Tamas Gombor

Based on: arXív:2110.07960 arXív:2311.04870 and recent unpublished work REN **GIGICI** 

## Exact overlaps for boundary states





• Motivation

Overlaps of two site states for gl(2)
Overlaps of two site states for gl(N)
Overlaps of matrix product states for

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### Motivation

### why boundary state overlaps?

### boundary state g

 $\langle \Psi | \overline{u} \rangle$ 

### Motivation

### Why boundary state overlaps?

In statistical physics

### boundary state h

Time evolution from initial state  $|\psi\rangle$ The overlaps are input for Quench Action

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Time evolution from initial state  $|\psi\rangle$ The overlaps are input for Quench Action

 $\langle \Psi | \overline{u} \rangle$ 

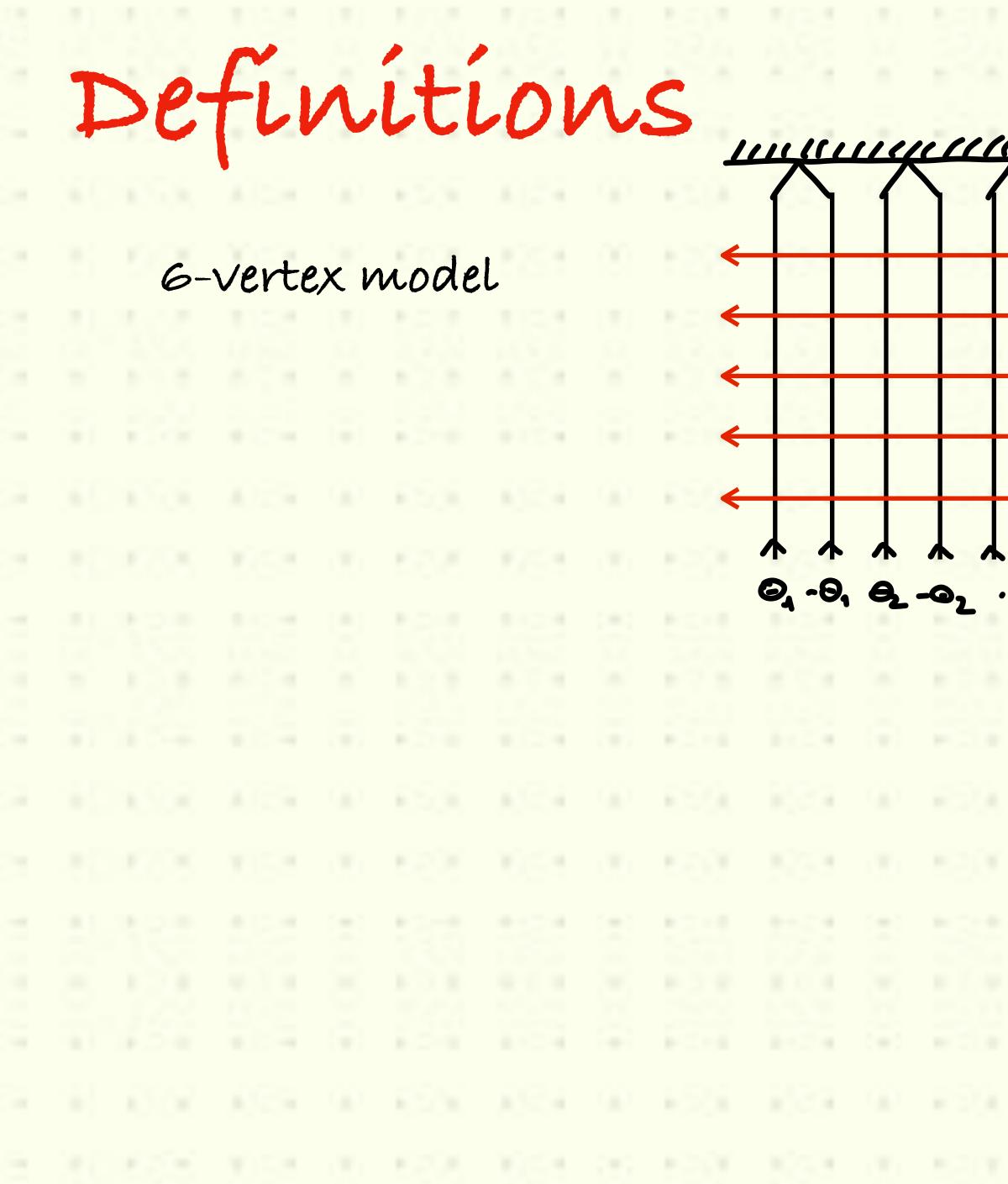
**G**Bethe state

The overlaps correspond to 1-pt functions of defect theories

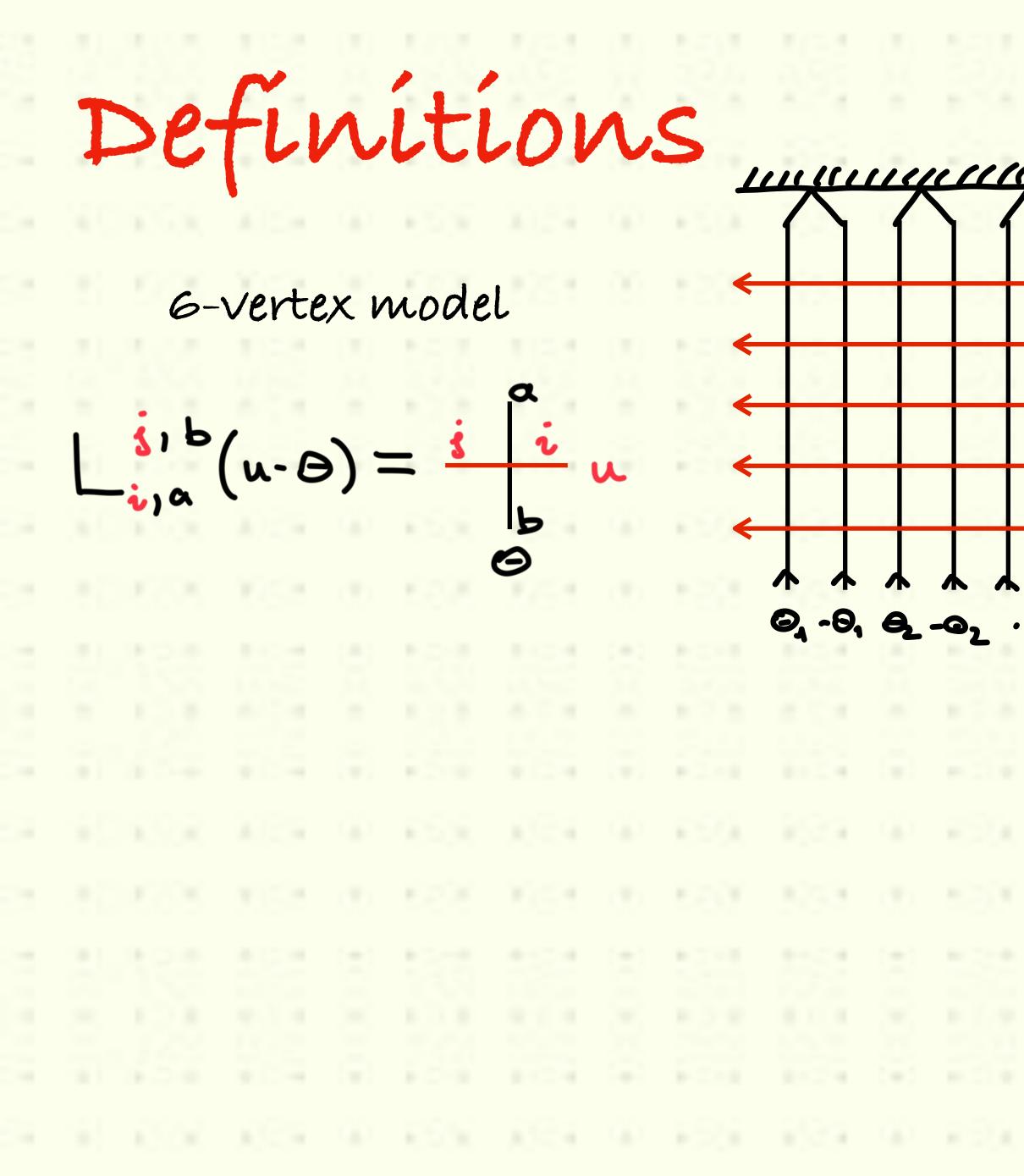


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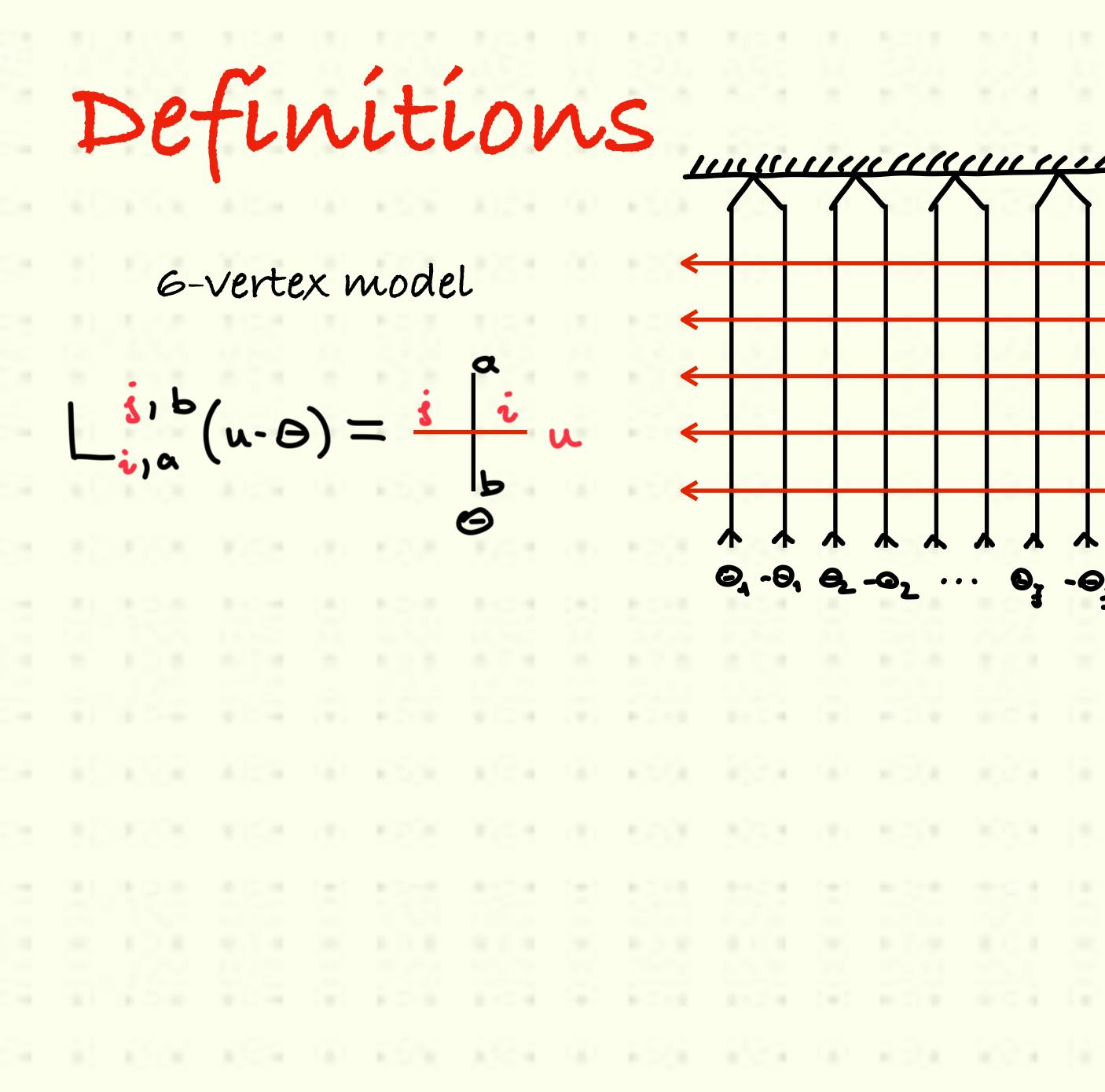
of gl(2) spin chains 4



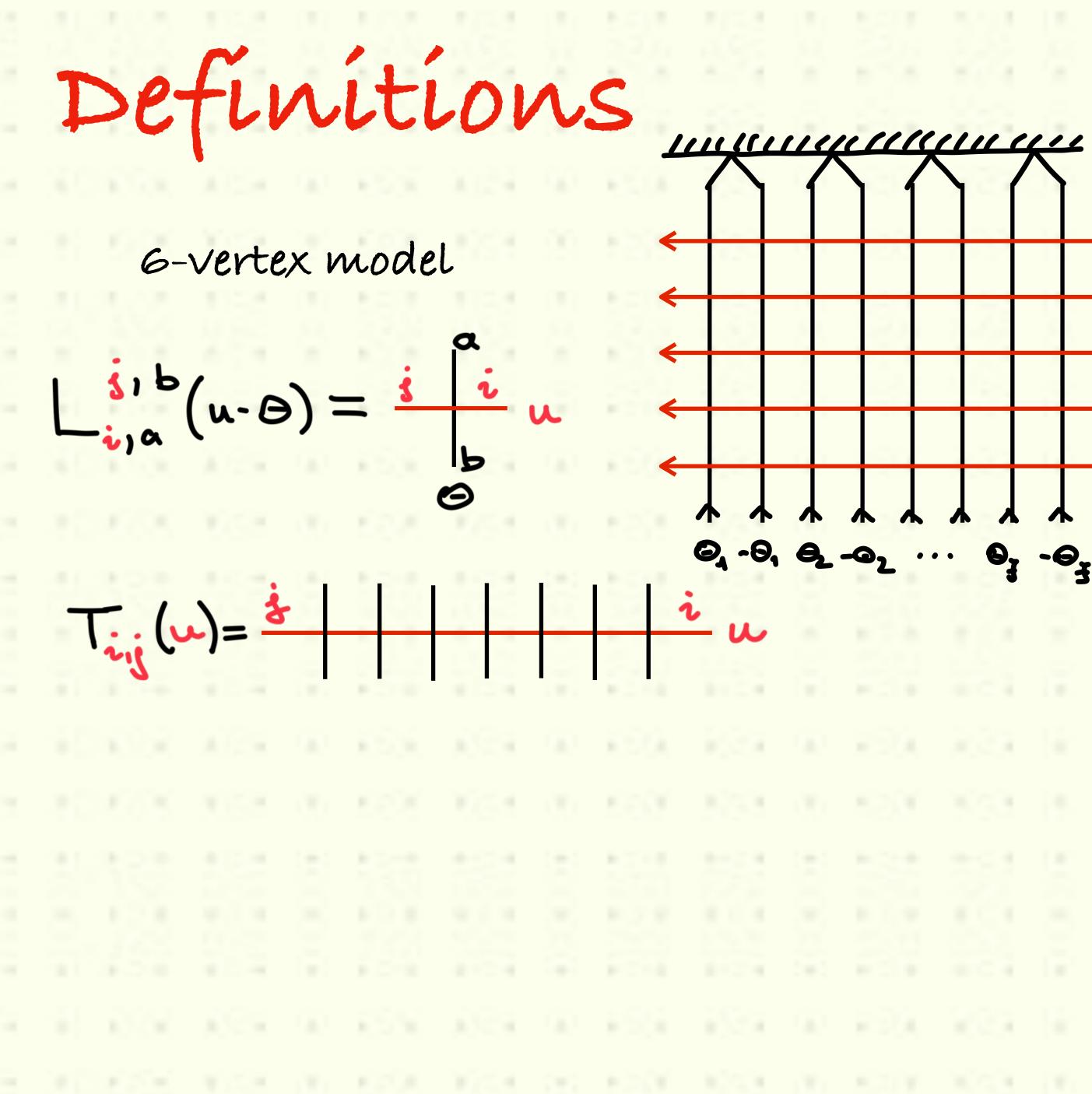
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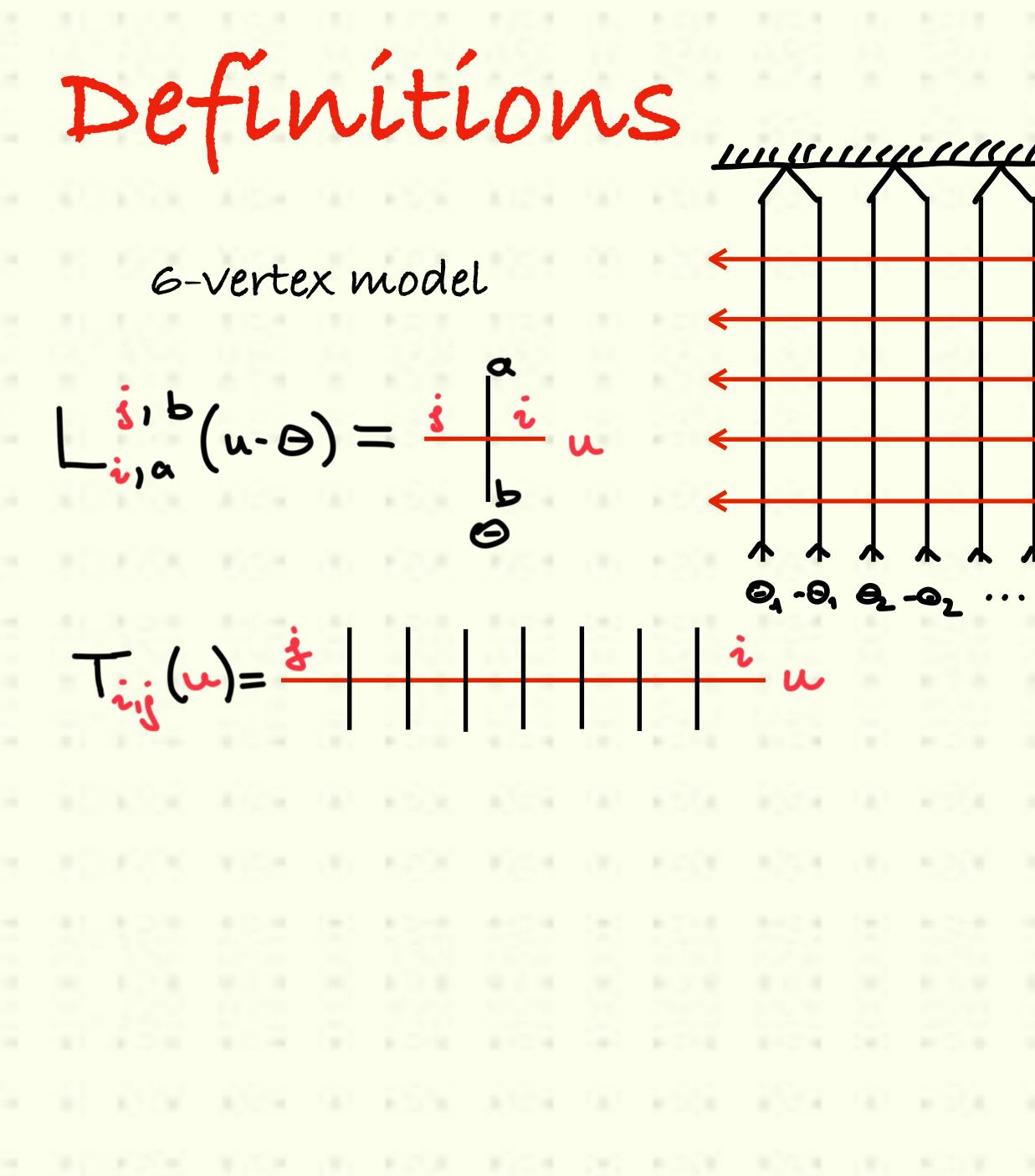
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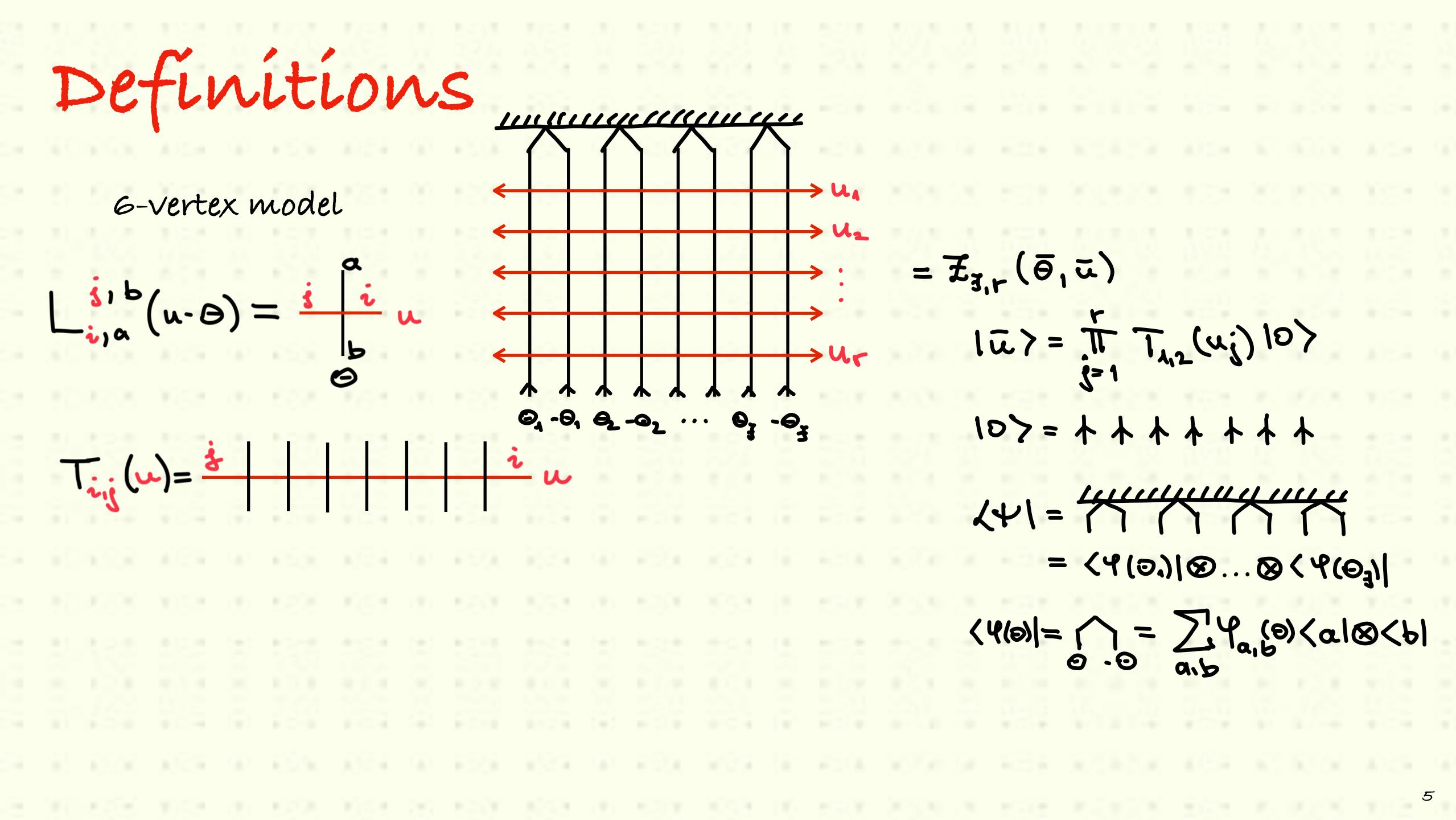
 $= \mathcal{I}_{3,r}(\bar{\Theta},\bar{u})$  $\Theta_1 - \Theta_1 \Theta_2 - \Theta_2 \cdots \Theta_r - \Theta_r$ -5



 $= \mathcal{I}_{3,r}(\bar{\Theta},\bar{u})$ -5

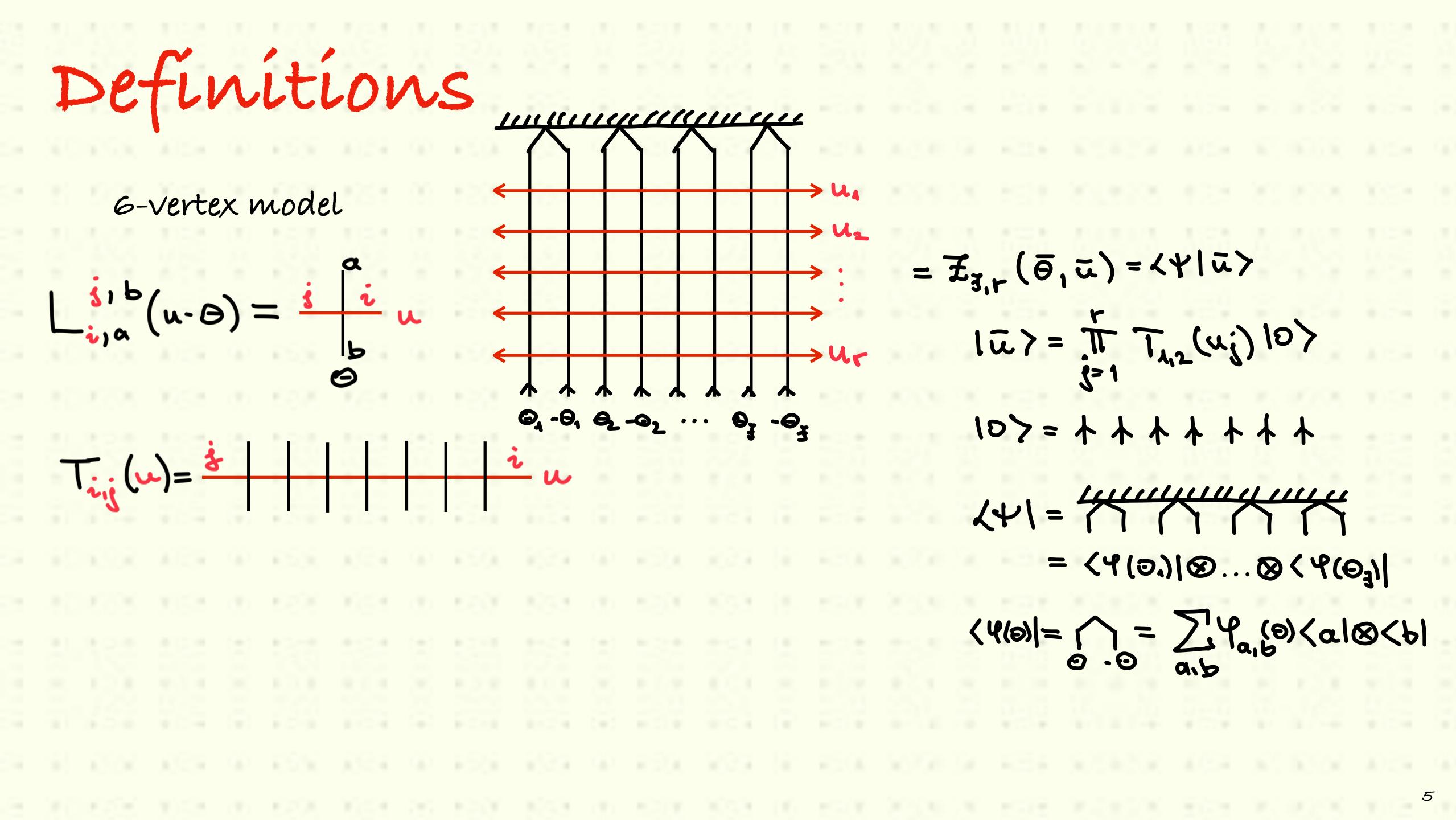


 $= \mathcal{I}_{\mathfrak{Z},r}(\bar{\mathfrak{O}},\bar{\mathfrak{u}})$  $|\bar{u}\rangle = \pi T_{1/2}(u_j)|0\rangle$ 10>= ト ト ト ト ト ト -5



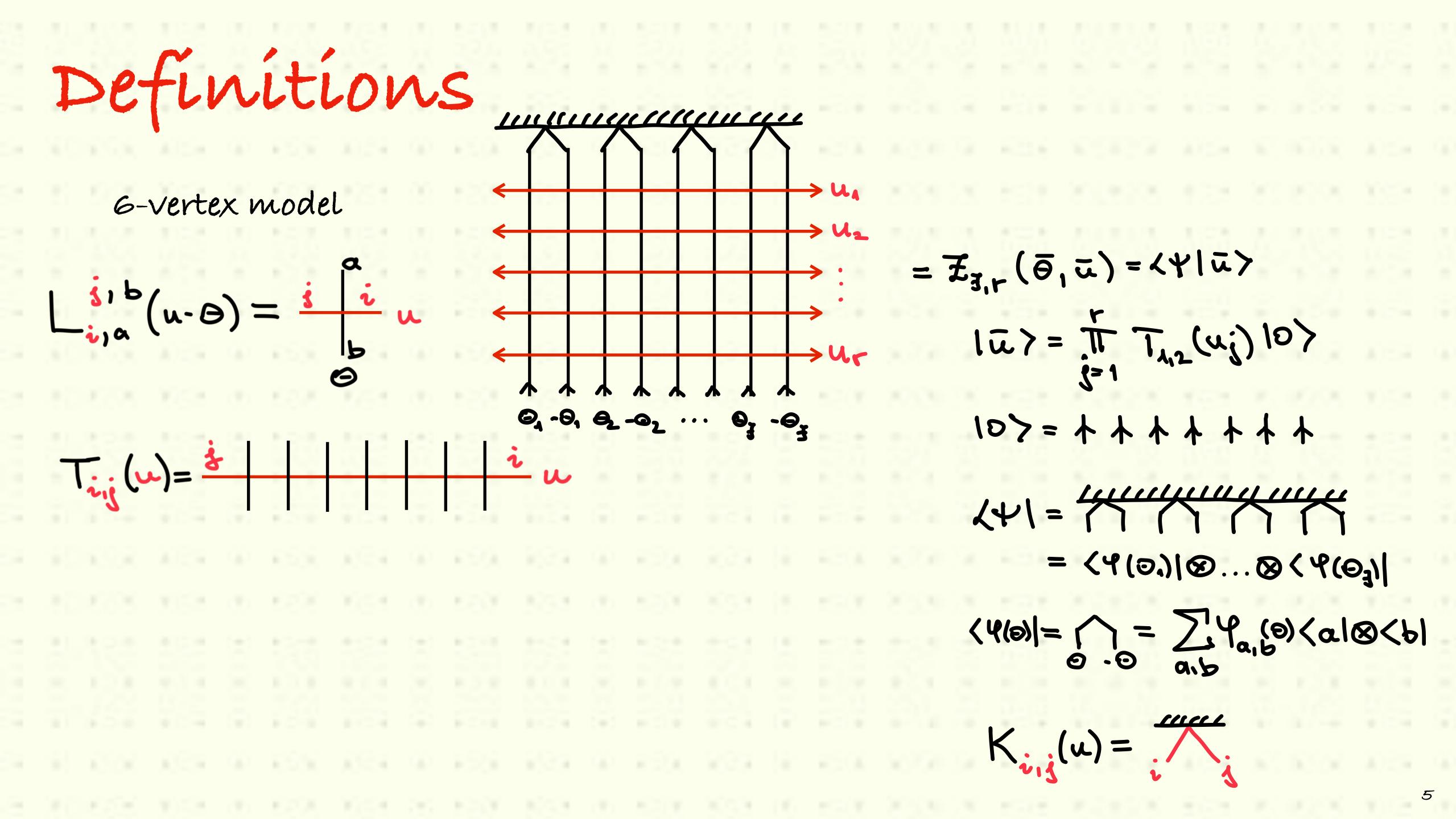
 $= \mathcal{I}_{\mathfrak{Z},r}(\bar{\mathfrak{O}},\bar{\mathfrak{u}})$  $|\bar{u}\rangle = \frac{r}{\pi} T_{1/2}(u_j)|0\rangle$ 10>= イイイイイイ Hund me 141=  $= \langle \Psi(\Theta_1) | \otimes \ldots \otimes \langle \Psi(\Theta_1) |$  $= \sum Y_{a,b}(0) \langle a | \otimes \langle b |$ < 4(6) =





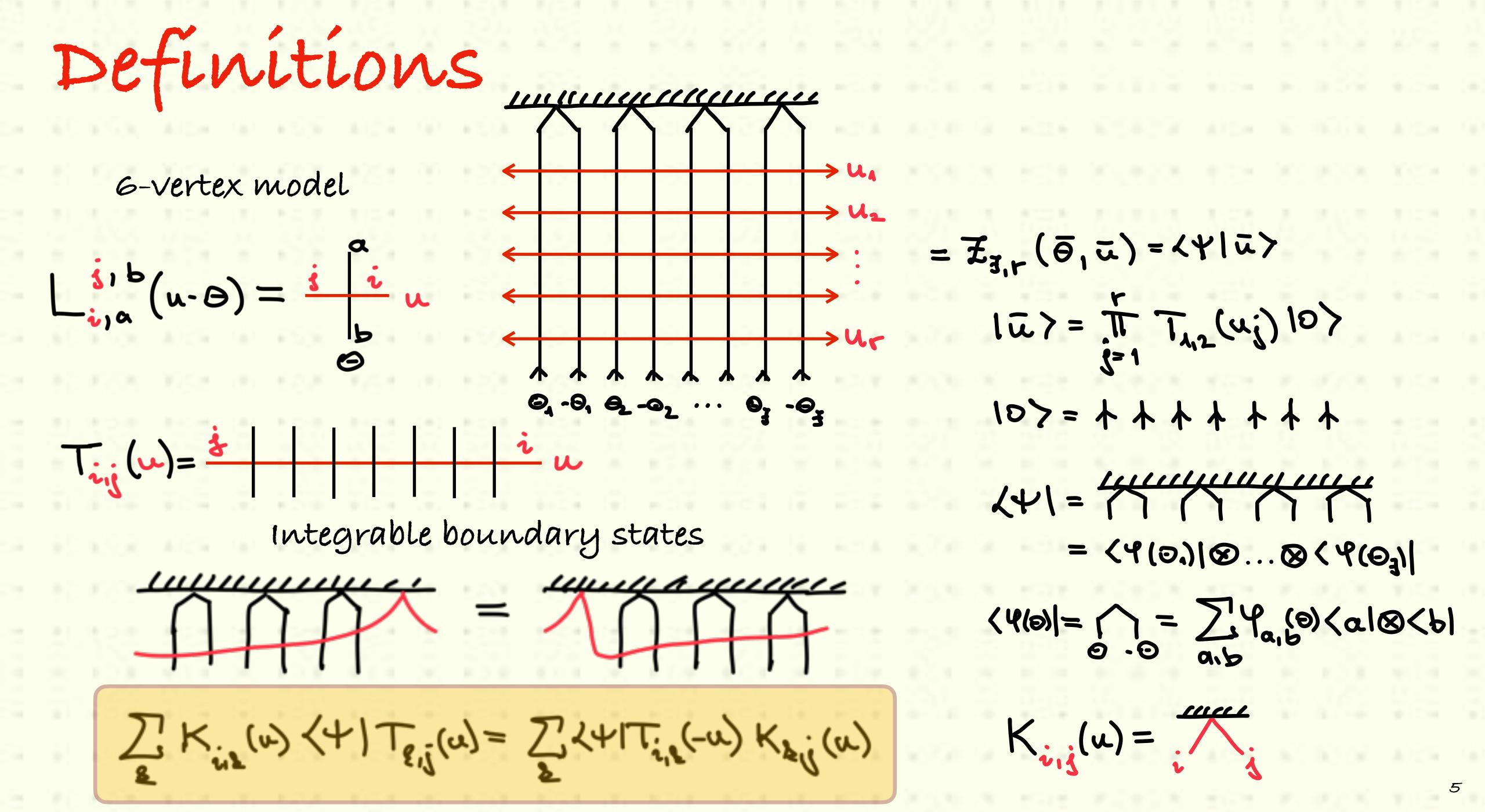
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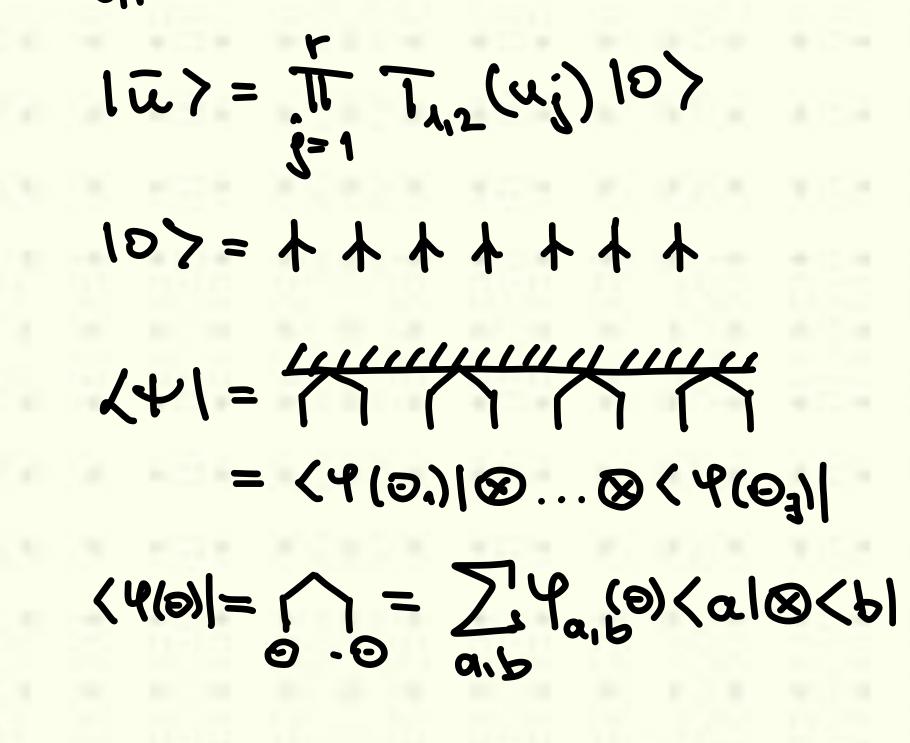




 $= \mathcal{I}_{3,r}(\bar{\Theta},\bar{u}) = \langle \Psi | \bar{u} \rangle$  $|u_{r}\rangle = \prod_{i=1}^{r} T_{u_{i}2}(u_{i})|0\rangle$ 10>= トトトトトトト Hun hun イヤ =  $= \langle \Psi(\Theta_1) | \otimes \ldots \otimes \langle \Psi(\Theta_1) |$  $= \sum Y_{a,b}(\theta) \langle a | \otimes \langle b |$ く 4(の) =







 $= \mathcal{I}_{3,r}(\bar{\Theta},\bar{u}) = \langle \Psi | \bar{u} \rangle$ 



### $K_{o}(z) < 4 | T_{o}(z) = < 4 | T_{o}(-z) K_{o}(z)$



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### Properties of the KT-relation <u>ummunner</u> $K_{o}(z)(4|T_{o}(z)) = (4|T_{o}(-z)K_{o}(z))$

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	1	1.5	$W := \mathbb{R}^{n}$	191	6295	$V \leq 0$	121	1201	1223	 520 V	10.00	11	$g \leq 1/\ell$	20200-22	0.803	1000	100	$(\mathbf{r}_{i})$	0.08

### Compatibility with the RTT-relation $R_{12}(u-v) T_{1}(w) T_{2}(v) = T_{2}(v) T_{1}(w) R_{12}(u-v)$



 $K_{0}(z)(4|T_{0}(z)) = (4|T_{0}(-z)K_{0}(z))$ 

 $\langle \Psi | T_{1}(z_{1}) T_{2}(z_{2}) = R \langle \Psi | T_{2}(z_{2}) T_{1}(z_{1}) R = ... = (...) \langle \Psi | T_{1}(z_{1}) T_{2}(z_{2}) (...)$ 

## Compatibility with the RTT-relation $R_{12}(u-v) T_{1}(w) T_{2}(v) = T_{2}(v) T_{1}(w) R_{12}(u-v)$

10.00	15.01	1000	193	1000	101	10.00

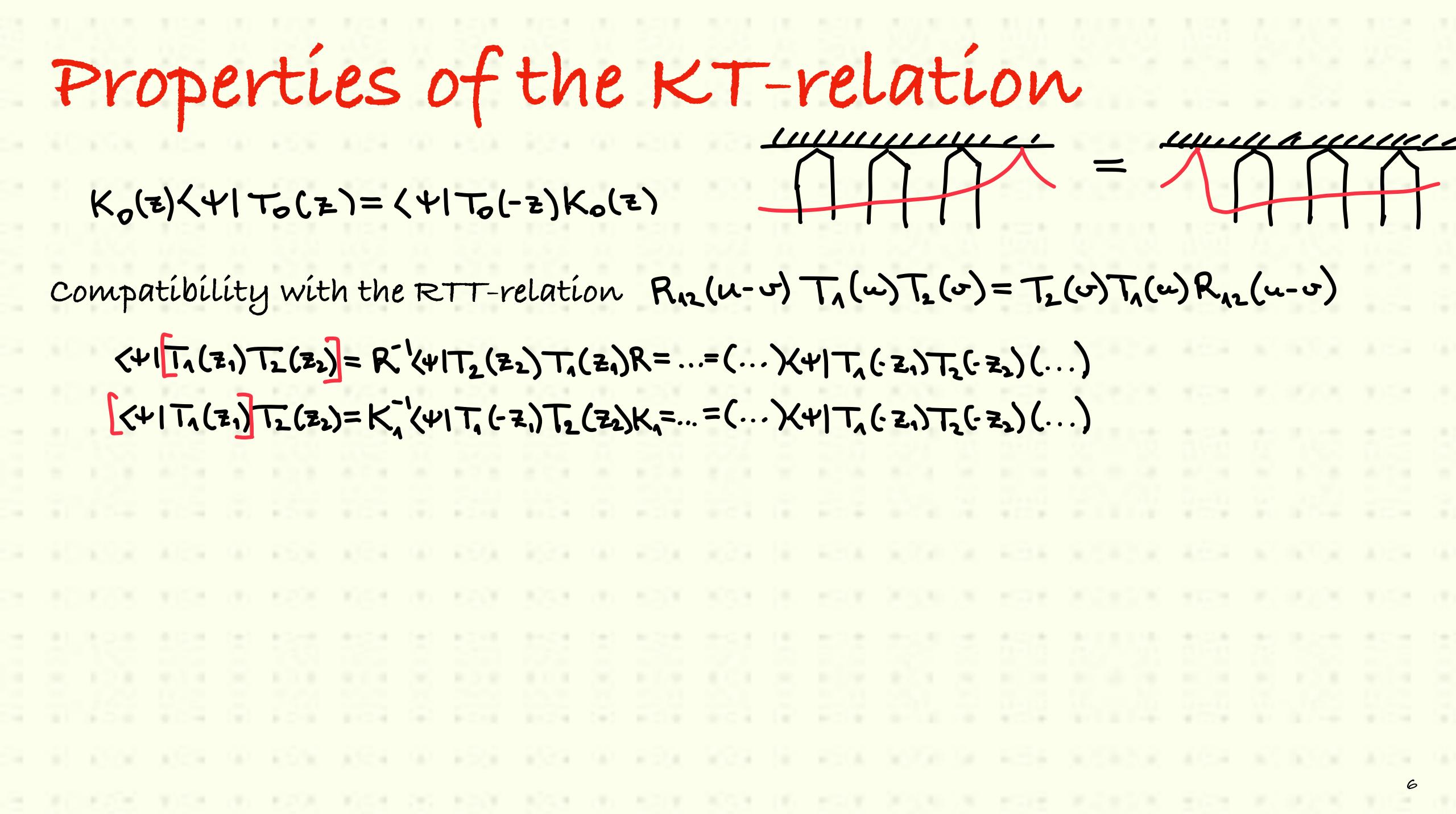


 $K_{0}(z)(4|T_{0}(z)) = (4|T_{0}(-z)K_{0}(z))$ 

 $\langle \Psi | T_{1}(z_{1}) T_{2}(z_{2}) = R \langle \Psi | T_{2}(z_{2}) T_{1}(z_{1}) R = ... = (... ) \langle \Psi | T_{1}(\cdot z_{1}) T_{2}(\cdot z_{2}) (...)$  $(+|T_{1}(z_{1})T_{2}(z_{2})=K_{1}'(+|T_{1}(-z_{1})T_{2}(-z_{2})K_{1}=...=(...)K_{1}|T_{1}(-z_{1})T_{2}(-z_{2})(...)$ 



# Compatibility with the RTT-relation $R_{12}(u-s) T_{1}(w) T_{2}(s) = T_{2}(s) T_{1}(w) R_{12}(u-s)$



 $K_{o}(z)(4|T_{o}(z)) = (4|T_{o}(-z)K_{o}(z))$ 

 $\langle \Psi | T_{4}(z_{1}) T_{2}(z_{2}) = R' \langle \Psi | T_{2}(z_{2}) T_{4}(z_{4}) R = ... = (...) \langle \Psi | T_{4}(\cdot z_{1}) T_{2}(\cdot z_{2}) (...)$  $(+|T_1(z_1)T_2(z_2)=K_1(+|T_1(-z_1)T_2(z_2)K_1=...=(...)X_1|T_1(-z_1)T_2(-z_2)(...)$ 

 $\Rightarrow$  reflection equation  $R_n(u \cdot \sigma)K_1(-u)R_n(u + \sigma)K_2(-\sigma)=K_2(-\sigma)R_n(u + \sigma)K_1(-u)R_n(u - \sigma)$ 

<u> Mullallelle</u>

- Compatibility with the RTT-relation  $R_{12}(u-v) T_{1}(w) T_{2}(v) = T_{2}(v) T_{1}(w) R_{12}(u-v)$





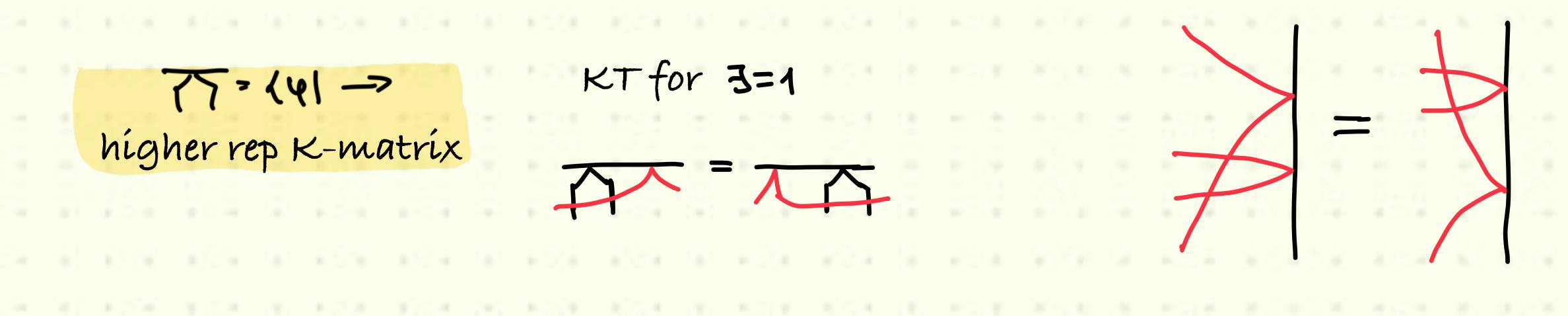
 $K_{0}(z)(4|T_{0}(z)) = (4|T_{0}(-z)K_{0}(z))$ 

 $\langle \Psi | T_{1}(z_{1}) T_{2}(z_{2}) = R' \langle \Psi | T_{2}(z_{2}) T_{1}(z_{1}) R = ... = (...) \langle \Psi | T_{1}(z_{1}) T_{2}(z_{2}) (...)$  $(+|T_{1}(z_{1})T_{2}(z_{2})=K_{1}'(+|T_{1}(-z_{1})T_{2}(z_{2})K_{1}=...=(...)X+|T_{1}(-z_{1})T_{2}(-z_{2})(...)$ 

KT for 3=1 フラーイターー

higher rep K-matrix

- Compatibility with the RTT-relation  $R_{12}(u-v) T_{1}(w) T_{2}(v) = T_{2}(v) T_{1}(w) R_{12}(u-v)$ 
  - $\Rightarrow$  reflection equation  $R_n(u \cdot \sigma)K_1(-u)R_n(u + \sigma)K_2(-\sigma)=K_2(-\sigma)R_n(u + \sigma)K_1(-u)R_n(u \sigma)$





# calculation of the Off-shell overlap (2,2) component of the KT-relation $K_{2,1}(z)(\psi|T_{1,2}(z) + K_{2,2}(z)(\psi|T_{2,2}(z) = \chi + |T_{2,1}(-z)K_{4,2}(z) + (\psi|T_{2,1}(-z)K_{2,2}(z))$ $\mathcal{F}$

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 $\mathcal{F}$ 

Creation diagonal annihilation

### calculation of the Off-shell overlap

(2,2) component of the KT-relation

 $K_{2,1}(z)(\psi|T_{1,2}(z) + K_{2,2}(z)(\psi|T_{2,2}(z) = \chi + |T_{2,1}(-z)K_{4,2}(z) + (\psi|T_{2,1}(-z)K_{2,2}(z))$ 

Assuming Kinto we can express 41 The with 2+1722 or 2+1721 Creation diagonal annihilation

off-shell overlap

 $S(\overline{u}) = \lambda + |\overline{u}\rangle$ 

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	10.000	$V \geq 0$	$(\mathbf{v})$	6.95	$Y \leq t$	20	120	1243	11	100

 $\mathcal{F}$ 

### calculation of the Off-shell overlap

(2,2) component of the KT-relation

 $K_{2,1}(z)(\psi|T_{1,2}(z) + K_{2,2}(z)(\psi|T_{2,2}(z) = \chi + |T_{2,1}(-z)K_{4,2}(z) + \langle \psi|T_{2,1}(-z)K_{2,2}(z)$ 

Assuming  $K_{2,1} \neq 0$  we can express  $4 + 1 T_{12}$  with  $2 + 1 T_{22}$  or  $4 + 1 T_{21}$ Creation diagonal annihilation

off-shell overlap

 $S(u) = \lambda + |u]$ 

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-										
	10.000	$V \geq 0$	$(\mathbf{v})$	6.95	$Y \leq t$	20	120	1243	11	100

 $|\overline{u}\rangle = \frac{m}{|\overline{u}|} T_{12}(u_j)|0\rangle$ 

### calculation of the Off-shell overlap

(2,2) component of the KT-relation

 $K_{2_{1}}(z)(\psi|T_{12}(z) + K_{22}(z)(\psi|T_{2_{12}}(z) = \chi+|T_{2_{1}}(-z)K_{42}(z) + (\psi|T_{2_{12}}(-z)K_{22}(z))$ 

Assuming  $K_{2,1} \neq 0$  we can express  $441T_{1,2}$  with  $2+1T_{2,2}$  or  $4+1T_{2,1}$ Creation diagonal annihilation

off-shell overlap

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	10.000	$V \geq 0$	$(\mathbf{v})$	6.95	$Y \leq t$	20	120	1243	11	100

 $|\overline{u}\rangle = \frac{n}{|\overline{u}|} T_{12}(u_j)|0\rangle$  $T_{i,i}(u) |0\rangle = \lambda_i(u) |0\rangle$  $\mathcal{F}$ 

### Calculation of the Off-shell overlap

(2,2) component of the KT-relation

 $K_{2_{1}}(z)(\psi|T_{12}(z) + K_{22}(z)(\psi|T_{2_{12}}(z) = \chi + |T_{2_{1}}(-z)K_{42}(z) + (\psi|T_{2_{12}}(-z)K_{22}(z))$ 

Assuming Kinto we can express 41712 with 2+1722 or 2+1721 Creation diagonal annihilation

off-shell overlap

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 $S_{A}(iz, u) = \langle + | T_{u2}(z) | u \rangle =$ 

 $|u\rangle = \frac{m}{\sqrt{2}} T_{42}(u_j)|0\rangle \qquad T_{i_ji}(u_j)|0\rangle = \lambda_i(u_j)|0\rangle$  $\mathcal{F}$ 

### Calculation of the Off-shell overlap

(2,2) component of the KT-relation

 $K_{2_{1}}(z)(\psi|T_{1,2}(z) + K_{2_{2}}(z)(\psi|T_{2_{1}}(z) = \chi + |T_{2_{1}}(-z)K_{4_{2}}(z) + (\psi|T_{2_{1}}(-z)K_{2_{2}}(z))$ 

Assuming  $K_{2,1} \neq 0$  we can express  $441T_{12}$  with  $2+1T_{2,2}$  or  $4+1T_{2,1}$ Creation diagonal annihilation

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off-shell overlap  $S(\overline{u}) = \lambda + |\overline{u}\rangle$ 

 $|\overline{u}\rangle = \frac{n}{|\overline{u}|} T_{12}(u_j)|_0\rangle \qquad T_{i,i}(u_j)|_0\rangle = \lambda_i(u_j)|_0\rangle$ 

 $S_{g}(i_{z_{1}}\bar{u}_{f}) = \langle \psi|T_{u_{2}}(z)|\bar{u}\rangle = \frac{K_{2,1}(z)}{K_{2,1}(z)} [\psi|T_{2,2}(-z)|\bar{u}\rangle - \langle \psi|T_{2,2}(z)|\bar{u}\rangle] + \frac{K_{4,2}(z)}{K_{2,1}(z)} \langle \psi|T_{2,2}(-z)|\bar{u}\rangle$ 

### Calculation of the Off-shell overlap

(2,2) component of the KT-relation

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Assuming  $K_{2,1} \neq 0$  we can express  $441T_{12}$  with  $2+1T_{2,2}$  or  $4+1T_{2,1}$ Creation diagonal annihilation

off-shell overlap  $S(\overline{u}) = \lambda + |\overline{u}\rangle$ 

 $S_{\lambda}(\{z,\bar{u}\}) = \sum(...)S_{\lambda}(\bar{w})$ 死こ {え-ころひん ¥ビッキ⊗

 $|\overline{u}\rangle = \frac{n}{|\overline{u}|} T_{12}(u_j)|_0\rangle \qquad T_{i,i}(u_j)|_0\rangle = \lambda_i(u_j)|_0\rangle$ 

 $S_{g}(i_{z_{1}}\bar{u}_{f}) = \langle \psi|T_{u_{2}}(z)|\bar{u}\rangle = \frac{K_{2,1}(z)}{K_{2,1}(z)} [\psi|T_{2,2}(-z)|\bar{u}\rangle - \langle \psi|T_{2,2}(z)|\bar{u}\rangle] + \frac{K_{4,2}(z)}{K_{2,1}(z)} \langle \psi|T_{2,2}(-z)|\bar{u}\rangle$ 

1) sum formula  $S_{\bar{\lambda}}(\bar{u}) = \sum_{\bar{u}=\bar{u}_{\bar{u}}\bar{v}\bar{u}_{\bar{u}}} W(\bar{u}_{\bar{u}}(\bar{u}_{\bar{u}})\lambda_{\bar{u}}(\bar{u}_{\bar{u}})\lambda_{\bar{u}}(\bar{u}_{\bar{u}}))$ 

 $\lambda_{k}(\bar{u}) = T \lambda_{k}(u_{j})$ 



1) sum formula  $S_{\bar{\lambda}}(\bar{u}) = \sum_{\bar{u}=\bar{u}_{I}\bar{v}\bar{u}_{I}} W(\bar{u}_{I}\bar{u}_{I}) \lambda_{I}(\bar{u}_{I}) \lambda_{2}(\bar{u}_{I})$ 

2) W( $\bar{u}_{r}|\bar{u}_{r}$ ) = f( $\bar{u}_{r},\bar{u}_{r}$ ) Z( $\bar{u}_{r}$ ) Z( $\bar{u}_{r}$ )

 $\lambda_{k}(\bar{u}) = T \lambda_{k}(u_{j})$ 

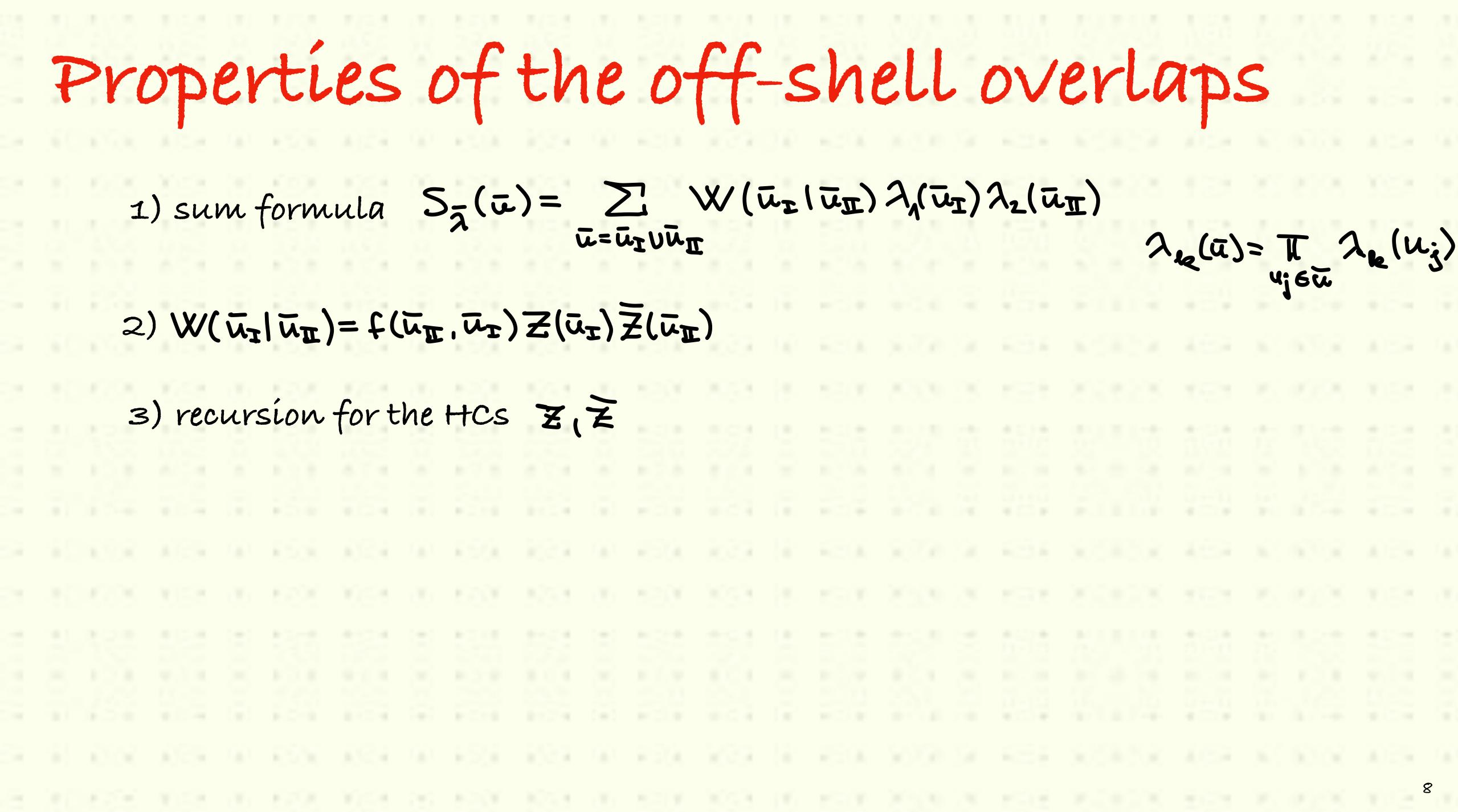


1) sum formula  $S_{\bar{\lambda}}(\bar{u}) = \sum_{\bar{u}=\bar{u}_{I}\bar{v}\bar{u}_{I}} W(\bar{u}_{I}\bar{u}_{I}) \lambda_{I}(\bar{u}_{I}) \lambda_{2}(\bar{u}_{I})$ 

2)  $W(\bar{u}_{I}|\bar{u}_{I}) = f(\bar{u}_{I},\bar{u}_{I})Z(\bar{u}_{I})\overline{Z}(\bar{u}_{I})$ 

3) recursion for the HCs Z,Z

 $\lambda_{k}(\bar{u}) = T \lambda_{k}(u_{j})$ 



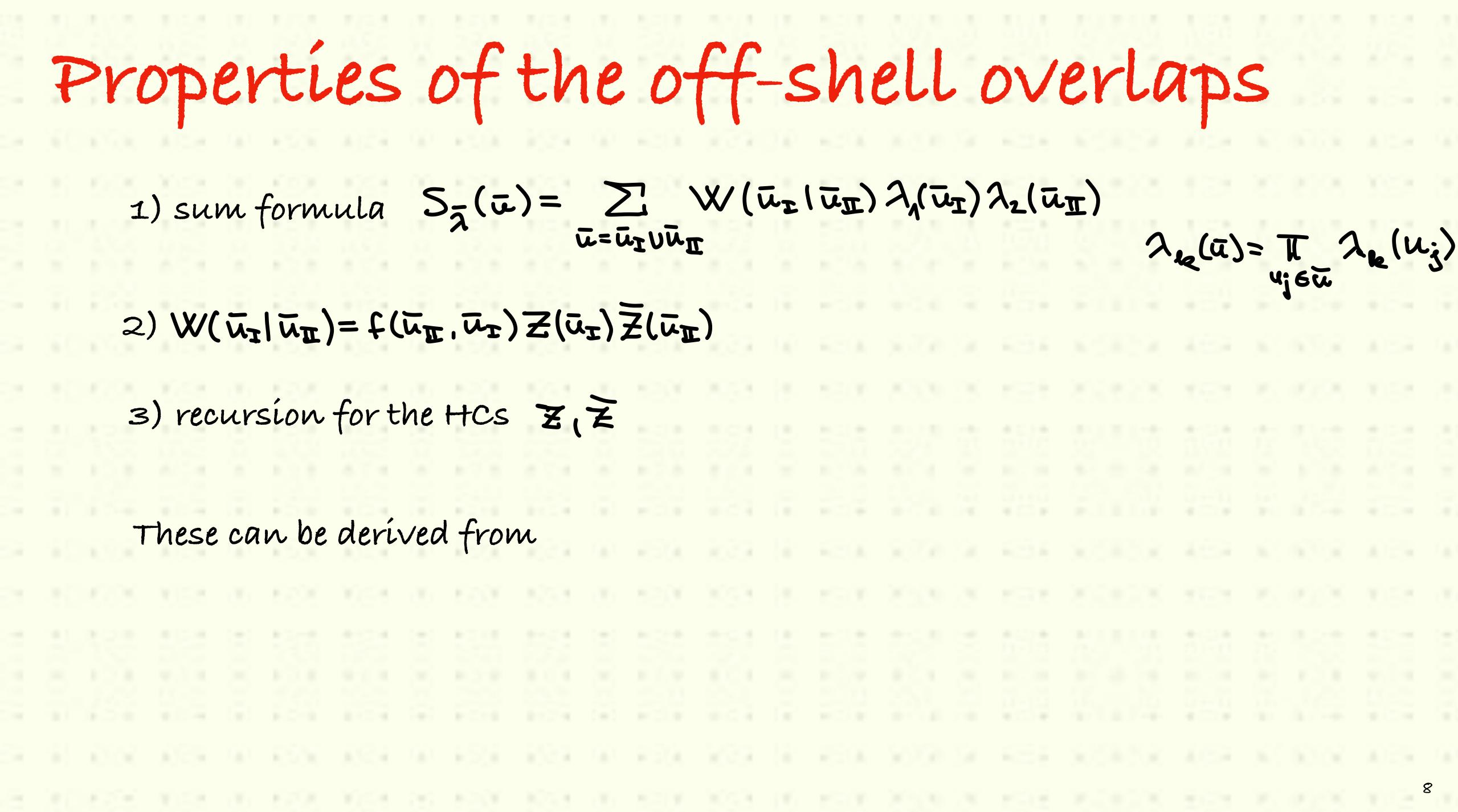
1) sum formula  $S_{\bar{\lambda}}(\bar{u}) = \sum_{\bar{u}=\bar{u}_{I}\bar{v}\bar{u}_{I}} W(\bar{u}_{I}\bar{u}_{I}) \lambda_{I}(\bar{u}_{I}) \lambda_{2}(\bar{u}_{I})$ 

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These can be derived from

 $\lambda_{k}(\bar{u}) = T \lambda_{k}(u_{j})$ 



1) sum formula  $S_{\bar{\lambda}}(\bar{u}) = \sum_{\bar{u}=\bar{u}_{I}} W(\bar{u}_{I} | \bar{u}_{I}) \lambda_{I}(\bar{u}_{I}) \lambda_{2}(\bar{u}_{I})$ 

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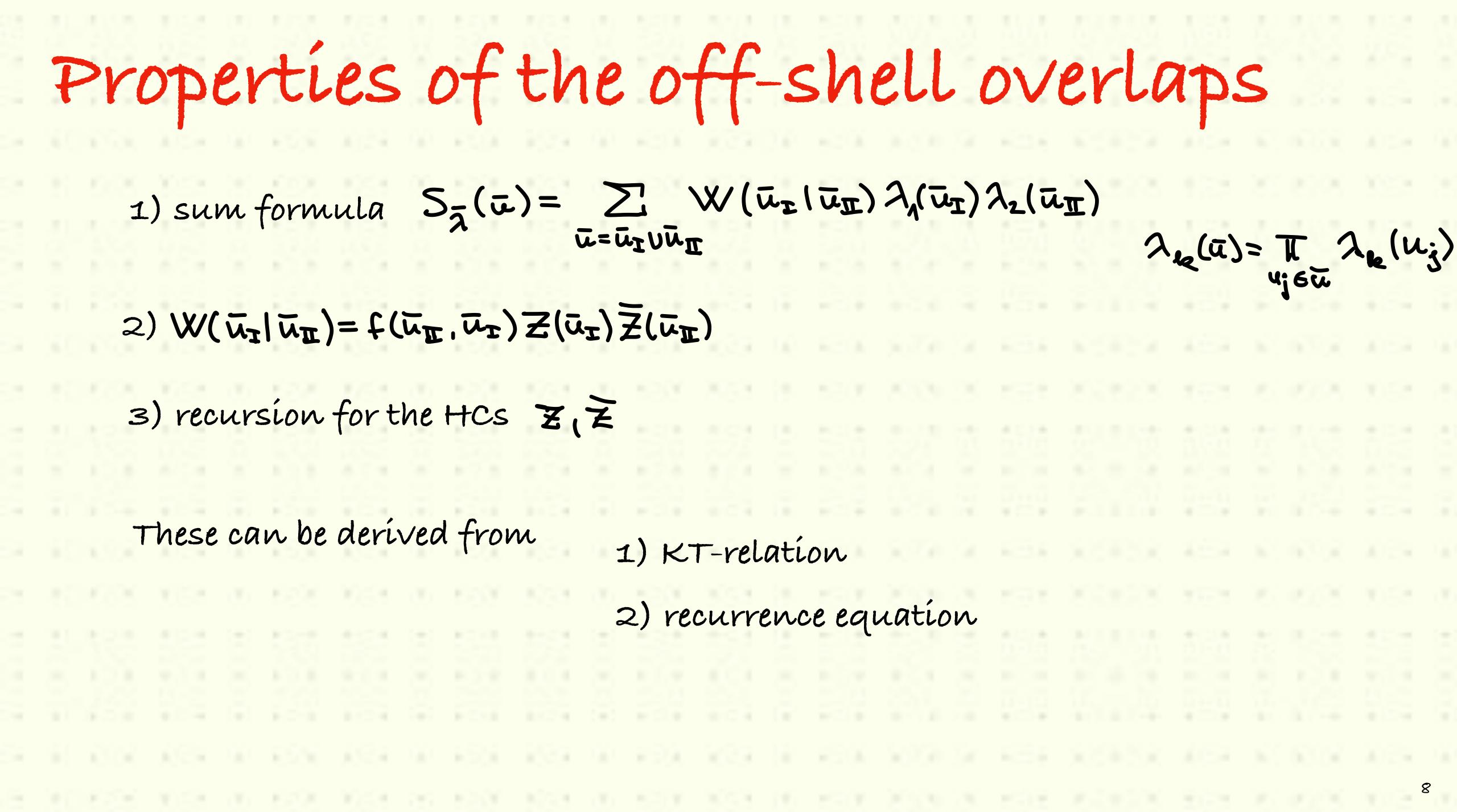
1) KT-relation



1) sum formula  $S_{\bar{\lambda}}(\bar{u}) = \sum_{\bar{u}=\bar{u}_{I}} W(\bar{u}_{I}(\bar{u}_{I})\lambda_{I}(\bar{u}_{I})\lambda_{I}(\bar{u}_{I}))$ 

2)  $W(\bar{u}_{I}|\bar{u}_{I}) = f(\bar{u}_{I},\bar{u}_{I})Z(\bar{u}_{I})Z(\bar{u}_{I})$ 3) recursion for the HCs Z,Z These can be derived from 1) KT-relation 2) recurrence equation

 $\lambda_{k}(\bar{u}) = T \lambda_{k}(u_{j})$ 

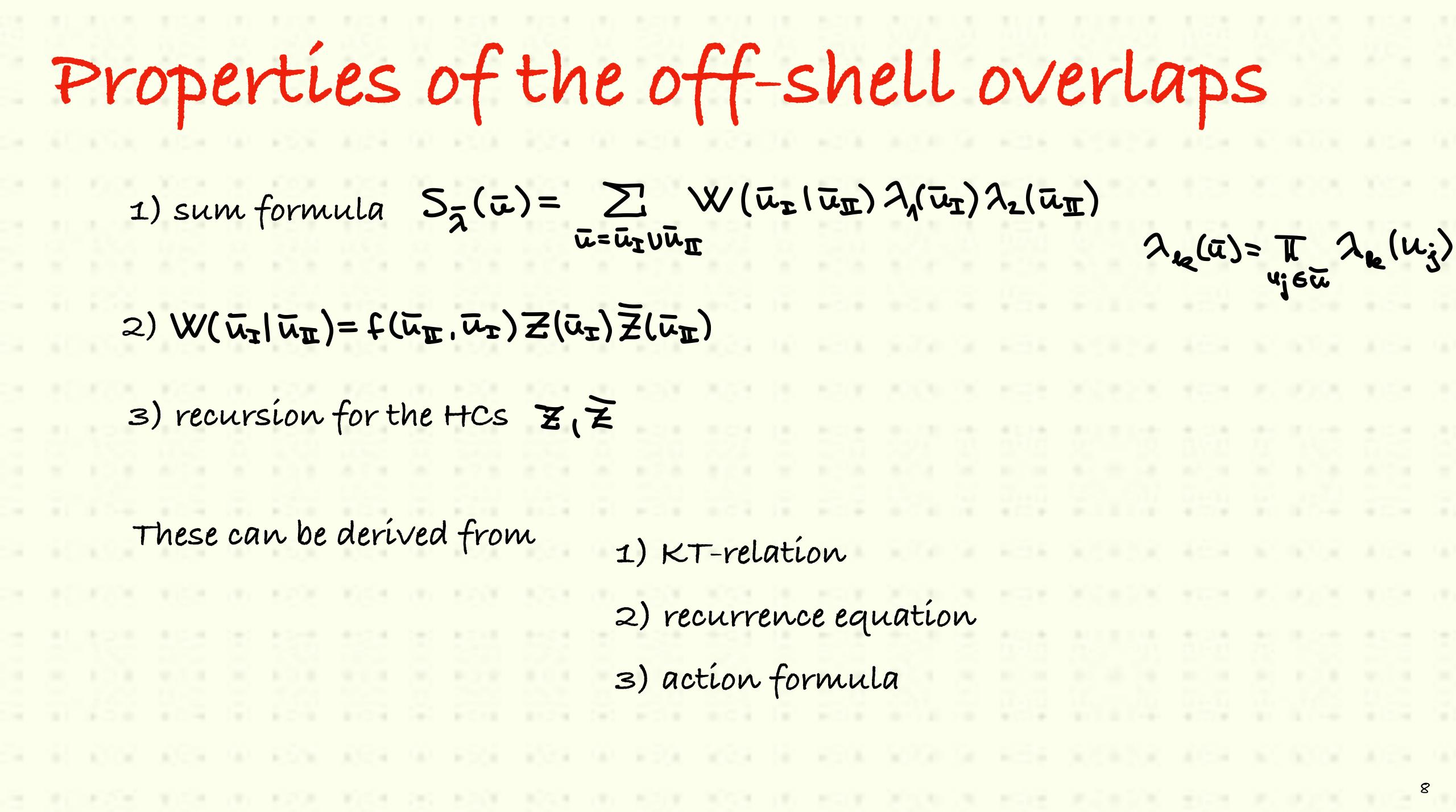


## Properties of the off-shell overlaps

1) sum formula  $S_{\bar{\lambda}}(\bar{u}) = \sum_{\bar{u}=\bar{u}_{\bar{u}}\bar{v}\bar{u}_{\bar{u}}} W(\bar{u}_{\bar{u}}(\bar{u}_{\bar{u}})\lambda_{\bar{u}}(\bar{u}_{\bar{u}})\lambda_{\bar{u}}(\bar{u}_{\bar{u}}))$ 

2)  $W(\bar{u}_{I}|\bar{u}_{I}) = f(\bar{u}_{I},\bar{u}_{I})Z(\bar{u}_{I})Z(\bar{u}_{I})$ 3) recursion for the HCs Z,Z These can be derived from 1) KT-relation 2) recurrence equation 3) action formula

 $\lambda_{k}(\bar{u}) = T \lambda_{k}(u_{j})$ 

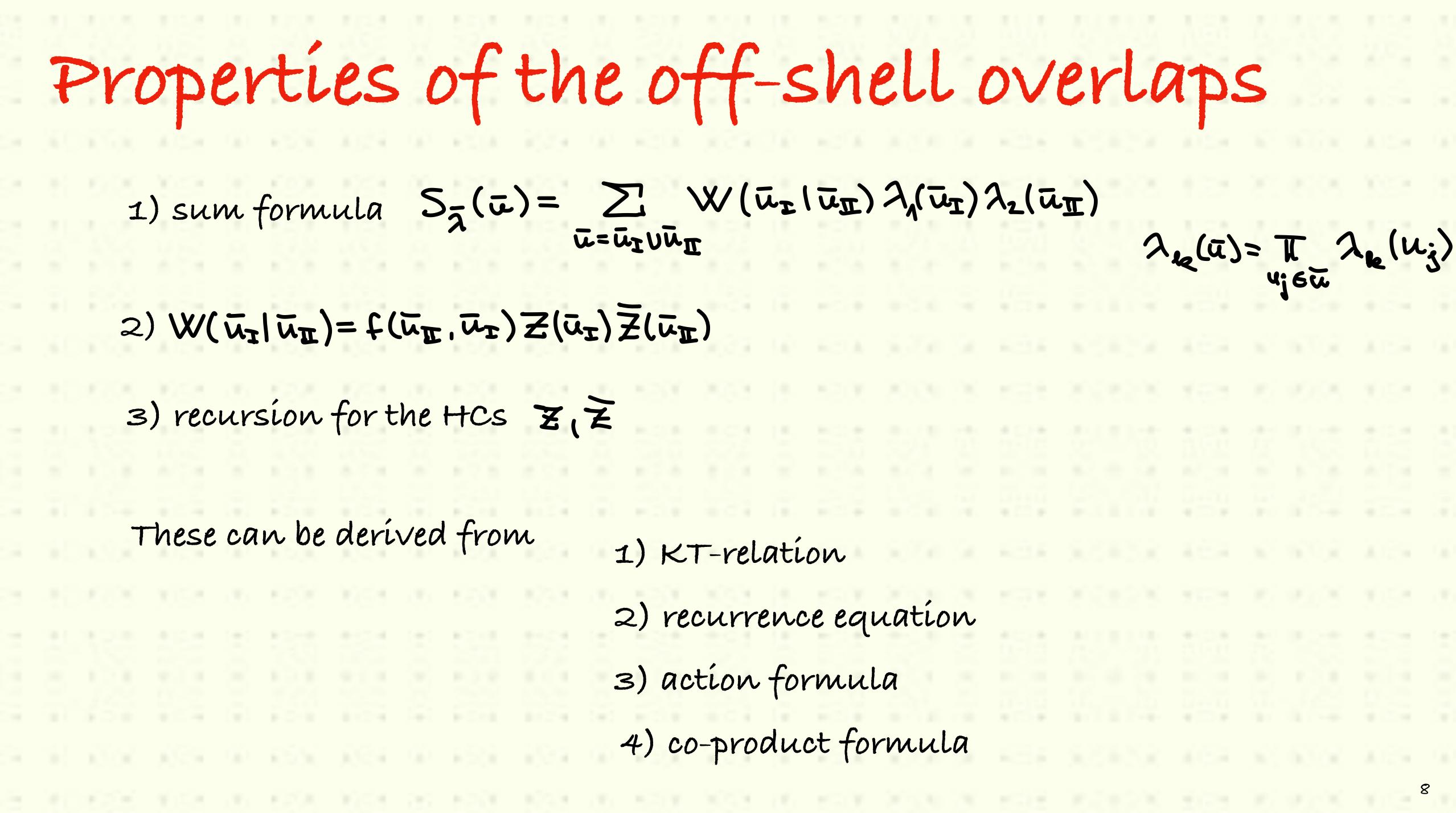


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 $\lambda_{k}(\bar{u}) = T \lambda_{k}(u_{j})$ 



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1) sum formula  $S_{\bar{\lambda}}(\bar{u}) = \sum_{\bar{u}=\bar{u}_{\bar{u}}\bar{v}\bar{u}_{\bar{u}}} W(\bar{u}_{\bar{u}} | \bar{u}_{\bar{u}}) \lambda_{\bar{\lambda}}(\bar{u}_{\bar{u}}) \lambda_{\bar{\lambda}}(\bar{u}_{\bar{u}})$ 

2)  $W(\bar{u}_{r}|\bar{u}_{r}) = f(\bar{u}_{r},\bar{u}_{r})Z(\bar{u}_{r})\overline{Z}(\bar{u}_{r})$ 

3) recursion for the HCs  $Z_{1}Z$ 

These can be derived from

 $\lambda_{k}(\bar{u}) = T \lambda_{k}(u_{j})$ 

1) KT-relation  $\langle \Psi | T_{A,2} \rightarrow \langle \Psi | T_{2,2} \& \langle \Psi | T_{2,n} \rangle$ 2) recurrence equation  $|\{z,\overline{w}\}\rangle = T_{A,2}(z)|\overline{w}\rangle$ Ting(2) (2) = 2(...) (...) 3) action formula  $|\overline{u}\rangle = \sum (...) |\overline{u}_{\mathrm{I}}\rangle \otimes |\overline{u}_{\mathrm{I}}\rangle^{(2)}$ 4) co-product formula



transfer matrix  $\Upsilon(u) = \sum_{i=1}^{2} T_{i}$ g

On-shell limit transfer matrix  $\langle \Psi | \Upsilon(u) = \langle \Psi | \Upsilon(-u)$  $\Upsilon(u) = \sum_{i=1}^{2} T_{i}$ g

transfer matrix  $\Upsilon(u) = \sum_{i=1}^{2} T_{i}$ 

on-shell Bethe states

 $T(z)|\overline{u}\rangle = \mathcal{L}(z|\overline{u})|\overline{u}\rangle$ 

 $\langle \psi | \Upsilon(u) = \langle \psi | \Upsilon(-u)$ g

transfer matrix  $\Upsilon(u) = \sum_{i=1}^{2} T_{i,i}$ 

> non-vaní on-shell o

on-shell Bethe states

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 $T(z)|u\rangle = f(z|u)|u\rangle$ 

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On-shell limit

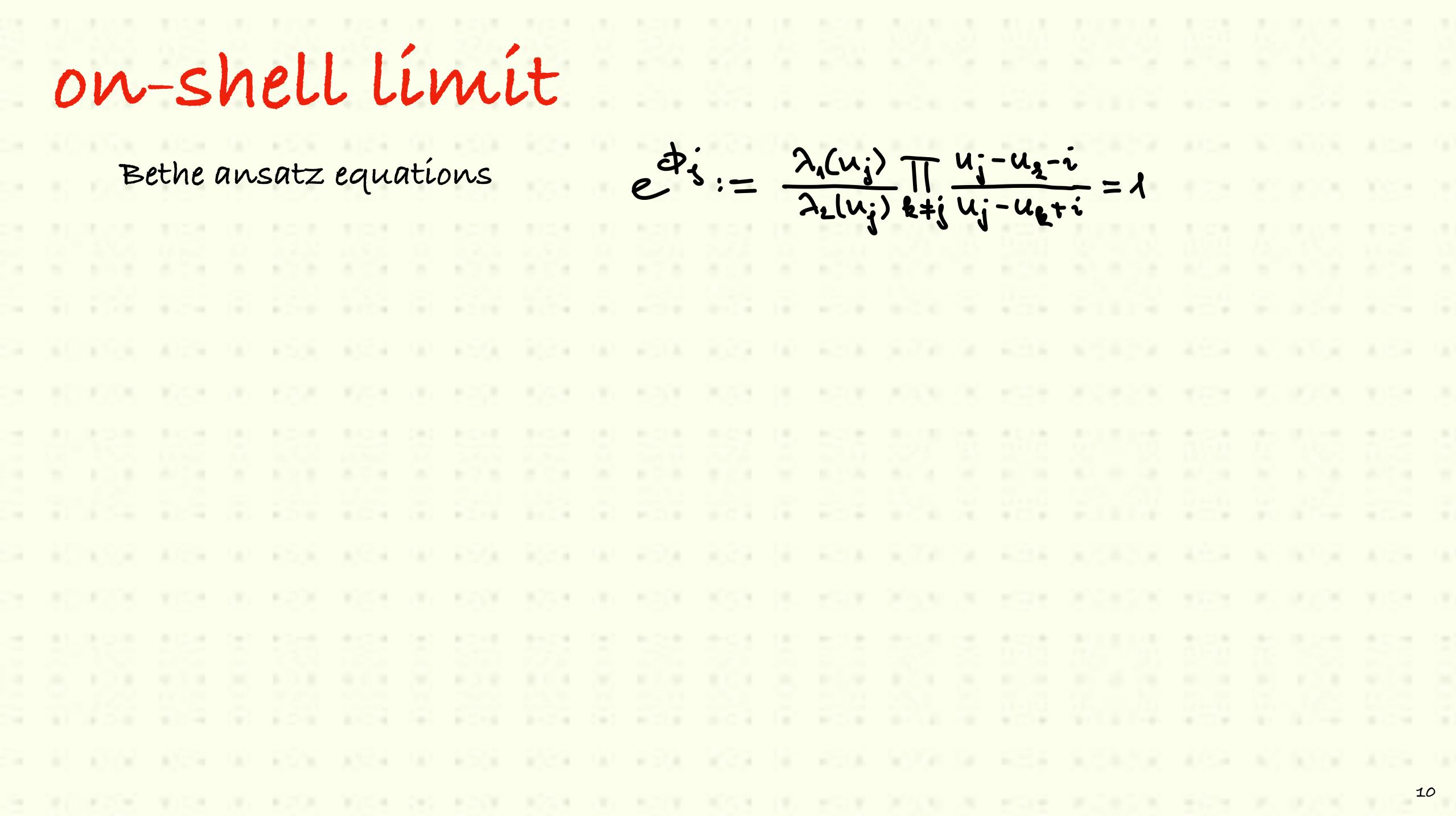
transfer matrix  $\Upsilon(u) = \sum_{i,i}^{2} T_{i,i}$ 

> non-vanishing on-shell overlaps

on-shell Bethe states

## $\langle \Psi | \Upsilon(u) = \langle \Psi | \Upsilon(-u)$ $\rightarrow$ $\mathcal{L}(z \mid \overline{u}) = \mathcal{L}(-z \mid \overline{u})$

	T (z)	)しょう	= /	£(z1	む)10	i>		くそうすう				
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								$ \rightarrow \left\{ u_{j} \right\}_{j=1}^{r} \left\{ - u_{j} \right\}_{j=1}^{r} $				
								Q(z) = (-i) Q(-z)				
24									604			
1												



 $e^{\frac{1}{2}i} = \frac{\lambda_i(u_j)}{\lambda_i(u_j)} \prod_{k\neq j} \frac{u_j - u_{k-1}}{u_j - u_{k+1}} = 1$ 

on-shell limit  $e^{\frac{1}{2}i} = \frac{\lambda_i(u_j)}{\lambda_i(u_j)} \prod_{\substack{\substack{i \neq j \\ i \neq j}}} \frac{u_j - u_{j-i}}{u_j - u_{j+i}} = 1$ Bethe ansatz equations Gaudín matrix Gjiz = Quz log \$ 10

Bethe ansatz equations

Gaudín matrix

Gjiz = Quzlog \$

on-shell norm

 $e^{\frac{1}{2}i} := \frac{\lambda_i(u_j)}{\lambda_i(u_j)} \prod_{\substack{k \neq j \\ i \neq j}} \frac{u_j - u_{k-1}}{u_j - u_{k+1}} = 1$ 10

Bethe ansatz equations

Gaudín matrix

Gjiz = Quz log Ø;

(uiu)~ detG

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 $e^{\varphi_{j}} := \frac{\lambda_{i}(u_{j})}{\lambda_{2}(u_{j})} \prod_{k \neq j} \frac{u_{j} - u_{k} - i}{u_{j} - u_{k} + i} = \lambda$ 

j Korepín's critería

Bethe ansatz equations

Gaudín matrix

Gjz= Quzlog Øj

on-shell norm

(ain)~ detG

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### pair structure

 $\frac{\lambda_i(u_j)}{\lambda_i(u_j)} \prod_{k\neq j} \frac{u_j - u_{k-1}}{u_j - u_{k+1}} = 1$ 

e :=

proved by Korepín'S critería

Bethe ansatz equations

Gaudín matrix

Gjiz = Quz log \$

on-shell norm

(uiu)~ detG

proved by Korepín's critería

10

### pair structure

 $e^{\varphi_{j}} := \frac{\lambda_{i}(u_{j})}{\lambda_{i}(u_{j})} \prod_{k\neq j} \frac{u_{j} - u_{k} - i}{u_{j} - u_{k} + i} = 1$ 

 $\overline{u} = \overline{u}^{\dagger} \vee \overline{u}^{-} \qquad G_{j,k}^{\dagger} = [\partial_{u_{k}} \pm \partial_{u_{k}}] \log \phi_{j}^{\dagger}$ 

Bethe ansatz equations

Gaudín matrix

Gjiz = Quz log \$

on-shell norm

(aiu)~ detG

pair structure

 $det G = det G^{+} det G$ 

factorisation

 $e^{\varphi_{j}} := \frac{\lambda_{i}(u_{j})}{\lambda_{1}(u_{j})} \prod_{\substack{u_{j} - u_{2} - i \\ \lambda_{2}(u_{j})}} \prod_{\substack{u_{j} - u_{2} + i \\ u_{j} - u_{2} + i}} = 1$ 

proved by Korepín's critería

## $\overline{u} = \overline{u}^{\dagger} \vee \overline{u}^{-} \qquad G_{\underline{j},\underline{v}}^{\pm} = [\partial_{u\underline{v}} \pm \partial_{u\underline{v}}] \log \varphi_{\underline{j}}^{\dagger}$

Bethe ansatz equations

Gaudín matrix

Gjiz = Quz log \$

on-shell norm

(aiu)~ detG

pair structure

factorisation

 $det G = det G^{+} det G$ 

Korepín's critería

 $e^{\varphi_{j}} := \frac{\lambda_{i}(u_{j})}{\lambda_{1}(u_{j})} \prod_{\substack{u_{j} - u_{2} - i \\ \lambda_{2}(u_{j})}} \prod_{\substack{u_{j} - u_{2} + i \\ u_{j} - u_{2} + i}} = 1$ 

proved by Korepín'S critería

## $\overline{u} = \overline{u}^{\dagger} \vee \overline{u}^{-} \qquad G_{\underline{j},\underline{v}}^{\pm} = \left[ \partial_{u\underline{v}} \pm \partial_{u\underline{v}} \right] \log \varphi_{\underline{j}}^{\dagger}$

Bethe ansatz equations

Gaudín matrix

Gjiz = Quz log \$

on-shell norm

<ūıū)∼ detG

proved by Korepín's critería

pair structure

factorisation

 $det G = det G^{+} det \overline{G}$ 

Korepín's crítería  $\longrightarrow$   $(4)u^7 \sim det G^4$ 

 $e^{\varphi_{j}} := \frac{\lambda_{i}(u_{j})}{\lambda_{1}(u_{j})} \prod_{\substack{u_{j} - u_{2} - i \\ \lambda_{2}(u_{j})}} \prod_{\substack{u_{j} - u_{2} + i \\ u_{j} - u_{2} + i}} = 1$ 

## $\overline{u} = \overline{u}^{\dagger} \vee \overline{u}^{-} \qquad G_{\underline{j},\underline{v}}^{\pm} = [\partial_{u\underline{v}} \pm \partial_{u\underline{v}}] \log \varphi_{\underline{j}}^{\dagger}$

# Normalized on-s $\frac{\langle \Psi | \overline{u} \rangle}{\langle \overline{u} | \overline{u} \rangle} = \frac{\mathcal{F}(\overline{u})}{\int \frac{d d d G}{d \sigma}}$

이는 것이 많이 좀 가지 않는 것이 같이 같이 많이 많이 많이 많이 많이 많이 많이 많이 했어?

	rel		01	lerl	.01	DS		
					124			
r.	10.0	$\{ \psi \}_{i \in \mathbb{N}}$	$g \sim 10^{-1}$	1000	r > 2	1000	100	10.00



## Normalized on-s

 $\frac{\langle \Psi | \overline{u} \rangle}{\langle \overline{u} | \overline{u} \rangle} = \mathcal{F}(\overline{u}) \int_{\overline{u}} \frac{d d G}{d d G}$ 

## universal part det det depends or

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n	the	Bet	the st	tate				
1	10.0	141	$\mathcal{T} = \mathcal{T}$	1000	1.12	1202	11.11	1000





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	uníve							tate			
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2											



## Normalized on-s

 $\frac{\langle \Psi | \overline{u} \rangle}{\langle \overline{u} | \overline{u} \rangle} = \frac{\mathcal{F}(\overline{u})}{\int \frac{d}{d} \overline{c}}$ 



boundary part  $\mp(\overline{u}) = \underbrace{\pi}_{f} \mp(u_{f}) de$ 

100	115.8	1.91	622	1.5.5	121	1200	10.00	1.91	1.11

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OVE	eauín	Jal	ently	y Kl	'u)			
			100 million (100 million)					



## Normalized on-s

 $\frac{\langle \Psi | \overline{u} \rangle}{\langle \overline{u} | \overline{u} \rangle} = \mathcal{T}(\overline{u}) \int_{\overline{u}}^{\overline{u}} \frac{d d c^{\dagger}}{\sqrt{u}}$ 







general solution of the reflection equ

$$K(u) = \frac{a}{b} + A = \begin{pmatrix} \frac{a}{b} + b_{1} \\ \frac{b}{b} \\ \frac{b}{b} \end{pmatrix}$$

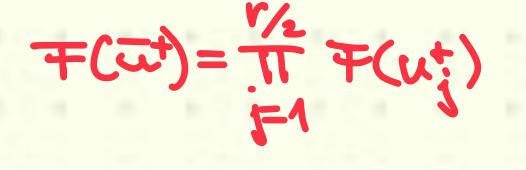
A2 = 81

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depends on the b.S.	
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8=-detA	







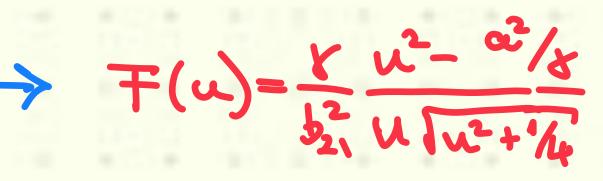


Normalized on-shell overlaps  $\frac{\langle \Psi | \overline{u} \rangle}{\langle \overline{u} | \overline{u} \rangle} = \mp (\overline{u}) \int_{\frac{d}{d}\overline{u}} \frac{d}{\overline{u}}$ universal part det det depends on the Bethe state boundary part  $\mp(\overline{u}) = \frac{\pi}{\pi} \mp(u_{1})$  depends on the b.S. or equivalently K(w) general solution of the reflection equation

a - 6 11

$$K(u) = \frac{a}{b} + A = \begin{pmatrix} \frac{a}{b} + b_{1} \\ \frac{b}{b} \\ \frac{b}{b} \end{pmatrix}$$

A2 = 81



8=-detA



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		ミンコ		ū <sup>4</sup> ū <sup>2</sup>		gl(					
		-la',i		- <b>ū</b> 3	-0-0	N)					
		x²,		•••	-0						
		., īc <sup>N-</sup> 'S	636	ūn-1	-0						

## Generalisation to gl(N) spin chains 이 승규에는 비교적 이번 승규에는 비교적 이번 승규에 가지 않고 한 것이에 가지 않고 한 바라에 비교적 것이 없다. two types of KT-relations 13

24										
	10.00	115.0	191	2.272	1.5.5	125	100	10.00	191	2011

## Generalisation to gl(N) spin chains

### two types of KT-relations

non-crossed K(u) (4) = (4) (-u) K(u)Crossed  $K(u)(t)(\tau(u) = (t)(\tau(u)))$ 13

## Generalisation to gl(N) spin chains

non-crossed  $K(u) \downarrow \downarrow T(u) = \downarrow \downarrow \downarrow (-u) K(u)$ 

### two types of KT-relations

Crossed  $K(w) \langle \psi | T(w) = \chi \psi | \hat{T}(-w) K(w)$ 

### inverse monodromy matrix

	 1128	w.	69.	1064	20	120	12.2	 507	10.0	11	2017	200	:03	1.1	28	3.03	7	1	1	13

 $\widehat{\tau}^{t}(\omega) T(\omega) = 4$ 

## Generalisation to gl(N) spin chains

two types of KT-relations

Crossed

ínverse monodromy matrix

compatibility conditions

- non-crossed K(u) (4) = (4) (-u) K(u)
  - $K(u) \langle \psi | T(u) = \langle \psi | \hat{T}(-u) K(u)$
  - $\widehat{T}^{t}(\omega) T(\omega) = 4$
- $R_{12}(u-\sigma)K_{(-u)}R_{n}(u+\sigma)K_{2}(\sigma) = K_{2}(\sigma)R_{n}(u+\sigma)K_{1}(-u)R_{n}(u-\sigma)$  $R_{12}(u-v)K_{(-u)}\overline{R_{n}(u+v)}K_{2}(v) = K_{2}(v)\overline{R_{n}(u+v)}K_{(-u)}R_{n2}(u-v)$
- 13



Symmetries and pair structures

### Non-crossed K-matrices



# Symmetries and pair structures Non-crossed K-matrices K(m)= = 1 + A A<sup>2</sup>=1



Symmetries and pair structures K(n)= = + A  $A^2 = 1$ Non-crossed K-matrices A~ diag( $+,+,\dots,+,-,-,\dots,-$ ) 



Symmetries and pair structures

### Non-crossed K-matrices

 $K(w) = \frac{2}{2}I + A$   $A^2 = I$ 

residual symmetry

 $A \sim diag(+,+,\dots,+,-,-,\dots,)$  $g(M) \oplus g(N-M)$ 

1000	11	$g \leq 1/\ell$	2010/01/01	t > 2	1000	3.03	10000



### Non-crossed K-matrices

			Yest in t														
			Crossed	l K-matri	ces												
2																	
2																	
	10	100	$V := \{v_1, \dots, v_n\} \in \mathbb{R}$	20. XXX 23	1000	1.253	(m. 823)	v - 10000	11	100	1000	$(-1)^{-1}$	1000	100	$(\mathbf{r}_{i})$	0.00	

 $A^2 = 4$ K(w)= = + A

residual symmetry

A~ diag(+,+,...,+,-,-,...)  $g(M) \oplus g(N-M)$ 



### Non-crossed K-matrices

### Crossed K-matrices

1									
	1000	10.52	0.000	1053	121	120	12.53	11	500

residual symmetry K(い)= 佘1 + A  $A^2 = 4$ 

 $A \sim diag(+,+,\dots,+,-,-,\dots,-)$  $g(M) \oplus g(N-M)$ 

 $\vee^{t} = \pm \vee$ K(w) = VNAMES AND ADDRESS AND ADDRES



### Non-crossed K-matrices

### Crossed K-matrices

1									
	1000	10.52	0.000	1053	121	120	12.53	11	500

residual symmetry  $A^2 = 4$ K(w)= = + A

 $A \sim diag(+,+,\dots,+,-,-,\dots,-)$  $g(M) \oplus g(N-M)$ 

O(N)  $\vee^{t} = \pm \vee$ K(w) = VSP(N) 

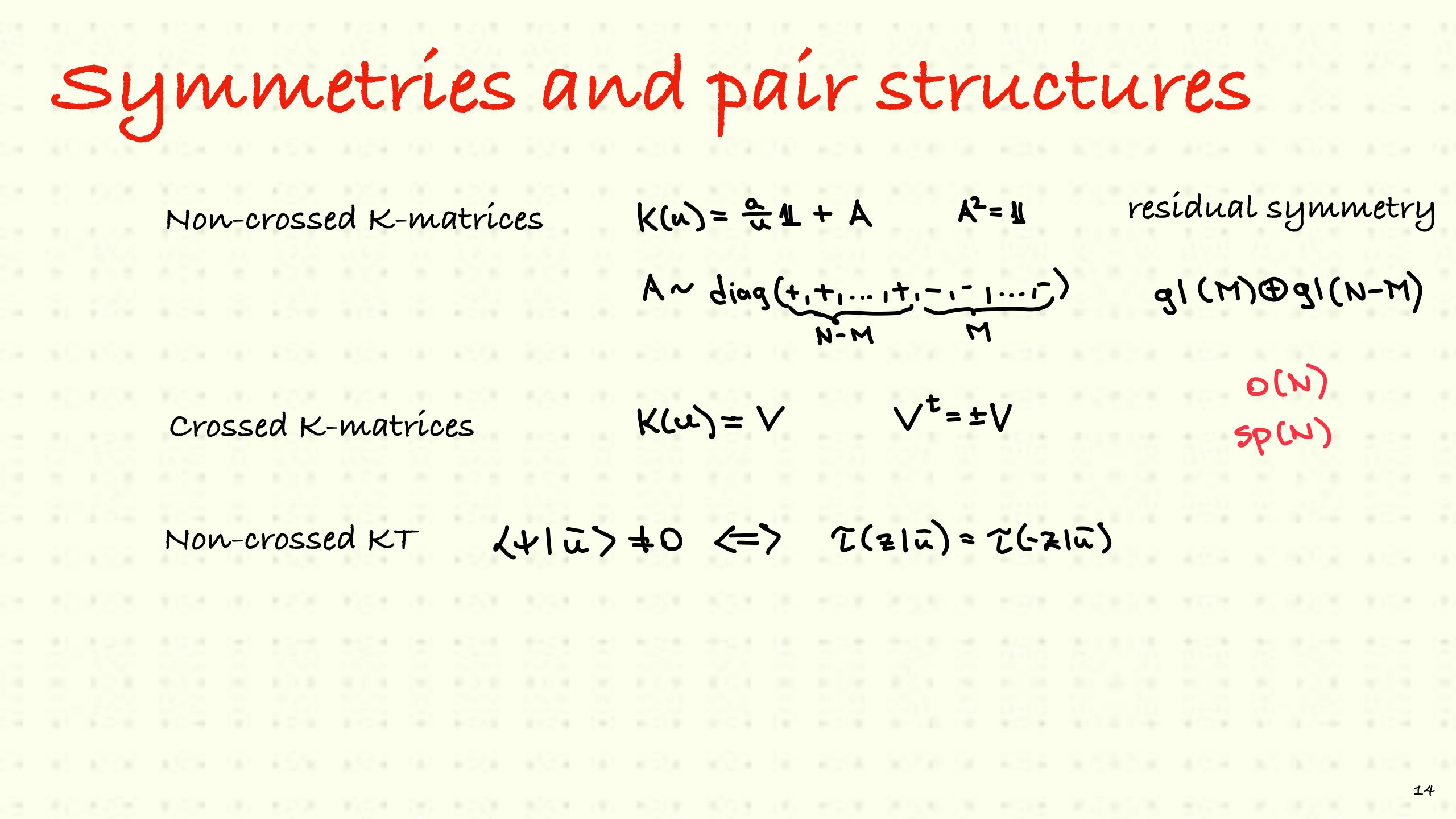


### Non-crossed K-matrices

### Crossed K-matrices

### Non-crossed KT 人112>キ0 (=>

- resídual symmetry K(w)= 유1+ A  $A^2 = 4$
- $A \sim diag(+,+,\dots,+,-,-,\dots,-)$
- K(u)=V  $\vee^{t} = \pm V$
- て(えし)= て(えし)



0(N)

SP(N)

# Symmetries and pair structures

#### Non-crossed K-matrices

Crossed K-matrices

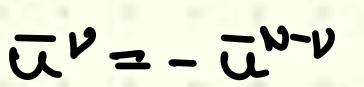
#### Non-crossed KT 人1112 +0

#### achiral pair structure

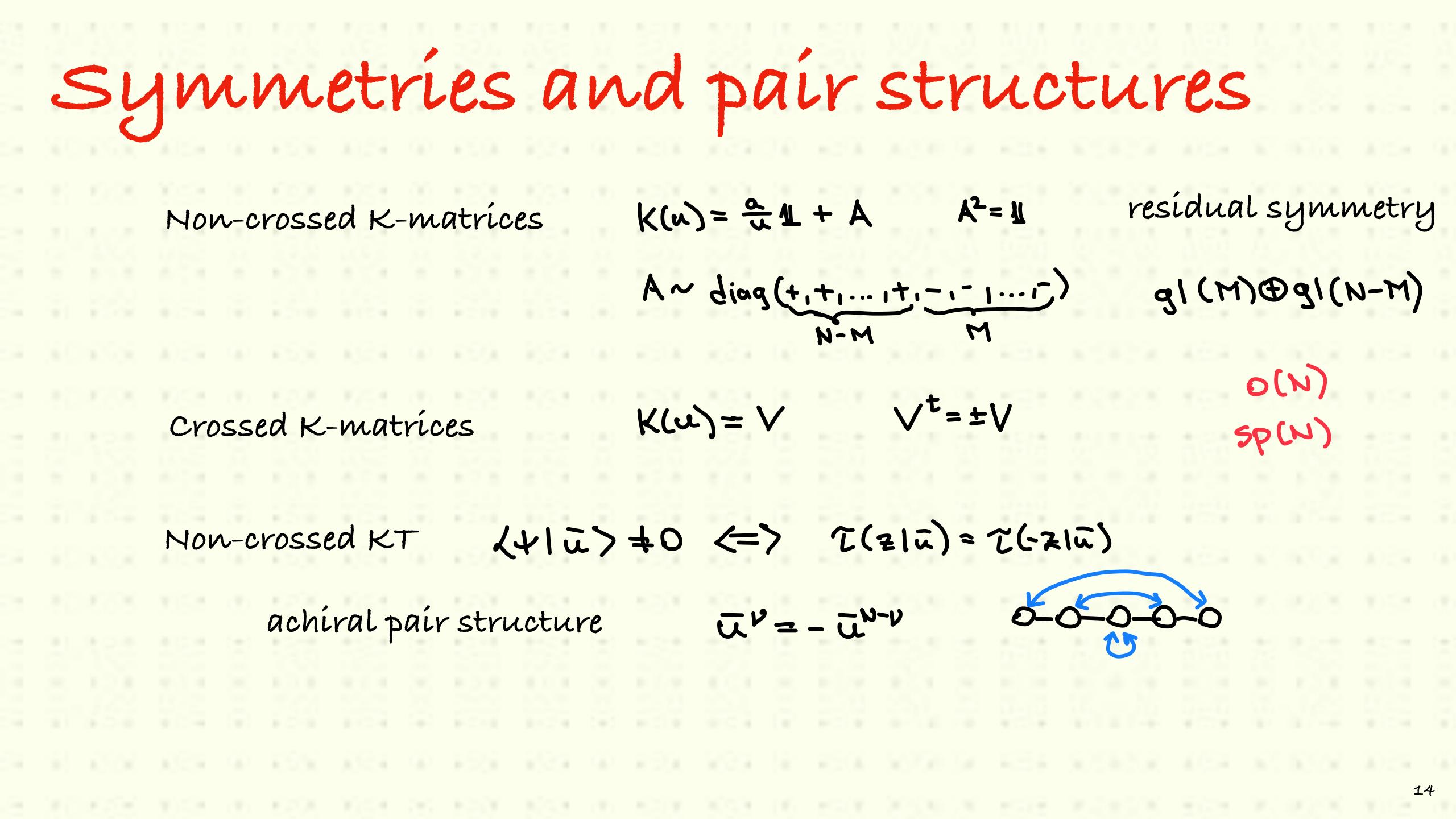
- residual symmetry K(い)= 유1+ A  $A^2 = 4$
- A~ diag(+,+,...,+,-,-,....) N-M M
- K(w)=V  $\vee^{t} = \pm \vee$

0(N) Sp(N)

 $\mathcal{I}(z|\overline{u}) = \mathcal{I}(-z|\overline{u})$ 







# Symmetries and pair structures

#### Non-crossed K-matrices

Crossed K-matrices

#### Non-crossed KT 人1112 +0

achiral pair structure

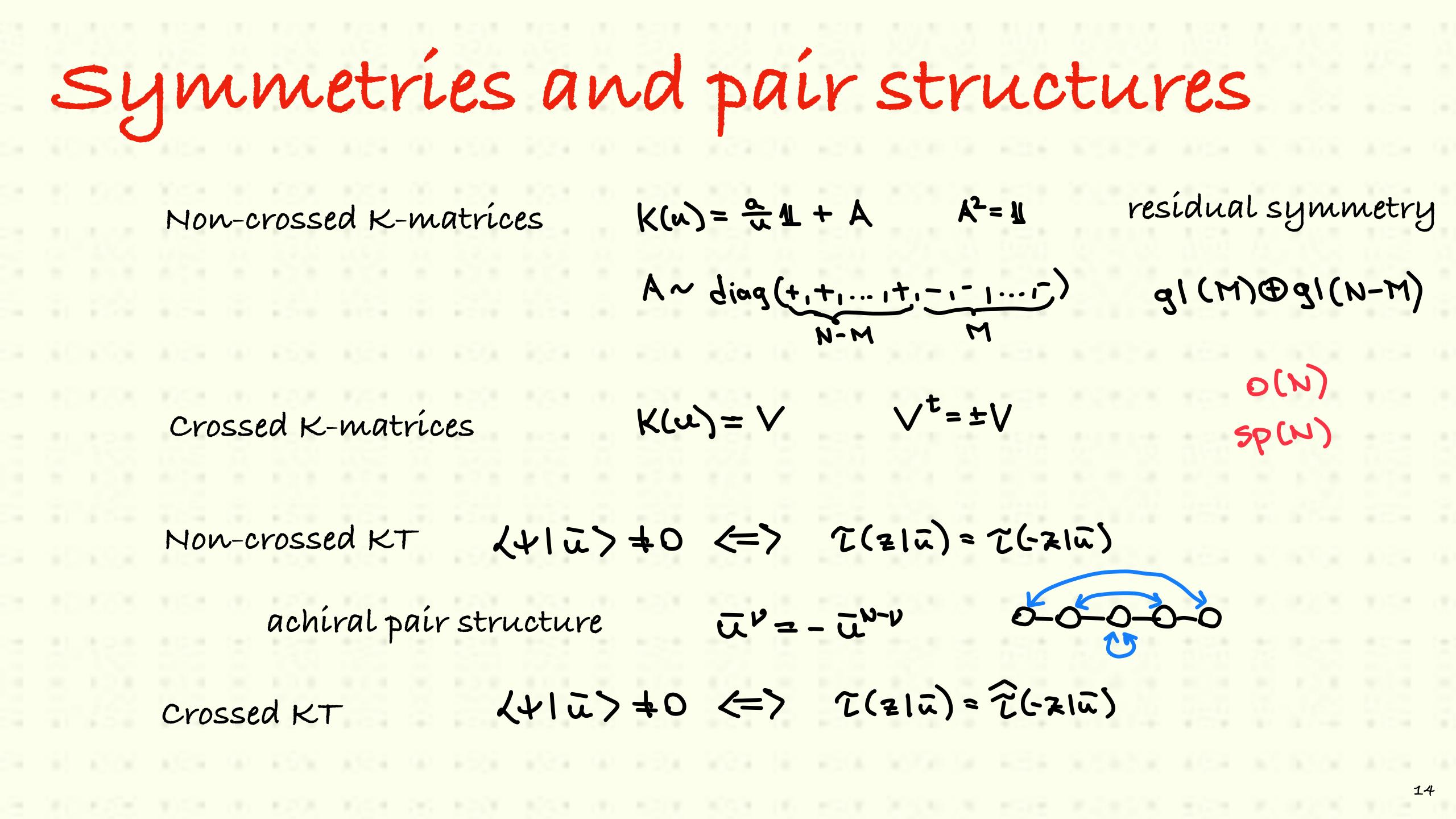
- residual symmetry K(い)= 유1+ A  $A^2 = 4$
- A~ diag(+,+,...,+,-,-,....) N-M M
- K(w)=V  $\vee^{t} = \pm \vee$

0(N) Sp(N)

- $\mathcal{I}(z|\overline{u}) = \mathcal{I}(-z|\overline{u})$



Crossed KT  $(41\pi) \neq 0 \iff \mathcal{I}(21\pi) = \widehat{\mathcal{I}}(-21\pi)$ 



# Symmetries and pair structures

#### Non-crossed K-matrices

Crossed K-matrices

#### Non-crossed KT 人1112 +0

achiral pair structure

chiral pair structure  $\overline{u}^{\prime} = -\overline{u}^{\prime}$ 

- residual symmetry K(い)= 유1+ A  $A^2 = 4$
- A~ diag(+,+,...,+,-,-,....) N-M M
- K(w)=V  $\vee^{t} = \pm \vee$

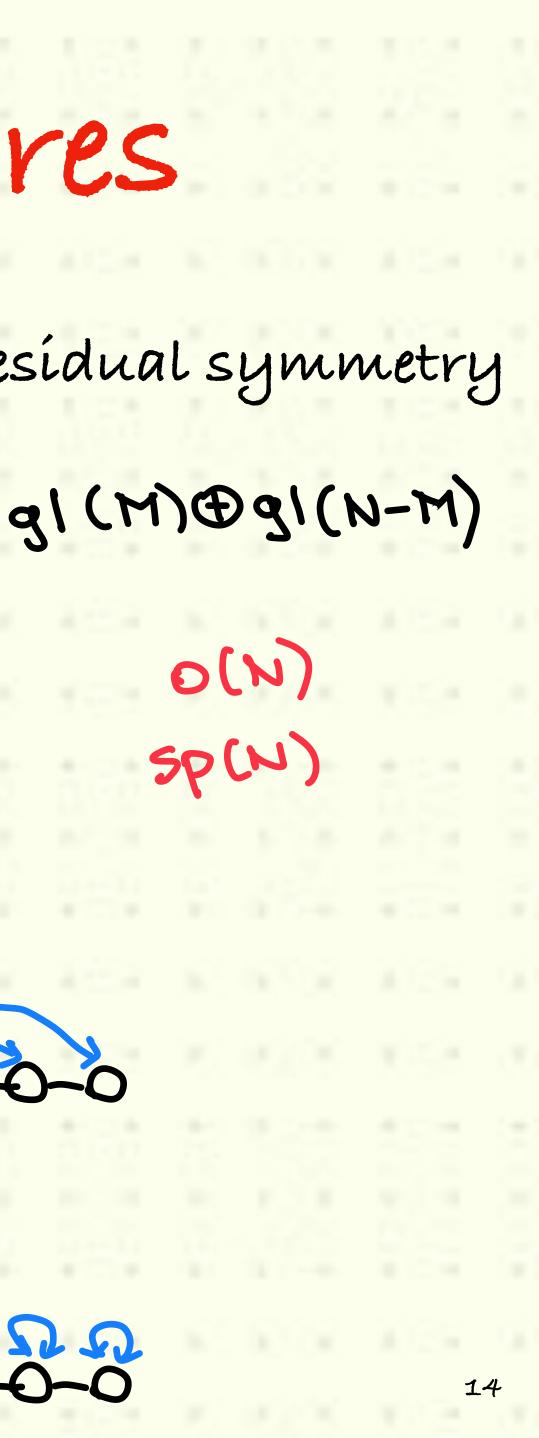
0(N) Sp(N)

- $\mathcal{I}(z|\overline{u}) = \mathcal{I}(-z|\overline{u})$



Crossed KT  $(41\pi) \neq 0 \iff \mathcal{I}(21\pi) = \widehat{\mathcal{I}}(-21\pi)$ 

19 20 20 20



# off-shell overlaps

#### List of criteria for off-shell overlaps

15

## $S_{\chi}(\bar{u}) = \sum W(\bar{u}_{r} | \bar{u}_{T}) \prod_{\mu=1}^{N-1} \lambda_{\nu}(\bar{u}_{r}) \lambda_{\mu_{\mu}}(\bar{u}_{T})$



# Off-shell overlaps

List of criteria for off-shell overlaps

1) KT-relation: creation to annihilation

 $S_{\chi}(\bar{u}) = \sum W(\bar{u}_{r}|\bar{u}_{r}) \prod_{\mu=1}^{N-1} \lambda_{\nu}(\bar{u}_{r}') \lambda_{\mu_{\mu}}(\bar{u}_{r}')$ 

#### ation (41 This > 241 The R>1

1	$\{ i, j \in I \}$	11	$g \leq 1/\ell$	20000	$t \in \{0, 0\}$	1000	$\widetilde{\mathcal{T}} \subset \mathcal{T}$	1000



# Off-shell overlaps

List of criteria for off-shell overlaps

1)KT-relation: creation to annihilation <+1 This > 2+1 There 2>1

2) recurrence formula  $|\{z, \bar{u}\}, \bar{u}^2, ... \rangle = Z(..., T_{ij}(z)|\bar{u}', \bar{v}^2, ... \rangle$ 

## $S_{\chi}(\bar{u}) = \sum W(\bar{u}_{I} | \bar{u}_{I}) \prod_{j=1}^{N-1} \lambda_{j}(\bar{u}_{I}) \lambda_{m_{j}}(\bar{u}_{I})$



# off-shell overlaps

List of criteria for off-shell overlaps 1)KT-relation: creation to annihilation <+1 This >> <+1 The 2>1 2) recurrence formula  $\{z,\bar{u}\},\bar{u}^2,...\}=\mathbb{Z}(...)T_{4,j}(z)|\bar{u}',\bar{v}^2,...\}$  $T_{i,j}(z)|u\rangle = Z(...)|w\rangle$ 3) action formula

 $S_{\chi}(\bar{u}) = \sum W(\bar{u}_{I} | \bar{u}_{I}) \prod_{\nu=1}^{N-1} \lambda_{\nu}(\bar{u}_{I}^{\nu}) \lambda_{\nu_{1}}(\bar{u}_{I}^{\nu})$ 

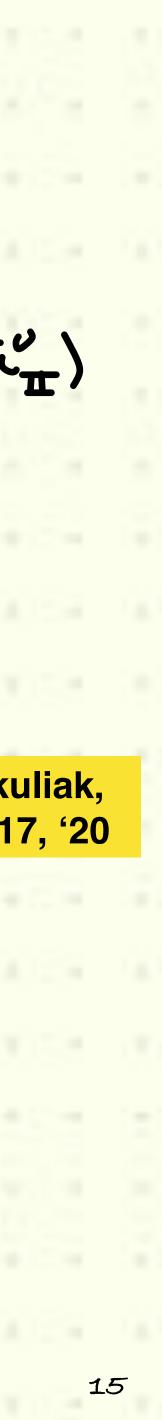


off-shell overlaps List of criteria for off-shell overlaps 1)KT-relation: creation to annihilation 2) recurrence formula з)action formula 4) co-product formula 15

 $S_{\chi}(\bar{u}) = \sum W(\bar{u}_{\pi} | \bar{u}_{\pi}) \prod_{\nu=1}^{N-1} \lambda_{\nu}(\bar{u}_{\pi}^{\nu}) \lambda_{\nu_{\eta}}(\bar{u}_{\pi}^{\nu})$ <+1 This >> <+1 The 2>1  $\{z,\bar{u}\},\bar{u}^2,...\}=Z(...)T_{ij}(z)|\bar{u}',\bar{v}^2,...\}$  $T_{i,j}(z)(i,z) = \sum_{i=1}^{\infty} (...)(i,z)$  $|\bar{u}\rangle = \sum (...) |\bar{u}_{r}\rangle^{(n)} \otimes |\bar{u}_{r}\rangle^{(2)}$ 



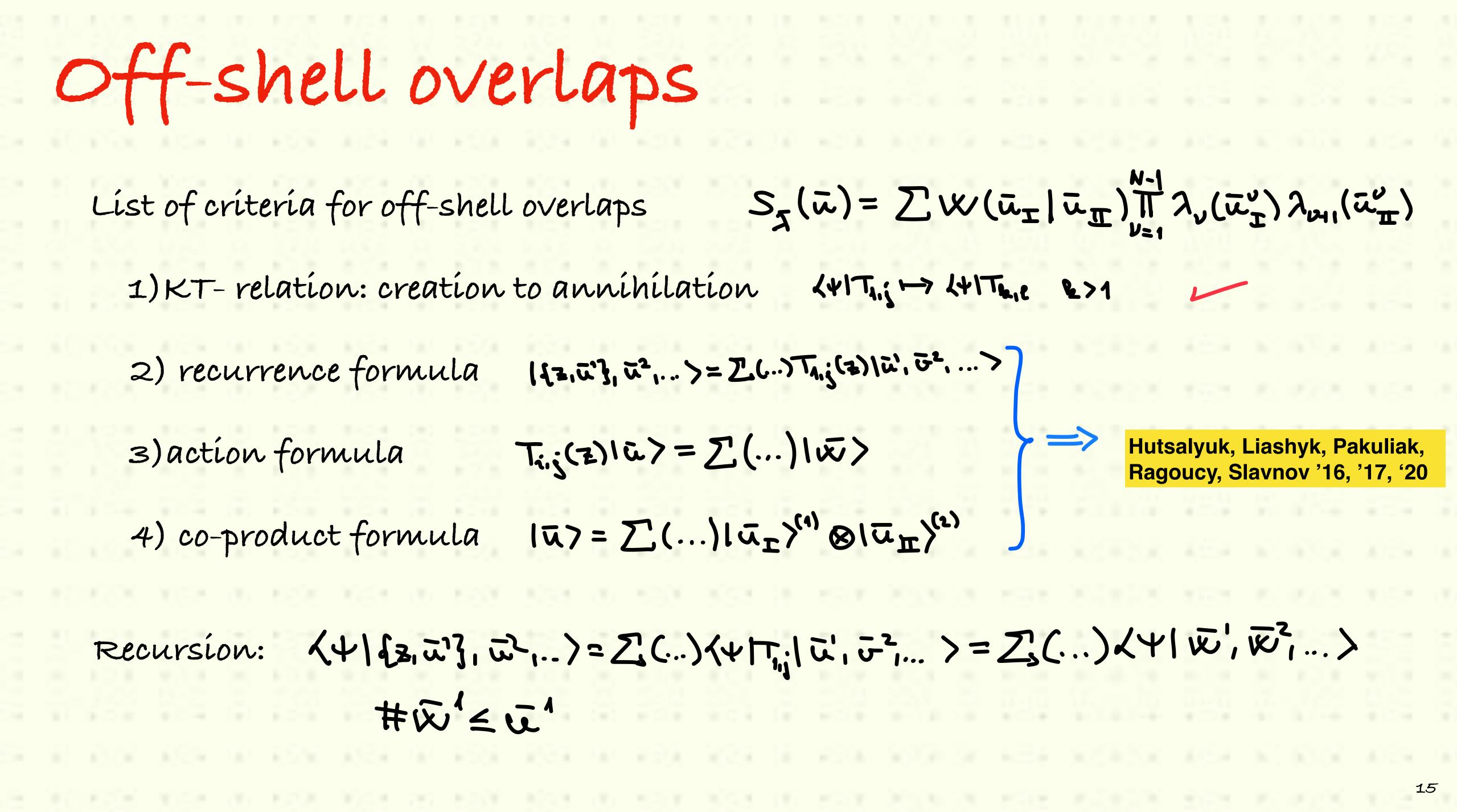
off-shell overlaps  $S_{\chi}(\bar{u}) = \sum W(\bar{u}_{I} | \bar{u}_{I}) \prod_{\nu=1}^{N-1} \lambda_{\nu}(\bar{u}_{I}^{\nu}) \lambda_{\nu_{1}}(\bar{u}_{I}^{\nu})$ List of criteria for off-shell overlaps 1)KT-relation: creation to annihilation <+ ITA > + ITA , E 2>1  $\{z,\bar{u}^{2}\},\bar{u}^{2},...\}=Z(...)T_{4,j}(z)|\bar{u}^{1},\bar{v}^{2},...\}$ 2) recurrence formula  $T_{i,j}(z)(i,z) = Z(...)(i,z)$ з)action formula Hutsalyuk, Liashyk, Pakuliak, Ragoucy, Slavnov '16, '17, '20  $|\overline{u}\rangle = \sum (...) |\overline{u}_{r}\rangle^{(n)} \otimes |\overline{u}_{m}\rangle^{(2)}$ 4) co-product formula 15



off-shell overlaps List of criteria for off-shell overlaps 1)KT-relation: creation to annihilation 2) recurrence formula  $T_{i,j}(z)|u\rangle = \sum_{i=1}^{\infty} (...)|w\rangle$ з)action formula 4) co-product formula Recursion:  $\#\omega' \leq \omega'$ 

- $S_{\chi}(\bar{u}) = \sum W(\bar{u}_{r}|\bar{u}_{r}) \prod_{\nu=1}^{N-1} \lambda_{\nu}(\bar{u}_{r}^{\nu}) \lambda_{\nu_{n}}(\bar{u}_{r}^{\nu})$
- <+ IT + + + The e 2>1
- $\{z,\bar{u}\},\bar{u}^{2},...\}=Z(...)T_{4,j}(z)|\bar{u}',\bar{v}^{2},...\}$
- $|\overline{u}\rangle = \sum (...) |\overline{u}_{r}\rangle^{(n)} \otimes |\overline{u}_{m}\rangle^{(n)}$

Hutsalyuk, Liashyk, Pakuliak, Ragoucy, Slavnov '16, '17, '20

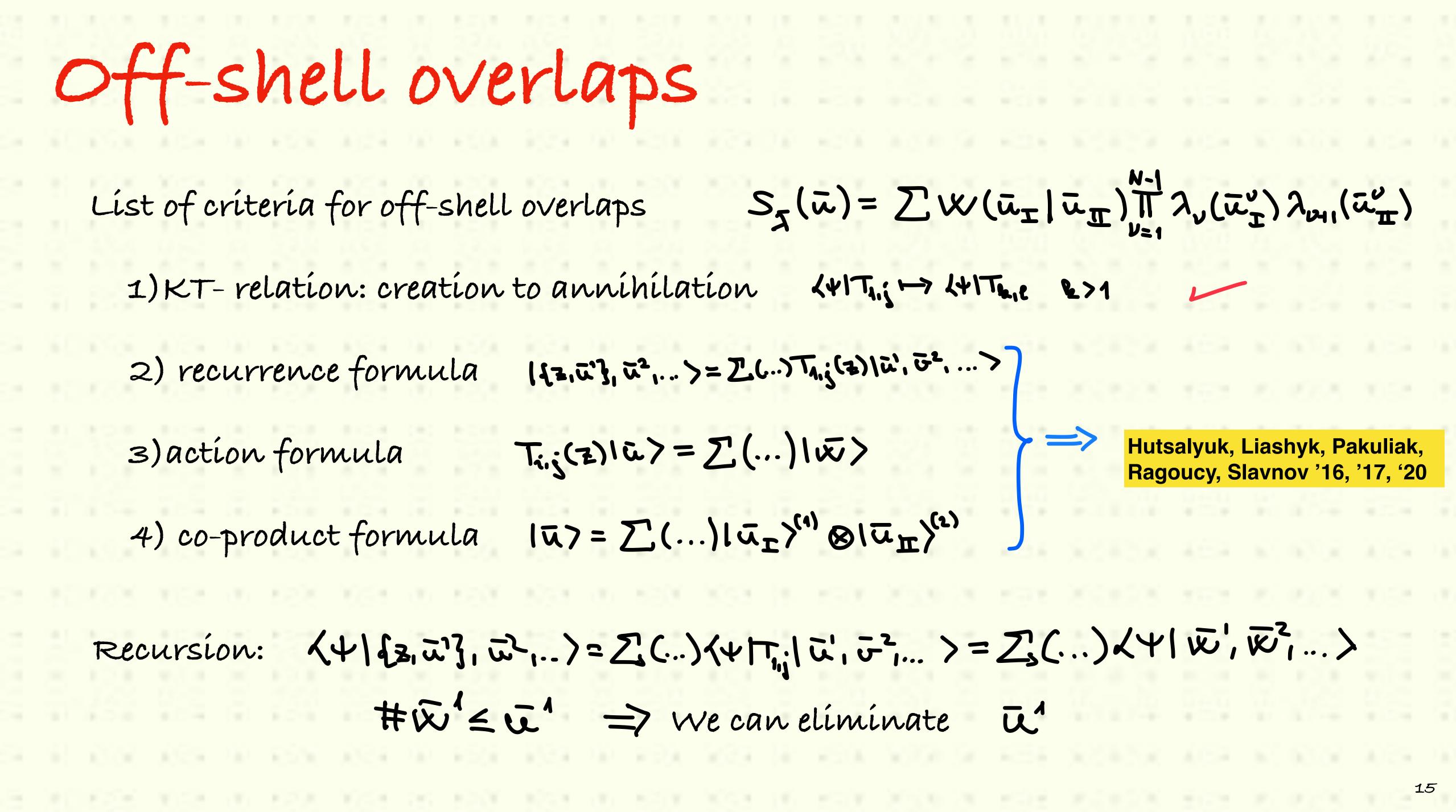


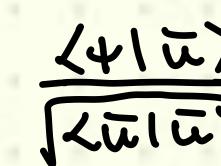
off-shell overlaps List of criteria for off-shell overlaps 1)KT-relation: creation to annihilation 2) recurrence formula  $T_{i,j}(z)|u\rangle = \sum_{i=1}^{\infty} (...)|w\rangle$ з)action formula 4) co-product formula Recursion:  $\#\overline{w}' \leq \overline{u}' \Rightarrow we can eliminate \overline{u}'$ 

- $S_{\chi}(\bar{u}) = \sum W(\bar{u}_{r}|\bar{u}_{r}) \prod_{\nu=1}^{N-1} \lambda_{\nu}(\bar{u}_{r}^{\nu}) \lambda_{\nu_{n}}(\bar{u}_{r}^{\nu})$
- <+ IT + + + The e 2>1
- $\{z,\bar{u}\},\bar{u}^{2},...\}=Z(...)T_{4,j}(z)|\bar{u}',\bar{v}^{2},...\}$
- $|\overline{u}\rangle = \sum (...) |\overline{u}_{r}\rangle^{(n)} \otimes |\overline{u}_{m}\rangle^{(n)}$

Hutsalyuk, Liashyk, Pakuliak, Ragoucy, Slavnov '16, '17, '20

- $\langle \Psi | \{z, \overline{\omega}\}, \overline{\omega}, \ldots \rangle = \Sigma(\ldots) \langle \Psi | \overline{\omega}, \overline{\omega}, \ldots \rangle = Z(\ldots) \langle \Psi | \overline{\omega}, \overline{\omega}, \ldots \rangle$







Korepin's criteria  $\longrightarrow \qquad \langle 4 | \overline{u} \rangle = T T T (\overline{u}) \int \frac{\det G}{\det G}$ 





G<sup>±</sup> depends on the pair structure

Korepín's crítería  $\longrightarrow \qquad \frac{\langle \psi | \overline{u} \rangle}{\langle \overline{u} | \overline{u} \rangle} = \overline{T} \overline{T} \overline{T} (\overline{u}) \sqrt{\frac{\det G}{\det G}}$ 

1000	$(\mathbf{v})$	$g \leq 0$	2000	100	1000	$\overline{\mathcal{C}}$	10.00





Korepín's crítería  $\longrightarrow \qquad \langle 4 | \overline{u} \rangle = T T T (\overline{u}) \int \frac{\det G}{\det G}$ 

G<sup>±</sup> depends on the pair structure

#### F, (w) given by the K-matrix

10.0	110	$g \leq 0$	2010.00	1018	1000	101	10.000







Fy (w)

 $Q_{M}(a)$  det  $G^{\dagger}$  $N = \frac{N}{2}$ Qn(D)Qn(=) det G 16

· g(M)@g((w·M)

2									
	12	19	6.95	1063	20	120	1223	19	100



Korepin's criteria  $\longrightarrow \qquad \frac{\langle \psi | \overline{u} \rangle}{\langle \overline{u} | \overline{u} \rangle} = \prod_{v} \overline{F_{v}(\overline{u}^{v})} \frac{\det G}{\det G}$ 

 $G^{\pm}$  depends on the pair structure

given by the K-matrix







F, (w)

Qm(a) det G<sup>+</sup>  $N = \frac{N}{2}$ Qn(D)Qn(z) det G 16

· gl(M)@gl(w·M)

2									
	12	19	6.95	1063	20	120	1223	19	100

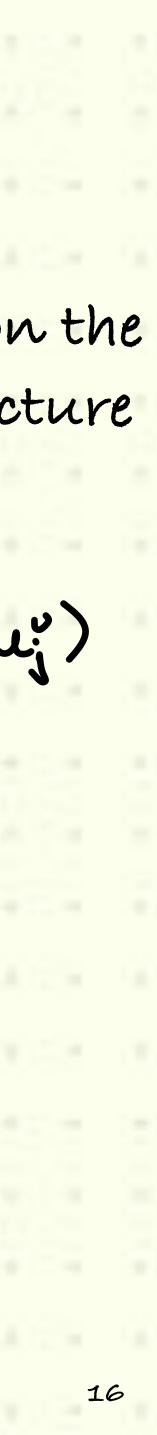


Korepín's crítería  $\longrightarrow \qquad \frac{\langle \psi | \overline{u} \rangle}{\langle \overline{u} | \overline{u} \rangle} = \prod_{v} \overline{F_{v}(\overline{u}^{v})} \frac{\det G^{t}}{\det G}$ 

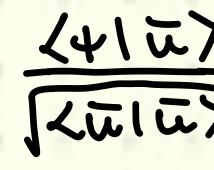
G<sup>±</sup> depends on the pair structure

given by the K-matrix

 $Q_{y}(z) = \prod_{j=1}^{r_{y}} (z - u_{j}^{y})$ 







F, (w)

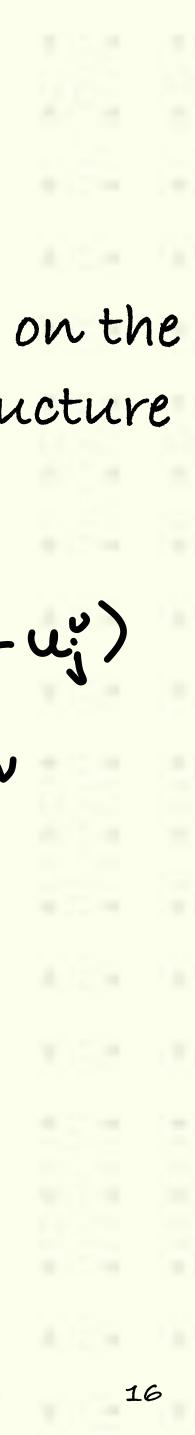
							$T_{y}(w)$ given by the K-matrix $Q_{y}$							<b>2</b> <sub>v</sub> ( <sub>₹</sub> )	$(z) = \prod_{j=1}^{r_{p}} (z - u)$				
	• 6		)⊕	g1(v	)-M)		6	$\frac{2}{n(0)Q}$	) い(注	de	tG <sup>+</sup>	٦		とえ		ichiva)	ī	(= - 1	
10	1.5	10.0	19	6.0	10<4	10	120	1.22.3									101	$[0,1] \in \mathcal{O}$	

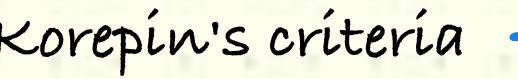


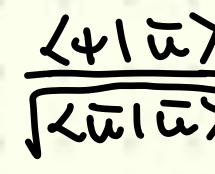
Korepín's crítería  $\longrightarrow \qquad \langle \psi | \overline{u} \rangle = T T(\overline{u}) \int \frac{\det G}{\det G}$ 

given by the K-matrix

depends on the pair structure



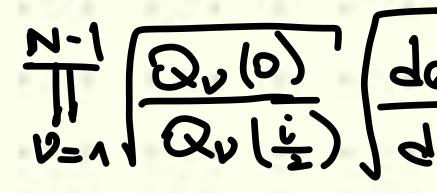




F, (u)

Qm(a) de 12、(の)の、(法)」

· gl(M)@gl(w·M)



 $\frac{Q_{2\nu}(3)Q_{2\nu}(\frac{i}{2})}{Q_{2\nu-1}(3)Q_{2\nu-1}(\frac{i}{2})}$ 

. 0(N)



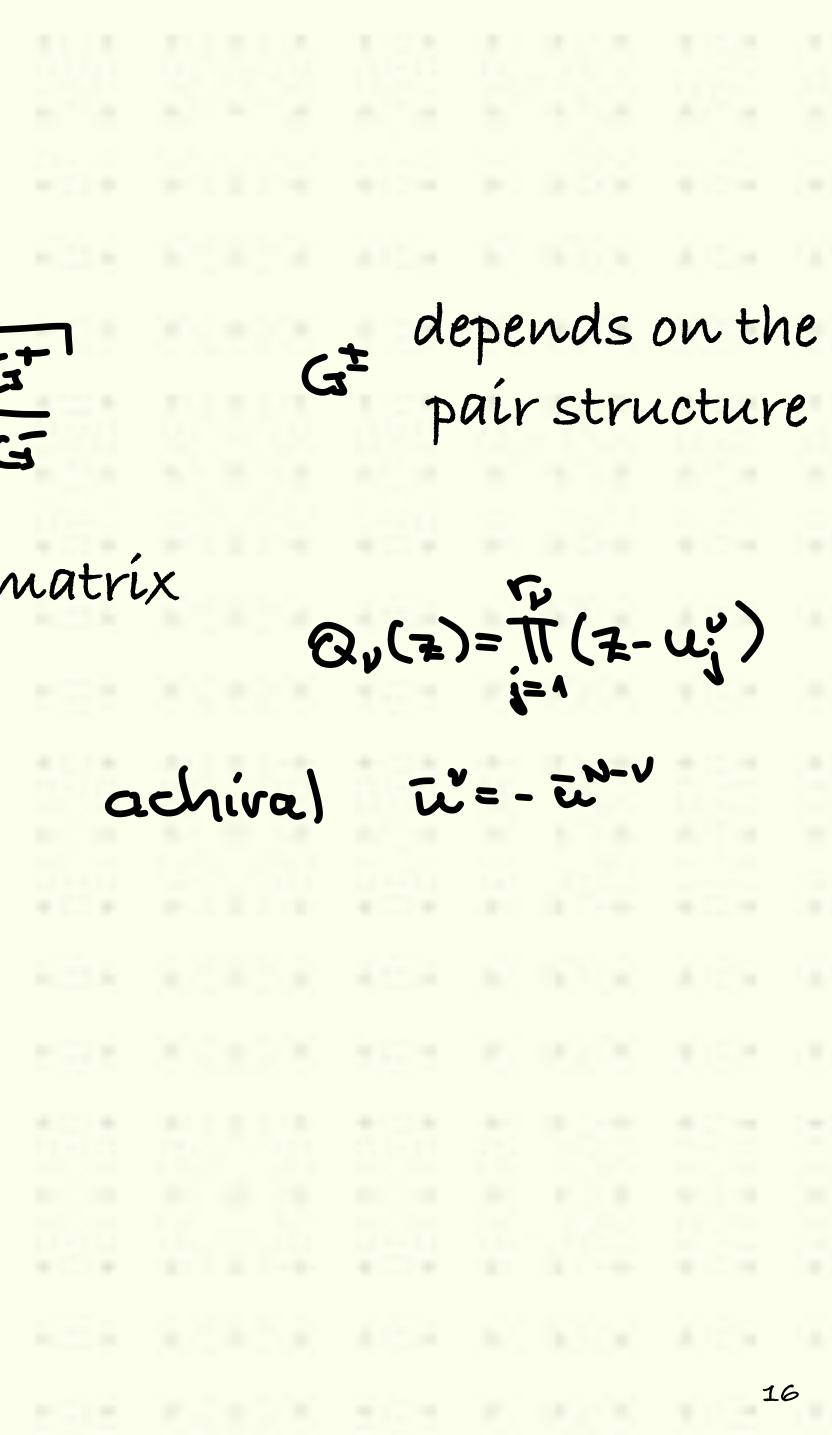
Korepin's criteria  $\longrightarrow \qquad \frac{\langle \psi | \overline{u} \rangle}{\langle \overline{u} | \overline{u} \rangle} = T_{\overline{v}} T_{\overline{v}} (\overline{u} ) \frac{\det G^{T}}{\det G}$ 

given by the K-matrix

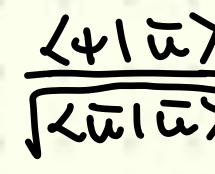
G<sup>±</sup> depends on the pair structure

			ビスノ(モ)= ハ(モー)						
etG <sup>+</sup> etG <sup>-</sup>	N=	N 2	C	ichiva)	ĩť	$= - \frac{N-V}{C}$			
0.04									
letG <sup>+</sup>									
det G									
$1 + c^+$									

det G<sup>T</sup> det G<sup>T</sup>

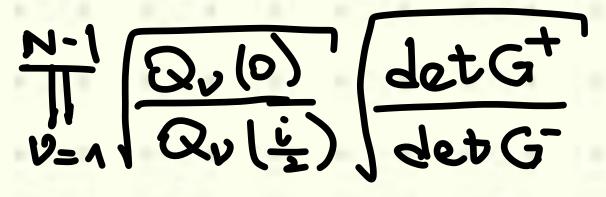






 $\frac{Q_{m}(\alpha)}{Q_{n}(0)Q_{n}(\frac{1}{2})} \det G$ 

 $\cdot g(M) \oplus g(N \cdot M)$ 



• SP(N)  $\prod_{v} \frac{Q_{2v}(s)Q_{2v}(\frac{1}{2})}{Q_{2v-1}(s)Q_{2v-1}(\frac{1}{2})} \frac{\det G^{\dagger}}{\det G^{\dagger}}$ 

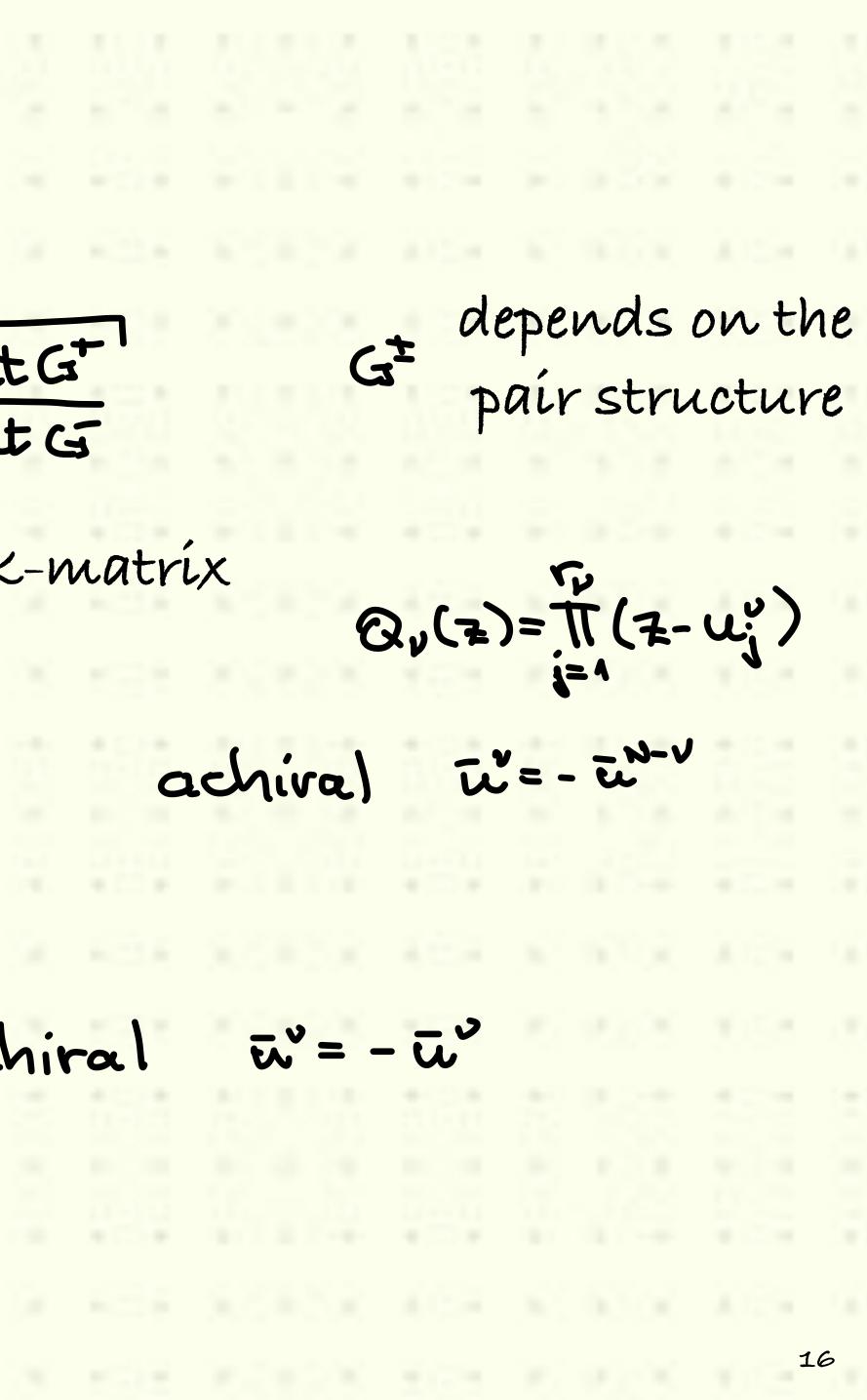
• 0(N)



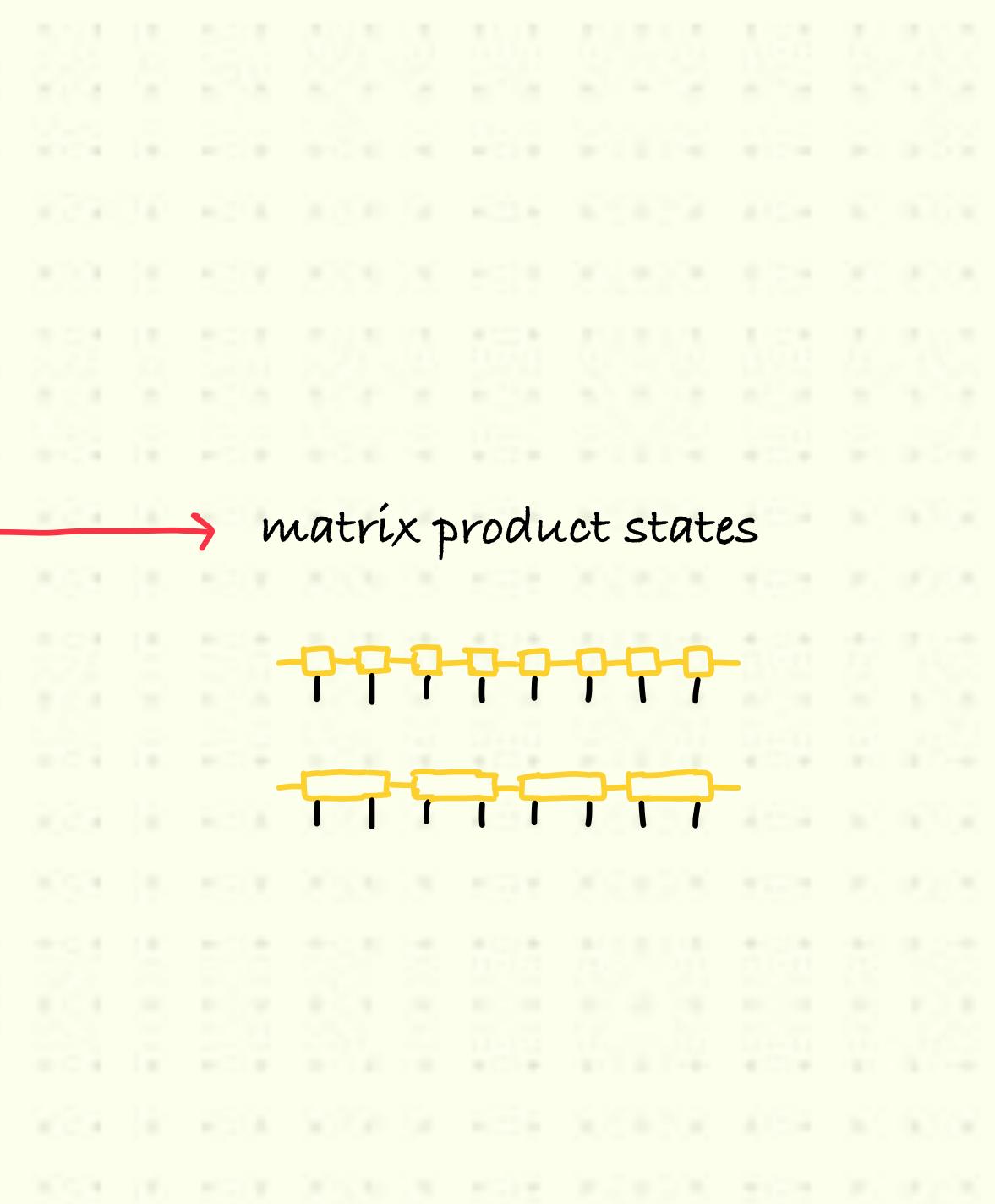
 $\frac{\langle \psi | \overline{u} \rangle}{\langle \overline{u} | \overline{u} \rangle} = \prod_{v} \overline{T}_{v} (\overline{u}^{v}) \frac{\det G}{\det G}$ 

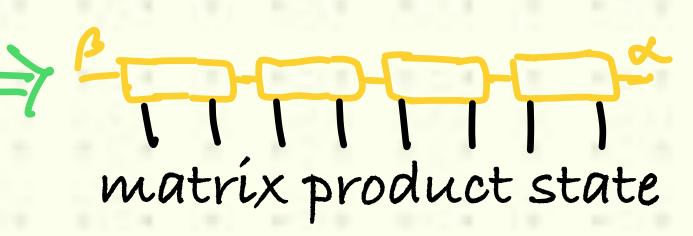
G<sup>±</sup> depends on the pair structure

F, (w) given by the K-matrix  $Q_{\nu}(z) = \prod_{j=1}^{r_{\nu}} (z - u_{j}^{\nu})$ achival  $\overline{u}^{*} = -\overline{u}^{N-v}$  $N = \frac{N}{2}$  $\overline{u}' = -\overline{u}'$ > chiral



two-site states — matrix product states  $1\mathcal{F}$ 





1										
2										
	1000	$\forall 1 \leq 2$	(V)	6.05	$V \leq t$	121	$t \geq t^{\prime}$	12.53	(V)	100

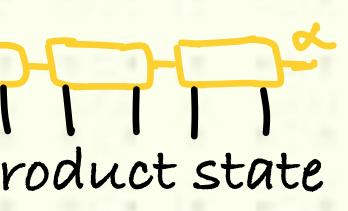


 $L + _{Lp} = \sum_{i=1, j_{23}} (M_{i}(\Theta_{i}) \dots M_{i}(\Theta_{i})) L_{i_{1}, j_{2}, \dots, j_{23}, j_{3}}$ 



 $\chi_{\mu} = \sum_{i=1}^{n} (M_{i}(\Theta_{i}) \dots M_{i}(\Theta_{i})) \chi_{\mu} \chi_$ i=1,...,d fl=[Cd]

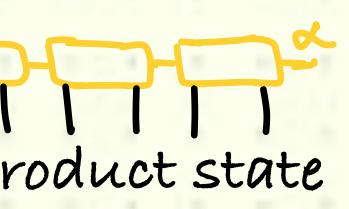




 $L + L = \sum_{i=1}^{n} (M_{i}(\Theta_{i}), M_{i}(\Theta_{i})) + L = \sum_{i=1}^{n} (M_$ 

<YI= ZKt apl⊗eupeft ⊗ End(H)







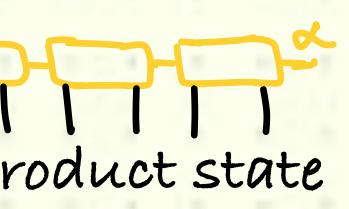
10.0	11	100	1000	1913	1000	101	10.00

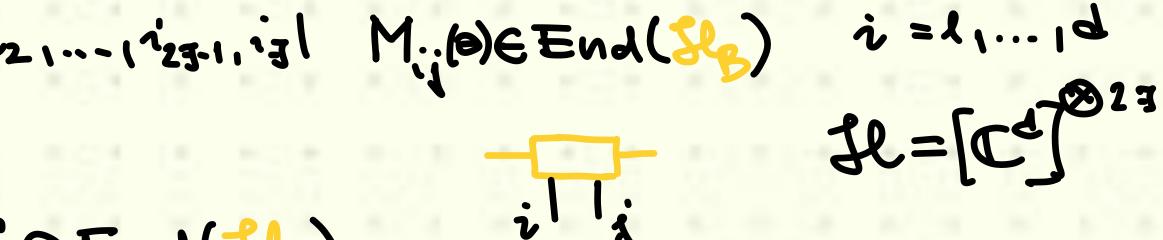
 $LY_{2} = \sum_{i_{2} \in I_{2}} (M_{i}(\Theta_{i}), M_{i}(\Theta_{i})) L_{i_{1}} L_{i_{1}} L_{i_{2}} L_{i_{2}} L_{i_{2}} L_{i_{2}} (M_{i_{2}}) = \sum_{i_{1} \in I_{2}} (M_{i_{2}}(\Theta_{i_{2}}), M_{i_{2}}(\Theta_{i_{2}})) L_{i_{1}} L_{i_{2}} L$ 

<YI= Z<+x, 1⊗ exp ∈ fet ⊗ End(H)

K(u) E End (CNOK)



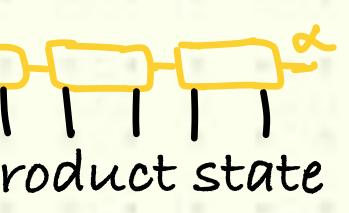


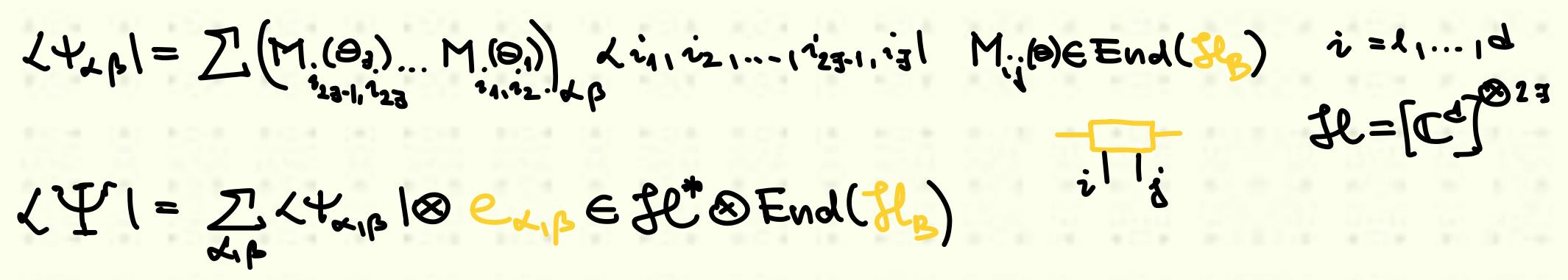


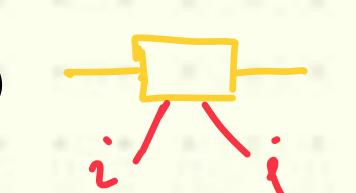
10.0	11	100	1000	1913	1000	101	10.00

K(u) E End (CNOLE) Kij (u) E End (Le)







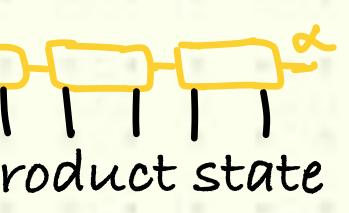


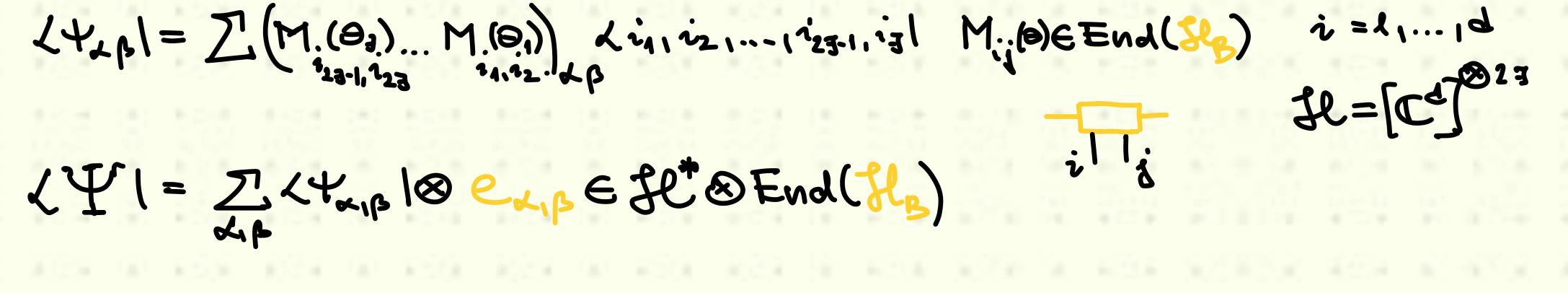
 $L + \mu = \sum_{i=1}^{n} (M_{i}(\Theta_{i}) M_{i}(\Theta_{i})) L_{i_{1}} L_{i_{1}} L_{i_{2}} L_{i_{$ 

K(u) E End (CNORB) Kij (u) E End (RB)

KT-relation







 $\sum_{j=1}^{N} K_{ij}(z) \langle \Psi|T_{j}(z) = \sum_{j=1}^{N} \langle \Psi|T_{ij}(-z) K_{j}(z) = \sum_{j=1}^{N} \langle \Psi|T_{ij}(-z) K_{j}(-z) K_{j}(z) = \sum_{j=1}^{N} \langle \Psi|T_{ij}(-z) K_{j}(-z) K_{j}(z) = \sum_{j=1}^{N} \langle \Psi|T_{ij}(-z) K_{j}(-z) K_{j}(z) = \sum_{j=1}^{N} \langle \Psi|T_{ij}(-z) K_{j}(-z) K_{j}($ 



# Integrability con the second s

1	di	ti	.01					
	10.0	$(\mathbf{v})$	$g \leq 1^{-1}$	2010/01/01	1993	1000	102	10.00



# Integrability con

## $\langle MPS | = \frac{1}{1} - \frac{1}{1} = \sum_{n=1}^{n} \langle \Psi_{nn} \rangle$

1	di	ti	.01					
	10.0	$(\mathbf{v})$	$g \leq 1^{-1}$	2000	:03	1000	303	10.00



Integrability con a law alter lat attend alter lat attend with a lat at

#### $\langle MPS | = \frac{1}{11} - \frac{1}{11} = \sum_{n=1}^{n} \langle \Psi_{nn} |$

 $\Rightarrow \langle MPS|T(u) = \langle MPS|T(-u)$  or

1	di	ti	.01						
(}	1PSIT	(n)	= < M	psite	- u)				
٢.,	1.5.5	121	2.12	20.5	1.1	2.12	1.11.1	21-21	1000



Integrability con

#### $\langle MPS | = \frac{1}{1} - \frac{1}{1} = \sum_{n=1}^{n} \langle \Psi_{nn} |$

#### $\Rightarrow \langle MPS|T(u) = \langle MPS|T(-u)$ or

homogeneous lím

1	di	ti	.01	1					
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ú	-								
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	1000	141	$\mathcal{C} \subseteq \mathcal{C}$	2010		$t \in \{1, 2\}$	10.000	$\overline{\mathcal{T}} \subset \mathcal{T}$	10.00



Integrability con

## $\langle MPS | = \frac{1}{1} - \frac{1}{1} = \sum_{n=1}^{n} \langle \Psi_{n,n} \rangle$

 $\Rightarrow \langle MPS|T(u) = \langle MPS|T(-u)$  or

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MPSIT	(u)	= < MF	psite	· u)		
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Integrability con

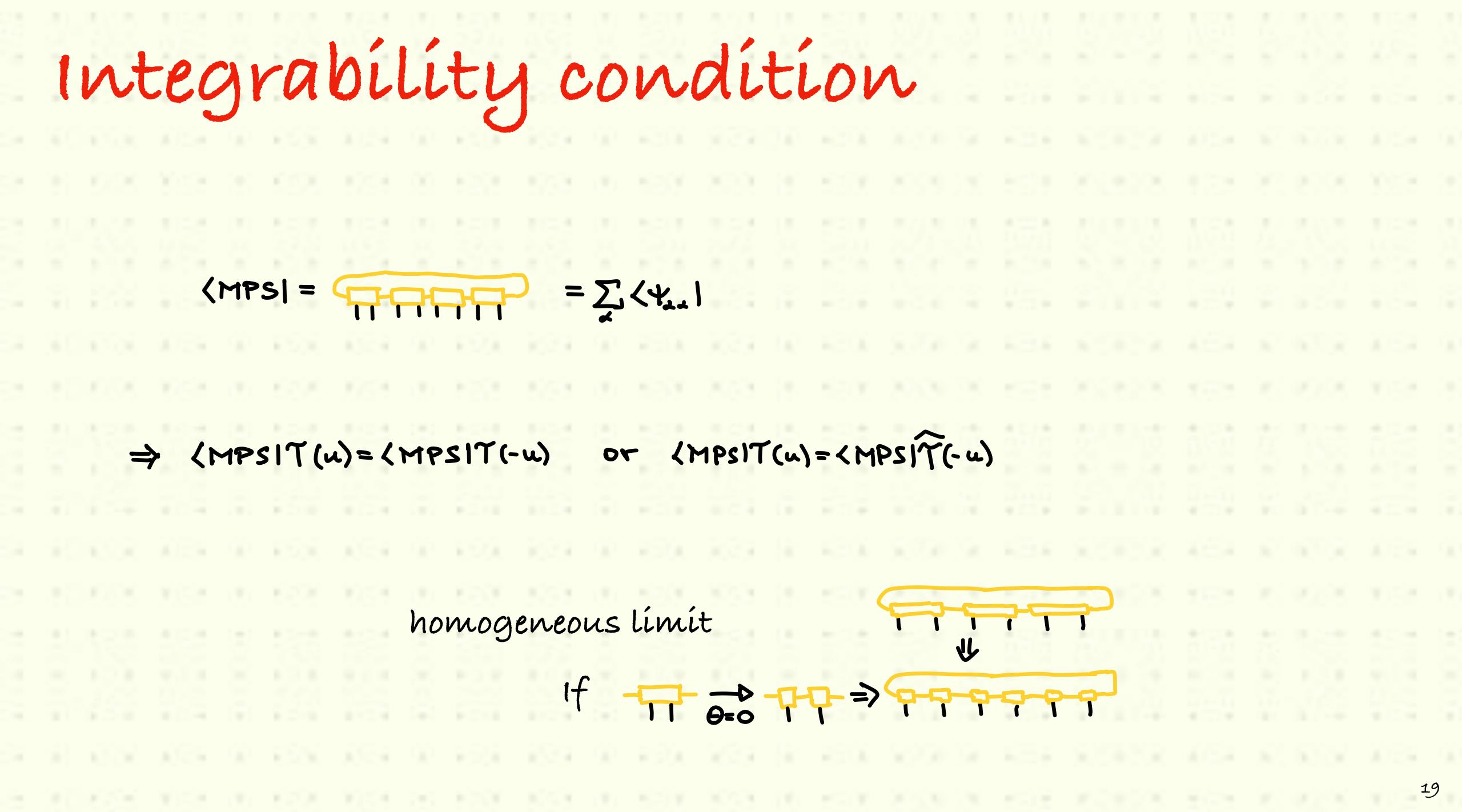
## $\langle MPS| = - - - - - - - - = \sum_{x} \langle \Psi_{xx} \rangle$

tal attra ditta tal attra attra ditta attra

 $\Rightarrow \langle MPS|T(u) = \langle MPS|T(-u)$  or

homogeneous lim

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## classification of

#### compatibility of the KT with RTT -

	8					Ne	
tn	e	K	-M	at	rici	es	
7							
$\{ i, j \in I \}$	$\{ \psi \}_{i}$	$g \leq 1^{-1}$	2010/01/01	$g(x) \in \mathbb{R}$	1000	10.1	10.000

# Classification of the K-matrices compatibility of the KT with RTT -> Reflection equation (byBe) 20

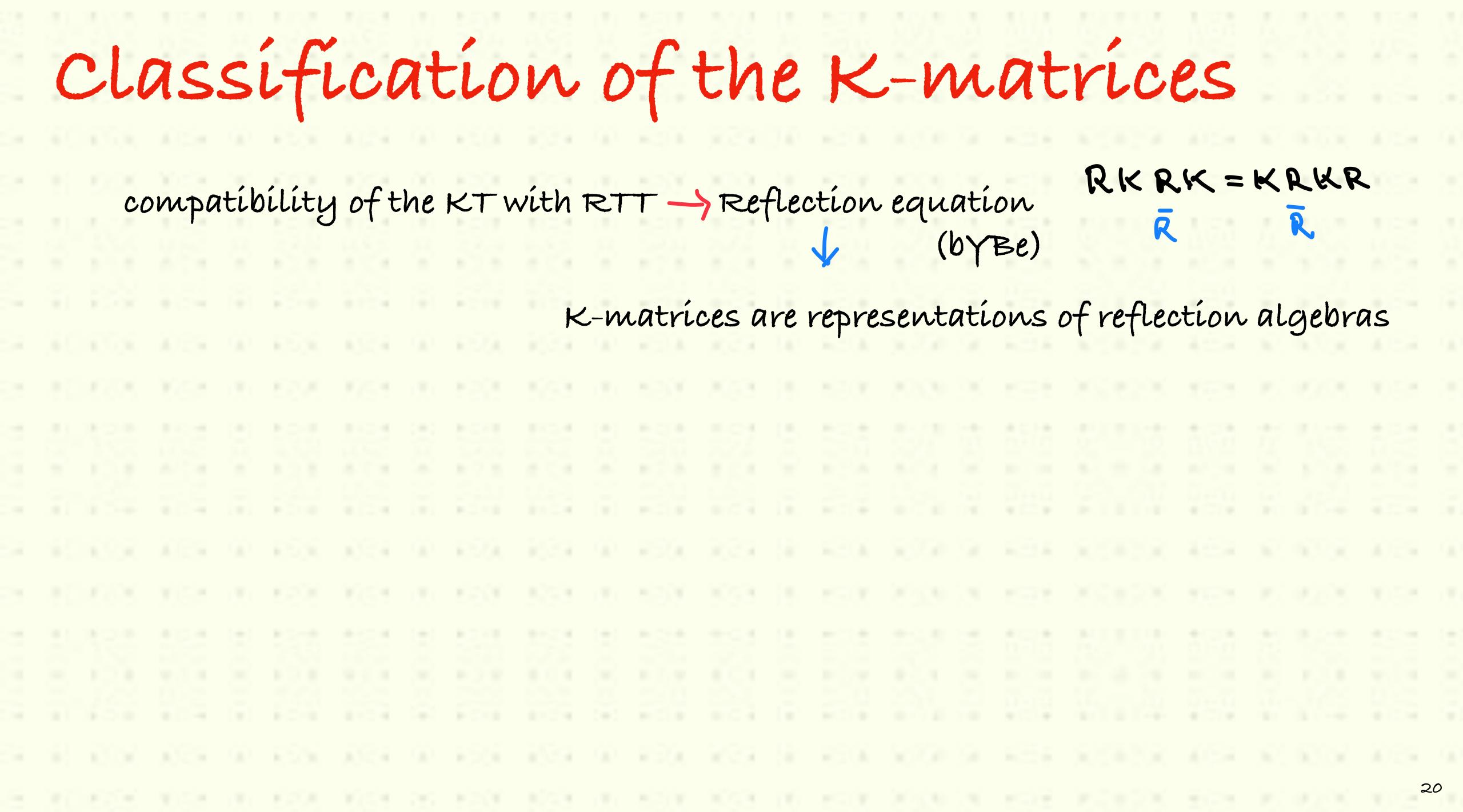
# Classification of the K-matrices

## compatibility of the KT with RTT $\rightarrow$ Reflection equation (bYBe) RKRK = KRKR

### compatibility of the KT with RTT -> Reflection equation RKRK = KRKR (byBe)



K-matrices are representations of reflection algebras



### compatibility of the KT with RTT $\rightarrow$ Reflection equation (by Be) RKRK = KRKR

K-matrices are representations of reflection algebras

### non-crossed by Be $\longrightarrow B(N,M)$ algebra

algebra 20



### compatibility of the KT with RTT -> Reflection equation

K-matrices are representations of reflection algebras

### non-crossed by Be $\longrightarrow B(N,M)$

 $\rightarrow$  Reflection equation (by Be) RKRK = KRKR

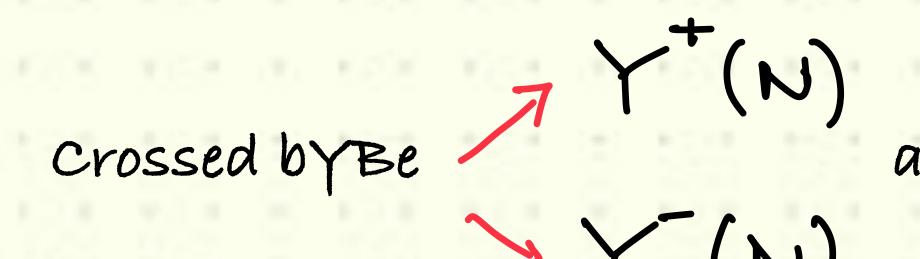
algebra → resídual symmetry: gl(M)⊕gl(N-M) 20



compatibility of the KT with RTT -> Reflection equation

K-matrices are representations of reflection algebras

### B(N,M) algebra non-crossed by Be



residual symmetry:  $g(M) \oplus g(N-M)$  $\rightarrow Y(N)$ 20

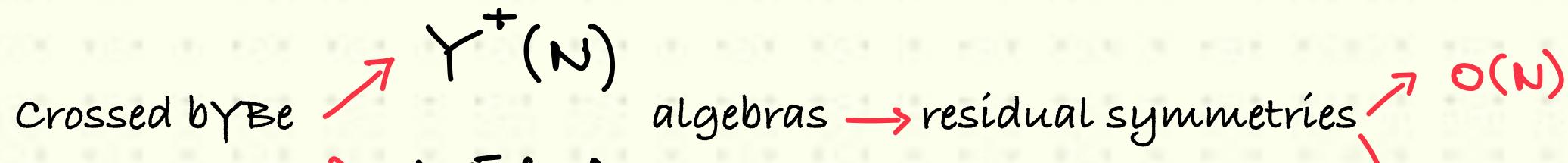
RKRK = KRKR (byBe)



compatibility of the KT with RTT -> Reflection equation

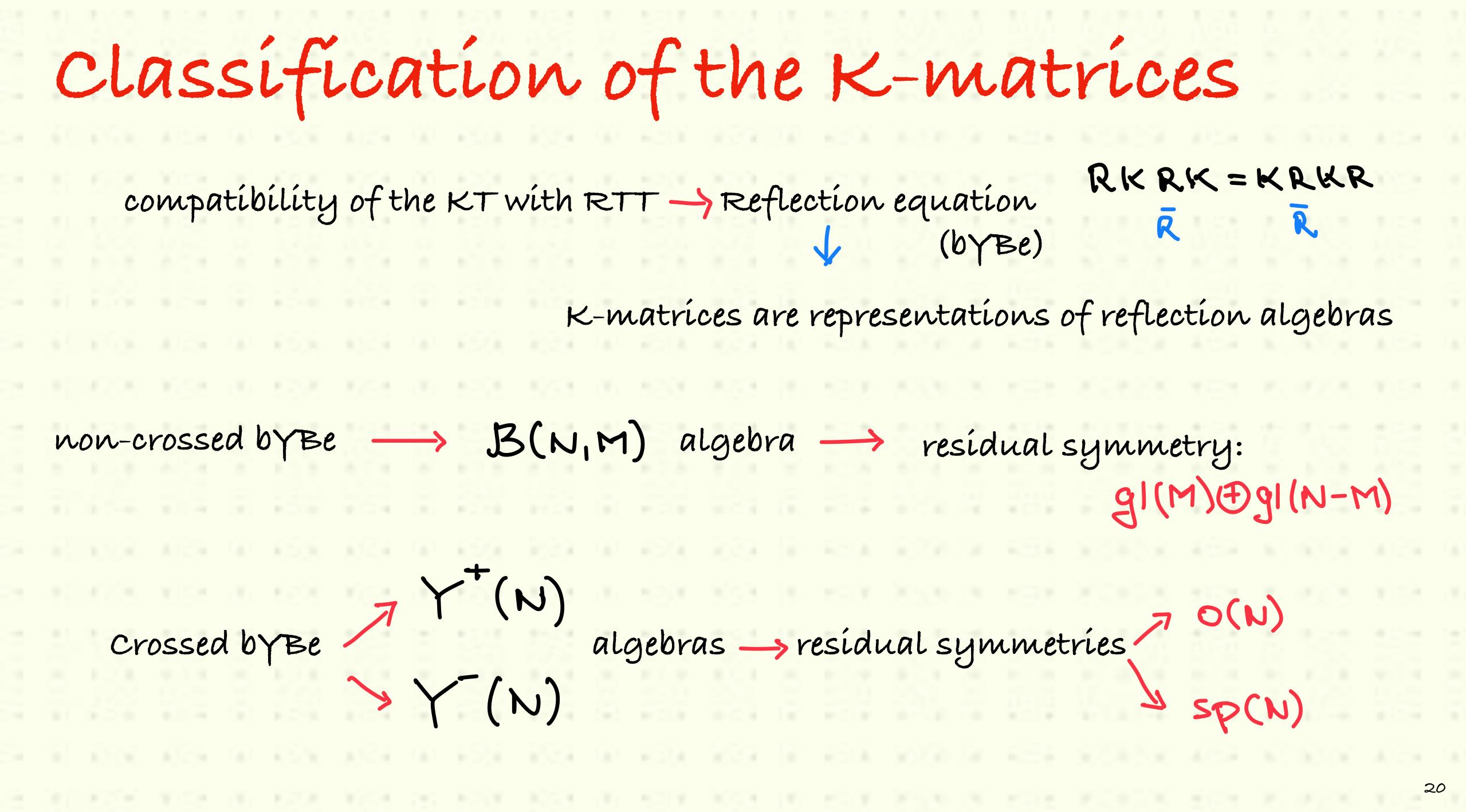
K-matrices are representations of reflection algebras

### non-crossed by Be B(N,M) algebra



RKRK = KRKR (bybe)

residual symmetry:  $g(M) \oplus g(N-M)$ 



Recursion for off-shell overlaps for crossed KT 21



# Recursion for off-shell overlaps for crossed KT -> 21



•				ing	k	6.11

21

### is invertible



•				ing	k	6.11

is invertible -recursion for u roots 21



for crossed KT -> assuming Kin is invertible -> recursion for u roots  $\langle \psi | \overline{\omega}^{*}, \overline{\omega}^{*}, \dots \rangle = \sum (\dots) \langle \psi | \varphi, \overline{\omega}^{*}, \dots \rangle$ 21



 $\langle \Psi | \tilde{u}, \tilde{u}, ... \rangle = \Sigma(...) \langle \Psi | \phi, \tilde{w}, ... \rangle \longrightarrow g(N-i)$  overlaps

21

for crossed KT -> assuming Kin is invertible -> recursion for u roots



 $\langle \Psi | \overline{u}, \overline{u}, ... \rangle = \Sigma(...) \langle \Psi | \phi, \overline{w}, ... \rangle \longrightarrow g(N-i) overlaps$ 

gl(N-1) KT-relation?

for crossed KT -> assuming Kin is invertible -> recursion for in roots



 $\langle \Psi | \tilde{u}, \tilde{u}, ... \rangle = \Sigma(...) \langle \Psi | \phi, \tilde{w}, ... \rangle \longrightarrow g(N-i)$  overlaps

gl(N-1) KT-relation?

for crossed KT -> assuming Kin is invertible -> recursion for u roots

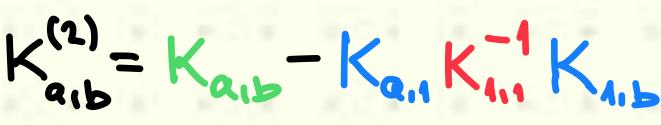


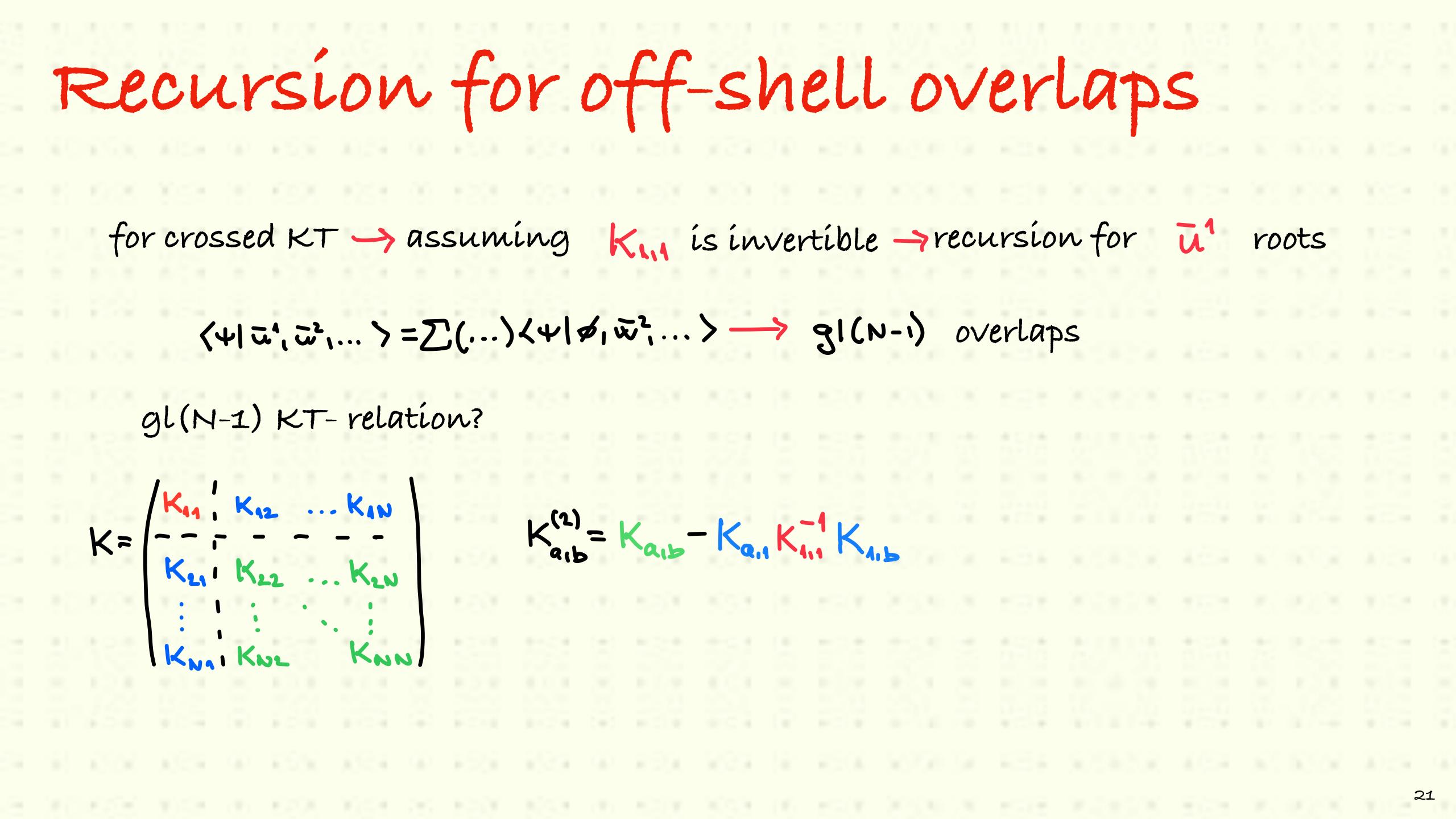
Recursion for off-shell overlaps

 $\langle \Psi | \tilde{u}, \tilde{u}, ... \rangle = \Sigma(...) \langle \Psi | \phi, \tilde{w}, ... \rangle \longrightarrow g(N-i)$  overlaps

gl(N-1) KT-relation?

for crossed KT -> assuming Kin is invertible -> recursion for I1 roots





Recursion for off-shell overlaps

 $\langle \Psi | \bar{\omega}^{*}, \bar{\omega}^{*}, ... \rangle = \Sigma(...) \langle \Psi | \phi_{1} \bar{\omega}^{*}, ... \rangle \longrightarrow g(N-1)$  overlaps

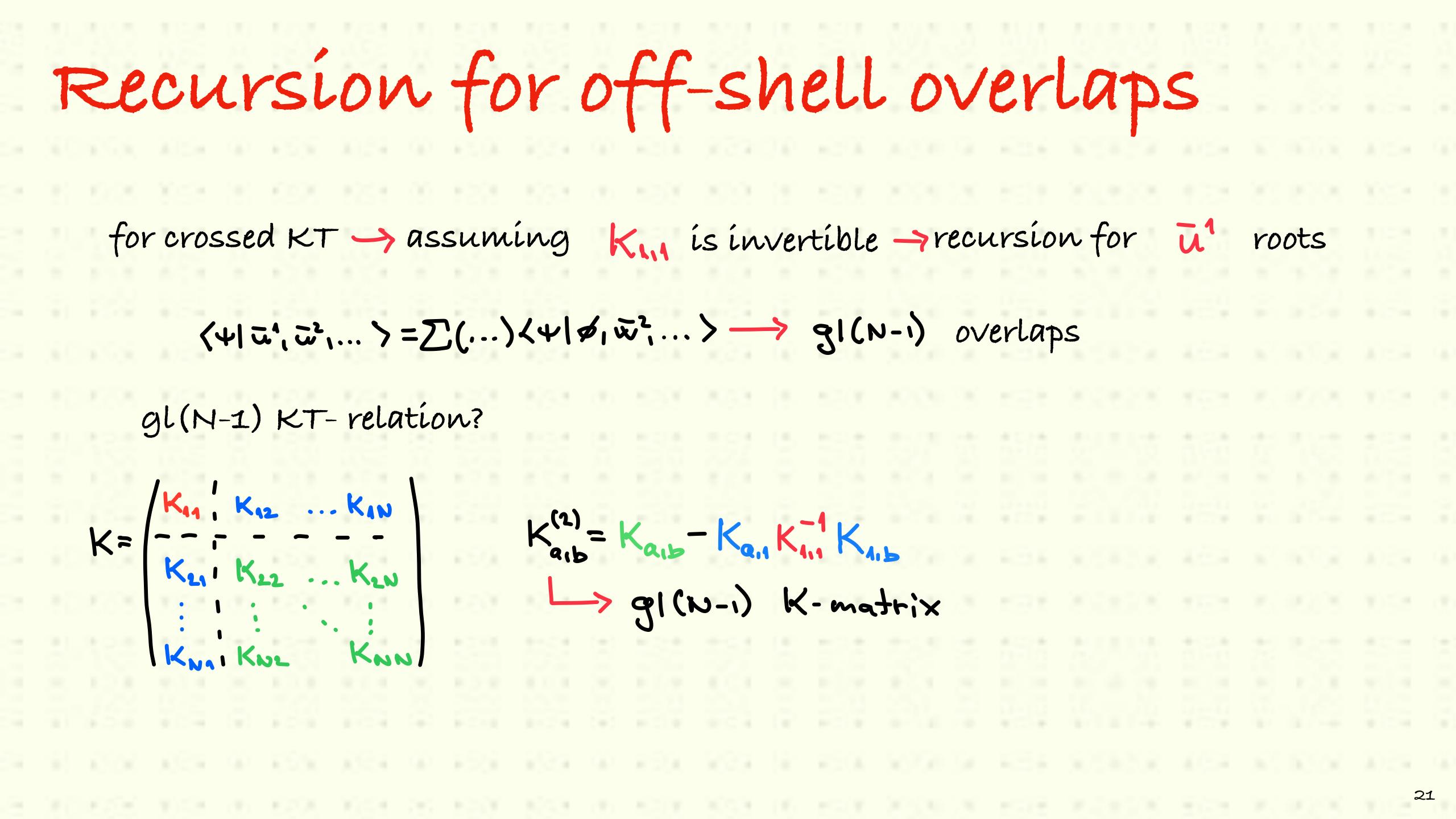
gl(N-1) KT-relation?

 $K = \begin{bmatrix} K_{11} & K_{12} & \dots & K_{1N} \\ - & - & - & - & - & - \\ K_{21} & K_{22} & \dots & K_{2N} \end{bmatrix}$ 

for crossed  $KT \rightarrow assuming K_{M}$  is invertible  $\rightarrow recursion$  for  $\overline{u}^{1}$ roots

 $K_{a,b}^{(2)} = K_{a,b} - K_{a,1} K_{a,1}^{-1} K_{a,b}$ 

-> g((N-i) K-matrix



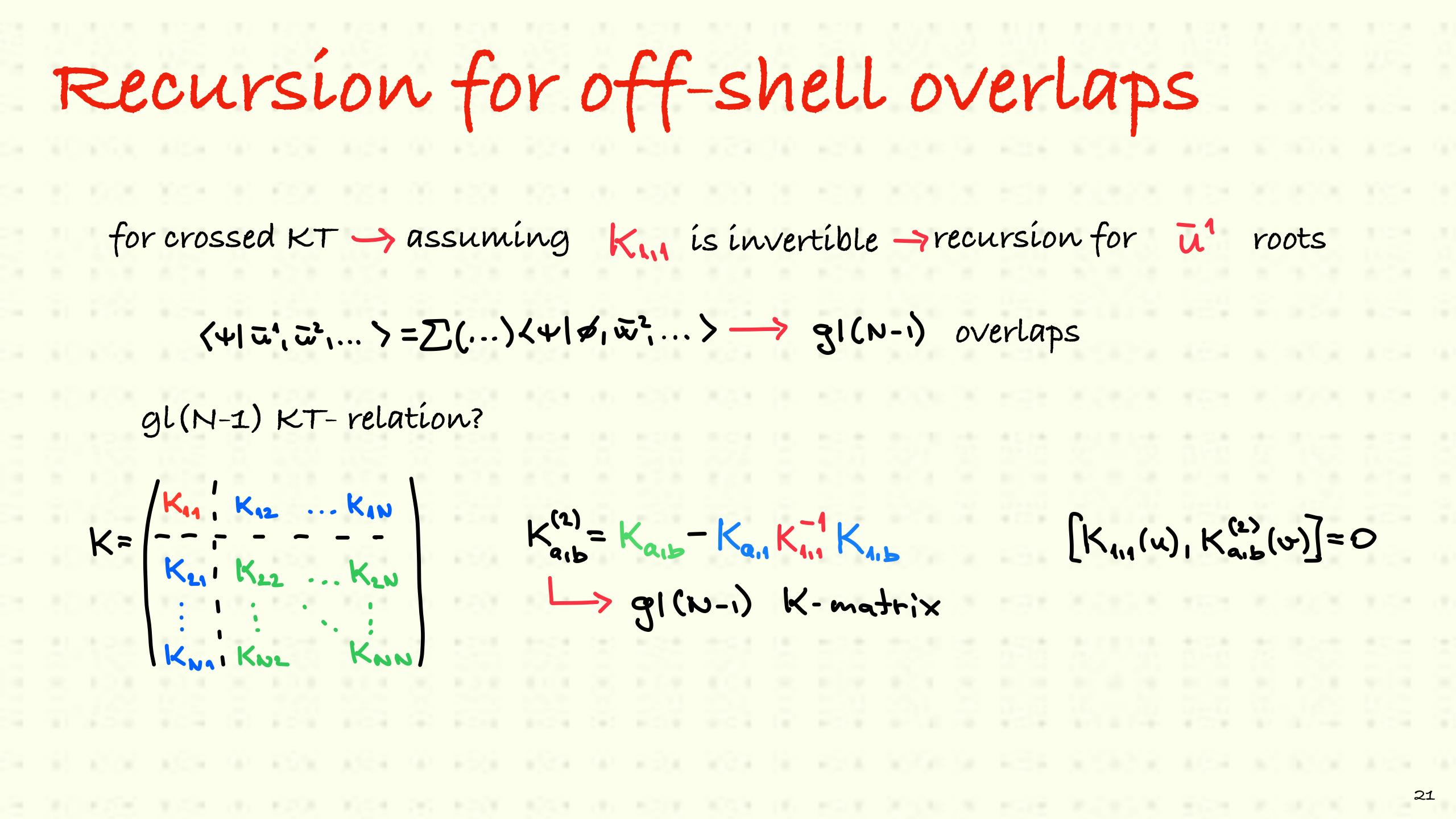
Recursion for off-shell overlaps

 $\langle \Psi | \tilde{\omega}^{*}, \tilde{\omega}^{*}, \dots \rangle = \Sigma(\dots) \langle \Psi | \phi, \tilde{\omega}^{*}, \dots \rangle \longrightarrow g(N-i)$  overlaps

gl(N-1) KT-relation?

 $K = \begin{bmatrix} K_{11} & K_{12} & \dots & K_{1N} \\ - & - & - & - & - & - \\ K_{21} & K_{22} & \dots & K_{2N} \end{bmatrix}$  $K_{a,b}^{(2)} = K_{a,b} - K_{a,i} K_{i,i} - K_{i,j}$ g(N-i) K-matrix 

 $[K_{1,1}(u), K_{a,b}^{(2)}(v)] = 0$ 



Recursion for off-shell overlaps

 $\langle \Psi | \tilde{\omega}^{*}, \tilde{\omega}^{*}, \dots \rangle = \Sigma(\dots) \langle \Psi | \phi, \tilde{\omega}^{*}, \dots \rangle \longrightarrow g(N-i)$  overlaps

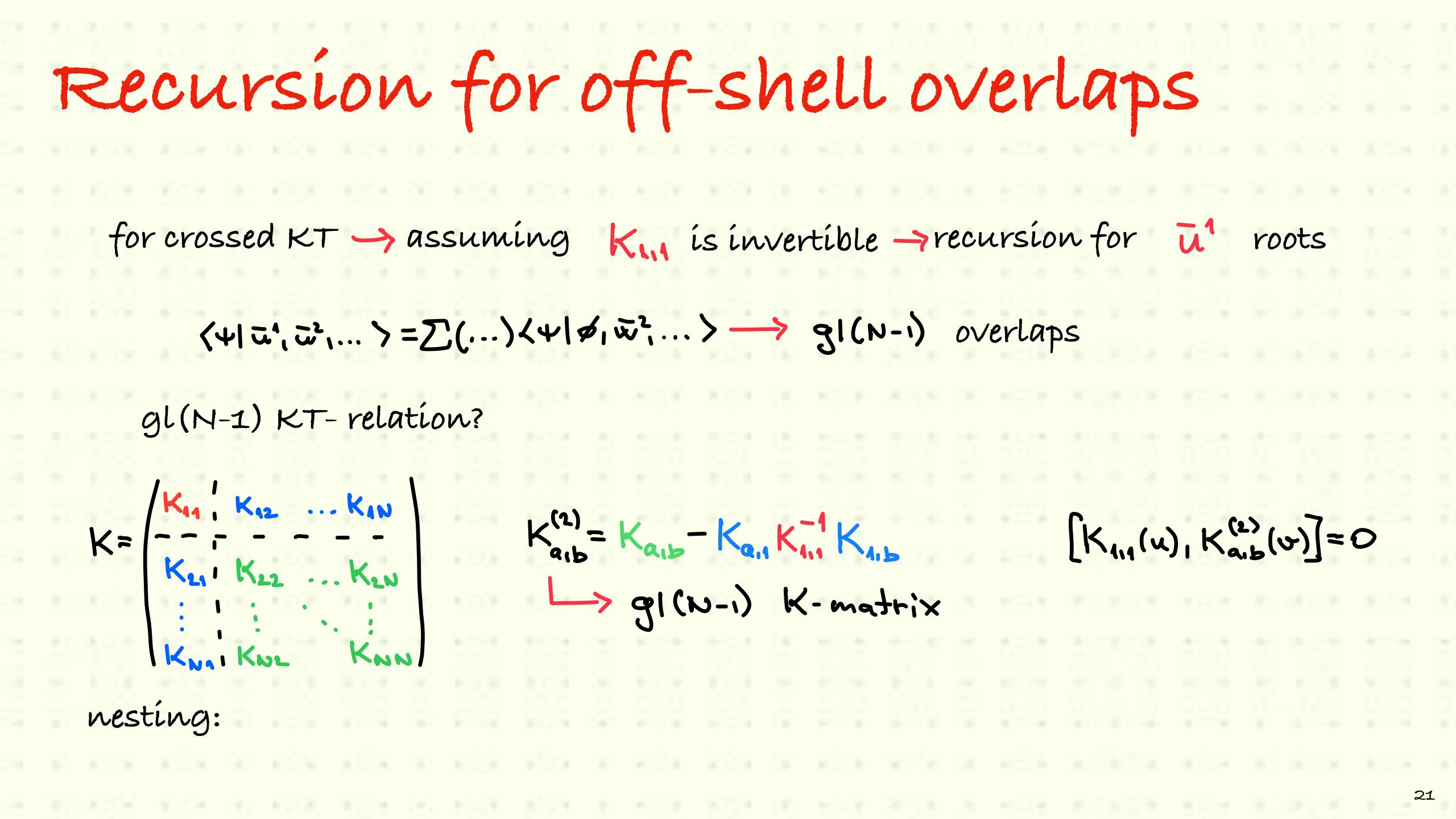
gl(N-1) KT-relation?

 $K = \begin{bmatrix} K_{11} & K_{12} & \dots & K_{1N} \\ - & - & - & - & - & - \\ K_{21} & K_{22} & \dots & K_{2N} \end{bmatrix}$ nesting:

 $K_{a,b}^{(2)} = K_{a,b} - K_{a,i} K_{i,i} - K_{i,j}$ 

g(N-i) K-matrix

 $[K_{1,1}(u), K_{a,b}^{(2)}(v)] = 0$ 



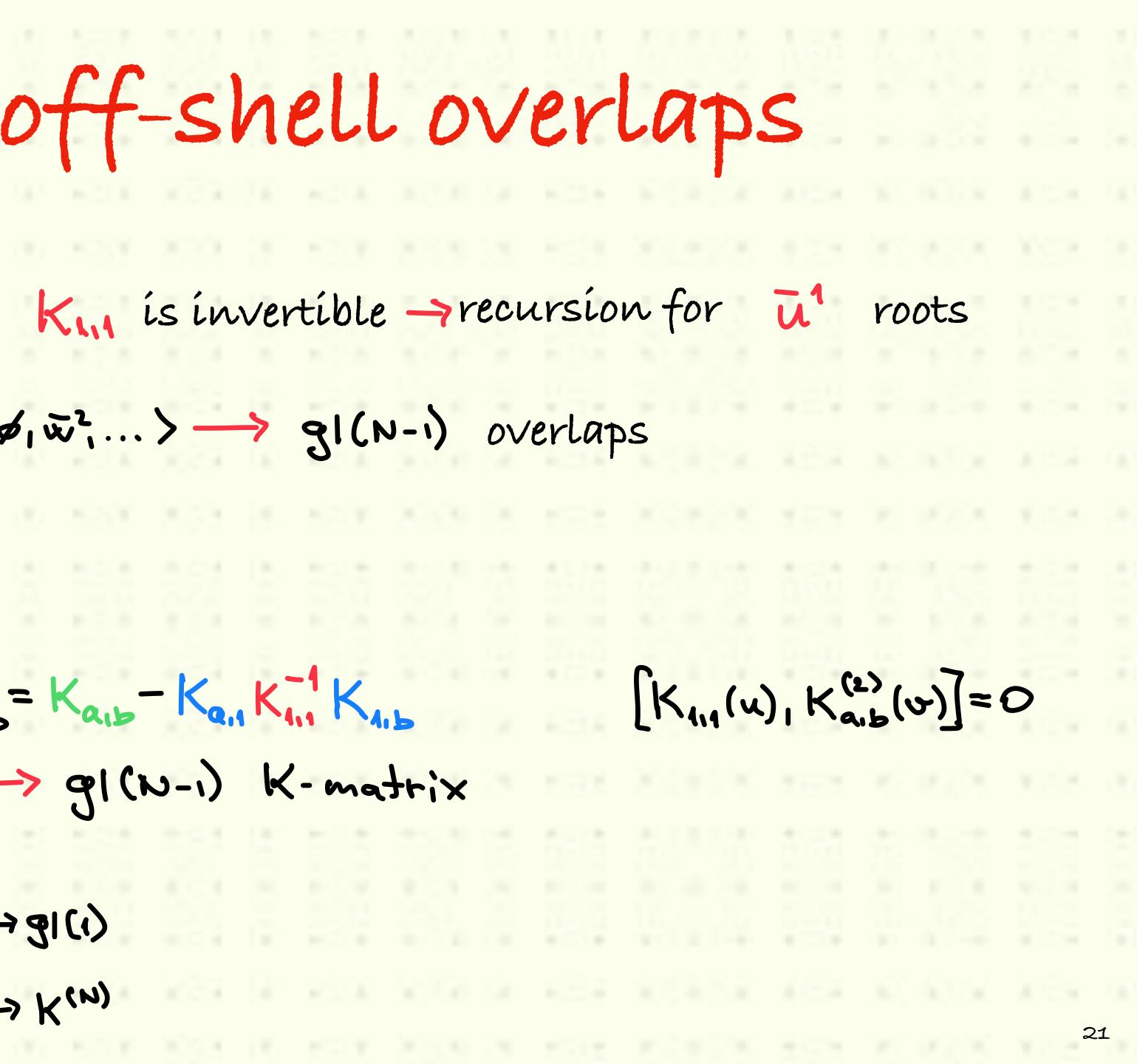
 $\langle \Psi | \tilde{\omega}^{*}, \tilde{\omega}^{*}, \dots \rangle = \Sigma(\dots) \langle \Psi | \phi, \tilde{\omega}^{*}, \dots \rangle \longrightarrow g(N-i)$  overlaps

gl(N-1) KT-relation?

 $K = \begin{bmatrix} K_{11} & K_{12} & \dots & K_{1N} \\ - & - & - & - & - & - \\ K_{21} & K_{22} & \dots & K_{2N} \end{bmatrix}$  $K_{a,b}^{(2)} = K_{a,b} - K_{a,i} K_{i,i} - K_{i,j}$ g(N-i) K-matrix nesting:  $g(N) \rightarrow g(N-1) \rightarrow ... \rightarrow g(N)$ 

 $K_{(n)} \equiv K \longrightarrow K_{(r)} \rightarrow \cdots \rightarrow K_{(N)}$ 

 $[K_{1,1}(u), K_{a,b}^{(2)}(v)] = 0$ 



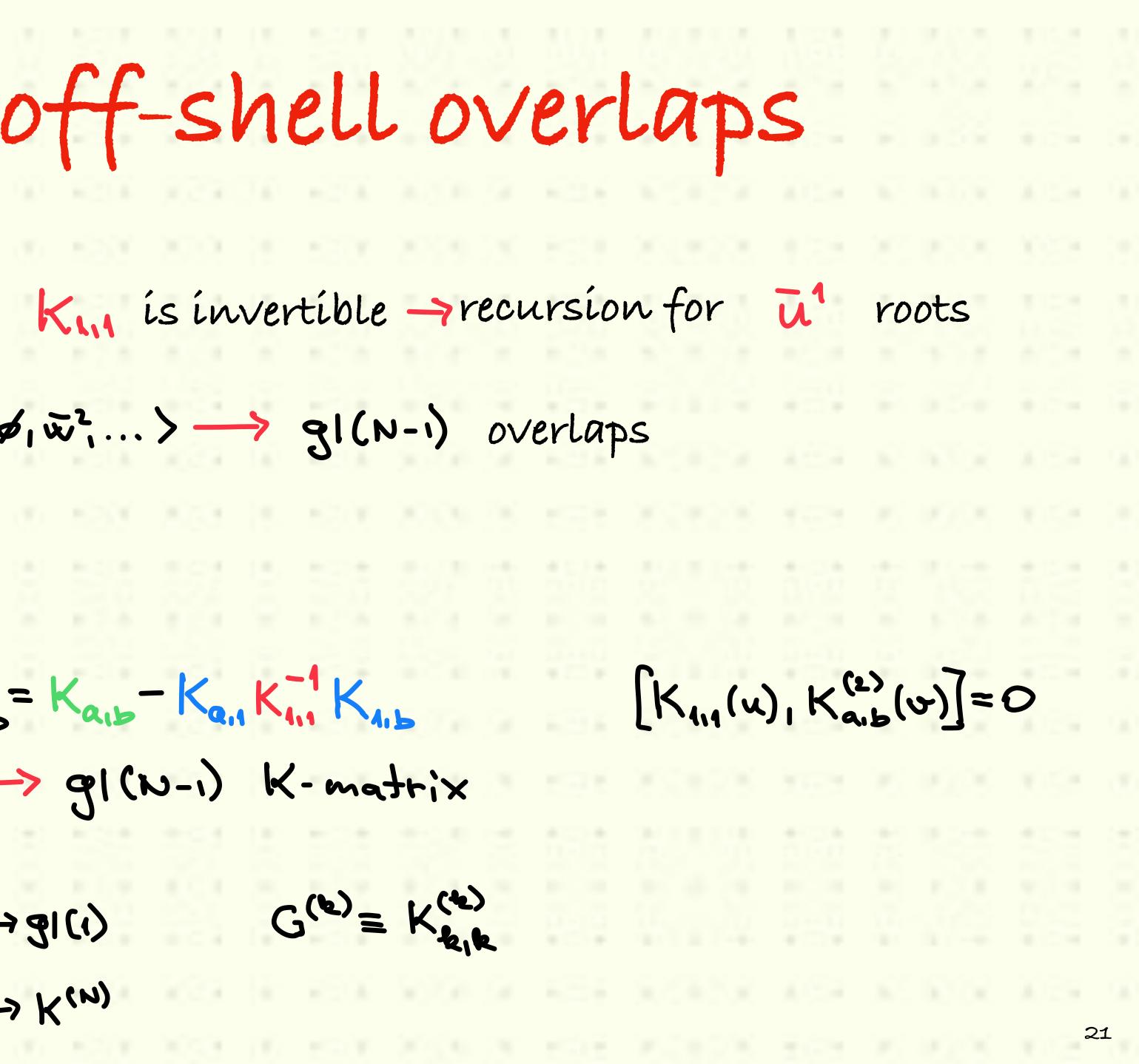
 $\langle \Psi | \tilde{\omega}^{*}, \tilde{\omega}^{*}, \dots \rangle = \Sigma(\dots) \langle \Psi | \phi, \tilde{\omega}^{*}, \dots \rangle \longrightarrow g(N-i)$  overlaps

gl(N-1) KT-relation?

 $K = \begin{bmatrix} K_{11} & K_{12} & \dots & K_{1N} \\ - & - & - & - & - & - \\ K_{21} & K_{22} & \dots & K_{2N} \end{bmatrix}$  $K_{a,b}^{(2)} = K_{a,b} - K_{a,i} K_{i,i} - K_{i,j}$ g(N-i) K-matrix  $G^{(k)} \equiv K_{k,k}^{(k)}$ nesting:  $g(N) \rightarrow g(N-1) \rightarrow ... \rightarrow g(I)$ 

 $K_{(n)} \equiv K \longrightarrow K_{(r)} \rightarrow \cdots \rightarrow K_{(n)}$ 

 $[K_{111}(u), K_{a,b}^{(2)}(v)] = 0$ 



 $\langle \Psi | \tilde{\omega}^{*}, \tilde{\omega}^{*}, \dots \rangle = \Sigma(\dots) \langle \Psi | \phi, \tilde{\omega}^{*}, \dots \rangle \longrightarrow g(N-i)$  overlaps

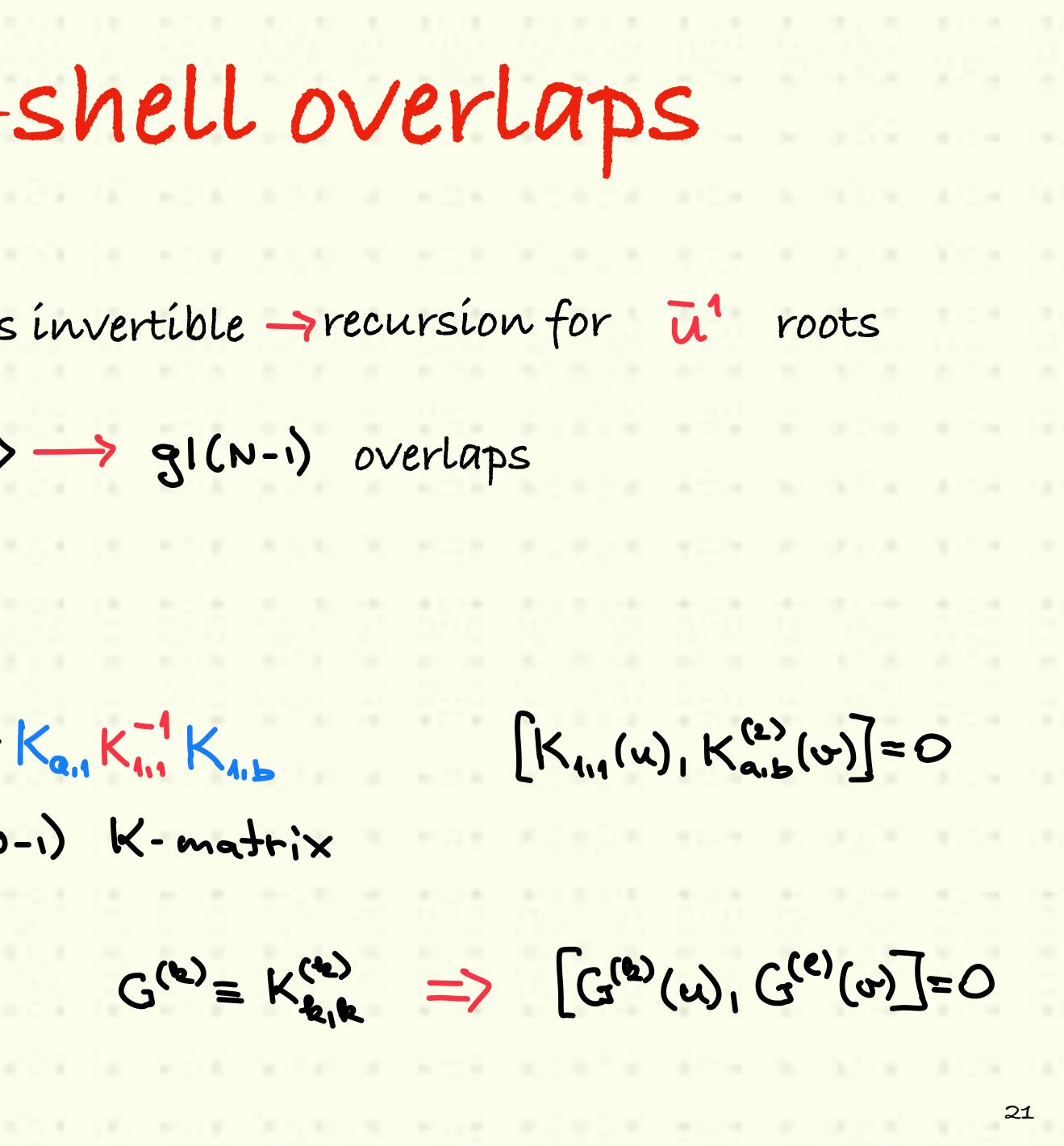
gl(N-1) KT-relation?

 $K = \begin{bmatrix} K_{11} & K_{12} & \dots & K_{1N} \\ - & - & - & - & - & - \\ K_{21} & K_{22} & \dots & K_{2N} \end{bmatrix}$  $K_{a_1b}^{(2)} = K_{a_1b} - K_{a_1} K_{a_1}^{-1} K_{a_1b}$ g(N-i) K-matrix nesting:  $g(N) \rightarrow g(N-1) \rightarrow ... \rightarrow g(I)$ 

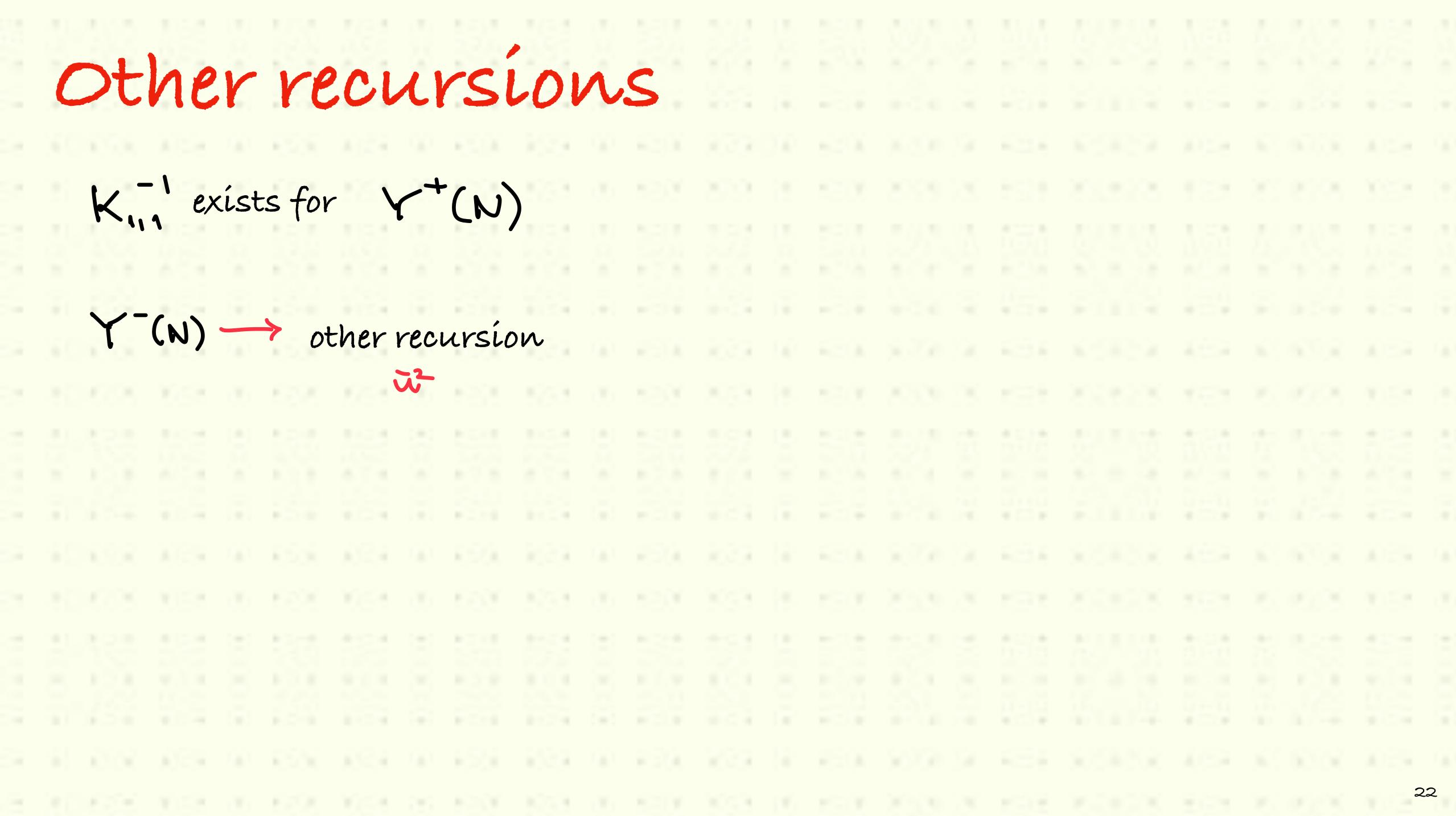
 $K_{(n)} \equiv K \longrightarrow K_{(r)} \rightarrow \cdots \rightarrow K_{(n)}$ 

 $[K_{1,1}(u), K_{a,b}^{(2)}(v)] = 0$ 

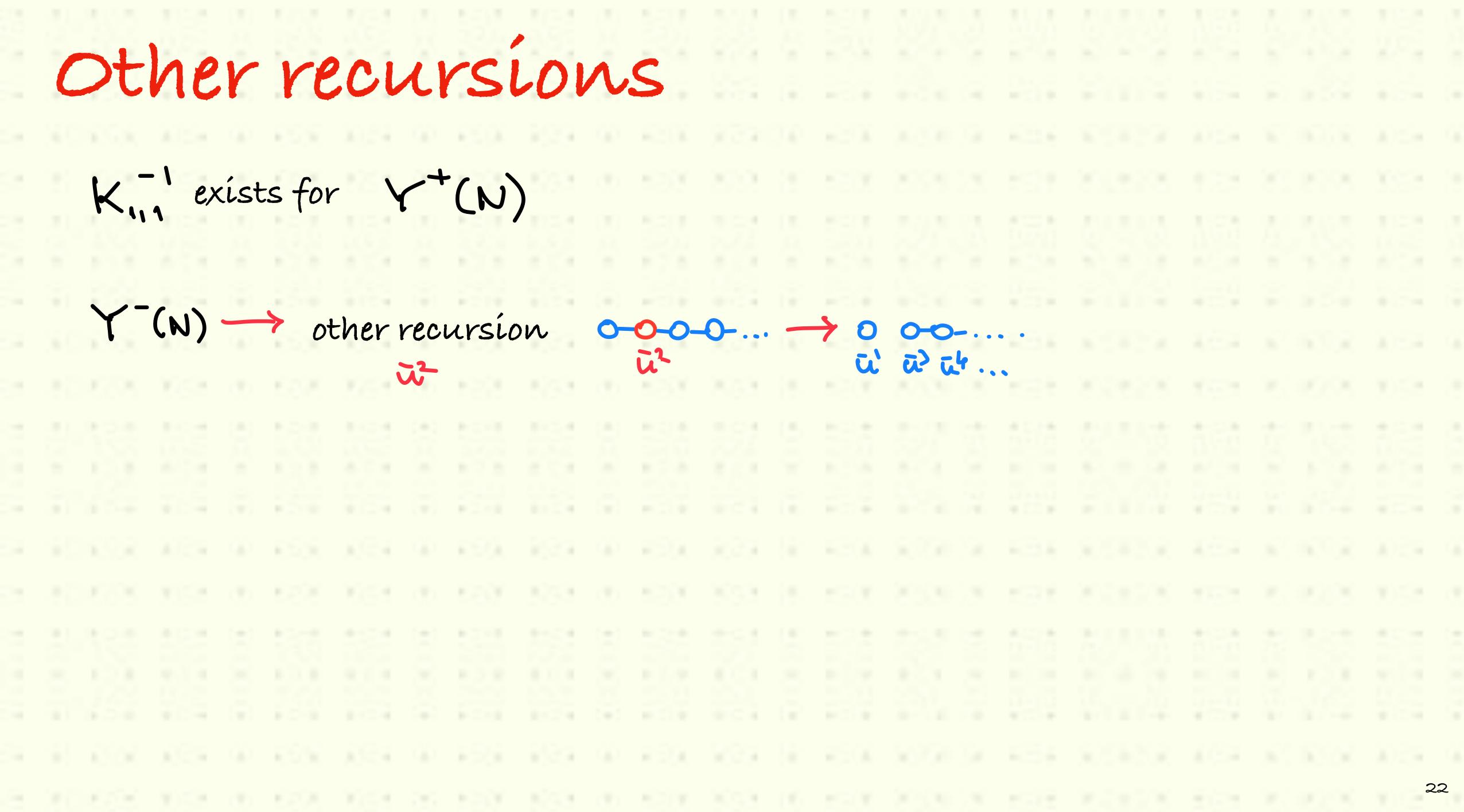
 $G^{(k)} = K_{k,k}^{(k)} \implies \left[G^{(k)}(u), G^{(e)}(v)\right] = 0$ 

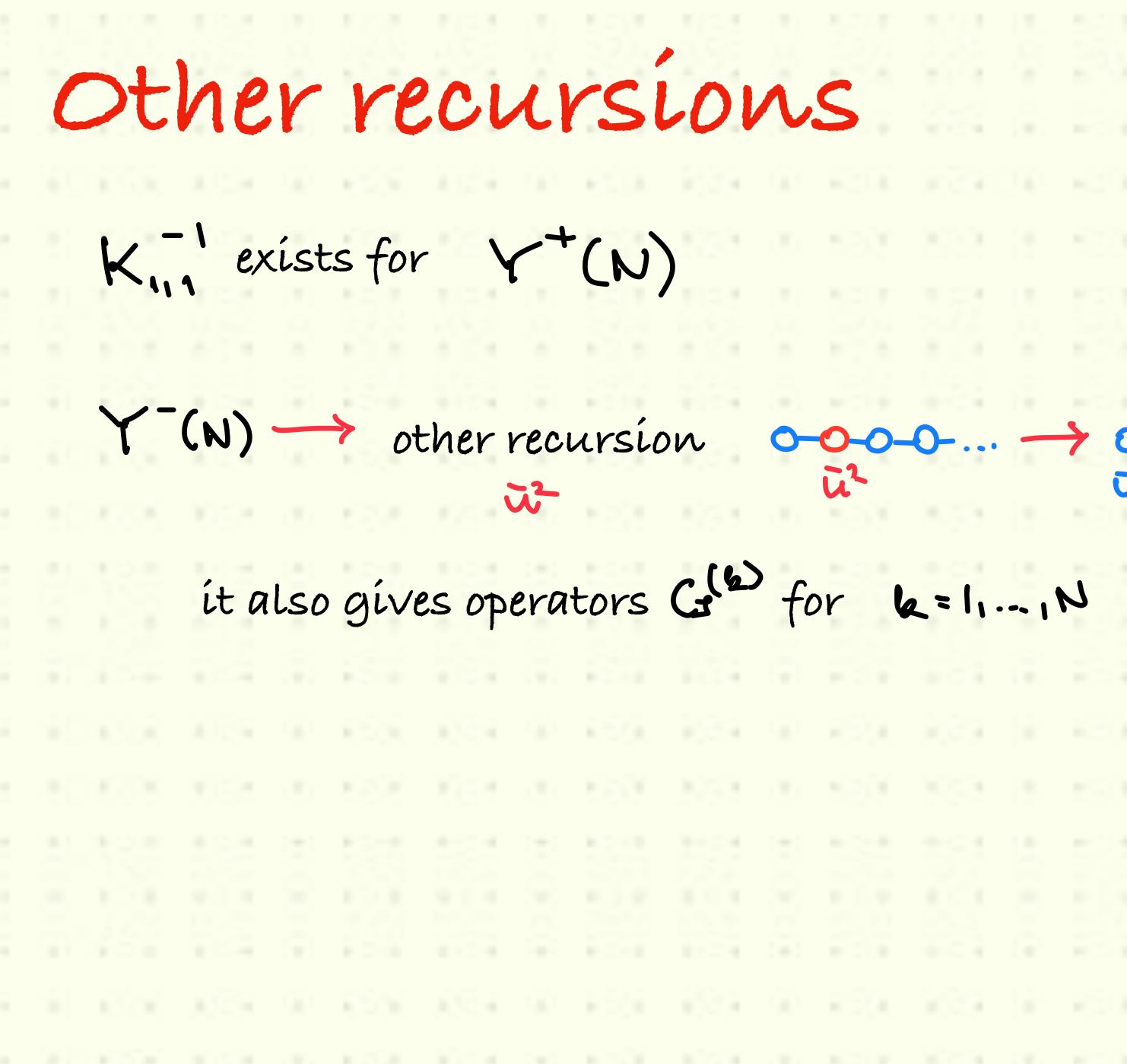


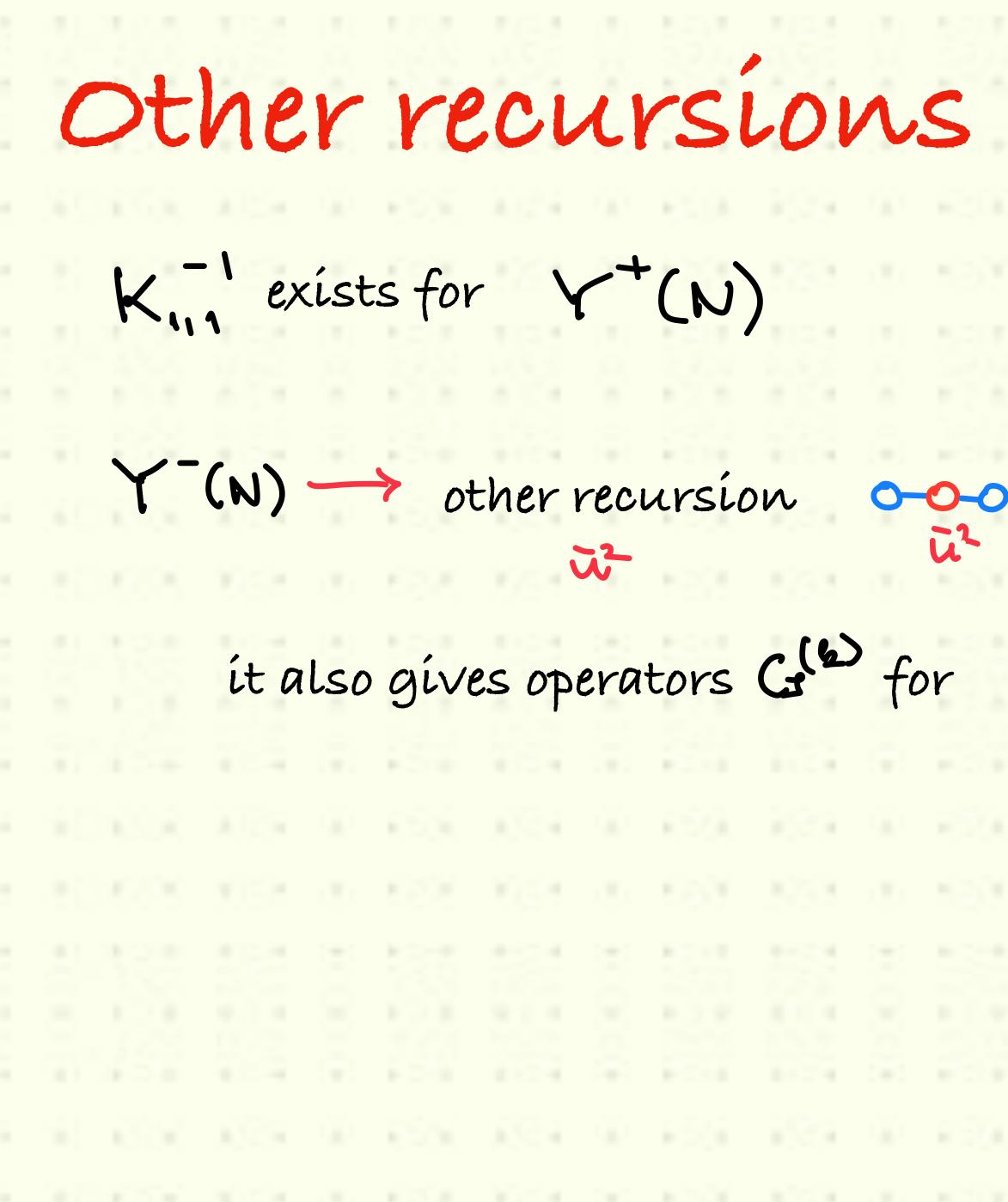
# Other recursions $K_{11}$ exists for $Y^+(N)$ 22



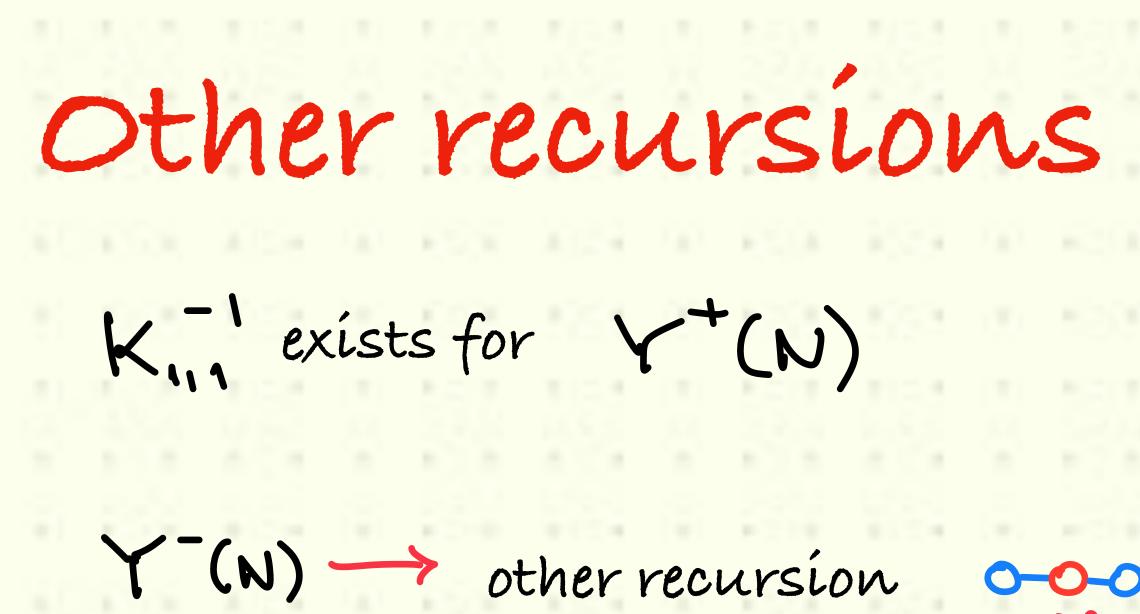
 $\Upsilon^{-}(N) \rightarrow$  other recursion







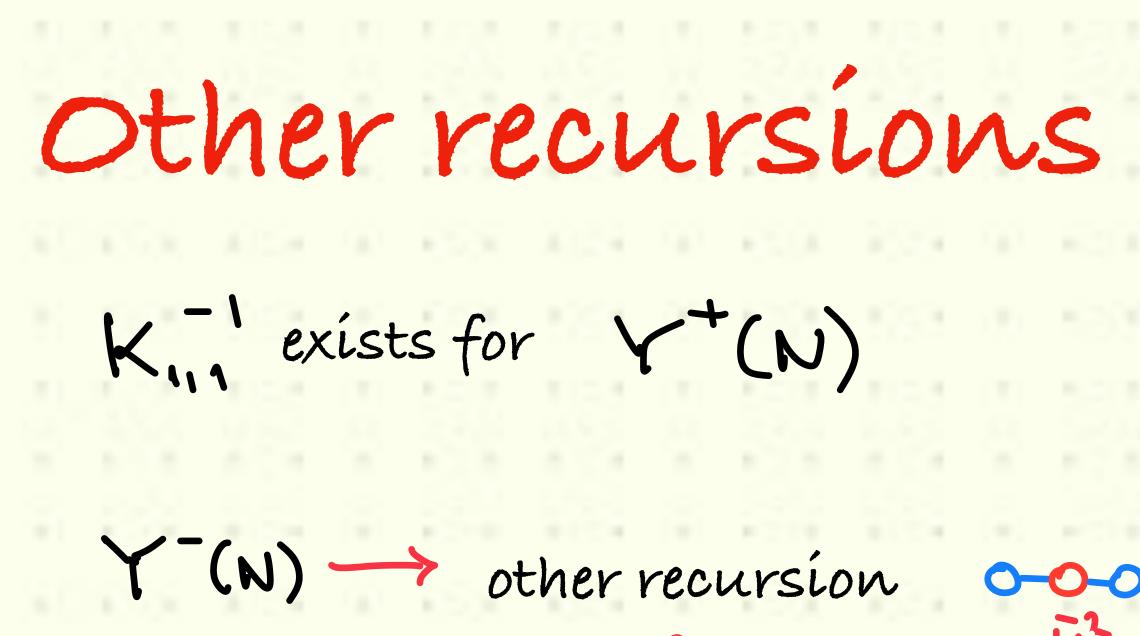
)-	0		> 0 ū	0-0 ū <sup>2</sup> ū4	• • • • •			
6	<b>ر = ۱</b> ر .	1	N	[G <sup>(e)</sup>	(w), (	ʒ <sup>(ℓ)</sup> (♂)]	= 0	



### it also gives operators G for

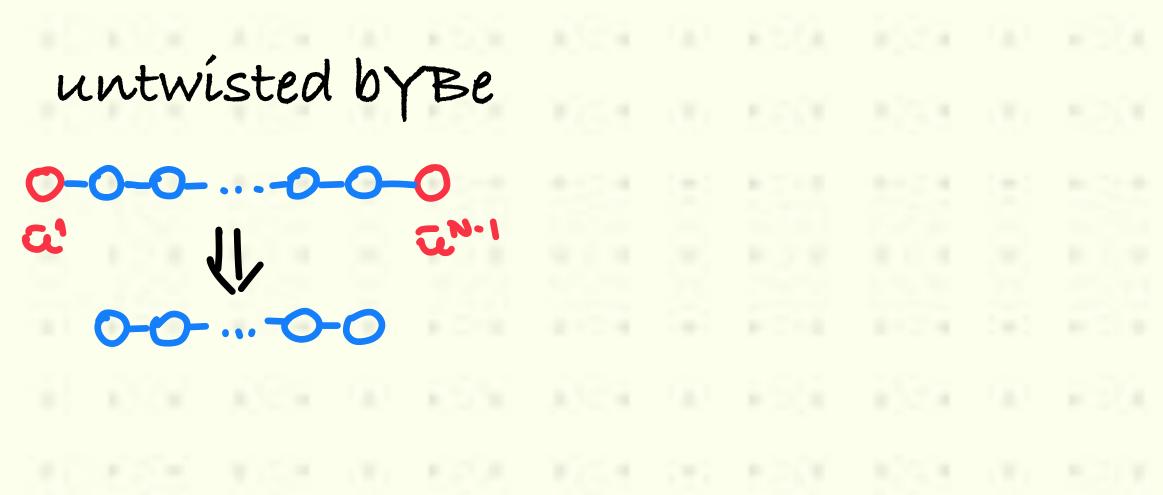
### untwisted by Be

)-	0		> 0 ū	0-0 ū <sup>)</sup> ū <sup>4</sup>	• • • • •			
6	<b>ر = ۱</b> ر .	1	N	[G <sup>(e)</sup>	(w), (	ʒ <sup>(ℓ)</sup> (♂)]	= 0	

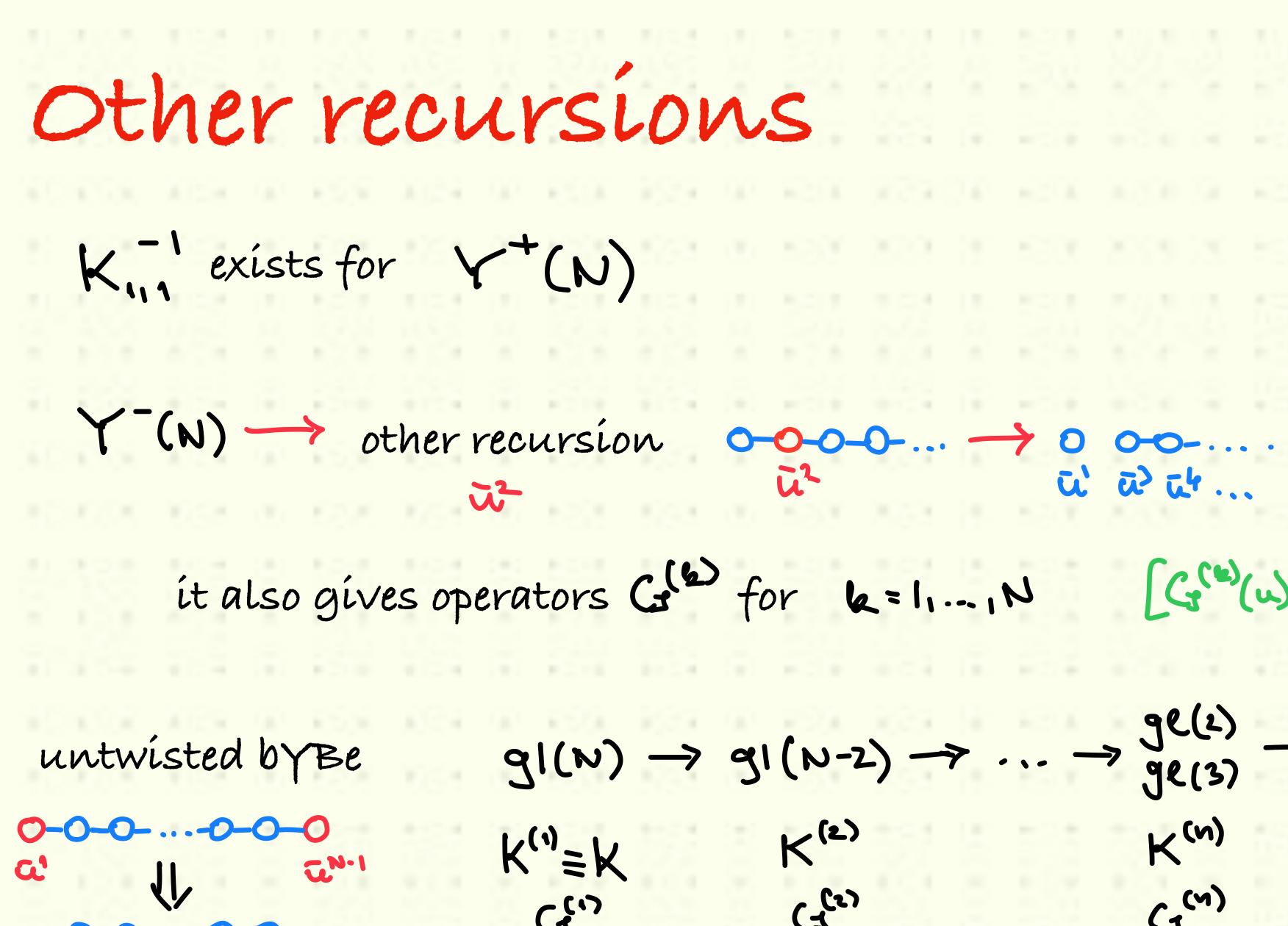


### it also gives operators G for

J.



)-	0		> 0 ū	0-0 ū <sup>)</sup> ū <sup>4</sup>	• • • • •			
6	<b>ر = ۱</b> ر .	1	N	[G <sup>(e)</sup>	(w), (	ʒ <sup>(ℓ)</sup> (♂)]	= 0	



it also gives operators  $G^{(b)}$  for k = 1, ..., N  $[G^{(b)}(w), G^{(c)}(w)] = 0$  $g(N) \rightarrow g(N-2) \rightarrow \cdots \rightarrow g(2) \rightarrow g(1)$ K<sup>(n)</sup> K<sup>(n+1)</sup> G(1+1) (M) n=LN/2

On-shell overlaps without twist

 $\mp^{(k)} = \left[ G^{(k)} \right]^{-1} \left[ G^{(k+1)} \longrightarrow \left[ \mp^{(k)} (\omega) \right] \mp^{(l)} (\omega) \right] = 0$ 

### On-shell overlaps without twist

 $\mp^{(k)} = \left[ G^{(k)} \right]^{-1} G^{(k+1)} \longrightarrow \left[ \mp^{(k)} (\omega) \right] \mp^{(l)} (\omega) = 0$ 

10.000	10.25	(V)	6.00	1044	121	1201	1223	(V)	1000

choosing diagonal basis  $F^{(b)} = diag(F_{1}, \dots, F_{d_{B}}^{(b)})$ 

## On-shell overlaps without twist

 $\mp^{(k)} = \left[ G^{(k)} \right]^{-1} G^{(k+1)} \longrightarrow \left[ \mp^{(k)} (\omega) \right] \mp^{(l)} (\omega) = 0$ 

 $\langle MPS|\bar{u}\rangle = \int_{a=1}^{b} T \widetilde{f}_{a}^{(s)}(u^{+s}) \times \frac{det G}{det G}$ 23

choosing diagonal basis  $F^{(b)} = diag(F_{1}^{(b)}, F_{d_B}^{(b)})$ 

On-shell overlaps without twist

 $\mp^{(k)} = \left[ G^{(k)} \right]^{-1} G^{(k+1)} \longrightarrow \left[ \mp^{(k)} (\omega) \right] \mp^{(l)} (\omega) = 0$ 

Crossed

choosing diagonal basis  $F^{(b)} = diag(F_{1}^{(b)}, F_{d_B}^{(b)})$ 

Non-crossed

 $\widetilde{T}_{\chi}^{(s)}(u) = \overline{T}_{\chi}^{(s)}(u-i\frac{s}{2})\sqrt{\frac{u^{2}}{u^{2}+1}/4r}} \qquad \widetilde{T}_{\chi}^{(s)}(u) = \begin{cases} \overline{T}_{\chi}^{(s)}(u-c_{s}) \\ \overline{T}_{\chi}^{(b_{2})}(u-c_{b_{1}})\sqrt{\frac{u^{2}}{u^{2}+1}/4r} \\ \overline{T}_{\chi}^{(b_{2})}(u-c_{b_{1}})\sqrt{\frac{u^{2}}{u^{2}+1}/4r} \end{cases}$ 23

# Other spin chains

Symmetry of the spin chain	Type of refl.	Resídual symmetry	Paír structure
	AI	0(N)	Chiral
gl(N)	All	sp(N)	Chiral
KGR (0) E228 (0) SS1 (0)	AIII	gl(M)+gl(N-M)	Achiral
o(2n+1)	BI	o(M) + o(N-M)	Chiral
sp(2n)	CI	sp(2m)+sp(2n-2m)	Chiral
Sp(~10)	CII	gl(n)	Chíral
	DI	o(M) + O(2n-M)	n-M=0 (mod 2) chíra
0(2n)		0(1+()+0(2+0+1+()))	n-M=1 (mod 2) achrío
0 (200)	DII	gl(n)	$n=0 \pmod{2}$ chiral
		90(10)	n=1 (mod 2) achrial

### The on-shell overlaps are extended to other rational spin chains without proofs

# Other spin chains

symmetry of the spin chain	Type of refl.	Resídual symmetry	Paír structure	
	AI	0(N)	Chíral	
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Sp(~/~)	CII	gl(n)	Chiral	
	DI	o(M)+O(2n-M)	n-M=0 (mod 2) chíral n-M=1 (mod 2) achríal	
o(2n)		O(1/1) + O(2/1-1/1)		
0(~~~)	DII	gl(n)	$n=0 \pmod{2}$ chiral	
		90(10)	n=1 (mod 2) achrial	

The overlaps are also conjectured for graded spin chains, including gl(mn) and osp(m 2n)

### The on-shell overlaps are extended to other rational spin chains without proofs

### Conclusions

1					
2					
-					
3					
2					
2					
-					
24					
1.1					

### 

# Conclusions Cartan subalgebra $F^{(s)}(u)$

### conclusions

### 

### Cartan subalgebra $F^{(s)}(u)$

On-shell overlaps for integrable MPS  $\langle MPS|\bar{u} \rangle = \int_{a}^{b} T \widetilde{f}_{a}^{(s)}(\bar{u}^{*s}) \times \frac{det G}{det G}$ 

10.0	16	$\leq 0$	1000	:03	1000	103	10.000

### conclusions

Cartan subalgebra  $F^{(s)}(u)$ 

On-shell overlaps for integrable MPS

where  $\widetilde{F}_{(u)}^{(s)}(u)$  are the eigenvalues of the operators  $F^{(s)}(u)$ 

### conclusions

Cartan subalgebra  $F^{(s)}(u)$ On-shell overlaps for integrable MPS

where  $\tilde{F}_{(u)}^{(s)}(u)$  are the eigenvalues of the operators  $F^{(s)}(u)$ 

1) KT-relation: creation to annihilation Proof is possible if

> 2) recurrence formula 3) action formula 4) co-product formula

 $|\overline{u}\rangle = \sum (...) |\overline{u}_{r}\rangle^{(n)} \otimes |\overline{u}_{m}\rangle^{(n)}$ 

 $T_{i,j}(z)(u) = Z(...)(w)$ 

 $\{\{z, \bar{u}\}, \bar{u}^2, ..., \} = \mathbb{Z}(...) T_{A,j}(z) | \bar{u}^1, \bar{v}^2, ... \}$ 

$$\frac{\langle MPS|\bar{u}\rangle}{\langle \overline{u}\overline{u}\overline{u}\rangle} = \begin{bmatrix} d_{e} \\ \sum_{\alpha \in \Lambda} T \\ d_{\alpha} \widetilde{f} \\ d_{\alpha} \widetilde{f}$$

MPS + K-matrix + Representation of a given reflection algebra