Integrable Quantum Field Theories, Irrelevant Perturbations and Minimal Form Factors

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My talk today is based on the following preprints/papers:

1. Olalla Castro-Alvaredo, Stefano Negro and Fabio Sailis, Completing the Bootstrap Program for $\mathrm{T\bar{T}}$-Deformed Massive Integrable Quantum Field Theories, J. Phys. **A57** 265401 (2024); ArXiv:2305.17068
2. Olalla Castro-Alvaredo, Stefano Negro and Fabio Sailis, Form Factors and Correlation Functions of $\mathrm{T\bar{T}}$-Deformed Integrable Quantum Field Theories, JHEP **09** 2023 048; ArXiv:2306.01640
3. Olalla Castro-Alvaredo, Stefano Negro and Fabio Sailis, Entanglement Entropy from Form Factors in $\mathrm{T\bar{T}}$-Deformed Integrable Quantum Field Theories, JHEP **11** 2023 129; ArXiv:2306.11064.
4. Olalla Castro-Alvaredo, Stefano Negro and István M. Szécsényi, On the Representation of Minimal Form Factors in Integrable Quantum Field Theory, Nucl. Phys. B1000 (2024) 116459. ArXiv: 2311.16955

As you can see, this is work with Stefano Negro, Fabio Sailis and István M. Szécsényi

Lecturer, University of York (UK)

PhD Student at City, University of London (UK)

Postdoctoral Researcher at University of Modena (Italy)

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C provides a nice way to interpret some of our results (see later)

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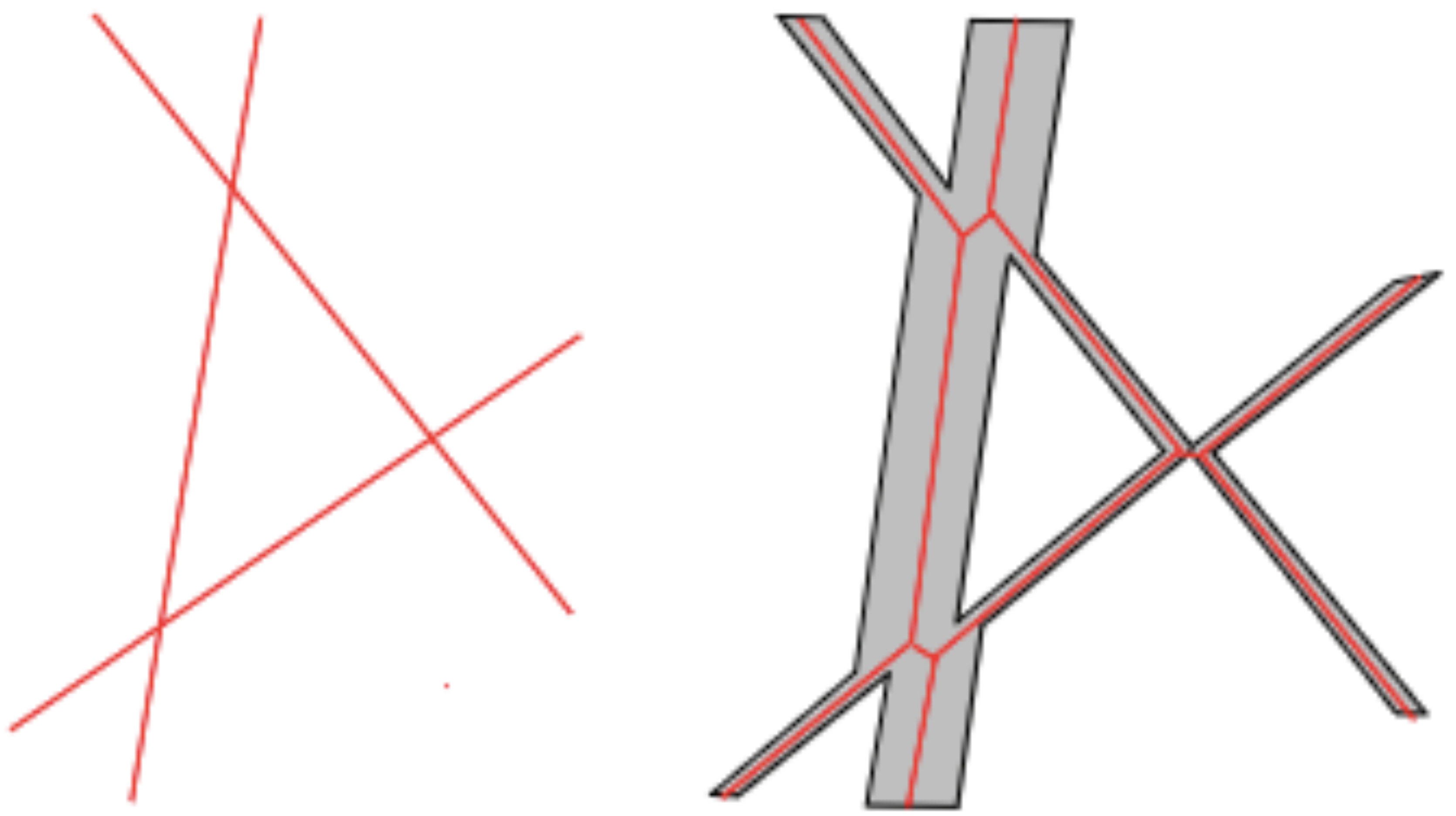

[Cardy & Doyon'10]

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$$F_n^{\mathcal{O}}(\theta_1,\ldots,\theta_i,\theta_{i+1},\ldots,\theta_n;\boldsymbol{\alpha}) = S_{\boldsymbol{\alpha}}(\theta_i-\theta_{i+1})F_n^{\mathcal{O}}(\theta_1,\ldots,\theta_{i+1},\theta_i,\ldots,\theta_n;\boldsymbol{\alpha})$$

$$F_n^{\mathcal{O}}(\theta_1+2\pi i,\theta_2\ldots,\theta_n;\boldsymbol{\alpha}) = \gamma_{\mathcal{O}}F_n^{\mathcal{O}}(\theta_2,\ldots,\theta_n,\theta_1;\boldsymbol{\alpha})$$

$$\lim_{\bar\theta\to\theta}(\bar\theta-\theta)F_{n+2}^{\mathcal{O}}(\bar\theta+i\pi,\theta,\theta_1,\ldots,\theta_n;\boldsymbol{\alpha}) = i\left(1-\gamma_{\mathcal{O}}\prod_{j=1}^{n}S_{\boldsymbol{\alpha}}(\theta-\theta_j)\right)F_n^{\mathcal{O}}(\theta_1,\ldots,\theta_n;\boldsymbol{\alpha})$$

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2. The solution depends on additional parameters not present in the S-matrix. In a sense, it contains its own CDD factor!
3. Understanding what the different choices of βs actually mean is still an open question (more on this later).

Higher-Particle Form Factors

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- For some theories and fields, it seems that higher particle form factors also factorise. We have shown this to be the case quite generally, and worked out the details for the Ising model.

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3. For local fields there can be an additional non-trivial multiplicative factor (like Θ).

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$$F^{\Theta}_{2n}(\theta_1,\ldots,\theta_{2n};\boldsymbol{\alpha}) = 2\pi i m^2\, i^n\frac{\sqrt{\prod_{i=1}^{2n}\sin\left(\sum_{s\in\mathcal{S}}\frac{\alpha_s}{2}\sum_{j=1}^{2n}\sinh(s\theta_{ij})\right)}}{\left|\sum_{s\in\mathcal{S}}\frac{\alpha_s}{4}\sqrt{\left(\frac{\sigma^{(2n)}_{1,s}\sigma^{(2n)}_{2n-1,s}}{\sigma^{(2n)}_{2n,s}}-4\right)\frac{\sigma^{(2n)}_{2n}\sigma^{(2n)}_{1,s}\sigma^{(2n)}_{2n-1,s}}{\sigma^{(2n)}_{2n,s}\sigma^{(2n)}_{1}\sigma^{(2n)}_{2n-1}}}\right|}\prod_{i<j}\tanh\frac{\theta_{ij}}{2}\varphi(\theta_{ij};\boldsymbol{\alpha})$$

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- Here $S_{\mathbf{0}}(\theta) = e^{i\delta(\theta)}$ and $\gamma^{\mathcal{O}} = e^{i\omega_{\mathcal{O}}}$.

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- These statements apply both to mr large and small, although for $\alpha^\star > 0$ and mr sufficiently large there is a rapidity cut-off that can make the series convergent.

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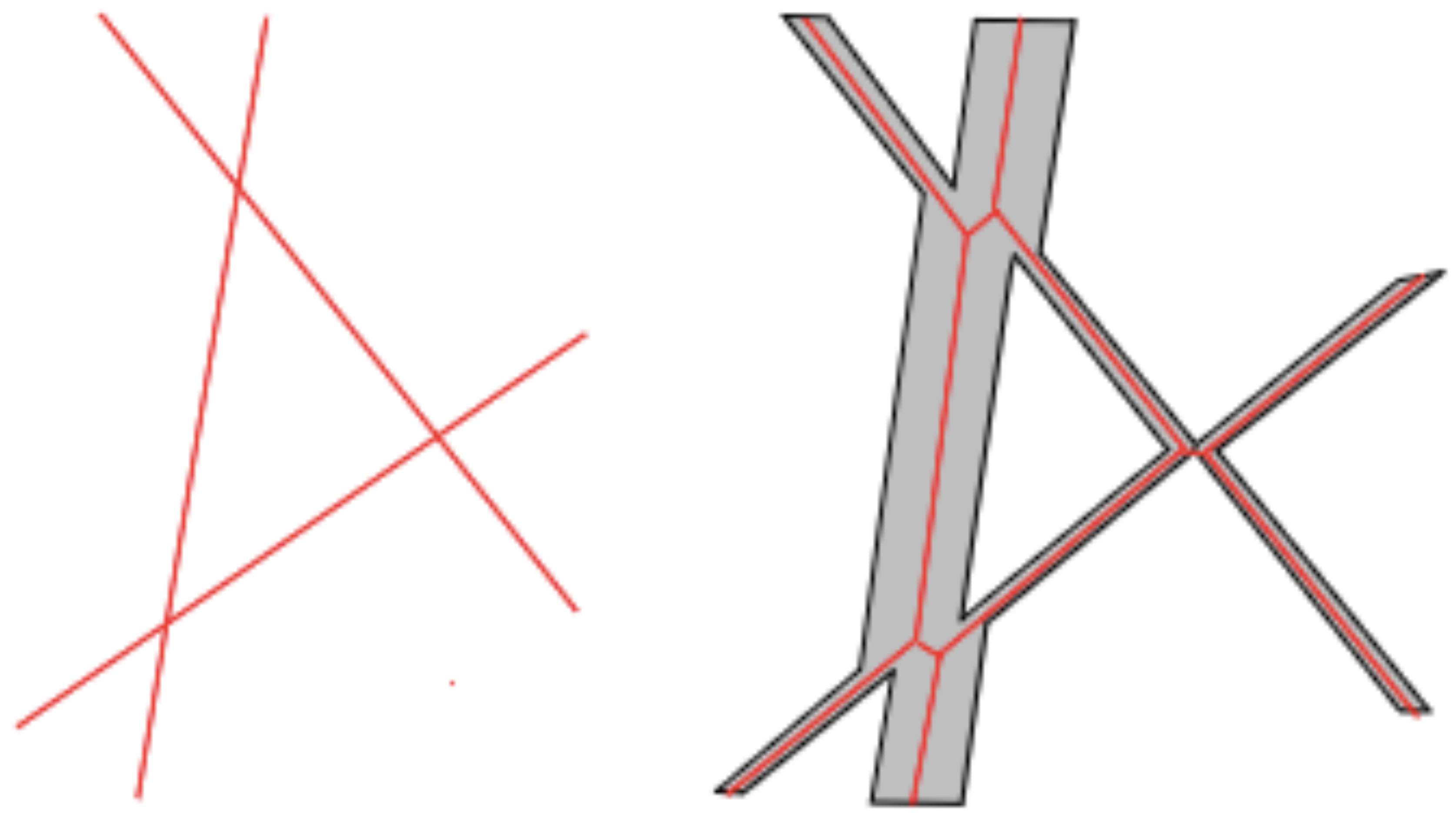

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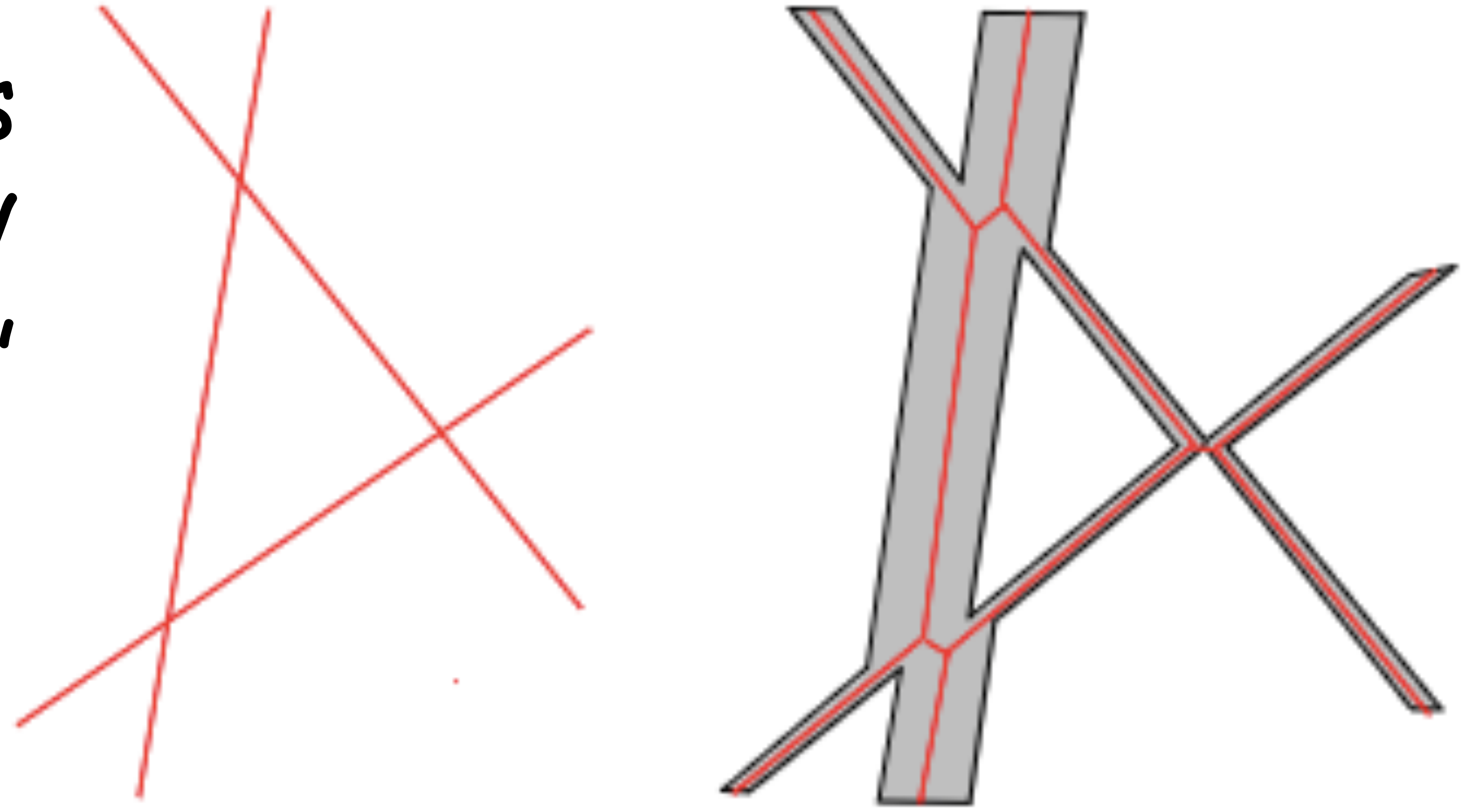
2. When $\alpha^\star < 0$ the fundamental excitations acquire a "negative" length so both the UV and IR can probed. In a sense "extra space" is created at short distances.



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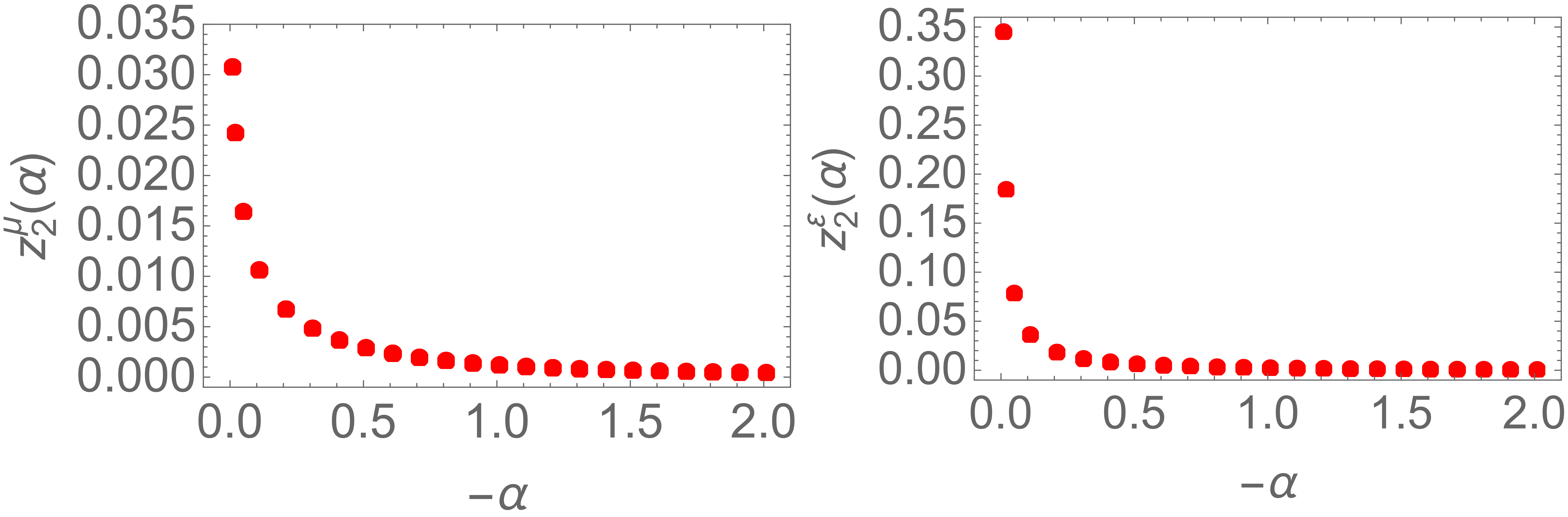
- For $\alpha < 0$ correlation functions exhibit power-law scaling at short distances, just like in CFT. The coefficients however, are now functions of α and there is no underlying CFT.

How do these Powers Behave?



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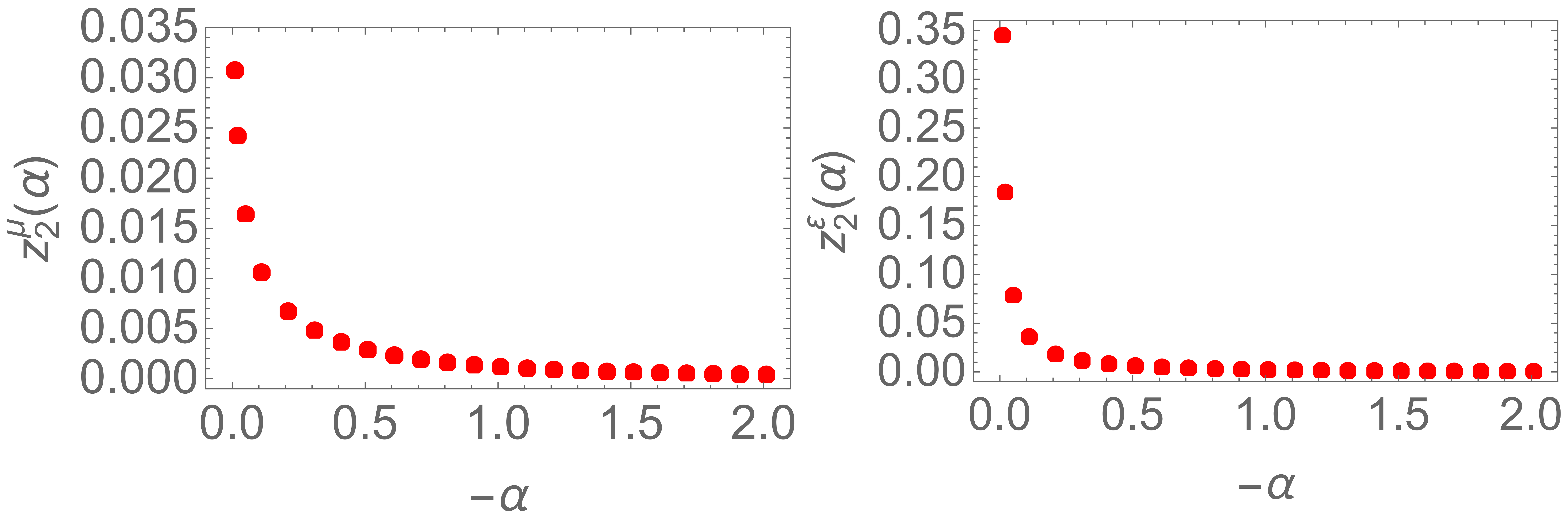

Here $z_2^{\mathcal{O}}(\alpha) = 4\Delta^{\mathcal{O}}(\alpha)$ in the two-particle approximation. The field ε is the "energy" field in the Ising model which is proportional to Θ.

How it All fits Together: Minimal Form Factors

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Is there a relationship between these two types of CDD factor?

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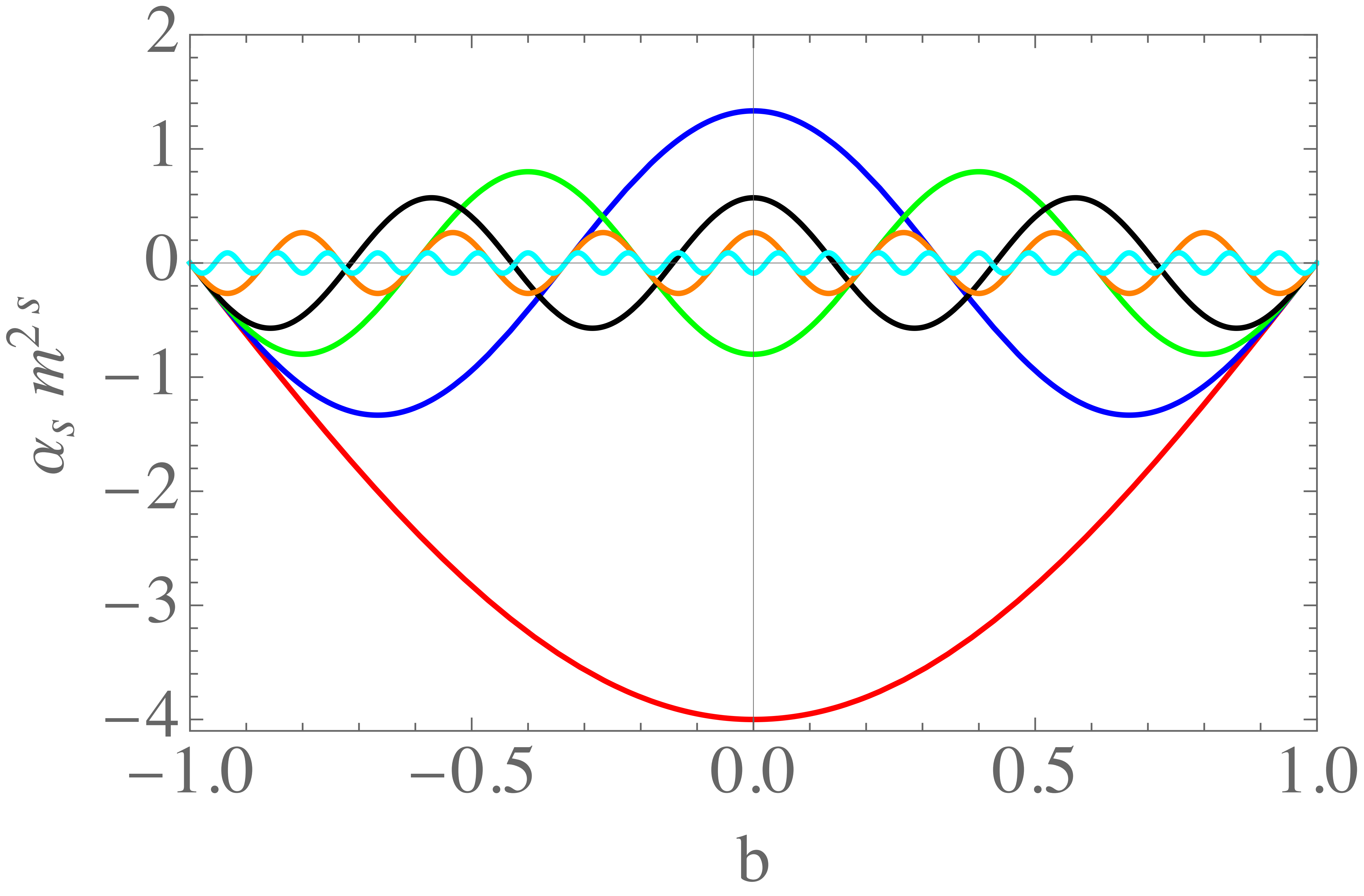

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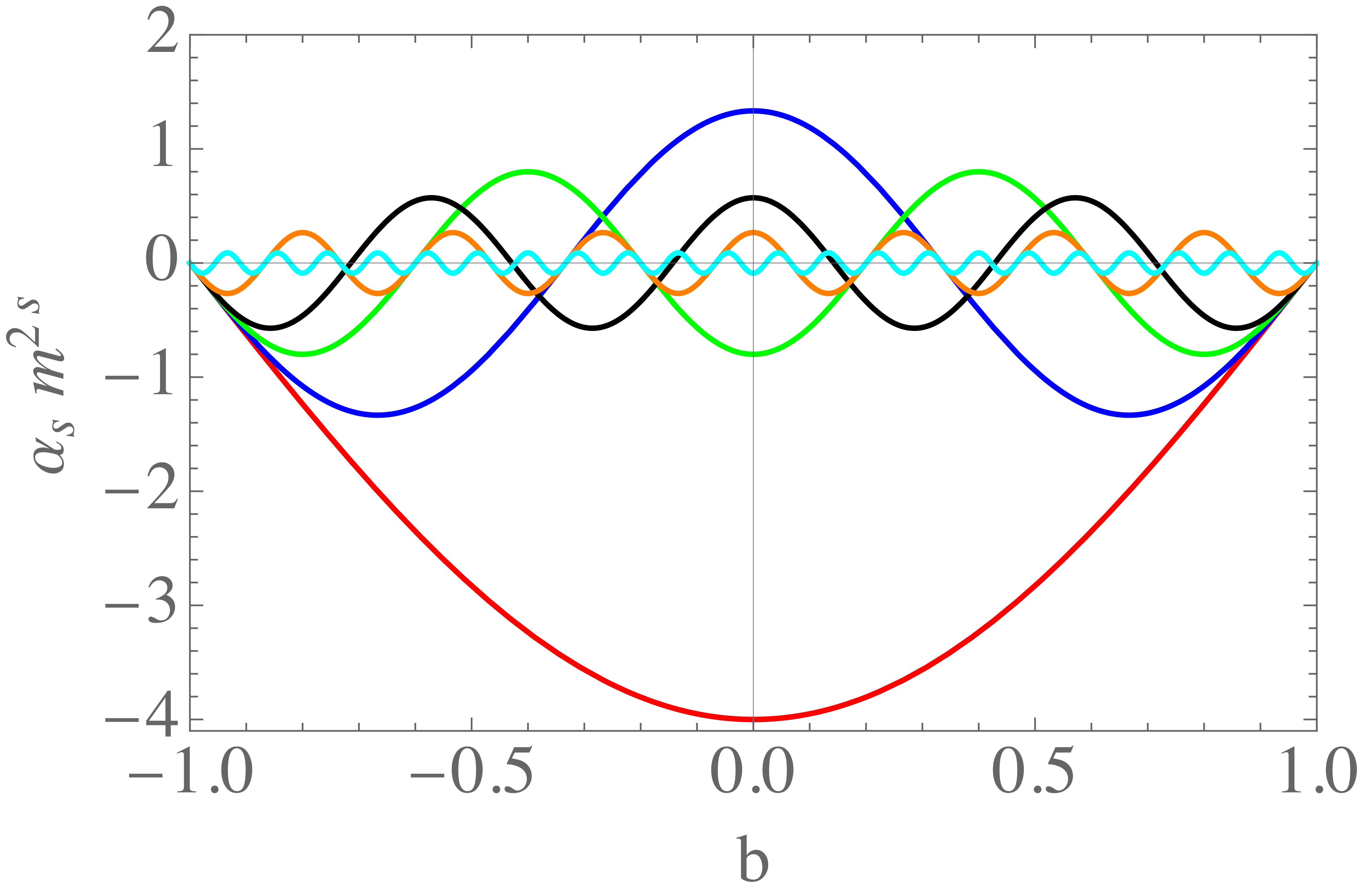

- The sinh-Gordon model can be seen as the Ising model perturbed by an infinite number of irrelevant perturbations for carefully chosen couplings [LeClair'21; Ahn & LeClair'22]

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1. The solution is very simple and general. The minimal form factors gets modified by a universal multiplicative factor, just like the S-matrix!
2. The solution depends on additional parameters not present in the S-matrix. In a sense, it contains its own CDD factor

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- Let's have a closer look at all these functions.....

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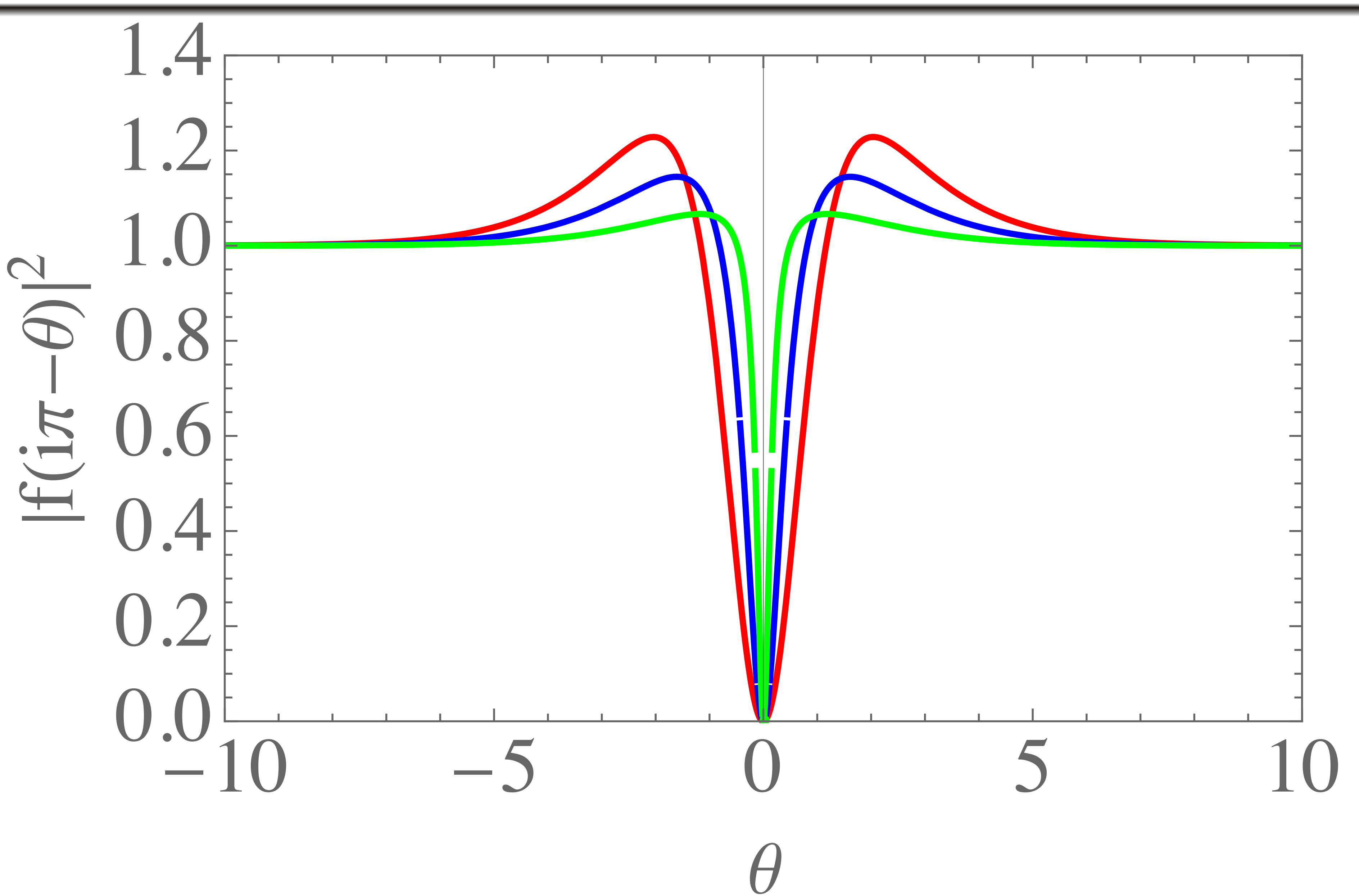

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- This work also proves that the MFF "CDD" factor plays a crucial role in standard theories. It actually ensures that the MFF is analytic and has the desired asymptotics. Understanding its role for $\mathrm{T\bar{T}}$-models remains an open question.


Thank you!