Dynamics with free fermions in disguise

Based on 2405.20832

Márton Mestyán[1], Balázs Pozsgay[1] and István Vona[1,2]

Annecy, September 2, 2024

[1]MTA-ELTE Momentum Integrable Quantum Dynamics Group, Budapest

[2]Wigner RCP Holographic Quantum Field Theory Research Group, Budapest

Outline

Exact results due to hidden free fermionic modes

Exact results due to hidden free fermionic modes

- (generalized) Jordan-Wigner transformation does not work here

Exact results due to hidden free fermionic modes

- (generalized) Jordan-Wigner transformation does not work here
- explicit forms of correlation functions are rare, even exact results are usually complicated [Kitanine et al.:0707.1995,0803.3305]

Exact results due to hidden free fermionic modes

- (generalized) Jordan-Wigner transformation does not work here
- explicit forms of correlation functions are rare, even exact results are usually complicated [Kitanine et al.:0707.1995,0803.3305]
- another example of a quantum many body system that can be simulated on a classical computer

Outline

Ising model as a motivation

TFIM on L sites with OBC

$$H = \sum_{k=1}^{L-1} J_k X_k X_{k+1} + \sum_{k=1}^{L} \mu_k Z_k$$

Ising model as a motivation

TFIM on L sites with OBC

$$H = \sum_{k=1}^{L-1} \overbrace{b_{2k} X_k X_{k+1}}^{h_{2k}} + \sum_{k=1}^{L} \overbrace{b_{2k-1} Z_k}^{h_{2k-1}}$$

Ising model as a motivation

TFIM on L sites with OBC

$$H = \sum_{k=1}^{L-1} \overbrace{b_{2k} X_k X_{k+1}}^{h_{2k}} + \sum_{k=1}^{L} \overbrace{b_{2k-1} Z_k}^{h_{2k-1}}$$

Algebra of densities

$$h_m^2 = b_m^2 \cdot \mathbb{1},$$

Ising model as a motivation

TFIM on L sites with OBC

$$H = \sum_{k=1}^{L-1} \overbrace{b_{2k} X_k X_{k+1}}^{h_{2k}} + \sum_{k=1}^{L} \overbrace{b_{2k-1} Z_k}^{h_{2k-1}}$$

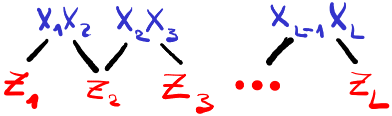
Algebra of densities

$$h_m^2 = b_m^2 \cdot \mathbb{1}, \qquad \{h_m, h_{m+1}\} = 0$$

Ising model as a motivation

TFIM on L sites with OBC

$$H = \sum_{k=1}^{L-1} \overbrace{b_{2k} X_k X_{k+1}}^{h_{2k}} + \sum_{k=1}^{L} \overbrace{b_{2k-1} Z_k}^{h_{2k-1}}$$



Algebra of densities

$$h_m^2 = b_m^2 \cdot \mathbb{1}, \qquad \{h_m, h_{m+1}\} = 0 = [h_m, h_n], \quad |m-n| > 1$$

TFIM on L sites with OBC

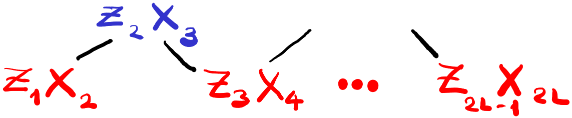
$$H = \sum_{k=1}^{L-1} \overbrace{b_{2k} X_k X_{k+1}}^{h_{2k}} + \sum_{k=1}^{L} \overbrace{b_{2k-1} Z_k}^{h_{2k-1}}$$

Algebra of densities

$$h_m^2 = b_m^2 \cdot \mathbb{1}, \qquad \{h_m, h_{m+1}\} = 0 = [h_m, h_n], \quad |m-n| > 1$$

Other rep. on a $2L$ chain

$$h_m = b_m Z_m X_{m+1}, \quad m = 1, 2, \ldots, 2L-1$$



FFD Hamiltonian [Fendley:1901.08078]

$M+2$ spins with OBC

$$H=\sum_{m=1}^{M} h_m, \quad h_m = b_m Z_{m-1} Z_m X_{m+1}$$

FFD Hamiltonian [Fendley:1901.08078]

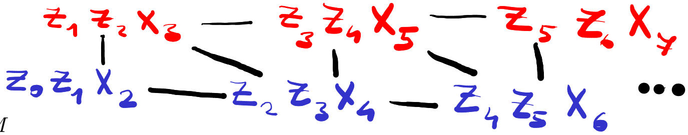
$M+2$ spins with OBC

$$H = \sum_{m=1}^{M} h_m, \quad h_m = b_m Z_{m-1} Z_m X_{m+1}$$

Algebra

$$h_m^2 = b_m^2 \cdot \mathbb{1}, \qquad \{h_m, h_{m+1}\} = \{h_m, h_{m+2}\} = 0$$

$M+2$ spins with OBC



$$H=\sum_{m=1}^{M} h_m, \quad h_m = b_m Z_{m-1} Z_m X_{m+1}$$

Algebra

$$h_m^2 = b_m^2 \cdot \mathbb{1}, \qquad \{h_m, h_{m+1}\} = \{h_m, h_{m+2}\} = 0 = [h_m, h_n], \quad |m-n|>2$$

FFD Hamiltonian [Fendley:1901.08078]

$M + 2$ spins with OBC

$$H = \sum_{m=1}^{M} h_m, \quad h_m = b_m Z_{m-1} Z_m X_{m+1}$$

Algebra

$$h_m^2 = b_m^2 \cdot \mathbb{1}, \qquad \{h_m, h_{m+1}\} = \{h_m, h_{m+2}\} = 0 = [h_m, h_n], \quad |m - n| > 2$$

SOLUTION?

$$[H, \Psi_k] = 2\epsilon_k \Psi_k$$

Integrable structures

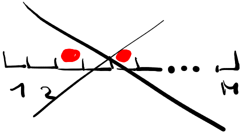
Conserved charges

$$Q^{(1)} = H = \sum_{n=1} h_n$$

Conserved charges

$$Q^{(1)} = H$$

$$Q^{(2)} = \frac{1}{2} \sum_{|m_2 - m_1| > 2} h_{m_1} h_{m_2}$$



$$\vdots$$

Integrable structures

Conserved charges

$$
\begin{aligned}
Q^{(1)} &= H \\
Q^{(2)} &= \frac{1}{2} \sum_{|m_2 - m_1| > 2} h_{m_1} h_{m_2} \\
&\vdots \\
Q^{(s)} &= \sum \overbrace{h_{m_1} h_{m_2} \dots h_{m_s}}^{\text{commuting densities}} \\
&\vdots
\end{aligned}
$$

Integrable structures

Conserved charges

$$Q^{(1)} = H$$

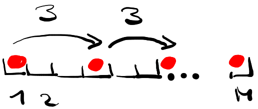
$$Q^{(2)} = \frac{1}{2} \sum_{|m_2 - m_1| > 2} h_{m_1} h_{m_2}$$

$$\vdots$$

$$Q^{(s)} = \sum \overbrace{h_{m_1} h_{m_2} \dots h_{m_s}}^{\text{commuting densities}}$$

$$\vdots$$

$$Q^{(S)} = \sum \dots h_m h_{m+3} h_{m+6} \dots , \quad S = \left[\frac{M+2}{3} \right]$$



Integrable structures

Conserved charges

$$Q^{(1)} = H$$

$$Q^{(2)} = \frac{1}{2} \sum_{|m_2 - m_1| > 2} h_{m_1} h_{m_2}$$

$$\vdots$$

$$Q^{(s)} = \sum \overbrace{h_{m_1} h_{m_2} \dots h_{m_s}}^{\text{commuting densities}}$$

$$\vdots$$

$$Q^{(S)} = \sum \dots h_m h_{m+3} h_{m+6} \dots, \quad S = \left[\frac{M+2}{3} \right]$$

Transfer matrix

$$T_M(u) = \sum_{s=0}^{S} (-u)^s Q^{(s)}, \qquad [T_M(u), T_M(u')] = 0$$

Special properties in the OBC case

Recursion

$$T_M(u) = T_{M-1}(u) - uh_M T_{M-3}(u),$$

Special properties in the OBC case

Recursion

$$T_M(u) = T_{M-1}(u) - u h_M T_{M-3}(u), \quad T_0 = T_{-1} = T_{-2} = \mathbb{1}$$

Special properties in the OBC case

Recursion

$$T_M(u) = T_{M-1}(u) - u h_M T_{M-3}(u), \quad T_0 = T_{-1} = T_{-2} = \mathbb{1}$$

Factorization

$$T_M(u) = G_M(u) G_M^T(u),$$

Special properties in the OBC case

Recursion

$$T_M(u) = T_{M-1}(u) - u h_M T_{M-3}(u), \quad T_0 = T_{-1} = T_{-2} = \mathbb{1}$$

Factorization

$$T_M(u) = G_M(u) G_M^T(u), \quad G_M(u) = g_1 g_2 \dots g_M$$

Special properties in the OBC case

Recursion

$$T_M(u) = T_{M-1}(u) - uh_M T_{M-3}(u), \quad T_0 = T_{-1} = T_{-2} = \mathbb{1}$$

Factorization

$$T_M(u) = G_M(u) G_M^T(u), \quad G_M(u) = g_1 g_2 \dots g_M \quad g_m = \cos\frac{\phi_m}{2} + \frac{h_m}{b_m}\sin\frac{\phi_m}{2}$$

Special properties in the OBC case

Recursion

$$T_M(u) = T_{M-1}(u) - uh_M T_{M-3}(u), \quad T_0 = T_{-1} = T_{-2} = \mathbb{1}$$

Factorization

$$T_M(u) = G_M(u) G_M^T(u), \quad G_M(u) = g_1 g_2 \dots g_M \quad g_m = \cos\frac{\phi_m}{2} + \frac{h_m}{b_m}\sin\frac{\phi_m}{2}$$

Inversion relation

$$T_M(u)T_M(-u) = \prod_{m=1}^{M} \cos^2\phi_m \cdot \mathbb{1} = P_M(u^2) \cdot \mathbb{1}$$

Properties of the polynomial $P_M(u^2)$

Recursion

$$P_m(u^2) = P_{m-1}(u^2) - u^2 b_m^2 P_{m-3}(u^2), \quad P_0 = P_{-1} = P_{-2} = 1.$$

Properties of the polynomial $P_M(u^2)$

Recursion

$$P_m(u^2) = P_{m-1}(u^2) - u^2 b_m^2 P_{m-3}(u^2), \quad P_0 = P_{-1} = P_{-2} = 1.$$

Roots

$$P_M(u_k^2) = 0$$

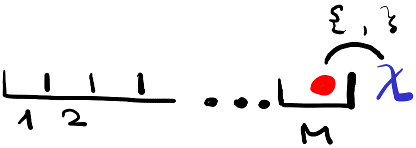
Properties of the polynomial $P_M(u^2)$

Recursion

$$P_m(u^2) = P_{m-1}(u^2) - u^2 b_m^2 P_{m-3}(u^2), \quad P_0 = P_{-1} = P_{-2} = 1.$$

Roots

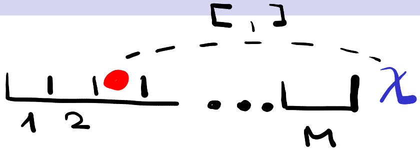
$$P_M(u_k^2) = 0 \quad \Rightarrow \quad P_M(u^2) = \prod_{k=1}^{S} \left(1 - u^2/u_k^2\right)$$



χ anticommutes with the density on the edge

$$\{h_M, \chi\} = 0$$

Edge operator and the eigenvalue problem



χ anticommutes with the density on the edge

$$\{h_M, \chi\} = 0 = [h_m, \chi] \quad \forall\, m < M$$

Edge operator and the eigenvalue problem

χ anticommutes with the density on the edge

$$\{h_M, \chi\} = 0 = [h_m, \chi] \quad \forall\, m < M$$

Solution to the eigenvalue problem

$$[H, T(-u_k)\chi T(u_k)] = \frac{2}{u_k} T(-u_k)\chi T(u_k)$$

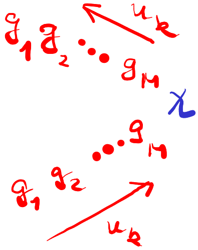
Disguised fermion modes

Definition

$$\Psi_{\pm k} = \frac{T_M(\mp u_k)\chi T_M(\pm u_k)}{N_k}$$

Definition

$$\Psi_{\pm k} = \frac{T_M(\mp u_k)\chi T_M(\pm u_k)}{N_k} = \frac{G_M(\mp u_k)\chi G_M(\pm u_k)}{\tilde{N}_k}$$



Disguised fermion modes

Definition

$$\Psi_{\pm k} = \frac{T_M(\mp u_k)\chi T_M(\pm u_k)}{N_k} = \frac{G_M(\mp u_k)\chi G_M(\pm u_k)}{\tilde{N}_k}$$

Eigenvalue problem

$$[H, \Psi_{\pm k}] = \pm \frac{2}{u_k} \Psi_{\pm k},$$

Disguised fermion modes

Definition

$$\Psi_{\pm k} = \frac{T_M(\mp u_k)\chi T_M(\pm u_k)}{N_k} = \frac{G_M(\mp u_k)\chi G_M(\pm u_k)}{\tilde{N}_k}$$

Eigenvalue problem

$$[H, \Psi_{\pm k}] = \pm\frac{2}{u_k}\Psi_{\pm k}, \quad \epsilon_k \equiv \frac{1}{u_k}$$

Disguised fermion modes

Definition

$$\Psi_{\pm k} = \frac{T_M(\mp u_k)\chi T_M(\pm u_k)}{N_k} = \frac{G_M(\mp u_k)\chi G_M(\pm u_k)}{\tilde{N}_k}$$

Eigenvalue problem

$$[H, \Psi_{\pm k}] = \pm\frac{2}{u_k}\Psi_{\pm k}, \quad \epsilon_k \equiv \frac{1}{u_k}$$

CAR

$$\Psi_k^\dagger = \Psi_{-k}, \quad \{\Psi_k, \Psi_{-k}\} = 1, \quad \{\Psi_k, \Psi_{k'}\} = 0, \quad k' \neq -k$$

Disguised fermion modes

Definition

$$\Psi_{\pm k} = \frac{T_M(\mp u_k)\chi T_M(\pm u_k)}{N_k} = \frac{G_M(\mp u_k)\chi G_M(\pm u_k)}{\tilde{N}_k}$$

Eigenvalue problem

$$[H, \Psi_{\pm k}] = \pm \frac{2}{u_k}\Psi_{\pm k}, \quad \epsilon_k \equiv \frac{1}{u_k}$$

CAR

$$\Psi_k^\dagger = \Psi_{-k}, \quad \{\Psi_k, \Psi_{-k}\} = 1, \quad \{\Psi_k, \Psi_{k'}\} = 0, \quad k' \neq -k$$

Hamiltonian

$$H = \sum_{k=1}^{S} \epsilon_k [\Psi_k, \Psi_{-k}],$$

Disguised fermion modes

Definition

$$\Psi_{\pm k} = \frac{T_M(\mp u_k)\chi T_M(\pm u_k)}{N_k} = \frac{G_M(\mp u_k)\chi G_M(\pm u_k)}{\tilde{N}_k}$$

Eigenvalue problem

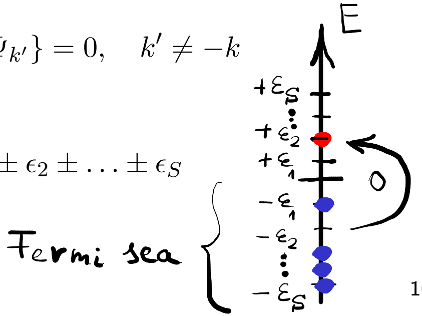
$$[H, \Psi_{\pm k}] = \pm\frac{2}{u_k}\Psi_{\pm k}, \quad \epsilon_k \equiv \frac{1}{u_k}$$

CAR

$$\Psi_k^\dagger = \Psi_{-k}, \quad \{\Psi_k, \Psi_{-k}\} = 1, \quad \{\Psi_k, \Psi_{k'}\} = 0, \quad k' \neq -k$$

Hamiltonian

$$H = \sum_{k=1}^{S} \epsilon_k [\Psi_k, \Psi_{-k}], \quad E = \pm\epsilon_1 \pm \epsilon_2 \pm \ldots \pm \epsilon_S$$



Graph theoretical construction

Extension for a large class of models [Chapman et. al.: 2003.05465, 2012.07857, 2305.15625, 2408.09684]

Graph theoretical construction

Extension for a large class of models [Chapman et. al.: 2003.05465, 2012.07857, 2305.15625, 2408.09684]

Frustration graph: $\mathcal{G}(H) \equiv (V, E)$

Extension for a large class of models [Chapman et. al.: 2003.05465, 2012.07857, 2305.15625, 2408.09684]

Frustration graph: $\mathcal{G}(H) \equiv (V, E)$

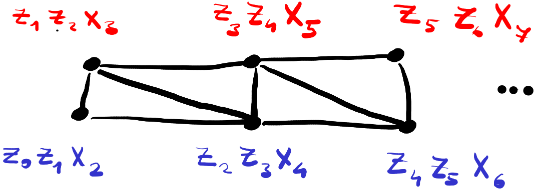
- vertices V - Pauli strings in H

Graph theoretical construction

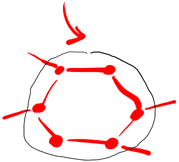
Extension for a large class of models [Chapman et. al.: 2003.05465, 2012.07857, 2305.15625, 2408.09684]

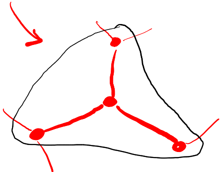
Frustration graph: $\mathcal{G}(H) \equiv (V, E)$

- vertices V - Pauli strings in H
- edges E - anticommuting strings



Even-hole-free and claw-free ⇒ the FFD construction can be generalized





Outline

Aims

The FFD model

Dynamics and correlation functions

Quantum circuits

Outlook

Decomposition of the edge operator

Decomposition of the edge operator

... into fermion modes

$$\chi = \sum_{j=-S}^{S} C_j \Psi_j$$

Decomposition of the edge operator

…into fermion modes

$$\chi = \sum_{j=-S}^{S} C_j \Psi_j$$

Exact zero mode

$$\Psi_0^2 = \mathbb{1}, \quad [H, \Psi_0] = 0, \quad \epsilon_0 = 0, \quad \{\Psi_0, \Psi_k\} = 0$$

Decomposition of the edge operator

…into fermion modes

$$\chi = \sum_{j=-S}^{S} C_j \Psi_j$$

Exact zero mode

$$\Psi_0^2 = \mathbb{1}, \quad [H, \Psi_0] = 0, \quad \epsilon_0 = 0, \quad \{\Psi_0, \Psi_k\} = 0$$

Coefficients expressed via P_M-s

$$C_{\pm k} = \sqrt{\frac{P_{M-1}(u_k^2)}{-u_k^2 P'_M(u_k^2)}}, \quad C_0 = \sqrt{\lim_{u\to\infty} \frac{P_{M-1}(u^2)}{P_M(u^2)}}$$

Local operator

Krylov-basis with $[H, \cdot]$ commutator

Local operator

Krylov-basis with $[H, \cdot]$ commutator

$$o_0 = \chi$$

Local operator

Krylov-basis with $[H,\cdot]$ commutator

$$o_0 = \chi \qquad \xrightarrow{\frac{1}{2}[H,\cdot]} \qquad o_1 = h_M \chi$$

Local operator

Krylov-basis with $[H, \cdot]$ commutator

$$o_0 = \chi \xrightarrow{\frac{1}{2}[H,\cdot]} o_1 = h_M \chi \xrightarrow{\frac{1}{2}[H,\cdot]} o_2 = \ldots$$

Local operator

Krylov-basis with $[H, \cdot]$ commutator

$$o_0 = \chi = \sum_{j=-S}^{S} C_j \Psi_j \quad \xrightarrow{\frac{1}{2}[H,\cdot]} \quad o_1 = h_M \chi = \sum_{j=-S}^{S} C_j \epsilon_j \Psi_j \quad \xrightarrow{\frac{1}{2}[H,\cdot]} \quad o_2 = \dots$$

Local operator

Krylov-basis with $[H,\cdot]$ commutator

$$o_0 = \chi = \sum_{j=-S}^{S} C_j \Psi_j \quad \xrightarrow{\frac{1}{2}[H,\cdot]} \quad o_1 = h_M \chi = \sum_{j=-S}^{S} C_j \epsilon_j \Psi_j \quad \xrightarrow{\frac{1}{2}[H,\cdot]} \quad o_2 = \dots$$

$$h_M = o_1 o_0$$

Local operator

Krylov-basis with $[H, \cdot]$ commutator

$$o_0 = \chi = \sum_{j=-S}^{S} C_j \Psi_j \quad \xrightarrow{\frac{1}{2}[H,\cdot]} \quad o_1 = h_M \chi = \sum_{j=-S}^{S} C_j \epsilon_j \Psi_j \quad \xrightarrow{\frac{1}{2}[H,\cdot]} \quad o_2 = \dots$$

Around the edge they remain local and bilinear in the fermions

$$h_M = o_1 o_0 = \sum_{j,k=-S}^{S} \epsilon_j C_j C_k \Psi_j \Psi_k$$

Local operator

Krylov-basis with $[H, \cdot]$ commutator

$$o_0 = \chi = \sum_{j=-S}^{S} C_j \Psi_j \quad \xrightarrow{\frac{1}{2}[H,\cdot]} \quad o_1 = h_M \chi = \sum_{j=-S}^{S} C_j \epsilon_j \Psi_j \quad \xrightarrow{\frac{1}{2}[H,\cdot]} \quad o_2 = \dots$$

Around the edge they remain local and bilinear in the fermions

$$h_M = o_1 o_0 = \sum_{j,k=-S}^{S} \epsilon_j C_j C_k \Psi_j \Psi_k$$

$$h_{M-1} + h_{M-2} = \left(o_0 - o_2/b_M^2\right) o_1$$

Local operator

Krylov-basis with $[H, \cdot]$ commutator

$$o_0 = \chi = \sum_{j=-S}^{S} C_j \Psi_j \quad \xrightarrow{\frac{1}{2}[H,\cdot]} \quad o_1 = h_M \chi = \sum_{j=-S}^{S} C_j \epsilon_j \Psi_j \quad \xrightarrow{\frac{1}{2}[H,\cdot]} \quad o_2 = \dots$$

Around the edge they remain local and bilinear in the fermions

$$h_M = o_1 o_0 = \sum_{j,k=-S}^{S} \epsilon_j C_j C_k \Psi_j \Psi_k$$

$$h_{M-1} + h_{M-2} = \left(o_0 - o_2/b_M^2\right) o_1 = \sum_{j,k=-S}^{S} \left(1 - \epsilon_j^2/b_M^2\right) \epsilon_k C_j C_k \Psi_j \Psi_k$$

Correlators

Time evolution

$$e^{iHt}\Psi_{\pm k}e^{-iHt} = e^{\pm 2i\epsilon_k t}\Psi_{\pm k}$$

Correlators

Time evolution

$$e^{iHt}\Psi_{\pm k}e^{-iHt} = e^{\pm 2i\epsilon_k t}\Psi_{\pm k} \quad \Rightarrow \quad \chi(t), h_M(t), \ldots$$

Correlators

Time evolution

$$e^{iHt}\Psi_{\pm k}e^{-iHt} = e^{\pm 2i\epsilon_k t}\Psi_{\pm k} \quad \Rightarrow \quad \chi(t), h_M(t), \ldots$$

Infinite temperature correlators

$$\langle O(t)O(0)\rangle = \frac{\mathrm{Tr}\left(O(t)O(0)\right)}{\mathrm{Tr}\left(\mathbb{1}\right)}$$

Correlators

Time evolution

$$e^{iHt}\Psi_{\pm k}e^{-iHt} = e^{\pm 2i\epsilon_k t}\Psi_{\pm k} \quad \Rightarrow \quad \chi(t), h_M(t), \ldots$$

Infinite temperature correlators

$$\langle O(t)O(0)\rangle = \frac{\mathrm{Tr}\,(O(t)O(0))}{\mathrm{Tr}\,(\mathbb{1})}$$

Building block

$$\langle \chi(t)\chi(0)\rangle = \sum_{k=-S}^{S} C_k^2 \cos(2\epsilon_k t)$$

Correlators

Time evolution

$$e^{iHt}\Psi_{\pm k}e^{-iHt} = e^{\pm 2i\epsilon_k t}\Psi_{\pm k} \quad \Rightarrow \quad \chi(t), h_M(t), \ldots$$

Infinite temperature correlators

$$\langle O(t)O(0)\rangle = \frac{\mathrm{Tr}\left(O(t)O(0)\right)}{\mathrm{Tr}\left(\mathbb{1}\right)}$$

Building block

$$\langle \chi(t)\chi(0)\rangle = \sum_{k=-S}^{S} C_k^2 \cos(2\epsilon_k t) \equiv B(t)$$

Correlators

Time evolution

$$e^{iHt}\Psi_{\pm k}e^{-iHt} = e^{\pm 2i\epsilon_k t}\Psi_{\pm k} \quad \Rightarrow \quad \chi(t), h_M(t), \ldots$$

Infinite temperature correlators

$$\langle O(t)O(0)\rangle = \frac{\mathrm{Tr}\left(O(t)O(0)\right)}{\mathrm{Tr}\left(\mathbb{1}\right)}$$

Building block

$$\langle \chi(t)\chi(0)\rangle = \sum_{k=-S}^{S} C_k^2 \cos(2\epsilon_k t) \equiv B(t)$$

$$\langle h_M(t)h_M(0)\rangle = \frac{1}{4}\left(\dot{B}^2(t) - \ddot{B}(t)B(t)\right)$$

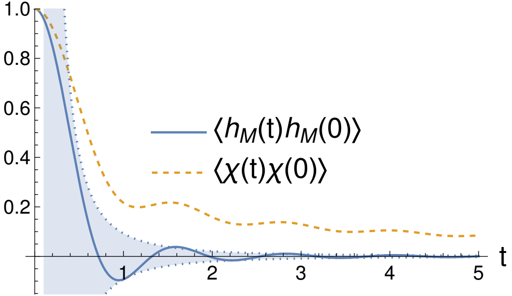
$$\vdots$$

Uniform case ($b_m = 1$) and thermodynamical limit ($M \to \infty$)

Uniform case ($b_m = 1$) and thermodynamical limit ($M \to \infty$)

$$B(t) = {}_2F_3\left(\tfrac{1}{3}, \tfrac{2}{3}; \tfrac{1}{2}, 1, \tfrac{3}{2}; -(\epsilon_{\mathsf{max}} t)^2\right)$$

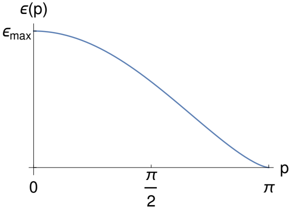
Uniform case ($b_m = 1$) and thermodynamical limit ($M \to \infty$)



$$B(t) = {}_2F_3\left(\tfrac{1}{3}, \tfrac{2}{3}; \tfrac{1}{2}, 1, \tfrac{3}{2}; -(\epsilon_{\mathsf{max}} t)^2\right)$$

Large t asymptotics

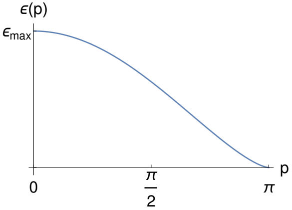
Dispersion relation in TDL [Fendley:1901.08078]



$$\epsilon^2(p) = \frac{\sin^3 p}{\sin(p/3)\sin^2(2p/3)}$$

Large t asymptotics

Dispersion relation in TDL [Fendley:1901.08078]

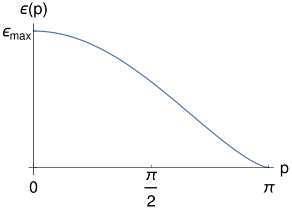


$$\epsilon^2(p) = \frac{\sin^3 p}{\sin(p/3)\sin^2(2p/3)}$$

$$B(t) = \sum_{k=-S}^{S} C_k^2 \cos(2\epsilon_k t)$$

Large t asymptotics

Dispersion relation in TDL [Fendley:1901.08078]

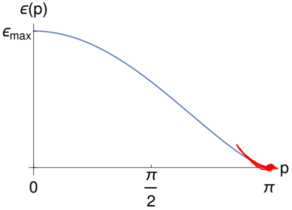


$$\epsilon^2(p) = \frac{\sin^3 p}{\sin(p/3)\sin^2(2p/3)}$$

$$B(t) = \sum_{k=-S}^{S} C_k^2 \cos(2\epsilon_k t) \quad \overset{\text{TDL}}{\Rightarrow} \quad \int_0^{\pi} \mathrm{d}p \; C^2(p) \cos(2\epsilon(p) t)$$

Large t asymptotics

Dispersion relation in TDL [Fendley:1901.08078]



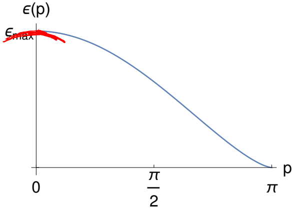
$$\epsilon^2(p) = \frac{\sin^3 p}{\sin(p/3)\sin^2(2p/3)}$$

Stationary Phase Approximation @ $p \simeq \pi$

$$B(t) = \sum_{k=-S}^{S} C_k^2 \cos(2\epsilon_k t) \overset{\text{TDL}}{\Rightarrow} \int_0^\pi \mathrm{d}p\ C^2(p)\cos(2\epsilon(p)t) \overset{\text{SPA}}{\Rightarrow} \frac{\alpha}{t^{2/3}} + \ldots$$

Large t asymptotics

Dispersion relation in TDL [Fendley:1901.08078]

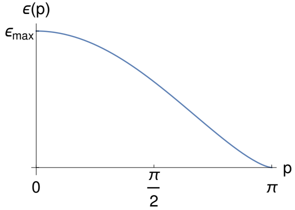


$$\epsilon^2(p) = \frac{\sin^3 p}{\sin(p/3)\sin^2(2p/3)}$$

Stationary Phase Approximation @ $p \simeq \pi$ and $p \simeq 0$

$$B(t) = \sum_{k=-S}^{S} C_k^2 \cos(2\epsilon_k t) \overset{\text{TDL}}{\Rightarrow} \int_0^{\pi} \mathrm{d}p\; C^2(p)\cos(2\epsilon(p)t) \overset{\text{SPA}}{\Rightarrow} \frac{\alpha}{t^{2/3}} + \frac{\beta \sin(2\epsilon_{\text{max}} t + \frac{\pi}{4})}{t^{3/2}} + \dots$$

Dispersion relation in TDL [Fendley:1901.08078]



$$\epsilon^2(p) = \frac{\sin^3 p}{\sin(p/3)\sin^2(2p/3)}$$

Stationary Phase Approximation @ $p \simeq \pi$ and $p \simeq 0$

$$B(t) = \sum_{k=-S}^{S} C_k^2 \cos(2\epsilon_k t) \overset{\text{TDL}}{\Rightarrow} \int_0^{\pi} \mathrm{d}p \; C^2(p)\cos(2\epsilon(p)t) \overset{\text{SPA}}{\Rightarrow} \frac{\alpha}{t^{2/3}} + \frac{\beta \sin(2\epsilon_{\text{max}} t + \frac{\pi}{4})}{t^{3/2}} + \dots$$

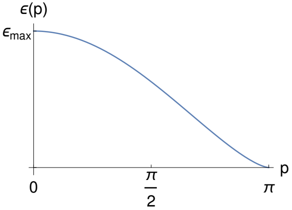
Combined

$$\langle h_M(t) h_M(0)\rangle \sim \frac{\sin(2\epsilon_{\text{max}} t + \frac{\pi}{4})}{t^{13/6}}$$

$2/3 + 3/2 = \frac{13}{6}$

Large t asymptotics

Dispersion relation in TDL [Fendley:1901.08078]



$$\epsilon^2(p) = \frac{\sin^3 p}{\sin(p/3)\sin^2(2p/3)}$$

Stationary Phase Approximation @ $p \simeq \pi$ and $p \simeq 0$

$$B(t) = \sum_{k=-S}^{S} C_k^2 \cos(2\epsilon_k t) \overset{\text{TDL}}{\Rightarrow} \int_0^\pi \mathrm{d}p\, C^2(p)\cos(2\epsilon(p)t) \overset{\text{SPA}}{\Rightarrow} \frac{\alpha}{t^{2/3}} + \frac{\beta\sin(2\epsilon_{\mathsf{max}}t + \frac{\pi}{4})}{t^{3/2}} + \ldots$$

Combined

$$\langle h_M(t) h_M(0)\rangle \sim \frac{\sin(2\epsilon_{\mathsf{max}}t + \frac{\pi}{4})}{t^{13/6}}$$

Ising at the boundary

$$\langle h_M(t) h_M(0)\rangle \sim t^{-3}$$

Outline

Trotterization via the factorized transfer matrix Pozsgay: 2402.02984

Unitary operator for time evolution

$$\mathcal{V}(\delta t) \sim T_M(i\delta t) \approx \mathbb{1} - iH\delta t$$

Trotterization via the factorized transfer matrix Pozsgay: 2402.02984

Unitary operator for time evolution

$$\mathcal{V}(\delta t) \sim T_M(i\delta t) \approx \mathbb{1} - iH\delta t$$

Approximating exact correlator

$$\langle h_M(t = N\delta t) h_M(0) \rangle \approx \mathrm{Tr}\left(\mathcal{V}^N(-\delta t) h_M \mathcal{V}^N(\delta t) h_M\right) / \mathrm{Tr}\left(\mathbb{1}\right)$$

Trotterization via the factorized transfer matrix Pozsgay: 2402.02984

Unitary operator for time evolution

$$\mathcal{V}(\delta t) \sim T_M(i\delta t) \approx \mathbb{1} - iH\delta t$$

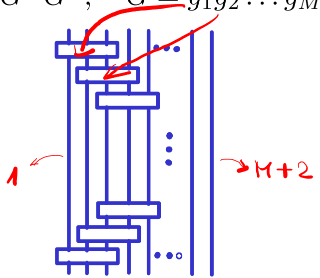
Approximating exact correlator

$$\langle h_M(t = N\delta t) h_M(0)\rangle \approx \mathrm{Tr}\left(\mathcal{V}^N(-\delta t) h_M \mathcal{V}^N(\delta t) h_M\right) / \mathrm{Tr}\left(\mathbb{1}\right)$$

Factorization to 3-site quantum gates g_j

$$\mathcal{V}(\delta t) = G \cdot G^T, \quad G = g_1 g_2 \dots g_M,$$

Staircase circuit



Trotterization via the factorized transfer matrix Pozsgay: 2402.02984

Unitary operator for time evolution

$$\mathcal{V}(\delta t) \sim T_M(i\delta t) \approx \mathbb{1} - iH\delta t$$

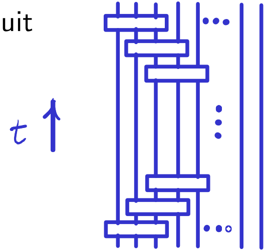
Approximating exact correlator

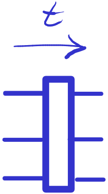
$$\langle h_M(t = N\delta t) h_M(0) \rangle \approx \mathrm{Tr}\left(\mathcal{V}^N(-\delta t) h_M \mathcal{V}^N(\delta t) h_M\right) / \mathrm{Tr}\left(\mathbb{1}\right)$$

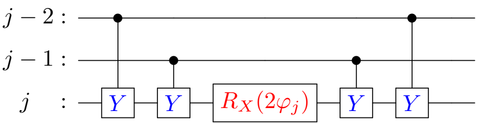
Factorization to 3-site quantum gates g_j

$$\mathcal{V}(\delta t) = G \cdot G^T, \quad G = g_1 g_2 \ldots g_M, \quad g_j = e^{i\varphi_j h_j} \quad \text{(recursion for } \varphi_j\text{)}$$

Staircase circuit







Operator-state mapping

String of ordered $(a_1 < a_2 < \ldots < a_n)$ densities

$$h_{a_1} h_{a_2} \ldots h_{a_n}$$

Operator-state mapping

String of ordered $(a_1 < a_2 < \ldots < a_n)$ densities $\rightarrow$ states

$$h_{a_1} h_{a_2} \ldots h_{a_n} \qquad \rightarrow \qquad \overbrace{|1001\ldots010\rangle}^{M}$$

$\uparrow$ a_1, $\uparrow$ a_2, $\ldots$, $\uparrow$ a_n

Operator-state mapping

String of ordered $(a_1 < a_2 < \ldots < a_n)$ densities $\rightarrow$ states

$$e^{-i\varphi_j h_j}[h_{a_1} h_{a_2} \ldots h_{a_n}] e^{i\varphi_j h_j} \quad \rightarrow \quad U_j |1001\ldots 010\rangle$$

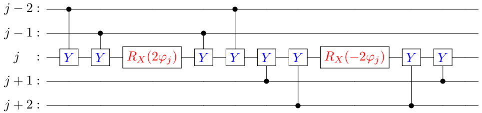
Conjugation with g_j acts as a unitary

Operator-state mapping

String of ordered ($a_1 < a_2 < \ldots < a_n$) densities $\rightarrow$ states

$$e^{-i\varphi_j h_j}[h_{a_1} h_{a_2} \ldots h_{a_n}]e^{i\varphi_j h_j} \quad \rightarrow \quad U_j|1001\ldots010\rangle$$

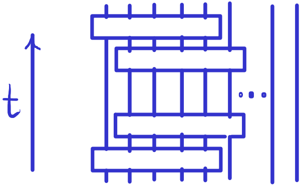
Conjugation with g_j acts as a unitary where U_j is a 5-site controlled gate

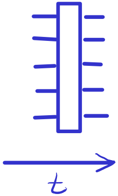


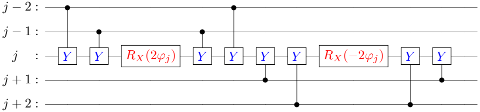
String of ordered $(a_1 < a_2 < \ldots < a_n)$ densities $\rightarrow$ states

$$e^{-i\varphi_j h_j}[h_{a_1} h_{a_2} \ldots h_{a_n}]e^{i\varphi_j h_j} \quad \rightarrow \quad U_j|1001\ldots010\rangle$$

Conjugation with g_j acts as a unitary where U_j is a 5-site controlled gate







Similar staircase structure

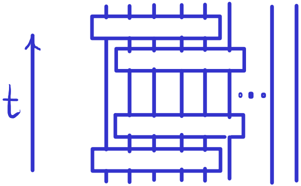
$$\mathcal{V}_5(\delta t) = G_5, \cdot G_5^T \quad G_5 = U_1 U_2 \ldots U_M$$

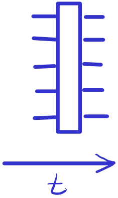
Operator-state mapping

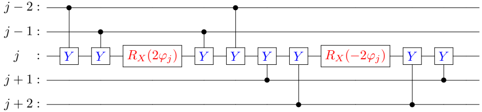
String of ordered $(a_1 < a_2 < \ldots < a_n)$ densities $\rightarrow$ states

$$e^{-i\varphi_j h_j}[h_{a_1} h_{a_2} \ldots h_{a_n}]e^{i\varphi_j h_j} \quad \rightarrow \quad U_j|1001\ldots010\rangle$$

Conjugation with g_j acts as a unitary where U_j is a 5-site controlled gate







Similar staircase structure

$$\mathcal{V}_5(\delta t) = G_5, \cdot G_5^T \quad G_5 = U_1 U_2 \ldots U_M$$

Correlator

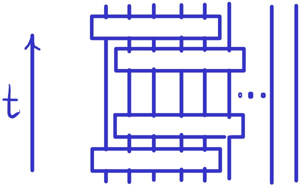
$$\langle h_M(t) h_M \rangle$$

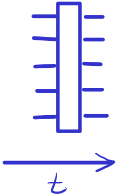
Operator-state mapping

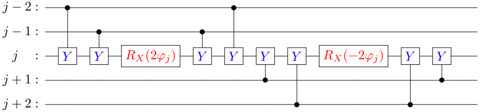
String of ordered $(a_1 < a_2 < \ldots < a_n)$ densities $\rightarrow$ states

$$e^{-i\varphi_j h_j}[h_{a_1} h_{a_2} \ldots h_{a_n}]e^{i\varphi_j h_j} \quad \rightarrow \quad U_j|1001\ldots010\rangle$$

Conjugation with g_j acts as a unitary where U_j is a 5-site controlled gate







Similar staircase structure

$$\mathcal{V}_5(\delta t) = G_5, \cdot G_5^T \quad G_5 = U_1 U_2 \ldots U_M$$

Correlator $\rightarrow$ Loschmidt-amplitude/fidelity

$$\langle h_M(t) h_M \rangle \approx \langle \psi_0 | \mathcal{V}_5^N(\delta t) | \psi_0 \rangle$$

Outline

Generalizations

Generalizations

- . . . for more local operators?

Generalizations

- . . . for more local operators?
- . . . for other models based on the graph-theoretic construction [Fendley and Pozsgay: 2310.19897, Chapman at al.: 2012.07857, 2305.15625 . . .]

Generalizations

- . . . for more local operators?
- . . . for other models based on the graph-theoretic construction [Fendley and Pozsgay: 2310.19897, Chapman at al.: 2012.07857, 2305.15625 . . .]
- . . . for parafermions [Fendley: 1310.6049; Pimenta et al.: 2108.04372; Chapman et al.: 2408.09684]

Generalizations

- . . . for more local operators?
- . . . for other models based on the graph-theoretic construction [Fendley and Pozsgay: 2310.19897, Chapman at al.: 2012.07857, 2305.15625 . . .]
- . . . for parafermions [Fendley: 1310.6049; Pimenta et al.: 2108.04372; Chapman et al.: 2408.09684]
- . . . for other type of correlators (zero and finite temperature, etc.)

Thank you!