

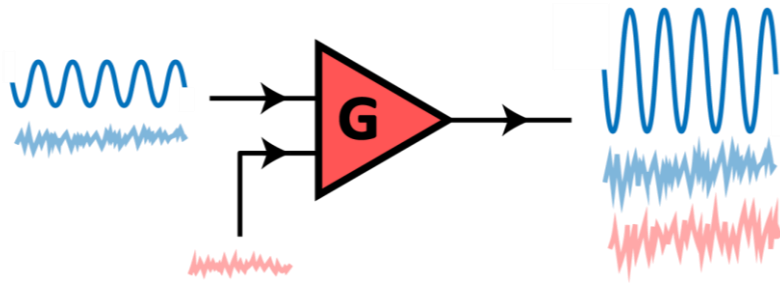
# Superconducting Josephson Traveling-Wave Parametric Amplifiers

Gwenael Le Gal, G. Butseraen, A. Ranadive, G. Cappelli, B. Fazlijj, A. Martin and N. Roch

Néel Institute, QuantECA team



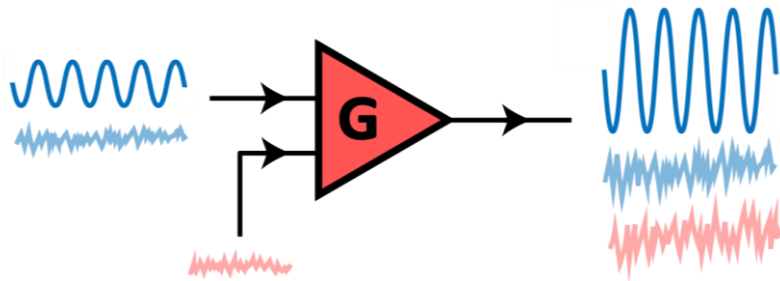
# Quantum limited amplification



An ideal amplifier adds white noise:

$$T_N \geq \frac{\hbar\omega}{2k_B} = T_{SQL} \quad \text{per mode amplified}$$

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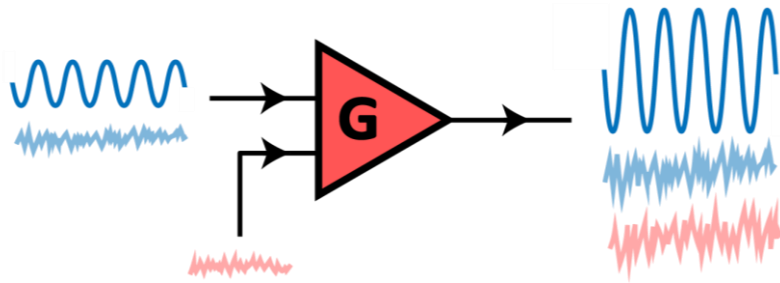
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A dissipative amplifier at  $T > 0$  K:  $T_N > T_{SQL}$ ,

HEMT amplifier:  $T_N \sim 10 T_{SQL}$

Superconducting amplifier

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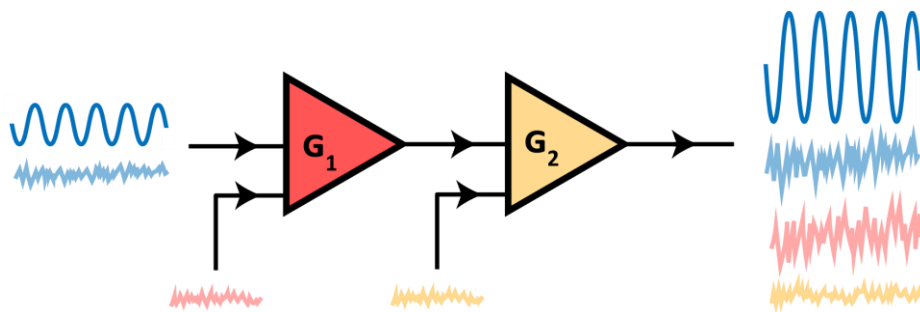
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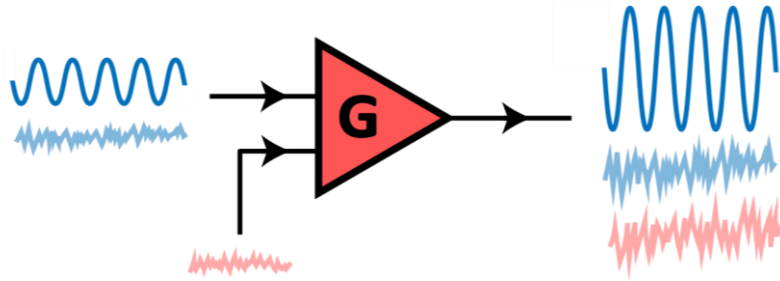
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Superconducting amplifier



$$T_N = T_{N,1} + \frac{T_{N,2}}{G_1}$$

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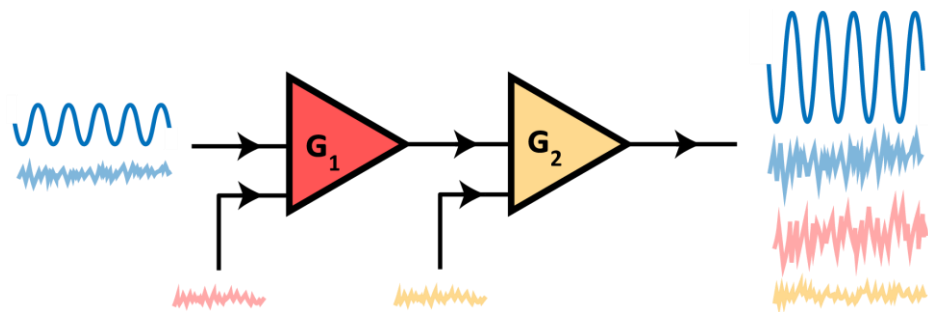
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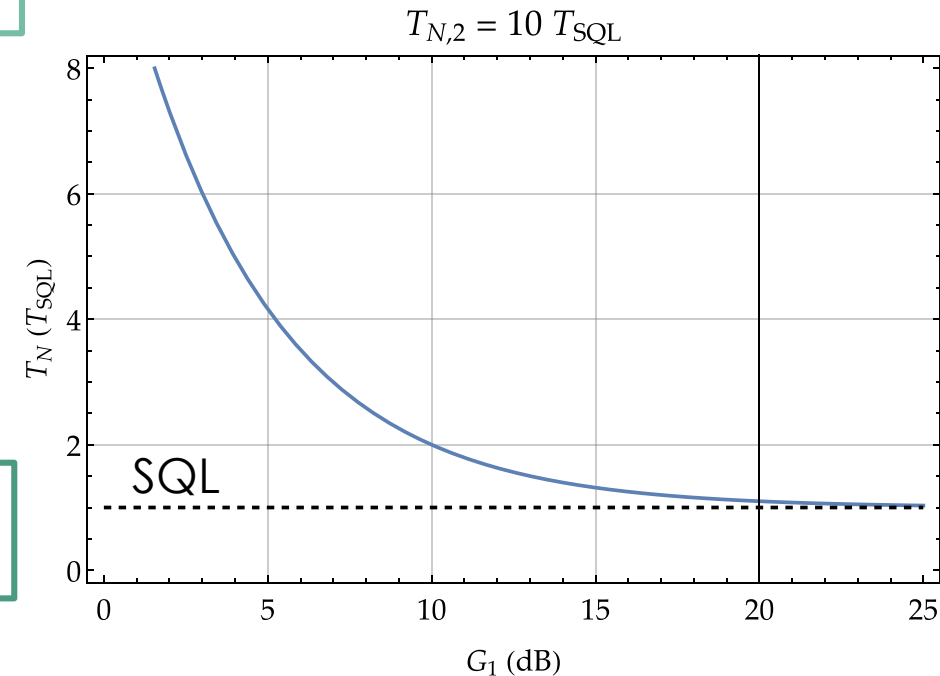
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How much minimum gain ?  
**20 dB**

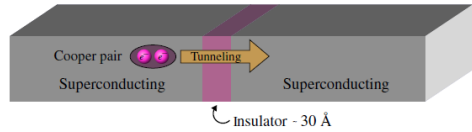


# Traveling Wave Parametric Amplification in a nutshell

$$I = I_c \sin(\phi) \approx I_c \phi + \chi^{(3)} \phi^3 + \dots \Rightarrow \text{4WM process} \propto \chi^{(3)} \hat{a}_i^\dagger \hat{a}_s^\dagger \hat{a}_p \hat{a}_p + h.c.$$

$$\text{Energy: } 2\omega_p = \omega_s + \omega_i$$

$$\text{Momentum: } 2k_p = k_s + k_i$$



S. E. Rasmussen *et al.*, PRX Quantum, 2021

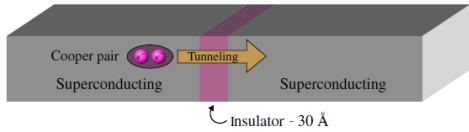


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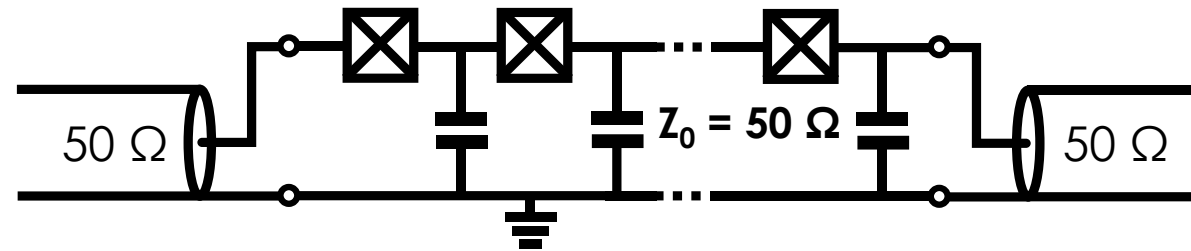
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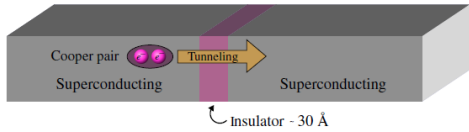


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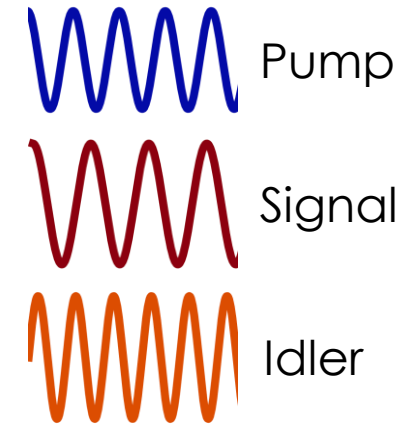
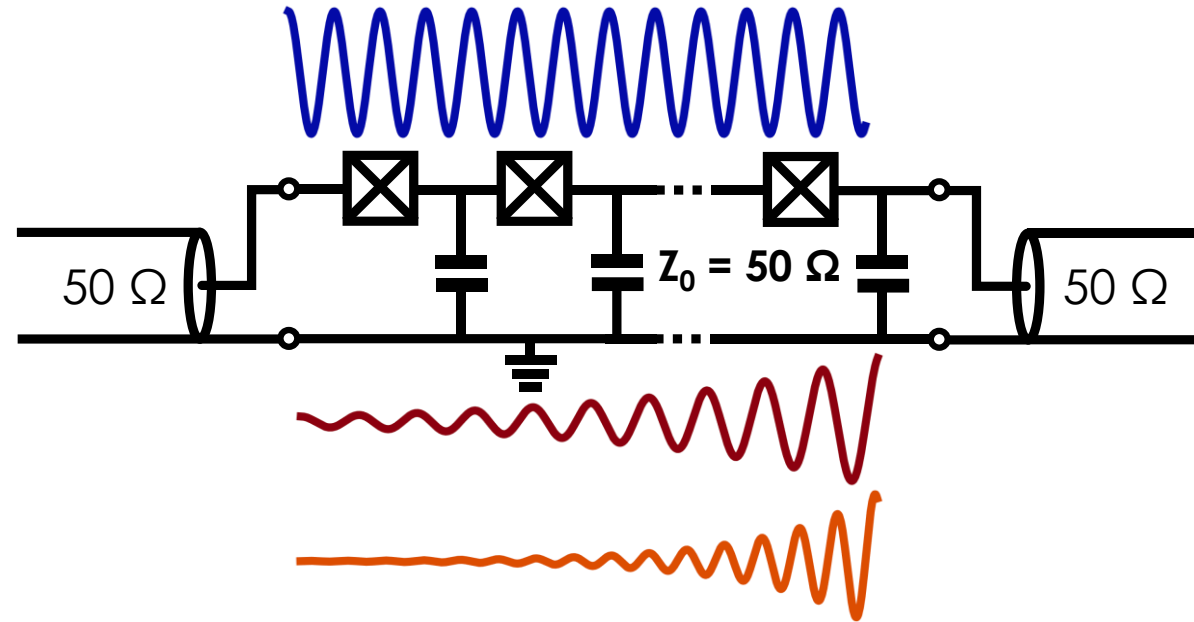
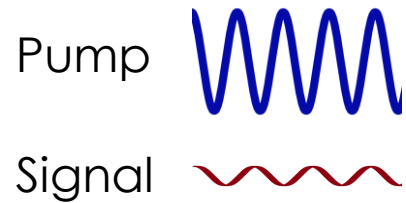
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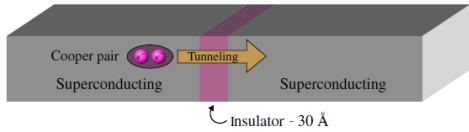


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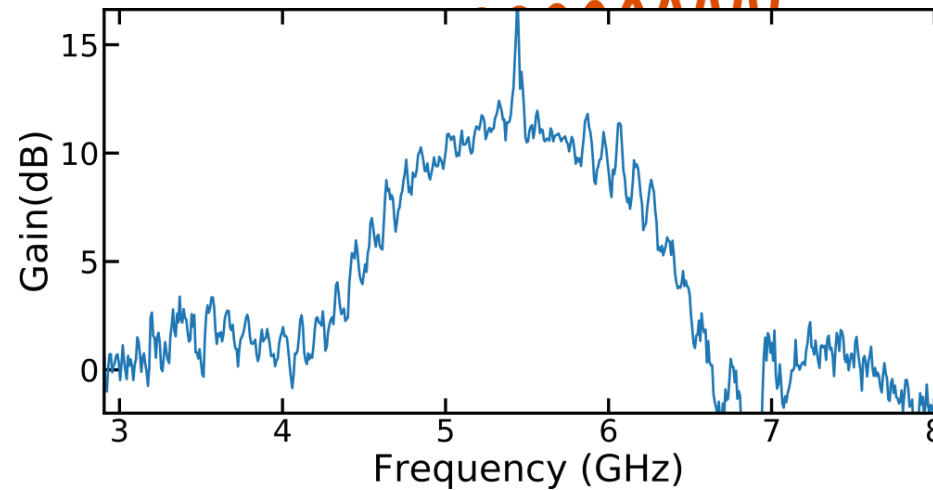
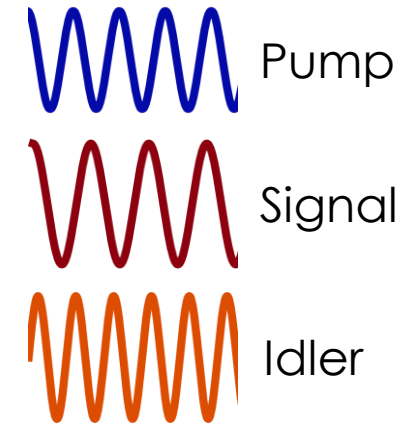
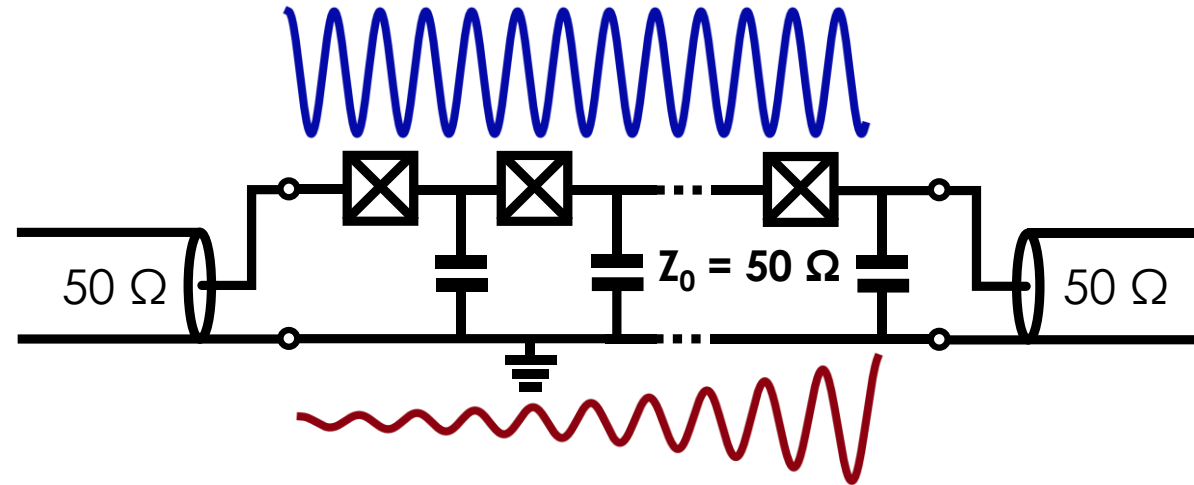
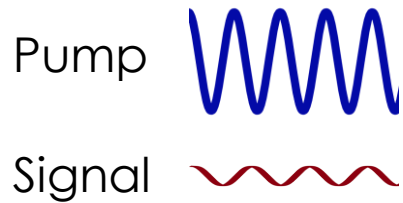
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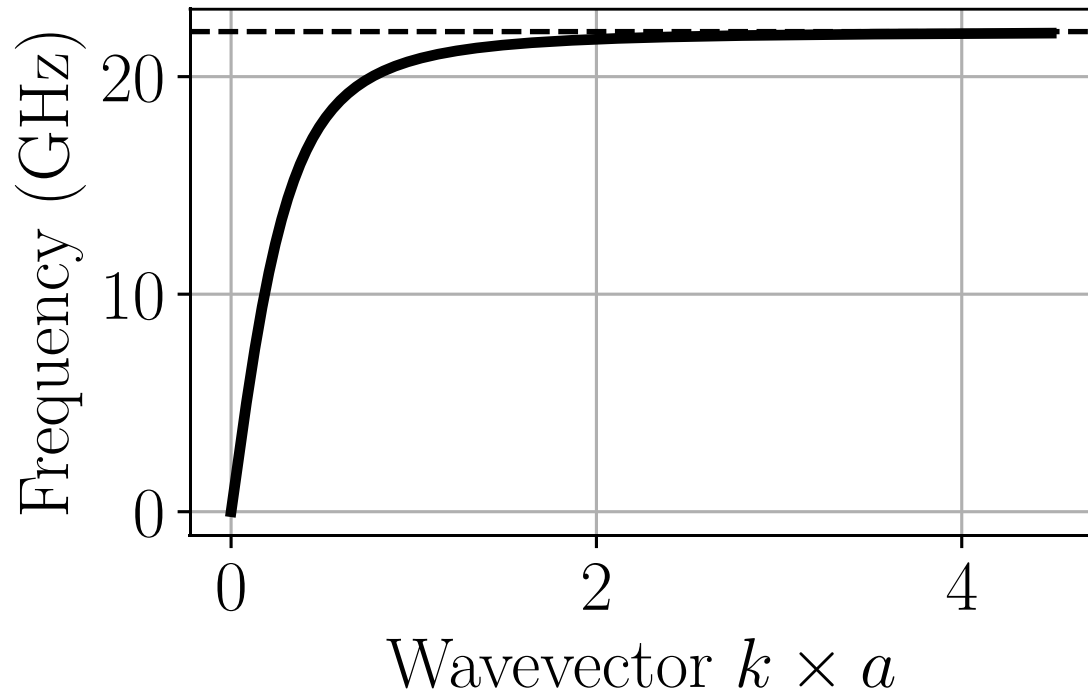


S. E. Rasmussen *et al.*, PRX Quantum, 2021



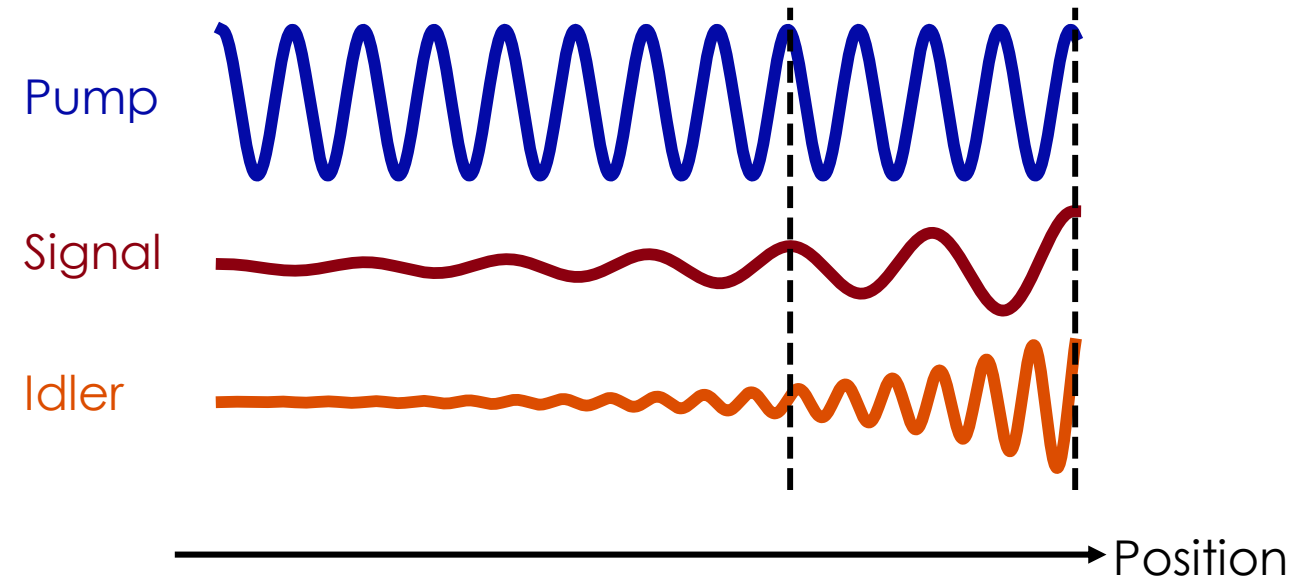
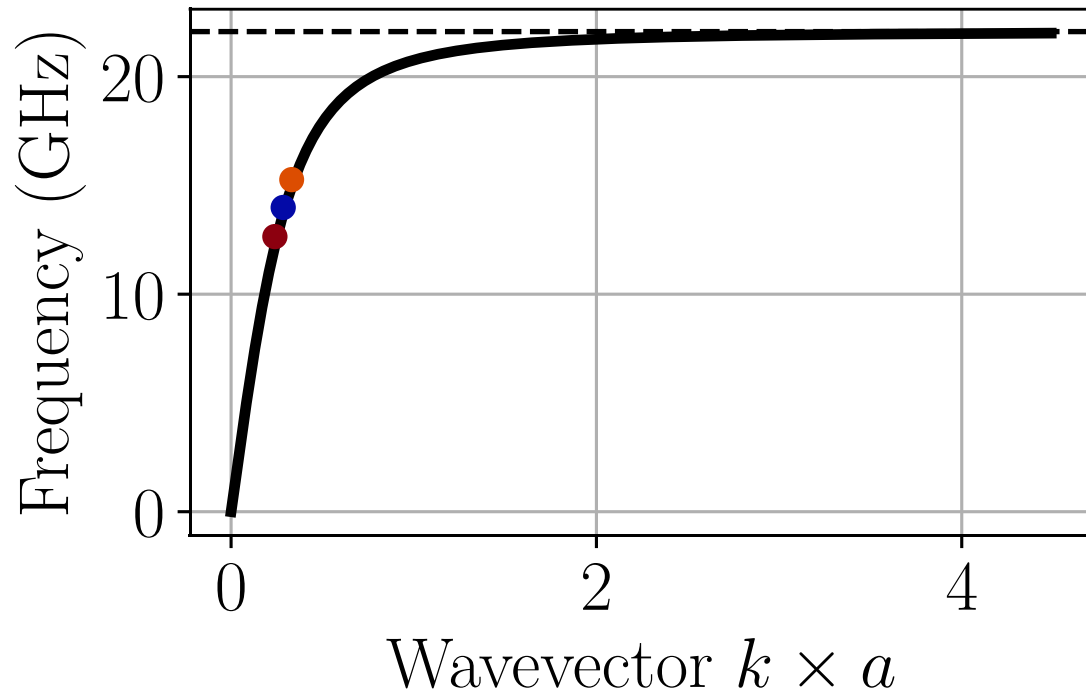
# Gain limitation: phase mismatch

4-wave mixing:  $\Delta k_{lin} = 2k_p - k_s - k_i$



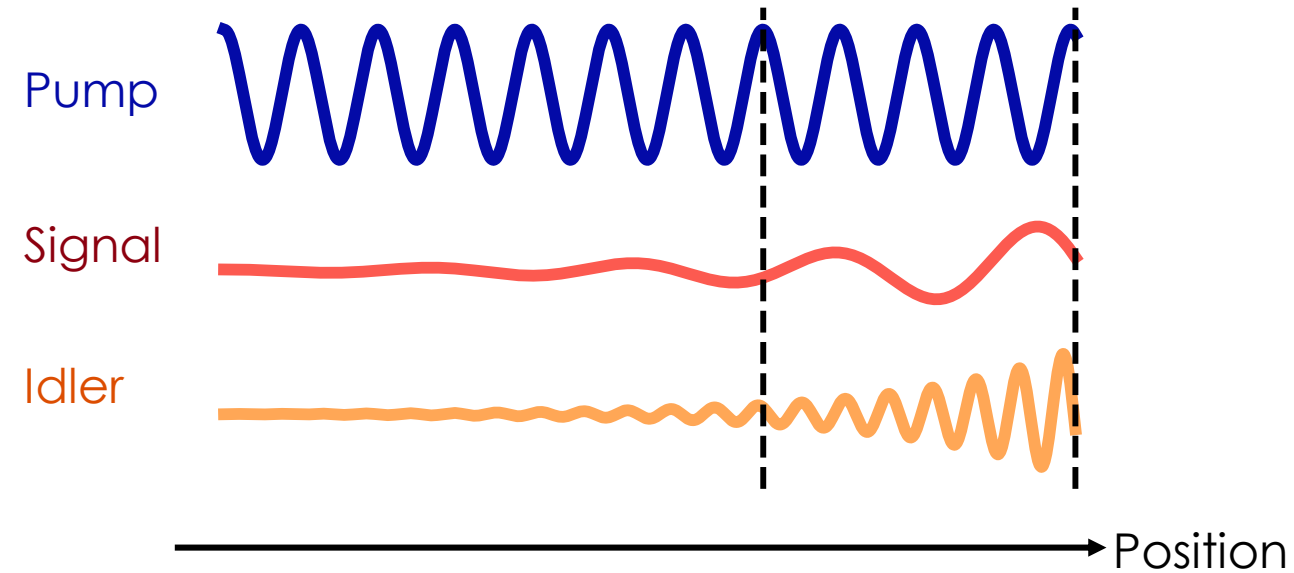
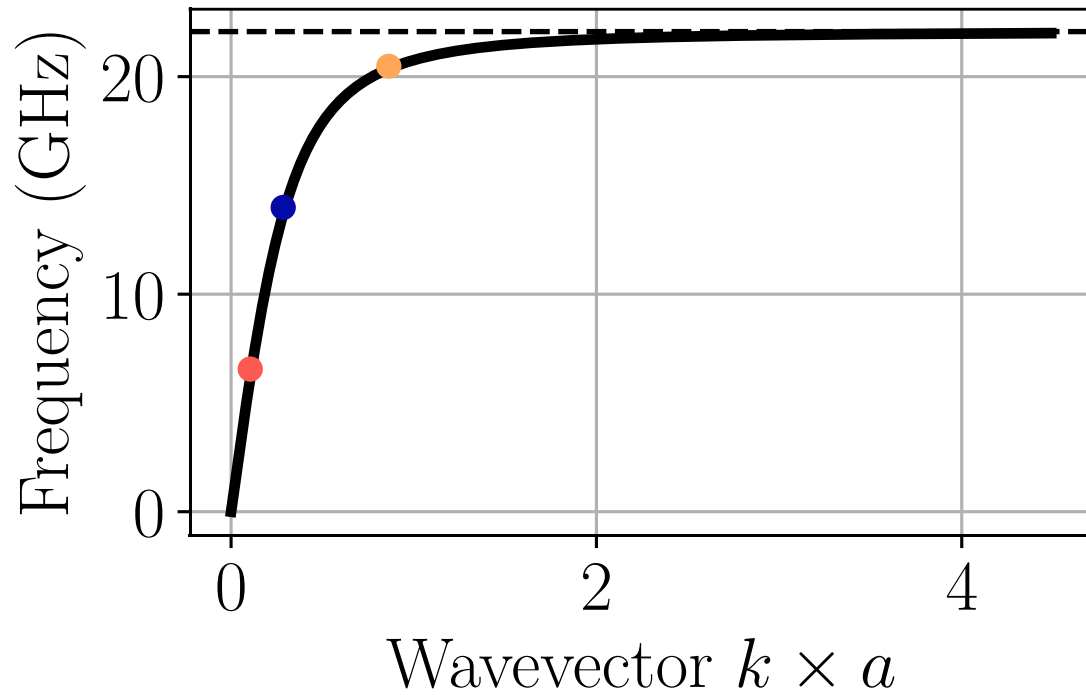
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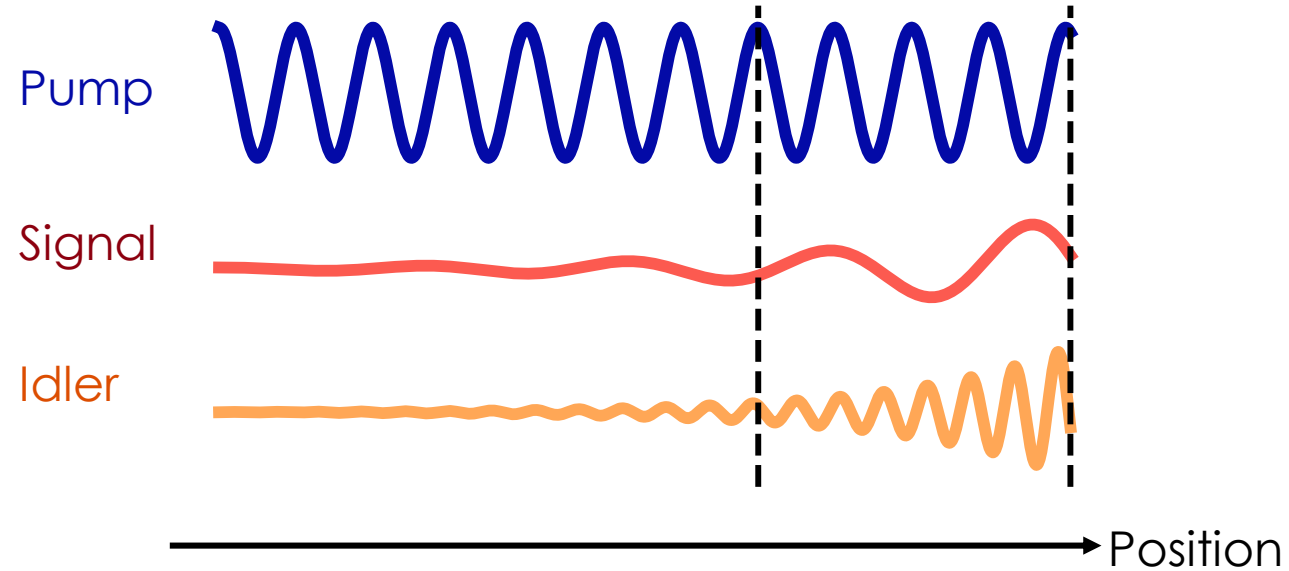
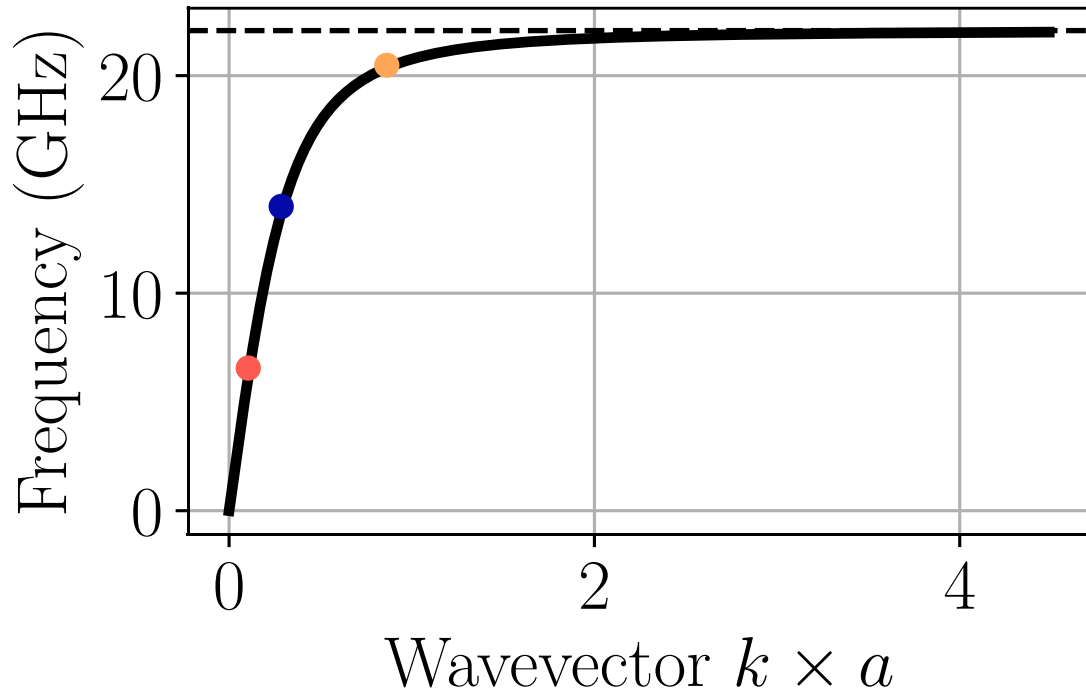
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# Gain limitation: phase mismatch

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Kerr effect: power dependent momentum

$$\Delta k_{nl} = 2\alpha_{pp} - \underbrace{\alpha_{sp} - \alpha_{ip}}_{\neq 0} \neq 0$$

SPM: self-Kerr  $\neq$  XPM: cross-Kerr

$$\Delta k = \Delta k_{lin} + \Delta k_{nl}$$

Rule of thumb:  $\Delta k \lesssim 1/L$

O. Yaakobi et al., Phys. Rev. B, 87, 144301 (2013)

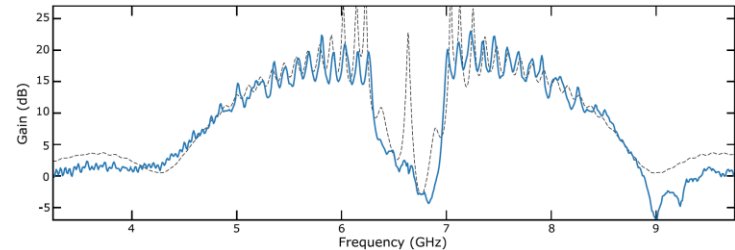
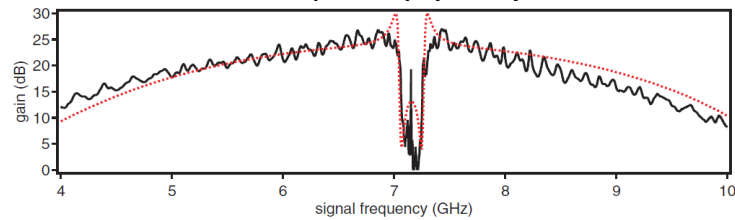
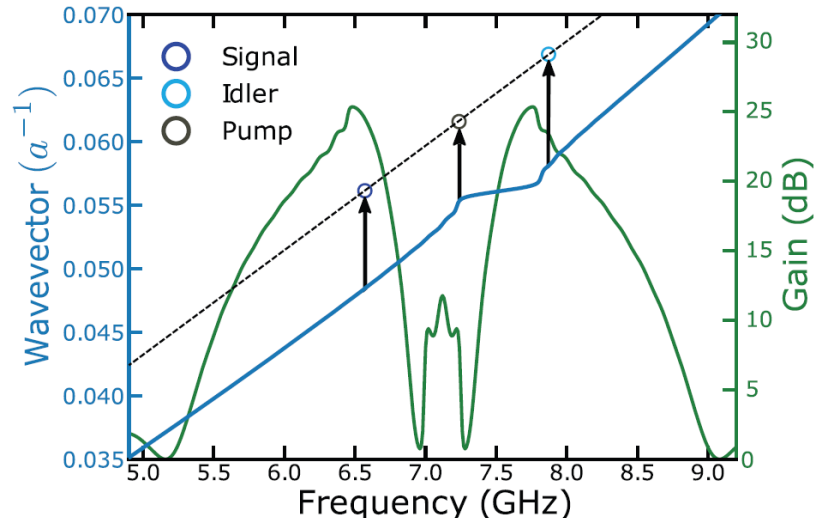
K. O'Brien et al., Phys. Rev. Lett., 113, 157001 (2014)

# Phase matching: implementation

## Dispersion engineering

C. S. Macklin et al., *Science*, 350, 6258 (2015)

L. Planat et al., *Phys. Rev. X*, 10, 021021 (2020)

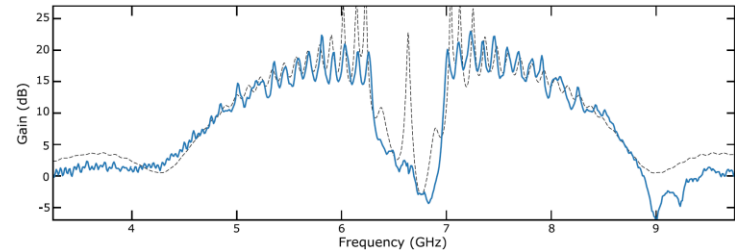
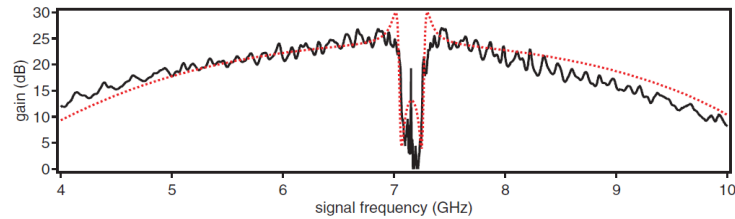
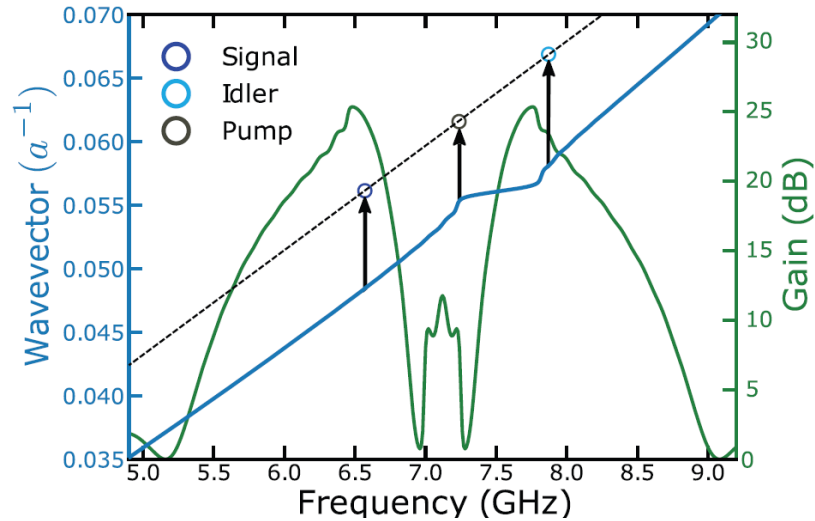


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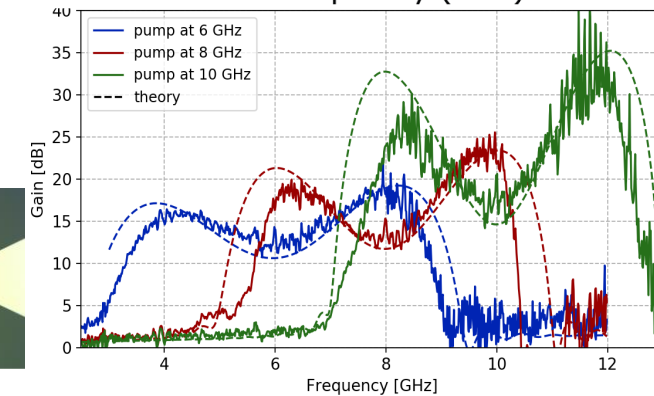
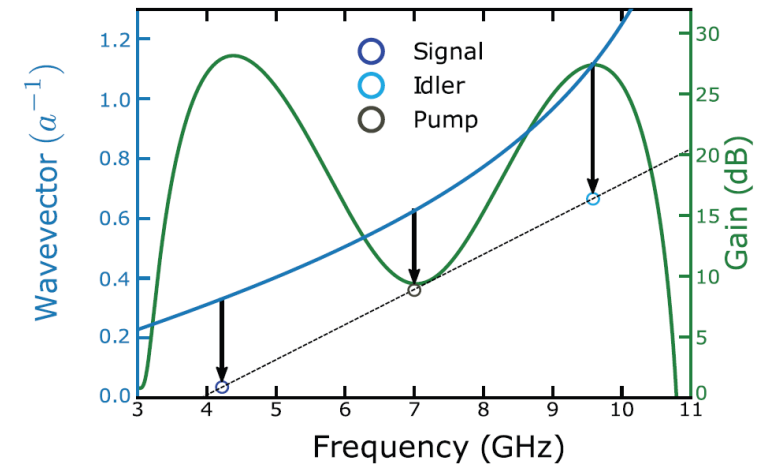
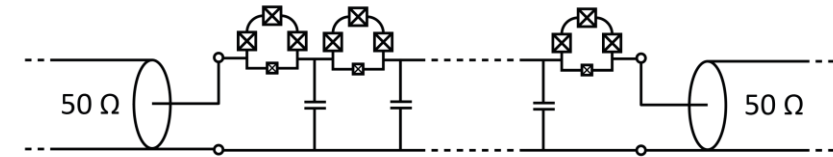
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## Kerr non-linearity engineering

A. Ranadive et al., *Nature Comm.*, 13, 1737 (2022)





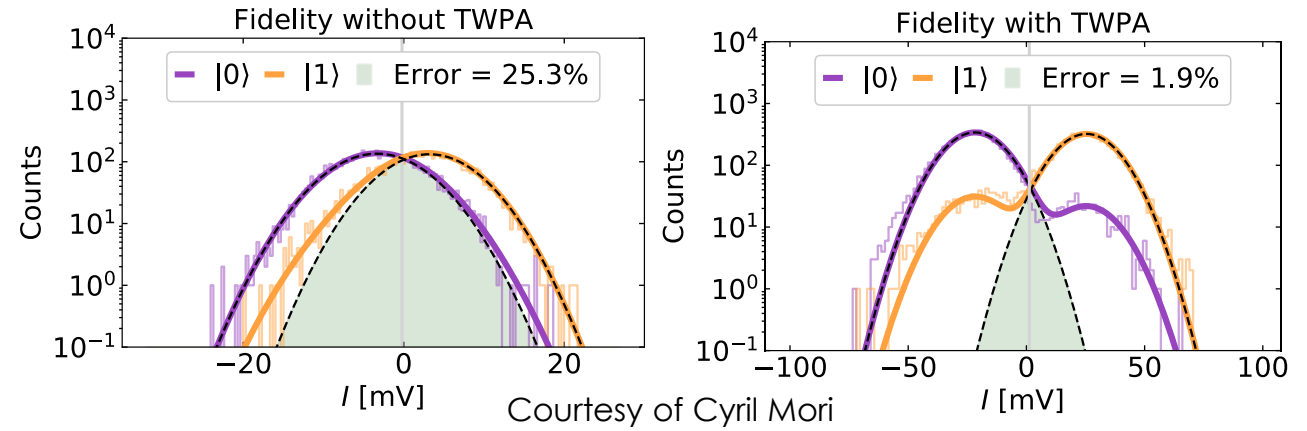
# TWPA Applications

- Superconducting qubit readout
- Multiplexed readout

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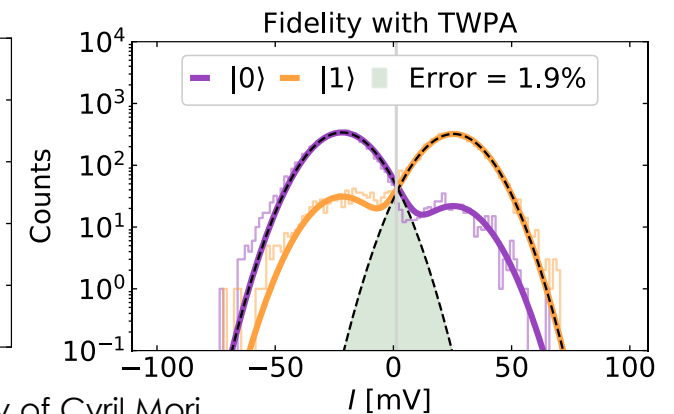
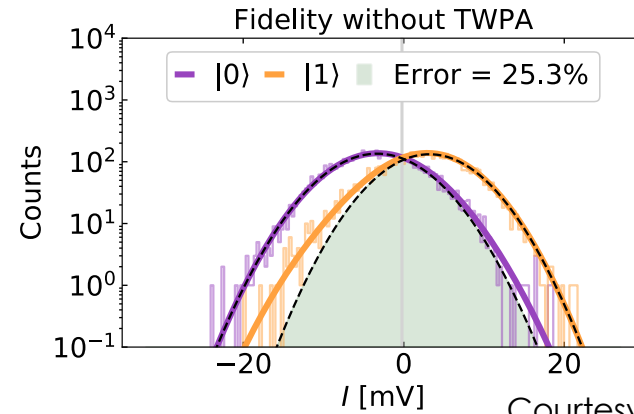
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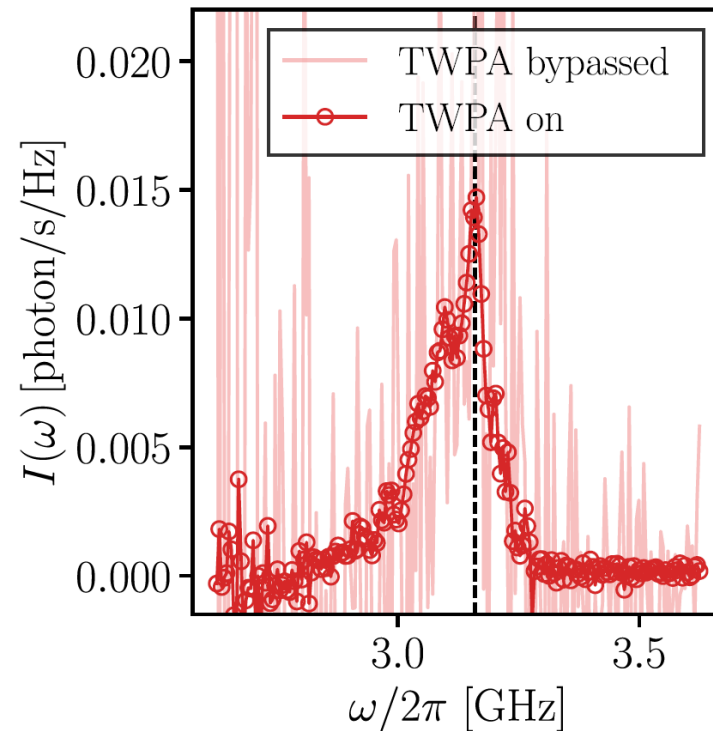
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- Broadband photo-detection

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Courtesy of Cyril Mori



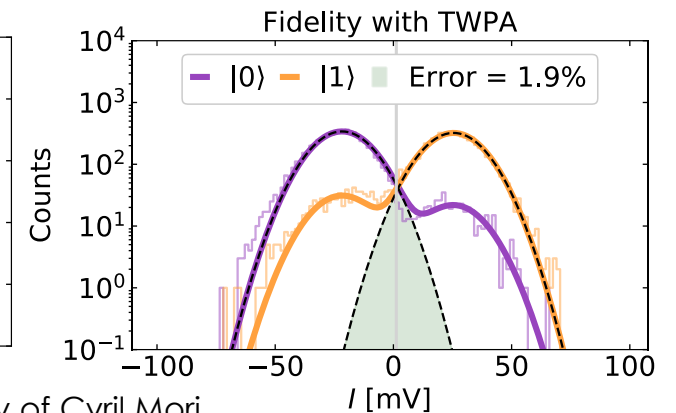
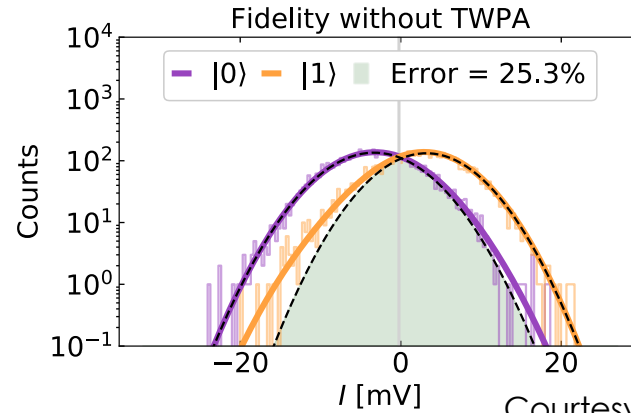
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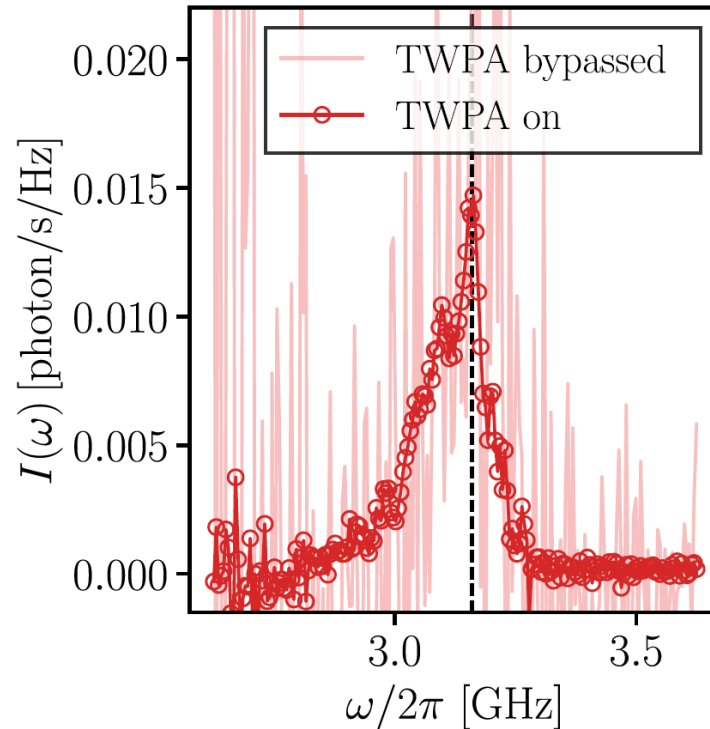
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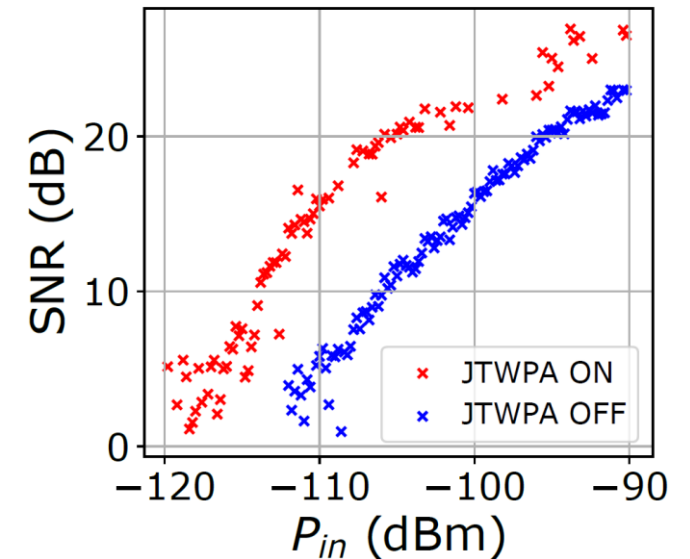
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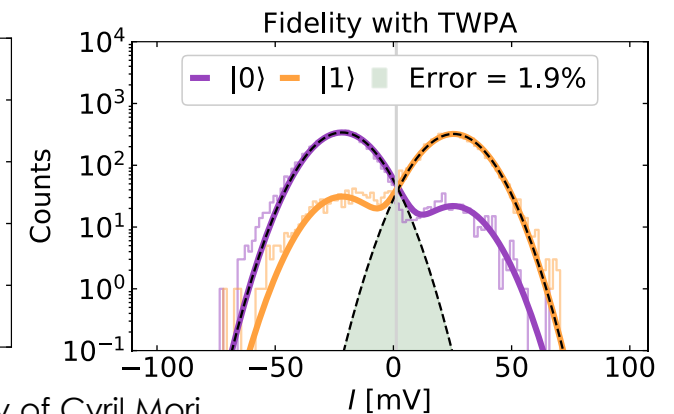
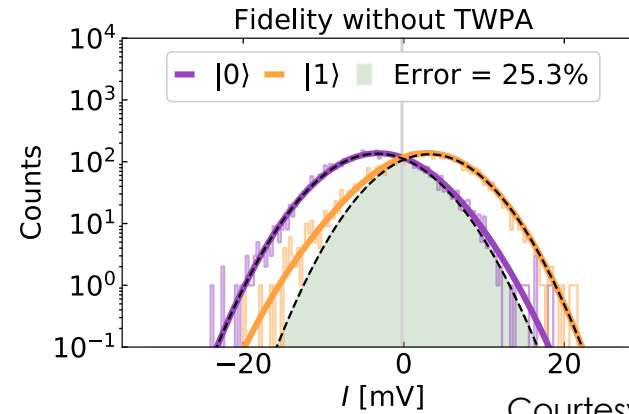
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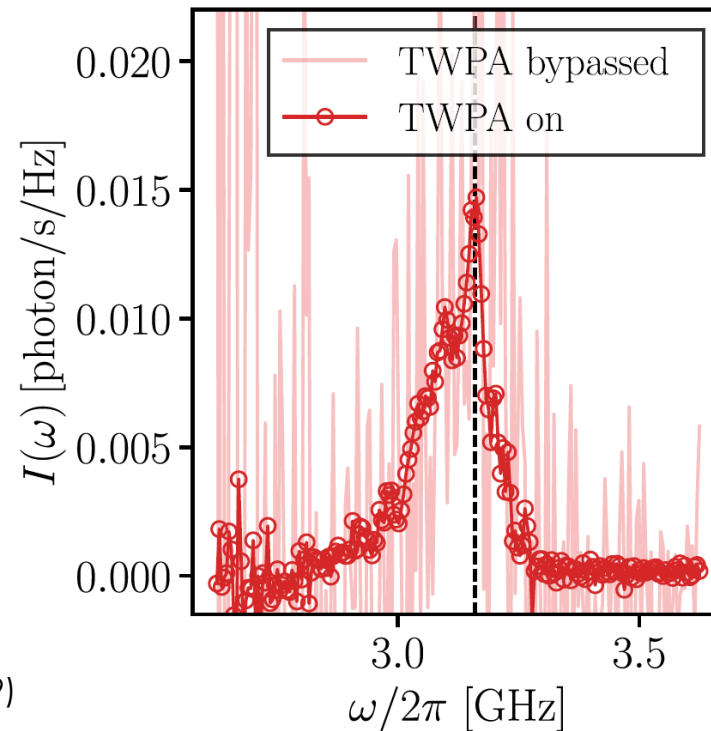
D. Fraudet et al., arXiv:2405.00411v1 (2024)

- High energy physics

R. Di Vora et al., Phys. Rev. D, 108, 062005 (2023)

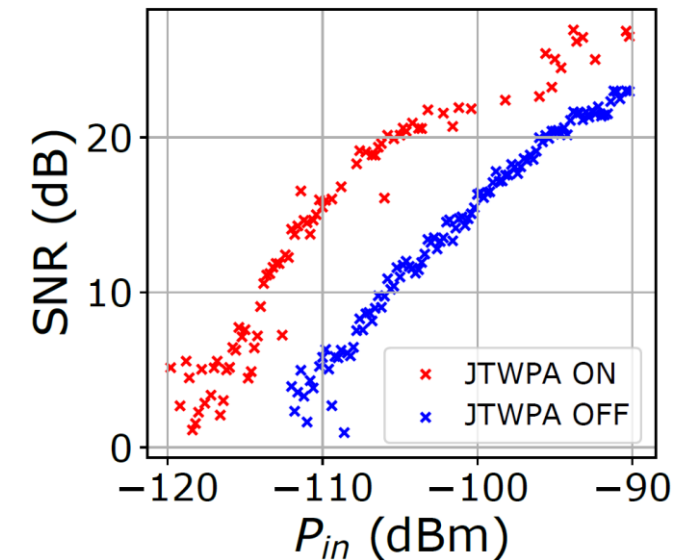
- MKID readout

N. Zobrist et al., Appl. Phys. Lett. 115, 042601 (2019)

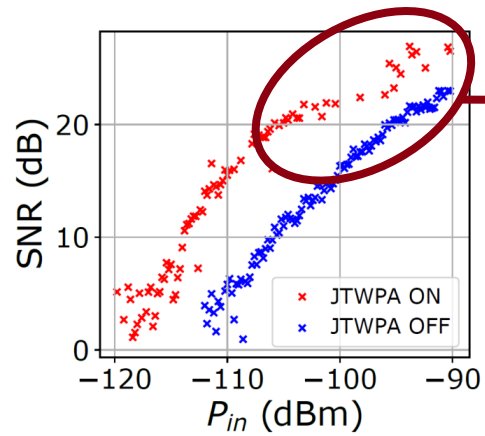


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# Why studying saturation in TWPAs?

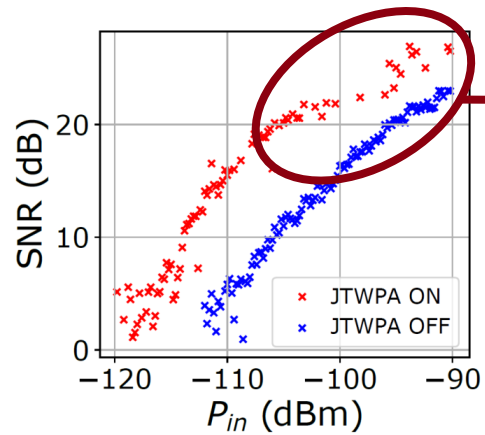


SNR saturates @ high  $P_{in}$

Can this be mitigated?

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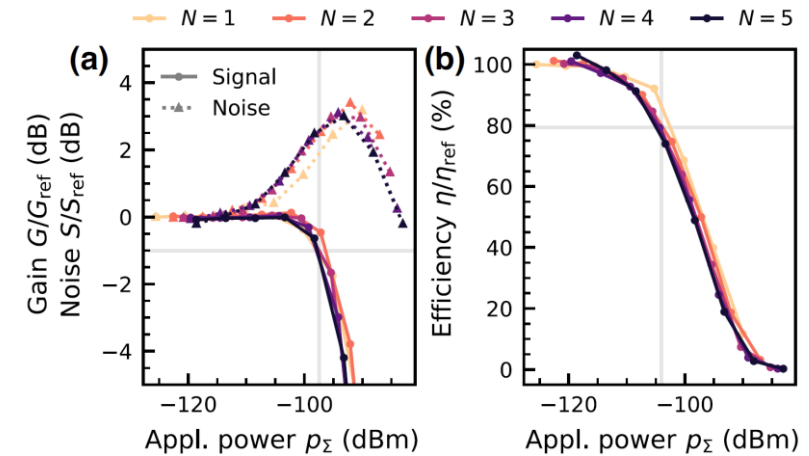


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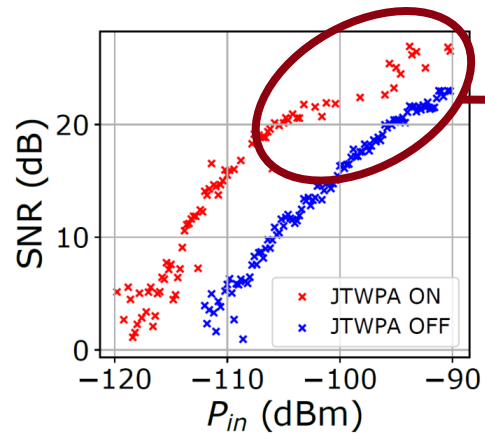
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## Intermodulation products generation



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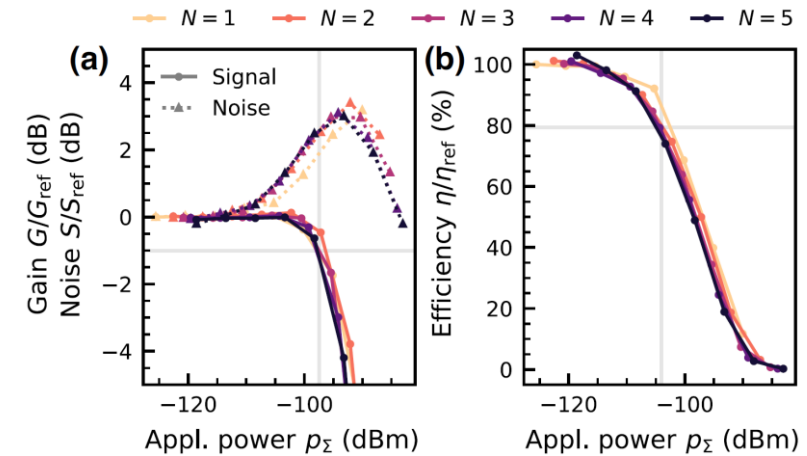


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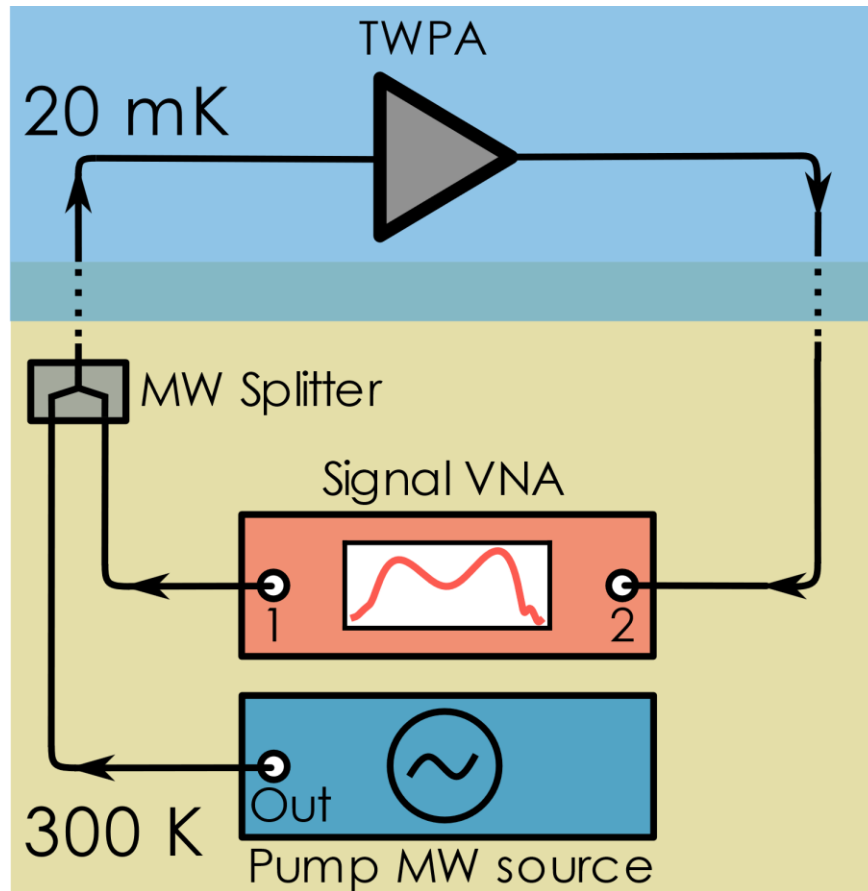
Understanding the causes of compression:

- Never studied in superconducting TWPAs
- Mitigate quantum efficiency reduction
- Understand implications on applications (readout signals correlations...)



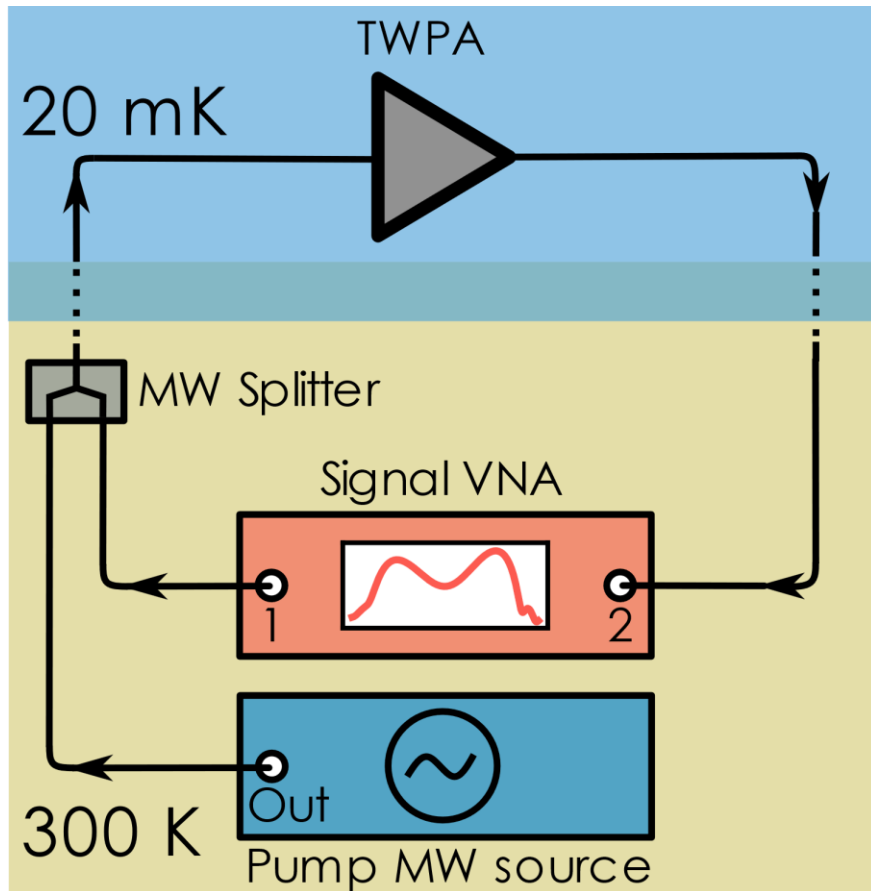
# Experimental study of saturation

Usual setup

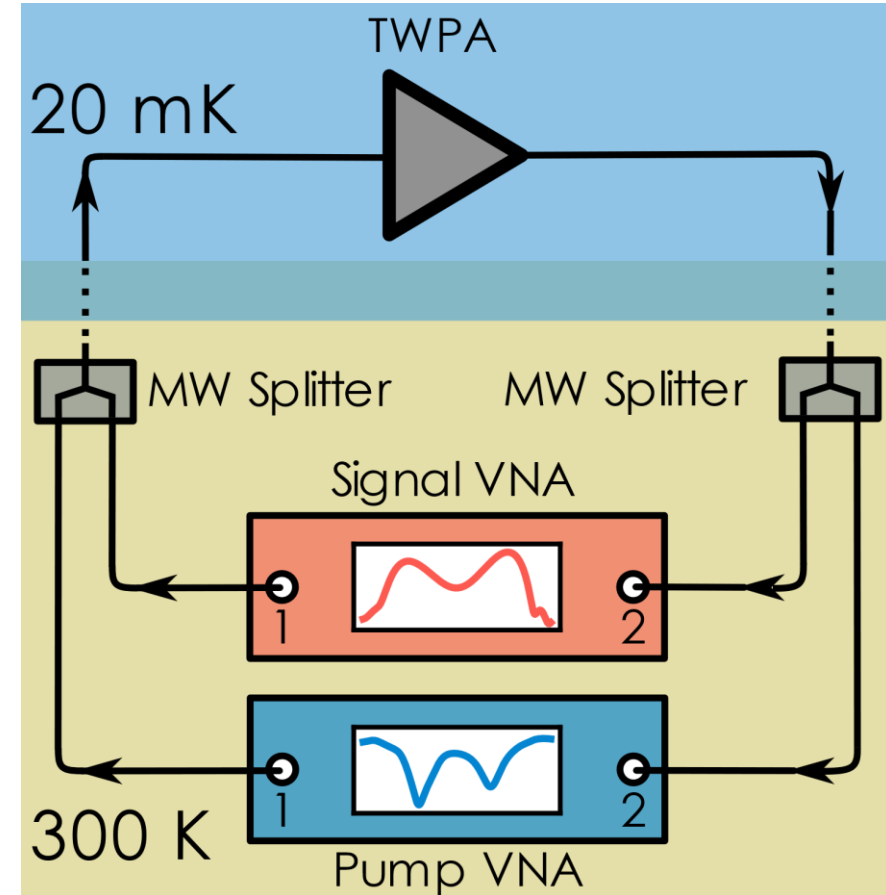


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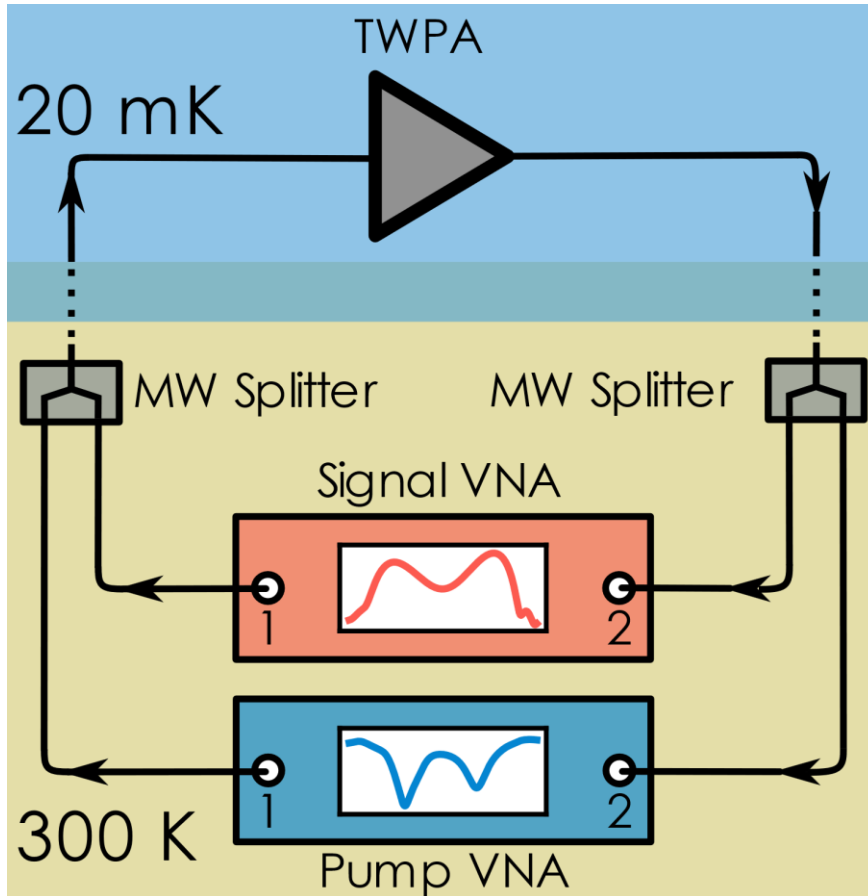


New setup

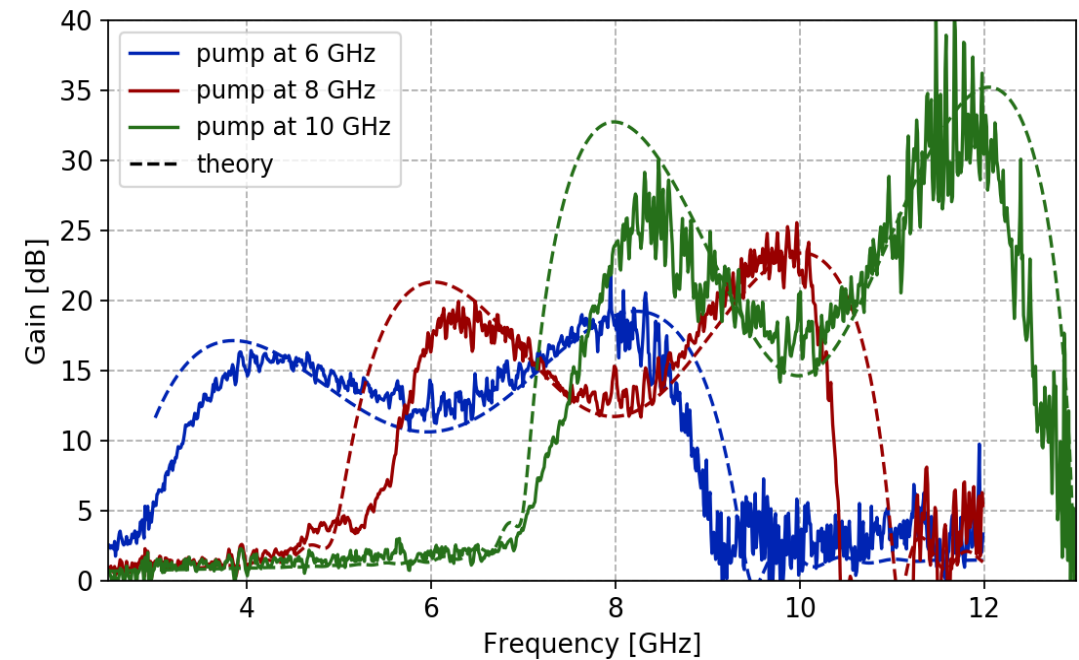
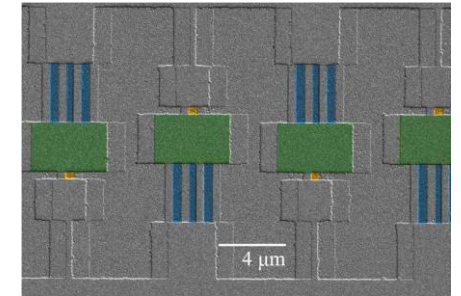
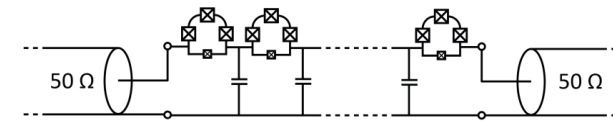


Monitoring signal & pump complex transmission

# Experimental study of saturation

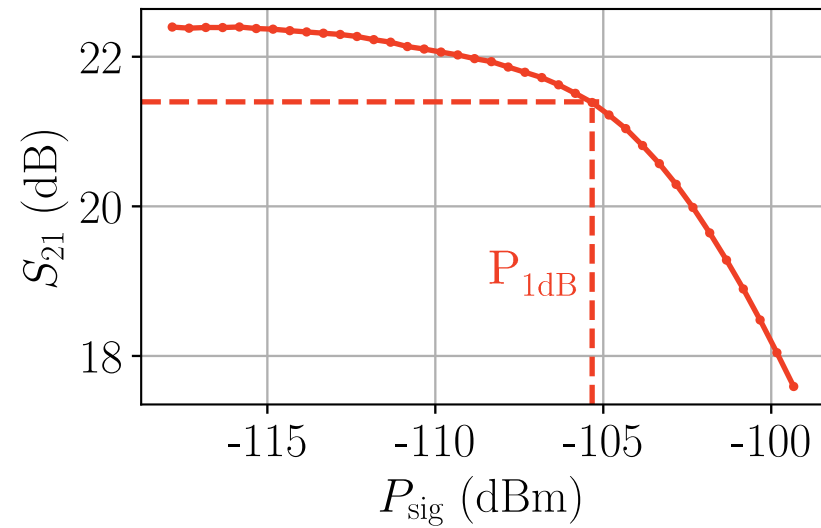


The device: SNAIL TWPA @  $\frac{1}{2}$  flux quantum

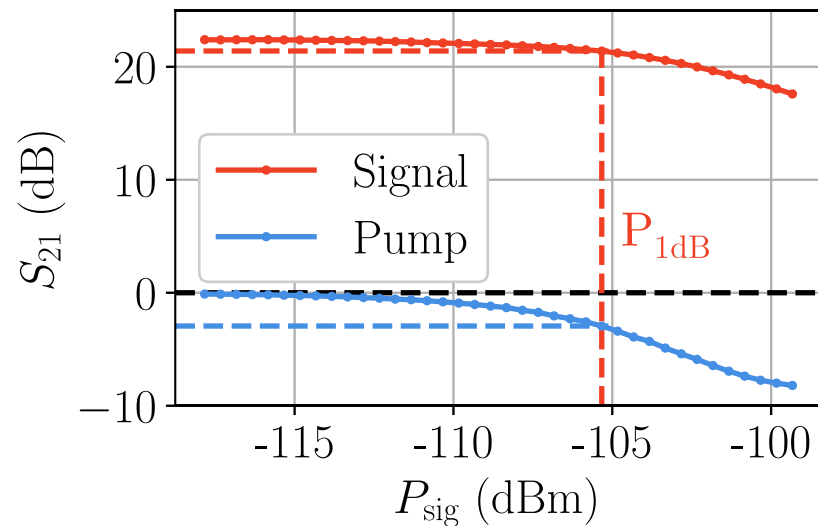
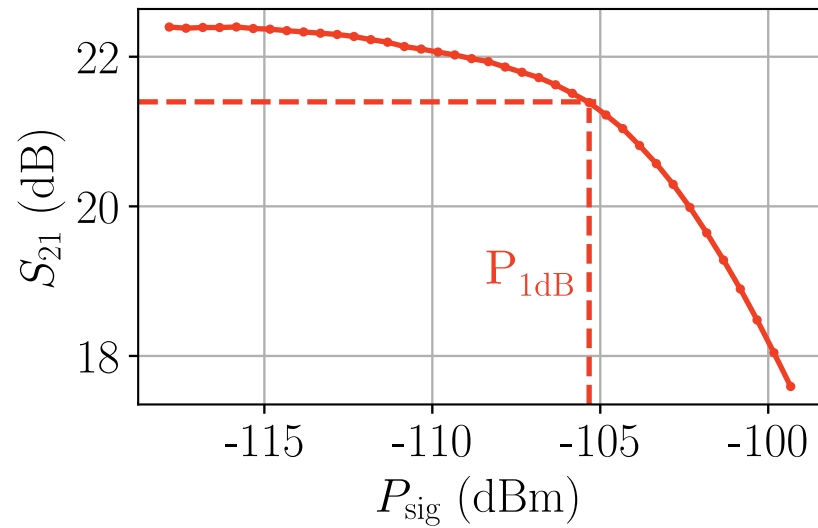


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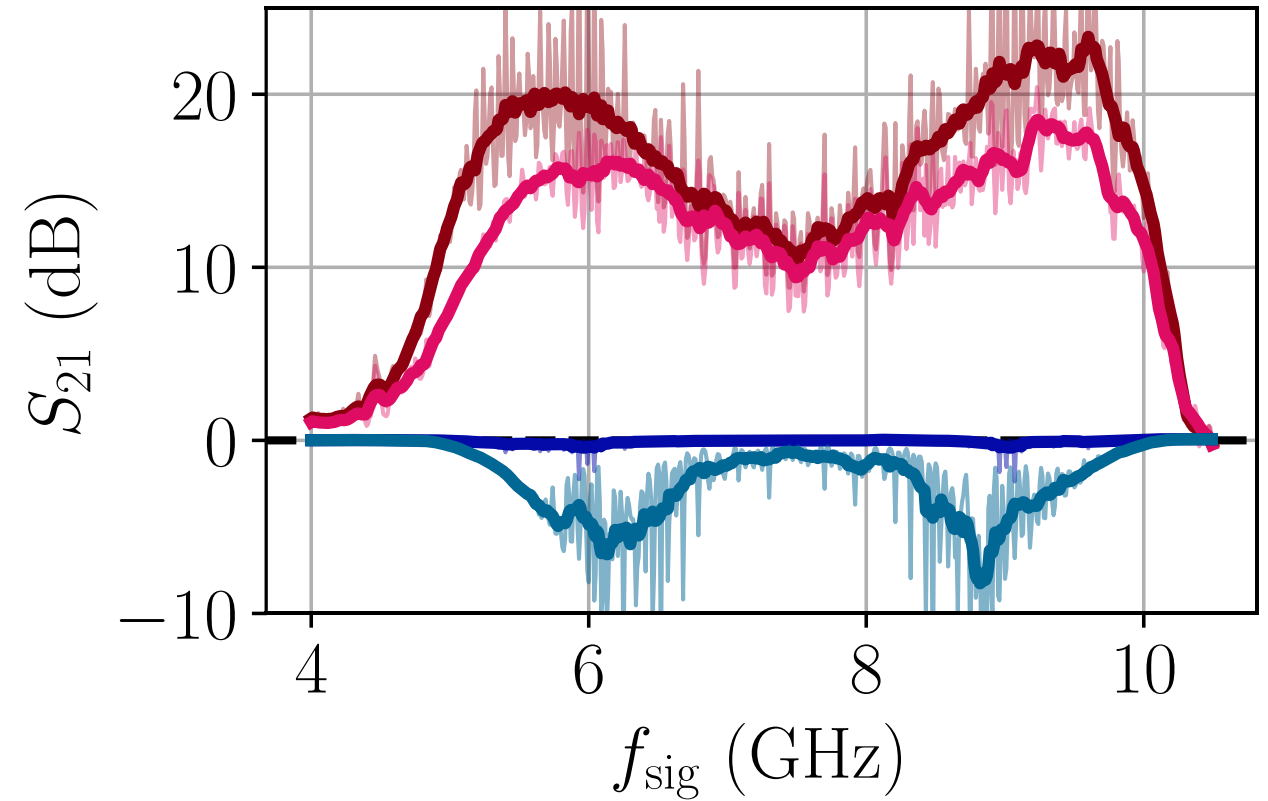
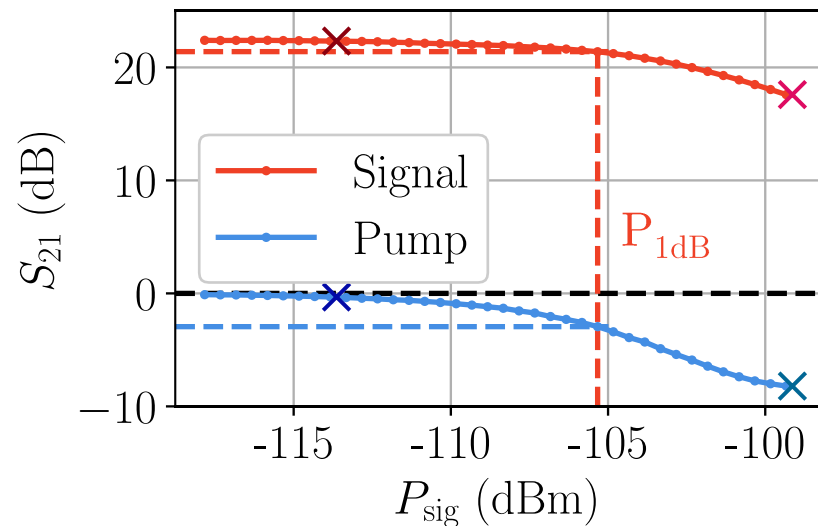
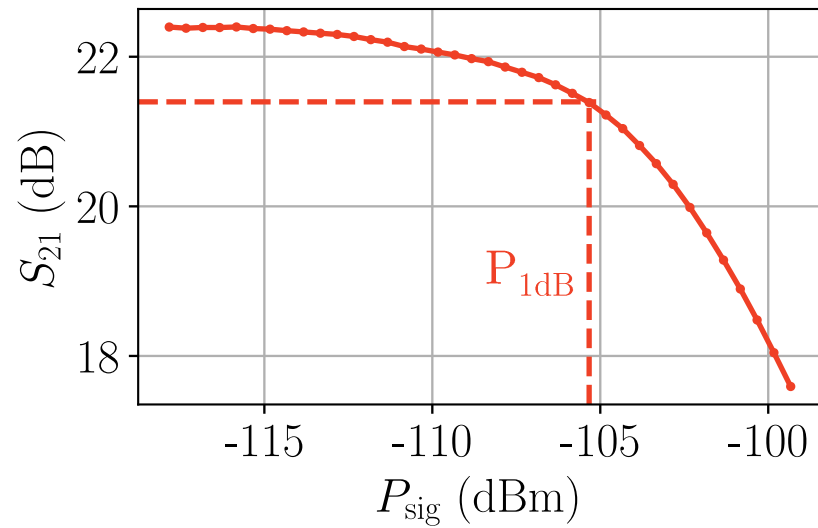
# 1-dB compression: Definition



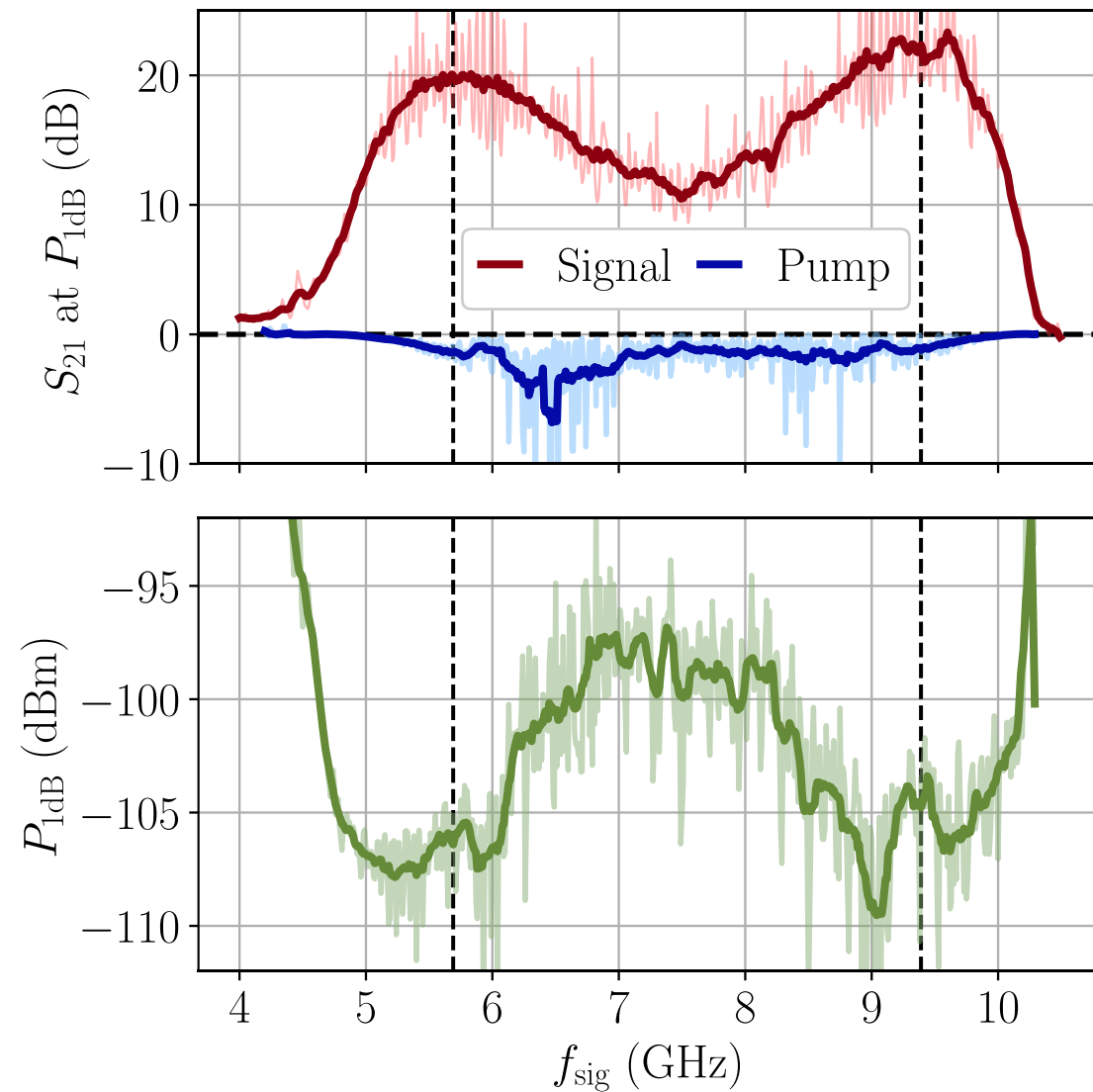
# Compression: what happens to the pump?



# Signal frequency influence

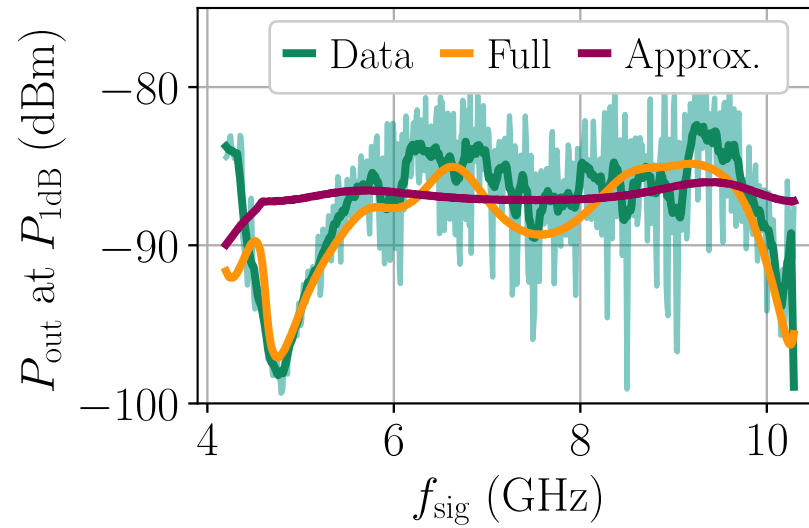
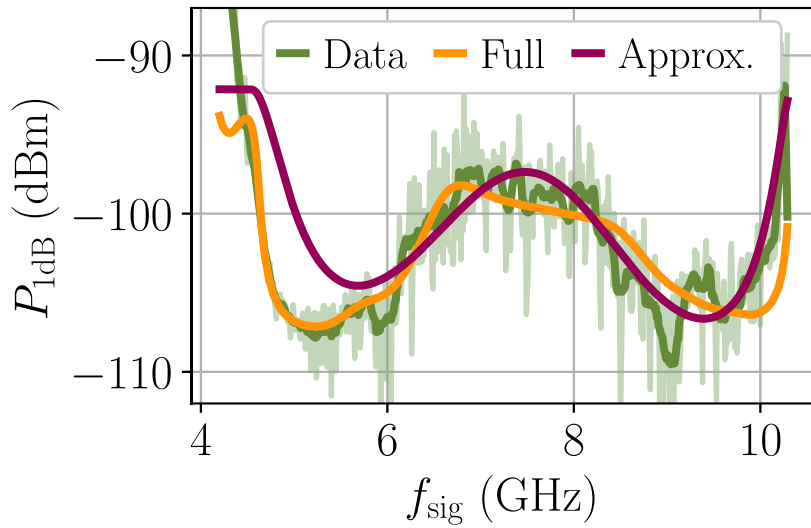


# Compression versus signal frequency

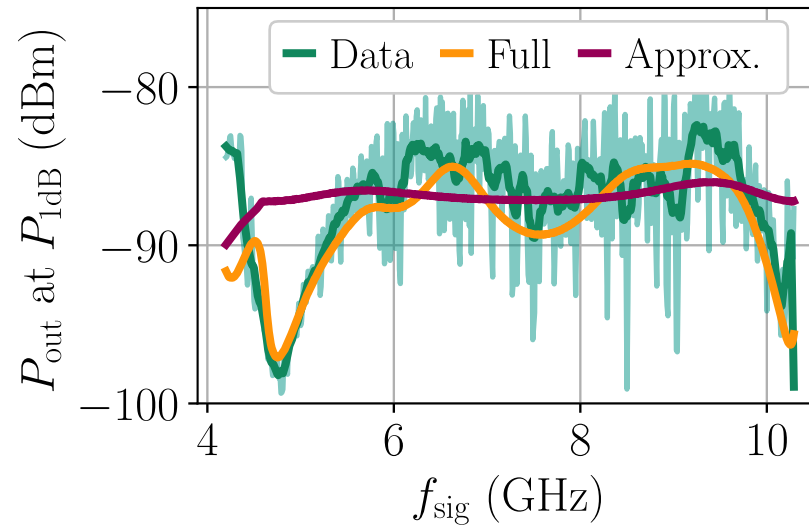
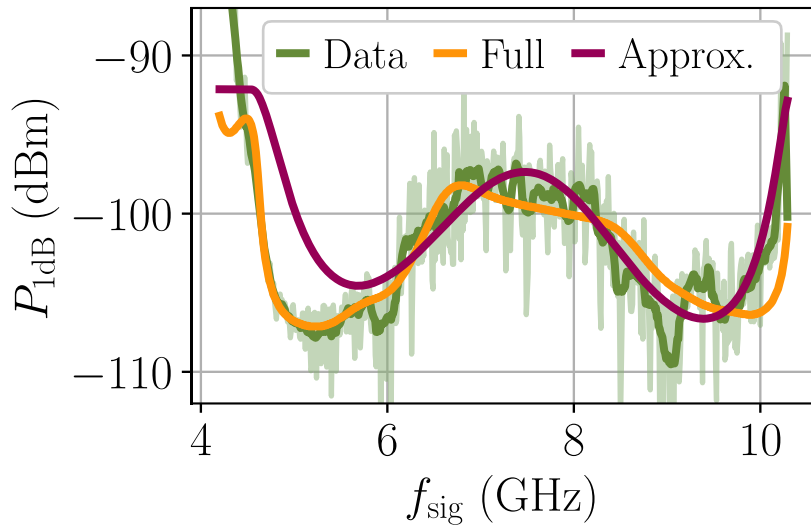




# Modeling the data



# Modeling the data



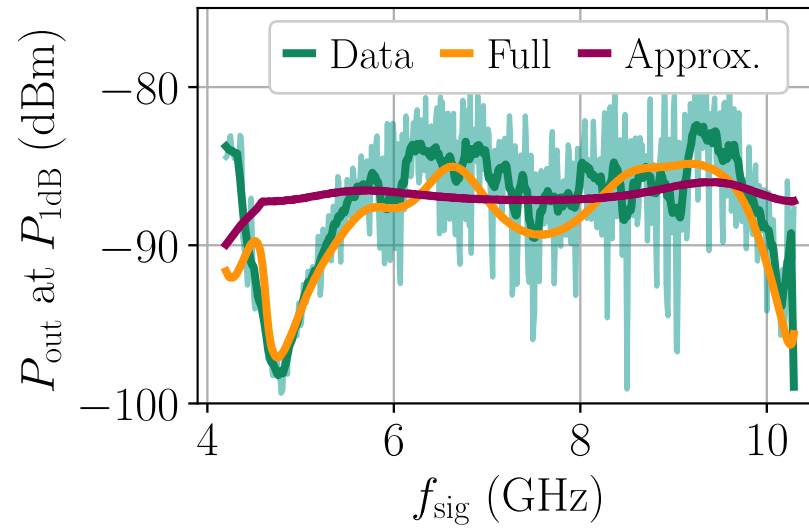
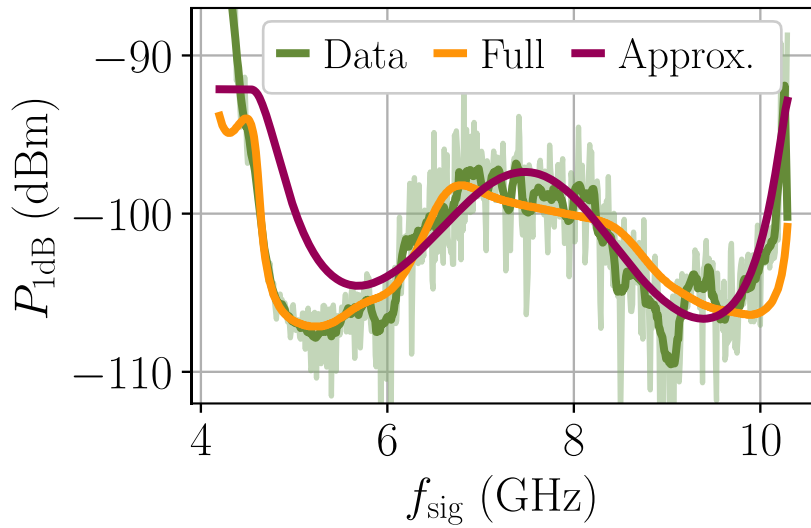
**Approx:** 
$$G(P_{sig}) = \frac{G_{lin}}{1 + 2G_{lin}P_{sig}/P_{pump}}$$

Assumes perfect energy exchange

K. O'Brien et al., Phys. Rev. Lett., 113, 157001 (2014)

P. Kylemark et al., J. Light. Technol., 24, 9 (2006)

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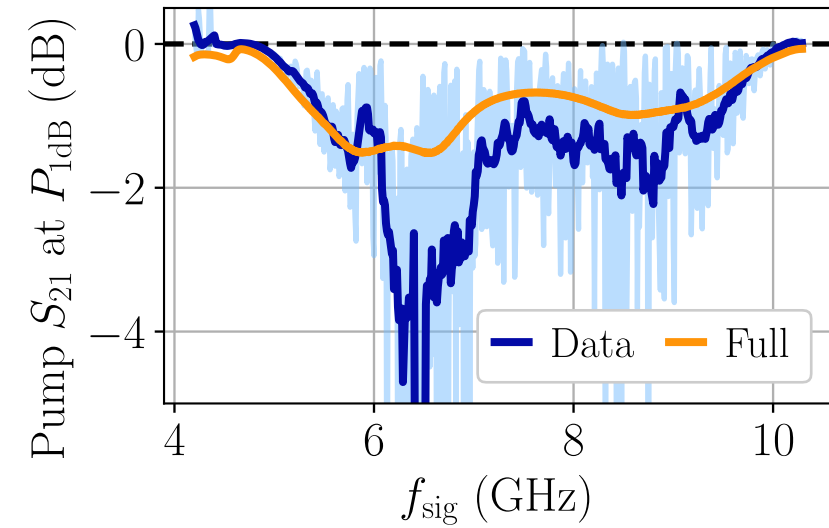
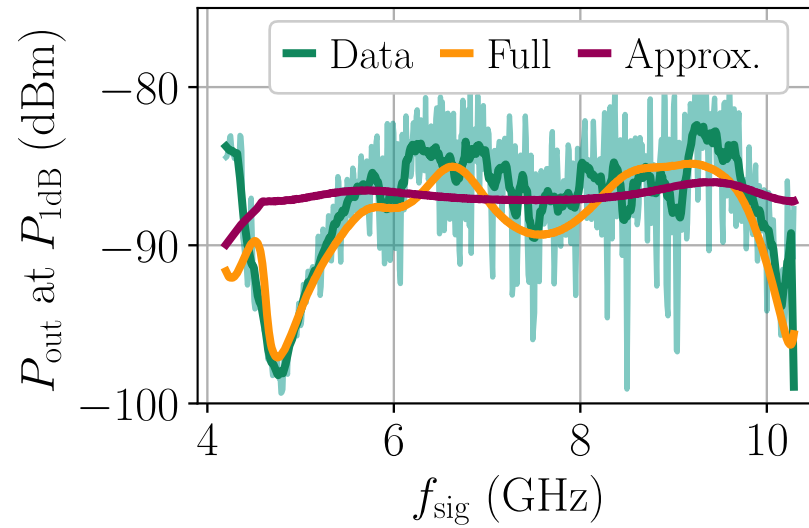
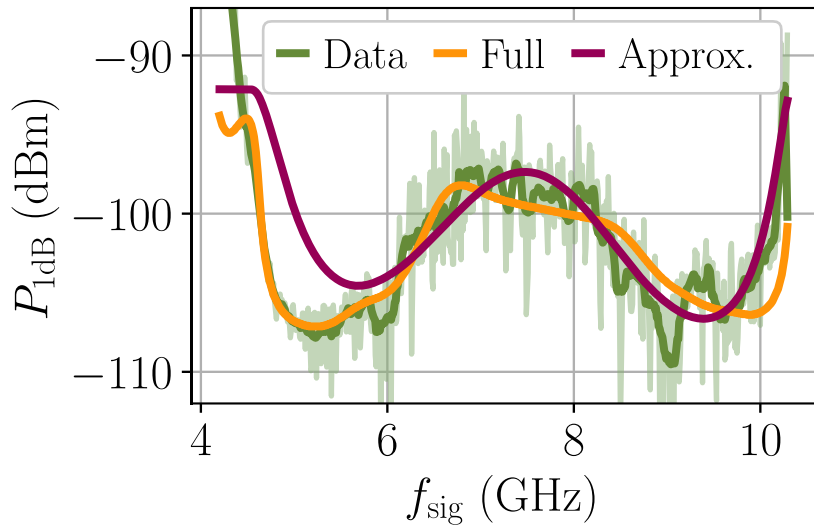
P. Kylemark et al., J. Light. Technol., 24, 9 (2006)

**Full model:** CME including losses & exchange of energy with pump tone

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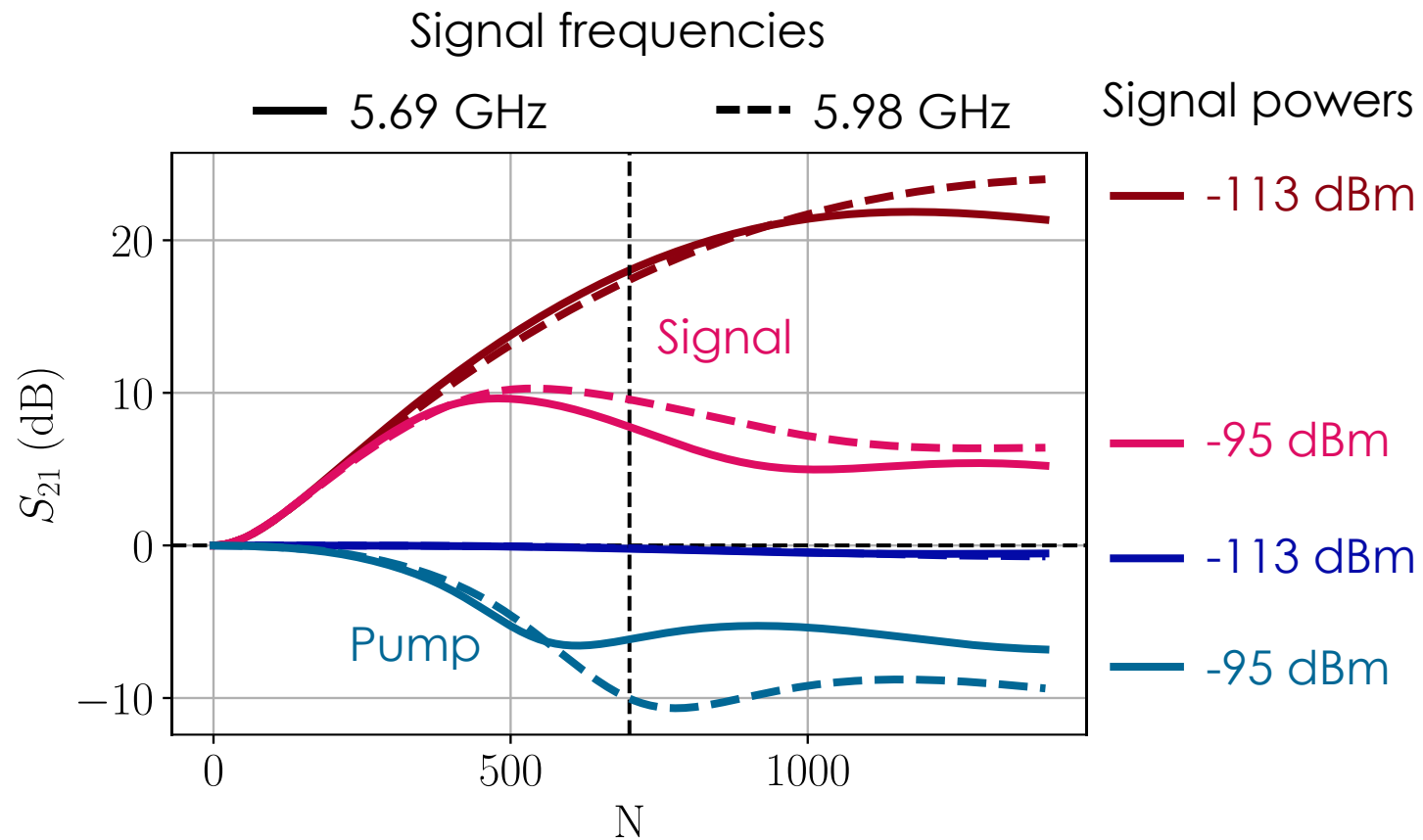
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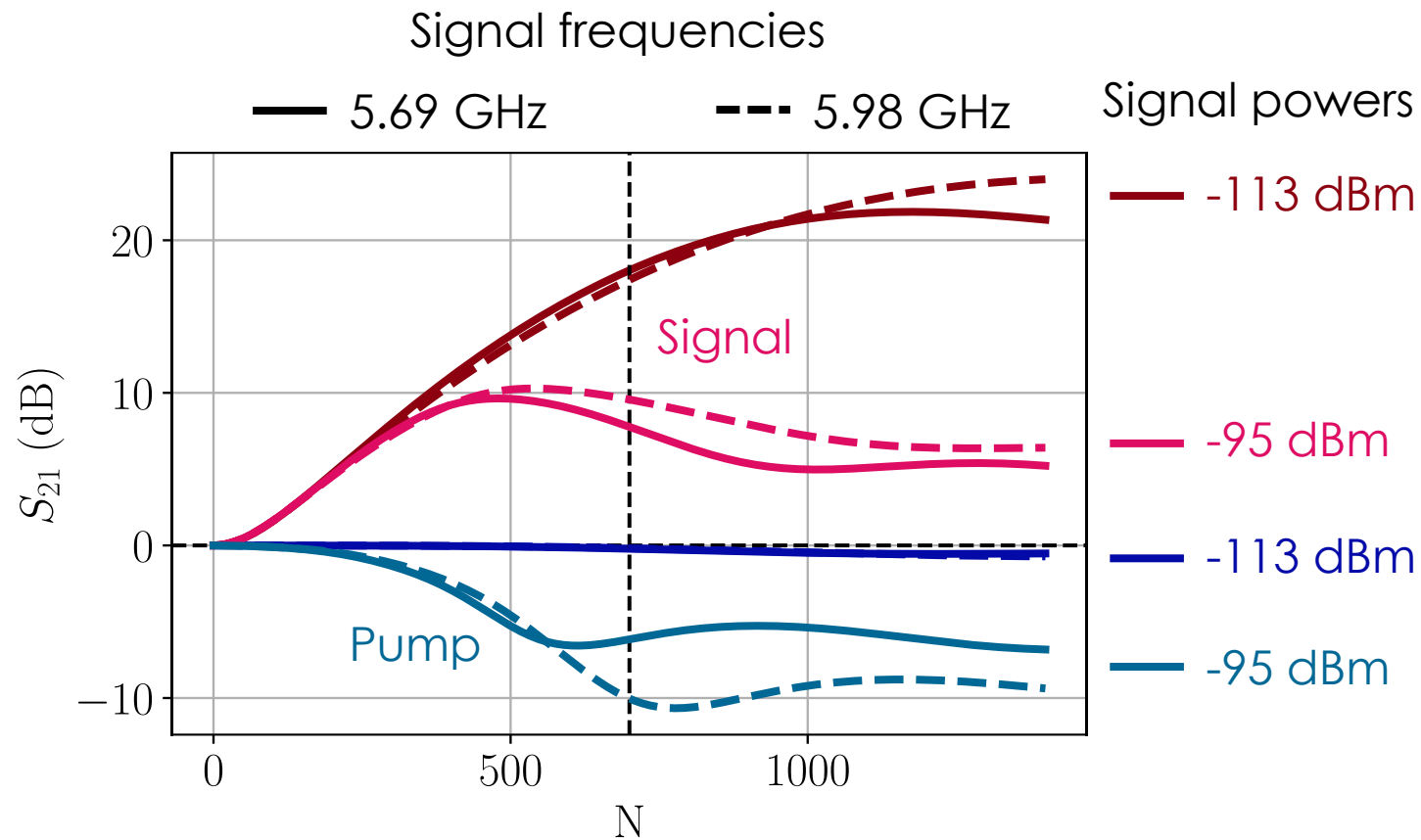
K. O'Brien et al., Phys. Rev. Lett., 113, 157001 (2014)

O. Yaakobi et al., Phys. Rev. B, 87, 144301 (2013)

# What gives the pump depletion profile?



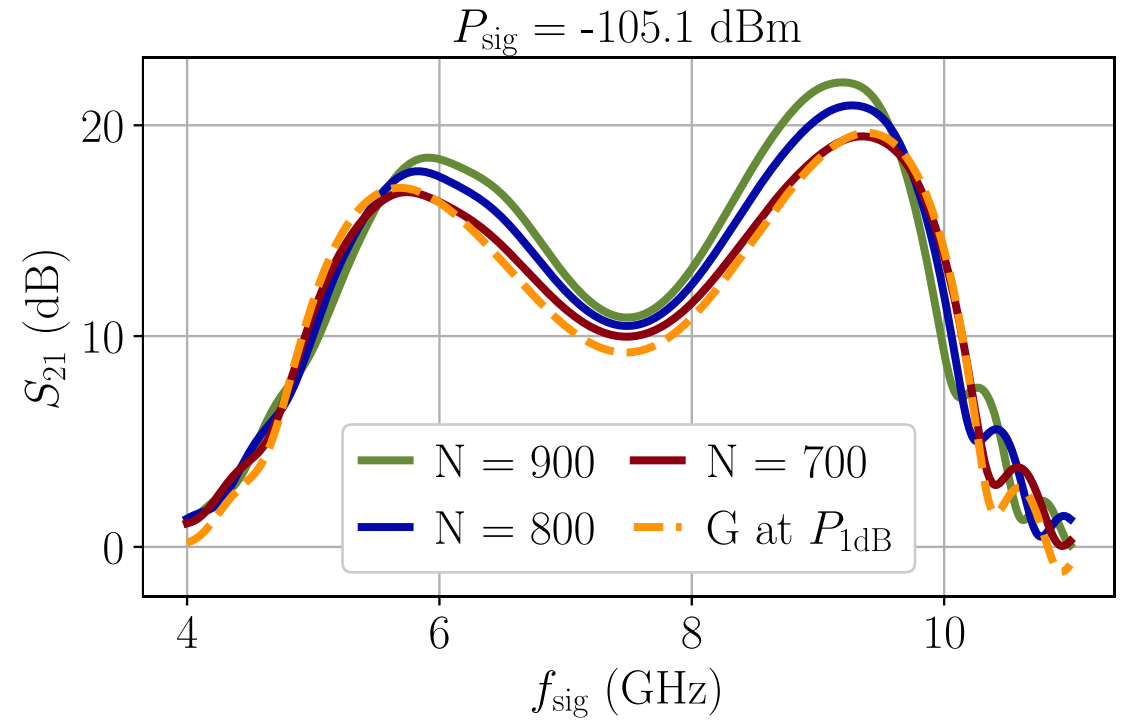
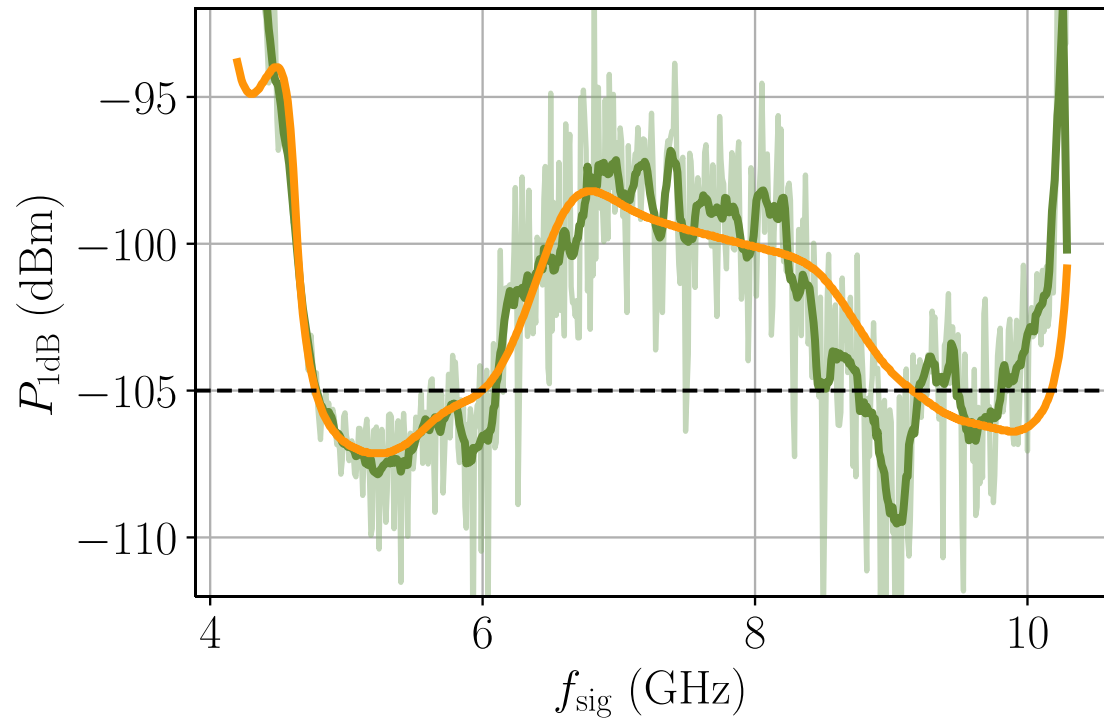
# What gives the pump depletion profile?



Pump transmission profile results from **different coherent lengths** and **conversion efficiencies**

Not related to linear gain profile

# Compressed gain vs TWPA length

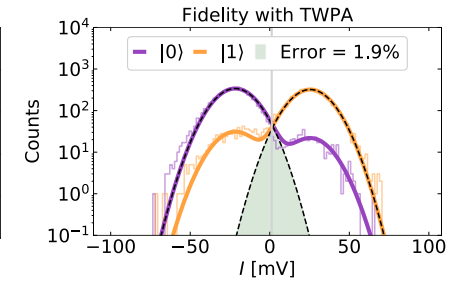
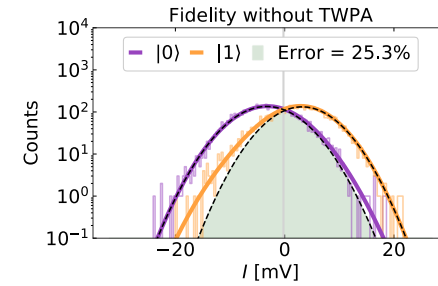
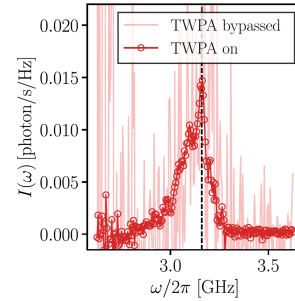


Mitigating compression by increasing device length



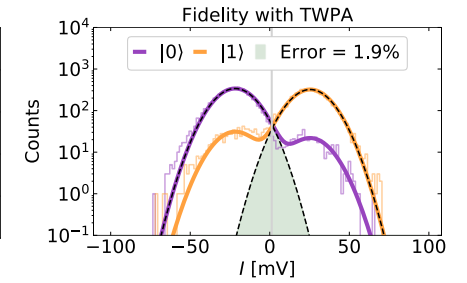
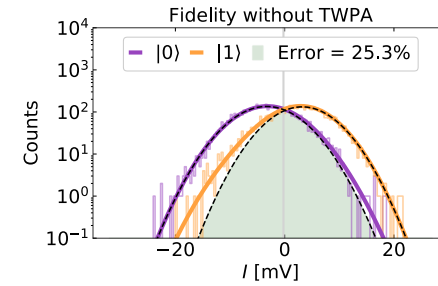
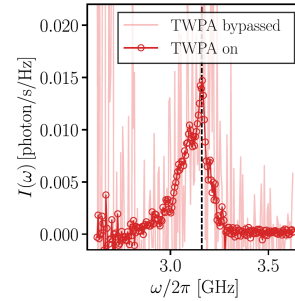
# Conclusion

- TWPAs can be useful for many applications

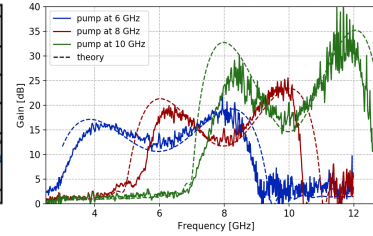
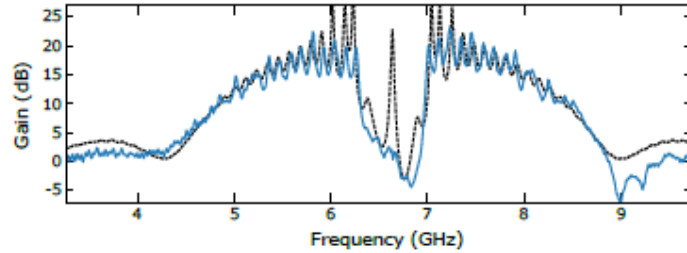


# Conclusion

- TWPAs can be useful for many applications



- Different flavours



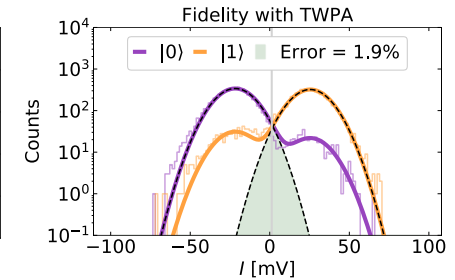
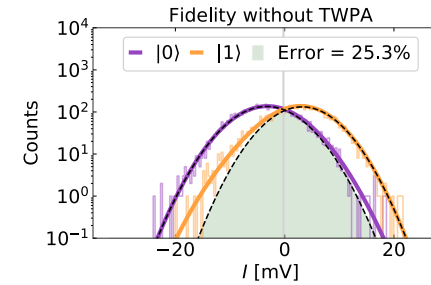
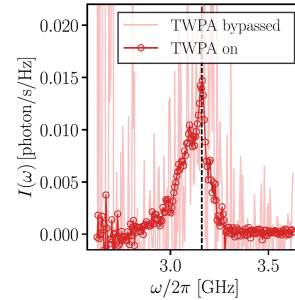
+ 3WM TWPAs

M. Malnou et al., PRX Quantum (2021)

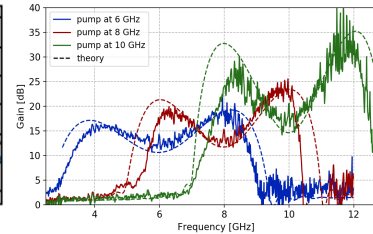
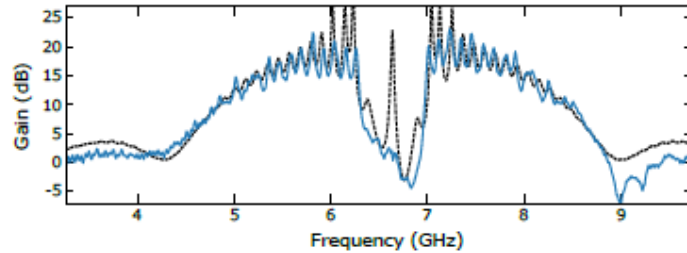
A. F. Roudsari et al., APL (2023)

# Conclusion

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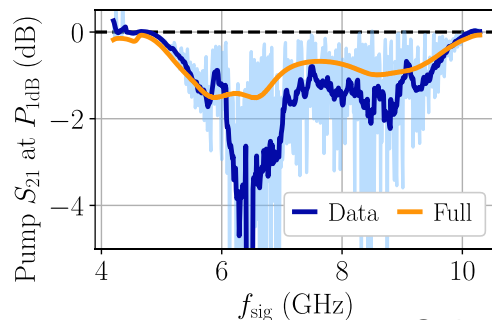
- Different flavours



+ 3Wm TWPAs

M. Malnou et al., PRX Quantum (2021)  
A. F. Roudsari et al., APL (2023)

- Interesting physics: understanding noise limitations, reciprocity, saturation
- Saturation: caused by pump depletion



G. Le Gal et al., To be published

- Mitigation strategies:

- Increase critical current
- Length engineering

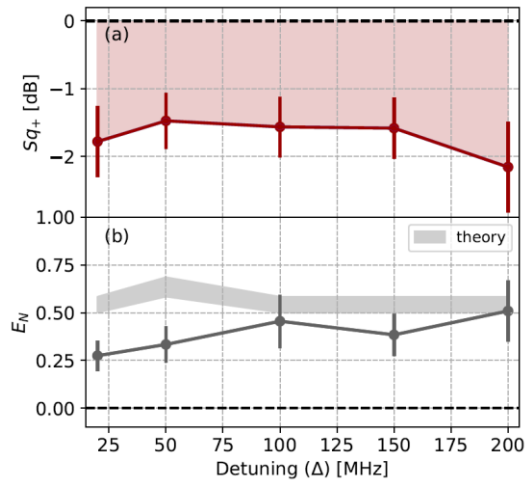
# Perspectives: understanding TWPA physics

Noise limitations

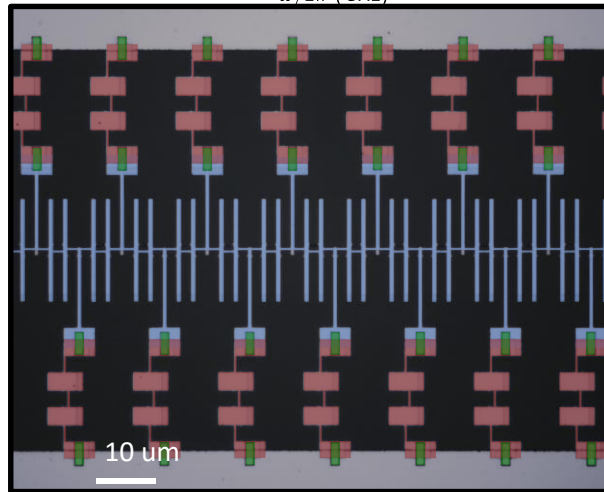
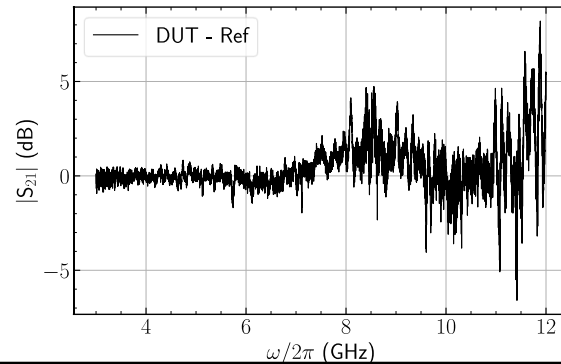
Reciprocity/directionality

Reducing losses

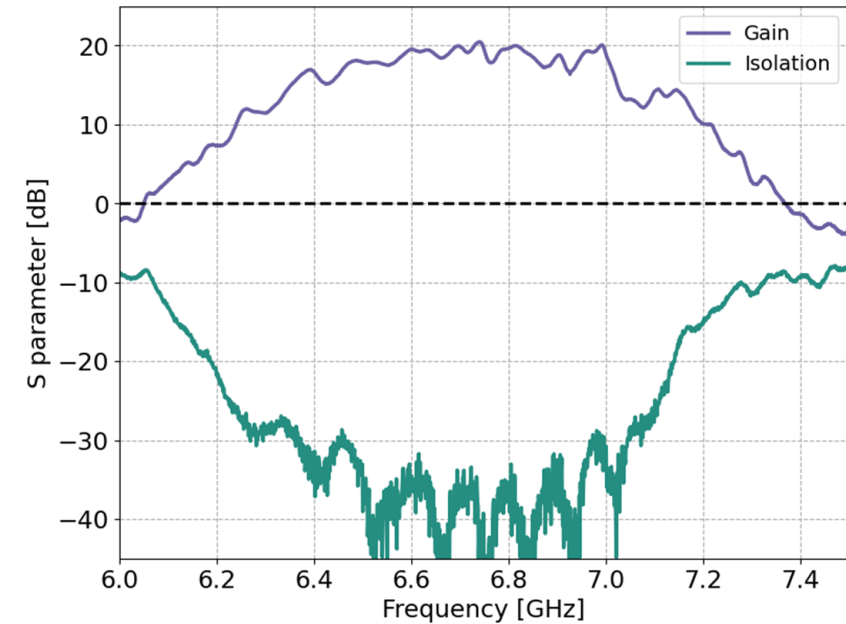
Squeezing



M. Esposito et al., Phys. Rev. Lett., 128, 153603 (2022)



Courtesy of Giulio Cappelli



Courtesy of Bekim Fazliji and Arpit Ranadive

A. Ranadive, B. Fazliji, et al., To be published



# The TWPA team



Nicolas  
Roch



Arpit  
Ranadive



Guilliam  
Butseraen



Gwenael  
Le Gal



Bekim  
Fazliji



Giulio  
Cappelli



Amaury  
Martin

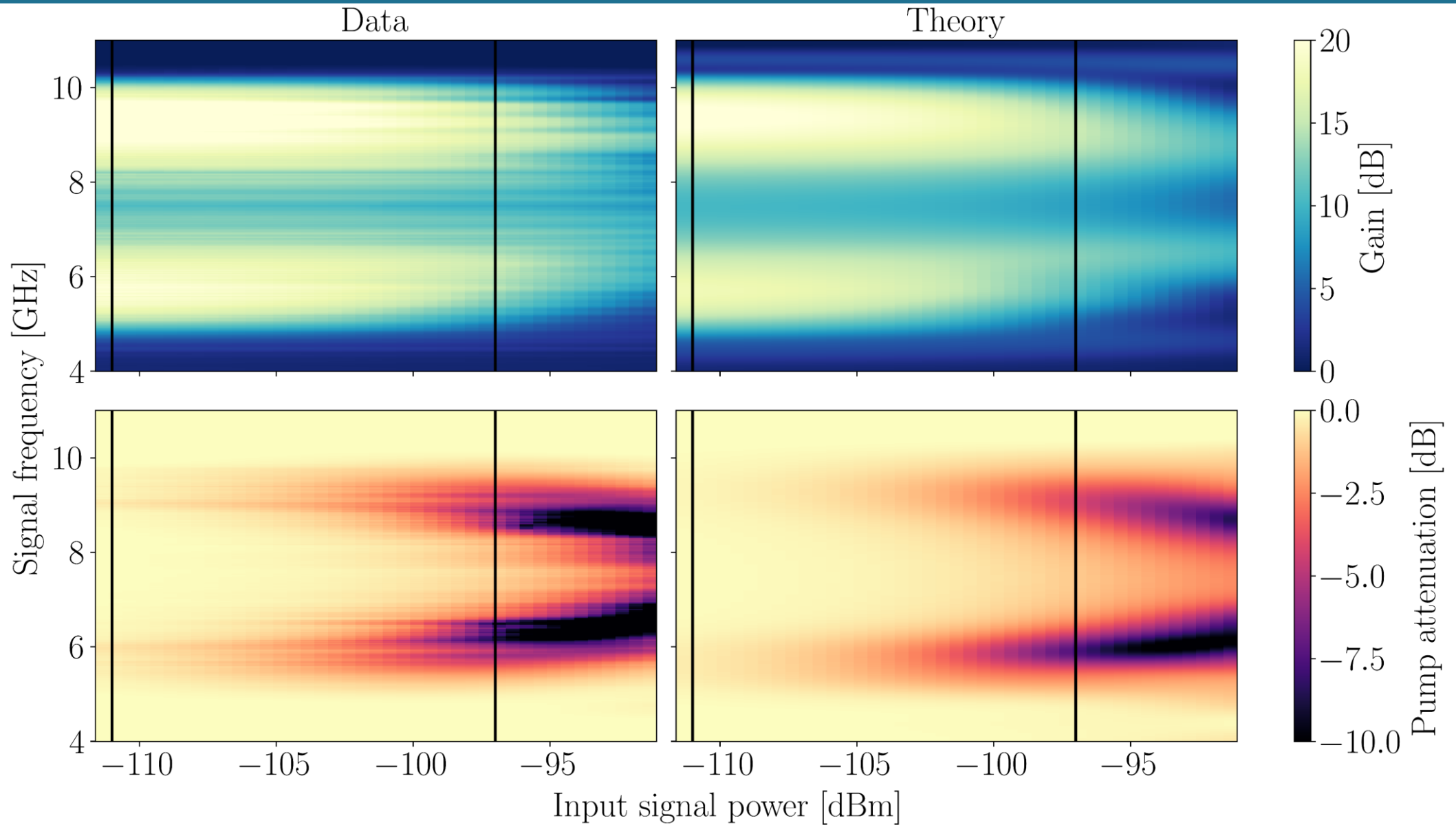
**SQC**  
[www.sqc.cnrs.fr](http://www.sqc.cnrs.fr)

# Thank you!



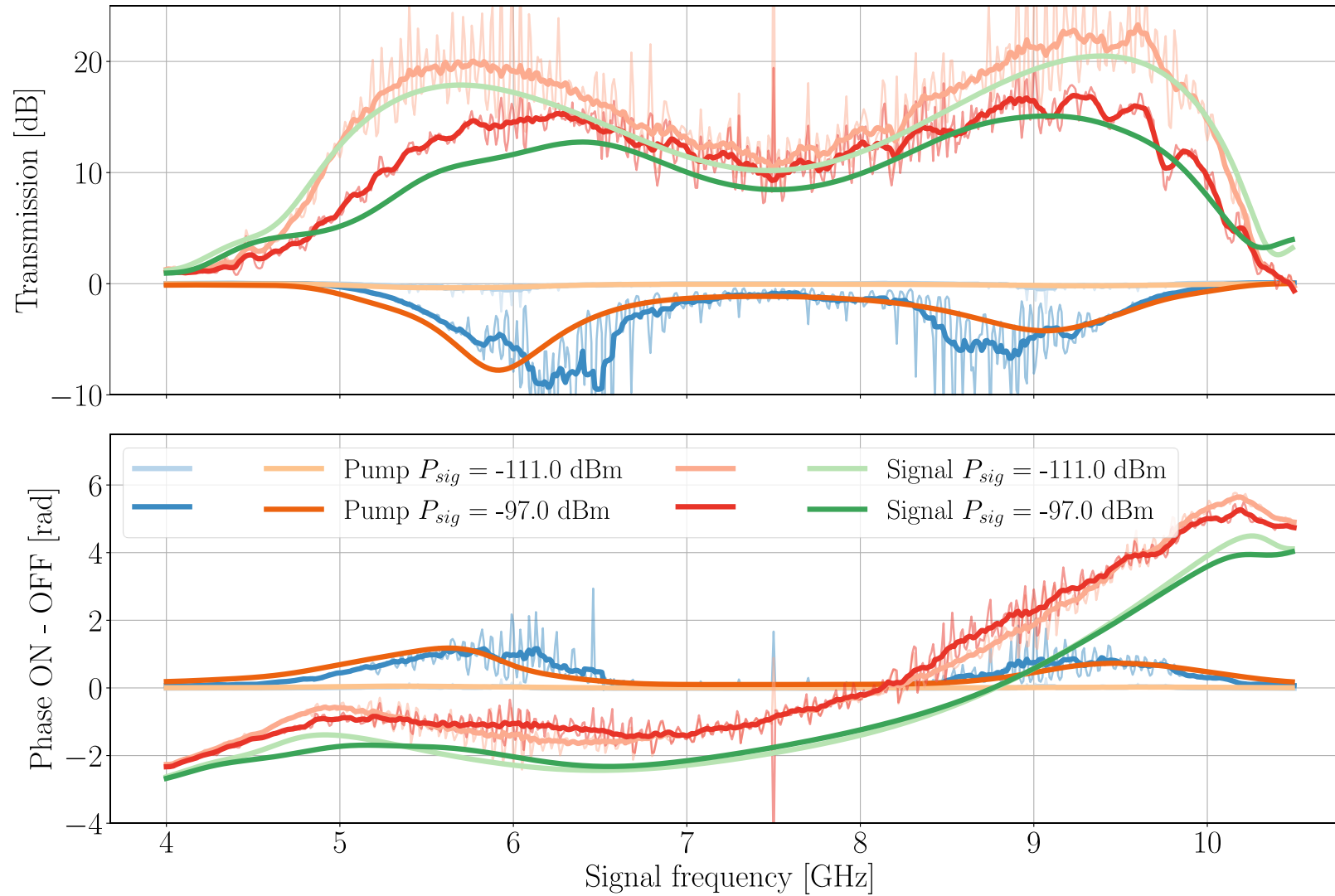


# Comparison with data



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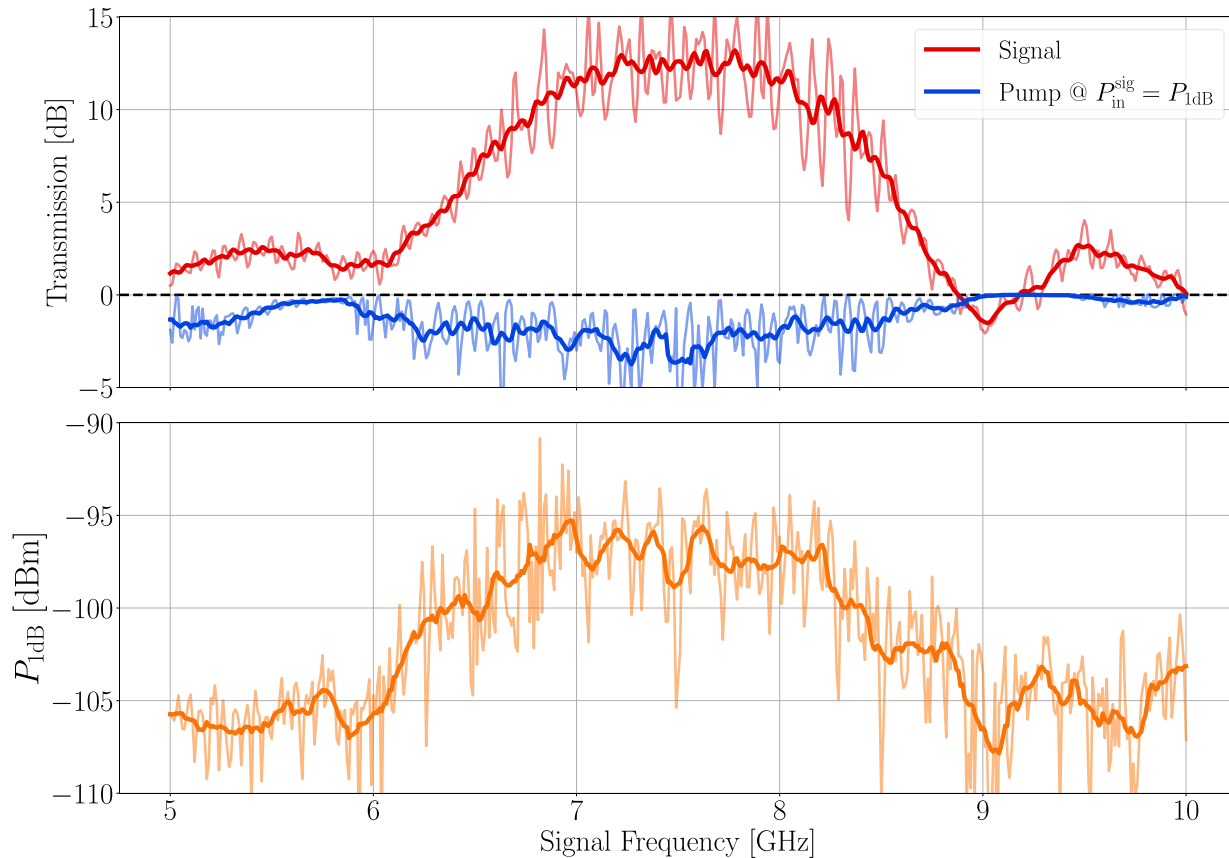
No fitting parameters



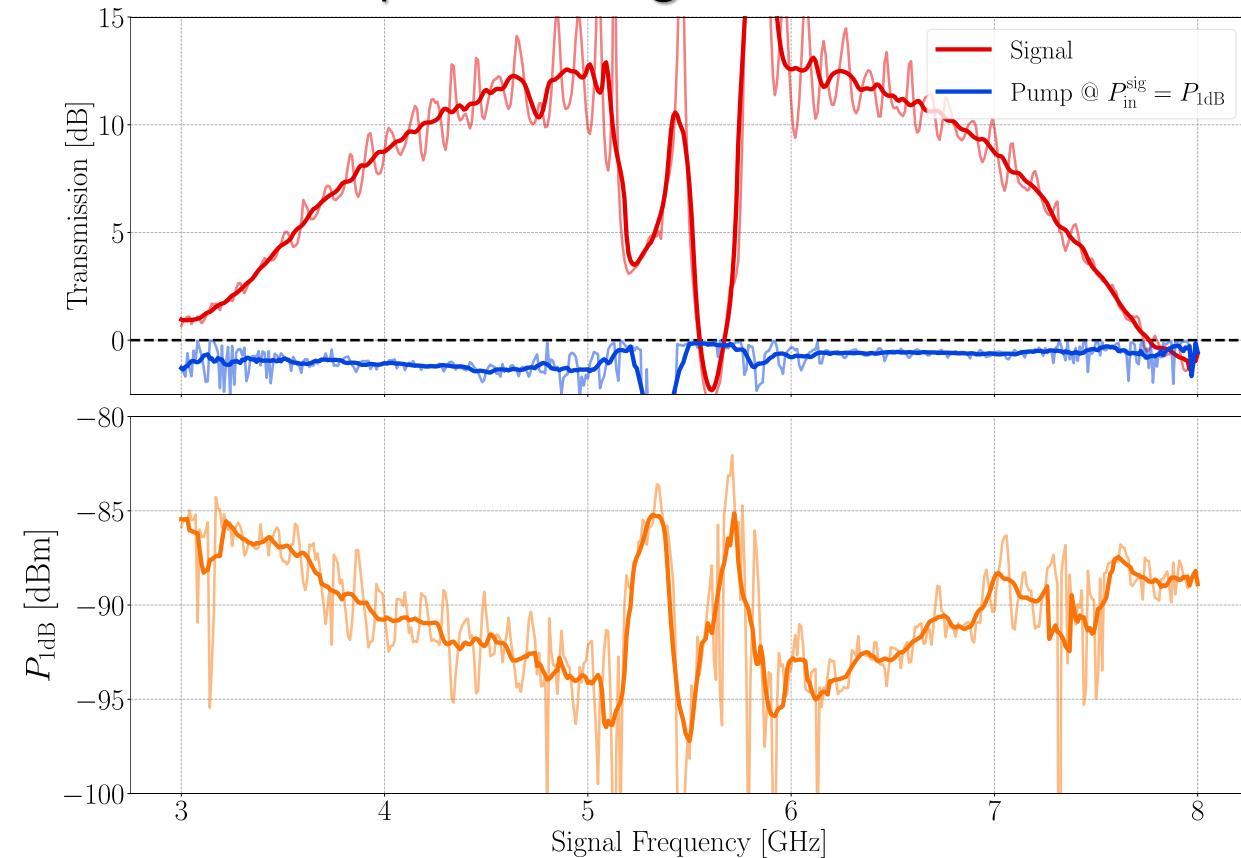
# Other devices

Is it proper to the SNAIL TWPA operated @ 1/2 flux ?

## SNAIL TWPA at 0 flux



## Dispersion engineered TWPA





# Comparing terms in theory

