Dynamics of nearly integrable quantum gases

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PRL 130 (2023)
PRB 109 (2024)
SciPost Physics 17 (2024)
arXiv 2404.14292
arXiv 2408.00593

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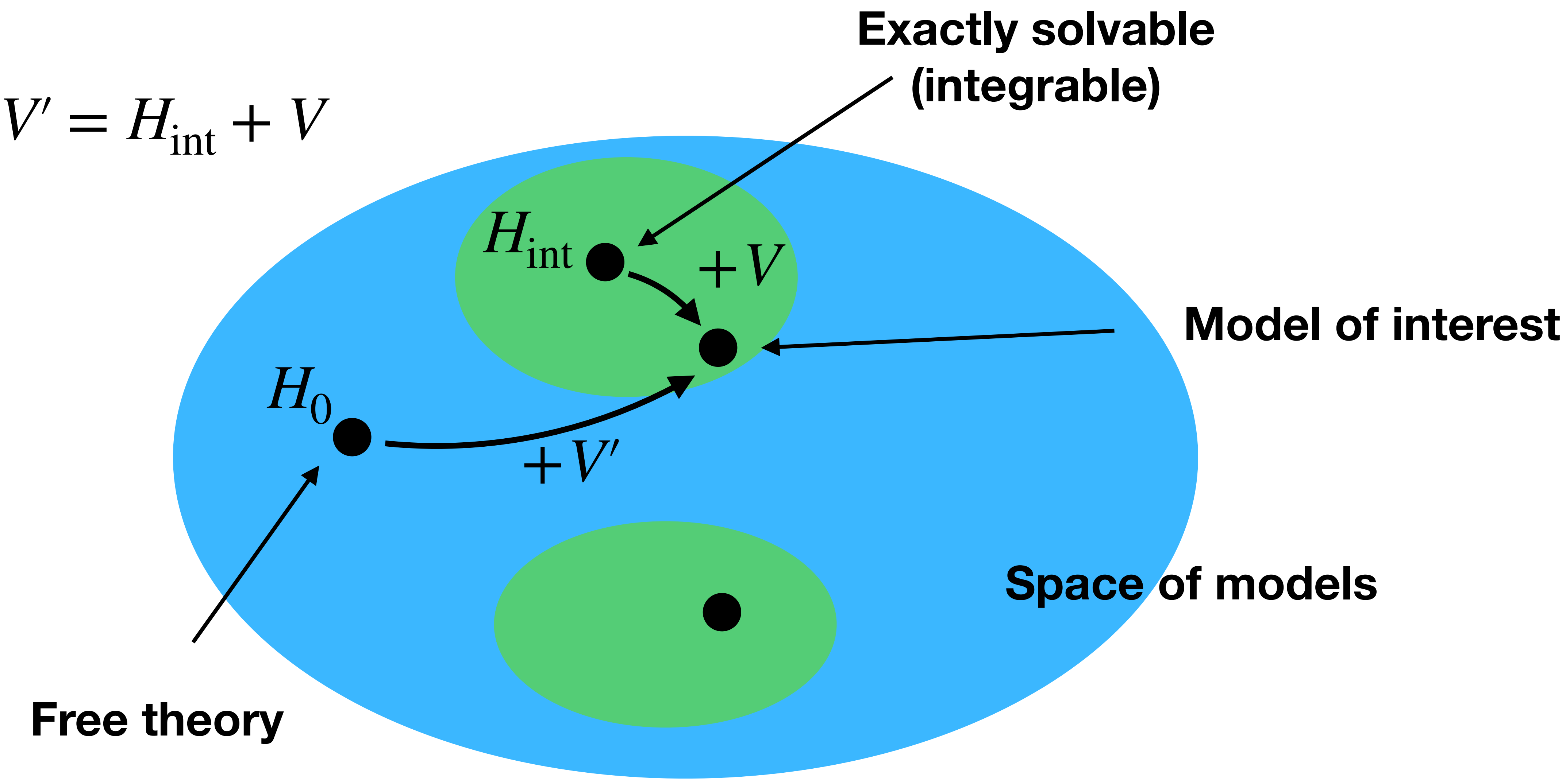
Plan

1. General idea and universality
2. Thermalisation and prethermalization in coupled 1d systems
3. Navier-Stokes equations from the generalized hydrodynamics
4. Generalized BBGKY hierarchy
5. Self-diffusion

General idea

$$H = H_0 + V' = H_{\rm int} + V$$

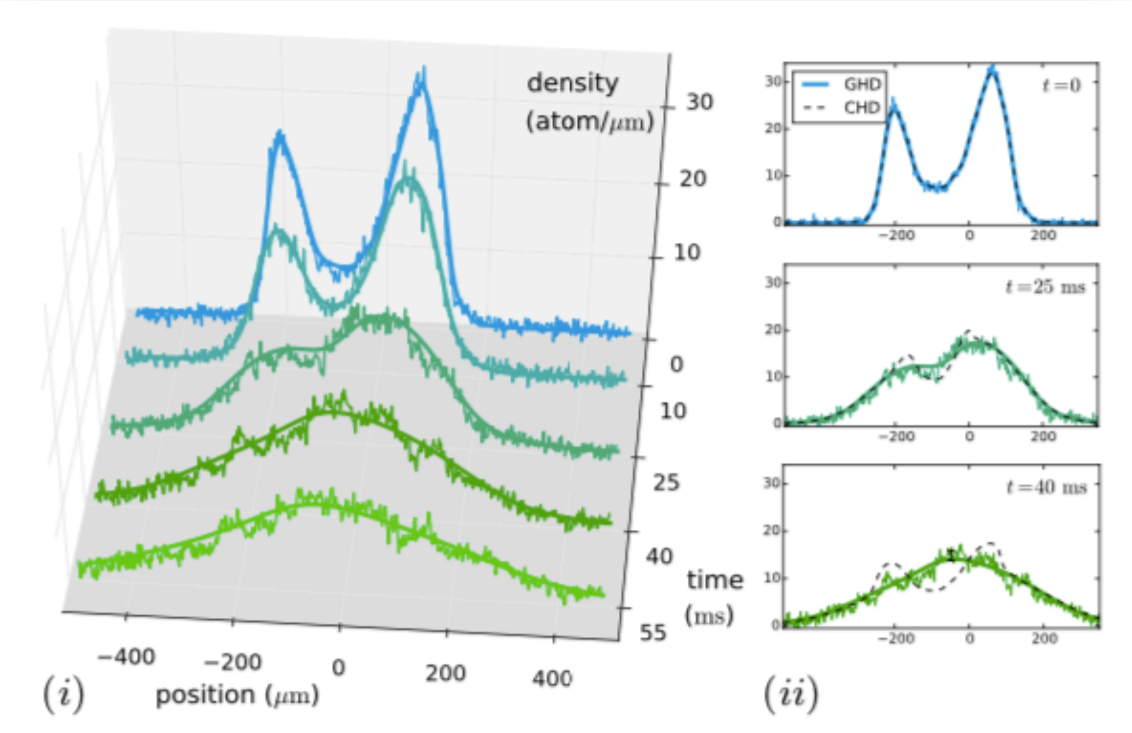


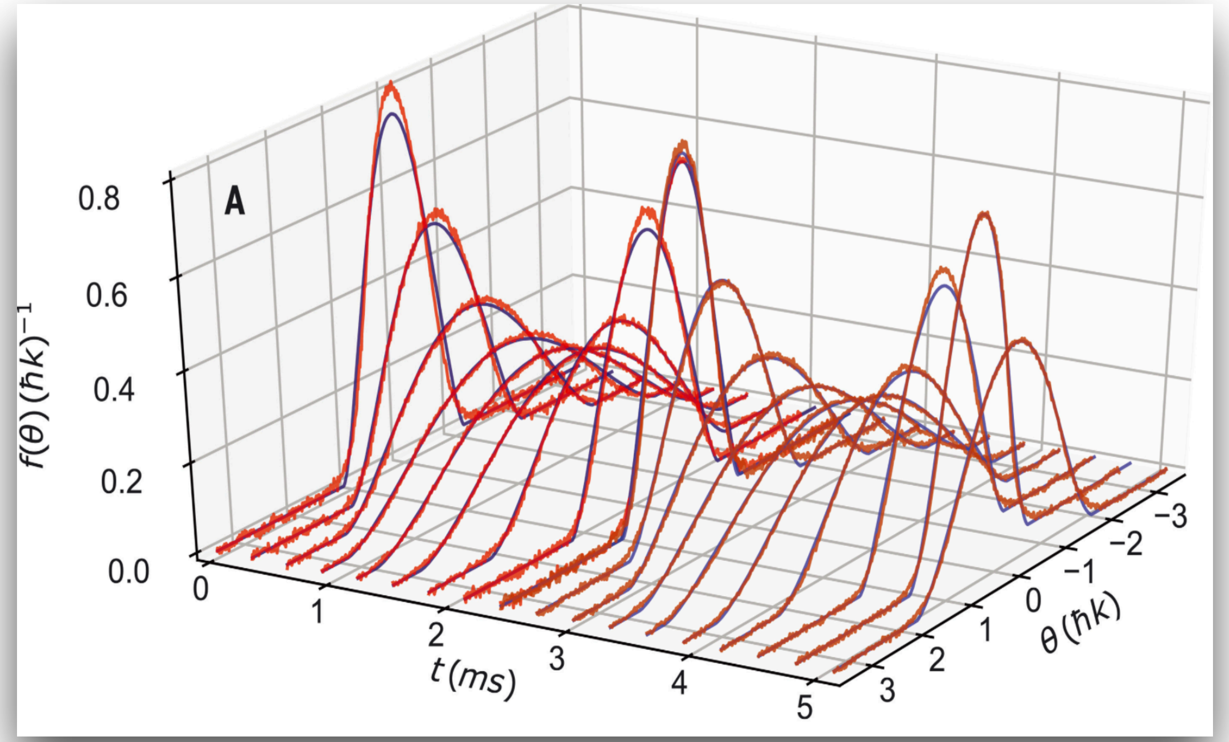
- perturbation theory: assumes V is weak
- local density approximation: gradient expansion in V
- our starting point is the generalized hydrodynamics (GHD)

Experiments

Lieb-Liniger model

$$\hat{H} = \sum_{j=1}^{N} \hat{p}_i^2 + 2c \sum_{1\leq i<j}^{N} \delta(\hat{x}_i - \hat{x}_j) + V_{\text{trap}} + V_{\text{tubes}} + \dots$$





M. Schemmer et al, PRL (2019)

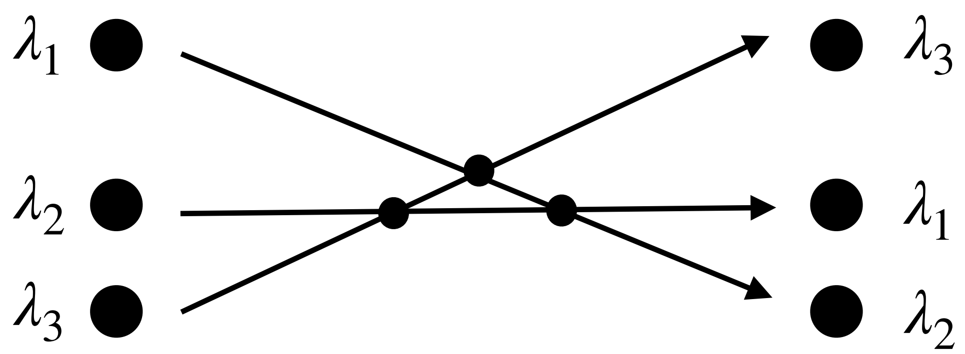
N. Malvania et al, Science (2021)

GHD developed by

O. A. Castro-Alvaredo, B. Doyon, and T. Yoshimura
B. Bertini, M. Collura, J. De Nardis, and M. Fagotti (2016)
and in further works

Kinetic equations

Quantum integrable theories → **Particle content conserved**



in large systems (TBA)

$\rho_{\rm p}(\lambda)$

in homogeneous systems $\rho_{\rm p}(\lambda, t)$

$$\partial_t \rho_{\rm p} = I[\rho_{\rm p}]$$

collision integral
non-integrable interactions

inhomogeneous systems $\rho_{\rm p}(\lambda, x, t)$

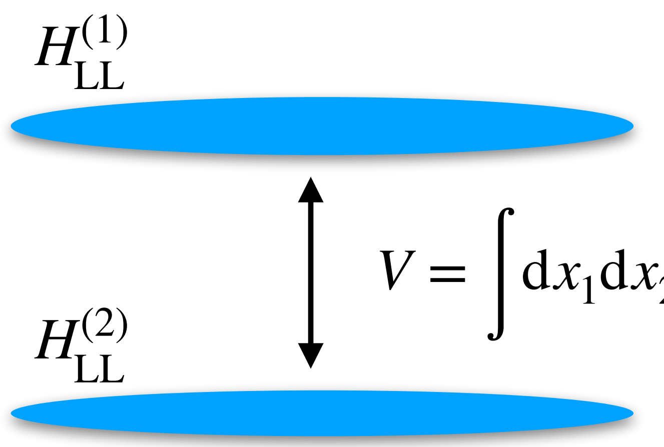
$$\partial_t \rho_{\rm p} + \partial_x(v_\rho \rho_{\rm p}) - \frac{1}{2}\partial_x(D_\rho \partial_x \rho_{\rm p}) = I[\rho_{\rm p}]$$

GHD (integrable interactions)

A.J. Friedman, S. Gopalakrishnan, R. Vasseur (2020)
J. Durnin, M. J. Bhaseen, B. Doyon (2021)
MP, S. Gopalakrishnan, R. Konik (2023)

Coupled 1d systems

Boltzmann equation



$$V = \int \mathrm{d}x_1 \mathrm{d}x_2 V(x_1, x_2) \hat{\rho}_1(x_1) \hat{\rho}(x_2)$$

Fermi's golden rule

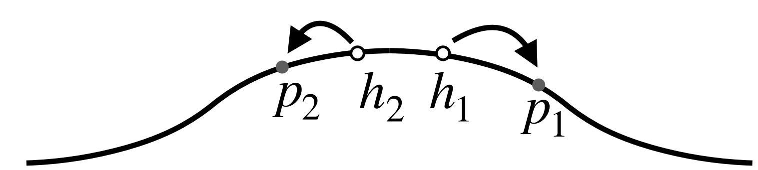
$$\Gamma \sim |\langle \rho_1', \rho_2' | \hat{\rho}(x_1) \hat{\rho}(x_2) | \rho_1, \rho_2 \rangle|^2$$

Product state

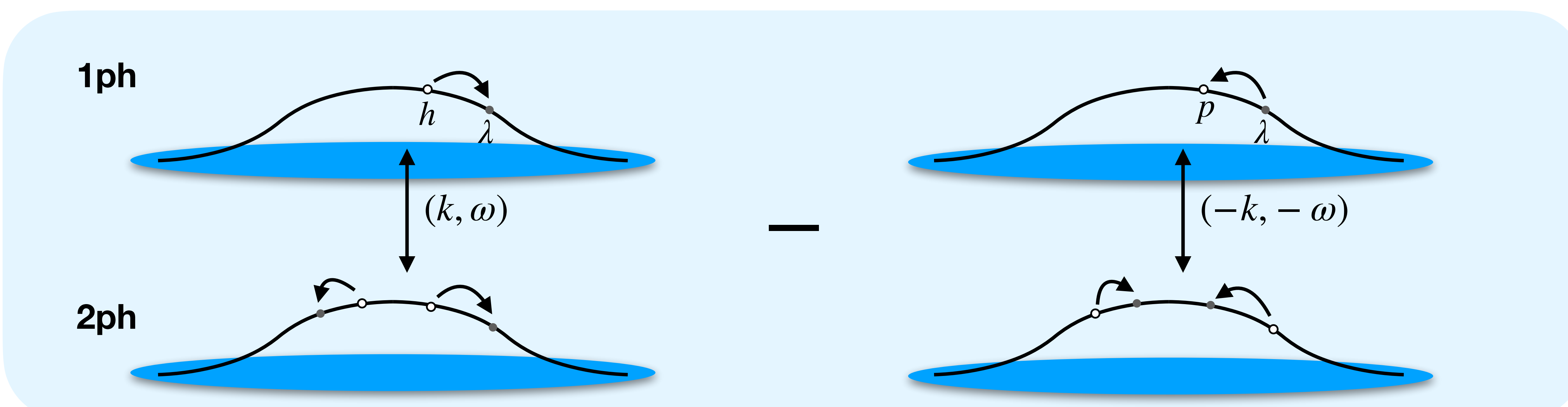
$$|\rho_1, \rho_2\rangle = |\rho_1\rangle | \otimes |\rho_2\rangle$$

Density operator

$$F^{\rho}(p_i; h_i) = \langle \rho, \{p_i, h_i\} | \hat{\rho}(x) | \rho \rangle$$



Kinetic equation - master equation



Boltzmann equation

Resulting equation

$$\partial_t \rho_{p1}(\lambda) = \sum_{m,n} I^{(m,n)}(\rho_{p1}, \rho_{p2})$$

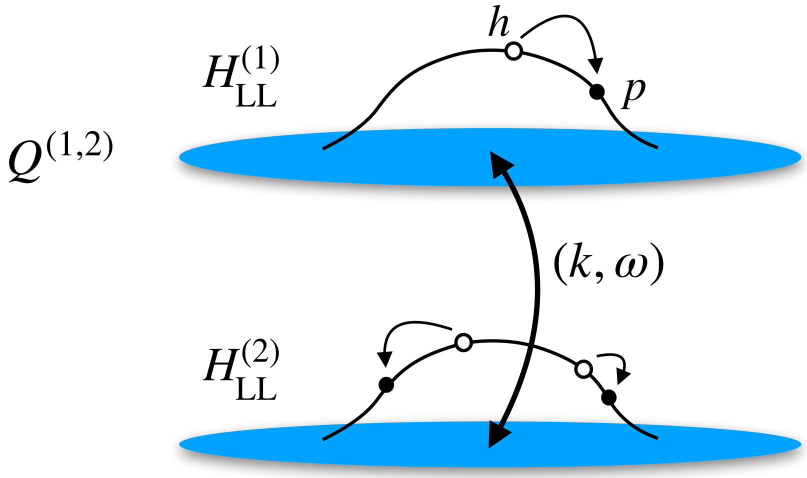
$$\partial_t \rho_{p2}(\lambda) = \sum_{m,n} I^{(m,n)}(\rho_{p2}, \rho_{p1})$$

$\xrightarrow[\text{distributions}]{\text{identical}}$

$$\partial_t \rho_{p}(\lambda) = \sum_{m,n} I^{(m,n)}(\rho_{p})$$

Features

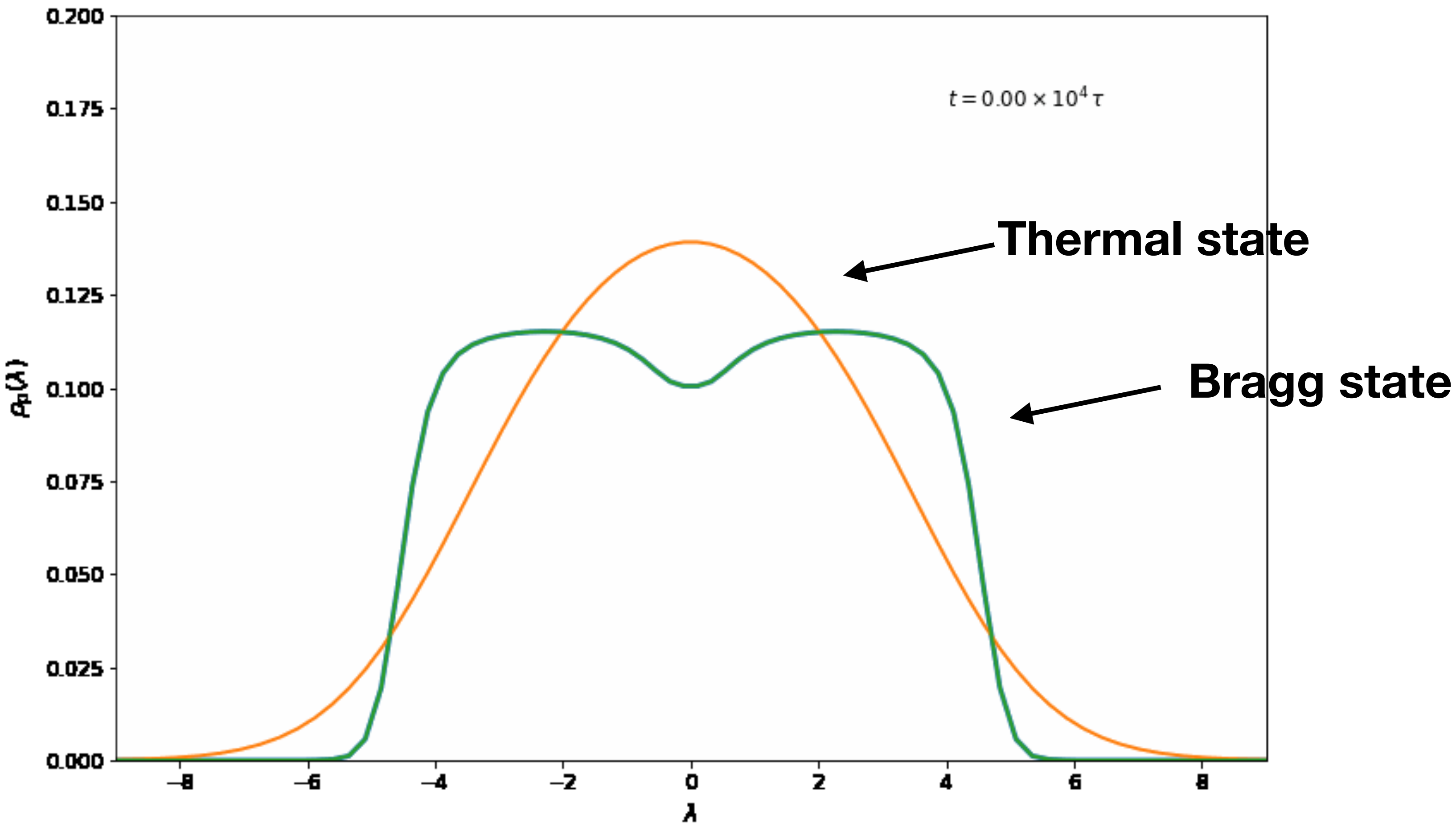
- conserves particle number, total momentum and total energy
- vanishes for a thermal distribution
- for identical distributions (1,1) vanishes



Example

dynamics following a Bragg pulse $$\rho_p(\lambda, t=0) = \frac{1}{2}\left(\rho_{T_0}(\lambda+p) + \rho_{T_0}(\lambda-p)\right)$$

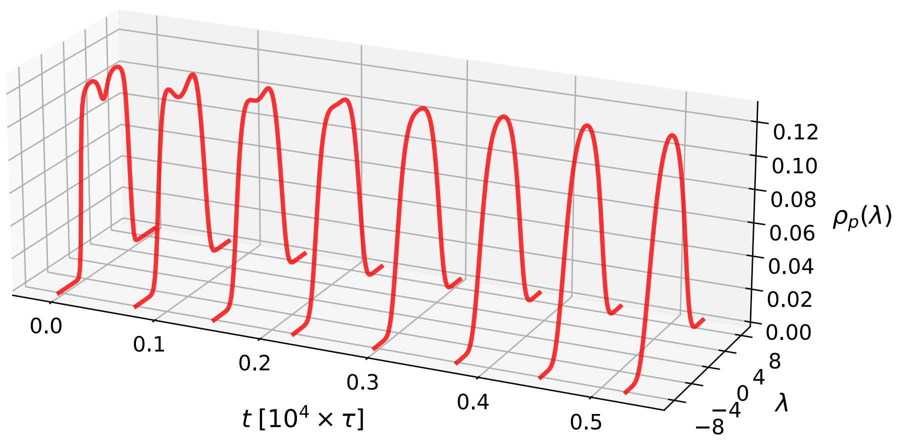
initial thermal state



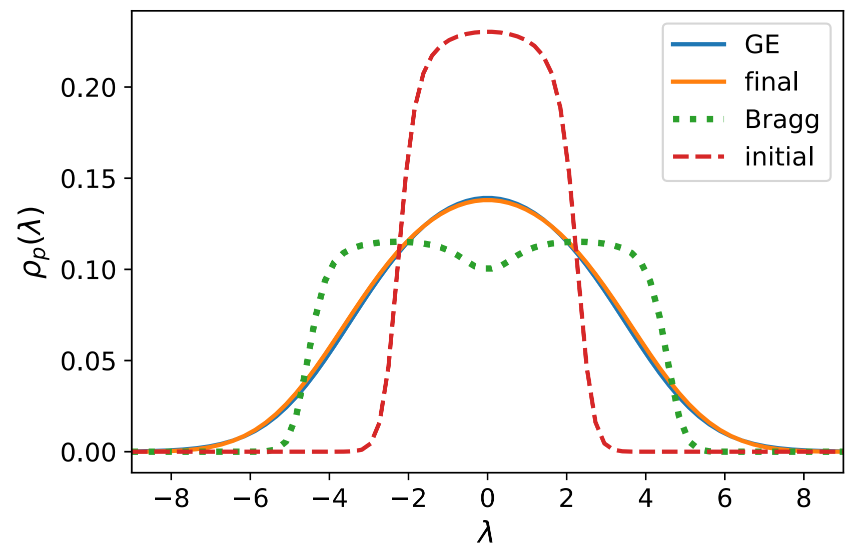
Thermal state $E_{\text{thermal}}(T) = E_{\text{Bragg}} \quad \longrightarrow \quad T$

Example

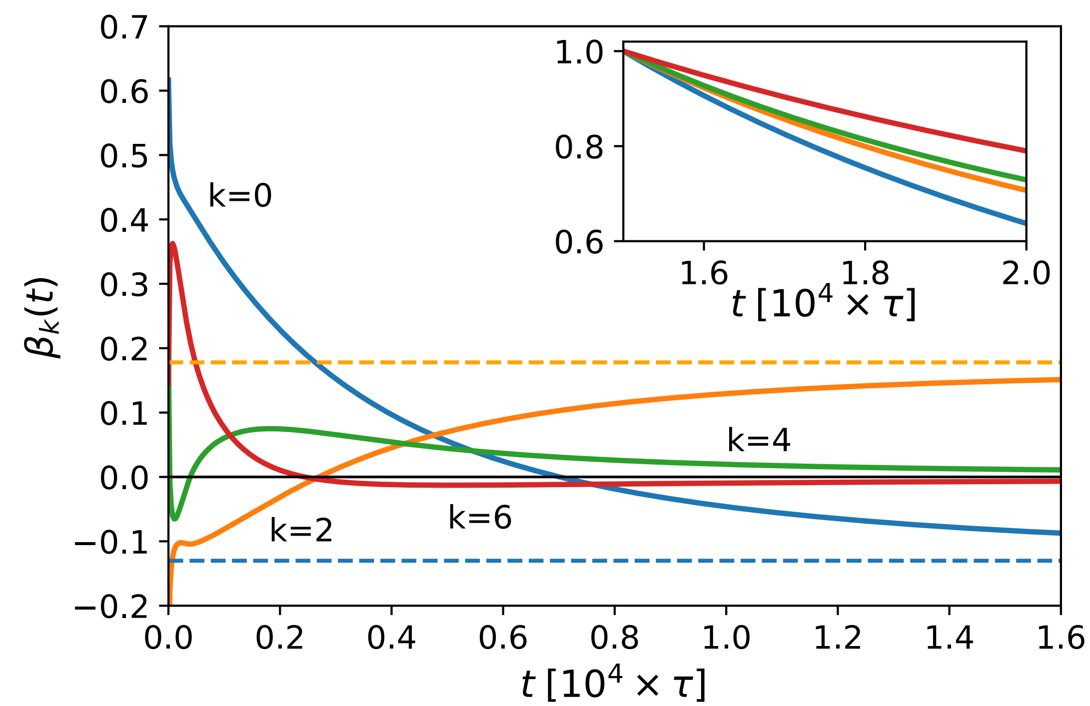
time evolution



initial and final distributions



chemical potentials



generalized Gibbs ensemble:

$$\rho_p(\lambda, t) \to \{\beta_k(t)\}$$

thermalisation:

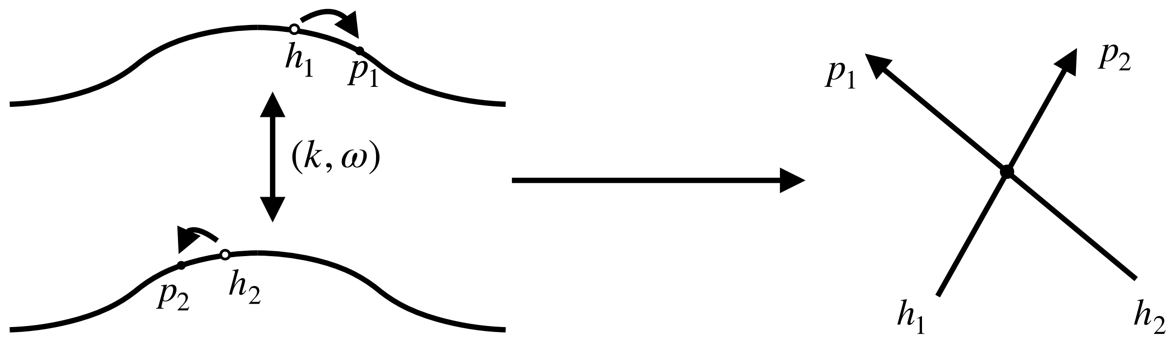
$$t \to \infty : \qquad \beta_0(t), \beta_2(t) \neq 0$$
$$\beta_4(t), \beta_6(t), \ldots = 0.$$

Prethermalization

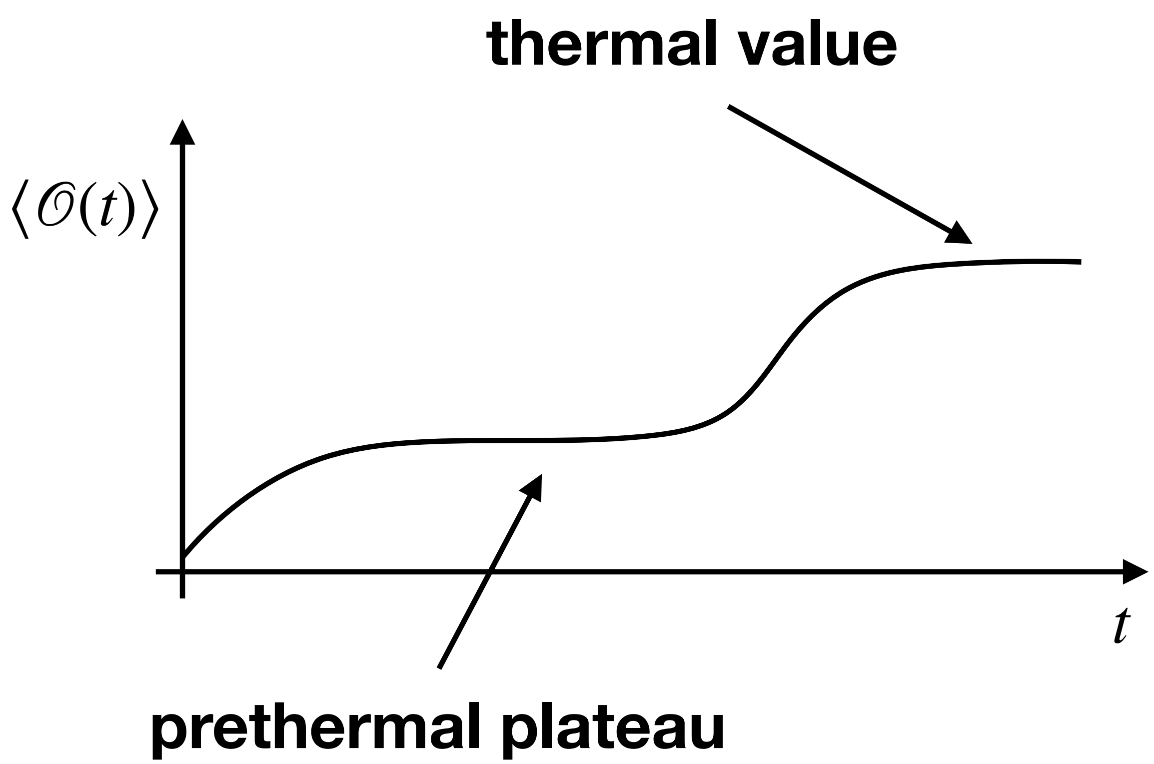
Prethermalization

different distribution in each tube -> (1,1) processes are leading

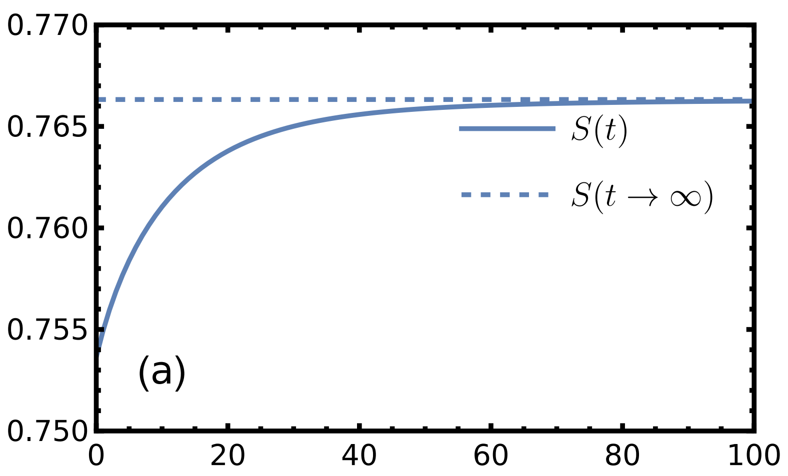
(1,1) process = 2-body scattering



not effective in redistribution of the densities -> athermal stationary state



entropy production



Conserved charges?

Are there some conserved charges surviving the integrability breaking?

At the level of kinetic equations: yes

Conserved charges of uncoupled tubes

$$Q_n^{(1)}, Q_n^{(2)}$$

Conserved charges of coupled system

$$Q_n^{(1)} + Q_n^{(2)}$$

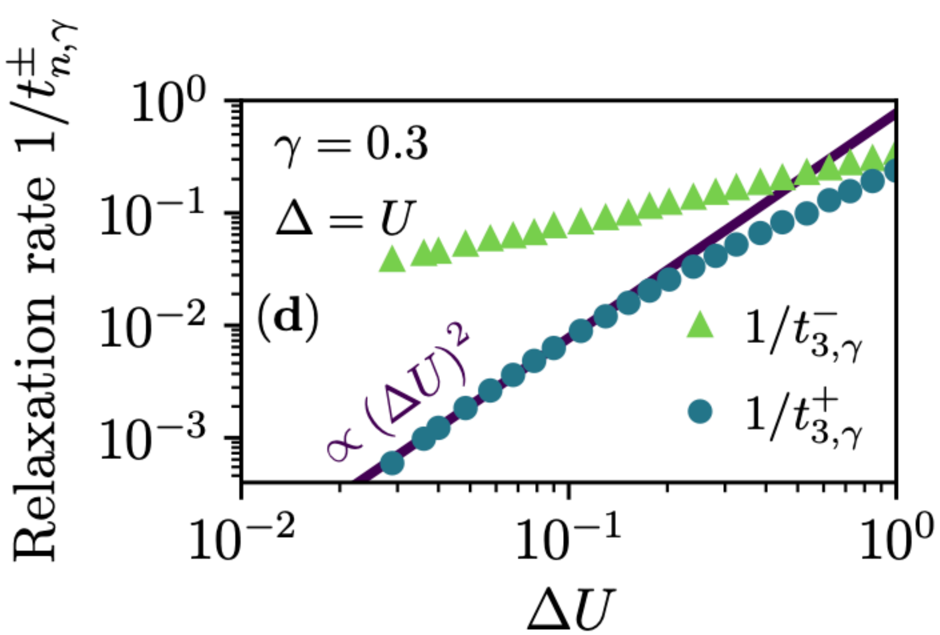
(only close to Tonks-Girardeau gas)

Similarly for spin chain ladders

$$H_{\text{ladder}} = H_{XXZ}^{(1)}(\Delta) + H_{XXZ}^{(2)}(\Delta) + V \sum_{j=1}^{N} s_{j,1}^z s_{j,2}^z$$

Also at the quantum level - DMRG

$$Q_n^{(1)} + Q_n^{(2)} = \text{const} + \mathcal{O}(\Delta^2 U^2)$$



Navier-Stokes equations

Navier-Stokes equations

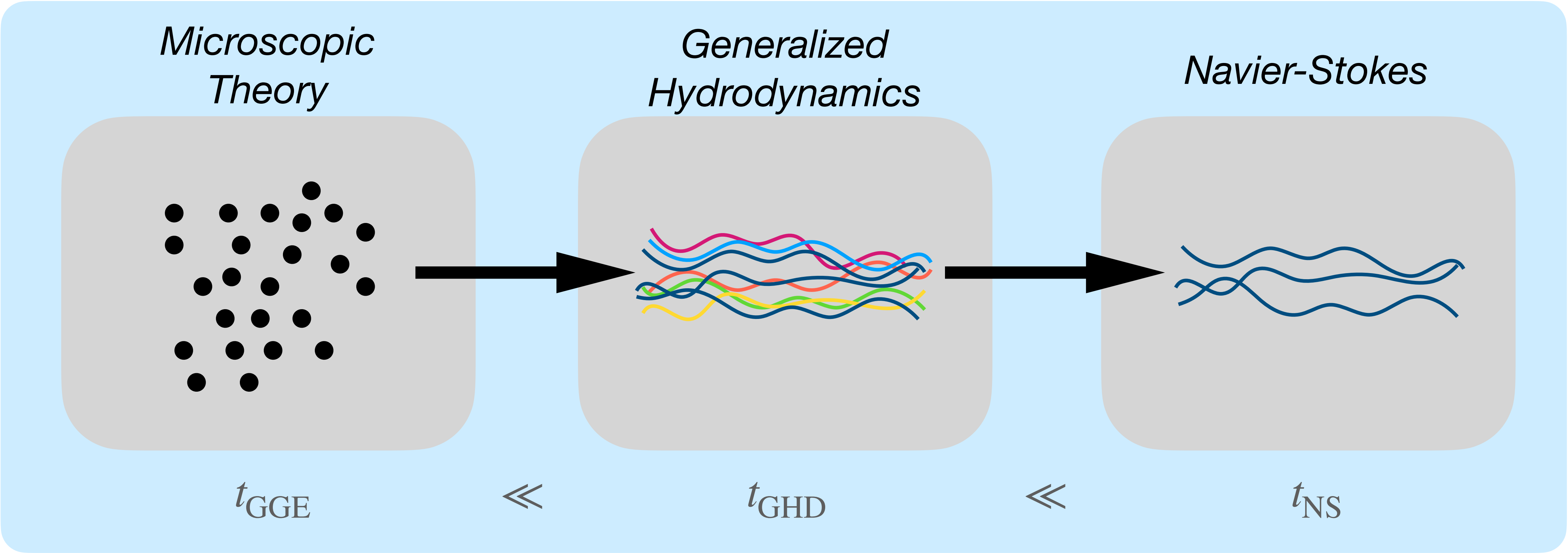
Continuity equations

$$\partial_t n = -\partial_x(nu)$$
$$\partial_t(nu) = -\partial_x(nu^2 + \mathscr{P})$$
$$\partial_t(ne) = -\partial_x(nue + \mathscr{J}) - \mathscr{P}\partial_x u,$$

Transport coefficients

$$\mathscr{P} = P - \zeta\partial_x u \qquad \mathscr{J} = -\kappa\partial_x T$$

viscosity **thermal conductivity**



Macroscopic conservation laws

Assume collision invariants

$I[\rho]$ **conserves the particle number, momentum and energy**

$n = \langle \rho \rangle, \quad nu = \langle v\rho \rangle, \quad ne = \langle \ldots \rangle$ **averages over momenta** $\langle f \rangle = \int \mathrm{d}\lambda\, f(\lambda)\rho_{\mathrm{p}}(\lambda, x, t)$

$$\partial_t \rho_{\mathrm{p}} + \partial_x (v_\rho \rho_{\mathrm{p}}) = \frac{1}{2}\partial_x (D_\rho \partial_x \rho_{\mathrm{p}}) + I[\rho_{\mathrm{p}}] \longrightarrow$$

$$\partial_t n = -\partial_x (nu)$$

$$\partial_t (nu) = -\partial_x (nu^2 + \mathscr{P})$$

$$\partial_t (ne) = -\partial_x (une + \mathscr{J}) - \mathscr{P}\partial_x u,$$

where $\mathscr{P} = \mathscr{P}_D + \mathscr{P}_v \qquad \mathscr{J} = \mathscr{J}_D + \mathscr{J}_v$

$$\mathscr{P}_{\mathfrak{D}} = h_1 D_\rho \partial_x \rho_{\mathrm{p}}/2 \qquad \mathscr{P}_v = \langle (\lambda - u) v(\lambda) \rangle$$

$$\mathscr{J}_{\mathfrak{D}} = h_2 D_\rho \partial_x \rho_{\mathrm{p}}/2 \qquad \mathscr{J}_v = \frac{1}{2}\langle (\lambda - u)^2 (v(\lambda) - u) \rangle$$

$$h_k(\lambda) = \lambda^k$$

Phenomenology

What we got

$$\mathscr{P}[\rho_\mathrm{p}] = \mathscr{P}_D + \mathscr{P}_v$$

$$\mathscr{J}[\rho_\mathrm{p}] = \mathscr{J}_D + \mathscr{J}_v$$

What we would like

$$\mathscr{P}(n, u, e) = P - \zeta \partial_x u$$

$$\mathscr{J}(n, u, e) = -\kappa \partial_x T$$

Problem:

$\mathscr{P}_\mathfrak{D}, \mathscr{P}_v, \mathscr{J}_\mathfrak{D}, \mathscr{J}_v$ **depend on full** ρ_p **rather than on** n, u, e

Solution:

$$\partial_t \rho_\mathrm{p} + \partial_x (v_\rho \rho_\mathrm{p}) = \frac{1}{2} \partial_x (D_\rho \partial_x \rho_\mathrm{p}) + I[\rho_\mathrm{p}] \quad \longleftrightarrow \quad \partial_t \rho + v \partial_x \rho = I[\rho]$$

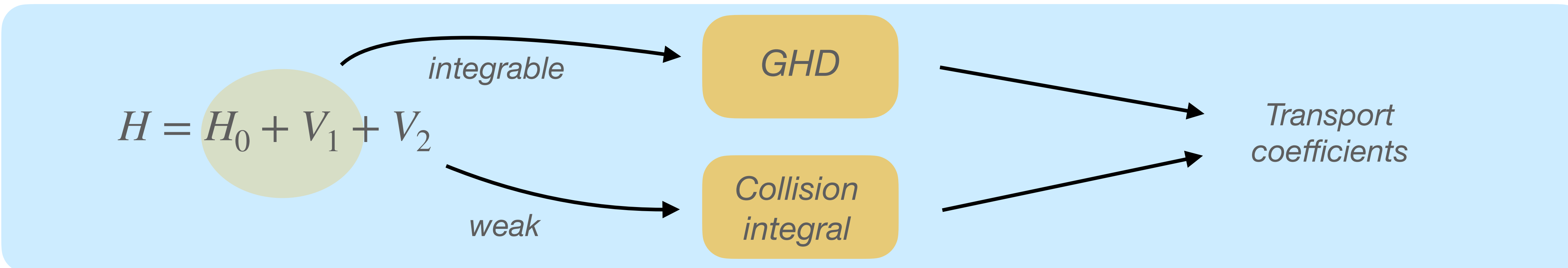
Boltzmann equation

Standard problem of the kinetic theory

linearisation or Chapman-Enskog method

central assumption: $\rho_\mathrm{p}(n, u, e)$

Transport coefficients



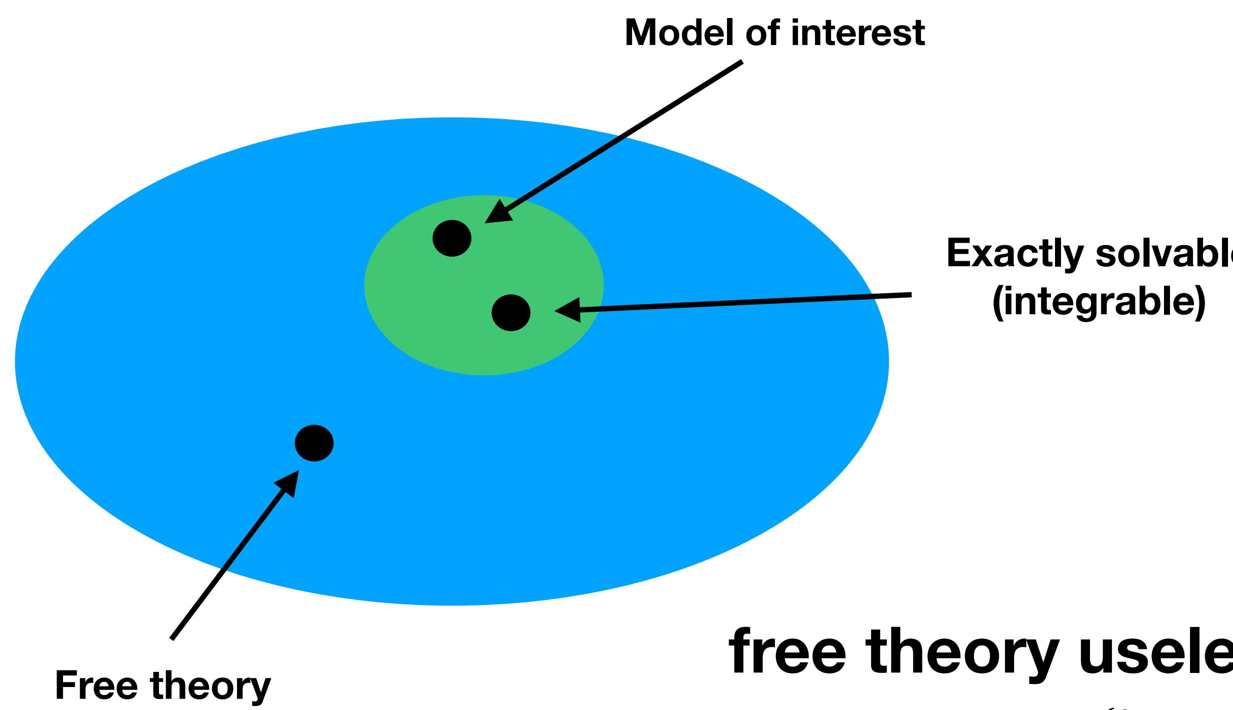
GHD diffusion

$\kappa = \kappa_I + \kappa_{\mathfrak{D}}$ $\quad$ $\zeta = \zeta_I + \zeta_{\mathfrak{D}}$

integrability breaking

Universal bounds:

$\kappa \geq \kappa_{\mathfrak{D}} \quad \zeta \geq \zeta_{\mathfrak{D}}$

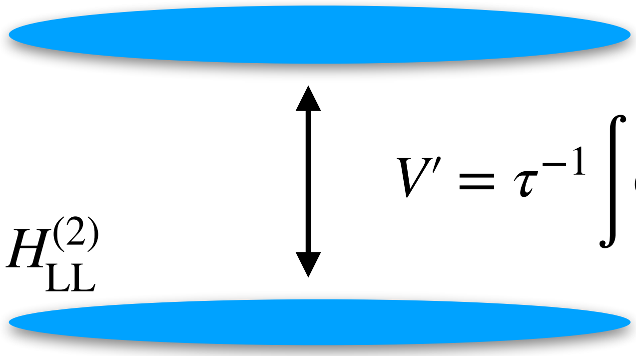


free theory useless

$\kappa_{\text{free}} = \infty \quad \zeta_{\text{free}} = 0$

Application

$H^{(1)}_{\mathrm{LL}}$



$H^{(2)}_{\mathrm{LL}}$

$$V' = \tau^{-1}\int \mathrm{d}x_1 \mathrm{d}x_2 V(x_1, x_2)\hat{\rho}_1(x_1)\hat{\rho}(x_2)$$

Lieb-Liniger model

$$\hat{H}_{LL} = \sum_{j=1}^{N} \hat{p}_i^2 + 2c \sum_{1\leq i<j}^{N} \delta(\hat{x}_i - \hat{x}_j)$$

$$\hat{H} = H^{(1)}_{LL} + H^{(2)}_{LL} + V'$$

initial states in two tubes identical

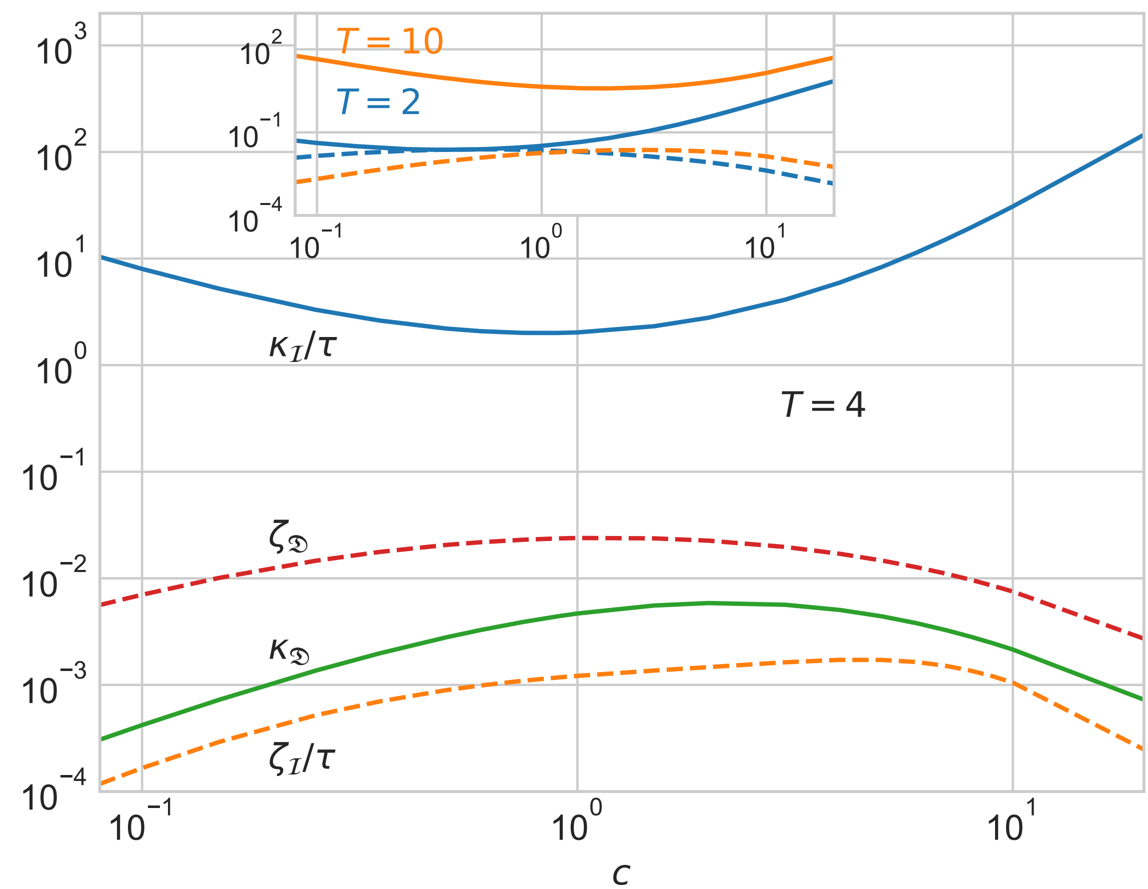
↓

really just one tube

collision integral for V' known

J. Durnin, M. J. Bhaseen, B. Doyon (2021)

MP, S. Gopalakrishnan, R. Konik (2023)



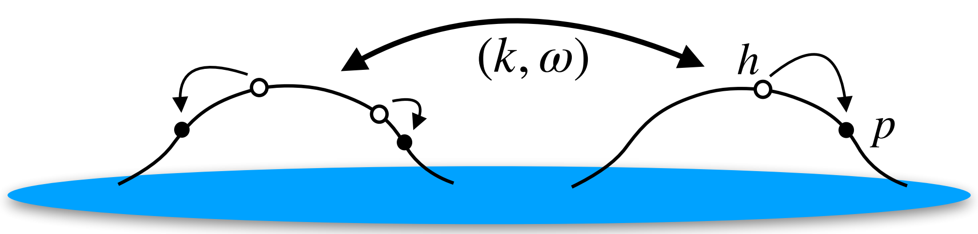
Generalized BBGKY

Previously: the only data was $\rho_{\mathrm{p}}(k, x, t)$

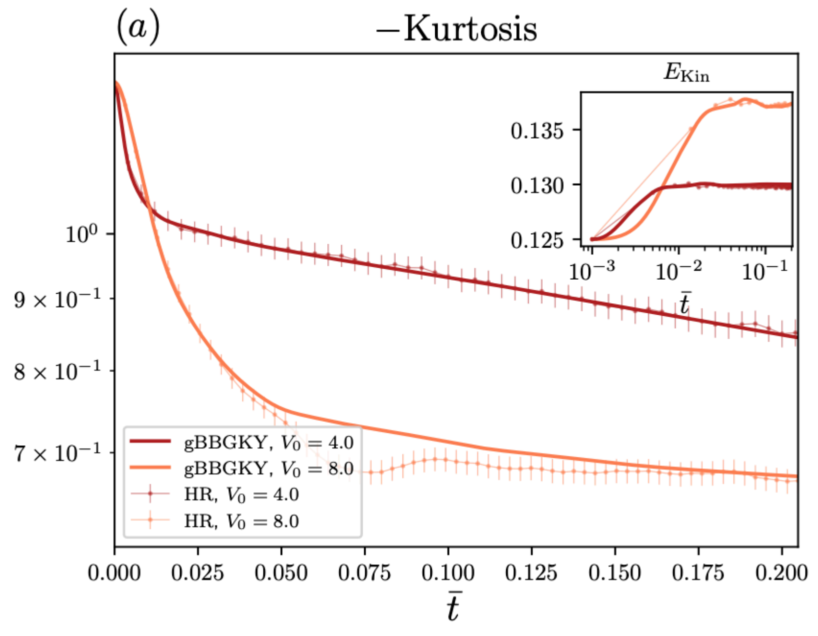
This neglects correlations induced by the perturbation

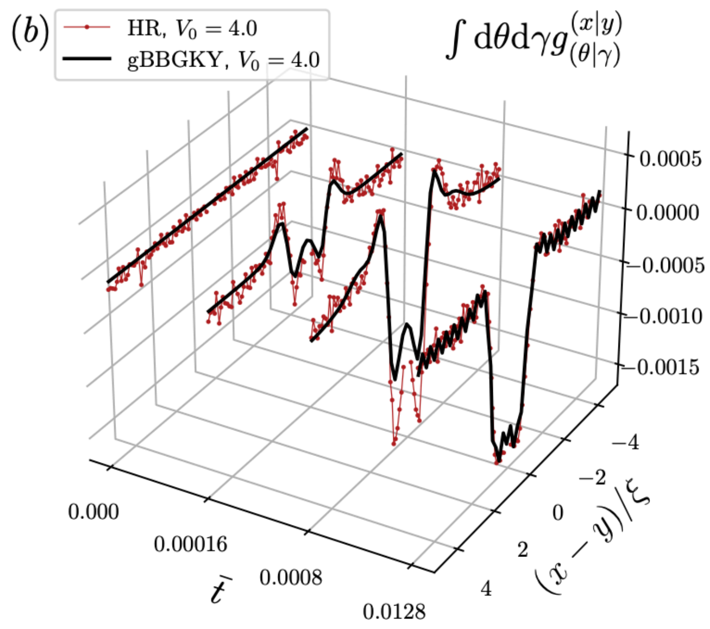
Counterexample: long-range interactions within a single tube

$$U(r) = V_{\mathrm{int}}(r) + V_{\mathrm{lr}}(r)$$



Results for hard-rods with long-range interactions





Self-diffusion

Integrability breaking that only conserves particle number

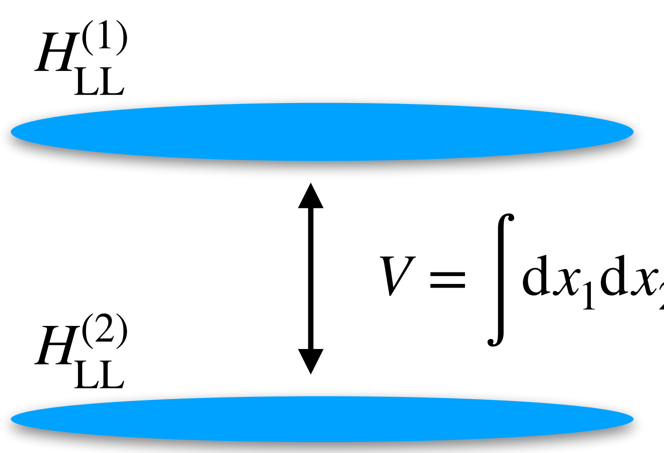
$$\partial_t \rho_{\mathrm{p}} + \partial_x (v_\rho \rho_{\mathrm{p}}) = \frac{1}{2}\partial_x (D_\rho \partial_x \rho_{\mathrm{p}}) + I[\rho_{\mathrm{p}}] \longrightarrow \partial_t n(x,t) + \partial_x (u(x,t) n(x,t)) = 0$$

By Chapman-Enskog method

$$\partial_t n(x,t) + \frac{1}{2}\partial_x \left(D[n] \partial_x n(x,t) \right) = 0$$

non-linear diffusion equation

contribution only from the integrability breaking



$$V = \int \mathrm{d}x_1 \mathrm{d}x_2 V(x_1, x_2) \hat{\rho}_1(x_1) \hat{\rho}(x_2)$$

freeze dynamics in of the tubes

Thank you!