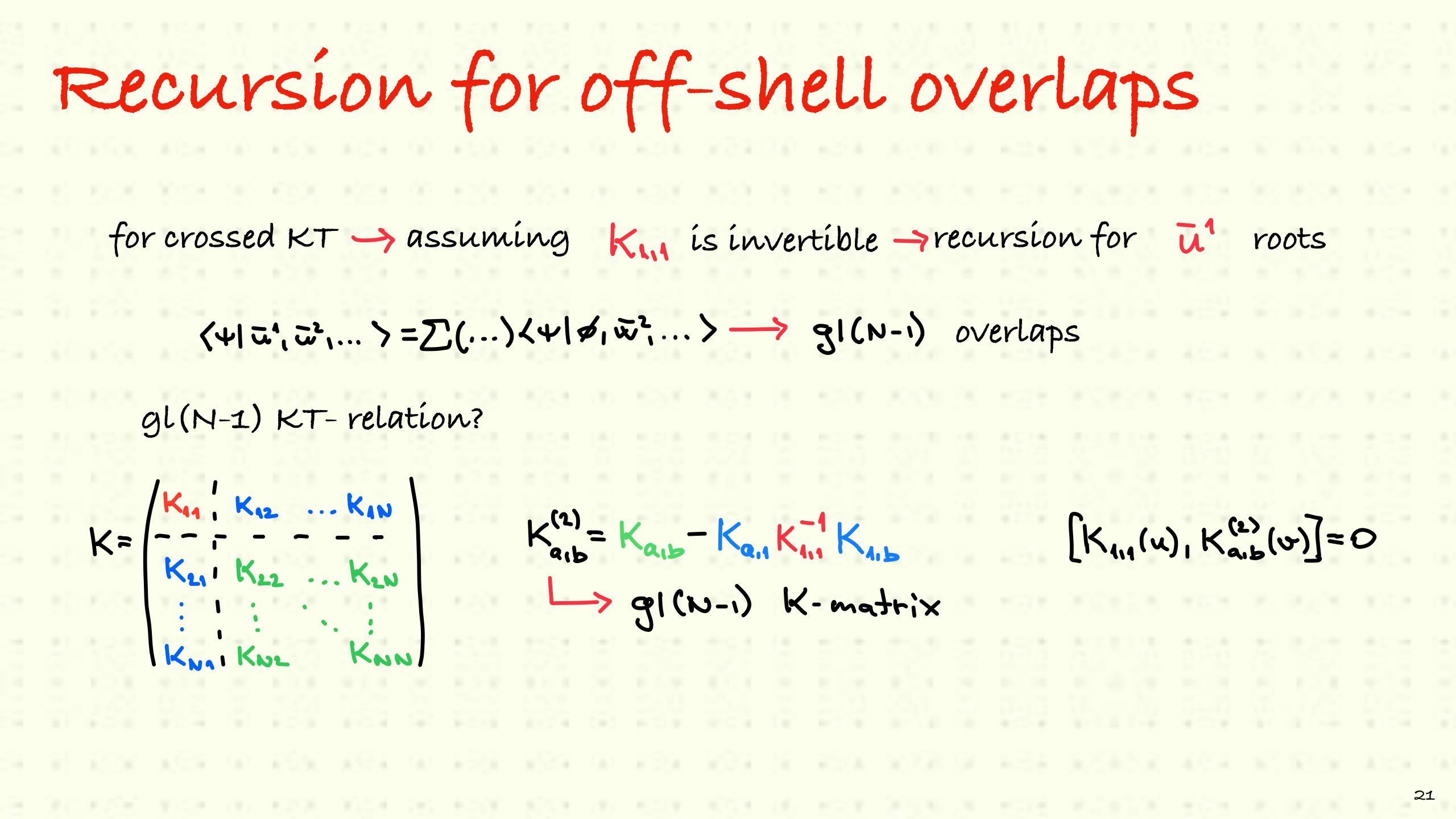
Recursion for off-shell overlaps

 $\langle \Psi | \tilde{\omega}^{*}, \tilde{\omega}^{*}, \dots \rangle = \Sigma(\dots) \langle \Psi | \phi, \tilde{\omega}^{*}, \dots \rangle \longrightarrow g(N-i)$ overlaps

gl(N-1) KT-relation?

 $K = \begin{bmatrix} K_{11} & K_{12} & \dots & K_{1N} \\ - & - & - & - & - & - \\ K_{21} & K_{22} & \dots & K_{2N} \end{bmatrix}$ $K_{a,b}^{(2)} = K_{a,b} - K_{a,i} K_{i,i} - K_{i,j}$ g(N-i) K-matrix

 $[K_{1,1}(u), K_{a,b}^{(2)}(v)] = 0$



Recursion for off-shell overlaps

 $\langle \Psi | \tilde{\omega}^{*}, \tilde{\omega}^{*}, \dots \rangle = \Sigma(\dots) \langle \Psi | \phi, \tilde{\omega}^{*}, \dots \rangle \longrightarrow g(N-i)$ overlaps

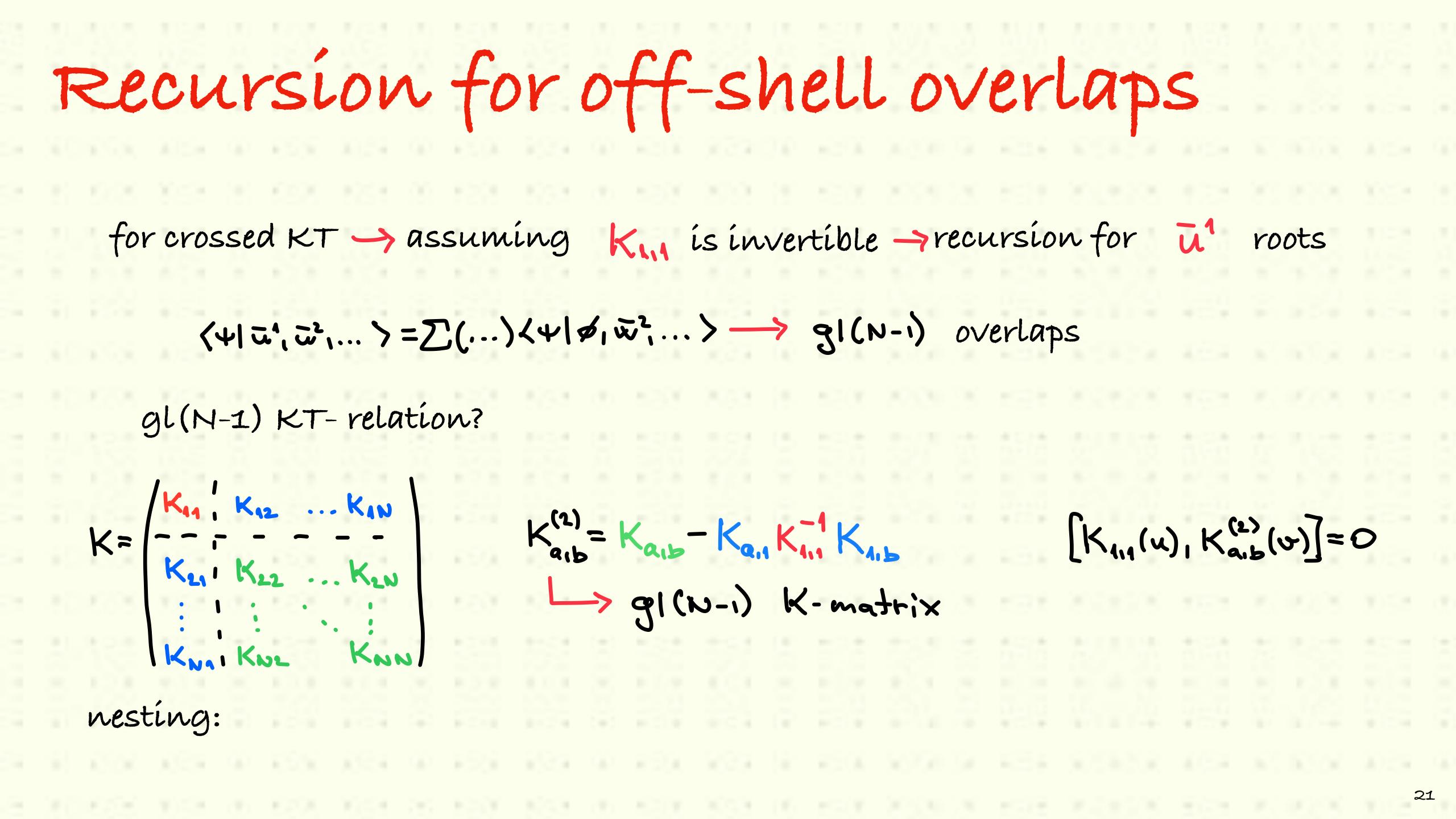
gl(N-1) KT-relation?

 $K = \begin{bmatrix} K_{11} & K_{12} & \dots & K_{1N} \\ - & - & - & - & - & - \\ K_{21} & K_{22} & \dots & K_{2N} \end{bmatrix}$ nesting:

 $K_{a,b}^{(2)} = K_{a,b} - K_{a,i} K_{i,i} - K_{i,j}$

g(N-i) K-matrix

 $[K_{1,1}(u), K_{a,b}^{(2)}(v)] = 0$



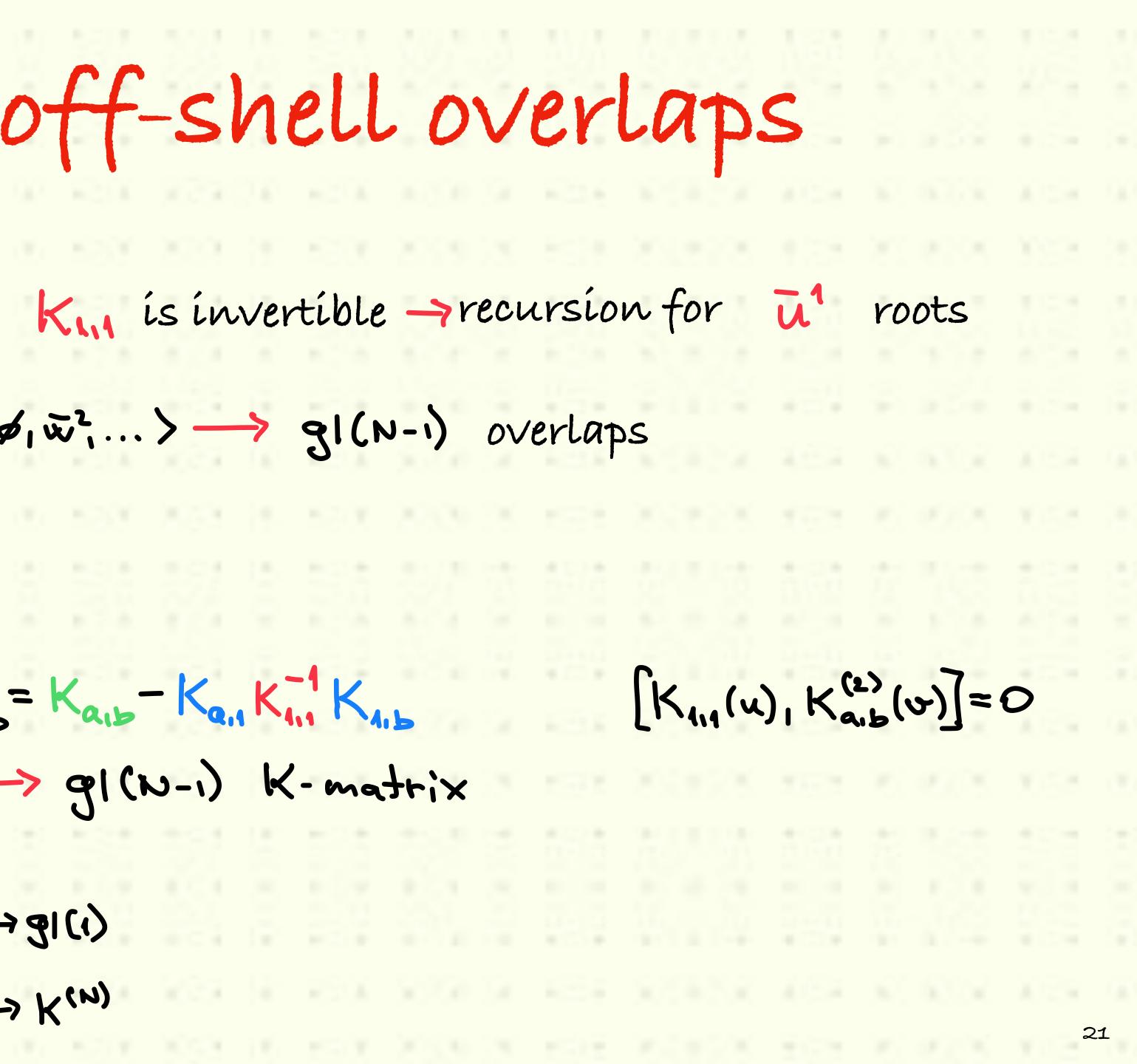
 $\langle \Psi | \tilde{\omega}^{*}, \tilde{\omega}^{*}, \dots \rangle = \Sigma(\dots) \langle \Psi | \phi, \tilde{\omega}^{*}, \dots \rangle \longrightarrow g(N-i)$ overlaps

gl(N-1) KT-relation?

 $K = \begin{bmatrix} K_{11} & K_{12} & \dots & K_{1N} \\ - & - & - & - & - & - \\ K_{21} & K_{22} & \dots & K_{2N} \end{bmatrix}$ $K_{a,b}^{(2)} = K_{a,b} - K_{a,i} K_{i,i} - K_{i,j}$ g(N-i) K-matrix nesting: $g(N) \rightarrow g(N-1) \rightarrow ... \rightarrow g(N)$

 $K_{(n)} \equiv K \longrightarrow K_{(r)} \rightarrow \cdots \rightarrow K_{(N)}$

 $[K_{1,1}(u), K_{a,b}^{(2)}(v)] = 0$



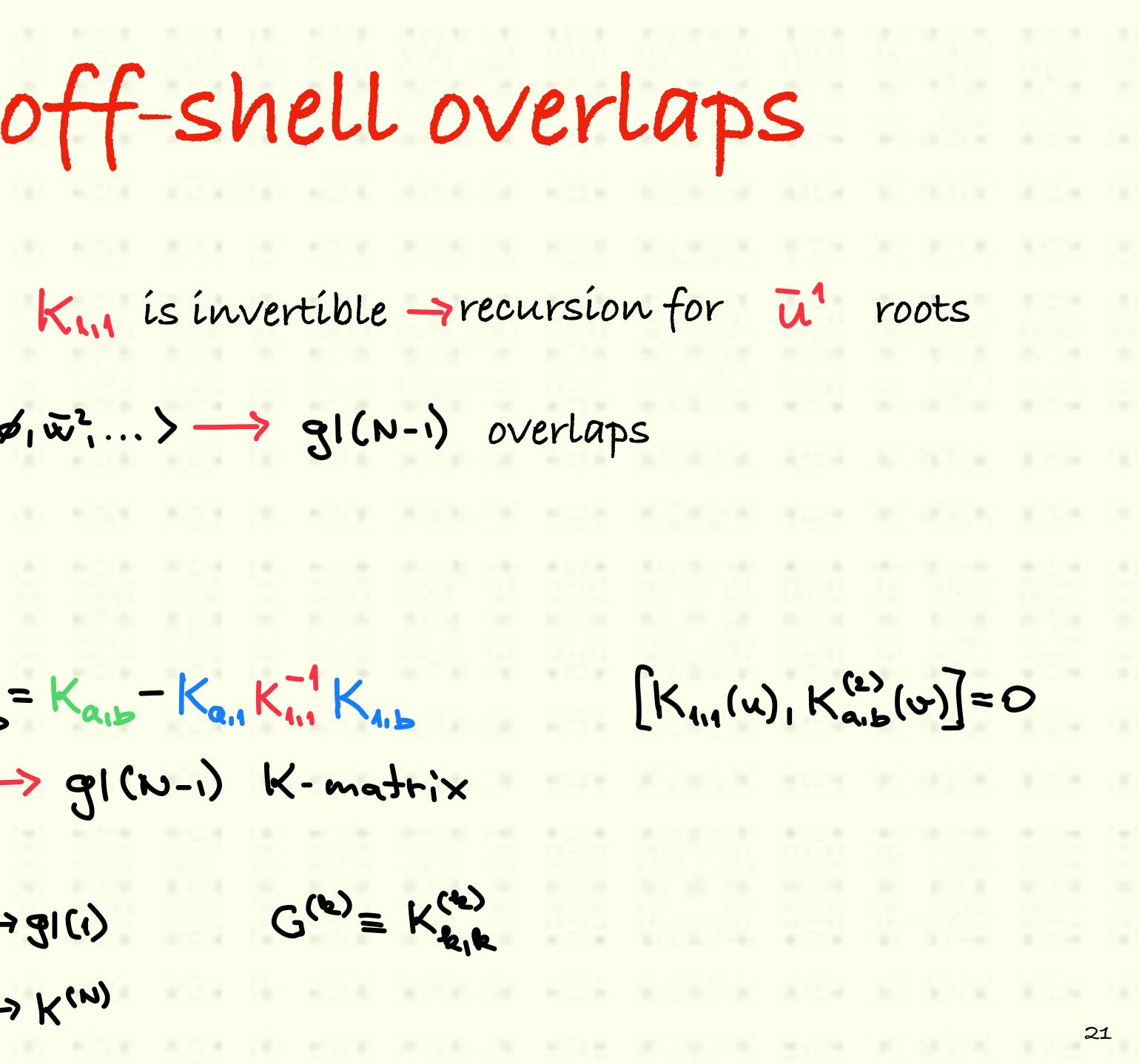
 $\langle \Psi | \tilde{\omega}^{*}, \tilde{\omega}^{*}, \dots \rangle = \Sigma(\dots) \langle \Psi | \phi, \tilde{\omega}^{*}, \dots \rangle \longrightarrow g(N-i)$ overlaps

gl(N-1) KT-relation?

 $K = \begin{bmatrix} K_{11} & K_{12} & \dots & K_{1N} \\ - & - & - & - & - & - \\ K_{21} & K_{22} & \dots & K_{2N} \end{bmatrix}$ $K_{a,b}^{(2)} = K_{a,b} - K_{a,i} K_{i,i} - K_{i,j}$ g(N-i) K-matrix $G^{(k)} \equiv K_{k,k}^{(k)}$ nesting: $g(N) \rightarrow g(N-1) \rightarrow ... \rightarrow g(I)$

 $K_{(n)} \equiv K \longrightarrow K_{(r)} \rightarrow \cdots \rightarrow K_{(n)}$

 $[K_{111}(u), K_{a,b}^{(2)}(v)] = 0$



 $\langle \Psi | \tilde{\omega}^{*}, \tilde{\omega}^{*}, \dots \rangle = \Sigma(\dots) \langle \Psi | \phi, \tilde{\omega}^{*}, \dots \rangle \longrightarrow g(N-i)$ overlaps

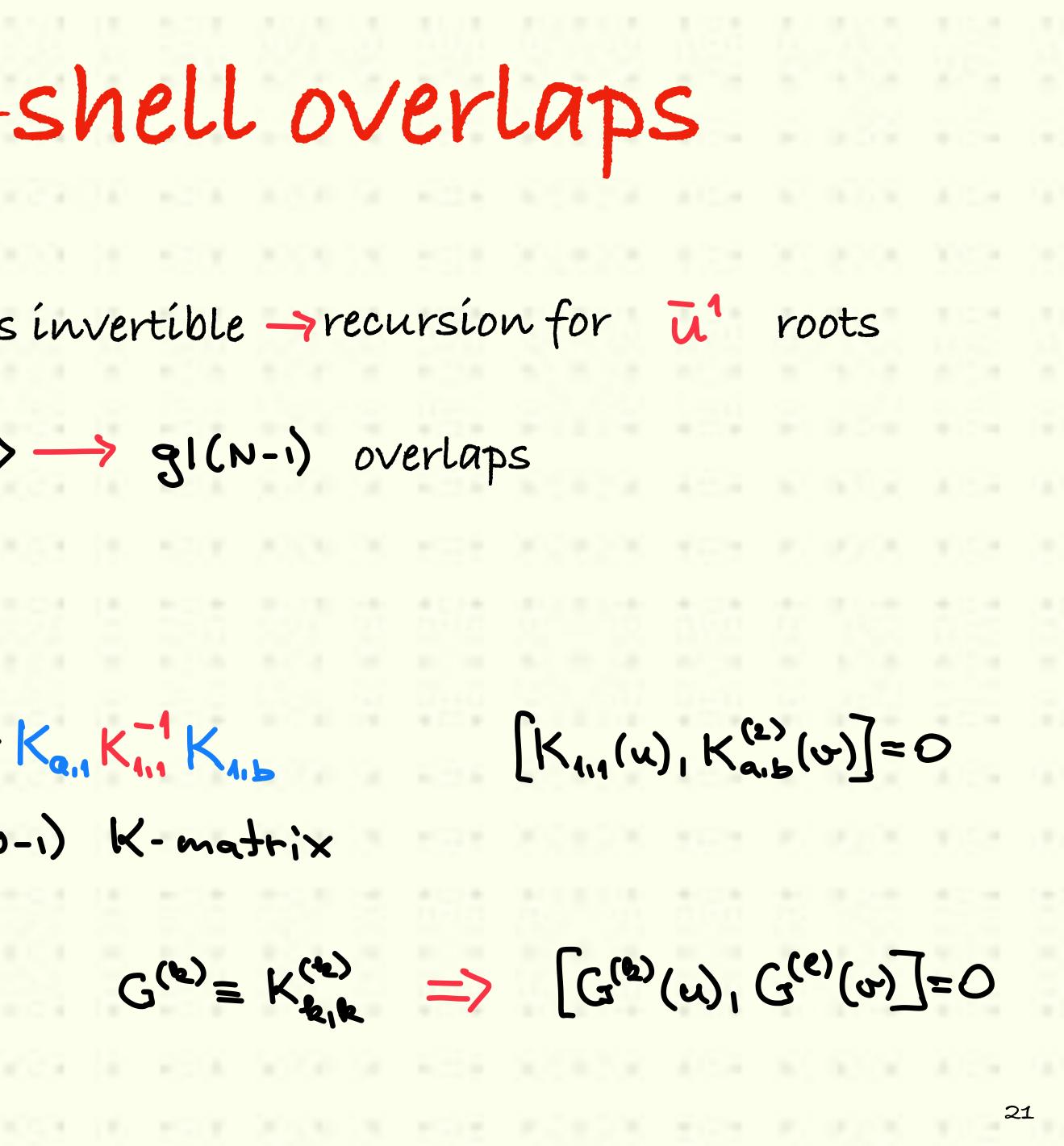
gl(N-1) KT-relation?

 $K = \begin{bmatrix} K_{11} & K_{12} & \dots & K_{1N} \\ - & - & - & - & - & - \\ K_{21} & K_{22} & \dots & K_{2N} \end{bmatrix}$ $K_{a_1b}^{(2)} = K_{a_1b} - K_{a_1} K_{a_1}^{-1} K_{a_1b}$ g(N-i) K-matrix nesting: $g(N) \rightarrow g(N-1) \rightarrow ... \rightarrow g(I)$

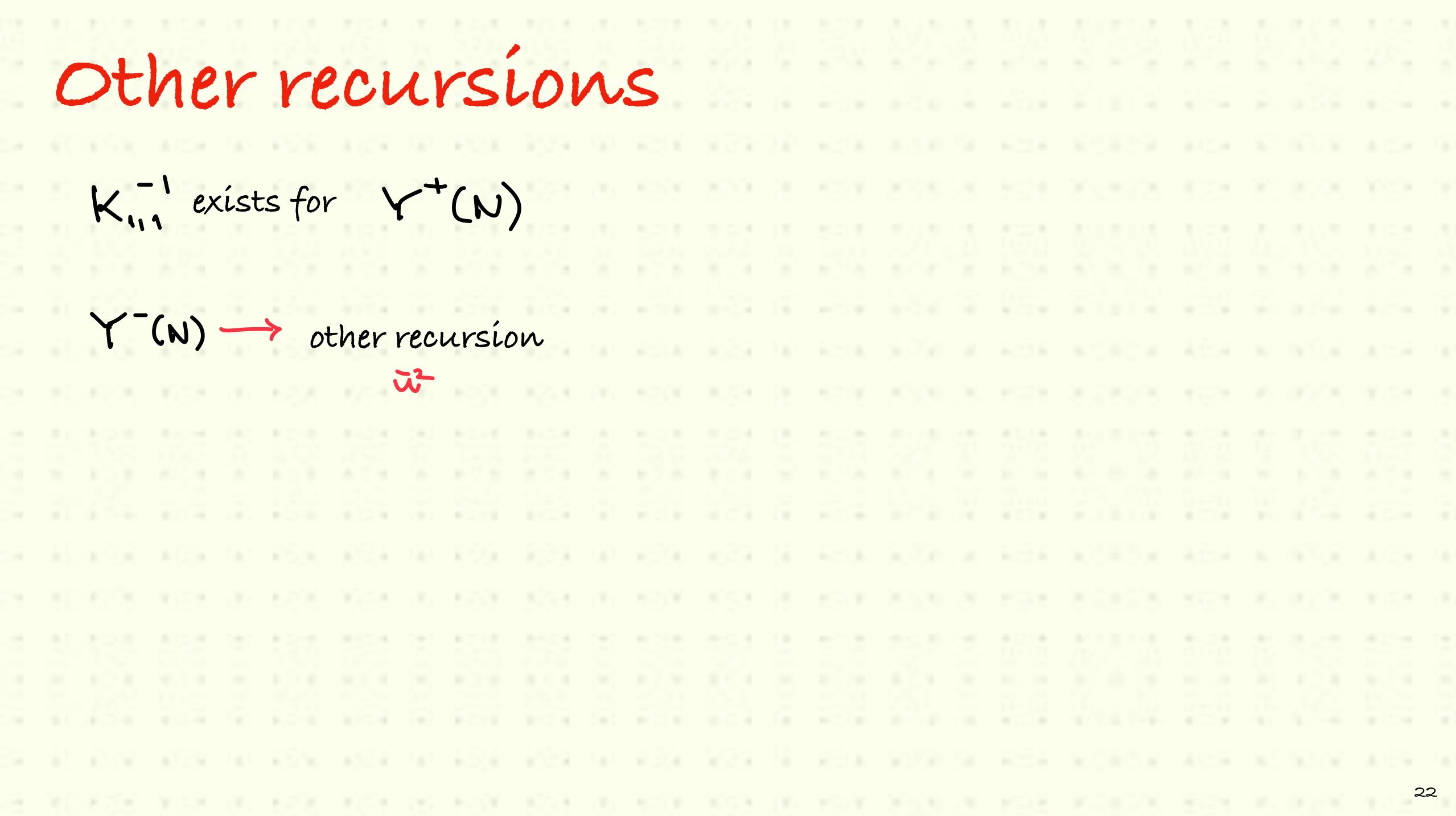
 $K_{(n)} \equiv K \longrightarrow K_{(r)} \rightarrow \cdots \rightarrow K_{(n)}$

 $[K_{1,1}(u), K_{a,b}^{(2)}(v)] = 0$

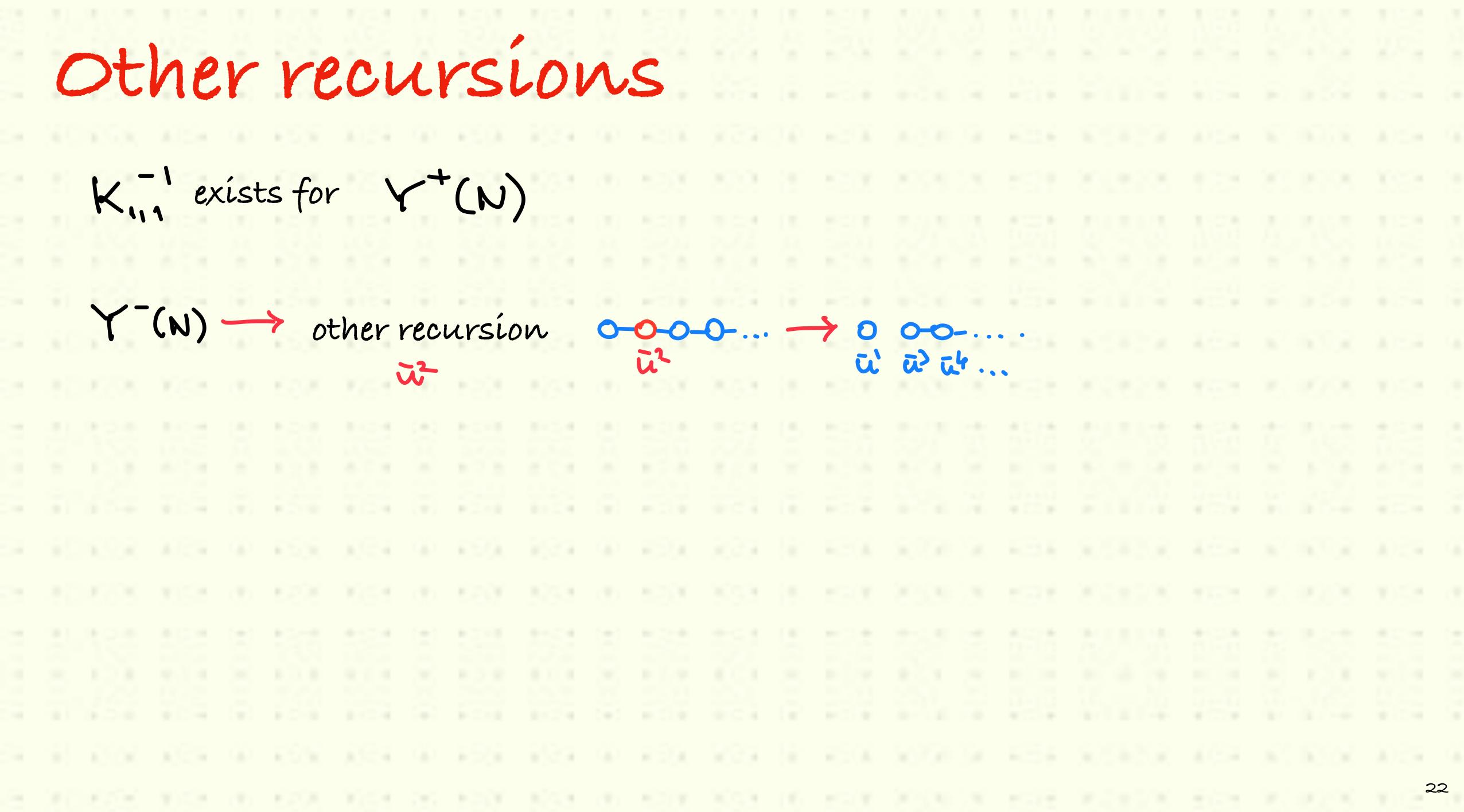
 $G^{(k)} = K_{k,k}^{(k)} \implies \left[G^{(k)}(u), G^{(e)}(v)\right] = 0$

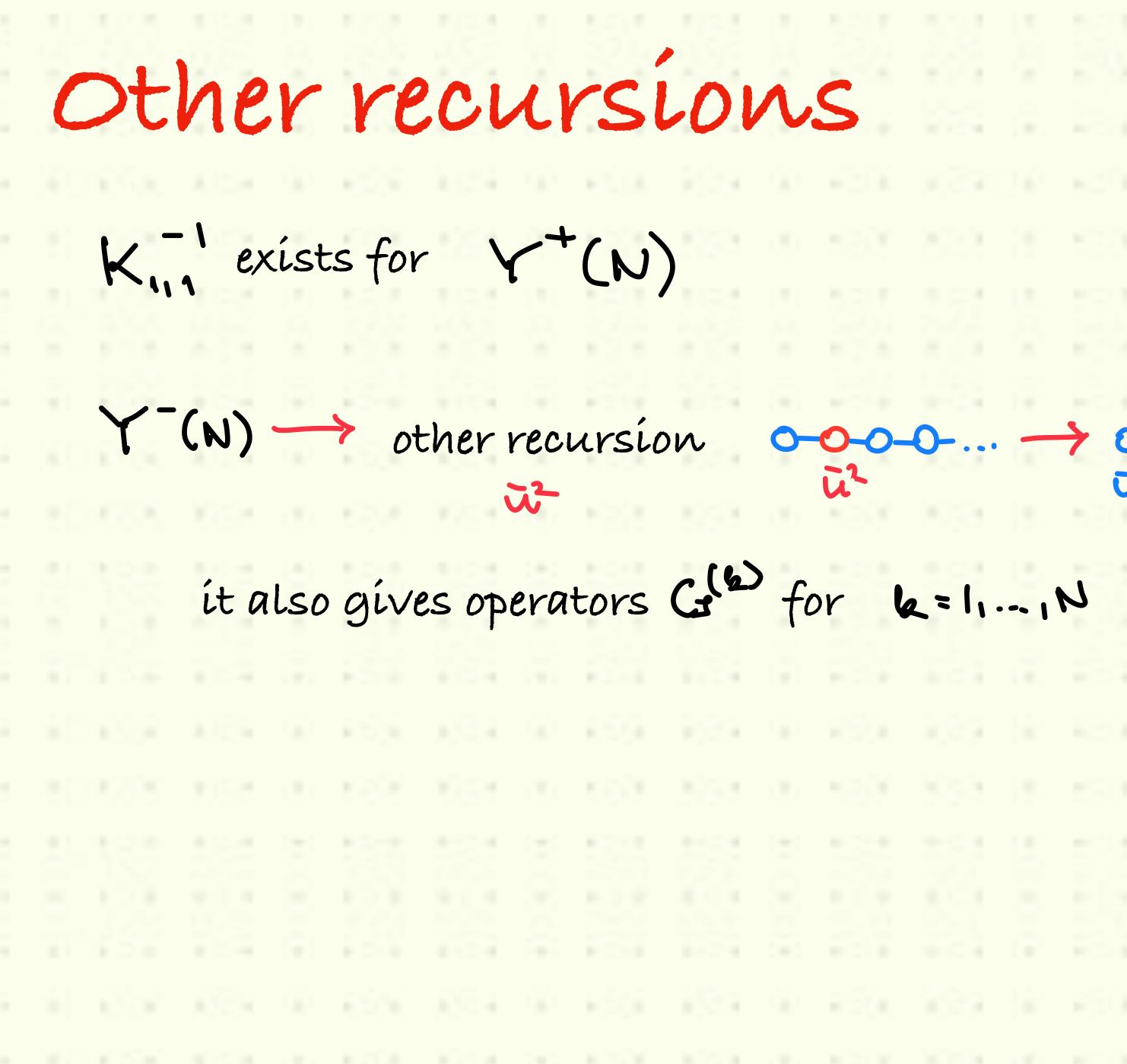


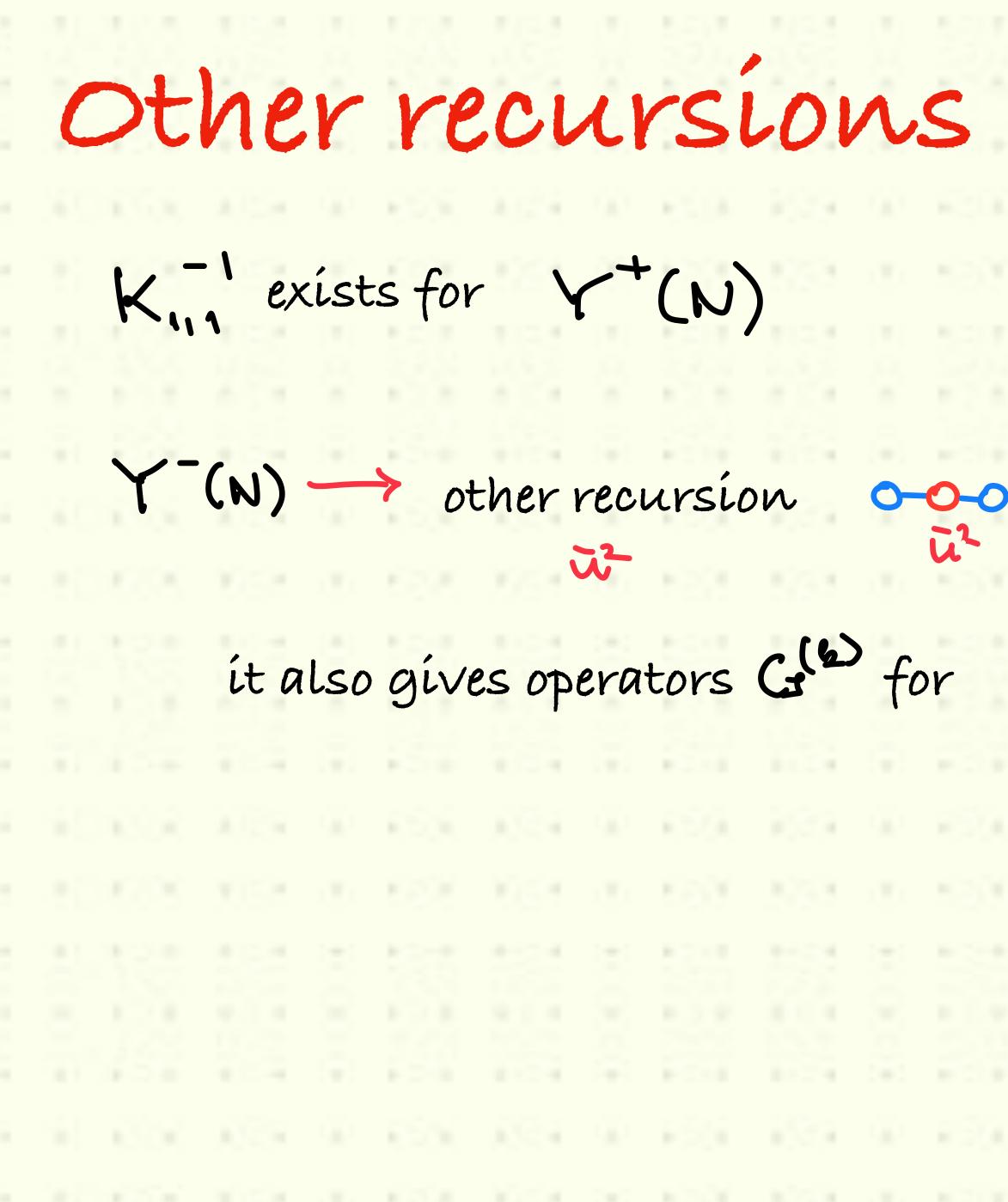
Other recursions K_{11} exists for $Y^+(N)$ 22



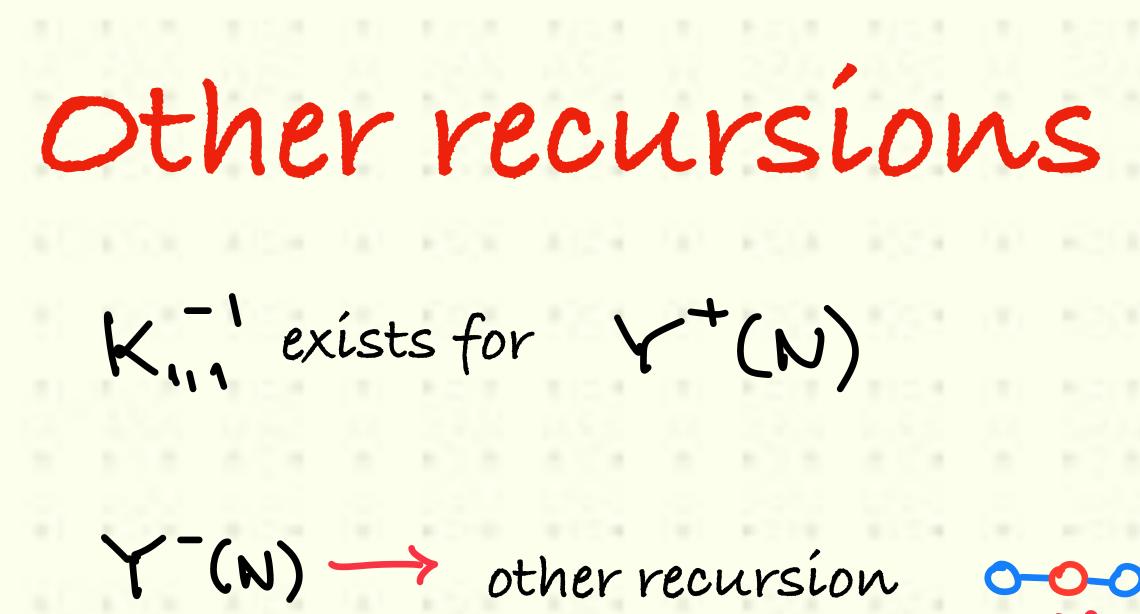
 $\Upsilon^{-}(N) \rightarrow$ other recursion







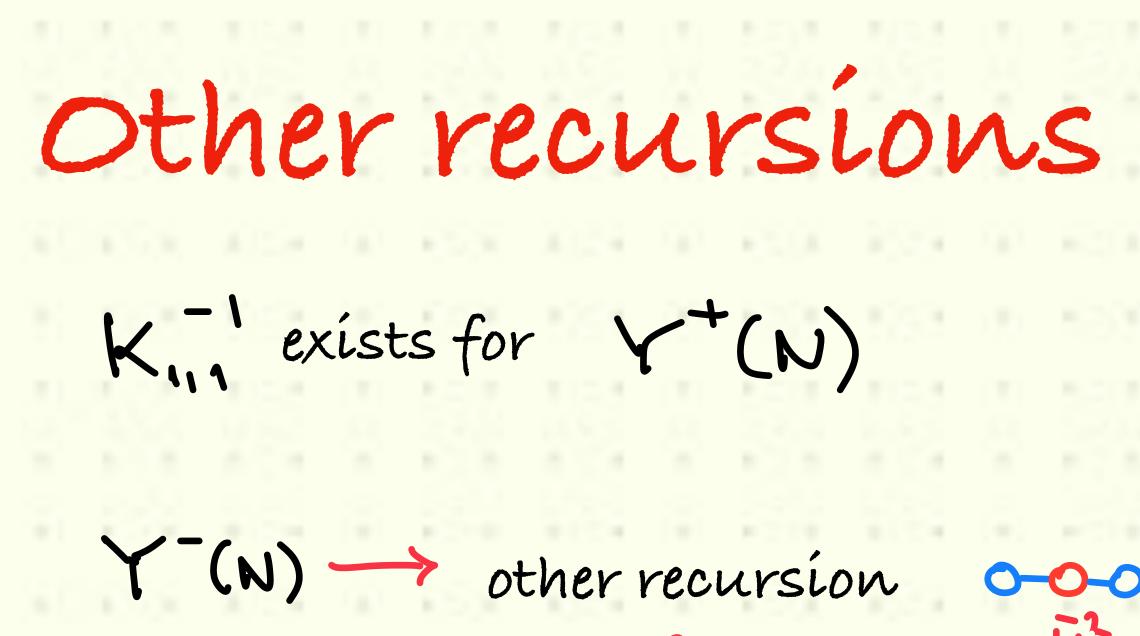
)-	0		> 0 ū	0-0 ū ² ū4	• • • • •			
6	ر = ۱ ر .	1	N	[G ^(e)	(w), (ʒ ^(ℓ) (♂)]	= 0	



it also gives operators G for

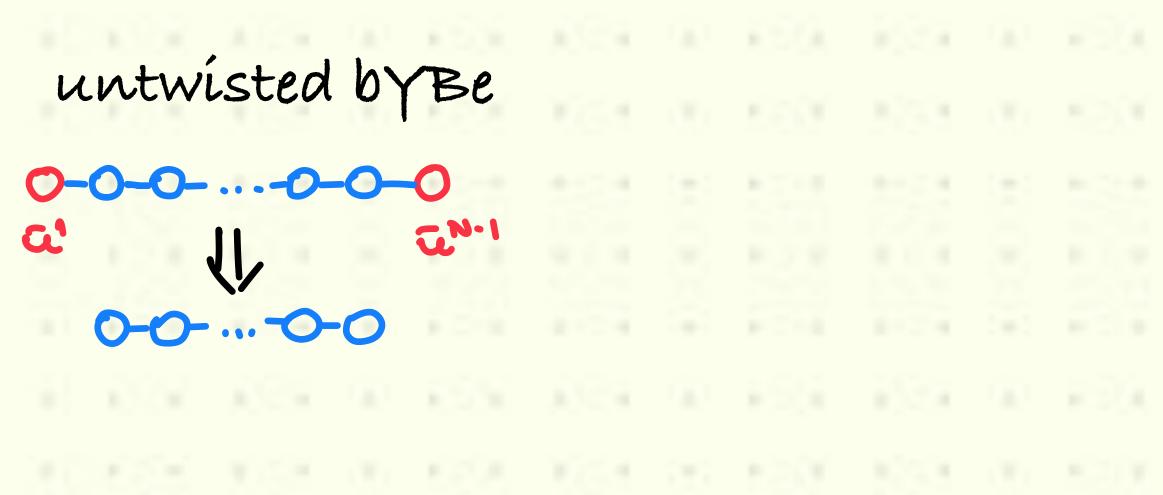
untwisted by Be

)-	0		> 0 ū	0-0 ū ⁾ ū ⁴	• • • • •			
6	ر = ۱ ر .	1	N	[G ^(e)	(w), (ʒ ^(ℓ) (♂)]	= 0	

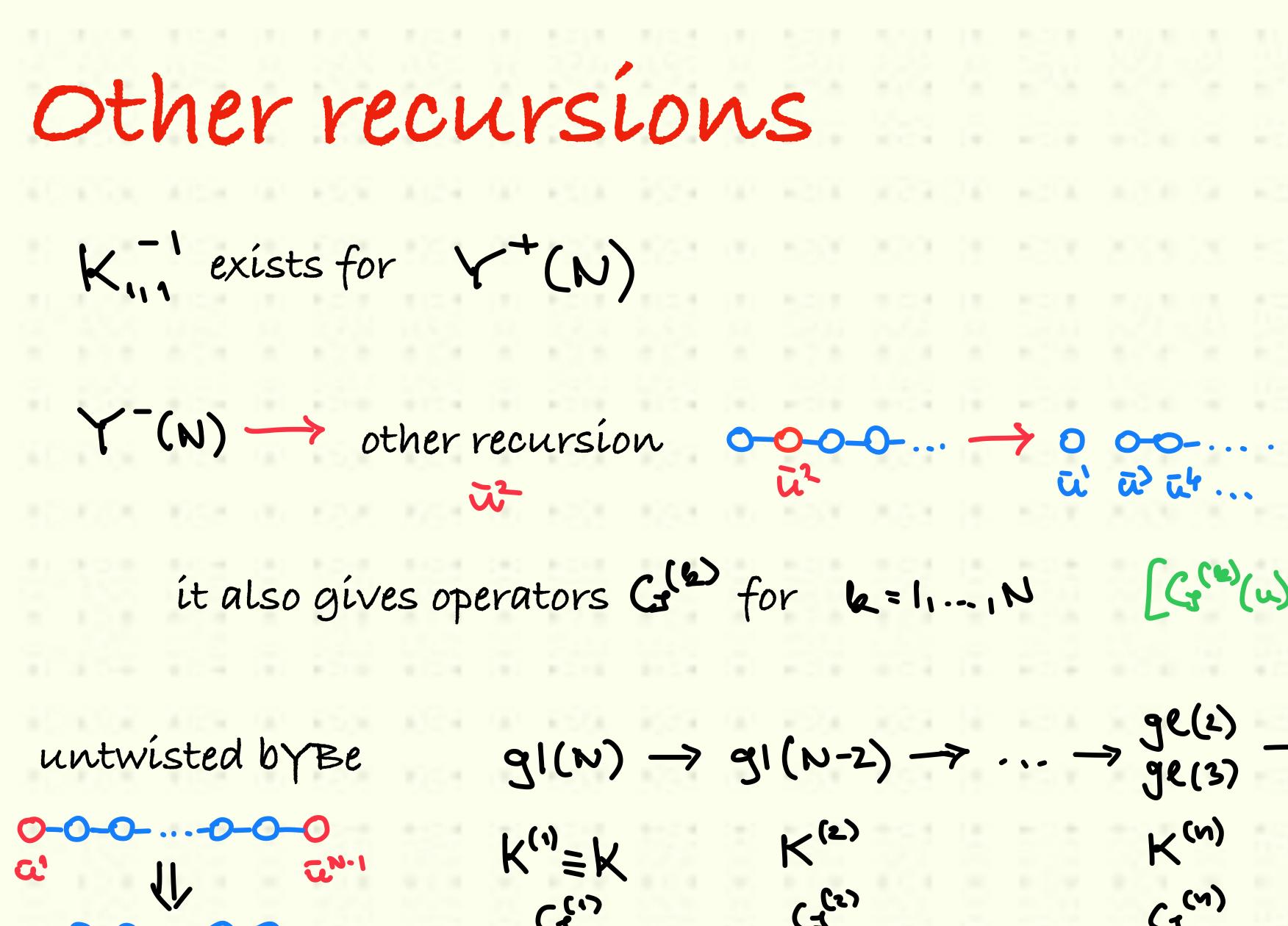


it also gives operators G for

J.



)-	0		> 0 ū	0-0 ū ⁾ ū ⁴	• • • • •			
6	ر = ۱ ر .	1	N	[G ^(e)	(w), (ʒ ^(ℓ) (♂)]	= 0	



it also gives operators $G^{(b)}$ for k = 1, ..., N $[G^{(b)}(w), G^{(c)}(w)] = 0$ $g(N) \rightarrow g(N-2) \rightarrow \cdots \rightarrow g(2) \rightarrow g(1)$ K⁽ⁿ⁾ K⁽ⁿ⁺¹⁾ G(1+1) (M) n=LN/2

On-shell overlaps without twist

 $\mp^{(k)} = \left[G^{(k)} \right]^{-1} \left[G^{(k+1)} \longrightarrow \left[\mp^{(k)} (\omega) \right] \mp^{(l)} (\omega) \right] = 0$

On-shell overlaps without twist

 $\mp^{(k)} = \left[G^{(k)} \right]^{-1} G^{(k+1)} \longrightarrow \left[\mp^{(k)} (\omega) \right] \mp^{(l)} (\omega) = 0$

10.000	10.25	(V)	6.00	1044	121	1201	1223	(V)	1000

choosing diagonal basis $F^{(b)} = diag(F_{1}, \dots, F_{d_{B}}^{(b)})$

On-shell overlaps without twist

 $\mp^{(k)} = \left[G^{(k)} \right]^{-1} G^{(k+1)} \longrightarrow \left[\mp^{(k)} (\omega) \right] \mp^{(l)} (\omega) = 0$

 $\langle MPS|\bar{u}\rangle = \int_{a=1}^{b} T \widetilde{f}_{a}^{(s)}(u^{+s}) \times \frac{det G}{det G}$ 23

choosing diagonal basis $F^{(b)} = diag(F_{1}^{(b)}, F_{d_B}^{(b)})$

On-shell overlaps without twist

 $\mp^{(k)} = \left[G^{(k)} \right]^{-1} G^{(k+1)} \longrightarrow \left[\mp^{(k)} (\omega) \right] \mp^{(l)} (\omega) = 0$

Crossed

choosing diagonal basis $F^{(b)} = diag(F_{1}^{(b)}, F_{d_B}^{(b)})$

Non-crossed

 $\widetilde{T}_{\chi}^{(s)}(u) = \overline{T}_{\chi}^{(s)}(u-i\frac{s}{2})\sqrt{\frac{u^{2}}{u^{2}+1}/4r}} \qquad \widetilde{T}_{\chi}^{(s)}(u) = \begin{cases} \overline{T}_{\chi}^{(s)}(u-c_{s}) \\ \overline{T}_{\chi}^{(b_{2})}(u-c_{b_{1}})\sqrt{\frac{u^{2}}{u^{2}+1}/4r} \\ \overline{T}_{\chi}^{(b_{2})}(u-c_{b_{1}})\sqrt{\frac{u^{2}}{u^{2}+1}/4r} \end{cases}$ 23

Other spin chains

Symmetry of the spin chain	Type of refl.	Resídual symmetry	Paír structure
	AI	0(N)	Chiral
gl(N)	All	sp(N)	Chiral
KGR (0) E228 (0) SS1 (0)	AIII	gl(M)+gl(N-M)	Achiral
o(2n+1)	BI	o(M) + o(N-M)	Chiral
sp(2n)	CI	sp(2m)+sp(2n-2m)	Chiral
Sp(~10)	CII	gl(n)	Chíral
	DI	o(M) + O(2n-M)	n-M=0 (mod 2) chíra
0(2n)		0(1+()+0(2+0+1+()))	n-M=1 (mod 2) achrío
0 (200)	DII	gl(n)	$n=0 \pmod{2}$ chiral
		90(10)	n=1 (mod 2) achrial

The on-shell overlaps are extended to other rational spin chains without proofs

Other spin chains

symmetry of the spin chain	Type of refl.	Resídual symmetry	Paír structure	
	AI	0(N)	Chíral	
gl(N)	All	sp(N)	Chiral	
Mich IV. BOX Mich IV.	AIII	gl(M)+gl(N-M)	Achiral	
0(2n+1)	BI	o(M) + o(N-M)	Chiral	
sp(2n)	CI	sp(2m)+sp(2n-2m)	Chiral	
Sp(~/~)	CII	gl(n)	Chiral	
	DI	o(M)+O(2n-M)	n-M=0 (mod 2) chíral n-M=1 (mod 2) achríal	
o(2n)		O(1/1) + O(2/1-1/1)		
0(~~~)	DII	gl(n)	$n=0 \pmod{2}$ chiral	
		90(10)	n=1 (mod 2) achrial	

The overlaps are also conjectured for graded spin chains, including gl(mn) and osp(m 2n)

The on-shell overlaps are extended to other rational spin chains without proofs

Conclusions

1					
2					
-					
3					
2					
2					
-					
24					
1.1					

Conclusions Cartan subalgebra $F^{(s)}(u)$

conclusions

Cartan subalgebra $F^{(s)}(u)$

On-shell overlaps for integrable MPS $\langle MPS|\bar{u} \rangle = \int_{a}^{b} T \widetilde{f}_{a}^{(s)}(\bar{u}^{*s}) \times \frac{det G}{det G}$

10.0	16	≤ 0	1000	:03	1000	103	10.000

conclusions

Cartan subalgebra $F^{(s)}(u)$

On-shell overlaps for integrable MPS

where $\widetilde{F}_{(u)}^{(s)}(u)$ are the eigenvalues of the operators $F^{(s)}(u)$

conclusions

Cartan subalgebra $F^{(s)}(u)$ On-shell overlaps for integrable MPS

where $\tilde{F}_{(u)}^{(s)}(u)$ are the eigenvalues of the operators $F^{(s)}(u)$

1) KT-relation: creation to annihilation Proof is possible if

> 2) recurrence formula 3) action formula 4) co-product formula

 $|\overline{u}\rangle = \sum (...) |\overline{u}_{r}\rangle^{(n)} \otimes |\overline{u}_{m}\rangle^{(n)}$

 $T_{i,j}(z)(u) = Z(...)(w)$

 $\{\{z, \bar{u}\}, \bar{u}^2, ..., \} = \mathbb{Z}(...) T_{A,j}(z) | \bar{u}^1, \bar{v}^2, ... \}$

$$\frac{\langle MPS|\bar{u}\rangle}{\langle \overline{u}\overline{u}\overline{u}\rangle} = \begin{bmatrix} d_{e} \\ \sum_{\alpha \in \Lambda} T \\ d_{\alpha} \widetilde{f} \\ d_{\alpha} \widetilde{f}$$

MPS + K-matrix + Representation of a given reflection algebra